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DEVELOPMENT OF COMPUTER MODELS FOR THE ASSESSMENT OF FOREIGN BODY IMPACT EVENTS ON COMPOSITE STRUCTURES

FINAL REPORT

by

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FORWARD

The structural efficiency requirements of the NASAs Reusable Launch Vehicle program has all but eliminated the use of conventional materials. Composite materials provide the strength, stiffness, density, and manufacturing flexibility that will make the RLV structure successful. However; the question of impact resistance still plagues this class of materials. The NASA Filament Wound Composite Pressure Vessel Damage Tolerance RTOP was an attempt to assess the effect of impact damage on composite structures through experimental and NDE techniques. Dr. Frank Ledbetter and Dr. Alan Nettles of the NASA Marshall Space Flight Center provided Dr. Ronald Bucinell of Union College with funds to investigate a methodology that can be used to analytically determine the formation of damage in a composite structure and to assess the effects this damage will have on the integrity of the composite structure.

The assigned task under this grant was to build a finite element model that could be used to gain insight into the formation of damage in the composite pressure vessels used in the NASA RTOP program. The technical question that needed to be answered is can a methodology be developed to predict the formation and propagation of damage in impacted composite structures. Additionally, can the methodology be used to predict the residual properties of the structure once the damage state is known.

The purpose of this report is to summarize the findings of this 12 month study and to recommend steps that should be taken to gain further insight into the problem of impact assessment in composite structures.

SUMMARY

The objective of this project is to develop a methodology that can be used to predict the formation of damage in composite structures subjected to impact events and to assess the effect of the impact damage on the integrity of the composite structure. The methodology used in this program is a global-local finite element model. The global model evaluates the performance of the entire structure while modeling the composite laminate as an orthotropic homogeneous material. The local model uses the displacements of the global model as boundary conditions for a more refined region around the impact site that models each composite layer as a specially isotropic homogeneous material. The stress states in these plies are then used as inputs to micromechanical models that provided the state of stress in the constituents of the plies. Experimental data of impacted 5-3/4 inch bottles was provided by NASA to compare with the analytical results.

As a result of this investigation the following conclusions can be made:

1) The global-local approach to the modeling of impact events on composite structures provides the three dimensional stress state that is required to predict the formation of damage in composites and to predict the residual properties that result from the presence of damage.

2) Carefully designed experimental data is required to isolate the phenomenon associated with the formation of damage in composite materials and the complicated confounding that occurs between the phenomenon present.

On the basis of the findings in this project it is recommended that an experimental program be developed, that is designed using statistical design of experiments, to isolate the phenomenon associated with the formation of damage that results from impact events. It is also recommended that a test analog be developed that has the flexibility to address size, scale, and various loading issues. It is also recommended that the global-local model methodology be integrated directly into the finite element model.

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INTRODUCTION

The use of composite materials in structural applications is becoming more widely accepted. The success of composites on the Delta and Titon solid rocket motor programs clearly indicates that composite materials can be used reliably in launch vehicle hardware. Even with this success, there is still concern related to the effect of foreign body impact events on the integrity of composite structures. The manufacturing, storage, transportation, and service environments are filled with foreign body impact event scenarios. Some of these scenarios can severely degrade the integrity of the composite structure; however, many will have little or no effect at all on structural integrity. A methodology for determining the severity and residual effects of foreign body impact events on composite structures has yet to be developed and accepted by the composite materials community. As a result, current flight hardware that is knowingly subjected to a significant impact event is removed from service.

This overly conservative approach to dispositioning composite structures subjected to impact events was recently used to remove a TOS-2 Kevlar motor case from service. This incident refocused NASA MSFC's attention on the need to develop a methodology for assessing the extent and effect of impact related damage on composite structures. The RTOP entitled *Filament Wound Composite Pressure Vessel Damage Tolerance Program* was conceived in March 1992 and funded in July 1992 at NASA MSFC to address this need. The scope of this RTOP includes development of Non-Destructive Evaluation (NDE) techniques, subscale testing, and analytical model development.

The test analog used in this RTOP program is the ASTM 2586 standard 5-3/4" pressure vessel shown in Figure 1. The geometry of the cylindrical section of this analog is $[\pm 115/90_2/\pm 115/90_2]$ laminate. The experimental program impacted several of these pressure vessels. The vessels were then subjected to several destructive and non-destructive evaluations in order to assess the effects of the impact event.

One aspect of the analytical model development task was to develop an analytical methodology that can be used to evaluate the experimental data, predict damage formation and modes, and predict the residual properties of an impacted composite structure. Several methodologies have successfully predicted the response of composite structures to impact events [1,2,3,4] and the scaling of impact events up to the point of damage initiation [5,6,7]. However, methodologies for predicting damage formation [8,9] to date have been either empirical or material specific. The intent of this project was to develop a methodology for assessing the effects of impact damage that was neither empirical or material specific.

In the remainder of the report the work performed under Grant NAG8-1144 will be reviewed. The discussion will start with an overview of the various levels of material models used to predict the properties in the various regions of the composite pressure vessel. This section will be followed by a description of the finite element model that was used to predict the stresses in the composite pressure vessels during the impact event. This will be followed by a discussion of the comparison between the analytical models and the experimental data that was provided by the NASA Marshall Space Flight Center.

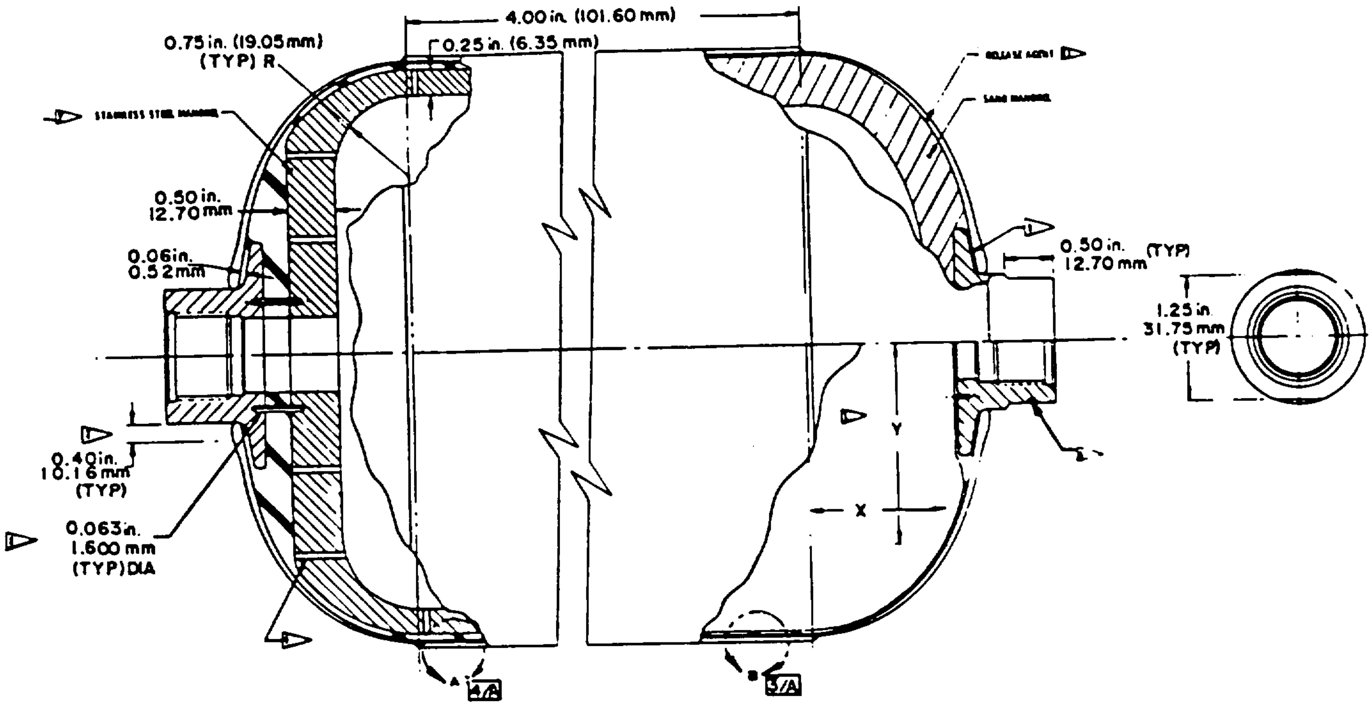

Figure 1: ASTM D2586 5-3/4 inch pressure vessel used as test analog in NASA RTOP program.

MATERIAL MODELING

Material properties for this project are modeled using basic theories of fiber/matrix/interface interactions and multidirectional material assemblages that are generally accepted within the composites community. These theories aid in determining the relations between the properties of the material and the residual stress state, the life of the structure under cyclic loading, and the effect of scale-up on the material properties. This approach allows the relative merits of all possible material configurations to be compared to determine the optimum material configuration and fabrication process for specified design requirements. To realize the benefits of this approach, material models must be compared with experimental data to insure that all of the phenomenon related to the residual properties of the composite are accurately and correctly modeled.

In the remainder of this section the various models used to predict the various levels of material properties are first discussed. This is followed by a discussion of the theory behind the determination of the laminate geometry in the dome region of the pressure vessel. The final subsection overviews the properties calculated for use in modeling the pressure vessels used in the NASA RTOP.

COMPOSITE MATERIALS

The material models that were used on this program include micromechanical, minimechanical, and macromechanical theories. The micromechanical models compute effective fiber bundle properties based upon the properties of the reinforcement, matrix, or equiaxial particle, etc. Voids, microcracks and disbonds can also be described along with other issues that are critical to composite material modeling at this level. These issues include material non-linearity, phase averaged stresses, and processing effects.

Micromechanical Models

The most detailed level of analytical magnification required by this program treats the reinforcement and matrix as separate and discrete materials. This level is termed micromechanical analysis and is the level at which the geometries and properties of the reinforcement, interphase, and matrix are modeled. The typical output of the micromechanical analysis is the anisotropic homogeneous bundle properties used in the minimechanics models described later in this section. The primary differences among the available micromechanical models are the reinforcement geometries which can be analyzed. Models are available to analyze fibers to spherical particles. The micromechanical modeling of the FRP material is described briefly in the following paragraphs along with a paragraph outlining a method for computing average stresses in the constituents once the properties of the composite are known.

The micromechanical analysis of continuous fiber bundles is based upon the composite cylinders assemblage model detailed in [10]. The composite cylinders assemblage uses the material properties of the fibers and matrix, and the fiber volume fractions to compute transversely isotropic properties of the fiber bundle. The model allows both the fibers and matrix to have transversely isotropic properties. The elastic properties of porous matrix materials are evaluated by introducing voids into the matrix material using a differential scheme [11] with the composite spheres assemblage model [12]. An interphase can also be included by first modeling a fiber within the interphase. The properties of that combination are then utilized with the matrix properties to determine effective bundle properties. Similar codes have been used to analyze the elastic properties of continuous fibers in organic, metallic, ceramic, and carbon matrices and have been extended to account for viscoelastic [13] and plastic matrix effects.

For specialty problems, such as complex voids, disbonds between fiber and matrix, and detailed interphase studies, periodic hexagonal array finite element models can be employed. This procedure identifies a repeating element of material, at the micro level, for detailed finite element evaluation. Appropriate symmetry boundary conditions at each boundary of the elements are required in this model. The interphase of the composite in this model is represented as a distinct phase with properties and thickness. This representation allows these features to be varied to determine the optimal interphase characteristics for a given application.

Material Nonlinearity

Nonlinearity in FRP Composite Materials is most significant in the matrix dominate properties (e.g., transverse moduli and shear moduli). Nonlinearity in fiber dominated properties tend to be less significant unless high modulus fibers, like T100 or IM9, are being utilized. In these cases, the fiber direction nonlinearity also becomes significant to the accuracy of the analysis. This requires nonlinear analysis procedures to be employed.

Matrix dominated material nonlinearity in an FRP material can be addressed in much the same way; however, much more complex issues are involved in the nonlinearity associated with the formation of transverse cracks. Approaches discussed in the literature include the application of fracture mechanics in conjunction with continuum constitutive behavior to model damage accumulation and stiffness reductions application of a negative tangent modulus following damage initiation. Eventually this leads to zero stiffness with increasing load where stiffness in the lamina transverse direction is assumed to be zero following damage initiation.

Less attention has been given to composite nonlinearity resulting from the fiber material nonlinear behavior when subjected to tension loads. As fiber strain-to-failure increases, this effect becomes more important in fiber dominated strength calculations. The phenomenon of elastic nonlinear stress-strain response of high-strength , high modulus graphite fiber is well documented. In the bulk of these investigations, this phenomenon is attributed to the changes in the preferred orientation of the graphite layers. These layers form long wrinkled ribbons along the fiber axis. As the fiber is strained in the axial direction, these wrinkles are stretched and their orientation changes, leading to hardening in the fiber. Graphite fibers with high strain-to-failure, such as AS4 and IM7, show up to a 15% increase in the secant modulus between the initial value and the value at which the fiber fails.

The nonlinear stress-strain response of graphite fibers can be modeled empirically by representing the nonlinear material response by a second-order Hooke's law which is consistent with a third-order polynomial expansion of the strain energy function. The elastic coefficients are determined by a least-squares fit of the quadratic model to tow data. Then the nonlinear laminate response is calculated using the fiber and matrix nonlinear relationships in an iterative solution scheme [14].

Phase Average Stress

The micromechanical models provide a means for computing effective stiffness for various combinations of reinforcement, interphase, and matrix. In order to estimate composite failure, it is necessary to have some measure of the stress state within the constituents. A method for determining average stresses within the fibers, matrix, and interphase is termed the phase average stress model. This concept is outlined in [15]. The phase average stress model can be applied to any continuous fibers or spherical particles. The phase average stress model can be used to compute the local average stresses in the constituents. These stresses can then be used to estimate the strength of the fiber bundle.

Process Modeling

The micromechanical analysis of FRP materials is complicated by the nature of the composite fabrication process. The fabrication process can introduce contaminants, imperfections, interphase regions, poorly consolidated fiber regions, poor fiber wetting, and residual stresses in the constituents. All of these effects will affect the resulting material properties. The determination of the physical effects on the fabrication process on the composite material properties requires a carefully designed set of process experiments. From these experiments the physical microstructural characteristics of the composite can be observed. These observations will be used to enhance the accuracy of the material models. The residual stress state can also have a dramatic effect on the composite level material behavior. For example, large changes in the stress free thermal expansion behavior can occur as a result of different stress free states of the constituents due to different thermal process histories. Process modeling can be used to predict detrimental stress states that result from the manufacturing process.

Structural Element

Performance requirements for composite pressure vessels will vary from place to place in a structure. The reinforcement architecture of the FRP material can be altered to meet the performance requirements. Identifying the appropriate architecture for the various regions of the pressure vessel is refereed to a minimechanical model. In the following subsections typical minimechanical models will be described. In addition, other minimechanical issues such as hygrothermal loading, creep, fracture, and thick cross-sections are discussed.

Minimechanical Models

The term minimechanical models is used here to refer to models that compute effective composite properties by treating fiber bundles as homogeneous anisotropic materials. This is the level of magnification that is typically used in analyzing a composite model. For example, in laminated plate analysis this is the level at which the orientation and stacking sequence of the plies is specified. The differences in the following models lie predominantly in the types of composite geometries and in the level of sophistication in the modeling assumptions.

The properties of unidirectional bundles (or plies) described in the previous section have been shown to be decidedly different from conventional metallic materials. The primary difference from an analytical viewpoint results from the material anisotropy. These materials typically have exceptional properties in the direction of the reinforcing fibers (axial) while properties perpendicular (transverse) to the fibers are poor to mediocre. Thus, with the exception of one-dimensionally loaded members (e.g., truss members), unidirectional composites would be expected to perform poorly with respect to conventional materials. By orienting the plies in a laminate configuration, the lesser properties of one ply are augmented by the axial properties of another ply.

The bonding together of individual plies is used with unidirectional composites to form laminates. The plies (often referred to as lamina) are oriented such that the effective properties of the laminate match the loading environment. The tailoring of laminate effective material properties to correspond to performance requirements is accomplished through the use of lamination theory.

Lamination theory[16] can be considered a form of structural analysis. Here, however, the structural material is being designed. This adds another level of effort to the design process, but at the same time allows the structural material to be tailored to match the loading. Thus, if a 2:1 biaxial loading environment is prescribed, the structural laminate used can be designed for a 2:1 strength. In this fashion, the amount of material is minimized in a way which is not possible with conventional materials.

For purposes of structural analysis, it is desirable to represent a laminate by a set of effective stiffnesses, just as a homogeneous plate is defined by its extensional and bending stiffnesses. The calculation of these laminate mechanical properties is illustrated in Figure 2.

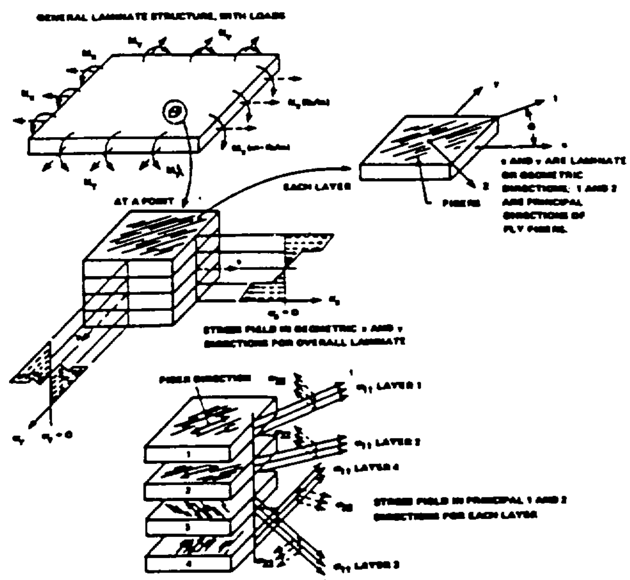

Figure 2. The FRP material properties are systematically built up from micromechanical to minimechanical to macromechanical to allow the designer to explore all constituent and reinforcement options

Hygrothermal Effects (Environmental Effects)

The elastic behavior of composite materials is concerned with deformations produced by stresses, thus by loads. Deformations are also produced by temperature changes, moisture absorption, and chemical shrinkage. These phenomenon are similar in that they all produce strains that can be superimposed on mechanical strains and therefore can be discussed together. A change of temperature in a free body produces thermal strains: moisture absorption produces swelling strains; and cross-link polymerization produces chemical shrinkage strains. The relevant physical parameters to quantify these phenomenon are thermal expansion coefficients, swelling coefficients, and chemical shrinkage coefficients.

The magnitude of there coefficients differ significantly for the fibers and the polymeric matrices. This mismatch leads to the situation in a unidirectional composite where free expansion of the constituents, as a result of the expansion phenomenon discussed above, is prevented. Thus internal or residual stresses develop which may be considerable. Additionally, residual stresses are affected by changes in environmental conditions, thus setting up the potential for a cyclic degradation in the material. The orthotropic coefficients for expansion of the unidirectional composite can be predicted using Levein's Theory [17]. These unidirectional expansion coefficients are indispensable information for stress analysis of laminates.

The expansion coefficients of unidirectional composites are directional (orthotropic). This is partly a consequence of the transversely isotropic nature of most unidirectional material. When laminates are constructed using various orientations of unidirectional layers, the layers are prevented from free expansion by the adjacent layer. Therefore, additional residual stresses are developed. The influence of these stresses can lead to microcracking, delaminations, and warpage.

The accumulation of residual stresses can be attributed to the processing and service environments. The stress free stage of a laminate, the stage where no residual stresses exist in the laminate, typically occurs at elevated temperatures during the processing stage of the structure. Beyond this point any change in environmental conditions (e.g., cooling to ambient conditions) leads to residual stresses. The stress free temperature at this stage can be as high as 500°F for FRP composite structures. As the thickness of the composite increases, temperature and degree of cure gradients through the thickness of the laminate can cause a heightening of residual stresses. In addition, volumetric shrinkage of thermosetting resins associated with the cross-link polymerization reaction also occurs. Outside the processing environments, service environmental changes in temperature and moisture can also have a significant effect on residual stress in the structure.

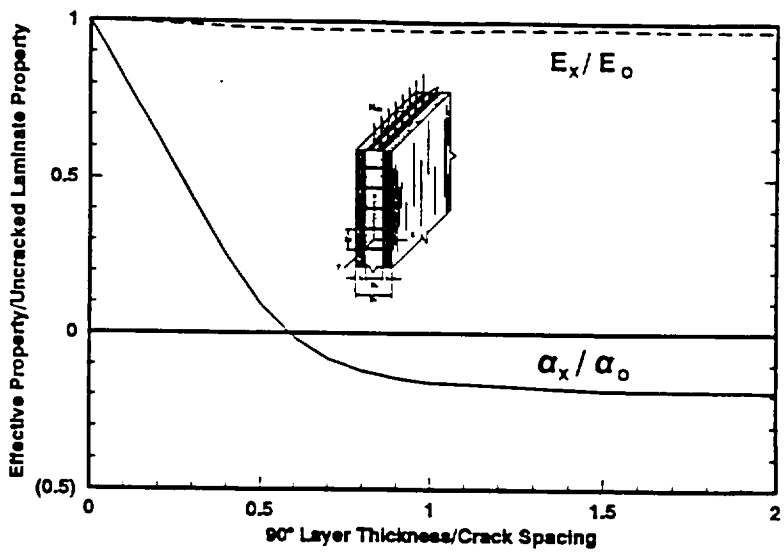

Figure 3: Hygrothermal stress can result in states of stress that cause microcracks that significantly alter the structural properties of a composite laminate.

The modeling of residual stress in a composite structure must account for all the above phenomenon. Residual stresses are important because they can have a significant effect on the integrity of a composite structure and can degrade the performance of a structure. Figure 3 illustrates how residual stress induced cracks in a laminate can cause a significant reduction in thermal expansion. Thermal expansion predictions will become important in the region where the dome meets the cylinder of the pressure vessel. A mismatch in thermal expansions can cause failure in this critical region.

Creep/Viscoelastic Properties (Sustained Load)

All polymers exhibit the phenomenon of time dependence. This manifests itself by the increase of deformations with time under constant load, which is called creep, and, conversely, by the decrease of stresses with time under deformation constraints, which is called relaxation. Another important effect of time dependence is the damping of vibrations due to energy dissipation in the polymeric matrix. The significance of all of these phenomena increases with rise in temperature.

Creep is defined as the change in a property over time when subjected to a constant forcing function. Creep should be considered if the end use involves high stress in the matrix-dominated direction, high temperature, or exposure to a harsh chemical environment - in other words, if there is a chance of matrix softening. In composites with a thermoplastic matrix, concern for creep is important, particularly if the service temperature is near or above the glass transition temperature. In a thermoset matrix, creep is expected to be small due to cross-linking. In general, creep testing does not provide primary design data. Designs should be checked for creep deformation if the working load involves major shearing action, e.g., short beam bending, etc. In composites, large shear stresses can be generated near a structural discontinuity; however, creep can be beneficial in some of the instances in relaxing the stress and avoiding catastrophic failure. In FRP materials creep will be more important when the composite is loaded in a matrix dominated manner than in a fiber dominated manner.

If the fiber and matrix complex moduli (storage and loss moduli) are known from experiments as a function of frequency and environmental conditions, it is a relatively straightforward task to obtain the lamina/laminate complex moduli by utilizing the dynamic viscoelastic correspondence principle. This principle states that the effective complex moduli of a viscoelastic composite are obtained by replacing the phase elastic moduli by the phase complex moduli in the expressions for the effective elastic moduli of identical phase geometry. Closed form expressions of the effective moduli or unidirectional fiber reinforced composites can be derived using the composite cylinder assemblage in a random fashion with the plane of isotropy perpendicular to the axis of the fibers. Exact expressions for four of the five independent elastic moduli and an upper bound estimate for the other modulus are utilized. It is an easy matter to obtain the required expressions in the case of transversely isotropic phases from the results in [15]. It should be mentioned that if complex moduli of unidirectional lamina materials are known, laminate complex moduli can be readily evaluated using lamination theories and the correspondence principle.

Fracture

Under loading FRP materials exhibit an accumulation of damage consisting of matrix cracking in the off-axis plies, delamination between layers, splitting (that is, matrix cracking interface). Failure is not due to the initiation and propagation of a single crack in a self-similar fashion as it is in metals, which raises the question of the validity of the application of fracture mechanics to FRP materials. However, fracture mechanics techniques can be applied to particular layers (intralaminar cracking). For these types of matrix cracks strain energy release rate techniques are employed. For fiber breakage probabilistic accumulation laws best explain the failure process.

Lamination theory and ply stress or strain failure criteria have generally given conservative predictions of matrix cracking. More successful methods of matrix crack analysis have employed the finite element technique, crack closure analysis, and an effective flaw concept to predict matrix cracking [18]. The effective flaw is synonymous with the term critical flaw size used in fracture mechanics approaches. The effective flaw is defined as a basic property that accounts for the collective interaction of inhomogeneities in the material's real microstructure. As such, it represents the combined effect of the microstructure's response to load rather than a measurable defect size. An effective flaw size distribution is assumed to account for variations in the microstructure.

Theoretical and experimental studies have shown that tensile fracture in unidirectional continuous fiber-reinforced composites occurs by random filament breakage, until a critical cluster of individual fiber breaks develop and from which catastrophic failure of the composite initiates, Figure 4. The irregular break of the filaments is due to the inherent randomly distributed flaws in the fibers that lead to premature break of the fibers well below the ultimate strength of the composite. As a fiber breaks, the matrix plays an important role in transferring the loads to the surrounding fibers and back into the broken fiber itself. Damage associated with such a fiber break can eventually propagate in directions either transverse or longitudinal to the fiber. Occurrence of either damage mode or a combination of these modes will finally lead to ultimate failure of the composite. However, the probability of one mode occurring over the other is strongly influenced by properties of the matrix, which may ultimately govern the strength of the composite.

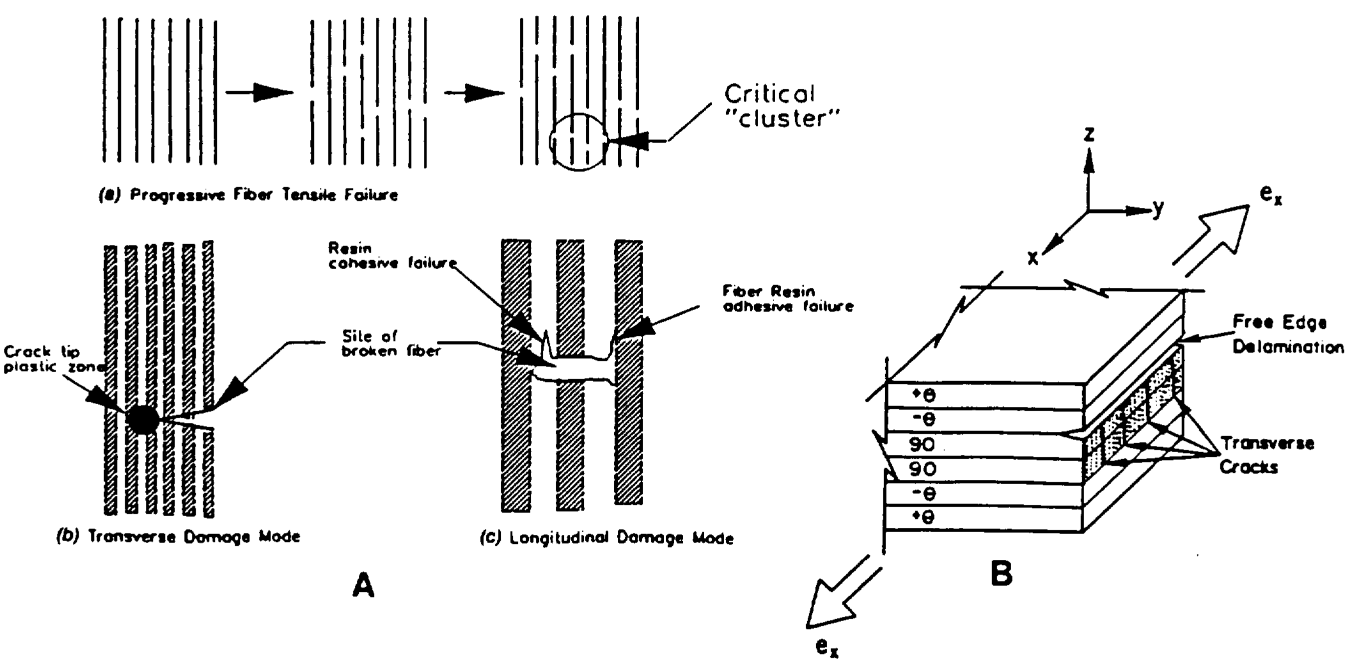

Figure 4: Crack propagation in composite materials occurs at the micromechanical level (A) and at the minimechanical level (B). Failure in a composite is due to the coalescence of multiple cracks and not to the propagation of a single crack.

Failure Criteria

The mathematical treatment of the relationships between the strength of a composite and the properties of its constituents is considerably less developed than the analyses for the other physical property relationships such as stiffness, thermal expansion, etc. One of the reasons for this is that failure is likely to initiate in a local region due to the influence of the local values of constituent properties and geometry in that region. Thus, the higher degree of variability of local geometry (e.g., relative locations of adjacent fibers) and the higher degree of variability of the local strength of the fibers both contribute to the onset of initial localized damage within the composite. This dependence upon the onset of initial localized damage within the composite failure mechanism is much more complex than the analyses of the physical properties discussed earlier.

The strength of a fiber composite clearly depends upon the orientation of the applied load with respect to the direction in which the fibers are oriented as well as upon whether the applied load is tensile or compressive.

Axial Tensile Strength. One of the most attractive properties of advanced fiber composites is their high tensile strength. The simplest model for the tensile failure of a unidirectional fiber composite subjected to a tensile load in the fiber direction is based upon the elasticity solution of uniform axial strain throughout the composite. Generally, the fibers have a lower strain to failure than the matrix, and composite fracture occurs at the failure strain of the fibers alone.

The problem with this approach is the variability of the fiber strength. Non-uniform strength is characteristic of most current high strength filaments. This is illustrated in Figure 5, which shows strength distributions for single filaments of two different types of commercial glass fibers. This statistical distribution of single filament strength is generally considered to result from a distribution of imperfections along the length of individual fiber strengths. First, all fibers will not equal the sum of the strengths of the individual fibers; nor will it equal the mean strength of these fibers. The second important factor is that those fibers which break earliest during the loading process will cause perturbations of the stress field in the vicinity to the break, resulting in localized high fiber-matrix interface shear stresses. These shear stresses transfer the load across the interface and also introduce stress concentrations into adjacent unbroken fibers. At each local break, the stress in the vicinity of the broken fiber changes so that the axial stress in the fiber vanishes at the fiber break and gradually builds back up along the fiber length to its undisturbed stress value.

This stress distribution may cause several possible failure events to occur. The shear stresses may cause a crack to progress along the interface. If the interface is weak, such propagation can be extensive; in this case, the strength of the composite material may differ only slightly from that of a bundle of unbonded fibers. This undesirable mode of failure can be prevented by the attainment of a strong fiber-matrix interface or by the use of a soft ductile matrix which permits the redistribution of the high shear stresses. When the bond strength is high

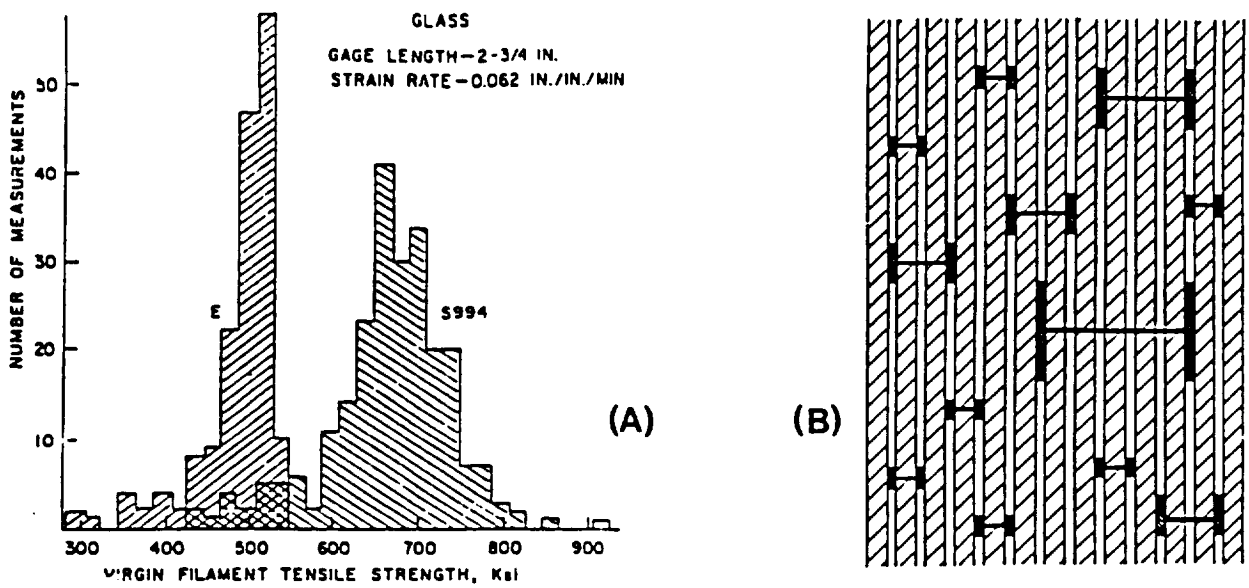

Figure 5: High levels of fiber variability observed in experimental tow tests (a) and the load transfer role of the matrix material cause the tensile failure mechanism to be a coalescence of dispersed damage (B) as apposed to a propagation of a single flaw as in metals

enough to prevent interface failure, the local stress concentrations may cause the fiber break to propagate through the matrix, to and through adjacent fibers. Alternatively, the stress concentration in adjacent fibers may cause one or more of such fibers to break prior to the occurrence of failure of the intermediate matrix. If such a crack or such fiber breaks continue to propagate, the strength of the composite may be no greater than that of the weakest fiber. This failure mode is defined as a weakest link failure. If the matrix and interface properties are of sufficient strength and toughness to prevent or arrest these failure mechanisms, than continued load increases will produce new fiber failure mechanisms, then continued load increases will produce new fiber failures at other locations in the material, resulting in an accumulation of dispersed internal damage as the loading continues.

It can be expected that all these effects will occur prior to material failure. That is, local fractures will propagate for some distance along the fibers and normal to the fibers. These fractures will initiate and grow at various points within the composite. Increasing the load will produce a statistical accumulation of dispersed damage regions until a sufficient number of such regions interact to provide a weak surface, resulting in composite tensile failure.

Axial Compressive Strength. For compressive loads applied parallel to the fibers of a unidirectional composite, both strength and stability failures must be considered. It was suggested that small wavelength micro-instability of the fibers occurs in a fashion analogous to the buckling of a column on an elastic foundation. It has been demonstrated that this will occur even for a brittle material, such as glass.

Analyses of this instability is analyzed by approximating the problem by treating a layered two-dimensional medium, as shown in Figure 6. The model consists of plates of thickness h separated by a matrix of dimension 2c. Each fiber is subjected to a compressive load, P, and the fiber length is given by the dimension, L. Two possibilities are for the instability failure mode.

First, the fibers may buckle in opposite directions in adjacent fibers, as shown on the left portion of Figure 6, and the so-called extension mode occurs. This mode receives its name from the fact that the major deformation of the matrix material is an extension of the direction perpendicular to the fibers. The analysis treats the fibers as stiff, relative to the matrix so that shear deformations in the fiber can be neglected relative to those in the matrix.

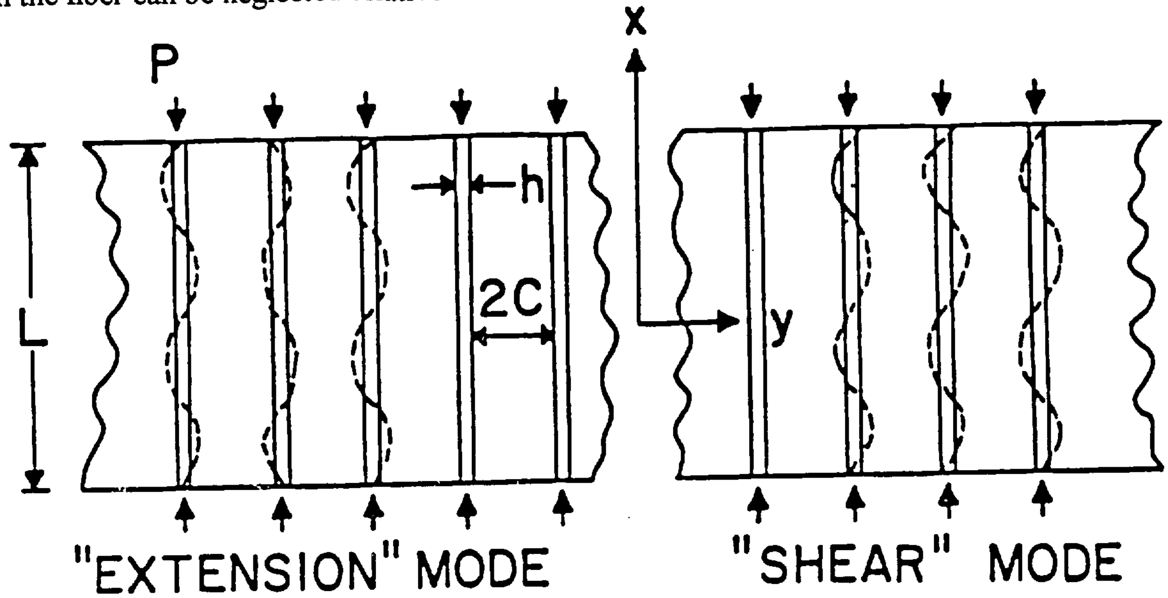

Figure 6: Compression in unidirectional material can cause micro-instabilities of the fibers

The second possibility is shown on the right portion of the Figure 6 where adjacent fibers buckle in the same wavelength and in phase with one another, so that the deformation of the matrix material between adjacent fibers is primarily a shear deformation. Hence, the shear mode label for this potential mode.

Matrix Mode Strength. The remaining failure modes of interest are transverse tension, transverse compression, and axial shear. For each of these loading conditions, it is possible to have material failure without fracture of the fibers, hence the terminology "matrix dominated" or "matrix modes" of failure.

Micromechanical Analyses of these failure modes are complex because, unlike the axial failure modes treated above, for these matrix modes the critical stress states are in the matrix, are highly non-uniform and are very dependent upon local details of the geometry. As a result, it appears that the most fruitful approaches will be those that consider average states of stress rather than local details.

Under transverse tensile stress, failure may occur within the matrix or along the fiber-matrix interface. It is not expected that composite transverse tensile strengths will be significantly in access of matrix tensile strength. Indeed, perhaps the addition of fibers will weaken the matrix in this direction due to local stress concentrations or weak interfaces, etc.

For transverse compression, failure may occur by shearing along a surface through the matrix parallel to the fiber axes in a fashion somewhat similar to the compressive failure of a homogeneous material. Thus, there are two types of shearing stresses which are of interest for matrix dominated failures: (1) in a plane which contains the filaments, and (2) in a plane normal to the filaments. In the first case, as the following discussion will show, the filaments provide very little reinforcement to the composite and the shear strength depends upon the shear strength of the matrix material. In the second case, some reinforcement may occur and, at high volume fractions of filaments, it may be substantial. Because the analysis shows that reinforcement does not take place in planes of surfaces parallel to the filaments, these planes may be considered planes of shear weakness. Surfaces of shear weakness do indeed exist in filamentary composites. It is important to recognize that filaments provide little resistance to shearing in any surface parallel to them.

The approach to the shear failure analysis is to consider that a uniaxial fibrous composite is comprised of strong and stiff fibers embedded in a matrix which is characterized by its initial elastic modulus and by a maximum stress level. Accordingly, the matrix is idealized so that its stress-strain relation is that of an elastic, perfectly plastic material. For homogeneous materials the existence of this plastic region generally signifies the possibility of unbounded structural deformations beyond some limiting load.

LAMINATE ANGLE CALCULATIONS

In order to determine material properties for the finite element model of the 5-3/4 inch diameter composite overwrapped pressure vessel shown in Figure 7, the angles the laminate makes with respect to "radial cuts" of the pressure vessel must be determined. The methodology presented herein was used in order to approximate the laminate angel, β. The laminate angel β is a constant 11.5 degrees in the cylindrical portion of the vessel, hence material properties can be easily determined. Determining β for the dome portion of the vessel involves the calculation of normal vectors and tangent planes at points of interest along the dome.

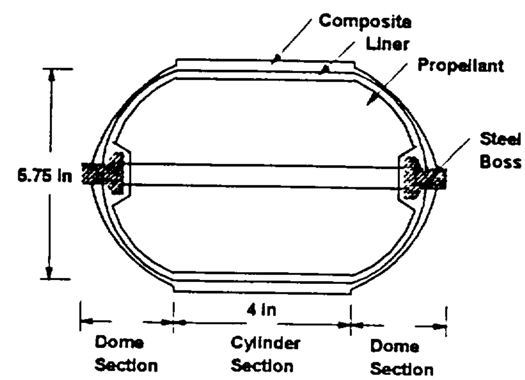

Figure 7: Illustration of the components of the 5-3/4 inch bottle used in the RTOP

First, vectors normal to the rubber surface at points of interest must be determined. Since only x,y coordinates for twenty-six points on the mandrel surface are known, normal angles to this surface will be required. The first step in the calculation of the normal angle is the determination of the surface equation. It was assumed that the mandrel surface is elliptical in the x-y plane and cylindrical in the x-z plane. Thus, the dome can be termed ellipsoidal, with the following parameterized equation:

$$\vec{r} = (a \cdot \cos v \cdot \cos u)\hat{i} + (b \cdot \cos v \cdot \sin u)\hat{j} + (c \cdot \sin v)\hat{k}$$

where a, b, and c are ellipse constants that were approximated by fitting an ellipse to data points describing the surface and u is the angle between the x-z plane and a line from the origin to the point of interest projected onto the x-y plane and v is the angle between the x-y plane and a line from the origin to the point of interest (see Figure 8).

For the 5-3/4 inch vessels, a and c are equivalent since the vessel is cylindrical in the x-z plane. The normal vector is found by taking the cross product of the partial derivative of the ellipsoid equation with respect to u with the partial derivative of the ellipsoid equation with respect to v, or

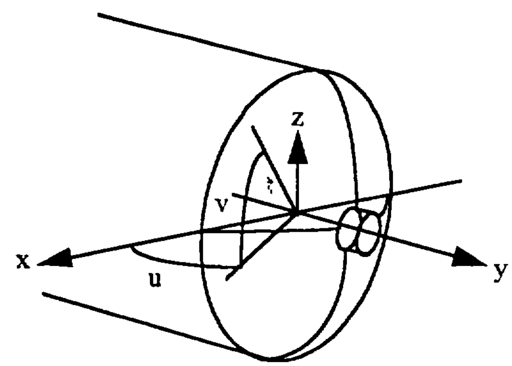

Figure 8: Coordinate system used in the calculation of the winding angles in the 5-3/4 inch pressure vessel.

$$\vec{N} = \vec{r}_u \times \vec{r}_v$$

with

$$\vec{r}_u = \partial\vec{r}/\partial u = -(a \cdot \cos v \cdot \sin u)\hat{i} + (b \cdot \cos v \cdot \cos u)\hat{j}$$

$$\vec{r}_v = \partial\vec{r}/\partial v = -(a \cdot \sin v \cdot \cos u)\hat{i} - (b \cdot \sin v \cdot \sin u)\hat{j} + (c \cdot \cos v)\hat{k}$$

Thus, by taking the cross product, we obtain

$$\begin{aligned}\vec{N} = {} & (b \cdot c \cdot \cos^2 v \cdot \cos u)\hat{i} + (a \cdot c \cdot \cos^2 v \cdot \sin u)\hat{j} \\ & +(a \cdot b \cdot \sin v \cdot \cos v \cdot \sin^2 u \ + \ a \cdot b \cdot \sin v \cdot \cos v \cdot \cos^2 u)\hat{k}\end{aligned} \tag{1}$$

Now, in order to simplify the calculations, we will concern ourselves with the x-y plane, knowing the results are fully applicable to the y-z plane and to any plane which is a rotation of the x-y plane about the y-axis due to the cylindrical symmetry of the vessel. Thus the following substitutions are made into Equation 1:

$$\begin{aligned} v &= 0 \\ \cos v &= 1 \\ \sin v &= 0 \\ a &= c \end{aligned}$$

Equation 1 can now be written:

$$\vec{N} = (a \cdot b \cdot \cos u)\hat{i} + (a^2 \cdot \sin u)\hat{j} \tag{2}$$

The unit normal vector is obtained by dividing the normal vector by its magnitude. The magnitude of Equation 2 is

$$\left\|\vec{N}\right\| = (a^2 \cdot b^2 \cdot \cos^2 u + a^4 \cdot \sin^2 u)^{1/2}$$

Therefore, the unit normal vector is

$$\hat{e}_n = \frac{\vec{N}}{\left\|\vec{N}\right\|} = \left[\frac{b \cdot \cos u}{\left(b^2 \cdot \cos^2 u + a^2 \cdot \sin^2 u\right)^{1/2}}\right]\hat{i} + \left[\frac{a \cdot \sin u}{\left(b^2 \cdot \cos^2 u + a^2 \cdot \sin^2 u\right)^{1/2}}\right]\hat{j} \quad (3)$$

The angle the unit normal takes with respect to the x-axis is given by

$$\gamma = \arctan\left[\frac{a}{b} \cdot \tan u\right] \quad (4)$$

This angle was used in conjunction with the mandrel surface data points and the rubber thickness(0.06 inches) in order to define data points on the surface of the rubber.

Second, a vector that describes the path or direction of a laminate as it passes through each successive point of interest must be determined. Since we took advantage of the vessel's symmetry about the y-axis, we define each point of interest as lying on the x-y plane. Now, let us consider that each point of interest lies on a particular fiber. Since the vessel is fabricated using polar windings, we know that the plane of one particular winding will be at an angle of 11.5 degrees to the x-y plane.

Therefore, each point of interest must be successively translated from its point on the laminate to the x-y plane by rotating through an angle v_i about the longitudinal axis of the vessel. Each point will have its own angle v_i determined as follows:

$$v_i = \arcsin\left[\frac{L_i \cdot \tan 11.5^\circ}{D_i/2}\right]$$

where L_i is the longitudinal location and D_i is the diametrical location of the point of interest I, as shown in Figure 9. Thus, the vector describing the direction of the laminate for any point I is

$$\hat{e}_I = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k} \quad (5)$$

where (as shown in Figure 9)

$$x_A = \frac{D_i}{2}$$
$$y_A = L_i$$
$$z_A = 0.0$$
$$x_B = \frac{D_{i+1}}{2} \cdot \cos(v_{i+1} - v_i)$$
$$y_B = L_{i+1}$$
$$z_B = \frac{D_{i+1}}{2} \cdot \sin(v_{i+1} - v_i)$$

Obviously, a laminate vector cannot be obtained for the last data point. Since this is the tangent point, through, we know that the angle of the laminate is 90°.

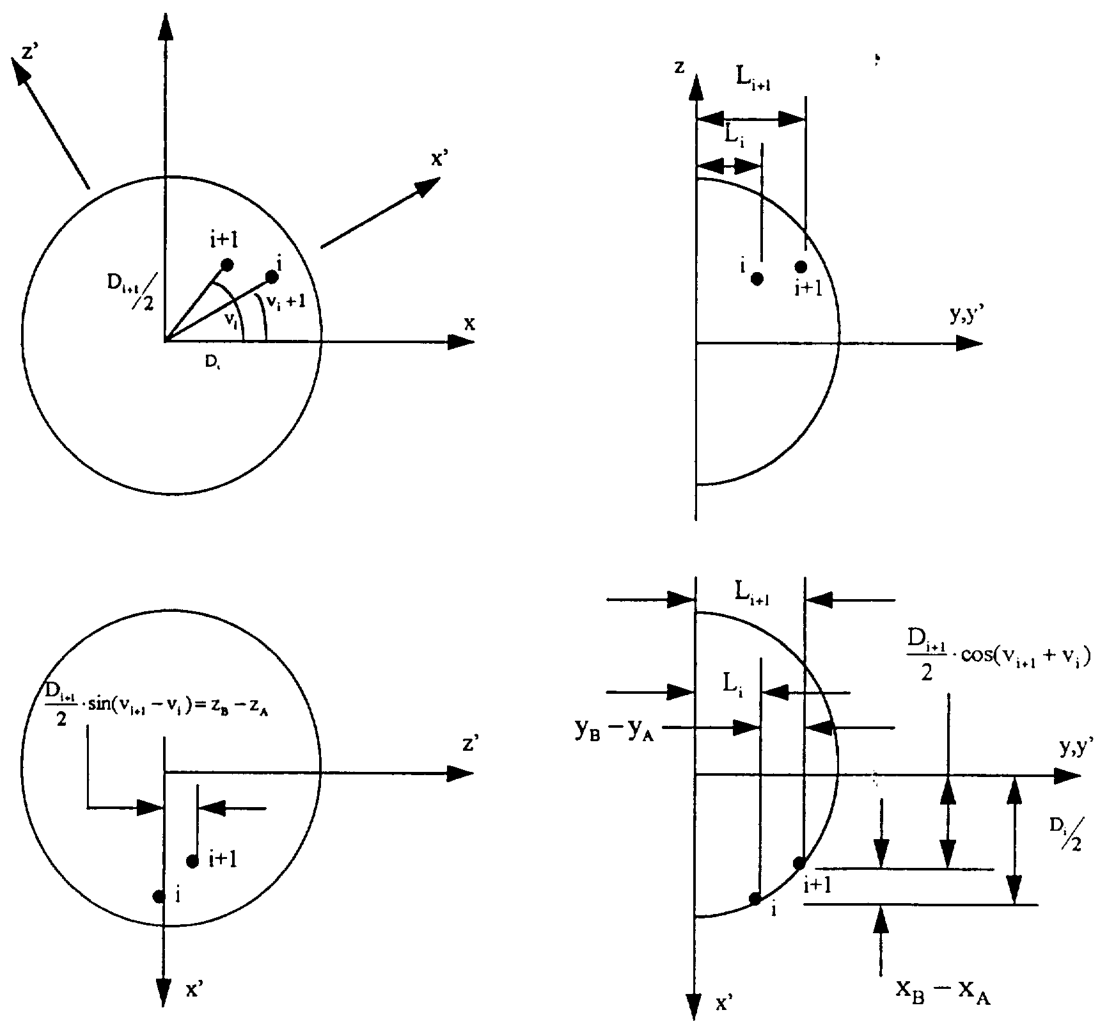

Figure 10: Illustration of critical distances on the dome of the pressure vessel.

Finally, enough information is generated to determine the laminate angle β. The laminate angle β is the angle between the laminate vector projected onto the tangent plane and the tangent vector. The tangent vector $\vec{r}_u$ is calculated in the first step. Next we must obtain the projection of the laminate vector onto the tangent plane. If we take a cross-product of the normal vector with the laminate vector, we will obtain a vector in the tangent plane that is perpendicular to the desired projection vector.

$$\vec{p} = \vec{N} \times \hat{e}_1$$

At this point, there are two ways to proceed that will obtain the same desired result. The first is to take a cross-product of the vector $\vec{p}$ with the normal vector $\vec{N}$. This will give the projected vector of interest

$$\vec{p}_r = \vec{p} \times \vec{N}$$

The angle β can be determined from the dot-product of the projected vector with the tangent vector $\vec{r}_u$.

$$\beta = \arccos\left[\frac{\vec{p}_r \bullet \vec{r}_u}{\|\vec{p}\| \cdot \|\vec{r}_u\|}\right]$$

Alternatively, taking the dot product of the cross-product $\vec{p}$ with the tangent vector $\vec{r}_u$ will give the angle (β+90°), or

$$\beta = \arccos\left[\frac{\vec{p} \cdot \vec{r}_u}{\|\vec{p}\| \cdot \|\vec{r}_u\|}\right] - 90^\circ$$

CALCULATED MATERIAL PROPERTIES

The components of the pressure vessel used in this program are illustrated in Figure 7. In modeling the pressure each component was considered; therefore, material properties for each component were determined using vendor supplied data. These data are summarized in Table 1. The boss, liner, and inert propellant are all isotropic materials. The composite constituents used in the composite overwrap are IM7 fibers in an 8553-45

Table 1: Constituent component properties in pressure vessel

Material	E_{11} (Msi)	E_{22} (Msi)	ν_{12}	ν_{23}
IM7-12k	40.5	2.80	0.22	0.28
8553-45	0.5	0.5	0.3	0.3
Boss	30.0	30.0	0.29	0.29
Liner	1.44	1.44	0.49	0.49
Propellant	0.80	0.80	0.49	0.49
Lamina vf=0.6	24.5	1.2	0.248	0.352

toughened epoxy matrix material. The matrix is isotropic; however, the fibers are transversely isotropic.

Using the theories and methodology described in previous sections, constituent material properties were used to calculate the structural properties in the various sections of the pressure vessels. The building block of each composite structural component is the lamina or unit cell. The composite lamina used in the pressure vessel is IM7-12k/8553-45 lamina that is reported to have a 60% fiber volume fraction. The resulting lamina material properties are also summarized in Table 1. The lamina properties are transversely isotropic.

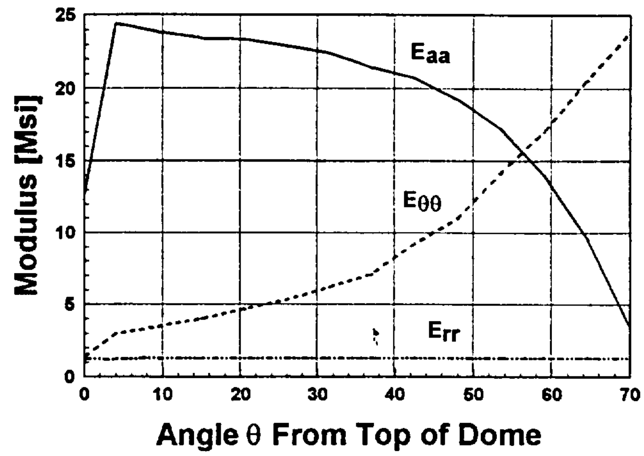

Figure 11: Youngs Moduli as a function of position on the dome of the pressure vessel.

In the cylinder section of the composite pressure vessel, the laminate geometry is $[\pm 11.5/90_2/\pm 11.5/90_2]$. This is not a symmetric winding configuration even accounting for the cylindrical structural configuration. The material properties computed for this section of the composite pressure vessel are anisotripic and are summarized below.

E_a=12.7 Msi	$\upsilon_{a\theta}$=0.002	$G_{a\theta}$=9.55 Msi
E_θ=13.6 Msi	$\upsilon_{\theta r}$=0.345	$G_{\theta r}$=5.32 Msi
E_r=1.34 Msi	υ_{ar}=0.345	G_{ar}=4.95 Msi

Where the "a" direction is along the longitudinal axis of the cylinder, the "r" direction is radically outward from the core of the cylinder, and the "θ" direction is circumferentually around the cylinder.

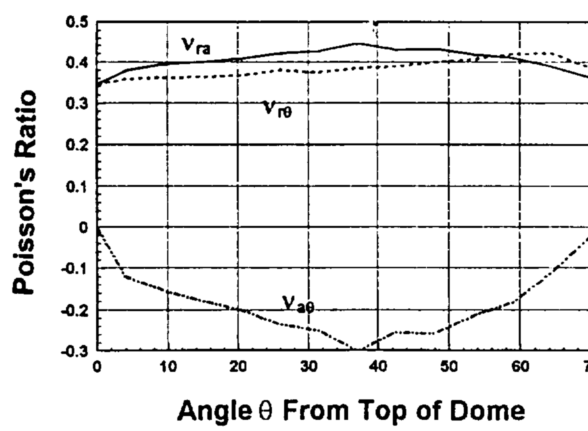

Figure 12: Poisson's Ratio as a function of position on the dome of the pressure vessel.

In the dome of the pressure vessel the laminate geometry changes continuously from the top of the dome to the steel boss. In the dome region, the 90° plies are removed. In addition, as discussed in a previous subsection, the orientation of the fibers in the helical plies also changes because of the winding geometry. Figures 11, 12, and 13 illustrate how the material properties change as a function of dome position. The top position being θ=0. The discontinuity at the beginning of these graphs is a result of the 90° layers being removed from the laminate.

The material properties of the boss, liner, inert propellant, and composite were input into the finite element model of the pressure vessel for evaluation. The composite properties in the dome region were changed in each element along perimeter of the dome.

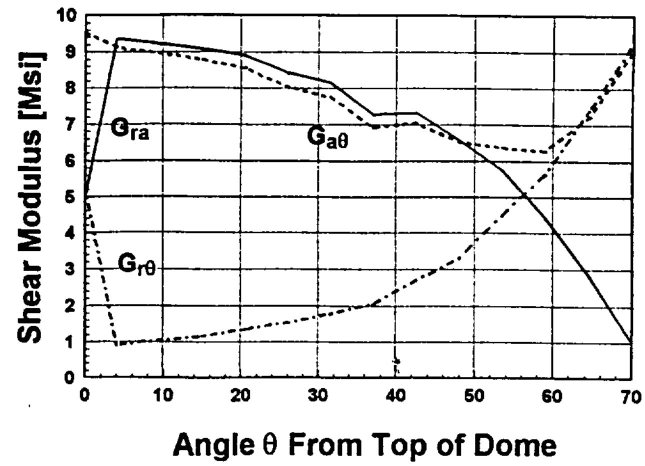

Figure 13: Shear Modulus as a function of position on the dome of the pressure vessel.

FINITE ELEMENT MODEL

The modeling of the 5-3/4" bottle shown in Figure 7 requires a systematic simplification of the complex phenomenon associated with the impact event. Because of the relatively high ratio of the impactor mass to effective bottle mass, the effect of the higher order vibration modes can be ignored enabling the use of a static analysis [3]. The static analysis employed involves various levels of modeling starting at the global or full structural level and ending at the micro-mechanics level. This step wise approach is taken in order to facilitate the determination of fiber and matrix phase averaged stresses in the composite layer that result from the impact event while at the same time making efficient use of computer resources. The phase averaged stresses in the composite are used to predict damage and eventually residual properties of the bottle.

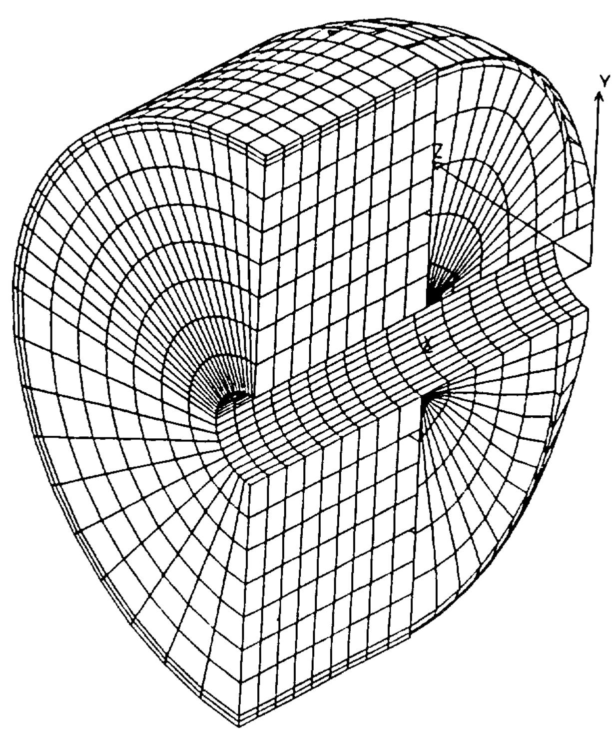

Figure 14: Finite Element Mesh used to model composite overwrapped pressure vessel used in NASA RTOP.

The global or full structural model employs the use of the finite element technique. The finite element mesh used in this investigation is shown in Figure 14. The finite element program used is COSMOS/M on the PC platform. 8 node isoparametric bricks were used through the model. A displacement bonding technique was employed between the cylinder and dome sections to facilitate the changes at the interface between these two regions. This discontinuity in the mesh was a result of element minimization efforts.

At this level the composite is modeled as a single layer of homogeneous material. The finite element model takes advantage of the symmetry associated with the bottle geometry. On the symmetric surfaces the displacements normal to the surface and rotations parallel to the surface are constrained from movement. In addition, a constraint representing the cradle used to hold the bottle is also imposed. This constraint restricts radial displacements in the cylinder section of the bottle along a 1/2" wide strip that adjoins the dome and circumferentially starts at the bottom of the bottle and ends half way up the bottle's side. The purpose of this model is to define displacements along a boundary region that is local to the impact event. Because this region will be chosen away from the actual impact event a point load is used to represent the force of the impactor tup on the bottle at the top intersection of the symmetric surfaces.

The results of the finite element investigation are illustrated in Figures 15 through 18. Figure 15 illustrates the axial stress distribution, Figure 16 the circumferential stress distribution, Figure 17 the radial stress distribution, and Figure 18 the total displacement of the structure. All of the figures include a global view of the field and a close-up of the field in the region of the impact event.

The second step in the modeling process models the region local to the impact event using a finer finite element mesh. At this level each layer of the composite is modeled as a unidirectional ply. The displacements calculated using the global finite element analysis are used as the boundary conditions for the local finite element mesh. Since the local mesh will have more nodal points along all of the boundaries, polynomials are fit through the results of the global analysis in order to assist in the estimation of the proper nodal constraints on the boundary surfaces of the local finite element mesh. The load resulting from the impact event in this model is modeled as a elliptical pressure distribution [19]. The resulting stress distributions in the individual plies are then calculated. These results are then used by a micro-mechanical analysis that computes the phase (constituent) averaged stresses. Knowing the phase averaged stresses, damage in the constituent phases are predicted. These predictions are then used to degrade the material properties in the damage regions in order to calculate the residual properties of the bottle.

This level of detail in the model allows the model results to be compared with the experimental results. Observations between stress fields predicted by the model and damage observed in the specimens can then be compared and theories can be postulated as to how impact damage forms, propagates, and what its residual effects are. Care must be taken not to draw generalized conclusion using limited sets of data. It is very important to insure that phenomenon being observed are not material system or geometry specific. This can be avoided through careful planning of the experimental program.

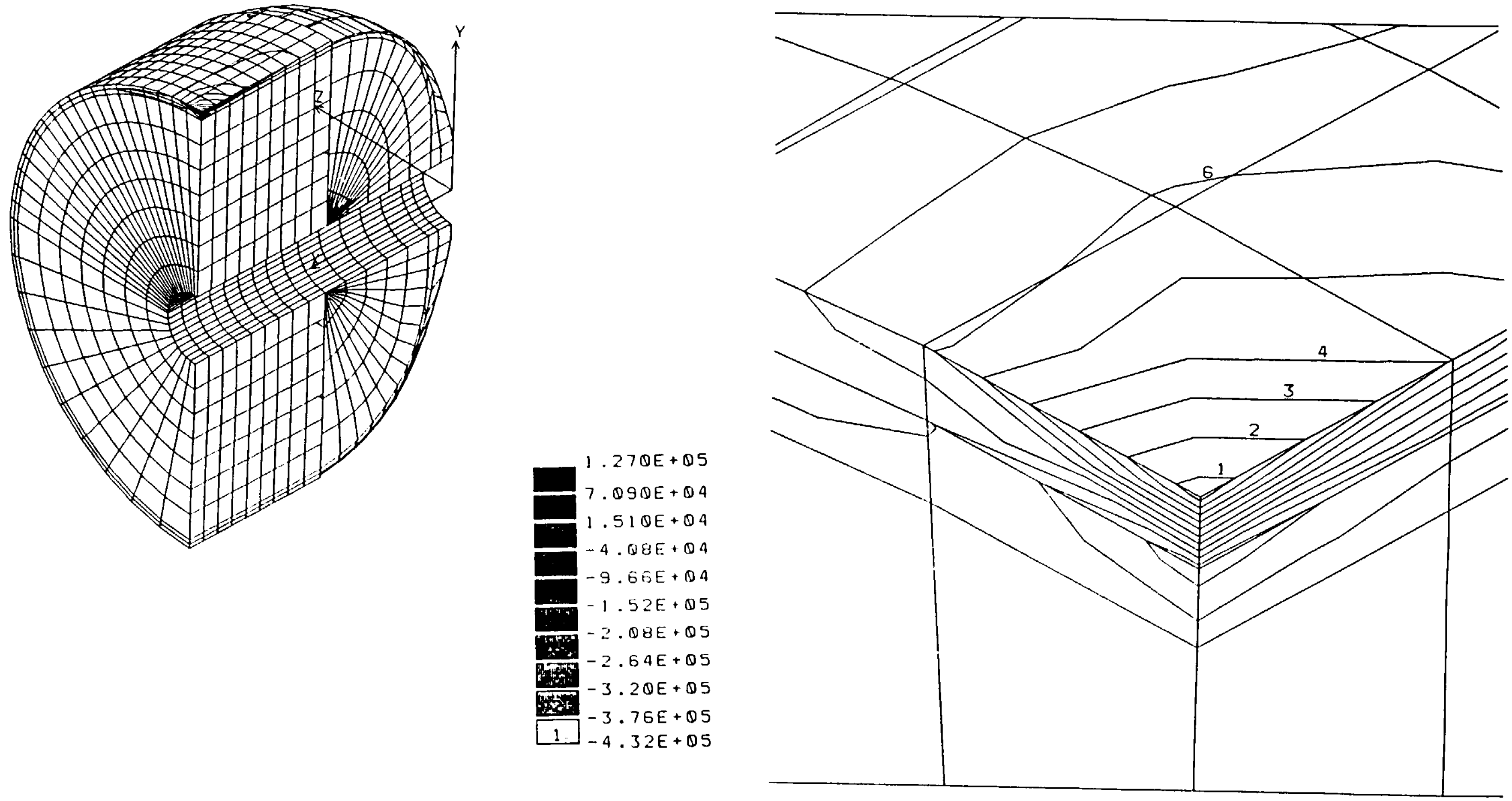

Figure 15: Axial Stress Distribution in composite pressure vessel as calculated using the finite element model

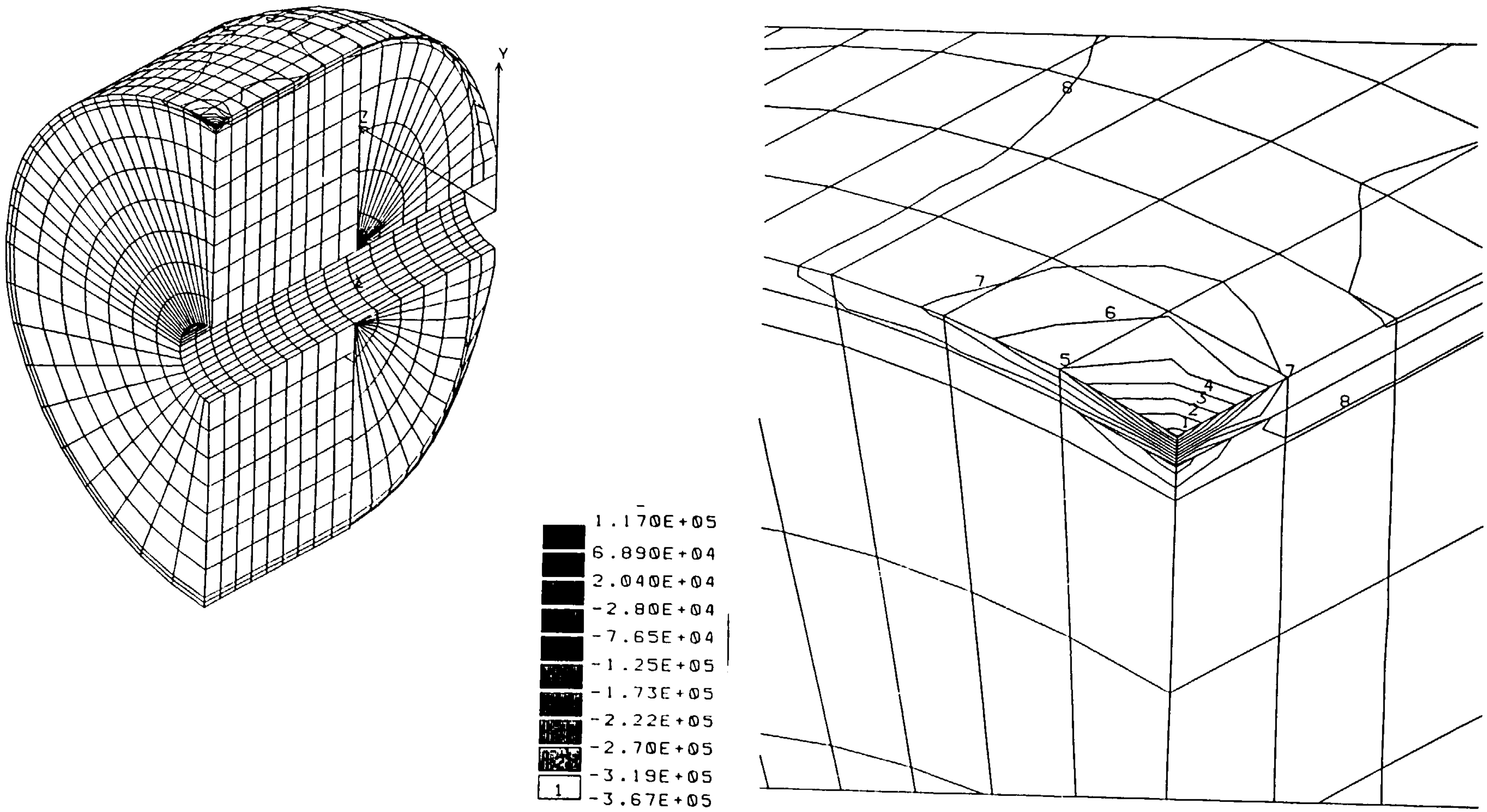

Figure 16: Circumferential stress distribution in composite pressure vessel as calculated using the finite element model

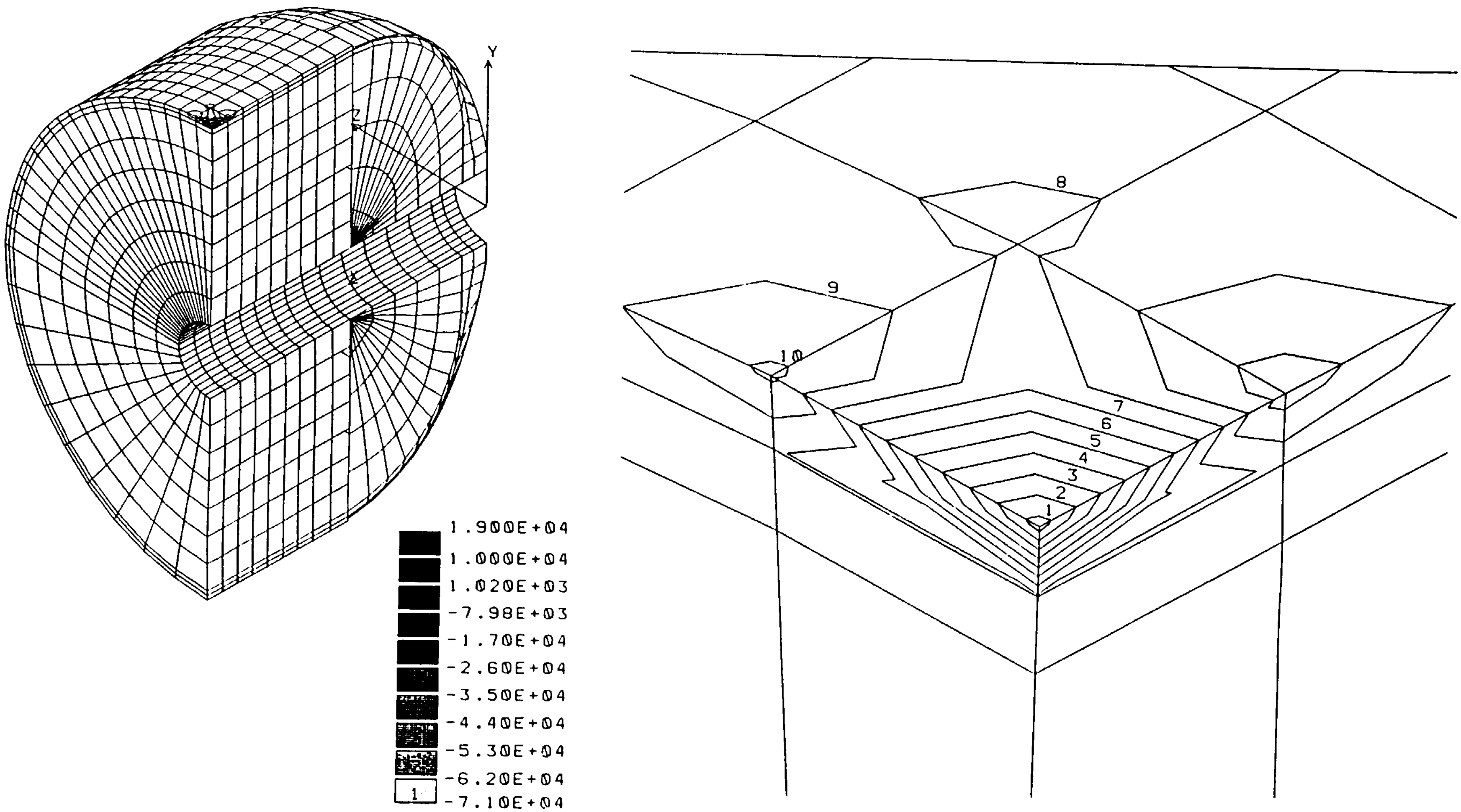

Figure 17: Radial stress distribution in composite pressure vessel as calculated using the finite element model

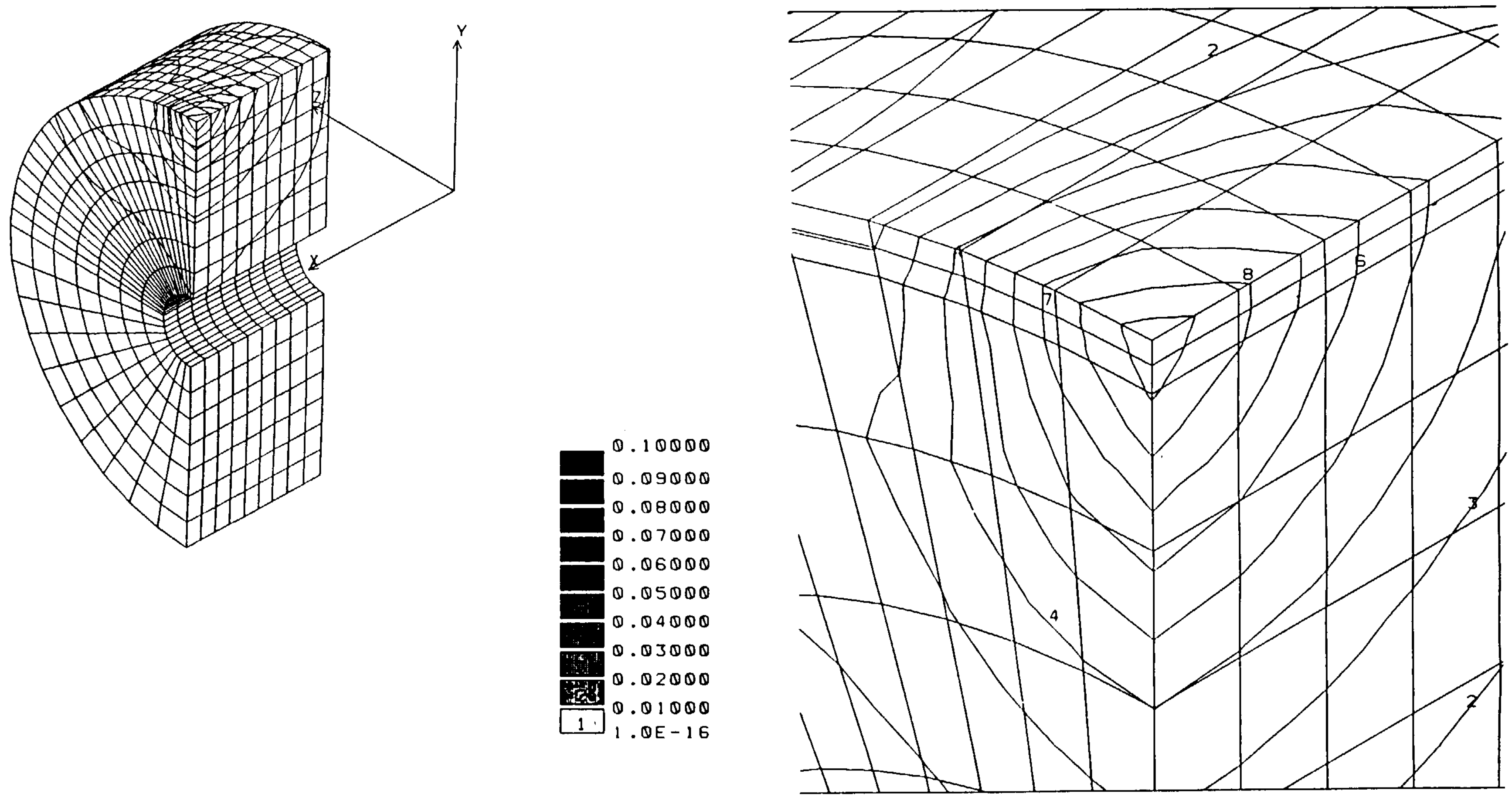

Figure 18: Displacements in composite pressure vessel as calculated using the finite element model

DISCUSSION

The effort being reported here was targeted specifically at modeling the impact of 5-3/4 inch filled composite overwrapped bottles used in the NASA RTOP program. Because of the mass ratio between the impactors and the bottles the dynamic effects could be ignored in this evaluation. Therefore the methodology that is presented in this report does not represent a generalized methodology for determining the state of stress in a structure impacted by a foreign object. It also needs to be noted at this point that this program was conceived and carried out well after all of the experimental work on the NASA RTOP was completed.

The objective of this project was to try and gain a better understanding of the mechanism that cause failure to occur in composite materials when subjected to foreign body impact events. This type of understanding will assist in the development of design methodologies that will assist in the design of damage resistant composite structures. Understanding damage mechanisms in materials is complicated often by the structural geometry and the constituent materials. Often observations of damage formation and the residual effects of the damage are confounded and are specific to a particular structure or material system. One must be careful when making generalized statements about damage formation and the residual effects of the damage.

On this project finite element models were used to determine the state of stress in the 5-3/4inch bottle structure. The components of the bottle are shown in Figure 7. First a global model of the structure was developed, Figure 14. The displacements in this model were used as input to a more refined local model. The difference between the global and local models is that the global model considered the composite laminate as a single homogeneous material with orthotropic properties and the local model modeled the composite on a layer by layer basis. This approach allowed the three dimensional ply level stress to be determined with a manageable number of elements. Another approach to the same end would have been to model the region local to the impact site with the layer size elements and then use the bonding technique discussed above to bond this section of the structure to the rest of the model.

Once the ply level stresses were determined, phase averaged stresses were calculated using micromechanical models. These provided an estimate of the state of stress at the constituent level. At this point various failure criteria can be considered and compared to the observed damage present in the impacted bottles.

This is the point at which the program started to run into problems. The data related to the impacted bottles consisted of time-load traces from the impactor and burst pressures. These provide a picture of how the damage started and what the effect of the damage was. Information related to the state of damage present in the structure after the impact event is critical to developing conclusion about damage formation mechanisms. Only when observations are made on the extent of matrix damage, the type of matrix damage, the extent of fiber damage, etc., can theories explaining the formation of damage be proposed. To insure that these theories are not biased by the material system or the structural geometry, various material systems and structural geometries must be included in the test matrix.

Adding to the problems related to the comparison of the experimental results with the analytical models is the test matrix that was employed on the experimental program. Mostly due to reasons of cost, each test condition was repeated only once. This raises the question of are the differences being reported in burst pressure a result of different states of damage or merely scatter in the experimental data. The author can not rule out the later conclusion because it was observed that the bottles used in this program showed signs of tremendous manufacturing variability. The surface of several bottles looked like strands of string rather than continuos layers. This is not typical of composite structures. This type of material system will have much different failure mechanism than composites with continuos plies. The stranded plies most likely contributed to the high amount of variability that was observed in the experimental data because the matrix was not able to redistribute the load off of broken strands as easily as if the plies were continuos.

The analytical approach does seem to be a reasonable approach to determining the amount of damage that will result from a given impact event. The more important question, and the most difficult, is what effect will this damage have on the integrity of the composite structure. Using the same global-local methodology, some work has been done in degrading material properties of constituents as a result of over stressing. These properties can then be fed back into the finite element code and a non-linear, sequential, analysis could continue with another time step. As in all non-linear finite element analyses several issues need to be addresses. The most obvious that each step through this model will require the redevelopment of the stiffness matrix and the solution of the model. This can be very time consuming depending on the time step. The time step is the second problem. What is the appropriate time step. Also contributing to the difficulties of this approach is the whole question related to including the dynamic effects in the analysis. Once again the accurate prediction of the effects of data will require a carefully engineered data set that will allow the competing effects to be isolated and the confound effects to be quantified.

SUMMARY AND CONCLUSION

The objective of this project was to model the 5-3/4 inch pressure vessels used on the NASA RTOP program in an attempt to learn more about how impact damage forms and what are the residual effects of the resulting damage. A global-local finite element model was developed for the bottle and the states of stress in the bottles were determined down to the constituent level. The experimental data that was generated on the NASA RTOP program was not in a form that enabled the model developed under this grant to be correlated with the experimental data. As a result of this exercise it is recommended that an experimental program be designed using statistical design of experiment techniques to generate data that can be used to isolate the phenomenon that control the formation of impact damage. This data should include residual property determinations so that models for post impact structural integrity can be developed. It is also recommended that the global-local methodology be integrated directly into the finite element code. This will require considerable code development.

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