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FINAL REPORT

ON

THE STUDY OF THE RELATIONSHIP

BETWEEN

PROBABILISTIC DESIGN
AND
AXIOMATIC DESIGN METHODOLOGY

NASA Grant No. NAG3-1479

Submitted By

Dr. Chinyere Onwubiko, P.E.
Principal Investigator
Dr. Landon Onyebueke
Research Associate

TENNESSEE STATE UNIVERSITY
DEPARTMENT OF MECHANICAL ENGINEERING

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FINAL REPORT ON THE STUDY OF THE RELATIONSHIP BETWEEN PROBABILISTIC DESIGN AND AXIOMATIC DESIGN METHODOLOGY

INTRODUCTION

This program report is the final report covering all the work done on this project. The goal of this project is technology transfer of methodologies to improve design process. The specific objectives are:

1. To learn and understand the Probabilistic design analysis using NESSUS.
2. To assign Design Projects to either undergraduate or graduate students on the application of NESSUS.
3. To integrate the application of NESSUS into some selected senior level courses in Civil and Mechanical Engineering curricula.
4. To develop courseware in Probabilistic Design methodology to be included in a graduate level Design Methodology course.
5. To study the relationship between the Probabilistic design methodology and Axiomatic design methodology.

STUDENTS PARTICIPATION

■ **Undergraduate Students**: Twelve undergraduate students were involved in various phase of the project, however, eight of them who were juniors and seniors actually completed projects using the probabilistic methodology. The projects they worked on are as follows:

1. Probabilistic Analysis of a Seismic Design Using NESSUS.
2. The Design of Shock Absorber using NESSUS.
3. Design of High Performance Spur Gear Using PDM.
4. Design of Helical Spring Using PDM
5. Application of PDM to the Analysis of A High Rise Truss Building
6. Application of Probabilistic Methodology in Shaft Design
7. Probabilistic Design of A High Performance Spur Gear with NESSUS As the Analysis code.
8. The development of a Probabilistic Piston-Rod Selector Guide

The students who actually completed their projects are shown along with their topics in Table 1.

■ **Graduate Students**: Seven graduate students were involved in the program and the projects they worked on are:

1. Probabilistic Design of Statistically indeterminate frame structures
2. Probabilistic Analysis of the Performance of a Shell and Tube Heat Exchangers

Table1. SUMARY OF UNDERGRADUATE STUDENTS PROJECTS AND STATUS

Name	Topic	Status
Douglas Crocker	Probabilistic Design of high performance spur gear with the application of NESSUS.	Completed Aug., 1994
Steve Bogard	The Design of Shock Absorber using PDM.	Completed Dec., 1995
Fred Higgs III	Probabilistic design of a helical spring with NESSUS as the design soft ware.	Completed Aug., 1995
Theresa Khayyam	Probabilistic design of a multi-stage truss with NESSUS as the computer code.	Completed May, 1995
Gregory Merriweather	Design of a shaft using probabilistic design methodology with NESSUS as the computer code.	Work in Progress
Daniel Ogbonna	Probabilistic Design of A High Performance Spur Gear with NESSUS as Analysis Code.	Completed May 1995
Sharon Claxton	. The Development of a Probabilistic Piston-Rod Selector Guide.	Completed May 1996

3. Comparative study of Safety Index Calculation

4. Probabilistic Design Methodology in Worm Gear Design

5. Comparative Study of the Use of AGMA Geometry Factor and PDM in the design of spur gear

6. Optimal Configuration of Gear Train Using PDM.

The graduate students who participated in the program with the topics of their Master's Projects are listed in Table 2.

Table 2. SUMARY OF GRADUATE STUDENTS PROJECT AND STATUS

Name	Topic	Status
Nitish P. Beri	System Reliability Studies Plane Frame of a Single Story Structure Under Cumulative Damage.	Completed Aug. 1996
Shiva M. Gangadharan	Comparative study of the use of AGMA geometry factors and PDM in the design of compact spur gear set.	Completed Dec. 1996
Muthuswamy E	Design of worm gears using probabilistic design methodology with NESSUS computer code.	Completed Aug. 1996
Weimin Zhang	Study of design of a gear train using reliability method based on optimization design method.	Completed Dec. 1996
Sharon Dixon	Probabilistic design of a shell-and-tube heat exchanger using NESSUS.	Completed Dec. 1996

RESULTS AND ACHIEVEMENTS

The note worthy achievements and the results of this work are summarized below:

Students Achievements

- ☞ 1st place in the ASME 1995 regional research competition
- ☞ 1st place in the 1995 College of Engineering capstone design presentation
- ☞ 2nd place in the 1996 College of Engineering capstone design presentation.
- ☞ 1st and 3rd place in the Tennessee State University 18th Annual University-Wide Research Day, March 25-26, 1996.

- ☞ Six capstone design projects have been completed and another one is near completion.
- ☞ Five masters thesis completed.
- ☞ Seven papers presented by students in different conferences

Faculty Achievements

- ☞ Nine publications were presented and published in conference proceedings.
- ☞ A course-work was developed and included in an undergraduate/graduate level design methodology course.

CONCLUSION:

We feel that we achieved all our objectives in the program. A major indication of the success of our work is the response of an industrial panel who judged one of our students work as the most outstanding among many Senior Projects that were presented to some industries in 1995. Further more, we have received requests to send the work of our students to Sverdrup Technology Inc. at Arnold AFB, Tennessee. More companies are beginning to show some interest on PDM. We have produced students with fundamental knowledge in Probabilistic Design Methodology. Therefore, we feel that we have started the process of Technology transfer which is the primary purpose of this grant. We have outlined a method for combining Probabilistic and Axiomatic design methodologies to form one comprehensive design method.

PUBLICATIONS/COMMUNICATIONS

Communications by Students

1. Fred Higgs III, "Probabilistic design of a helical spring with NESSUS as the design soft ware". **Presented at the National Conference on Undergraduate Research,** April 20-22, 1995, Union College Schenectady, NY

2. Daniel Ogbonna, "Probabilistic Design of A High Performance Spur Gear with NESSUS as Analysis Code". **Presented at the National Conference on Undergraduate Research,** April 20-22, 1995, Union College Schenectady, NY

3. Sharon D. Dixon, "The Redesign of a Shell and Tube Heat Exchanger Using The Probabilistic Design Methodology". **Presented at Tennessee State University 18th Annual University-Wide Research Day,** March 25-26, 1996.

4. Shiva M. Gangadharan, "Comparative Study of the Use of AGMA Geometry Factors and PDM in the Design of Compact Spur Gear Set". **Presented at Tennessee State University 18th Annual University-Wide Research Day,** March 25-26, 1996.

5. Nitish P. Beri, "Structural Reliability Studies of a Single Story Plane Framed Structure Under Cumulative Damage". **Presented at Tennessee State University 18th Annual University-Wide Research Day,** March 25-26, 1996.

6. Muthuswamy E, "Design of worm gears using probabilistic design methodology with NESSUS computer code". **Presented at Tennessee State University 18th Annual University-Wide Research Day,** March 25-26, 1996.

7. Weimin Zhang, "Design of a Gear Train Using Reliability Method Based on Optimization Design Method". **Presented at Tennessee State University 18th Annual University-Wide Research Day,** March 25-26, 1996.

Publications and Communications by Faculty/Staff

Onyebueke, C. Onwubiko. " The Participation of Students in the Transfer of Technology with regard to Probabilistic Design Methodology." **Proceedings of the 1996 ASEE Annual Conference,** Washington, DC, June 1996.

F.C. Chen, C. Onwubiko, L. Onyebueke. "Design of a Framed Building using a Probabilistic Fault Tree Analysis Method." **Proceedings of the 37th AIAA/ASCE/AHS/ASC structures, structural dynamics and material conference, part 4, pp. 2504-2510**, Salt Lake City, Utah; April 1996.

L. Onyebueke, C. Onwubiko. " The Future Role of Probabilistic Design Methodology in Engineering Education." **Proceedings of the World Conference on Engineering Education, vol. 4, pp. 126-130**, Minneapolis-St. Paul, Minnesota, October 1995.

L. Onyebueke, C. Onwubiko, F.C. Chen. "Probabilistic Design Methodology and Application of Probabilistic Fault Tree Analysis to Machine Design." **Proceedings of the ASME 11th Biennial Conference on Reliability, Stress Analysis and Failure Prevention vol. 2, pp. 125-133**, Boston, Massachusetts; September 1995.

Onwubiko, L. Onyebueke, Chen F.C. "Probabilistic Optimum Design of Compact Spur Gear sets." **Proceedings of the ASME 11th Biennial Conference on Reliability, Stress Analysis and Failure Prevention, vol. 2, pp. 115-124,** Boston, Massachusetts; September 1995.

L. Onyebueke, C. Onwubiko. "Probabilistic Design Methodology as a Tool for Improving Engineering Education." **Proceedings of the 1995 Annual Int'l Conf. of the ASEE pp. 964-969**, Anaheim, California; June 1995.

F.C. Chen, C. Onwubiko, L. Onyebueke. "Design of a Multistory Framed Building using System Reliability Method.". **Proceedings of the 36th AIAA/ASCE/AHS/ASC structures, structural dynamics and material conference, pp. 950-956**, New Orleans, Louisiana; April 1995.

L. Onyebueke, C. Onwubiko. "Probabilistic Design Methodology and Application to Machine Design." **Presented at the 3rd Int'l Conference on Stochastic Structural Dynamics; San Juan, Puerto Rico, January 1995. To be published in the Probabilistic Engineering Mechanics Journal.**

L. Onyebueke, C. Onwubiko. "Probabilistic Design Methodology in Engineering Education." **Proceedings of the ASEE 1994 Southeastern Section meeting, pp. 380-391.**

Appendix A

Copies of communications and publications made by Faculties. The copies enclosed in the phase II report are not included.

1. Combination of Axiomatic and Probabilistic Design Methodologies for Efficient Design Analysis
2. Design of a Framed Building Using A Probabilistic Fault Tree Analysis.
3. The Participation of Students in the Transfer of Technology with regard to Probabilistic Design Methodology

COMBINATION OF AXIOMATIC AND PROBABILISTIC DESIGN METHODOLOGIES FOR EFFICIENT DESIGN ANALYSIS.

L. C. Onyebueke and C. Onwubiko

Tennessee State University

Abstract

Presented here is a methodology that combines the Probabilistic and Axiomatic design methodologies in one. This method, when fully developed, is expected to produce a more efficient design analysis method.

Introduction:

In engineering, the word design has a different meaning to different people. Engineering design could be taken to mean the design of things and systems of engineering nature, such as, machines, products, structures, etc. For the most part, engineering design utilizes mathematics, physics, material sciences, chemistry, thermal and fluid sciences, etc.

There was a time when engineers tackled many problems by cut and try methods. Most designs were based on trial and error. Much emphasis was on workability not on efficiency nor reliability. Cost was out of the question since competition was not an issue. History shows that mankind has made a lot of progress in the area of engineering design. However, much still remains to be done before we can ever get to the desired perfect point.

Almost every engineering design book gives the steps that lead to achieving an output in a design problem. What is mainly lacking in most of these steps is knowing how to choose between the many information that could result from the steps. Increasing complexity of design problems militates against undefined and non-scientific approaches to choice in design. It is now a human error for a designer to automatically exclude certain parts of the solution space simply because it does not seem to contain a feasible solution. The main problem that faces every designer is that of coping with the complexity of a huge search space filled with millions of alternative combinations of possible sub-components. Traditionally, this type of situation is dealt with by concentrating on one sub-problem at a time. This still does not always give a satisfactory solution. The search space in which we have to look for feasible new systems, composed of radically new products and components, is too big for rational search and too unfamiliar to be penetrated and simplified by the judgments of those whose education and experience has been limited to the existing design and planning professions.

The need for a scientific and systematic method in the choice of design alternatives has led to the development of different methodologies such as the deterministic, axiomatic, probabilistic, etc. The deterministic design methodology has been applied in so many areas of machine design. Suh and others [1-2] have demonstrated and applied the axiomatic design methodology to reaction injection molding machinery, manufacturing and manufacturing systems. There is a growing evidence that the probabilistic design methodology is beginning to attract more attention. The evidences include the growing number of reliability-oriented specialty conferences, short courses, sponsored research, research papers, and technical books [3-5]. The advancement in design as a result of the above design methodologies cannot be over-emphasized. Though, a lot has been achieved using these methods separately, is it not possible to combine these different methods into one comprehensive method?

Developing a comprehensive design system that incorporates the deterministic, axiomatic and probabilistic design methodologies will give the designer a better scientific and systematic ability to choose between several design alternatives.

Overview:

1.1 Deterministic Design Methodology:

In deterministic design approach, the designer considers the design parameters to be known with certainty. The uncertainties in the response functions are not quantified, and the actual safety margin remains unknown. The contingency of failure is totally discounted, and this leads to the use of high factor of safety. Deterministic design is reliable in situations where the design is tolerant to the environment, is insensitive to material properties, and is characterized by simple geometry, redundancy and fail-safe features. Under the deterministic approach, external loading and the properties of the structure are represented as though they are fully determined. Though much importance is placed on factor of safety, this does not give a direct account of uncertainties in design parameters.

1.2 Axiomatic Design Methodology

This design methodology is based on axioms. The two basic axioms state that:

(a) Each functional requirement of a product should be satisfied independently by some aspect, feature or component within the design.

(b) Good designs are minimally complex.

These two concepts are formalized as the working set of design and manufacturing axioms [1].

Axiom 1: Maintain the independence of functional requirements.

Axiom 2: Minimize the information content.

The functional requirement (FR) is the statement of the design task. This method demands that the FRs be stated such that any of the FRs can be specified without regard to any other FR. A necessary and sufficient condition for a set of functional requirements to be acceptable is that none of the FRs are redundant or inconsistent with the other FRs. The second axiom requires minimization of information content. The information is given in terms of design parameters (DPs). The design process then involves relating the FRs of the functional domain to the DPs of the physical domain. This is illustrated in figure 1.

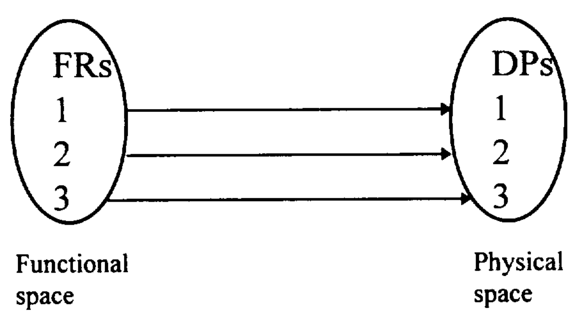

Figure 1: Mapping the FRs in the functional space to the DPs in the physical space to satisfy the designer specified FRs.

The relationship between functional requirements and design parameters can be written in a matrix form as follows:

$$F = \begin{bmatrix} FR_1 \\ FR_2 \\ . \\ . \\ FR_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & a_{nn} \end{bmatrix} \begin{bmatrix} DP_1 \\ DP_2 \\ . \\ . \\ DP_n \end{bmatrix}$$

The axiomatic approach classified designs under coupled, quasi-coupled and coupled. In an uncoupled design, only the diagonal elements of the design matrix are non-zero e.g.

$$F = \begin{bmatrix} FR_1 \\ FR_2 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} DP_1 \\ DP_2 \end{bmatrix}$$

In a quasi-coupled design, the matrix is represented as

$$F = \begin{bmatrix} FR_1 \\ FR_2 \\ FR_3 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{bmatrix}$$

In this case the independence of the FRs can be assured if the DPs are adjusted in a particular order. The last and most common is the coupled design which can be represented with the following matrix

$$F = \begin{bmatrix} FR_1 \\ FR_2 \\ FR_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{bmatrix}$$

In a coupled design, all elements of the design matrix may be non-zero.

Considering the statements of the two axioms, an uncoupled design is considered the best design, followed by a quasi-coupled design. The coupled design is considered to be a bad design. The axioms in this method serve more as analytical tools to evaluate the design decisions. They simplify the design process by eliminating at any stage of the design process many alternatives that prove to be unsatisfactory.

1.3 Probabilistic Design Methodology

Probabilistic design is concerned with the probability of non-failure performance of structures of machine elements. It is much more useful in situations where design is characterized by complex geometry, possibility of catastrophic failure or sensitive loads and material properties. In a probabilistic structural analysis, the primitive (random) variables that affect the structural behavior have to be identified. These variables, which include temperature, material properties, structural geometry and loading conditions, must be described by their respective probability distributions. A probabilistic analysis requires substantially more computation than the corresponding deterministic analysis. Some of the probabilistic analysis methods that have been developed described [6]. The methods are:

1. Approximate evaluation of the mean and variance of functions through Taylor series expansion.
2. Monte Carlo simulation and variation.
3. Limit state function approach.
4. Hybrid approach in which the most probable point or directional information approaches is used to reduce the sample space in simulation methods.

The foundation theory of the PDM using the limit state approach can be expressed as follows: In the limit state approach the designer is required to define the limit state functions applicable to a given design problem. The limit state function or g-function is a function of a vector of basic random variables, $\mathbf{X} = (X_1, X_2, ---X_n)$, with regions, namely, the failure ($g \leq 0$) and the safe ($g>0$)regions. Given the probability density function (PDF), $f_X(X)$, the probability of failure domain Ω

$$P_f = \int_{\Omega} \cdots \int f_x(X)dx$$

This multiple integral is difficult to evaluate directly for complicated g-function. It can be computed using a straight forward standard Monte Carlo procedure which is usually time consuming. The limit state function method applies the most probable point (MPP) search approach. Several approaches are available to search for the MPP. One efficient method in use is the advanced mean value method (AMV). This method provides efficient cumulative distribution function (CDF) analysis as well as reliability analysis [6]. Another method that is considered efficient as well is the adaptive importance sampling method (AIS) [7]. This method focuses on minimizing the sampling domain in the search apace after the MPP is identified. The AIS method is generally used for system reliability analysis. The PDM accounts for uncertainties in design variables. It quantifies the effects of uncertainties for structural variables and the evaluation of failure probability.

1.4 Comprehensive Approach

The method being proposed here is a method that incorporates all the three methods discussed above. The flow-chart for this method is represented on figure 3.

In tackling any particular phase of the design problem as spelled out in a design morphology, the designer is unable to make a start until he has defined the problem to the best of his ability. This requires a statement of need and a clear formation of the goals to be achieved. Once the problem has been stated as completely as possible, the designer collects and organizes all the information available to him that appears to have bearing on the problem and then proceeds with its solution. Stating the problem as completely as possible will require a clear and precise problem statement and a clear definition of the functional requirements. Having defined the FRs, the designer can now choose DPs to satisfy the FRs by conceiving a physical solution in the physical domain. At this stage of the design, certain constraints can be defined as those factors which establish the boundaries on acceptable solutions. The difference between functional requirements and constrains is that functional requirements are negotiable final characteristics of a product while constraints are not.

The mapping stage is followed by test to verify if the design axioms are met. If they are met, the designer then determines if the design requires a deterministic approach or a probabilistic approach.

Conclusion:

The method presented demands a lot of steps but will certainly give a more efficient design analysis.

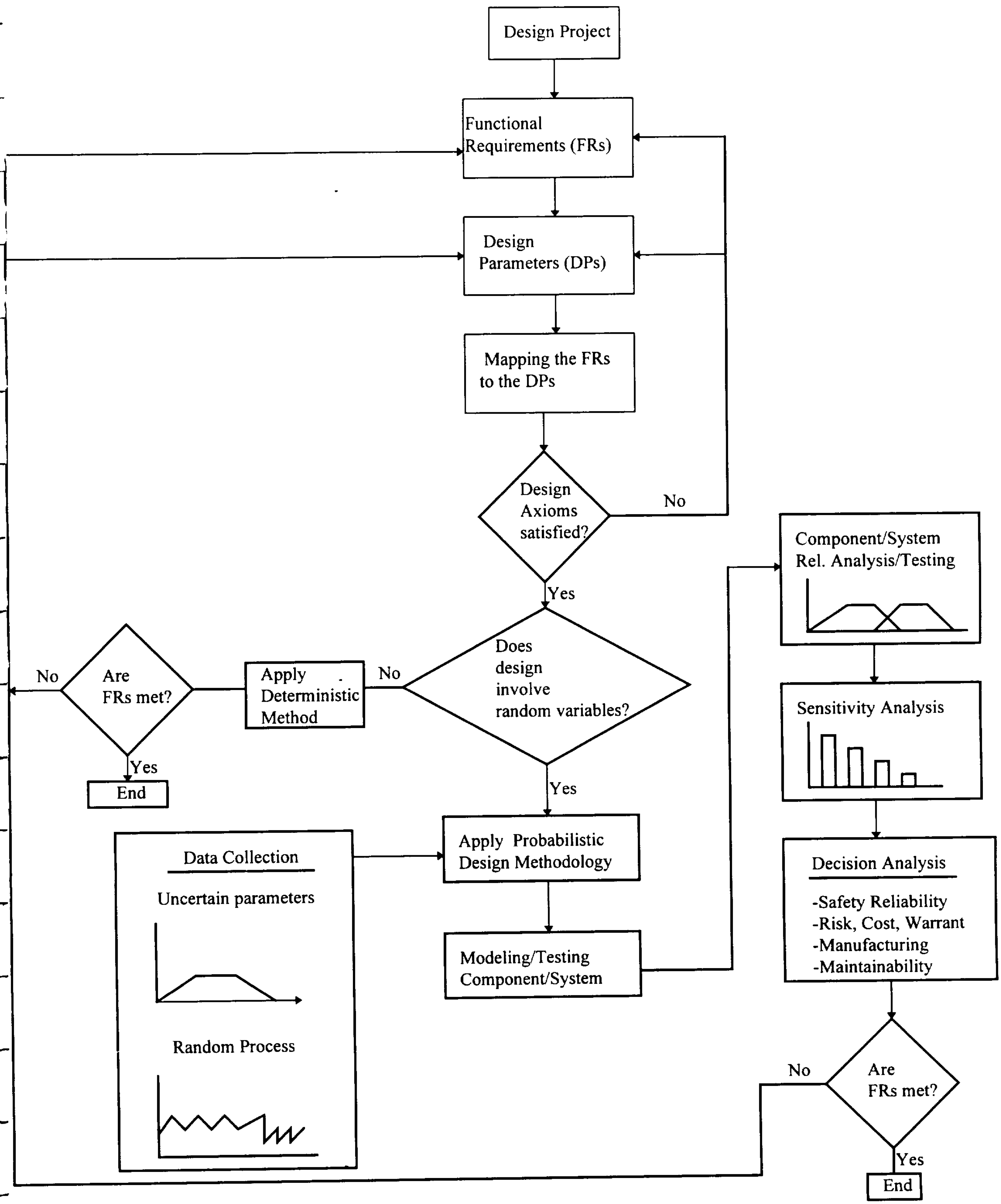

Figure 3: Combination Design Methodology Flow Chart.

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AIAA-96-1608-CP

DESIGN OF A FRAMED BUILDING USING A PROBABILISTIC FAULT TREE ANALYSIS METHOD

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Abstract

This paper shows the application of probabilistic fault tree analysis (PFTA) method to the design of a framed structure. The PFTA includes the development of a fault tree to represent the system, construction of an approximation function for bottom events, computation of sensitivity factors of design variables, and the calculation of the system reliability. The effect of uncertainty in the design parameters is quantified by changing the standard deviation of some of the design parameters and recomputing the probability of failure. The computer code employed for the analyses is NESSUS (Numerical Evaluation of Stochastic Structure Under Stress). A design example is presented. The importance of considering geometry as random variables in structural design is quantified.

Introduction

Probabilistic structural analysis methods (PSAM)[1] have been developed at the NASA Lewis Research Center to analyze the effects of fluctuating loads, variable material properties, uncertainties in analytical models and geometry, and other factors. NESSUS (Numerical Evaluation of Stochastic Structure Under Stress)[2-4] is a probabilistic structural analysis computer code developed under the PSAM program. This code can predict the scatter of structural response variables due to structural and environmental uncertainties. These predictions are subsequently compared with their probable failure modes to assess the risk of component fracture.

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The method of probabilistic structural analysis has been applied by Shiao and Chamis[5] to determine structural reliability and to assess the associated risk due to various uncertainties in design variables. Chamis and Hopkins[6] have applied this methodology to hot engine structures. Other applications also have been reported by Shantaram and Chamis[7]. They studied the effect of combined mechanical and thermal loads on space trusses.

This paper demonstrates the application of probabilistic analysis procedure in the design of a framed building. The software tool utilized in this example is NESSUS[8].

Framed Building Analysis

A framed building is a system whose main components are beams and columns. Different failure modes of these components can be identified by various collapse mechanisms. For each collapse mode, the virtual work principle is applied to relate the applied loads to the structural resistance (plastic moment). The difference between the structural resistance and the applied loads is defined as the response function (g-function) or limit state function. The failure of a system is due to the failure of its components. Thus, a probabilistic fault tree analysis is applied on the components to assess the probability of the failure of a building.

Design Example

A building design involves several steps, which has been illustrated by Chen et al.[9] Presented in this paper is the application of the PFTA to the design of a three-story two-bay rigid framed building as shown in Figure 1(a). The span, SP, and the height, HT, are considered to have normal distribution with the mean of 24 ft and 12 ft on each floor, respectively. The design dimensions are assumed to have 1% variation, or scatter.

1. Load estimation

The loads are estimated as horizontal force H=4 k, vertical load V=40 k and distributed load W=2 k/ft. The loads are assumed to have lognormal distribution with 20% variation and are shown in Table 1.

2. Collapse mechanisms

Five possible collapse modes are shown in Figures 1(b) to 1(f). To simplify the diagram, all the loads with zero virtual displacement are omitted.

3. Response function or g-function

A response function or the margin of safety is defined as the difference between the internal energy stored and the external work done on the collapse mechanism. Applying the virtual work principle to Figures 1(b) to 1(f) and rearranging the equation yields the following response functions, respectively,

g1 = PM1 - 1/2*H*HT
g2 = PM2 - 1/3*H*HT
g3 = PM3 - 1/6*H*HT
g4 = PM4 - 1/16*W*SP**2
g5 = PM5 - 1/8*V*SP

where PM's are the plastic moments of steel sections.

4. Fault tree

Since the failure of each mode will cause the failure of the building, an **OR** gate is selected according to NESSUS format[3,10]. Figure 2 is a representation of the fault-tree.

5. Preliminary design

From the AISC table, a set of steel sections is selected to provide positive values for the response functions g1 to g5. It is assumed that the plastic moment of steel section has lognormal distribution with 10% variation. The values of plastic moments and their statistics are shown in Table 1. An input file is prepared according to the NESSUS format and the output of the safety indexes is listed in column 2 of Table 2.

6. Revisions

Assume that the safety index is 3.0 as recommended by Galambos et al.[11], the results of the first revision and the second revision are recorded in column 3 and column 4 of Table 2, respectively. Since all the safety indexes are greater than 3.0, the results of final steel selections are shown in the last column of Table 2.

Discussion

The safety indexes in Table 2 are the output of NESSUS using the curvature-based adaptive importance sampling method (AIS)[12], since it is generally used for system reliability analysis. For comparison purposes, the conventional Monte Carlo method (MCM) is also performed. Safety indexes by both methods are identically the same for each performance function. However, the computer time for the MCM is much higher than that of the AIS. A slight difference in the system's safety index, by both methods, is shown in Table 3.

Figure 3 shows the sensitivity factors for the design parameters in the response function g4. It indicates that the dimension of the structure, the span, is more sensitive than other parameters in terms of the failure probability of the structure. To further quantify the effect of the uncertainty of the dimension, three (3) tables are presented: Table 4 shows the effect of the variation of the span alone, Table 5 for height alone, and Table 6 for both. The dimension scatter is assumed to have 1% increment from the mean value. The associated failure probability and the percentage change, based on the mean value, are recorded in Table 4 to 6. Table 6 shows that the failure probability increases almost 60% for 5% variation of dimensions. These relationships of geometry uncertainty vs. failure probabilities are also plotted in Figure 4.

This paper shows an example of designing a framed building with the possible variations in applied loads, resistant moments, and geometry. The results presented in Figures 3 and 4 illustrate the importance of considering geometry as random variables in structural design. With the help of the PFTA the designer is able to compute the joint effect of the failure modes.

Acknowledgment

This paper is based upon work supported by NASA Lewis Research Center under grant NAG3-1479 and it is gratefully acknowledged.

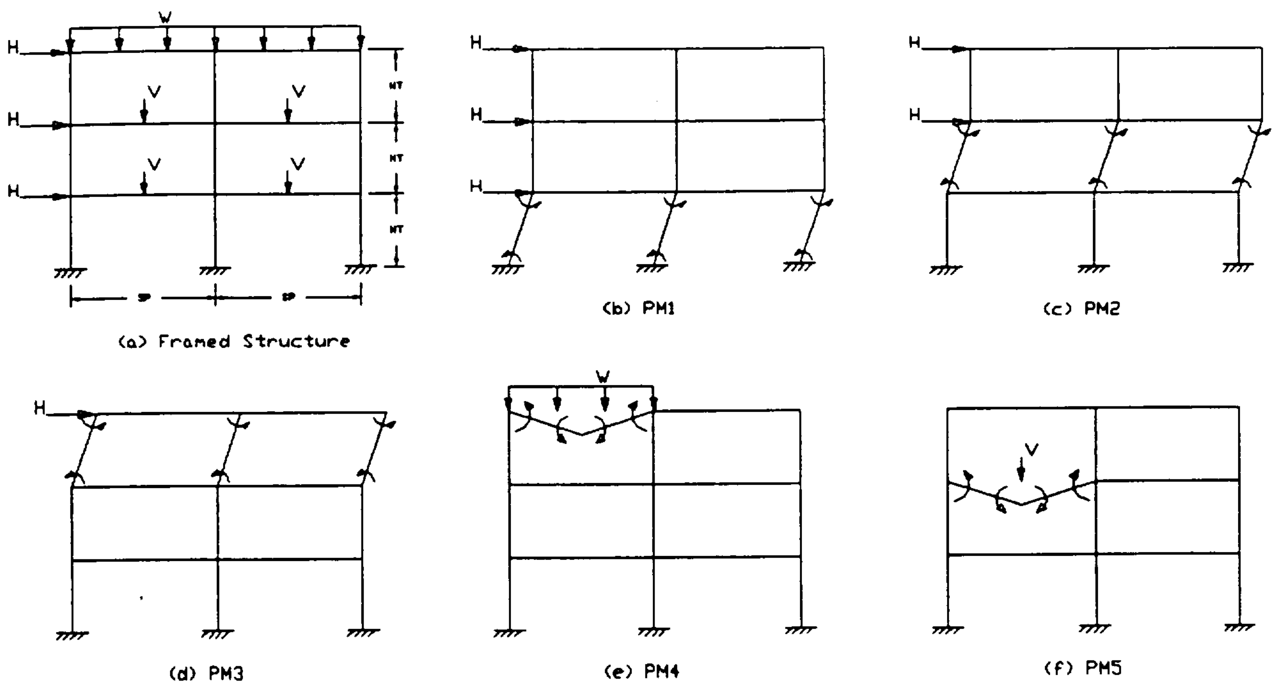

Fig. 1 (a) Framed Structure, (b)-(f) Collapse Mechanisms

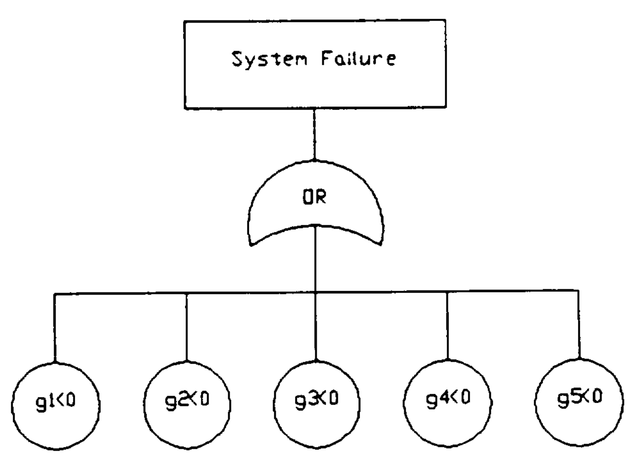

Fig. 2 Probabilistic fault tree representing 5 failure modes

Table 1. Random Variables and Their Statistics

Variable	Distribution	Mean	Std. Dev.	Scatter, Percentage
PM1(Col.1), k-ft	Lognormal	48.0	4.80	10
PM2(Col.2), k-ft	Lognormal	48.0	4.80	10
PM3(Col.3), k-ft	Lognormal	34.2	3.42	10
PM4 (Roof), k-ft	Lognormal	99.3	9.93	10
PM5 (Beam), k-ft	Lognormal	162.0	16.20	10
HT (Height), ft	Normal	12.0	0.12	1
SP (Span), ft	Normal	24.0	0.24	1
H (Wind), k	Lognormal	4.0	0.80	20
V (Gravity), k	Lognormal	40.0	8.00	20
W (G-Roof), k/ft	Lognormal	2.0	0.40	20

Table 2. Plastic Moments vs. Safety Indexes for Various Trial

Response Function	Preliminary		1st Revision		2nd Revision	
	Plastic Moment	Safety Index	Plastic Moment	Safety Index	Plastic Moment	Safety Index
g1, PM1 (Col.1)	48.0 W10x15	3.1886	same	same	same W10x15	same
g2, PM2 (Col.2)	48.0 W10x15	5.0155	34.2 W8x13	3.4882	same W8x13	same
g3, PM3 (Col.3)	34.2 W8x13	6.6122	same	same	same W8x13	same
g4, PM4 (Roof)	99.3 W14x22	1.5107	132.0 W16x26	2.7882	162.0 W16x31	3.7080
g5, PM5 (Beams)	162.0 W16x31	1.4179	200.0 W18x35	2.3673	286.0 W21x44	3.9786
System	---	1.042	---	2.224	---	3.083

Table 3. Comparison of Different Solution Methods

Methods	Number of g-function Calculations	Safety Index	Error, Percentage
Curvature-Based Adaptive Importance Sampling (AIS2)	66 (55+)	3.083	2
Conventional Monte Carlo*	100000 (83+)	3.145	0

* This method is used as the "exact" for comparison.
+ Number of failure points.

Table 4. The Effect of the Uncertainty of Span on the Failure Probability of the Structure

Dimension Scatter, Percentage	Number of g-function Calculations*	Failure Probability	Change,# Percentage
0	65(54+)	0.10180E-2	0
1	66(55+)	0.10254E-2	0.7
2	67(56+)	0.10481E-2	3.0
3	70(59+)	0.10930E-2	7.4
4	74(62+)	0.11498E-2	12.9
5	83(71+)	0.13468E-2	32.3

* Adaptive importance sampling method (AIS2) is used.
# The 0% scatter (mean value) is used as the base for comparison.
+ Number of failure points.

Table 5. The Effect of the Uncertainty of Height on the Failure Probability of the Structure

Dimension Scatter, Percentage	Number of g-function Calculations*	Failure Probability	Change,# Percentage
0	66(55+)	0.10137E-2	0
1	66(55+)	0.10254E-2	1.2
2	67(55+)	0.10448E-2	3.1
3	67(55+)	0.10922E-2	7.7
4	66(54+)	0.11598E-2	14.4
5	72(58+)	0.13511E-2	33.3

* Adaptive importance sampling method (AIS2) is used.
# The 0% scatter (mean value) is used as the base for comparison.
+ Number of failure points.

Table 6. The Effect of the Uncertainty of both Span and Height on the Failure Probability of the Structure

Dimension Scatter, Percentage	Number of g-function Calculations*	Failure Probability	Change,# Percentage
0	66(55+)	0.10068E-2	0
1	66(55+)	0.10254E-2	1.8
2	68(56+)	0.10675E-2	6.0
3	69(57+)	0.11574E-2	15.0
4	71(59+)	0.12964E-2	28.8
5	81(66+)	0.15821E-2	57.1

* Adaptive importance sampling method (AIS2) is used.
# The 0% scatter (mean value) is used as the base for comparison.
+ Number of failure points.

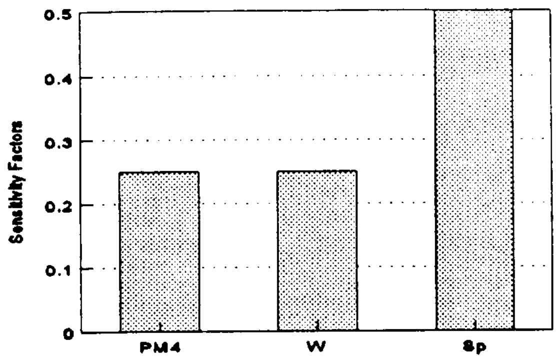

Fig. 3 Sensitivity Factors for the Design Parameters in g4 (Probability of failure=0.000105)

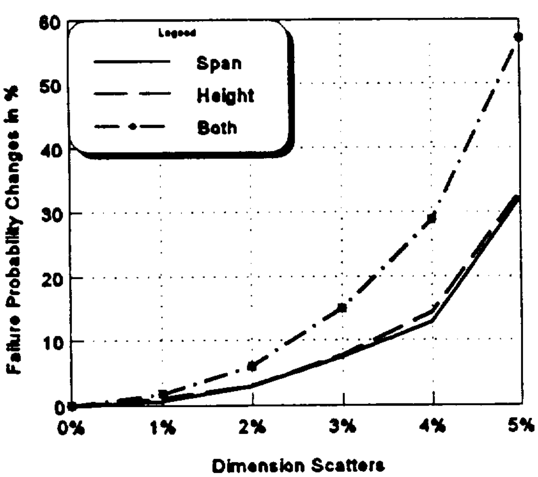

Fig. 4 Geometry Uncertainty vs. Failure Probability Change

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The Participation of Students in the Transfer of Technology with Regard to Probabilistic Design Methodology.

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Abstract:

The variability and complexity of human needs and demands always lead to advancement in technology, new discoveries and hence, the need for technology transfer.

Probabilistic Design Methodology (PDM) is a computational simulation method based on simple engineering models. It is concerned with the probability of non-failure performance of structures or machine elements. This technology has been successfully applied to various loading conditions encountered during space flights. However, this technology is yet to be accepted in the industries. This paper uses PDM to demonstrate the role of students in the transfer of technology.

Included in the paper are overview of PDM, the different stages that are necessary in preparing students for effective technology transfer. Projects carried out by engineering students at Tennessee State University are used to illustrate the features of PDM and how students can be used as a means for the transfer of the technology to industries.

Introduction:

The effectiveness of any new technology is reflected in its usefulness. Its popularity depends on the application and success in transferring the technology.

One of the most important areas in engineering that requires precision and accuracy is engineering design. Engineering design mostly depended on deterministic design methodology. As a result, deterministic design has reached a very high level of sophistication to the point that modern computational techniques make it possible to determine the stresses, strain and displacement of complex structures. In deterministic design the contingency of failure is totally discounted, which leads to the choice of a high factor of safety.

Unfortunately, the design of structures are really clouded with uncertainties. The fact that deterministic design methodology does not account for uncertainties in a direct manner makes it impossible to know when a system is over-designed or vice versa. This fact has led to a more focus on PDM. PDM is increasing in popularity among researchers due to the fact that it takes into consideration reliability, optimization, cost parameters and the sensitivity of design parameters. Deterministic method, which is the most common design method in the design of machine elements lacks most of these features. Probabilistic design approach is concerned with the probability of non-failure performance of structures or machine elements. It is much more useful in situations where the design is characterized by complex geometry, possibility of catastrophic failure or sensitive loads and material properties. The PDM normally requires a lot of computation but the advancement in technology has reduced the rigors that normally accompany

most of the analyses involved. This method is used at the moment in a limited way due to the following facts:
1. Most people are unaware of the capabilities of the PDM and the available computer codes.
2. There is very little information available on most design parameters.

However there is a growing evidence that the PDM is beginning to attract more attention. The evidences include the growing number of reliability-oriented specialty conferences, short courses, sponsored research, research papers, and technical books[1-5]. The PDM and the information it provides are becoming more widely understood and better appreciated.

This technology is not yet common in the industries. In order to transfer the technology effectively to the industries, students must participate in the program. The stages necessary for students participation in technology transfer can be outlined as follows:
1. Recruitment of students.
2. Teaching the students the new technology and equipping them with required tools.
3. Test their understanding of the technology by involving them in projects that will require the application of the technology.
4. Sending students to industries that may need the technology.

Overview of PDM:

Probabilistic design approach is concerned with the probability of failure or preferably, reliability, the probability that a structure will realize the function assigned to it without failure. In a probabilistic structural analysis, the primitive (random) variables that affect the structural behavior have to be identified. These variables, which include temperature, material properties, structural geometry and loading conditions, must be described by their respective probability distributions. The amount of information contained in the output data is equivalent to the amount of information required for the input data. Some of the probabilistic analysis methods that have been developed are described by Rajagopal et al[6]. These methods are:
1. Approximate evaluation of the mean and variance of functions through Taylor series expansion
2. Monte Carlo simulation and variation
3. Limit State function approach
4. Hybrid approach in which the most probable point or directional information from limit state function approach is used to reduce the sample space in simulation methods.

The analysis method applied in this demonstration is the limit state approach. This method requires the designer to define the limit state functions applicable to a given design problem. The limit state function or g-function is a function of a vector of basic random variables, $\mathbf{X} = (X_1, X_2, \text{---} X_n)$, with $g(\mathbf{X}) = 0$ being the limit state surface that separates the design space into two regions, namely, the failure ($g \leq 0$) and the safe ($g > 0$) regions. Given the point probability density function (PDF), $f_X(X)$, the probability of failure in the failure domain Ω is,

$$p_f = \int_\Omega \text{---} \int f_x(X)\, dx \qquad (1)$$

This multiple integral is difficult to evaluate directly for complicated g-function. It can be computed using a straight forward standard Monte Carlo procedure which is usually time consuming. The limit state function method applies the Most Probable Point (MPP) search approach (see figure 1). Several approaches are available to search for the MPP. One efficient method in use is the Advanced Mean Value method

(AMV). This method provides efficient cumulative distribution function (CDF) analysis as well as reliability analysis[7]. Another method that is considered efficient as well is the Adaptive Importance Sampling Method (AIS). This method focuses on minimizing the sampling domain in the search space after the MPP is identified. The AIS method is generally used for system reliability analysis[8].

The analytical process involved in the limit state approach has been illustrated elsewhere[9]. For any given g-function, there exists one or more solutions that satisfy the condition of $g(\mathbf{X})=0$ with locally or globally maximum joint probability density. One of these solutions is the MPP in the X-space. In some computational codes (e.g. NESSUS) the MPP is defined in a transformed space called the u-space (in which the u's are independent) to facilitate probability computations[10]. By transforming $g(\mathbf{X})$ to $g(\mathbf{u})$, the most probable point, u* on the limit state, $g(\mathbf{X})=0$, is the point which defines the minimum distance from the origin to the limit state surface. This value is referred to as the safety index, β. Figure 1 illustrates a MPP diagram.

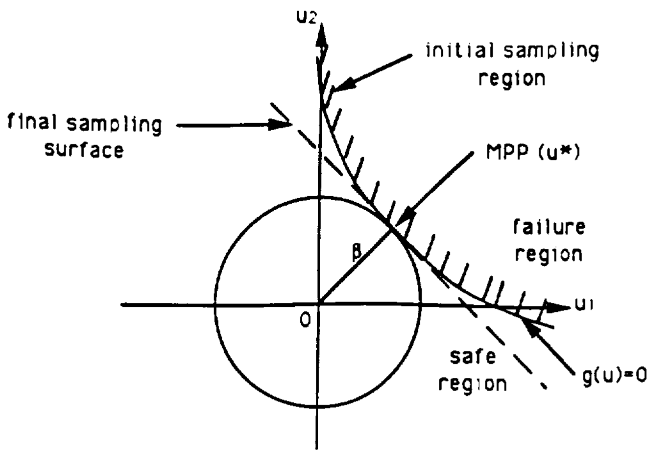

Figure 1: Illustration of the Most Probable Point (MPP).

Computational Tool:

One of the software tools available for determining MPP and p_f is NESSUS. It has three different modules known as NESSUS/PRE, NESSUS/FEM and NESSUS/FPI. These modules are described by Shah et al[11]. Incorporated into the NESSUS/FPI module is a sensitivity analysis program. The sensitivity analysis factors computed by this module indicate which random variables are crucial and require special attention. The equation employed for the sensitivity analysis is the Multi-factor Interaction Equation (MFIE) given by Shah et al[11]. The general form of the equation is given by:

$$\frac{M_p}{M_{po}} = \prod_{i=1}^{N}\left[\frac{V_f - V}{V_f - V_0}\right]_i^a \qquad (2)$$

Where

M_p represents degraded material property, M_{po} represents reference material property, V denotes an effect and the subscripts 0 and f represent conditions at reference and final stages respectively. a and i are user defined.

In order to prepare the students adequately for the technology transfer, they are taught how to write a simple program that will compute the approximate value of the MPP. The program is based on the following analysis:

Define a performance function for a system,

$$g(\mathbf{X}) = g(X_1,X_2,..,X_n) \qquad (3)$$

where $\mathbf{X} = (X_1,X_2,..,X_n)$ is a vector of basic state variables of the system. The limit state function is defined as

$$g(\mathbf{X}) = 0 \qquad (4)$$

In terms of reduced variates, X_i', the limit-state equation is

$$g(\sigma_{X1}X_1' + \mu_{X1},...,\sigma_{Xn}X_n' + \mu_{Xn}) = 0 \qquad (5)$$

where

$$X_i' = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad ; \qquad i=1,2,...,n \qquad (6)$$

First order interpretation[12]:
The performance function $g(\mathbf{X})$ is expanded in a Taylor series at a point x^*, which is on the failure surface $g(\mathbf{x}^*) = 0$; that is,

$$g(X_1,X_2,...,X_n) = g(x_1^*,x_2^*,...,x_n^*) + \sum_{i=1}^{n} (X_i - x_i^*)\left(\frac{\partial g}{\partial X_i}\right)_*$$

$$+ \sum_{j=1}^{n}\sum_{i=1}^{n} (X_i - x_i^*)(X_j - x_j^*) / \left(\frac{\partial^2 g}{\partial X_i \partial X_j}\right)_* + ... \qquad (7)$$

where the derivatives are evaluated at $(x_1^*, x_2^*,.., x_n^*)$. But $g(x_1^*,x_2^*,..,x_n^*) = 0$ on the surface; therefore

$$g(X_1,X_2,...,X_n) = \sum_{i=1}^{n} (X_i - x_i^*)\left(\frac{\partial g}{\partial X_i}\right)_*$$

$$+ \sum_{j=1}^{n}\sum_{i=1}^{n} (X_i - x_i^*)(X_j - x_j^*) / \left(\frac{\partial^2 g}{\partial X_i \partial X_j}\right)_* + ... \qquad (8)$$

Note that,

$$X_i - x_i^* = (\sigma_{Xi}X_i' + \mu_{Xi}) - (\sigma_{Xi}x_i'^* + \mu_{Xi}) = \sigma_{Xi}(X_i' - x_i'^*)$$

and

$$\frac{\partial g}{\partial X_i} = \frac{\partial g}{\partial X_i'} \left(\frac{\partial X_i'}{\partial X_i} \right) = \frac{1}{\sigma_{X_1}} \left(\frac{\partial g}{\partial X_i'} \right)$$

Then

$$g(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} (X_i' - x_i'^{*}) \left(\frac{\partial g}{\partial X_i'} \right)_{*} + \ldots \qquad (9)$$

In first-order approximation, that is, truncating the above series at the first-order term, the approximate value of the mean and the variance of the function g(**X**) (for uncorrelated variate) are:

$$\mu_g \simeq -\sum_{i=1}^{n} x_i'^{*} \left(\frac{\partial g}{\partial X_i'} \right)_{*} \qquad (10)$$

$$\sigma_g^2 \simeq \sum_{i=1}^{n} \sigma_{X_i'}^2 \left(\frac{\partial g}{\partial X_i'} \right)_{*} = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i'} \right)_{*}^2 \qquad (11)$$

From equations (10) and (11) we obtain,

$$\frac{\mu_g}{\sigma_g} = \frac{-\sum_{i=1}^{n} x_i'^{*} \left(\frac{\partial g}{\partial X_i'} \right)_{*}}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i'} \right)_{*}^2}} \qquad (12)$$

The most probable failure point of this equation is

$$x_i'^{*} = -\alpha_i^{*} \beta \qquad (13)$$

in which α_i are the direction cosines and is given by

$$\alpha_i^* = \frac{\left(\frac{\partial g}{\partial X_i'}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X_i'}\right)_*^2}} \tag{14}$$

The above analysis is summarized in the following simple numerical algorithm by Rackwitz[12].

1. Assume initial values of xi*; i=1, 2, .., n and obtain

$$x_i'^* = \frac{x_i^* - \mu_{x_i}}{\sigma_{x_i}}$$

2. *Evaluate* $(\partial g/\partial X_i')$ *and* α_i^* *at* x

3. From $x_i^* = \mu_{xi} - \alpha_i^* \sigma_{Xi} \beta$
4. Substitute above xi* in $g(x_1^*, x_2^*, .., x_n^*) = 0$ and solve for β
5. Using β obtained in step 4, reevaluate $x_i'^* = -\alpha_i^* \beta$
6. Repeat steps 2 through 5 until convergence is obtained.

The above analysis can be adjusted for non-normal distributions and correlated variables.

Students' Involvement:

Students were given projects that will require the application of NESSUS. They were also given assignments that required them to write a computer program based on the above numerical algorithm and to compare the results obtained with that of NESSUS. Tables 1 and 2 are the design input data for design examples 1 and 2. Figures 2 to 5 show the results obtained by students for defined performance functions. The results reflect some of the features of the PDM. Table 4 represents a comparison of the results obtained using NESSUS and the program written by students.

Design example 1. Design of a helical spring considering two failure modes where the limit state functions are defined as follows [13]:

$$g_\tau = S - \frac{(8 * D_m * F_o * k)}{\pi D_w^3} \tag{15}$$

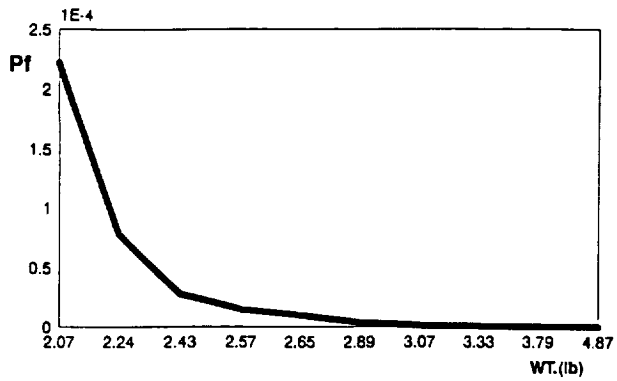

Figure 2: Probability of failure as a function of the weight of spring

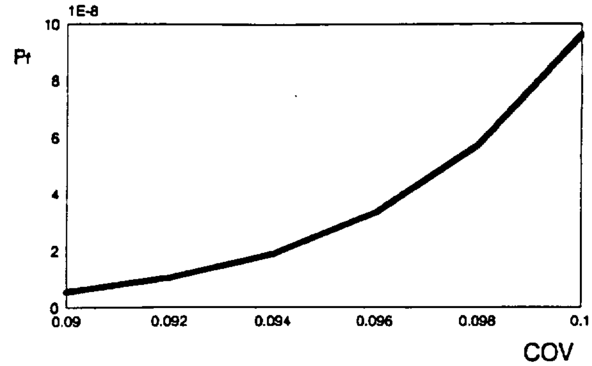

Figure 3: Probability of failure versus coefficient of variation of the wire diameter.

Design example 2. Design of a spur gear considering two failure modes where the limit state functions are defined as follows[14]:

$$g_1 = S_L - \frac{W_t * P_d}{f_w * Y_f} \tag{17}$$

$$g_2 = S_c - \left[\frac{W_t * P * M_c}{\pi * F_w * L_a} * \frac{\frac{1}{R_1} + \frac{1}{R_2}}{\frac{1-V_p^2}{E_p} + \frac{1-V_g^2}{E_g}} \right]^{\frac{1}{2}} \tag{18}$$

where,

g_1 = limit state function for bending stress
S_L = design strength limit of the gear material
W_t = tangential load
P_d = Pitch diameter
Y_f = form factor
g_2 = limit state function for contact stress
S_c = design endurance limit of gear material
P = circular pitch
R_1 = radius of curvature of pinion tooth
R_2 = radius of curvature of gear tooth
E_p = Young's modulus of the pinion material
E_g = Young's modulus of the gear material
V_p = Poisson's ratio of the pinion material

$$g_{\Delta} = (L_f*2.48e^{-0.315(\frac{L_f}{D_m})}) - \frac{8*F_o*D_m^3*N_a}{G*D_w^4} \tag{16}$$

Table 1 is the design input data for equations (15) and (16).

Table 1: Input data

Variable	Mean	Standard deviation	Distribution type
S (psi)	1.30E+4	3.25E+2	normal
F_o (lbs)	20	5	normal
D_m (in)	7.50E-1	7.50E-2	normal
D_w (in)	2.30E-1	2.30E-2	normal
L_f (in)	6	1	normal
G (psi)	1.12E+7	6.40E+4	normal
N_a	5	1	normal

where,

g_{τ} = limit state function for torsional shear stress failure mode
S = design limit ultimate strength of the spring material (Chromium-Vanadium)
D_m = mean diameter
F_o = Force on spring
k = Wahl constant
D_w = wire diameter
g_{Δ} = limit state function for deflection failure mode
L_f = free length
N_a = Number of active coils
G = spring modulus of elasticity

The student studied the relationship between the probability of failure and the weight of the spring. This is presented in figure 2. Figure 3 is the curve of the probability of failure as a function of the coefficient of variance of the wire diameter. In figure 2 it is observed that the probability of failure decreases with increase in the weight of spring. It also illustrates that after a certain amount of increase of the weight, any further increase becomes ineffective. Figure 3 shows that the probability of failure increases with increase of the coefficient of variance of the wire diameter. In other words, the probability of failure increases as the uncertainty increases.

V_g = Poisson's ratio of the gear material
M_c = contact ratio
L_a = length of the line of action
Table 2 is the design input data for equations (17) and (18).

Table 2: Input data

Variables	Mean	Standard deviation	Distribution type
S_L (psi)	30E+3	1.0E+3	normal
W_t (lb)	650	20	normal
F_w (in)	1.5	0.07	lognormal
Y_f	0.409	0.02	lognormal
P_d (teeth/in)	8	1	normal
E_p (psi)	30E+6	1.5E+6	normal
E_g (psi)	14.5E+6	7.25E+5	normal
R_1 (in)	0.342	0.081	lognormal
R_2 (in)	1.069	0.20	lognormal
L_a (in)	0.736	0.11	lognormal
M_c	2.5	0.1	normal
P (in)	0.392	0.1	lognormal
S_c (psi)	55E+3	2.0E+3	normal
V_p	0.292	0	
V_g	0.211	0	

The student presented the sensitivity analysis and the probability of failure for each of the limit state functions as illustrated in figures (4) and (5).

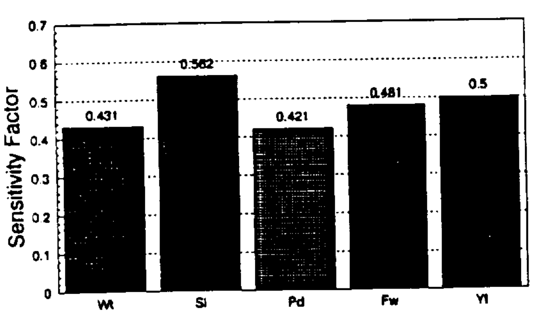

Figure 4: Sensitivity factors for the design parameters in g1 (Probability of failure, Pf = 0.0455)

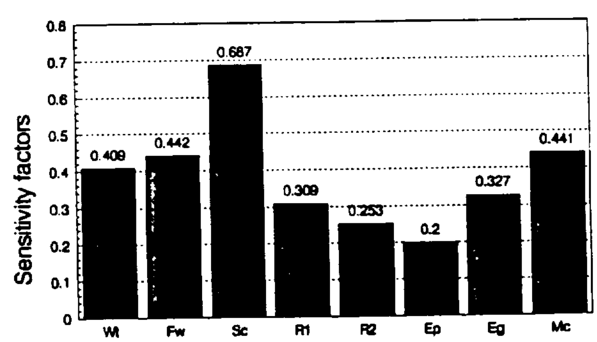

Figure 5: Sensitivity factors for the design parameters in g2 (Probability of failure, Pf = 0.0672)

A probability fault tree analysis method[15] was applied in the computation of the probability of failure of the system. The system probability of failure, P_f, was found to be 0.0701.

Design example 3. Design of a cantilever beam considering only one failure mode where the limit state function is defined as:

$$g_D = \Delta_L - \frac{4*Q*l^3}{E*b*h^3} \tag{19}$$

Table 3 is the design input data for equation (19).

Table 3: Input data

Variable	Mean	Standard deviation	Distribution type
Δ_L (in)	0.75	0.042	normal
Q (kips)	1.888	0.282	normal
L (in)	18	0.9	lognormal
E (ksi)	30E+3	0	
b (in)	1	0.05	lognormal
h (in)	1.5	0	

where,

Δ_L = deflection limit of beam
Q = concentrated load on beam
L = length of beam
E = Young's modulus
b = with of beam
h = height of beam

Example 3 is analyzed using NESSUS and an equivalent program written by students. The results are summarized in table 4.

Table 4: Table of Comparison

Method	Safety index	Prob. of Failure
Students' program	2.53936	0.56880E-2
NESSUS	2.53557	0.56132E-2

Strategy:

The aim of this exercise is to involve the students in the program in such a way that they will have a full knowledge of the technology. The fact that the students can write their own programs means that they can demonstrate the technology in any industry even when the industry has no standard PDM software. It gives the students the opportunity to use any available computer hardware. It also makes it possible for industries to have a good idea of the features of the technology without much financial commitment.

Conclusion:

The steps necessary for preparing students to participate in the transfer of technology has been illustrated. The effectiveness of this participation depends on how well the students are trained in the new technology. In the case of PDM, the fact that the students are trained to write their own program reduces their limitations when they go to industries that do not have any standard PDM software. Most industries will be more willing to try a new technology that requires little or no capital commitment than when a huge capital is involved.

Acknowledgement:

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Appendix B

Copies of Capstone Design Projects. The copies enclosed in the phase II report are not included.

1. The Development of a Probabilistic Piston-Rod Selector Guide.
2. The Design of Shock Absorber using PDM.
3. Probabilistic Design of a helical Spring with NESSUS as the Design Software.

The Development of a Probabilistic Piston-Rod Selector Guide

by

Sharon Y. Claxton

Design Project Report

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S.Y.C.

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NOMENCLATURE

P_{cr}	critical load, lbs
A	area, in^2
S_y	yield strength, psi
L	length of column, in
n	coefficient of end conditions, dimensionless
E	modulus of elasticity, psi
r	least radius gyration= $(I/A)^{1/2}$, in
I	moment of inertia of area, in^4
$G(x)$	G-function

CHAPTER I

INTRODUCTION

Uncertainties are unavoidable in the design and planning of engineering systems. Therefore, the tools of engineering analysis should include methods and concepts for evaluating the significance of uncertainty on system performance and design. In this regard, the principles of probability (and its allied fields of statistics and decision theory) offer the mathematical basis for modeling uncertainty and the analysis of its effect on engineering design [1].

Probabilistic design methodology have significant roles in all aspects of engineering planning and design. Under conditions of uncertainty, probability and statistical decision theory allows modeling of engineering problems and evaluation of system performances. Relative to decision making, probabilistic design methodology outlines the logical framework for risk assessment and risk-benefit.

The objective of this project is to develop a piston-rod selector guide using probabilistic design methodology. This selector guide will allow a quick design selection of three design parameters; diameter, length and load at a specified reliability level. Any one parameter can be found if the other two are known. The results obtained are compared to an existing deterministic selection guide.

The following chapters deal with certain topics that form the basis of the design. In CHAPTER II, a discussion of the theory used in designing a piston-rod was developed. CHAPTER III discusses the numerical simulation of the design. The discussion and results of the piston-rod selector guide are presented in CHAPTER IV and CHAPTER V outlines the conclusion of the design.

CHAPTER II

BACKGROUND INFORMATION

A. Piston-Rod

A piston-rod is the connecting rod attached to a piston and its crankshaft. Piston-rods are compression members; therefore, the theory of columns is applicable.

B. Critical Loading

When an applied load places the connecting rod in an unstable condition (critical load) a failure occurs. The four modes of failure investigated that normally occur with a connecting rod are compression, wear, fatigue and buckling. To determine the critical load for long and medium columns Euler's equation (Equation 2-1) must be applied [2]. Johnson's equation (Equation 2-2) is utilized for short columns [2].

$$P_{cr}=AS_y(1-Q/4r^2) \tag{2-2}$$

$$P_{cr}=S_yA\frac{r^2}{Q} \tag{2-1}$$

$$Q=(S_yL^2)/(n\pi^2E) \tag{2-3}$$

TABLE 2-1: **Coefficients of End Conditions Defined** [3]

	Theoretical	Conservative	Recommended
Fixed-Free	1/4	1/4	1/4
Rounded-Rounded	1	1	1
Fixed-Rounded	2	1	1.2
Fixed-Fixed	4	1	1.2

The factor n is called the end-condition constant as shown in TABLE 2-1, and it may have any one of the theoretical values ¼, 1, 2, and 4, depending upon the manner in which the load is applied. To fix the column ends so the factor n = 2 or n = 4 would apply is very difficult, if not impossible. Because of this, some designers never use a value of n greater than unity.

To determine whether the column is long, medium or short a condition must be met. If $\mathbf{Q/r^2 < 2}$ then Johnson equation applies, otherwise Euler equation is used. Q, which is mathematically expressed in Equation 2-3, has no physical significance; it is a means of simplifying the expression for critical load [2]. The critical load is the load necessary to place the column in a condition of unstable equilibrium. In this state any small crookedness of the member, or slight movement of the support or load will cause the column to collapse.

C. Column Failure

A column failure is always sudden, total, and unexpected, and hence dangerous. There is no advance warning. A beam will bend and give visual warning that it is overloaded; but not so for a column [3]. Therefore, probabilistic design methodology is very important in the development of a piston-rod.

Short columns subjected to centrally applied loads may be analyzed or designed on the basis of direct compression [4]. The compression is formed by the load stress applied. The load stresses are formed within the elastic limit and induced by external loads. A compressive stress is the reverse of a tensile load. Tensile stress is defined as:

$$\sigma = P/A \qquad (2\text{-}5)$$

where,

σ = tensile stress

P = axial load

A = cross-sectional area.

Wear is the gradual abrading of surfaces in contact as a result of relative motion between them. Some rule of thumbs with wear are as follows:

1. Wear increases with time of running.
2. Wear is less with hard surfaces than with soft surfaces.
3. Below a certain load, wear is low and above this load it rises greatly

causing severe wear and this holds true for both clean and lubricated surfaces.

More than 90% of all mechanical fractures are a result of fatigue failures, it is important for the engineer to know how materials will react to fatigue conditions. Fractured surfaces correspond to discontinuities in the surface, machining marks, "metallurgical notches" such as a abrupt changes in the metal structure. Once started, a crack propagates through the metal upon repeated application of load, crack growth due to the stress at the tip of the crack exceeding the strength of the material.

The primary failure mode focus of this project is buckling. Theoretical equations for the determination of the buckling load of columns were first developed by Euler . The Euler formula (Equation 2-1) for buckling load or critical load of a column was derived on the assumption that the column bows sideways while the stresses are within the elastic limit. This type of failure is the result of elastic instability. If the column is of less slender proportions, the maximum stress may reach the yield point before sideways bowing occurs; the Euler formula does not predict the critical load. This type of failure is the result of plastic instability and J. B. Johnson's Formula as seen in Equation 2-2, which predicts this critical load.

CHAPTER III

SELECTOR GUIDE DEVELOPMENT

An existing design of a piston-rod selector guide was utilized. The existing design was based on deterministic design methodology. Therefore, this design is the redesign of an existing piston-rod selector guide implementing probabilistic design methodology. With this design the advantages of probabilisitic design over deterministic methodology are to be pointed out. As mentioned earlier, the main objective of this project is to develop a piston-rod selector guide using probabilistic design methodology and compare it to an existing deterministic design.

To arrive at the final design goal a sequential design methodology was utilized. These steps are as follows:

1. Problem Definition
2. Selection of Design Parameters
3. Data Assembly
4. Probabilistic Analysis
5. Interpreting Results

A. Problem Definition

In this design there is only one functional requirement, which is to design a reliable machine element to carry a load under compression. In order to accomplish this a selector guide was designed to allow a quick design selection of three parameters; diameter, length and load at a specified reliability level.

B. Selection of Design Parameters

To design a piston-rod acceptable design parameters must be established to meet the functional requirement. Also, the objective must be clearly defined.

In order to develop a probabilistic piston-rod selector guide the design parameters and failure mode(s) must be identified. The primary design parameters associated with the designing of the piston-rod are load, length, diameter and modulus of elasticity. The objective of the piston-rod selector guide is to allow a quick selection of one design parameter (load, length and diameter) if the other two are known. Also, the selector guide indicates the reliability of the piston-rod as a result of buckling failure.

C. Data Assembly

This project is the redesign of a current deterministic piston-rod selector guide (TABLE 3-1). The same material and some of the design variables of the deterministic design were used in the probabilistic design. However, the redesign allows for uncertainty in the diameter, load and length, indicating the reliability of the piston-rod.

A computer program was written (Figure 3.-1) to design the piston-rod and calculate the diameter. Given the load and length the program indicates the column type (long or short) and diameter size. From equations (2-1) and (2-3) the diameter of a long column is expressed as follows:

$$d=[(64P_{cr}L^2)/(n\pi^3E)]^{1/4} \qquad (3\text{-}1)$$

The diameter of a short column from equations (2-2) and (2-3) is given by

$$d=[\frac{4P_{cr}}{\pi S_y}+\frac{4S_yL^2}{n\pi^2E}]^{1/2} \qquad (3\text{-}2)$$

D. **Probabilistic Analysis**

Numerical Evaluation of Stochastic Structures Under Stress (NESSUS) computer code was used to preform the probabilistic analysis of this design. Using Nessus, the probability of failure and the sensitivity of the diameter in the design were obtained. NESSUS has three different modules; NESSUS/PRE, NESSUS/FEM, NESSUS/FPI (Figure 3-2).

NESSUS/PRE is a pre-processor. It prepares the statistical data needed for probabilistic design analysis. In this module, the user describes the uncertainties in the random variables. Knowledge of statistical theory is required to define random 0variable characterization. In this module the G-function must be defined as shown in Equation 4-1, which is the limit state function for buckling. It specifies

$$G(x)=P_{cr}-F \qquad (4\text{-}1)$$

the boundary between the safe and failure region, which is defined as follows:

$$G(x)>0 \ \textit{safe}$$

$$G(x)<0 \ \textit{failure}$$

NESSUS/FEM is a general purpose finite element code which is used to perform structural analysis and evaluate the sensitivity of the design parameters. The sensitivity indicates which design parameter effects the design most.

NESSUS/FPI is the fast probability integrator. It extracts information from the FEM module and calculates the probability of failure.

E. Interpreting Results

As mentioned probabilistic analysis indicates the sensitivity of the design parameters, meaning it specifics the parameter that effects the probability of failure most. From the study of the sensitivity analysis associated with each piston-rod design the diameter was the most sensitive of the design parameters. Figure A-1 illustrates an example of the sensitivity analysis.

The probability of failure was determined for each design increasing the diameter 1/16 inch. As the diameter increased the probability of failure readily decreased. The information obtained was organized into a selector guide.

TABLE 3-1: Deterministic Piston-Rod Selector Guide [2]

	PISTON ROD DIAMETERS							
	⅝	1⅜	2	3	4	5	7	10
[illegible]	38	180						
[illegible]	34	161						
[illegible]	31	147						
[illegible]	28	137	288					
[illegible]	27	128	270					
[illegible]	25	120	254					
[illegible]	24	114	241					
[illegible]	22	104	220					
[illegible]	20	97	204					
[illegible]	19	90	191					
[illegible]	18	85	180					
[illegible]	17	81	171					
[illegible]	16	77	163					
[illegible]	15	71	150					
[illegible]	14	66	139					
[illegible]	12	57	121	271				
[illegible]	11	50	108	243				
[illegible]	10	47	99	222				
[illegible]	9	43	91	205	365			
[illegible]	8	41	85	192	341			
[illegible]		38	81	181	322			
[illegible]		36	76	172	305	477		
[illegible]		32	68	154	273	426		
[illegible]		30	63	140	249	389		
[illegible]		27	58	130	231	360		
[illegible]		26	54	121	216	337		
[illegible]		23	48	109	193	302		
[illegible]		21	44	99	176	275	530	
[illegible]		18	38	86	153	238	460	
[illegible]			34	77	137	213	429	
[illegible]			31	70	125	195	375	
[illegible]			27	61	108	169	322	
[illegible]			24	55	97	151	291	
[illegible]				50	88	138	266	
[illegible]					79	123	238	493
[illegible]						107	206	427
[illegible]						96	184	382
[illegible]							168	348
[illegible]							156	322
[illegible]							146	302
[illegible]							137	284
[illegible]							132	270
[illegible]							126	257
[illegible]							120	246

PISTON-ROD DESIGN

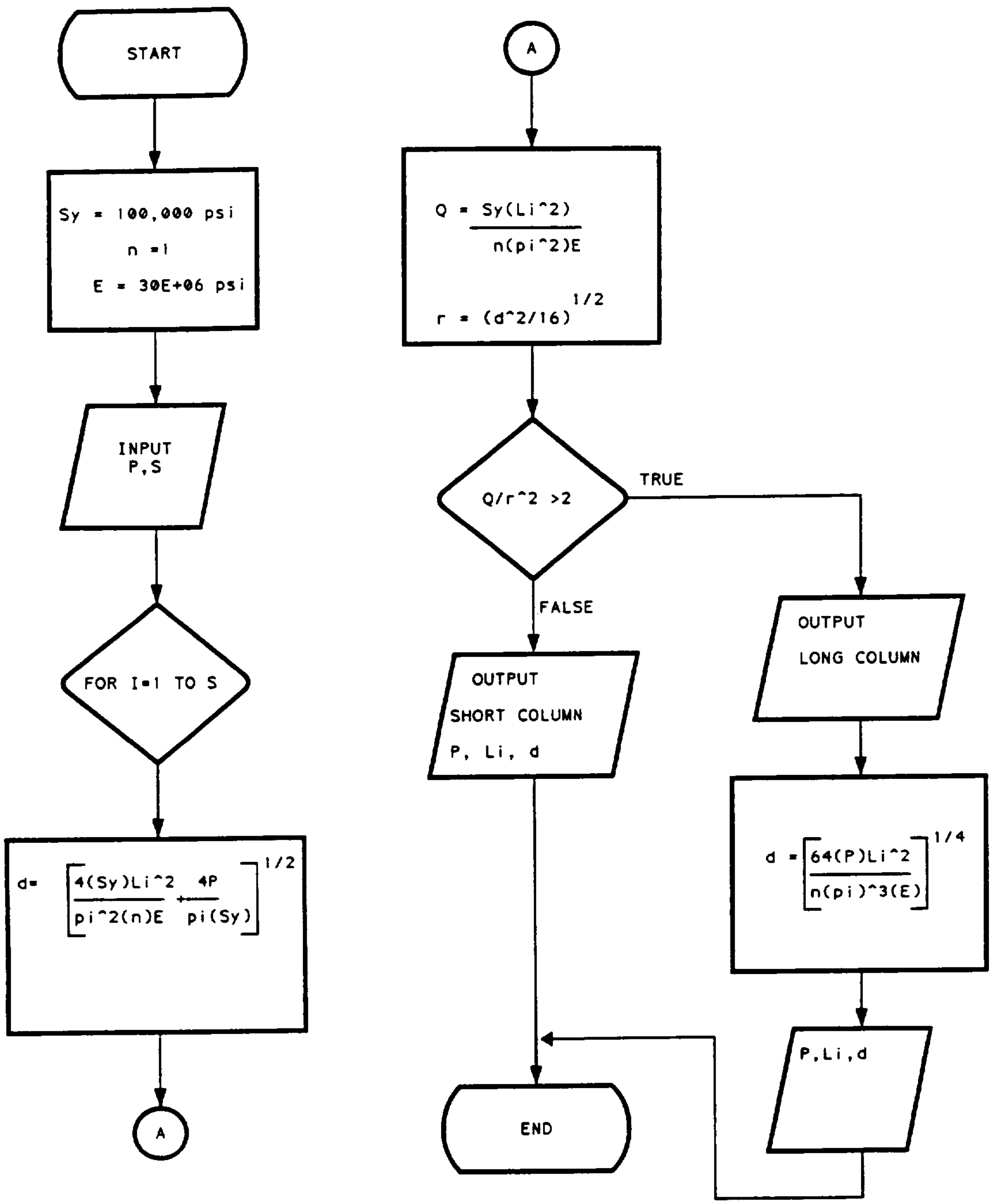

Figure 3-1: FLOWCHART

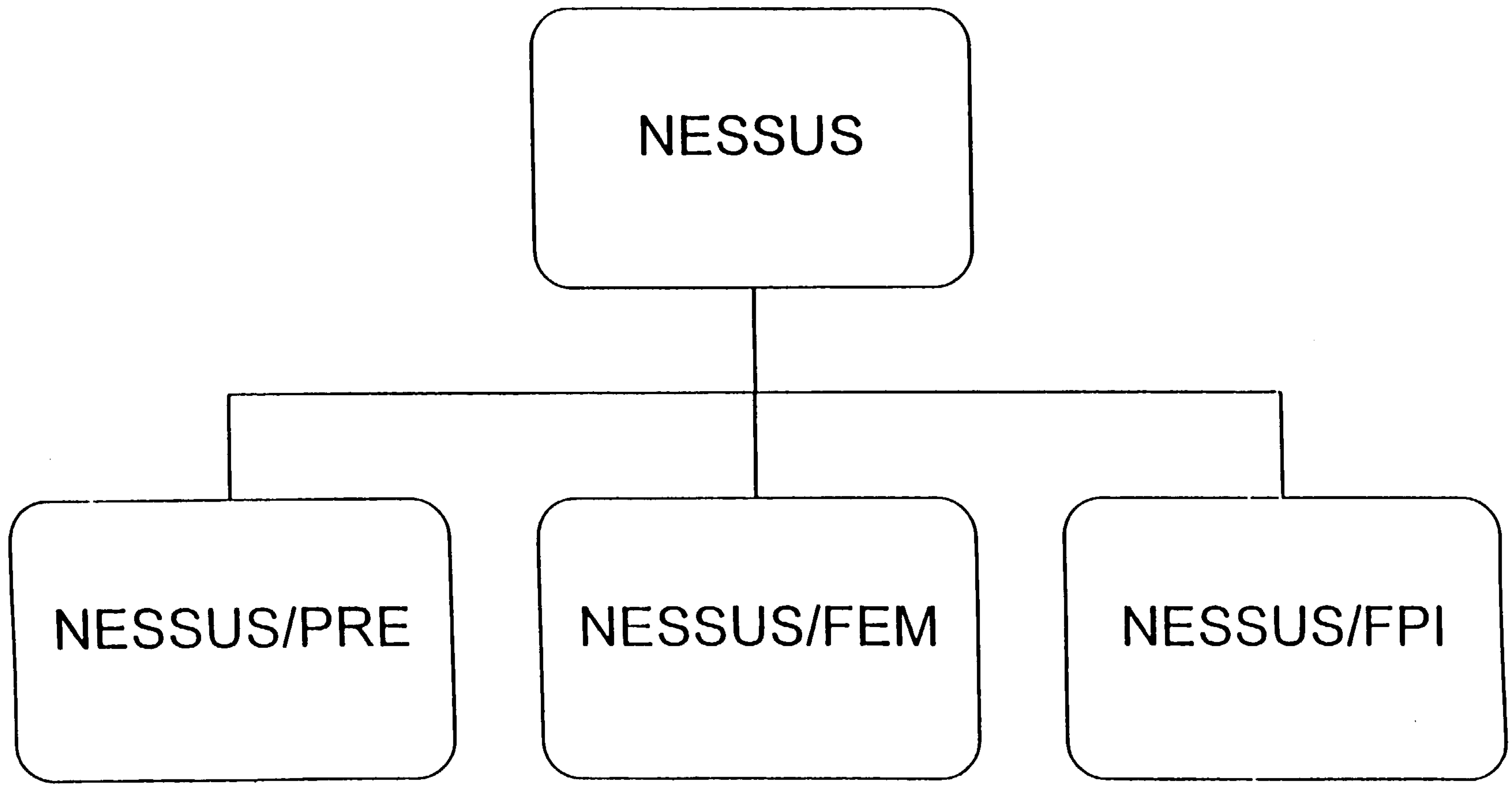

Figure 3-2 NESSUS Tree

CHAPTER IV

RESULTS

The piston-rod is made from a medium carbon steel, chrome plated and polished. The yield strength is 100,000 psi and a modulus of elasticity of 30,000,000 psi. The rod is well protected against wear and corrosion. The ends of this piston-rod design are rounded; therefore, the coefficient of the end condition has a value of 1. The relationship of the design parameters and probability of failure as a result of buckling is outlined in TABLE 4.1.

TABLE 4-1: Probabilistic Piston-Rod Selector Guide

Load (lbs)	Length (in)	Deterministic Diameter (in)	Probability of Failure	Probabilistic Diameter (in)	Probability of Failure
400	38	0.625	0.0023104	0.57166	0.0193529
				0.63416	0.0015492
400	96	1.000	0.0019302	0.88416	0.0330739
				0.94666	0.0078143
				1.00916	0.0014986
400	180	1.375	0.0017219	1.22212	0.0277050
				1.28462	0.0096361
				1.34712	0.0029909
				1.40962	0.0008483
400	292	1.750	0.0017573	1.61585	0.0127632
				1.67835	0.5251890
				1.74085	0.0020290
				1.80335	0.0007450
500	34	0.625	0.0023212	0.57173	0.0193731
				0.63423	0.0015194
500	86	1.000	0.0019721	0.89799	0.0249538
				0.96049	0.0056138
				1.02299	0.0010416
500	161	1.375	0.0017223	1.22213	0.0277060
				1.28463	0.0096365
				1.34713	0.0029909
				1.40963	0.0008483
500	261	1.750	0.0017413	1.55024	0.0297830
				1.61274	0.0131094
				1.67524	0.0054528
				1.73774	0.0021108
				1.80024	0.0007762
600	31	0.063	0.0028356	0.57146	0.0193196
				0.63396	0.0015441
600	79	1.000	0.0021473	0.90022	0.0252742
				0.96272	0.0057267
				1.02522	0.1070508
600	147	1.375	0.0017268	1.22220	0.0277726
				1.28472	0.0096419
				1.37220	0.0029932
				1.40972	0.0008491
700	28	0.063	0.0016274	0.56598	0.0182599
				0.62848	0.0013936
				0.69098	0.0000749
700	72	1.000	0.0017320	0.89464	0.2446953
				0.95714	0.0054463
				1.01964	0.0009991
700	137	1.375	0.0018918	1.22544	0.0280848
				1.28794	0.0098220
				1.35044	0.0030666
				1.41294	0.0008750
700	221	1.750	0.0017870	1.55141	0.0298876
				1.61391	0.0132734
				1.67641	0.0054847
				1.73891	0.2126458
				1.80141	0.0007829

TABLE 4-1: Probabilistic Piston-Rod Selector Guide Continued

Load (lbs)	Length (in)	Deterministic Diameter (in)	Probability of Failure	Probabilistic Diameter (in)	Probability of Failure
700	288	2.000	0.0017320	1.78929	0.0244668
				1.85179	0.0054459
				1.97679	0.0023794
				2.03929	0.0009990
800	27	0.625	0.0024638	0.57274	0.0195632
				0.63524	0.0015803
800	68	1.000	0.0019765	0.89805	0.0024961
				0.96055	0.0056164
				1.02305	0.0010422
800	128	1.375	0.0018612	1.22487	0.0280160
				1.28737	0.0097887
				1.34987	0.0030530
				1.41237	0.0008703
800	207	1.750	0.0018197	1.55220	0.0299746
				1.61472	0.0133200
				1.67722	0.0055108
				1.73972	0.0021374
				1.80222	0.0007878
800	270	2.000	0.0017861	1.72836	0.0476084
				1.79086	0.0248517
				1.85336	0.0119360
				1.91586	0.0054853
				1.97836	0.0023999
				2.04086	0.0010090
900	25	0.625	0.0019302	0.56871	0.0187810
				0.63121	0.0014668
900	64	1.000	0.0019302	0.89743	0.0248745
				0.95993	0.0055853
				1.02243	0.0010345
900	120	1.375	0.0017219	1.22212	0.0277053
				1.28462	0.0096360
				1.34712	0.0029909
				1.40962	0.0008483
900	195	1.750	0.0017992	1.55171	0.0299184
				1.61421	0.0132914
				1.67671	0.0054943
				1.73921	0.0021308
				1.80171	0.0007847
900	254	2.000	0.0017323	1.72681	0.0474414
				1.78931	0.0244691
				1.85181	0.0118672
				1.91431	0.0054463
				1.97681	0.0023796
				2.03931	0.0009999
1000	24	0.625	0.0022676	0.57135	0.0192925
				0.63385	0.0015404
1000	61	1.000	0.0020573	0.89909	0.0251153
				0.96159	0.0056699
				1.02409	0.0010559

TABLE 4-1: Probabilistic Piston-Rod Selector Guide Continued

Load (lbs)	Length (in)	Deterministic Diameter (in)	Probability of Failure	Probabilistic Diameter (in)	Probability of Failure
1000	114	1.375	0.001755167	1.22279	0.0277831
				1.28529	0.0096739
				1.34779	0.0030063
				1.41029	0.0009537
1000	187	1.750	0.0020855	1.68341	0.0056878
				1.74591	0.0022219
				1.80841	0.0008244
1000	241	2.000	0.0017362	1.72691	0.0474526
				1.78941	0.0244767
				1.85191	0.0118716
				1.91441	0.0054489
				1.97691	0.0023110
				2.03941	0.0009976
2000	17	0.625	0.0023212	0.58469	0.0197408
				0.64719	0.0008715
2000	43	1.000	0.0019721	0.91433	0.0172635
				0.97683	0.0036722
				1.03933	0.0006555
2000	81	1.375	0.0018757	1.18643	0.0504251
				1.24893	0.0190726
				1.31143	0.0063700
				1.37393	0.0019163
				1.43643	0.0005321
2000	131	1.750	0.0018354	1.52281	0.0428594
				1.58531	0.0198301
				1.64781	0.0084857
				1.71031	0.0033880
				1.77281	0.0012781
3000	14	0.063	0.0026027	0.57365	0.0197513
				0.63615	0.0000200
3000	35	1.000	0.0018896	0.89688	0.0247930
				0.95938	0.0055785
				1.02188	0.0010274
3000	66	1.375	0.0018232	1.22414	0.0279326
				1.28664	0.0097479
				1.34914	0.0030363
				1.41164	0.0008643
3000	107	1.750	0.0018446	1.55284	0.0300209
				1.61534	0.0133507
				1.67784	0.0055261
				1.74034	0.0021456
				1.80284	0.0007912
3000	139	2.000	0.0171181	1.72619	0.0473764
				1.78869	0.0244248
				1.85119	0.0118401
				1.91369	0.0054312
				1.97619	0.0023731
				2.03869	0.0009953

TABLE 4-1: Probabilistic Piston-Rod Selector Guide Continued

Load (lbs)	Length (in)	Deterministic Diameter (in)	Probability of Failure	Probabilistic Diameter (in)	Probability of Failure
3000	218	2.500	0.0018025	2.14542	0.0540247
				2.20792	0.0322305
				2.27042	0.0185690
				2.33292	0.0102802
				2.39542	0.0054964
				2.45792	0.0028485
				2.52042	0.0014369
4000	12	0.625	0.0022676	0.57135	0.0192925
				0.63385	0.0015404
4000	30	1.000	0.0016381	0.89324	0.0242632
				0.95574	0.0053765
				1.01824	0.0009815
4000	57	1.375	0.0017552	1.22279	2.7782906
				1.28529	0.0096739
				1.34779	0.0030063
				1.41029	0.0008537
4000	93	1.750	0.0019384	1.55508	0.0302308
				1.61758	0.0134736
				1.68008	0.0055905
				1.74258	0.0021761
				1.80580	0.0008044
4000	121	2.000	0.0018380	1.72984	0.0477653
				1.79234	0.0246885
				1.85484	0.0120013
				1.91734	0.0055216
				1.97984	0.0024190
				2.04234	0.0010183
4000	189	2.500	0.0018297	2.14639	0.0541119
				2.20889	0.0322959
				2.27139	0.0186150
				2.33389	0.0103107
				2.39639	0.0055155
				2.45889	0.0028601
				2.52139	0.0014434
4000	271	3.000	0.0017251	2.55863	0.0584025
				2.63113	0.0382359
				2.68363	0.0244525
				2.74613	0.0151909
				2.80863	0.0092020
				2.87113	0.0054405
				2.93363	0.0031482
				2.99613	0.0017877
				3.05863	0.0009978
5000	11	0.625	0.0031414	0.57686	0.0203903
				0.63936	0.0017048
5000	27	1.000	0.0017861	0.89543	0.0248201
				0.95793	0.0054851
				1.02043	0.0010090

TABLE 4-1: Probabilistic Piston-Rod Selector Guide Continued

Load (lbs)	Length (in)	Deterministic Diameter (in)	Probability of Failure	Probabilistic Diameter (in)	Probability of Failure
5000	50	1.375	0.0013370	1.15087	0.0685626
				1.20337	0.0267224
				1.27587	0.0091615
				1.33837	0.0027989
				1.40087	0.0007813
5000	83	1.750	0.0018812	1.55372	0.0624661
				1.61622	0.0133990
				1.67872	0.0055515
				1.74122	0.0021576
				1.80372	0.0007964
5000	108	2.000	0.0017861	1.72836	0.0476083
				1.79086	0.0245816
				1.85336	0.0119361
				1.91586	0.0054852
				1.97836	0.0023999
				2.04086	0.0010090
5000	169	2.500	0.0018227	2.14614	0.0540904
				2.20864	0.0322809
				2.27114	0.0186037
				2.33640	0.0103032
				2.39614	0.0055108
				2.45864	0.0028572
				2.52114	0.0014417
5000	243	3.000	0.0017861	2.56130	0.0586139
				2.62380	0.0384034
				2.68630	0.0245803
				2.74880	0.0152841
				2.81130	0.0092722
				2.87380	0.5484629
				2.93630	0.0031769
				2.99880	0.0018059
				3.06130	0.0010089
5000	330	3.500	0.0017310	3.03745	0.0432893
				3.09950	0.0297572
				3.16245	0.0200159
				3.22495	0.0131966
				3.28745	0.0085561
				3.34995	0.5445124
				3.41245	0.0034114
				3.47495	0.0021076
				3.53745	0.0012844
6000	25	1.000	0.0021685	0.83798	0.0884258
				0.90048	0.0253087
				0.96298	0.0057394
				1.02548	0.1073796
6000	47	1.375	0.0020087	1.16508	0.0712483
				1.22758	0.0283299
				1.29008	0.0099417
				1.35258	0.0031160
				1.41508	0.0008925

TABLE 4-1: Probabilistic Piston-Rod Selector Guide Continued

Load (lbs)	Length (in)	Deterministic Diameter (in)	Probability of Failure	Probabilistic Diameter (in)	Probability of Failure
6000	76	1.750	0.0019615	1.49312	0.0627293
				1.55562	0.0302822
				1.61812	0.0135031
				1.68062	0.0056407
				1.74312	0.0021832
				1.80562	0.0008076
6000	154	2.500	0.0017786	2.14456	0.0539466
				2.20706	0.0321724
				2.26956	0.0185280
				2.33206	0.0102529
				2.39456	0.0054794
				2.45706	0.0028386
				2.51956	0.0014311
6000	222	3.000	0.0018053	2.56212	0.0586815
				2.62462	0.0384569
				2.68712	0.0246204
				2.74962	0.0153137
				2.81212	0.0092877
				2.87462	0.0054986
				2.93712	0.0031860
				2.99962	0.0017967
				3.06212	0.0010124
6000	302	3.500	0.0017916	2.97804	0.0621568
				3.04054	0.0434675
				3.10304	0.0299008
				3.16554	0.0201270
				3.22804	0.0132795
				3.29054	0.0086178
				3.35304	0.0054889
				3.41554	0.0034418
				3.47804	0.0021282
				3.54054	0.0012981
7000	23	1.000	0.0019904	0.83573	0.0877990
				0.89823	0.0249887
				0.96073	0.0056259
				1.02323	0.0010447
7000	43	1.375	0.0017062	1.15930	0.0700920
				1.22180	0.0276693
				1.28430	0.0096192
				1.34680	0.0029838
				1.40930	0.0008458
7000	70	1.750	0.0018273	1.48991	0.0622841
				1.55241	0.0299840
				1.61491	0.0133297
				1.67741	0.0055143
				1.73991	0.0021400
				1.80241	0.0007888

TABLE 4-1: Probabilistic Piston-Rod Selector Guide Continued

Load (lbs)	Length (in)	Deterministic Diameter (in)	Probability of Failure	Probabilistic Diameter (in)	Probability of Failure
7000	91	2.000	0.0017126	1.72622	0.0473767
				1.78872	0.0244245
				1.85122	0.0118399
				1.91372	0.0054314
				1.97622	0.0023718
				2.03872	0.0009954
7000	143	2.500	0.0018526	2.14719	0.0541860
				2.20960	0.0323764
				2.27219	0.0186539
				2.33469	0.0103366
				2.39719	0.0055316
				2.45969	0.0028697
				2.52219	0.0014488
7000	205	3.000	0.0017419	2.55937	0.5846314
				2.62187	0.0382826
				2.68437	0.0244879
				2.74687	0.0152168
				2.80937	0.0092202
				2.87187	0.0054529
				2.93437	0.0031563
				2.99687	0.0017928
				3.05937	0.0010008
7000	279	3.500	0.0017395	3.03789	0.0433144
				3.10039	0.0297786
				3.16289	0.0200314
				3.22539	0.0132090
				3.28789	0.0085649
				3.35039	0.0054512
				3.41289	0.0034158
				3.47539	0.0021105
				3.53789	0.0012864
7000	365	4.000	0.0017789	3.39382	0.0648747
				3.45632	0.0475831
				3.51882	0.0343912
				3.58132	0.0245654
				3.64382	0.0172429
				3.70632	0.0119259
				3.76882	0.0081419
				3.83132	0.0054795
				3.89382	0.0036445
				3.95632	0.0023971
				4.01882	0.0015606
8000	22	1.000	0.0026870	0.90621	0.0261499
				0.96871	0.0060367
				1.03121	0.0011513
8000	41	1.375	0.0022168	1.23114	0.0287366
				1.29364	0.0101419
				1.35614	0.0031990
				1.41864	0.0009220

TABLE 4-1: Probabilistic Piston-Rod Selector Guide Continued

Load (lbs)	Length (in)	Deterministic Diameter (in)	Probability of Failure	Probabilistic Diameter (in)	Probability of Failure
8000	66	1.750	0.0020369	1.55734	0.0304427
				1.61984	0.0135976
				1.68234	0.0056555
				1.74484	0.0022067
				1.80734	0.0008178
8000	85	2.000	0.0016788	1.72520	0.0472681
				1.78770	0.0243520
				1.85020	0.0117956
				1.91270	0.0054068
				1.97520	0.0023602
				2.03770	0.0009891
8000	133	2.500	0.0017119	2.14212	0.0537236
				2.20462	0.0320873
				2.26712	0.0184108
				2.32962	0.0101755
				2.39212	0.0054310
				2.45462	0.0028096
				2.51712	0.0014147
8000	192	3.000	0.0017722	2.56070	0.0585669
				5.62320	0.0383658
				2.68570	0.0245515
				2.74820	0.0152631
				2.81070	0.0092552
				2.87320	0.0054747
				2.93570	0.0031704
				2.99820	0.0018018
				3.06070	0.0010064
8000	261	3.500	0.0017413	3.03789	0.0433199
				3.10048	0.0297825
				3.16298	0.0200349
				3.22548	0.0132113
				3.28798	0.0085668
				3.35048	0.0054526
				3.41298	0.3416609
				3.47548	0.0021112
				3.53798	0.0012868
8000	341	4.000	0.0017485	3.45455	0.0474896
				3.51705	0.0343129
				3.57955	0.0245016
				3.64205	0.0171926
				3.70455	0.0118871
				3.76705	0.0081125
				3.82955	0.0054577
				3.89205	0.0036285
				3.95455	0.0023856
				4.01705	0.0015527
9000	20	1.000	0.0016381	0.89324	0.0242632
				0.95574	0.0053765
				1.01824	0.0009815

CHAPTER V

CONCLUSION

The functional requirement of the design was met. Probabilistic design methodology has been developed to design a piston-rod. The information obtained was organized into a selector guide. NESSUS was used to evaluate the probability of failure. Probabilistic design methodology has an important design feature over a deterministic method. It evaluates the risk in the design indicating a probability of failure.

This design used no factor of safety. It gives the designer a range of diameters to select from for each piston-rod design. Therefore, the designer is able to select a diameter depending on the failure tolerance specified. Using a probabilistic design can eliminate the waste of material on a deterministic design. Also, probabilistic design methodology indicates the sensitivity of the design parameters, which can aid in readily reducing failure.

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General Motors Saturn Corporation, Springhill, TN
Process Engineer, Summer Intern
Designed and modified the steel staging area layout; found a system that will enable virgin and regrind material to be mixed prior to the injection molding machines; and documented processes for steel process control plans.

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General Motors Corporation, Powertrain Division, Ypsilanti, MI
Product Engineer, Summer Intern
Developed databases for the torque converter clutch pressure plate and spring cage for the GM Council of Experts; assisted in design layouts including numerical stack analysis to determine where the torque converter bottoms and updated design layout files for tooling release.

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American Society of Automotive Engineers Car Design Winner 1995
Engineering Saturday Academy Counselor 1991-1993
Miss Tennessee State University First Attendant 1995-1996
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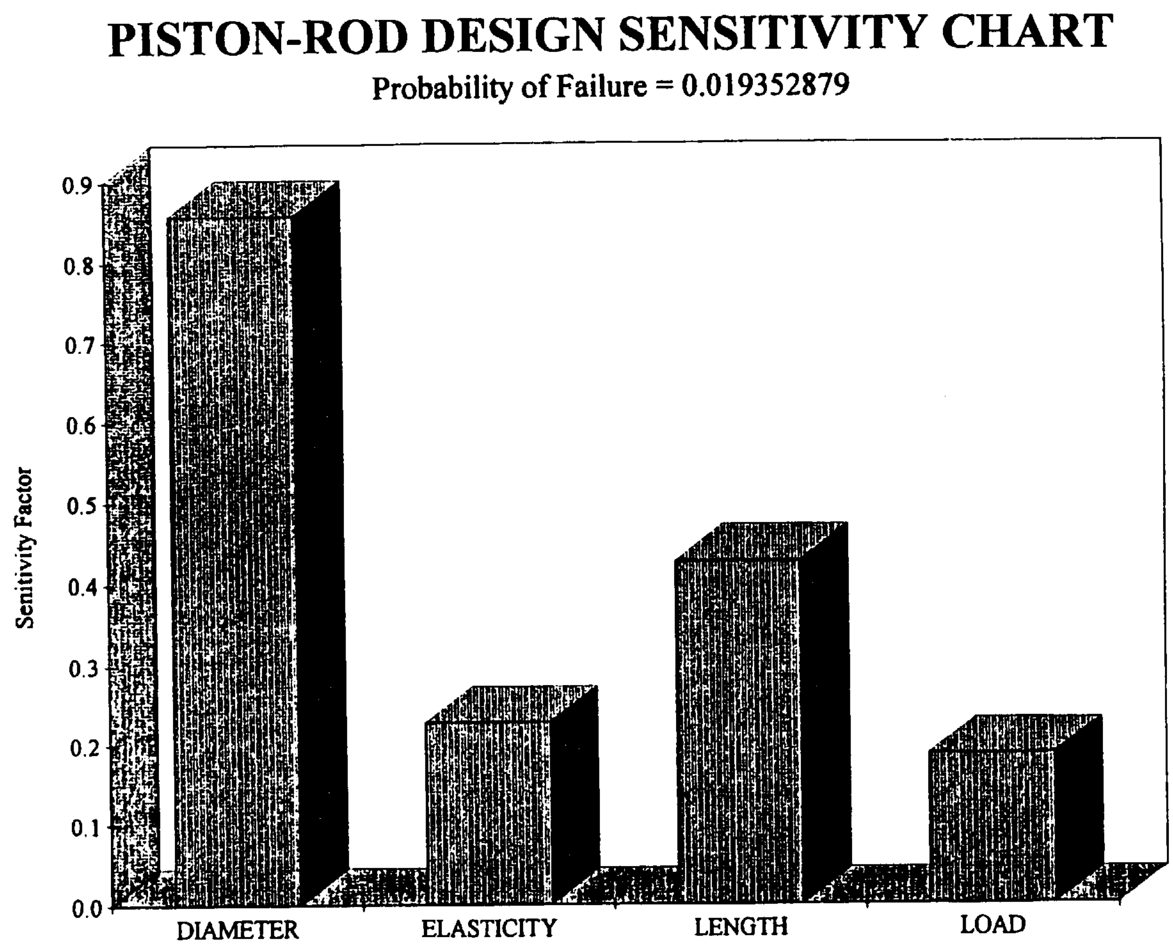
Delta Sigma Theta Sorority, Inc. Financial Secretary 1995-1996

REFERENCES Available upon request.

APPENDIX



Design Load = 400 lbs Length = 38 in Diameter 0.57166 in

Figure A-1: Piston-Rod Sensitivity Chart

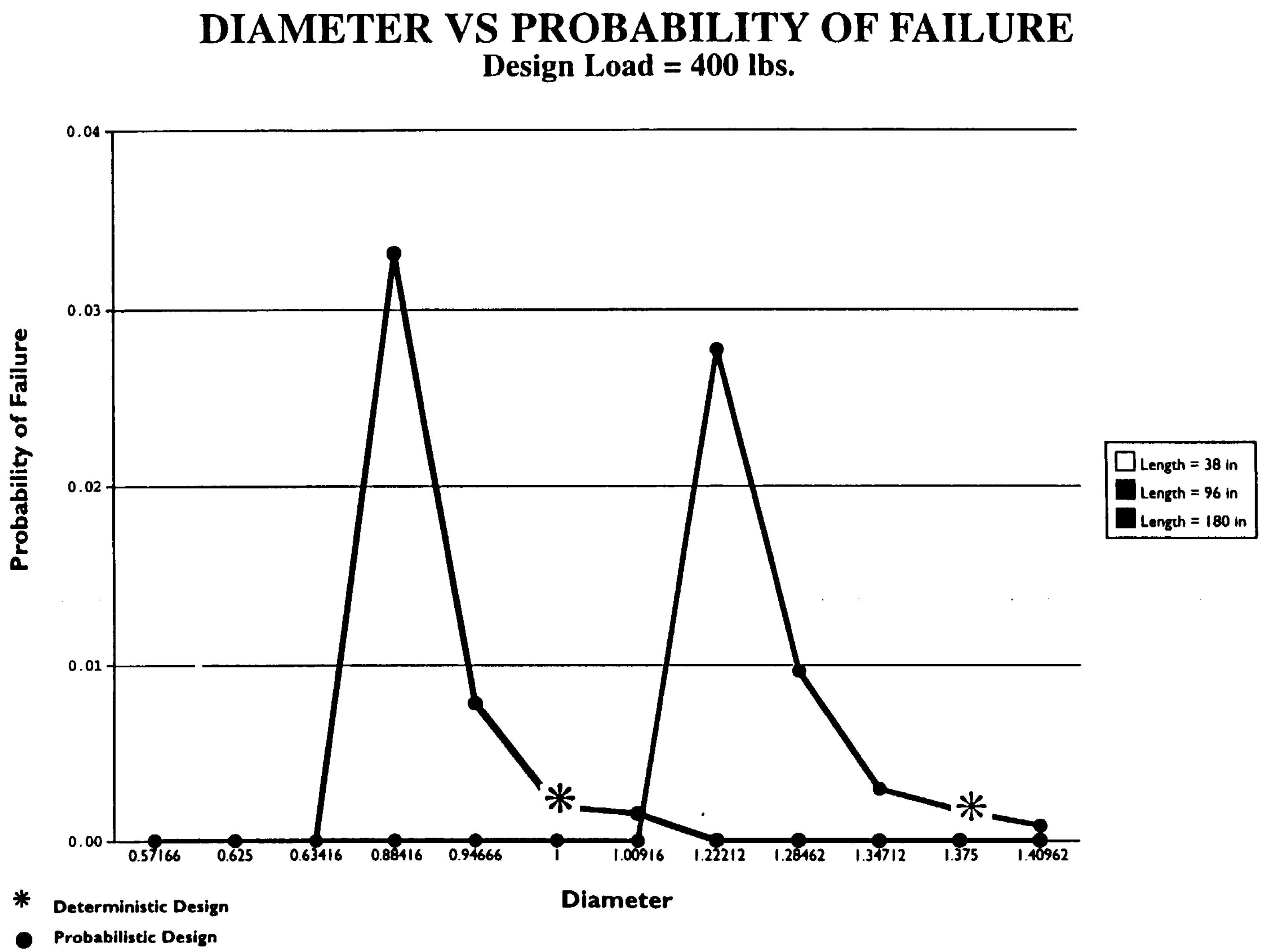


Figure A-2: Diameter Vs Probability of Failure

DESIGN OF A SHOCK ABSORBER USING PROBABILISITIC DESIGN METHODOLOGY

by

STEVE BOGARD

Design Project Report

Submitted to the Faculty

of the

College of Engineering and Technology

in

Partial Fulfillment of the Requirements

for the Degree of

Bachelor of Science

in

Mechanical Engineering

December, 1995

College of Engineering and Technology

Tennessee State University

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S.E.B.

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NOMENCLATURE

D	mean diameter of coil(inches).
d	diameter of wire(inches).
K_w	Wahl factor
τ	torsional shear stress(lb/in^2)
τ_{max}	maximum torsional shear stress(lb/in^2)
G	Modulus of rigidity(Mpsi)
E	Modulus of elasticity(Mpsi)
k	spring rate(lb/in)
t	time(sec)
ω	angular frequency(rads/sec)
n	number of coils
c	damping constant(lb.s/in)
Lf	free length of spring(ft)
δ	deflection in springs (inches)
δ_{max}	maximum deflection in springs(inches)
T	torque on spring(lb.ft)
J	polar moment of inertia(in^4)
μ	dynamic viscosity of fluid(lb.s/in^2)

CHAPTER I

INTRODUCTION

This project has a specific purpose, which is to use Probabilistic Design Methodology (PDM) to design a shock absorber. The shock absorber is analyzed by pin pointing certain failure modes in an attempt to maximize the life expectancy and optimize the performance.

There are certain failure modes that are very critical in the design of a shock absorber. The analysis for quantifying these failure modes and the design approach is discussed in the next Chapters.

Chapter two deals with the design approach and how Probabilistic Design Methodology was initially implemented into this design project. Also, this chapter introduces phases of the design and all the necessary components such as a brief discussion of helical compression springs, viscous damping, and how they are interfaced.

Chapter three deals with the design parameters and how the shock absorber can fail due to stress over loads, buckling, and vibrations. Through stress analysis, this chapter provides some detail to how failure can occur in a shock absorber by analyzing compression springs under static loads and fatigue. Also, it covers the topic of deflection, and how failure can result due to buckling. The most critical part of this chapter is the vibration analysis, which deals with how automotive suspensions react under dynamic loading. Chapter four involves the actual design procedure for the shock absorber. Also, it deals with the probabilistic and deterministic results involved in a design.

Chapter five concludes this project by giving a brief discussion of all the different techniques that are used to obtain the final results.

CHAPTER II

DESIGN APPROACH

This chapter focuses on the design approach for the shock absorber. The first task begins with understanding the theory behind Probabilistic Design Methodology, and how to incorporate this method into designing a shock absorber.

Also, attention is confined to different components that make up the shock absorber. The sections in this chapter provide some detail of what the design components are and their functional requirements. An overview of how PDM is implemented in the project is also discussed.

2.1 Overview of Probabilistic Analysis

As a design approach, it was crucial to develop a concise understanding of how to apply Probabilistic Design Methodology (PDM) and use the computer code NESSUS.

The NESSUS code served as a tool to perform all the necessary calculations. NESSUS has three different modules known as NESSUS/PRE, NESSUS/FEM and NESSUS/FPI [1].

NESSUS/PRE is simply a pre-processor that prepares statistical data needed for the probabilistic analysis and allows the user to describe the uncertainties in the random variables.

NESSUS/FEM (Finite Element Module) is used to perform structural analysis and evaluation of sensitivity due to variation in different uncorrelated variables. It also contains an algorithm to compute the sensitivity of random variable, to be stored and used later [1].

NESSUS/FPI is a Fast Probability Integrator that has several analysis methods, with the Fast Probability Integrator being the fastest. The FPI uses data created by the NESSUS/FEM, to develop a statistical distribution of the random variables and to compute the cumulative distribution function and sensitivity of the variables.

While using the computer code NESSUS, it is important to have a general knowledge of how the input data is incorporated into the NESSUS code. Therefore, one must understand the meaning of limit state functions. This function defines the boundary between the safe and failed regions of a design, as shown below.

$$G(\Delta)=\Delta_L-\Delta=0 \qquad (2\text{-}1)$$

Where,

Δ_L = Allowable design variable

Δ = Actual design variable

and the limit state function can be defined as the following:

$G(\underline{\mathbf{X}})>0$ implies safe set of $\underline{\mathbf{X}}$

$G(\underline{\mathbf{X}})<0$ implies failed set of $\underline{\mathbf{X}}$

Where $\underline{\mathbf{X}}$ is any set of random variables $(X_1, X_2, X_3, ..X_n)$.

Also, to deal with the problems of system reliability, fault tree analysis is incorporated into the NESSUS code. This method has three major characteristics such as bottom events, combination gates, and the connectivity between the bottom events and gates.

2.2 Defining The System

For this project, the shock absorber is used to generate smoother rides in vehicles by reducing the amount of vibrations that a vehicle undergoes during dynamic loading such as a bump on a road as shown in FIGURE 2-1. The bumpy road can also be represented as a harmonic function which is shown later in Chapter three.

In FIGURE 2-2, one can see that the shock absorber consists of two components which are a compression spring and viscous damper. This model shows a clear and defined representation of how the components are interfaced. Due to the fact that these two components will work simultaneously, it was crucial to grasp a clear and concise knowledge of compression springs and viscous dampers to understand the design criteria and failure modes.

2.2.1 Helical Compression Springs.

The helical compression spring serves as an energy absorbing component. By selecting the appropriate wire, it can be made to resist tensile, compressive, or torsional loads.

In FIGURE 2-3, one can see that there are many factors that must be considered such as coils, pitch or lead, free length, and space between the coils.

The material that was used for this particular compression spring is chrome vanadium. This is the most popular alloy steel spring for conditions involving higher stresses than can be used with the high-carbon steels and for use where fatigue and long endurance are needed. Also, it is good for shock and impact loads [2].

2.2.2 Viscous Damper

Viscous damping serves a very important purpose for this part of the design process. Without any damping, the system will fail due to extreme oscillation. The damper is shown in FIGURE 2-1, and it is represented by the dashpot symbol. The viscous damper is characterized by the resistive force exerted on a body moving in a viscous fluid [3]. Also, this device serves as an energy-dissipating element.

To include the effects of viscous damping in a vibrating system, it is assumed that the mass element is directly connected to the piston of a dashpot. The velocity of the piston is the same as that of the vibrating mass subjected to a damping force, is explained in more detail in Chapter three.

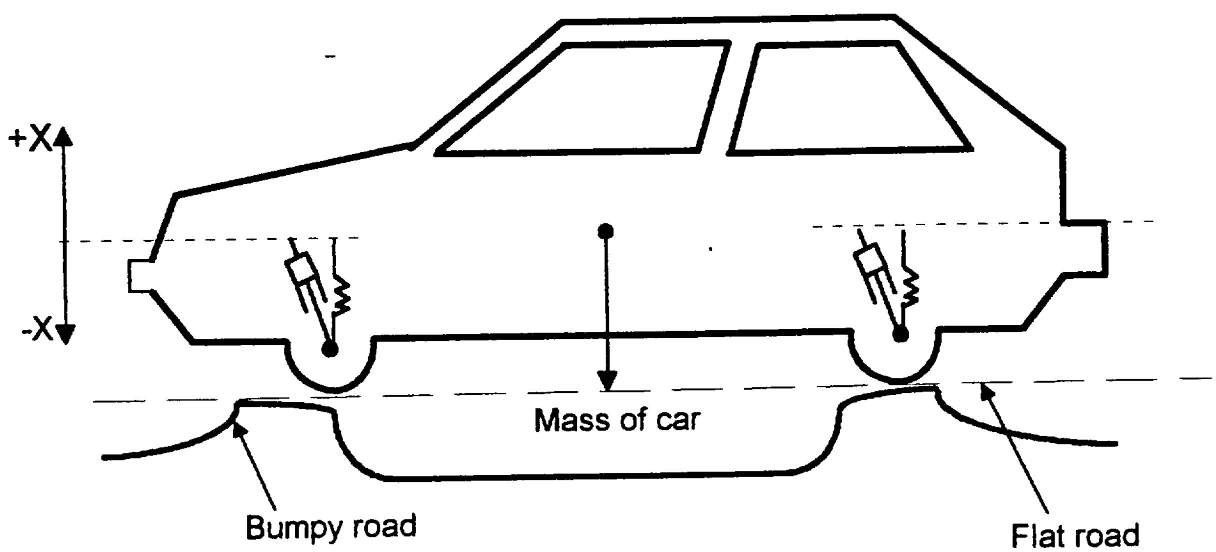

Figure 2-1: Automotive Suspension Under Dynamic Loading.

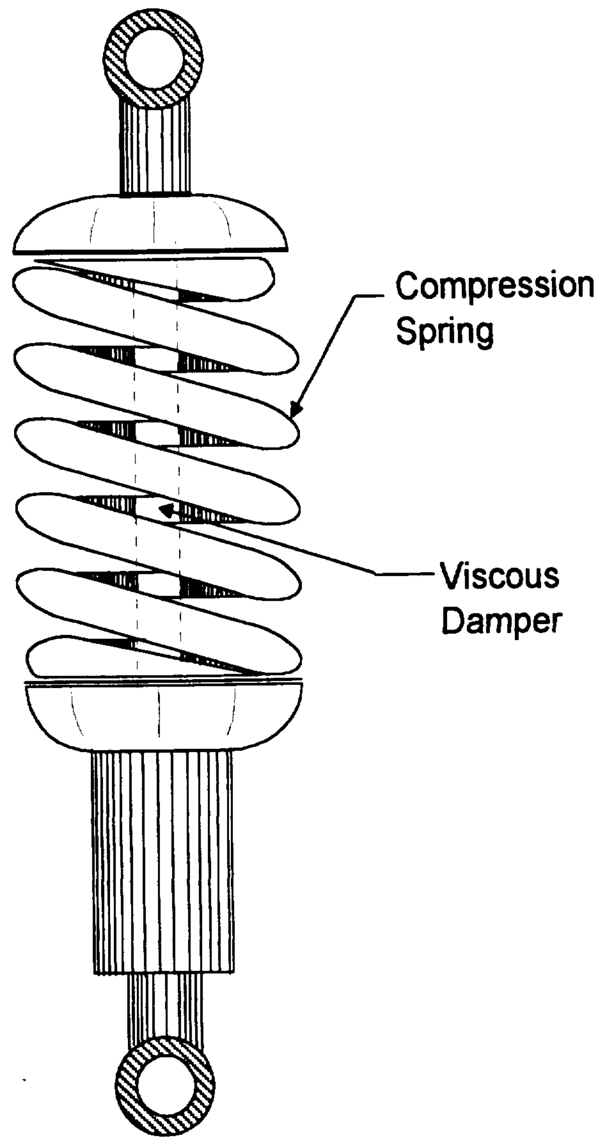

FIGURE 2-2: Idealized View of Shock Absorber.

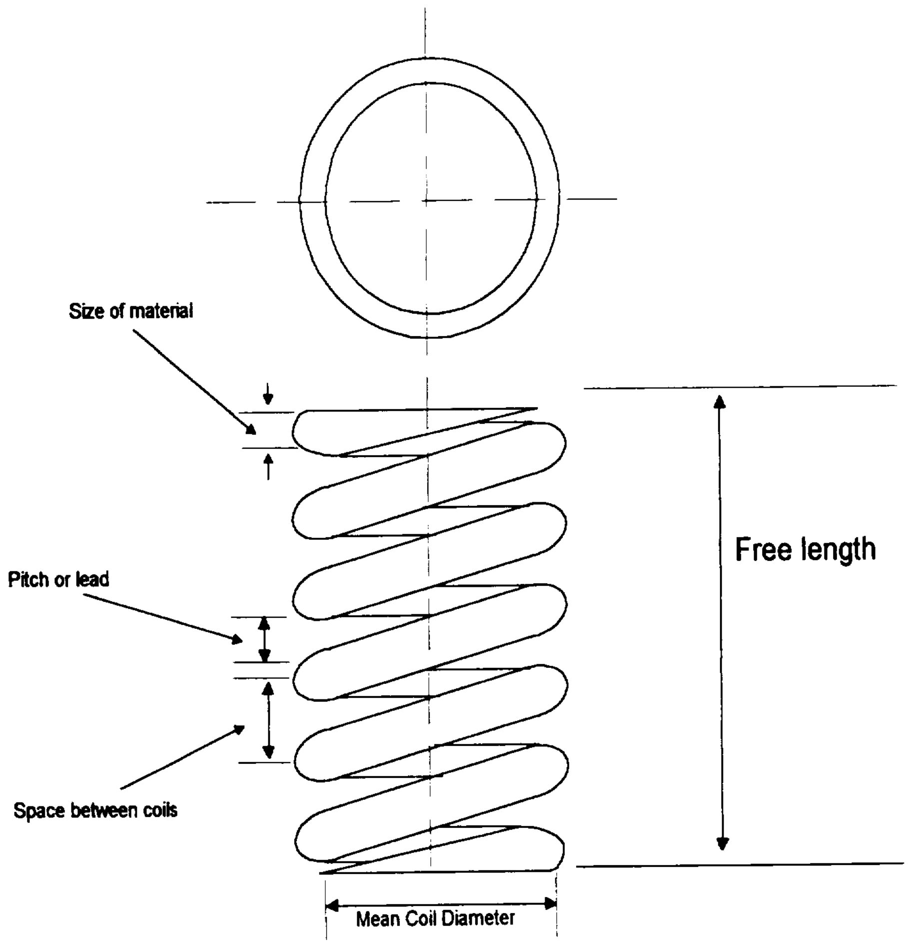

FIGURE 2-3: Compression Spring.

CHAPTER III

DEFINING THE DESIGN PARAMETERS

This chapter discusses the Design Parameters that are acceptable for meeting the functional requirement of the system. Later in this Chapter, the design parameters that are affected under stress, deflection, and vibration are defined, and equations are generated which are pertinent to qauntifying failures. These equations are called limit state functions. With limit state functions and defining the random variables, it is possible to use the NESSUS code to design the components of the shock absorber such as determining the appropriate spring stiffness under stress and deflection, and to minimize the vibrations. Also, the limit state function for vibration is used in determining the viscous damping constant. This criteria is explained in more detail during the next two sections in this Chapter.

3.1 Stress Analysis

In designing the compression spring as a component of the shock absorber, stress is analyzed as one of the failure modes for this design. Therefore, defining the design parameters and random variable will require some simple analysis. This analysis can be visualized by taking a round-wire helical compression spring loaded by an axial load [4]. A helical compression or tension spring can be thought of as a torsion bar wound into a helix. Since the spring used in this design is made of solid round wire, the resulting torsional stress is

$$\tau=\frac{Tr}{J}=\left(\frac{2D+d}{2D}\right)\left(\frac{8FD}{\pi d^3}\right) \qquad (3\text{-}1)$$

where, D is the mean coil diameter, d is the wire diameter, and F is the static load.

Using Equation 3-1, the limit state function can be defined as

$$g=\tau_{max}-\left(\frac{2D+d}{2D}\right)\left(\frac{8FD}{\pi d^3}\right) \qquad (3\text{-}2)$$

where, τ_{max} is maximum torsional stress. This limit state function can be incorporated into NESSUS to determine the probability of failure for stress. From Equation 3-2, the random variables are mean coil diameter, applied load, and wire diameter. The maximum torsional stress is derived as

$$\tau_{max}=0.51\tau_{ut} \qquad (3\text{-}3)$$

Where τ_{ut} is an estimate of the minimum tensile strength as shown in Equation 3-4 below.

$$\tau_{ut} = A/d^m \tag{3-4}$$

A and m are constants found in TABLE 3-1 for chrome vanadium.

While looking at stresses in more detail, there are shear stresses represented by the inner surfaces of a coil spring, but there are two additional shear stress components [4]. A transverse shear stress resulting from force F, applied to the arbitrary cutting plane and torsional shear stress. At the inner coil surface, the direction of this stress coincides with that of torsional stress because of the curvature of the coil.

Also, since the shock absorber is subjected to dynamic loading, fatigue failure most be considered during the stress analysis. Therefore, the goodman criterion is employed to find the factor of safety as shown in Chapter four. A factor of safety in the range of 2.5 to 3 is acceptable for the spring, since it is used in uncertain environments or subjected to uncertain stresses [4].

3.2 Deflection Analysis

This portion of the design serves the purpose of deriving the limit station function as an expression of deflection. This function is used to determine an appropriate spring stiffness (k) for the design of a shock absorber. During this point of the design stage, it is important to design a compression spring that prevents buckling from occurring in the system [4].

Using FIGURE 3-1, the maximum deflection is determine by selecting the appropriate curve. Curves A and B are for two different types of compression spring ends. Curve A is for a compression spring where one end of the plate is free at the tip. Curve B is for a compression spring where the ends are constrained and parallel such as plain ground ends. The equation for deflection can be derived from strain

$$\delta=\frac{8FD^3n}{d^4G} \tag{3-5}$$

energy methods [4]. The deflection is

which consist of the applied load, n number of coils, d wire diameter, G modulus of rigidity, D the mean coil diameter. Using Equation 3-3, the limit state function is derived as

$$g=\delta_{max}-\frac{8FD^3n}{d^4G} \tag{3-6}$$

where, F,D, d, G are random variables. For this equation, the maximum deflection is determined from FIGURE 3-1.

3.3 Vibration Analysis

After deriving all the expressions for the design or selection of a spring, the most critical part of the design is decreasing the vibrations by using viscous damping. The importance of decreasing the vibrations lies in the fact that an automotive suspension system is to isolate the car body from road irregularities. Thus while the axles may undergo fairly violent motions in response to bumps, the car body is not to be affected by them. Therefore, the use of vibration analysis is implemented into this design process.

The shock absorber was designed for a worst case scenario, a harmonic force was used to represent irregularities in a road as shown in Equation 3-7.

$$F(t) = F \sin \omega t \qquad \textbf{(3-7)}$$

The function, F(t) is an external force that is time dependent time. F(t) is used to represent the forces caused by irregularities or bumps that a vehicle encounters while driving over a road. In Equation 3-7, ω is the angular frequency in units of rad/sec. The angular frequency is simply a representation of how often the bumps will occur. The damping is represented by the dashpot, which exerts a force Fd on the mass m, given by Fd = -cx, where x is the vertical displacement of the mass in units of ft or meters. The damping constant c has units of lb-s/ft or N.s/m. The negative sign indicates that if dx/dt > o, then Fd is in the negative x direction, and vice versa [5]. The stiffness is represented by a linear spring, so that the static force, Fs, exerted on the mass m by the spring is defined by Fs = -kx; k is the spring constant, in units of lb/ft or N/m. The negative sign in Fs is important since it states that the force opposes the displacement[6]. Using the relationships from above, the equation of motion is derived, see Equation 3-8.

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+mx=F(t) \tag{3-8}$$

Since the objective is to design the shock absorber elements so that the vertical motion of the car body in FIGURE 2-1, remain very small even if the axle motions are fairly violent. An expression for analyzing the vertical displacement can be derived, by which this equation is used in the NESSUS code. Solving for x in Equation 3-8, an expression for vertical displacement is obtained as

$$X_p=\frac{F}{m[(\frac{k}{m}-\omega^2)^2+\frac{\omega^2c^2}{m^2}]}[(\frac{k}{m}-\omega^2)\cos\omega t+\frac{c}{m}\sin\omega t]$$

and,

$$X_c = e^{\frac{c}{2m}} \left(c_1 \sin\sqrt{\frac{k}{m} - \frac{c^2}{m^2}}\, t + c_2 \cos\sqrt{\frac{k}{m} - \frac{c^2}{m^2}}\, t\right)$$

where,

$$X = X_c + X_p \qquad (3\text{-}9)$$

Since Equation 3-8 is referred to as a nonhomogeneous second-order differential equation. The general solution consists of a complementary solution, X_c, plus a particular solution, X_p. The complementary solution X_c defines the free vibration. The particular solution X_p describes the forced vibration of a mass caused by the applied force in Equation 3-7 [6].

Equation 3-9 is implemented into the NESSUS code as a limit state function. This equation is used to determine the maximum displacement for the mass and shock absorber while it is subjected to a harmonic motion. Therefore, a limit state function is generated, as shown below.

$$G = X_{max} - (X_c + X_p) \qquad (3\text{-}10)$$

There are many random variables that can be used as design parameters in Equation 3-10. But, the most critical ones are ω which is related directly to the frequency of bumpy roads or irregularities, the harmonic force $F(t)$ which changes with respect to time, and the damping constant c.

TABLE 3-1: Constants Used To Estimate The Tensile Strength[4].

Material	Size range, in	Size range, mm	Exponent, m	Constant, A	
				kpsi	MPa
Music wire	0.004-0.250	0.10-6.5	0.146	196	2170
Oil-tempered wire	0.020-0.500	0.50-12	0.186	149	1880
Hard-drawn wire	0.028-0.500	0.70-12	0.192	136	1750
Chrome vanadium	0.032-0.437	0.80-12	0.155	173	2000
Chrome silicon	0.063-0.375	1.6-10	0.112	202	2000

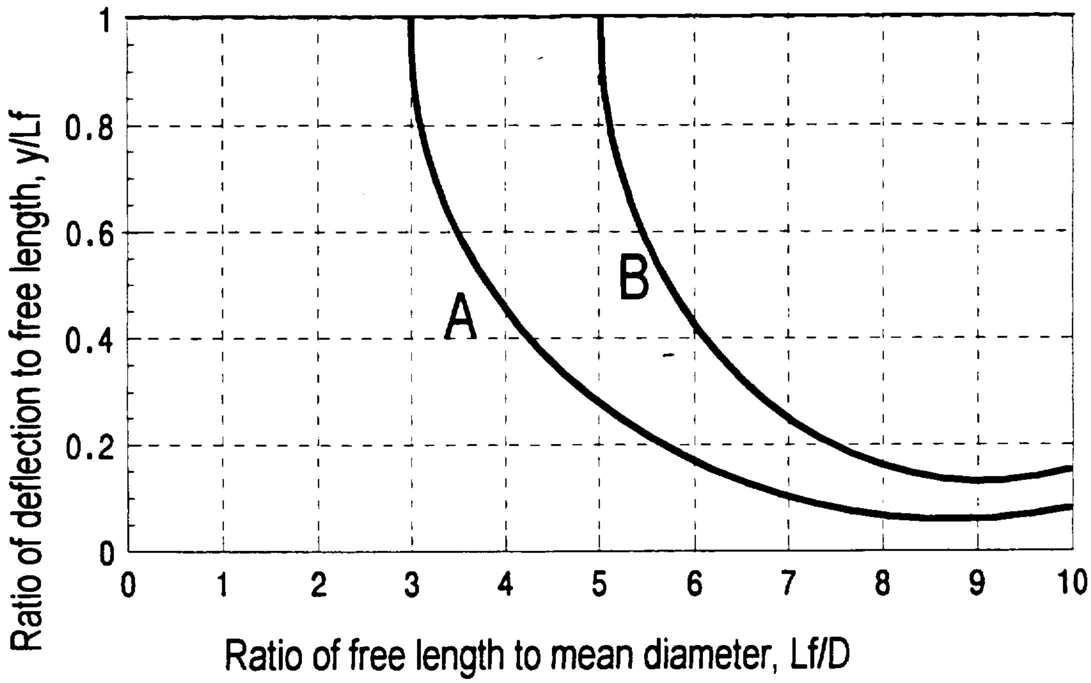

FIGURE 3-1: **Buckling Conditions For Helical Compression Springs[4].**

CHAPTER IV

DESIGN CRITERIA FOR SHOCK ABSORBER

This chapter provides detail information for the actual design of the shock absorber and how to determine the mean values and standard deviation which are used as input data for the NESSUS code. The probability of failure is determine by analyzing the stress, deflection, and vibrations.

The designer must assume the following are known:

(a) The material that is used.

(b) The outside diameter of the helical spring.

(c) Wire diameter

(d) Vehicle weight

(e) An understanding of how the shock absorber will be subjected in its environment.

Using the assumption above, the designer can determine the following necessary information.

(a) The maximum torsional shear stress for the spring.

(b) The maximum deflection

(c) The free length for a helical compression spring that is used with the shock absorber.

(d) The spring stiffness which is used in conjunction with vibrational analysis.

(e) The damping constant.

After determining these values above, the NESSUS Code is used in the design procedure to quantify the probability of failure for the shock absorber.

PROBLEM STATEMENT:

A shock absorber is to be designed for a vehicle that has a body weight of 2700 lbs. The shock absorber must fit into an area where the diameter will not exceed 3.6 inches and total length will not exceed 28 inches in height. The spring that aids the shock absorber will have a wire diameter of 0.60 inches and the total number of coils will not exceed 30. The each shock absorber is subjected to an angular frequency of 3 rad/sec and a 900 lb maximum load.

Find:

Determine the mean values for the NESSUS code. Use PDM to improve the design by analyzing stress, deflection, and vibrations.

Given:

1. Car weight=2600 lb
2. Wire diameter d=.60 inches
3. Outside diameter D_o=3.6 inches
4. Material- Chrome vanadium
5. Ends are plain and ground
6. Total number of coils Nt=30
7. Angular frequency, ω = 3rad/sec
8. Maximum force F_{max}=900 lbs

Properties:

1. $G = 11\times10^6$ psi, Modulus of Rigidity for Chrome vanadium.

Solution:

I. The first step involves making all the necessary calculations for the helical compression spring.

(a) Minimum Tensile Strength using Equation 3-3

τ_{ut}=173,000 psi/(.60)$^{.155}$

τ_{ut}=187,254.7 psi

(b) Maximum Torsional Shear Stress using Equation 3-4

τ_{max} =(.51)x(187,254.7)=95,499.9 psi

(c) Spring Index **C=D/d** **(4-1)**

Diameter D=Do-d=3.0 inches

C = 3.0/0.6 =5

(d) Stress Correction Factor

Ks=(2C+1)/(2C) **(4-2)**

Ks=(2x5+1)/(2x5)=1

(e) Number of Active Coils

Na=Nt-1 **(4-3)**

N=Na=30-1=29

(f) Free Length

$$L_0=d(Na+1) \tag{4-4}$$

$L_f=(.6)(29+1)$

$L_f=18$inches

(g) Maximum Force $F_s=\pi d^3\tau/8K_sD$ (4-5)

$F_s=(3.14)(.6)^3(95,499.9)/8(1.1)(3.0)$

$F_s=2,453.47$ lb

(h) Spring Rate

$$K=d^4G/8D^3N \tag{4-6}$$

$K=(.60)^4(11x10^6)/(8(3)^3(29))$

$K=227.5862$ lb/in or 2,731.03 lb/ft

(i) Maximum Deflection is determined from FIGURE 3-1. Using $L_f/D = 6$, the ratio of deflection to free length is determined.

$\delta/L_f=.4$

(j) Therefore, the maximum deflection without buckling is

$\delta_{max}=(.4)(L_f)=7.2$ inches

II. Fatigue Loading

(a) Bergstrasser Factor

$$KB= (4C+1)/(4C-3) \tag{4-7}$$

$KB=(4(5)+1)/(4(5)-3)= 1.24$

(b) Alternating Force

$$F_a = (F_{max} - F_{min})/2 \quad (4\text{-}8)$$

$F_a = (900-650)/2$ lbs

$F_a = 125$ lbs

(c) Midrange Force

$$F_m = (F_{max} + F_{min})/2 \quad (4\text{-}9)$$

$F_m = (650+900)/2$ lbs

$F_m = 775$ lbs

(d) Alternating shear-stress

$$\tau_a = (K_B 8 F_A D)/(\pi d^3) \quad (4\text{-}10)$$

$\tau_a = (1.24)(8)(125)(3)/(3.14)(.6)^3$

$\tau_a = 5{,}484.78$ psi

(e) Midrange shear stress

$$\tau_m = (K_s 8 F_m D)/(\pi d^3) \quad (4\text{-}11)$$

$\tau_m = (1)(8)(775)(3)/(3.14)(.6)^3$

$\tau_m = 27{,}423.92$ psi

(f) Endurance limit for plain and grounded ends[4].

$S_{se} = 45.0$ kpsi

(g) Factor of safety using Goodman's criterion.

$$n = (S_{se}\tau_{max})/(\tau_a\tau_{max} + \tau_m S_{se}) \quad (4\text{-}12)$$

$n = (45.0)(95.5)/(5.5)(95.5) + (27.4)(45.0)$

$n = 2.4$

III. The next step involves determining input data for vibration analysis.

(a) The mass acting on the shock absorber

$m = w/g = (650 \text{ lbs})/(32.175 \text{ft/s}^2)$

$m = 20.2$ slugs

(b) The critical damping coefficient is determine from mass and spring stiffness.

$$c = 2m(k/m)^{1/2} \tag{4-13}$$

$$c = 2(20.2)(2731.034/20.2)^{.5}$$

$$c = 469.75 \text{ lb.s/ft}$$

(c) The maximum vertical displacement, X_{max}, shown in Equation 3-10 is obtained after plotting Equation 3-9, as shown in FIGURE 4-1.

After going through the deterministic results for the shock absorber, the following mean values were obtained in TABLE 4-1. Using these values, many trial experiments were performed using the NESSUS code to reduce the probability of failure in the design. In each trial experiment, the sensitivity of each random variable is displayed on bar charts as shown in FIGURE(4-2 through 4-4). The random variable with the highest sensitivity is the most crucial variable for reducing the probability of failure and vice versa. Also, fault tree analysis was performed to find reliability of the system as shown in TABLE 4-11.

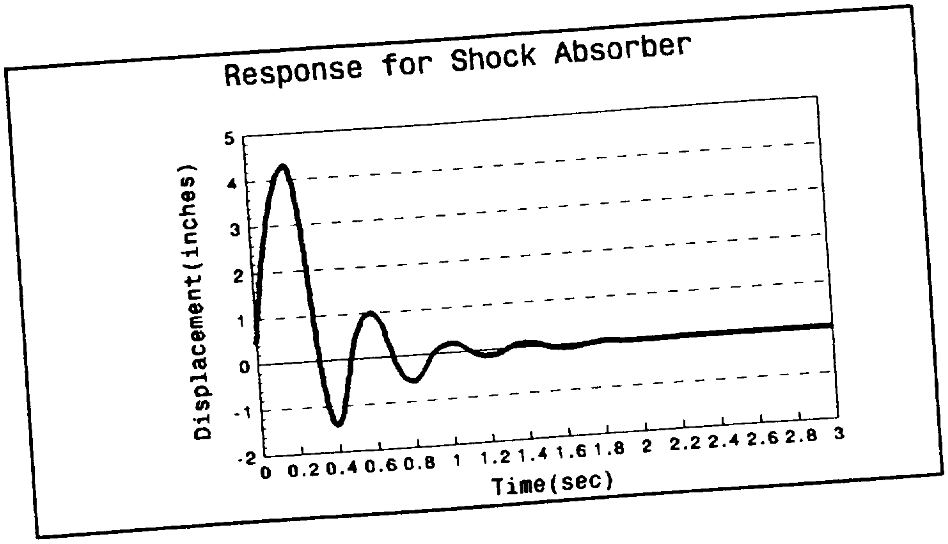

FIGURE 4-1: Displacement Verses Time For Shock Absorber.

TABLE 4-1: Mean Values Obtained From Calculations.

τ_{max} torsional shear stress	95,499.9 psi
D, mean diameter of spring	3.0 inches
d, wire diameter	0.60 inches
N, number of active coils	29
F load	650 lb
K spring constant	2731.03 lb/ft
X_{max}, displacement	4.35 inches
δ_{max} deflection	7.2 inches
c damping constant	469.75 lb.s/ft
Factor of Saftey for fatigue	2.4

INPUT DATA FOR TRIAL 1

TABLE 4-2: Random Variables For Deflection, Trial 1.

Random Var	Mean	Std	Dist Type
d	0.60 inches	.014 inches	normal
D	3.0 inches	.045 inches	normal
N	29	4.87	normal
F	650 lbs	124 lbs	normal

TABLE 4-3: Random Variables For Stress, Trial 1.

Random Var	Mean	Std	Dist Type
d	0.60 inches	.014 inches	normal
D	3.0 inches	.045 inches	normal
τ_{max}	95,499.9 psi	8,000 psi	normal
F	650 lbs	124 lbs	normal

TABLE 4-4: Random Variables For Vibrations, Trial 1.

Random Var	Mean	Std	Dist Type
c	469.7 lb.s/ft	61.06 lb.s/ft	normal
ω	3 rad/s	.39 rad/s	normal
F	650 lbs	124 lbs	normal

INPUT DATA FOR TRIAL 2

TABLE 4-5: Random Variables For Deflection, Trial 2.

Random Var	Mean	Std	Dist Type
d	0.60 inches	.010 inches	normal
D	3.0 inches	.025 inches	normal
N	29	3.07	normal
F	650 lbs	90 lbs	normal

TABLE 4-6: Random Variables For Stress, Trial 2.

Random Var	Mean	Std	Dist Type
d	0.60 inches	.010 inches	normal
D	3.0 inches	.025 inches	normal
τ_{max}	95,499.9 psi	6,500 psi	normal
F	650 lbs	90 lbs	normal

TABLE 4-7: Random Variables For Vibrations, Trial 2.

Random Var	Mean	Std	Dist Type
c	469.7 lb.s/ft	45.0 lb.s/ft	normal
ω	3 rad/s	.30 rad/s	normal
F	650 lbs	90 lbs	normal

INPUT DATA FOR TRIAL 3

TABLE 4-8: Random Variables For Deflection, Trial 3.

Random Var	Mean	Std	Dist Type
d	0.60 inches	.058 inches	normal
D	3.0 inches	.015 inches	normal
N	29	2.07	normal
F	650 lbs	50 lbs	normal

TABLE 4-9: Random Variables For Stress, Trial 3.

Random Var	Mean	Std	Dist Type
d	0.60 inches	.058 inches	normal
D	3.0 inches	.015 inches	normal
τ_{max}	95,499.9 psi	4,500 psi	normal
F	650 lbs	50 lbs	normal

TABLE 4-10: Random Variables For Vibrations, Trial 3.

Random Var	Mean	Std	Dist Type
c	469.7 lb.s/ft	34.0 lb.s/ft	normal
ω	3 rad/s	.23 rad/s	normal
F	650 lbs	50lbs	normal

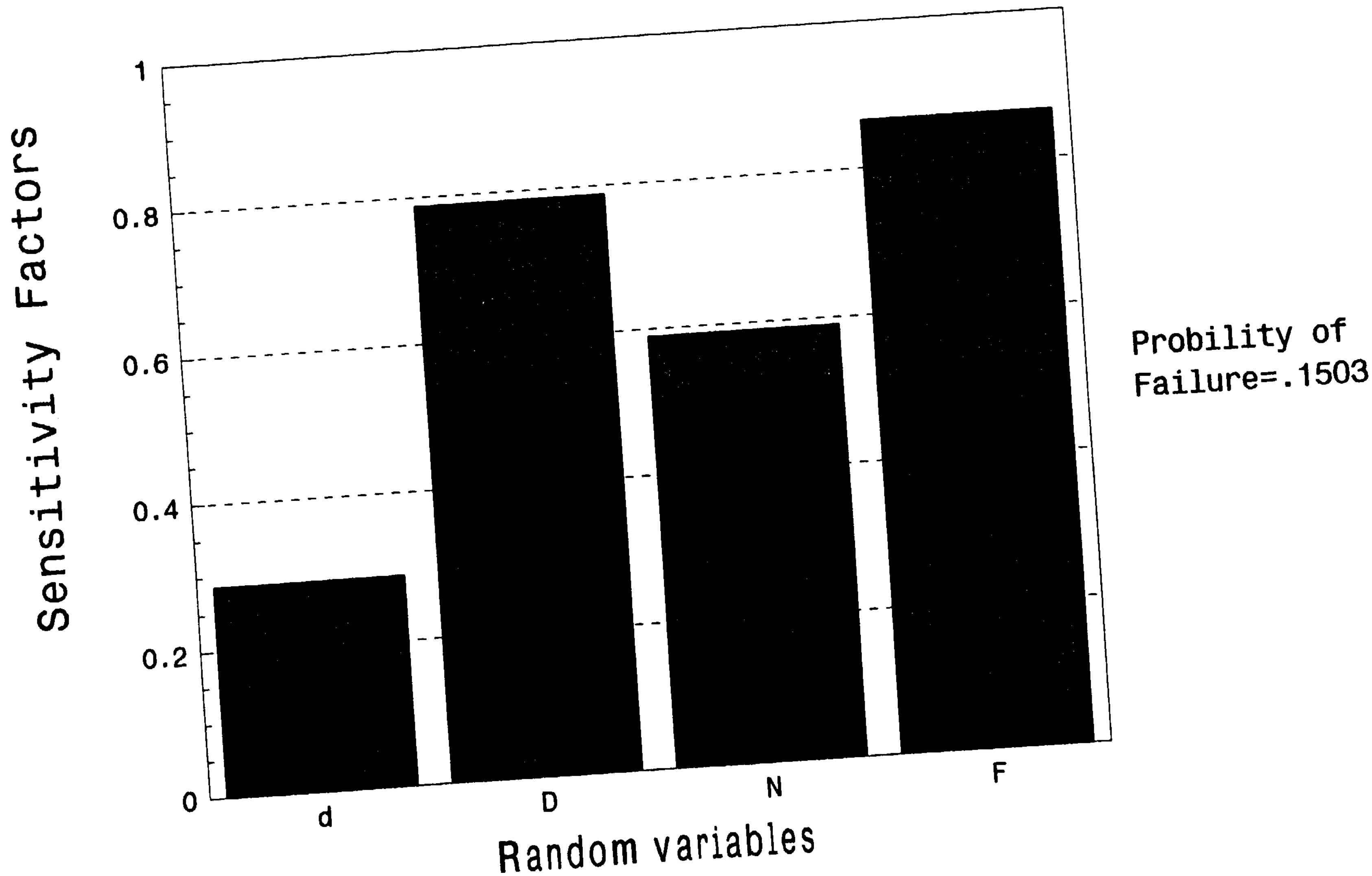

Figure 4-2A: Sensitivity For Deflection, Trial 1.

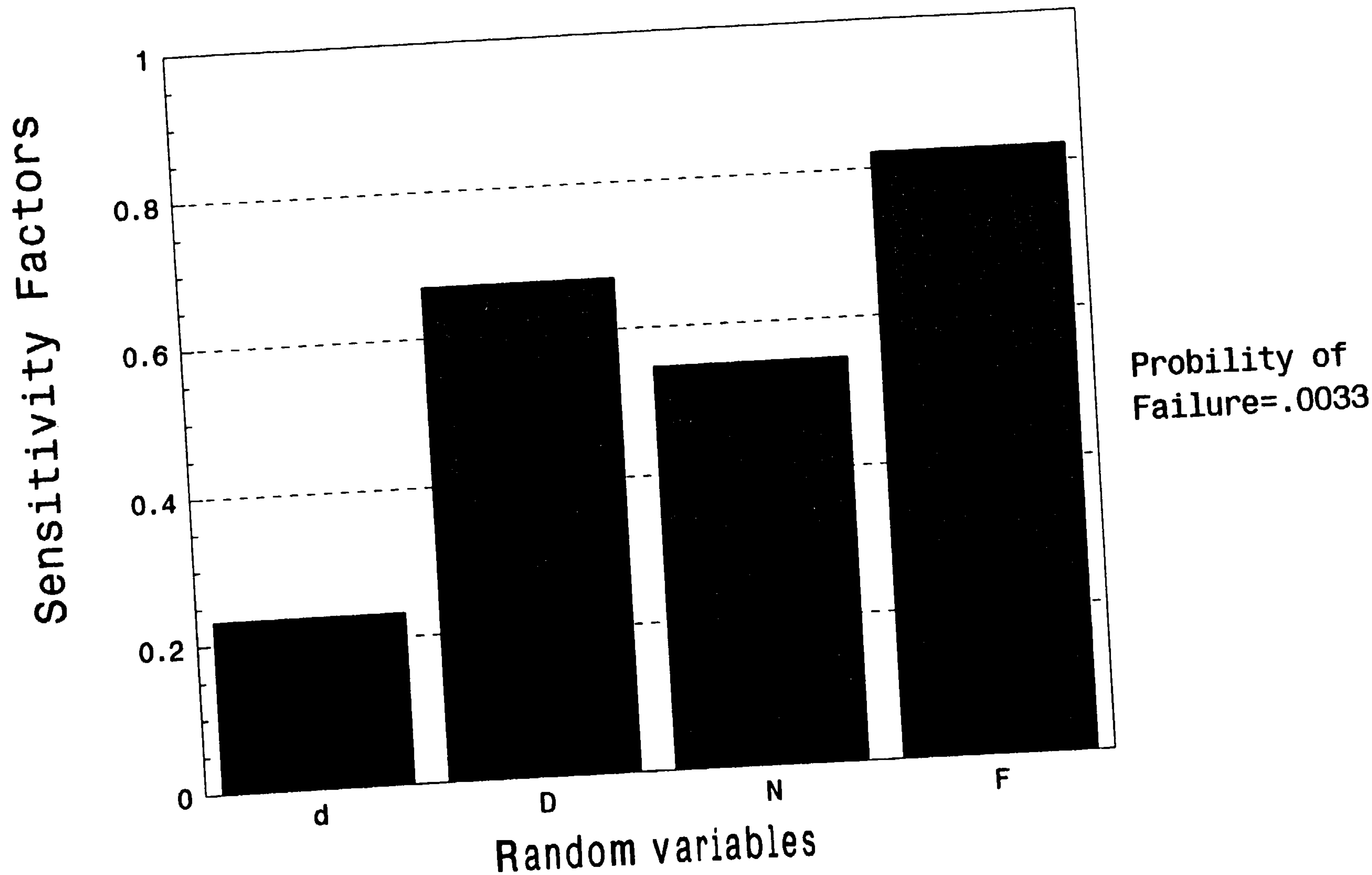

Figure 4-2B: Sensitivity For Deflection, Trial 2.

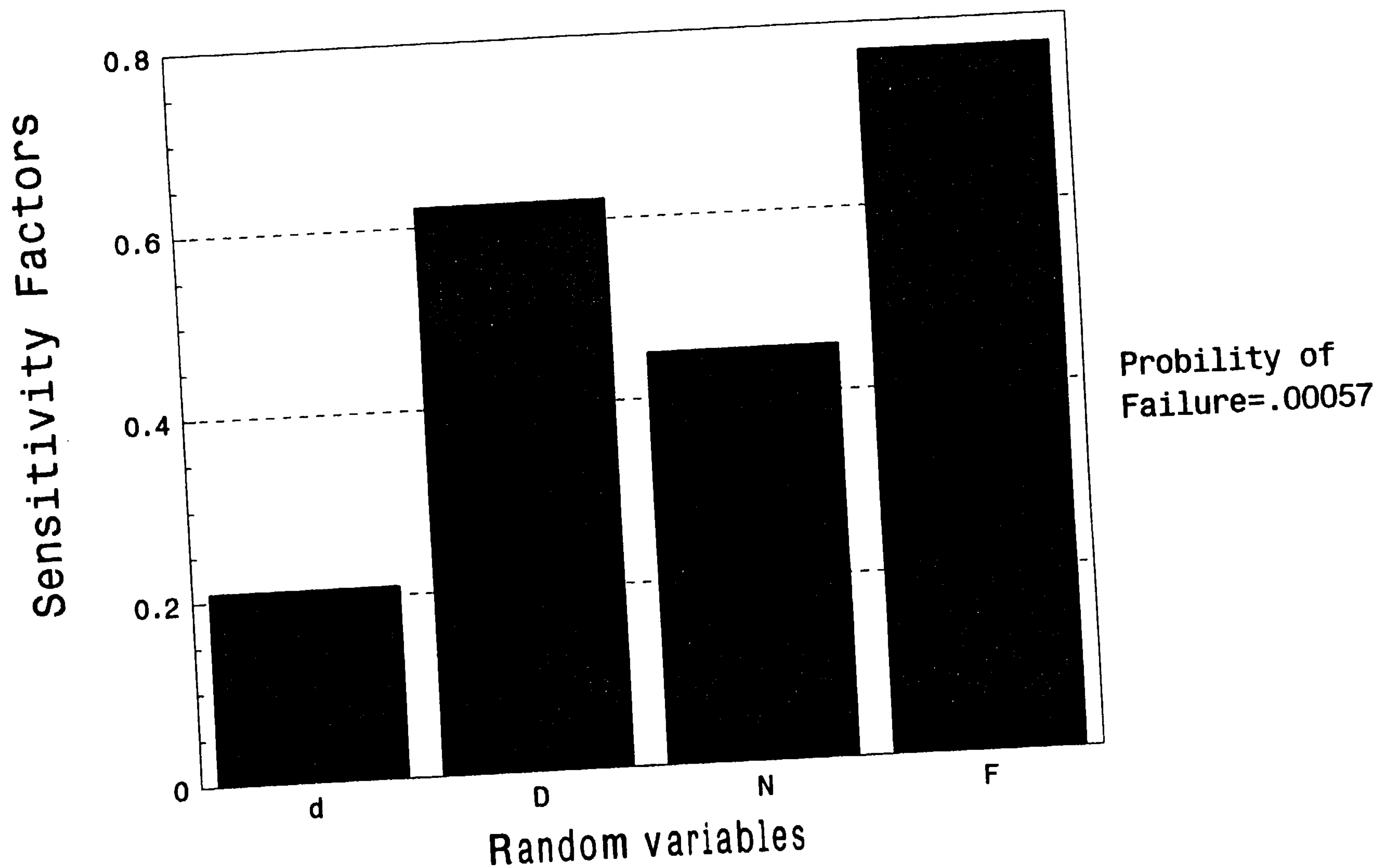

Figure 4-2C: Sensitivity For Deflection, Trial 3.

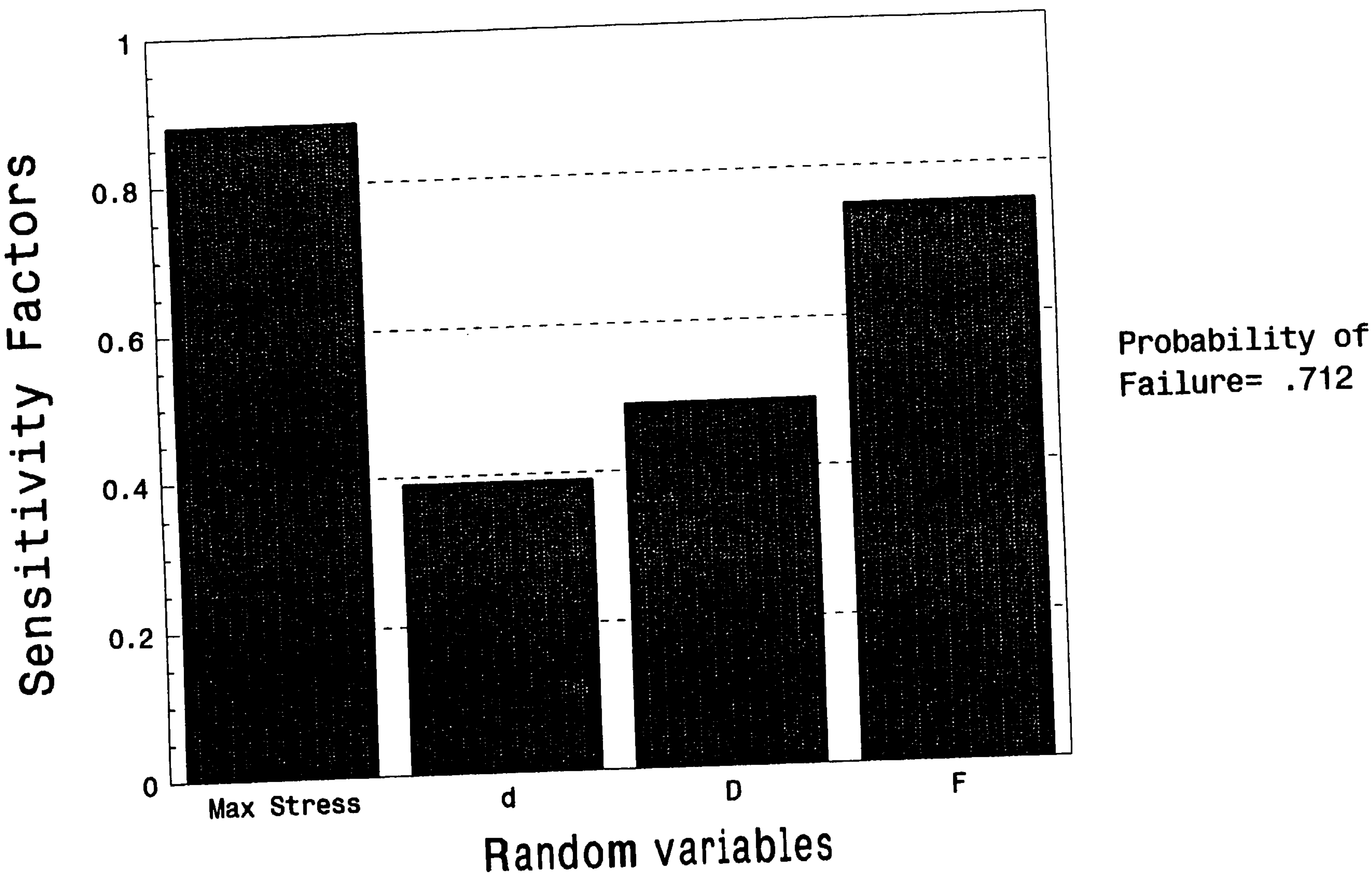

Figure 4-3A: Sensitivity For Stress, Trial 1.

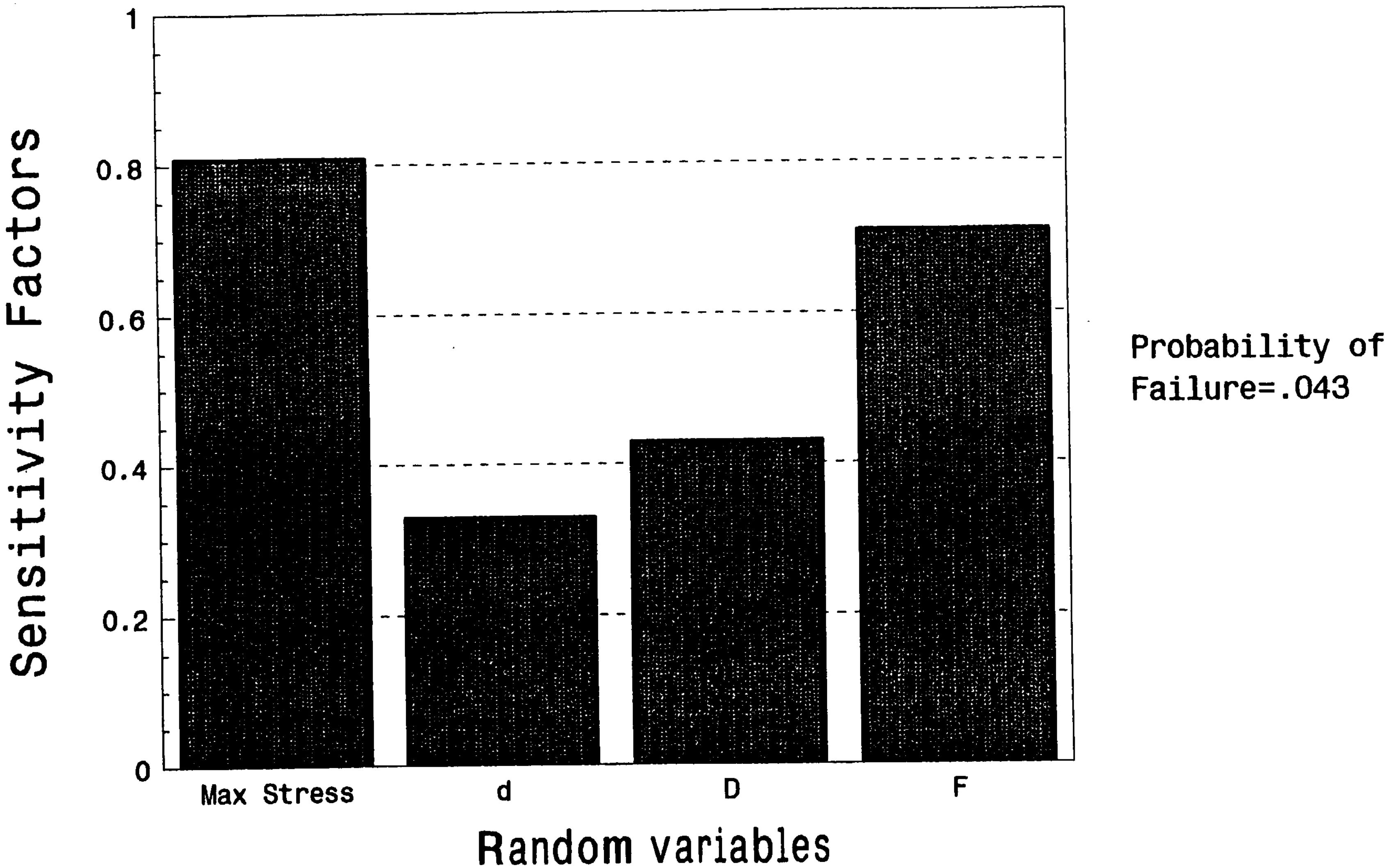

Figure 4-3B: Sensitivity For Stress, Trial 2.

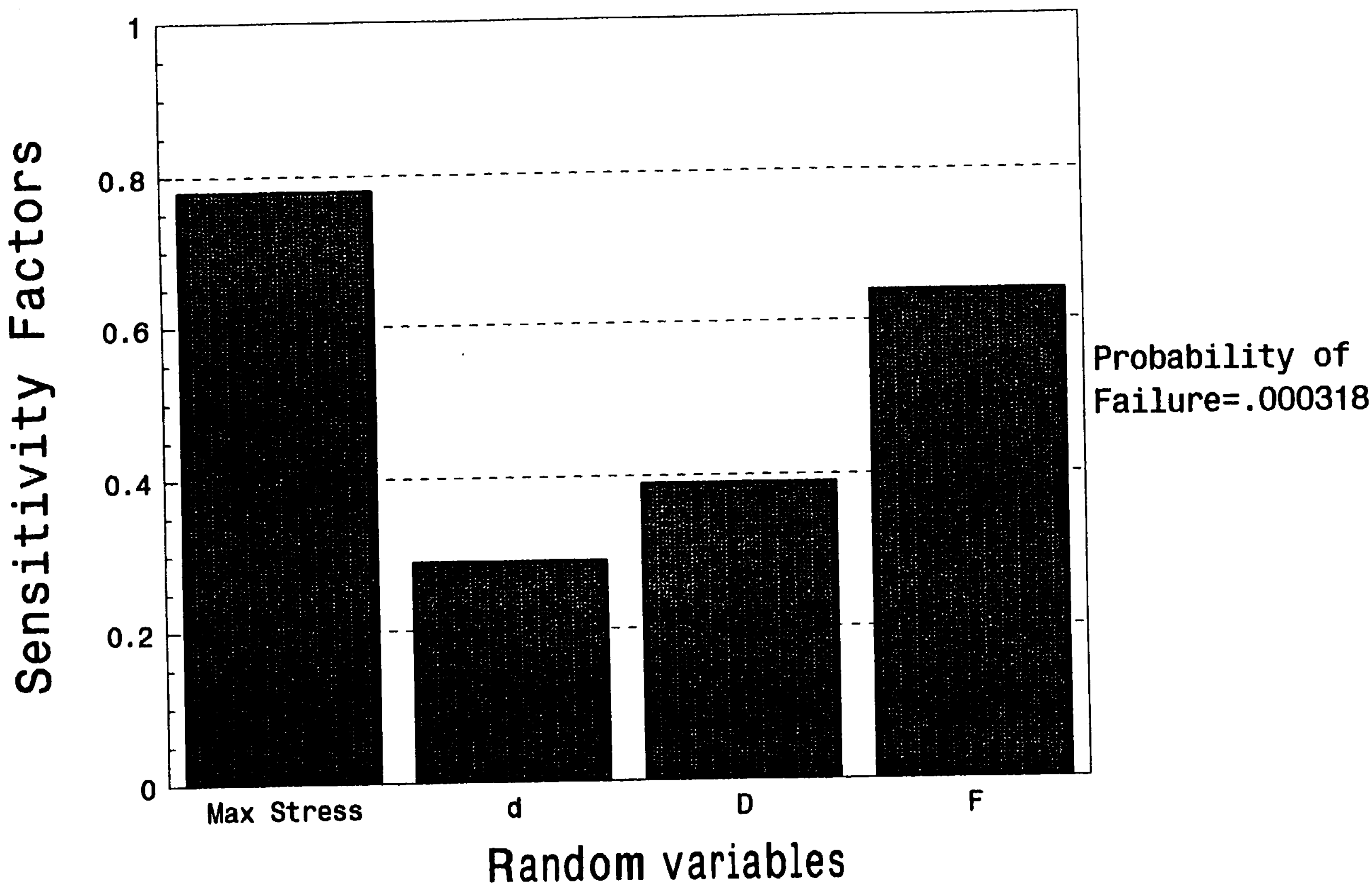

Figure 4-3C: Sensitivity For Stress, Trial 3.

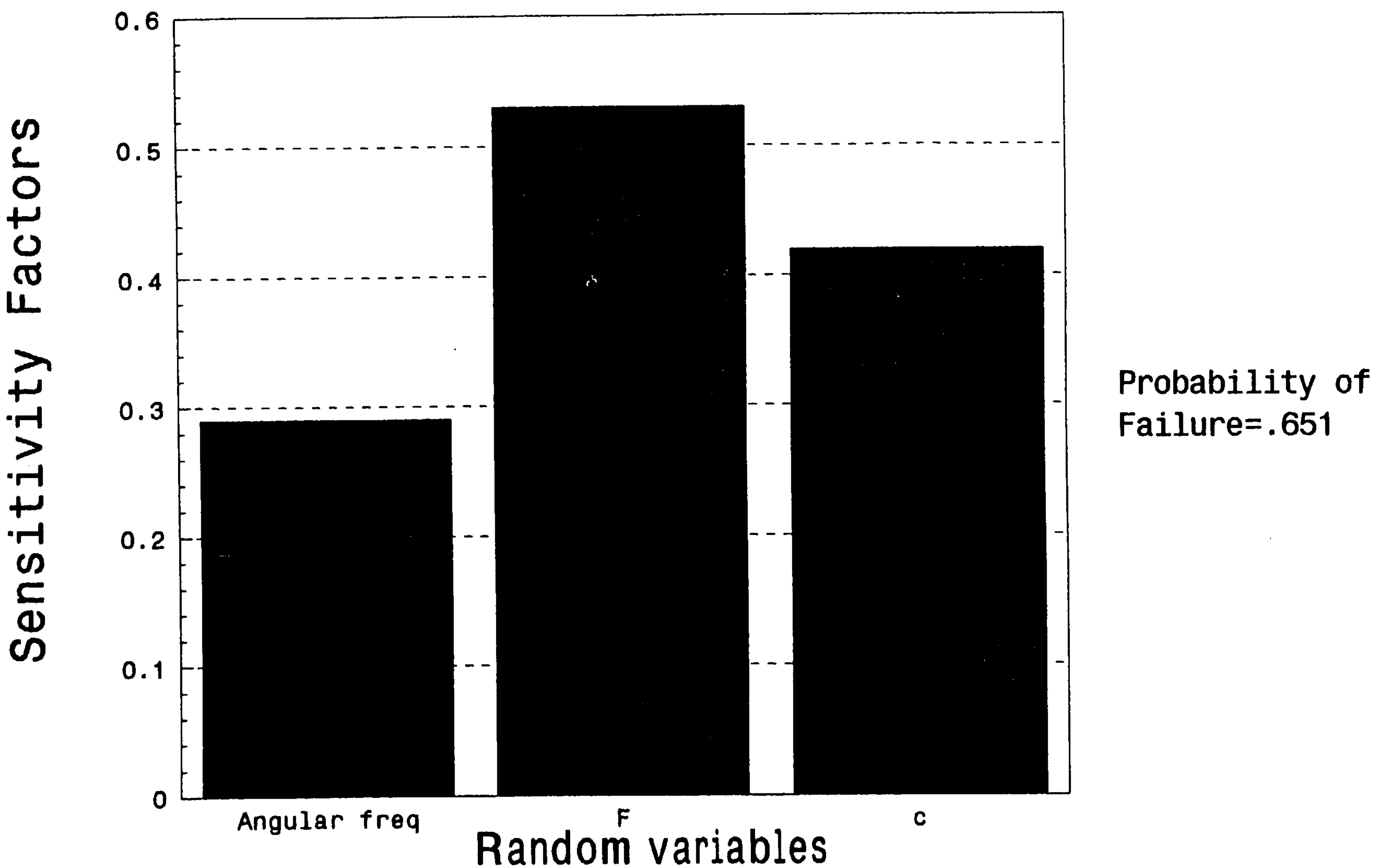

Figure 4-4A: Sensitivity For Vibration, Trial 1.

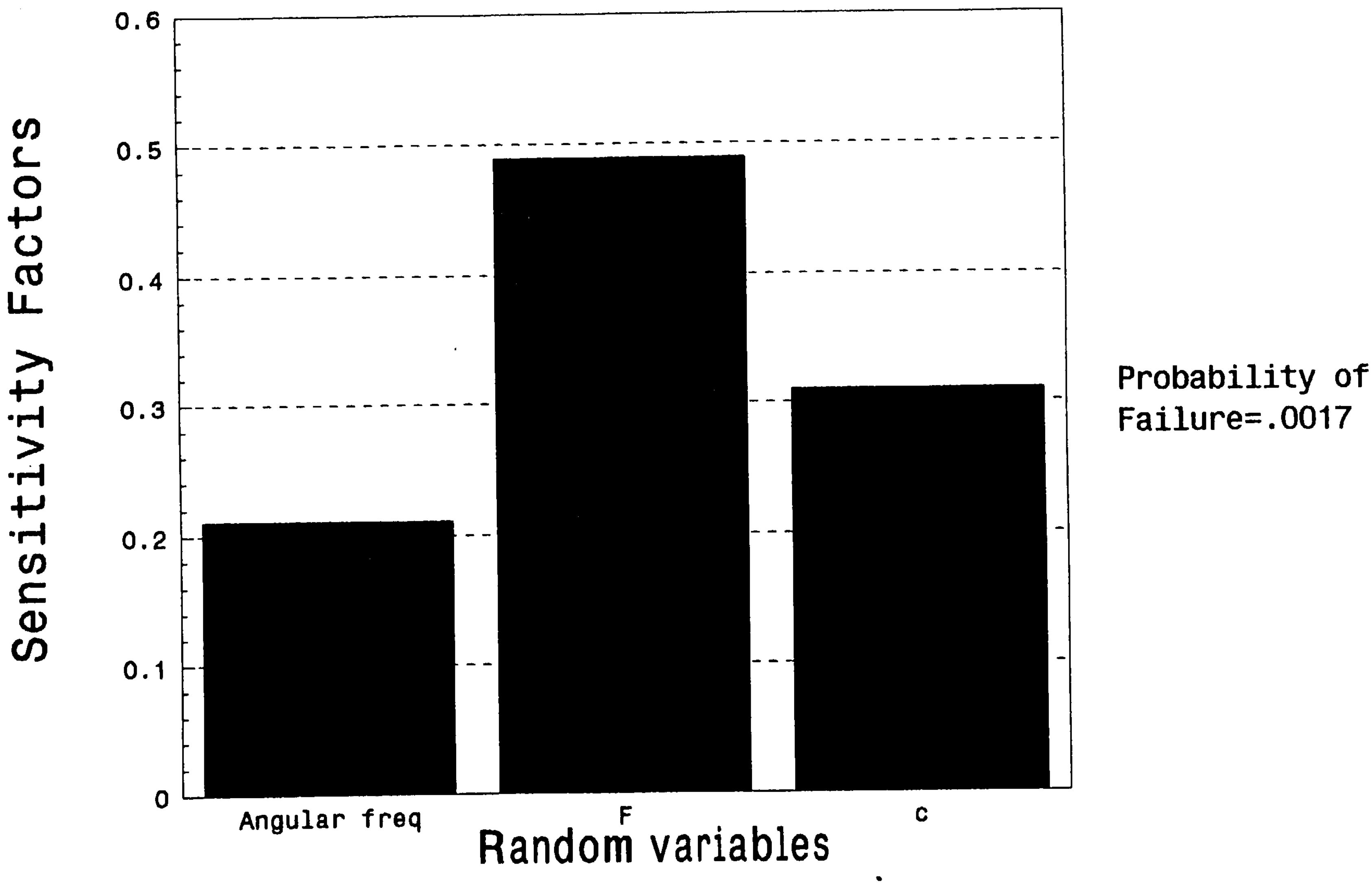

Figure 4-4B: Sensitivity For Vibration, Trial 2.

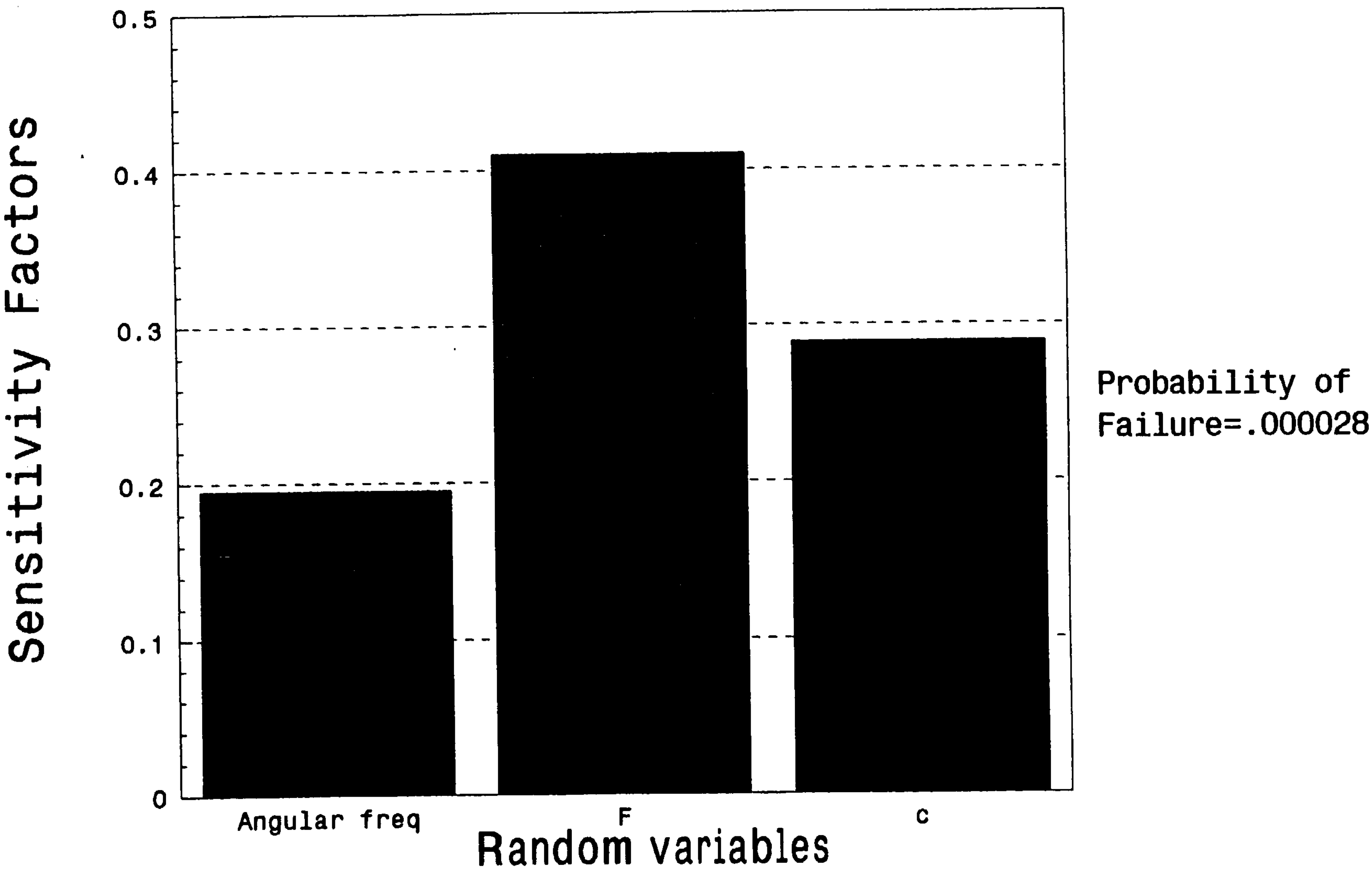

Figure 4-4C: Sensitivity For Vibration, Trial 3.

TABLE 4-11: Probabilistic Results.

Stress Analysis	$3.2\text{x}10^{-4}$ Prob of Failure
Deflection Analysis	5.7x10-4 Prob of Failure
Vibration Analysis	$2.8\text{x}10^{-5}$ Prob of Failure
Fault Tree Analysis	$8.6\text{x}10^{-4}$ Prob of Failure
Factor of Safety For Fatigue Failure	2.48

TABLE 4-12: Design Specifications.

D, mean wire diameter	3.0 ± .015 inches
d, wire diameter	.60 ± .058 inches
Ls, Free length	18 inches
c, damping constant	469.7 ± 34 lb. s/ft
N, number of coils	29 ± 2.07

CHAPTER V

CONCLUSION

In concluding this project, the design parameters and functional requirements were clearly defined. An overview of Probabilistic Design Methodology was outlined along with the computer code NESSUS. The equations for the design parameters were generated for the three failure modes defined in Chapter three such as stress, deflection, and vibrations, and the probabilistic results were obtained.

From the results obtained, as shown in Chapter four, the stress analysis was used to obtain the probability of failure under torsional shear stress and the factor of safety for fatigue failure. Along with this analysis, deflection was analyzed to design the spring component so that buckling will not occur in the system. Under the vibration analysis, the spring component was lumped with the damper to analyze the vibrations in the complete system. Also, the reliability of the system was determined using fault tree analysis, and the design specifications were generated for the shock absorber.

It is clear that Probabilistic Design Methodology(PDM) is very effective in determining the reliability of a system by quantifying the probability of failure due to stress, deflection, and vibration.

Finally, this project has provided all the steps of designing a shock absorber using PDM. From the results obtained in Chapter four, one can see that PDM is a very effective means of designing a system which is reliable.

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CAREER OBJECTIVE

To make a planned progression into a position utilizing the fundamental principals of engineering

EDUCATION

Major: Mechanical Engineering, Tennessee State University, graduation Dec 1995 (Expected Date).Also,passed the EIT.

Bachelor of Science in Electrical Engineering, UNIVERSITY OF MISSISSIPPI,Oxford,Mississippi,May 1992

EMPLOYMENT

NASA RESEARCH STUDENT, Tennessee State (1/94-5/95) University, Used Probablistic Design Methodology to design an amplifier circuit with minimum noise output.(8/94-5/95), Designed a Shock Absorber as a Senior Design Project.

MISSISSIPPI POWER & LIGHT COMPANY, MS, December 1989-(June 1990 Electrical Engineer, Assisted with the design, development,and implementation function of system installations throughout assigned State district. Worked in a support role coordinating tasks and completing assignments for three Senior Engineers.

Special Skills:

Ability Utilizing Computers
(1) WordPerfect to write lab reports
(2) Pspice application software to solve circuits
(3) AUTOCAD

Electrical Engineering
Have skill using electrical instruments including: analog and digital,controls,PLC's, multimeters,transistor curve tracers,logic probes, signal generators, frequency counters, and oscilloscopes.

Mechanical Engineering
Possess a very strong knowledge in machine design,Heating Ventilation and Airconditioning

Computer Skills
(1) C++ Programming
(2) Fortran Programming
(3) Q-basic
(4) Developed numerous software for Mechanical Engineering Applications

REFERENCES: Available upon request

USING PROBABILISTIC DESIGN METHODOLOGY IN THE DESIGN OF A HELICAL SPRING

by

C. Fred Higgs III

Design Project Report

Submitted to the Faculty

of the

College of Engineering and Technology

in

Partial Fulfillment of the Requirements

for the Degree of

Bachelor of Science

in

Mechanical Engineering

August 1995

College of Engineering and Technology

Tennessee State University

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C. F. H. III

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SYMBOLS

C_i	INITIAL COST
C_m	MAINTENANCE COST
C_T	TOTAL COST
COV	COEFFICIENT OF VARIATION
D_m	MEAN DIAMETER
D_w	WIRE DIAMETER
Dps	DESIGN PARAMETERS
D-TYPE	DISTRIBUTION TYPE
δ	STANDARD DEVIATION
f	DEFLECTION
f_{CRIT}	CRITICAL DEFLECTION
F_o	FORCE ON SPRING
FS	FACTOR OF SAFETY
G	SPRING MODULUS OF ELASTICITY
g	LIMIT STATE FUNCTION
K	WAHL CONSTANT
L_f	FREE LENGTH
μ	MEAN VALUE OF A SET OF DATA
Na	NUMBER OF ACTIVE COILS
P_f	PROBABILITY OF FAILURE
R	RELIABILITY
s	STANDARD DEVIATION
S_y	YIELD STRENGTH
σ	STRESS
τ	TORSIONAL SHEAR STRESS OR SHEAR STRESS
x	MEASUREMENT IN A SET OF DATA

CHAPTER I

INTRODUCTION

The evaluation of an engineering design is done by deterministic methods. This conventional form of evaluation leads to the determination of a factor of safety. In the preliminary design phase, the engineer could specify a factor of safety in order to ensure a durable design. The factor of safety is determined as the ratio of the failure stress to the stress incident on the structure or element. Some deterministic methods involve usage of the worst case scenario. This ultra-conservative method utilizes the assumption that the combination of the worst possible design parameters in a design produces a design void of probable failure. The deterministic approach to design totally discounts any possible variations in the component's dimensions, material properties and any loads which may be externally applied. This sets down the path for probabilistic design as a valuable alternative to deterministic design methods.

Probabilistic Design Methodology (PDM) concerns itself with the reliable performance of a machine element or structure. It differs from deterministic methods in that it seeks to quantify the uncertainties between the safe and failed region. The fact that the design parameters are statistical in nature is a consideration of probabilistic design. PDM makes it simpler to predict the behavior of structural performance. This is because the reduction in uncertainties is conducive to producing reliable results.

Consider a helical spring given to an engineer for performance testing. The engineer could apply a non-constant load to the spring of about 20 lbs. Stress will occur as this load is being applied. The mean diameter could contract or expand. The wire diameter could react to the load in the same manner. The amount that each particular design parameter increases or decreases can be specified by some standard deviation from their mean value. The design parameters are the variables used to completely model the failure modes. If the type of distribution and standard deviation for each design parameter and the yield strength (S_y) of the material is known, the probability of failure for the helical spring can be predicted from [1]

$$g = S_y - \sigma \qquad \textbf{(1-1)}$$

Where,

g = variable defining limit state function
s = standard deviation (psi)
σ = stress incident on spring (psi)

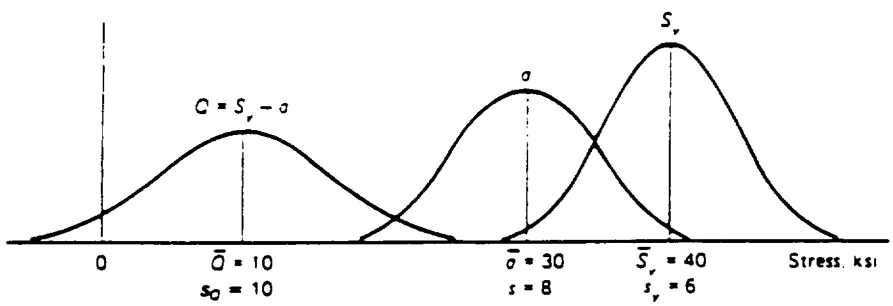

FIGURE 1-1: DISTRIBUTION OF YIELD STRENGTH AND STRESS [1]

The limit state function or g-function, which was defined by Q in FIGURE 1-1, is the equation which defines the boundary between the safe and failed regions. It is the distribution curve determined from the area where the s distribution curve overlaps the yield strength distribution curve. When the g-function, Q, becomes negative, the spring will become unreliable because the stress on the spring will have exceeded it yield strength. Thus the probability of failure, P_f, can be defined by [1]

$$P_f = P(\sigma > S_y) \tag{1-2}$$

The terms probability of failure and reliability are used interchangeably to define a good and bad design respectively. The reliability, R, is the probability that the strength exceeds the stress or the stress margin is greater than zero. It is a function of the probability of failure and is defined as [2]

$$R = 1 - P_f \tag{1-3}$$

CHAPTER II

BACKGROUND INFORMATION

A spring is a flexible element utilized to exert a force, torque or store energy. The force is either a push or pull in linear fashion. The spring possesses rebounding capabilities for returning to its original shape and dimension.

2.1 Helical Springs

Helical springs are basically made from round wire, wrapped into a straight cylindrical form. Helical compression springs have a constant pitch between adjacent coils. There are four end treatments which are shown in FIGURE 2-1. The free length is the length of the spring when no load is applied to it. The coils of the spring are compressed together when the force is applied. When the coils are touching completely, this minimum length is called the solid length. The spring undergoes deflection as it is pushed linearly to its solid length.

Helical extension springs are almost identical to the helical compression spring in shape, but a force must be applied to it in tension. It undergoes deflection as the tensile force is applied linearly. The fundamental difference between the compression spring and the extension spring is that the coils of the extension spring are touching or at least closely spaced when in its free length state. An example of this type of spring is that used in a ball point pen. As the tensile force is applied, the spring's energy is converted into a pulling force. There are several end configurations for the helical extension spring shown in FIGURE 2-2.

In this paper, a helical compression spring is used to demonstrate the applicability of PDM in determining the reliability of a helical spring. When the spring is compressed, the wrapped wire which makes up the spring twists and undergoes torsional shear stress, τ. equated by [3]:

$$\tau = \frac{8KF_oD_m}{\pi D_w^{\ 3}} \qquad \textbf{(2-1)}$$

Where,

K = Wahl constant
F_o = Force on spring
D_m = Mean diameter
D_w = Wire diameter

The Wahl constant accounts for the curvature in the spring. FIGURE 2-3 shows the points out each design parameter on the spring. The compression spring can be compressed to a point exceeding its maximum deflection. If this occurs, the spring will then fail due to deflection.

The equation for deflection is [3]

$$f = \frac{8 F_o D_m^3 N_a}{G D_w^4} \tag{2-2}$$

Where,

Na = number of active coils
G = spring modulus of elasticity

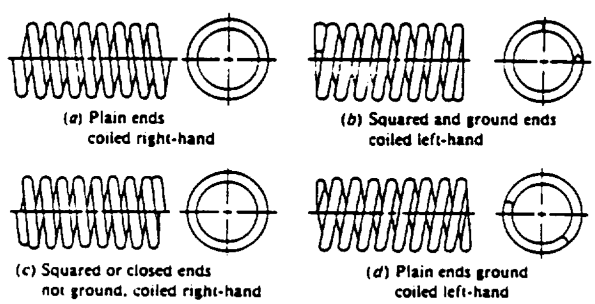

FIGURE 2-1: HELICAL COMPRESSION SPRING END TREATMENTS [3]

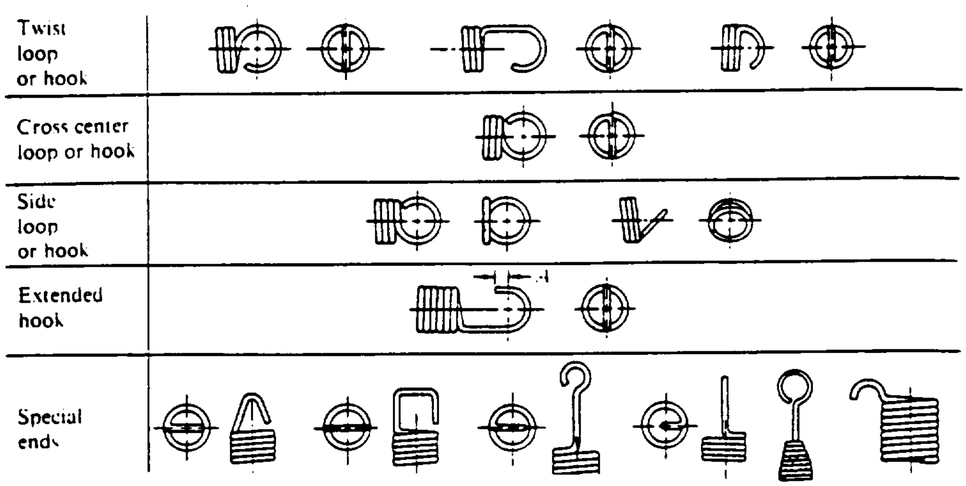

FIGURE 2-2: HELICAL EXTENSION SPRING END CONFIGURATION [3]

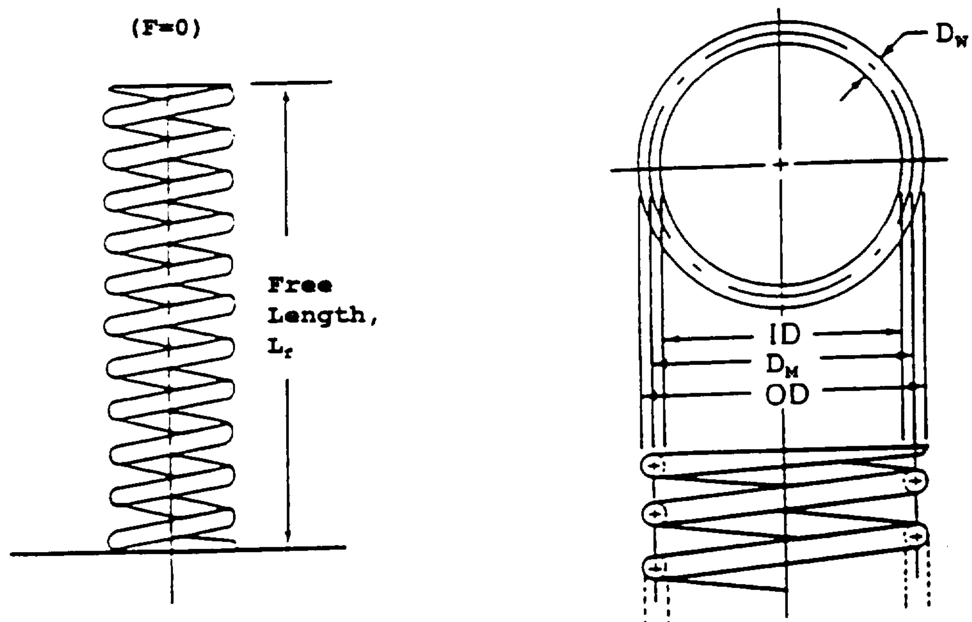

FIGURE 2-3: HELICAL COMPRESSION SPRING FRONT AND TOP VIEW

CHAPTER III

APPLICATION OF PROBABILISTIC DESIGN METHODOLOGY

Probabilistic Design Methodology establishes an organized format for designing durable products using probabilistic design. The sequential stages of this methodology can be outlined as:

1. Problem Definition
2. Creating acceptable design parameters
3. Related the defined problem to design parameters
4. Data assembling and application
5. Probabilistic Analysis
6. Interpreting results

The objective is to design a reliable helical spring using probabilistic design methodology and to identify the critical design parameters of the spring. For the sake of contrast, a deterministic approach will be taken *first*. And finally, the problem is approached probabilistically by applying PDM.

Problem Statement

Evaluate the design described by the parameters of an A231 chromium-vanadium steel helical compression spring. Identify the critical design variables.

Given:

L_f = 6 in. D_m = 0.75 in.
F_o = 20 lb. D_w = 0.15 in.
G = 11.2×10^6 psi Na = 5

The torsional shear stress, τ, which occurs on the helical spring is shown as equation 2-1, but is repeated here for clarity [3]

$$\tau = \frac{(8*D_m*F_o*k)}{\pi D_w^{\ 3}} \tag{2-1}$$

The spring could buckle if the deflection on the spring exceeds its critical deflection. The deflection, f, is equated by [3]

$$f = \frac{8*F_o*D_m^{\ 3}*Na}{GD_w^{\ 4}} \tag{2-2}$$

Both of these equations characterize the failure modes of the spring. If this problem is looked at deterministically, the worst possible values are taken from the given information. For this problem, the force will be taken as 25 lbs. as opposed to 20 lbs. The shear stress on the spring is computed as 18.6×10^3 psi. Noting that the ultimate strength of the vanadium-chromium spring is 130×10^3 psi, the factor of safety is 7. This is a high factor of safety and should be indicative of a reliable design.

Results

Deterministic Method:

Given: F = 25 lb.

Stress, S

S_{max}= 130×10^3 psi

τ = 18.6×10^3 psi

FS = 7

When the deflection is calculated, it is computed as 0.07 in. Using FIGURE 3-1, the critical deflection is determined to be 0.6 in. This means that the spring will buckle if it is deflected more than the critical deflection. There is a 88% difference in the occurring deflection. This means the spring can deflect 88% percent more than the actual deflection before it buckles. This is also indicative of a successful design outcome. These values do not account for the fact that the load will not always be 25 lbs. This means the design is over-designed and will be costly to the customer. The results from the deterministic methods for deflection are printed below as:

Deflection, f

f_{crit}	= 0.6 in.
f	= 0.07 in.
% Difference	= 88.3%

When PDM is applied to this problem, all of the given design

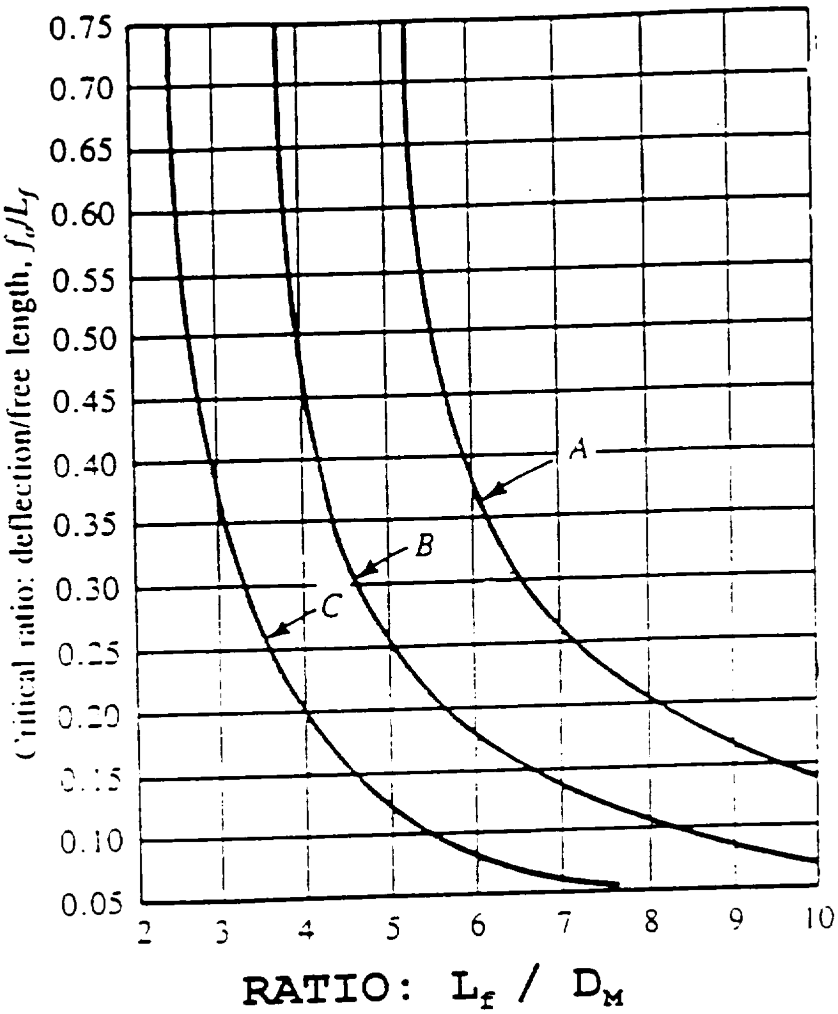

FIGURE 3-1: CRITICAL DEFLECTION GRAPH [3]

parameters are treated as random variables. The stages of the methodology are then applied to this problem.

3.1 Problem Definition

In order to evaluate the design described by the parameters, the reliability must be determined from the failure modes. The failure modes are torsional shear stress and deflection. They are also referred to as response functions or responses [4]. These response functions are noted as equations 3-1 and 3-2. Since some parameters are more crucial to the design than others, special attention should be given to them. These particular variables are critical and can be identified from the sensitivity analysis performed by NESSUS.

3.2 Creating Acceptable Design Parameters

The design parameters are defined from the responses. The design parameters of the helical compression spring are D_m, D_w, F_o, S, G, L_f, Na. FIGURE 2-3 shows the design parameters on the spring. These variables completely characterize the possible failure of the design.

3.3 Relating Problem Definition to Design Parameters

The design task is to design a reliable spring. Considerations should take probability of failure, size and weight provisions, and economics into account. Since the cube of the wire diameter, D_w, is inversely proportional to the shear stress, τ, it was expected to be a critical design parameter. The force is not a parameter which can be varied because it is a specified condition of the system. The system is taken to mean the helical spring and the compressive force on the spring. Varying the parameter, G, means changing the material. S is the ultimate strength and must be varied by selecting a stronger material (one with higher strength) or using some material processing technique to increase the strength of the material. Designing a reliable spring is dependent on the factors which characterize the design parameters and the parameter magnitudes.

3.4 Data Assembling

This stage of the methodology involves utilizing the computer code NESSUS to assemble data which is not accounted for in the response functions. The code provides the designer with the ability to generate more details conducive to analyzing the system probabilistically. At this point in the paper, information about the computer code called NESSUS is imparted.

NESSUS has three different modules used to perform the analysis. NESSUS/PRE generates statistical data necessary for probabilistic design analysis. Uncertainties in the failure modes are quantified from the design parameters, which are considered as random variables.

3.4.1 NESSUS Computer Code

NESSUS/FEM is the portion of the code which enacts the finite element module. This module is used for to analyze structure and perform a sensitivity analysis of the random variables. The sensitivity analysis uses mathematical modeling to indicate which DPs are critical and have the most crucial effect on the probability of failure.

NESSUS/FPI is the module with a fast probability integrator. Data from the NESSUS/FEM module is needed to utilize the third module of NESSUS. This module also develops the cumulative density function, which aid in determining the median of the values generated between the range of the lower and higher standard deviation from the mean value of a specific design parameter [5]. The cumulative density function is used for computational purposes in this module also.

The designer must select the probability distribution which best describes the each random variable. There are many type of distribution. Since all of the design parameters were assumed to take a normal distribution, this type of distribution is the only distribution of concern in this paper.

3.4.2 Normal Distribution

This type of distribution is also known as the Gaussian distribution. Many sets of engineering data have normal type distribution. The measurements which form the mean usually form a bell shaped curve. The types of measurements which follow this type of curve is usually the length and diameter of the bar, or the strength of the material. The probability density function or equation of the normal curve is [2]

$$f(x) = \frac{1}{\delta\sqrt{2\pi}} e^{(-0.5\frac{x-\mu}{\delta})^2} \tag{3-1}$$

Where,

x = measurements in a set of data
μ = mean value of data
δ = standard deviation from mean

3.5 Probabilistic Analysis

The limit state function or g-function is defined at this point of the design methodology. The first limit state function is formulated from the shear stress equation 3-1 as [5]:

$$g_\tau = S - \frac{(8*D_m*F_o*K)}{\pi D_w^3} \tag{3-2}$$

Where,

g_τ = limit state function for torsional shear stress
S = ultimate strength of the Chromium - Vanadium (psi)

The data which was input into the NESSUS code from the shear stress failure mode as shown in TABLE 3-1 was from the first trial of the sensitivity analysis. The sensitivity analysis from this trial is depicted in FIGURE 3-2. Each design parameter has a standard deviation which covers the maximum to minimum range of possible values for the variable. The mean value represents the midpoint of the range. The probability of the spring failing because of shear stress can be determined according to the inequality equation [5]

$$P_f = g_\tau \le 0 \qquad \textbf{(3-3)}$$

TABLE 3-1: TORSIONAL SHEAR STRESS FAILURE MODE DATA INPUT TABLE TRIAL 1

DP	MEAN	s	D-TYPE
S (psi)	$130x10^2$	$32.5x10^2$	NORMAL
F_o(lbs)	20	5	NORMAL
D_m(in.)	0.75	0.075	NORMAL
D_w(in.)	0.15	0.015	NORMAL

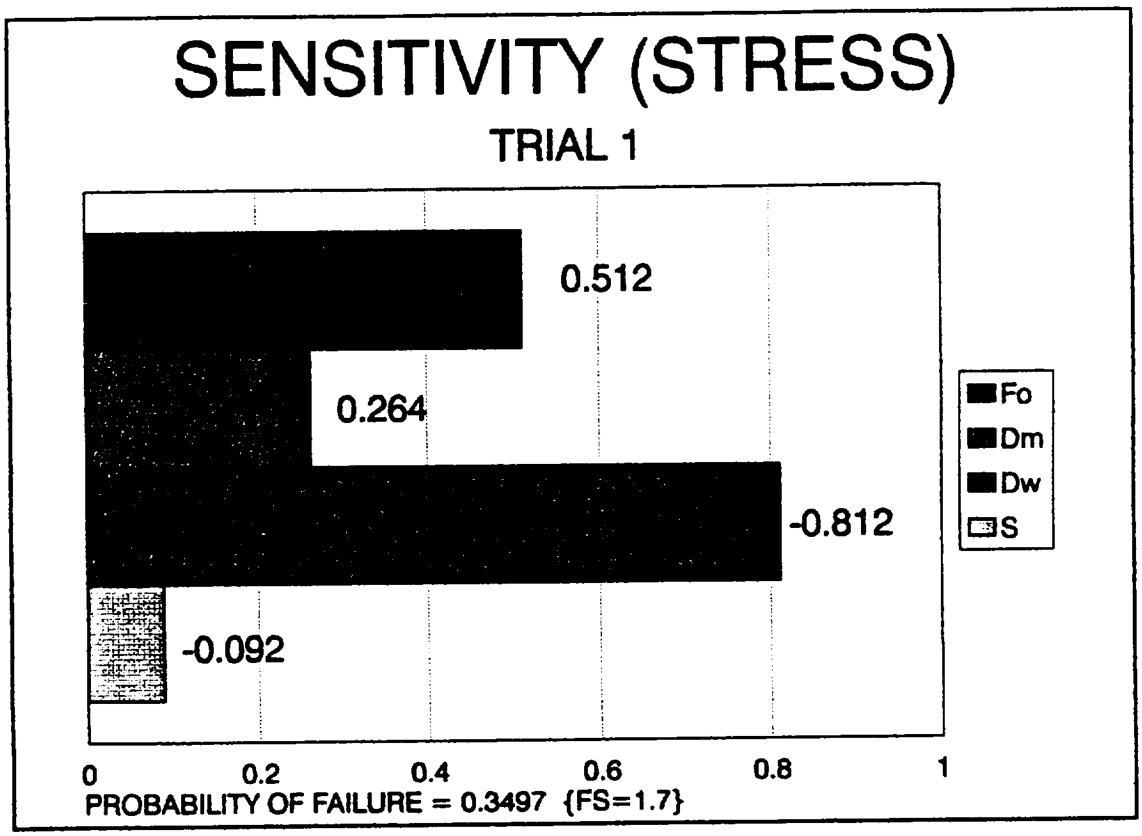

FIGURE 3-2: TORSIONAL SHEAR STRESS SENSITIVITY ANALYSIS TRIAL 1

TABLE 3-2: TORSIONAL SHEAR STRESS FAILURE MODE DATA INPUT TABLE TRIAL 2

DP	MEAN	s	D-TYPE
S (psi)	130×10^2	32.5×10^2	NORMAL
F_o (lbs)	20	5	NORMAL
D_m (in.)	0.75	0.075	NORMAL
D_w (in.)	0.23	0.023	NORMAL

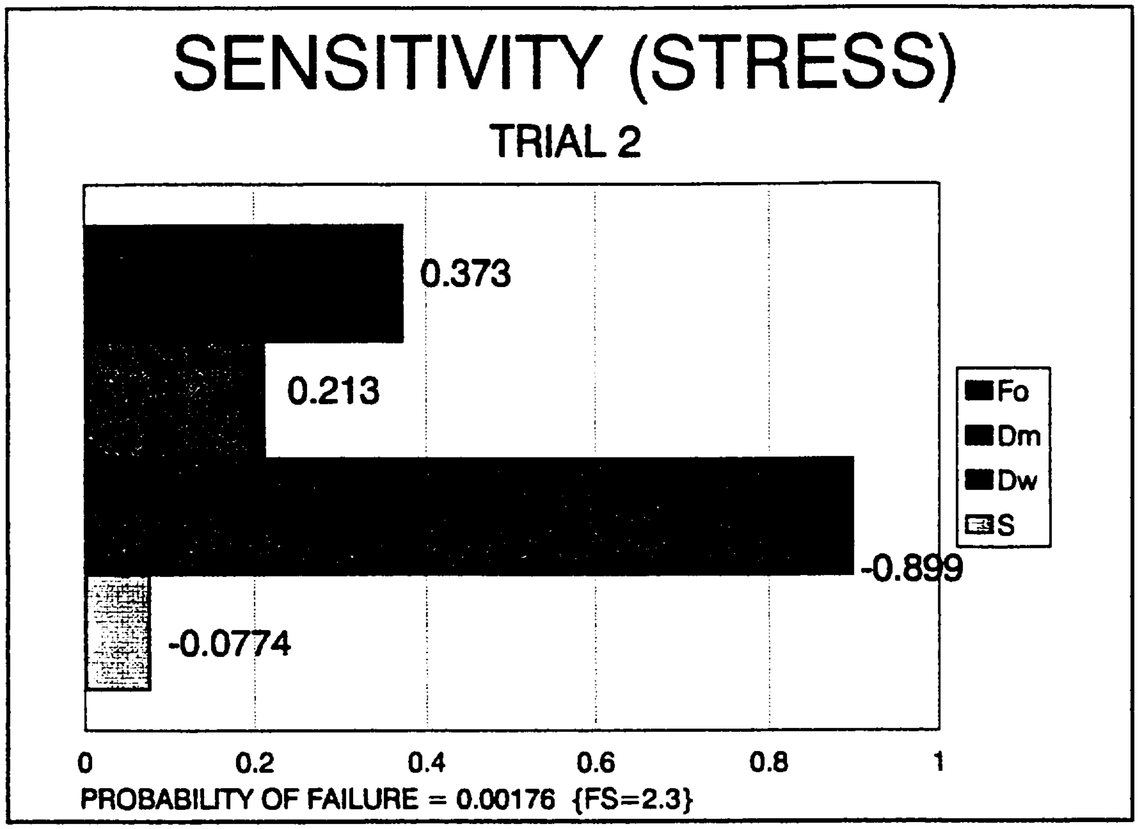

FIGURE 3-3: TORSIONAL SHEAR STRESS SENSITIVITY ANALYSIS TRIAL 2

The second limit state function relates to the spring failing by buckling or exceeding its maximum deflection. The g-function for this occurrence is represented by:

$$g_{\Delta} = (L_f * \frac{f_{crit}}{L_f}) - \frac{8 * F_o * D_m^{\,3} * Na}{GD_w^{\,4}} \tag{3-4}$$

f_{crit} is the deflected length at which the spring will buckle. The critical ratio, f_{crit} to the free length, L_f, is determined from the chart shown in FIGURE 3-1 [3]. The critical ratio is read from FIGURE 3-1 as a function of the free length and mean diameter, D_m. The curve in FIGURE 3-1 was curve fit to determine the relationship for the deflection ratio. Each curve, denoted as A, B, and C represents the critical ratio for a helical spring which is fixed at both ends, fixed at one end, or pinned at both ends.

A relationship was determined by fitting each curve with an exponential equation, thus representing the critical ratio. The equations for curves A, B, and C are as follows:

<u>Curve A</u>

$$\frac{f_{crit}}{L_f} = 2.48e^{-0.315(\frac{L_f}{D_m})} \tag{3-5}$$

Curve B

$$\frac{f_{crit}}{L_f} = 1.848e^{-0.353(\frac{L_f}{D_m})} \tag{3-6}$$

Curve C

$$\frac{f_{crit}}{L_f} = 1.944e^{-0.553(\frac{L_f}{D_m})} \tag{3-7}$$

This makes the g-function for deflection, g_Δ, different. For example, for curve A:

$$g_\Delta = (L_f * 2.48e^{-0.315(\frac{L_f}{D_m})}) - \frac{8F_o D_m^3 N_a}{GD_w^4} \tag{3-8}$$

which is obtained by substituting equation 3-5 for the critical ratio in equation 3-4. The input for the second deflection failure mode is shown on TABLE 3-6. In this table, the distribution for each design variable was taken as normal. Equations 3-4 and 3-8 define the two failure modes for a helical spring.

The spring will fail when it extends beyond its critical deflection. This can be seen in equation 3-9 when the probability of failure is determined from the deflection incident on the helical compression spring, defined by g_Δ.

When g_Δ is less than or equal to zero [5]

$$P_f = g_\Delta \leq 0 \tag{3-9}$$

The sensitivity analysis for the first trial of the deflection failure mode is depicted in FIGURE 3-4.

For the final results of the probabilistic analysis, the failure modes are run simultaneously through NESSUS. This gives consideration to the possibility that the spring may under go detrimental stress and buckle simultaneously. These occurrences are the consequences of shear stress and deflection. For the sake of analogy, the deterministic and probabilistic design results will be shown on consecutive pages as TABLES 3-3 and 3-4 respectively.

3.6 Explanation of Results

There are two failure modes for which a sensitivity analysis must be executed. The sensitivity analysis identifies the most critical design parameters and yields a probability of failure for the defined limit state function. From the sensitivity analysis of the stress limit state function, it can be interpreted that the wire diameter, D_w, is the most critical parameter as seen in FIGURE 3-2. The wire diameter has the greatest magnitude of sensitivity. The negative sign is discarded because it only represents the direction the parameter should be varied in order to decrease the probability of failure. Since there is a

TABLE 3-3: FINAL RESULTS OF HELICAL SPRING DESIGN FOR DETERMINISTIC METHOD

DP	MEAN
L_f	6 (in.)
D_m	0.75 (in.)
D_w	0.15 (in.)
Na	5

THE DESIGN HAS A FACTOR OF SAFETY OF 7
IT CAN DEFLECT 88.3% MORE BEFORE IT BUCKLES

TABLE 3-4: FINAL RESULTS OF HELICAL SPRING DESIGN FOR PROBABILISTIC METHOD

DP	MEAN
L_f	6 (in.)
* D_m	0.9 (in.)
* D_w	0.40 (in.)
Na	5

* MOST CRITICAL DESIGN VARIABLES
THIS DESIGN HAS A 3.34×10^{-4}% CHANCE OF FAILING

negative sign in front of the sensitivity for the wire diameter, the design parameter should be increase in order to lower the probability of failure. The force, F_o, is not considered a design parameter which can be varied by the designer because it is a given. The probability of failure is calculated at this point using NESSUS. For trial two, the dimension of the wire diameter is increased, therefore, decreasing the probability of failure. FIGURE 3-4 follows TABLE 3-3 and shows the sensitivity and the probability of failure which corresponds to data in the input TABLE 3-3, likewise FIGURE 3-5 follows TABLE 3-4. The sensitivity results for trials 3, 4 and 5 are shown in the APPENDIX as FIGURES A-1, A-2 and A-3 respectively.

From the first trial of the sensitivity analysis for deflection, the magnitudes of the sensitivity for each design parameter show that the mean diameter is the most critical parameter that affects deflection (see FIGURE 3-4). Since the mean diameter from trial 1 has a negative sign in front of its sensitivity factor, its mean value is increased in the second trial as seen in TABLE 3-4. The sensitivity factors and the probability of failure for the second trial is shown in FIGURE 3-4. The deflection sensitivity results for trials 3, 4, and 5 are shown in the APPENDIX as FIGURES A-4, A-5, and A-6.

TABLE 3-5: DEFLECTION FAILURE MODE DATA INPUT TABLE TRIAL 1

DP	MEAN	s	D-TYPE
L_f(in.)	6	1	NORMAL
F_o(lbs)	20	5	NORMAL
D_m(in.)	0.75	0.075	NORMAL
D_w(in.)	0.15	0.015	NORMAL
G(psi)	11.2×10^6	64.00×10^3	NORMAL
Na	5	1	NORMAL

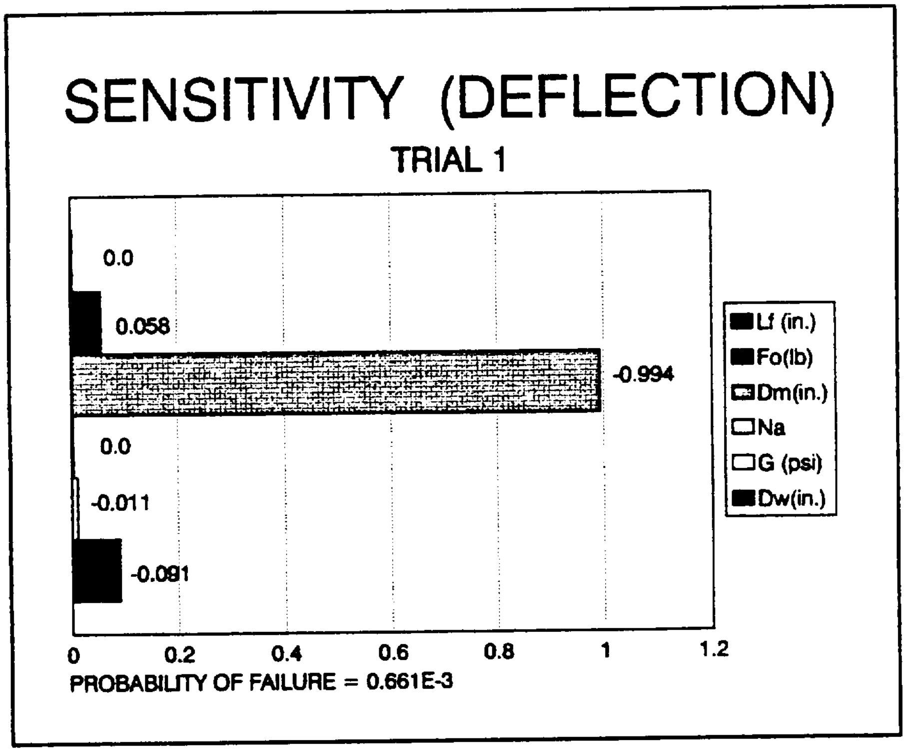

FIGURE 3-4: DEFLECTION SENSITIVITY ANALYSIS TRIAL 1

TABLES 3-6: DEFLECTION FAILURE MODE DATA INPUT TABLE TRIAL 2

DP	MEAN	s	D-TYPE
L_f (in.)	6	1	NORMAL
F_o (lbs)	20	5	NORMAL
D_m (in.)	0.8	0.080	NORMAL
D_w (in.)	0.15	0.015	NORMAL
G (psi)	11.2×10^6	64.00×10^3	NORMAL
Na	5	1	NORMAL

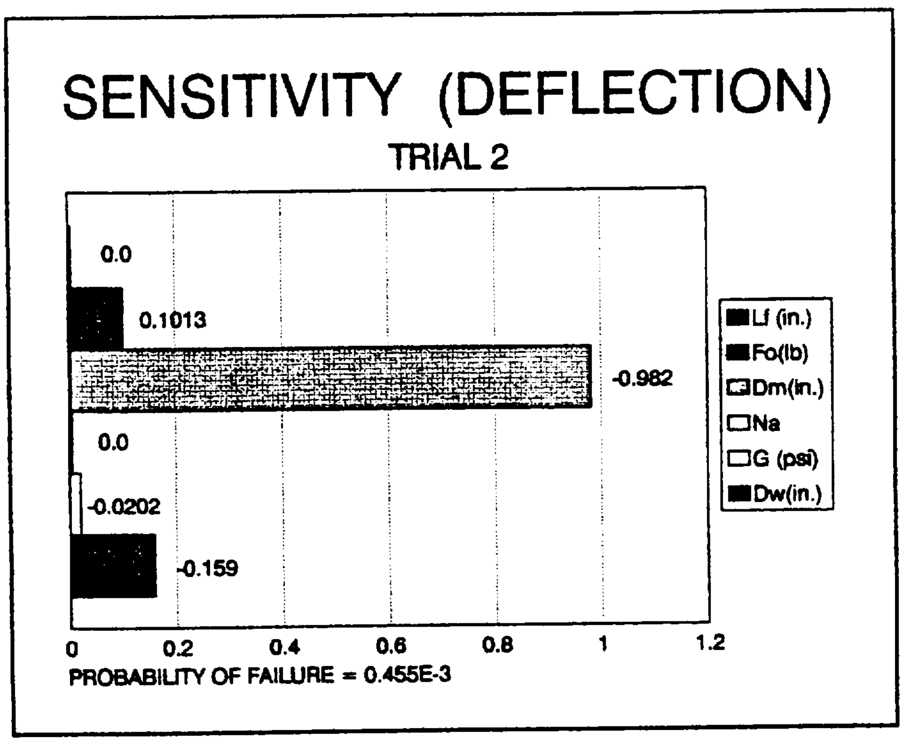

FIGURE 3-5: DEFLECTION SENSITIVITY ANALYSIS TRIAL 2

TABLE 3-7 shows the dimensions of the design parameters at a probability of failure of 3.34×10^{-4}. This probability of failure occurred when the failure modes were run simultaneously. FIGURE 3-6 shows that the wire diameter and the mean diameter are clearly the most sensitive and therefore most critical design parameters which can be varied. A relationship between the spring weight and the reliabilty can be determined. The weight, W, was computed as

$$W = \gamma V \tag{3-10}$$

Where,

γ = specific weight of vanadium-chromium (lb/ft^3)
V = volume of spring (ft^3) [6]

The weight is plotted versus the probability of failure and shown in FIGURE 3-7. The graph shows that as the weight increases, the probability of failure decreases.

FIGURE 3-8 shows the coefficient of variation (COV) plotted against the probability of failure. COV relates the standard deviation to the mean of the design parameter by the equation [2]

$$COV = \frac{\delta}{\mu} \tag{3-11}$$

Where,

δ = standard deviation
μ = mean value of design parameter

This plot corresponds with the wire diameter at a mean value of 0.15 in. The standard deviation for the plot is varied, while the mean value is kept constant to generate the plot.

The reliability and the total cost to achieve the reliability of the spring can be computed from [4]

$$C_t = C_i + (P_f \times C_M) \tag{3-12}$$

Where,

C_t = total cost
C_i = initial cost
C_M = maintenance cost

This equation implies that the probability of failure is used to determine the chances that the spring will incur 100% of the maintenance cost on the spring. This weighs the maintenance cost by the reliability.

TABLE 3-7: NESSUS DATA INPUT TABLE FOR SHEAR STRESS AND DEFLECTION FAILURE MODES OCCURRING SIMULTANEOUSLY

DP	MEAN	s	D-TYPE
D_m (in.)	0.9	0.09	NORMAL
D_w (in.)	0.4	0.04	NORMAL
L_f (in.)	6	0.06	NORMAL
F_o (lbs.)	20	5	NORMAL
G (psi)	11.2×10^6	2.8×10^5	NORMAL
Na	5	1	NORMAL
S (psi)	130×10^3	3250	NORMAL

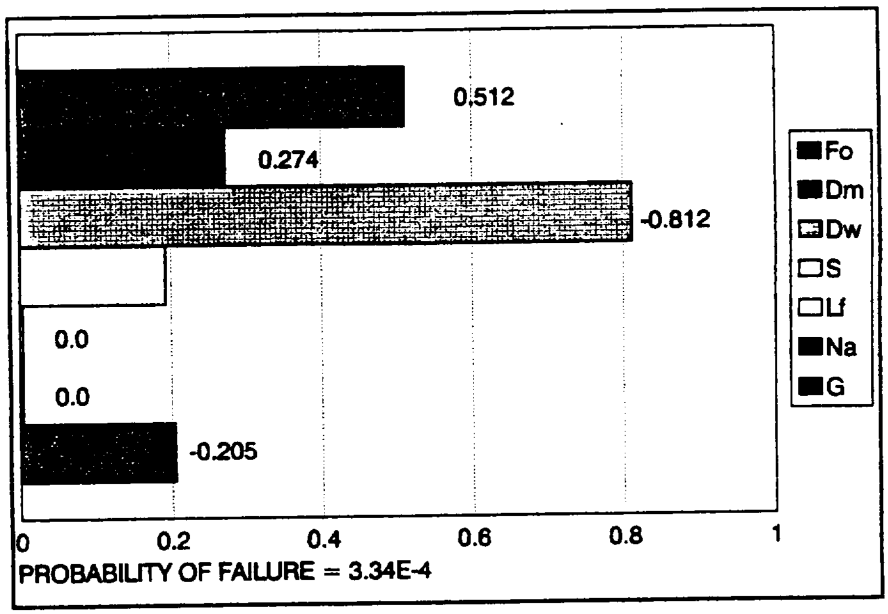

FIGURE 3-6: SENSITIVITY ANALYSIS FOR SHEAR STRESS AND DEFLECTION OCCURRING SIMULTANEOUSLY

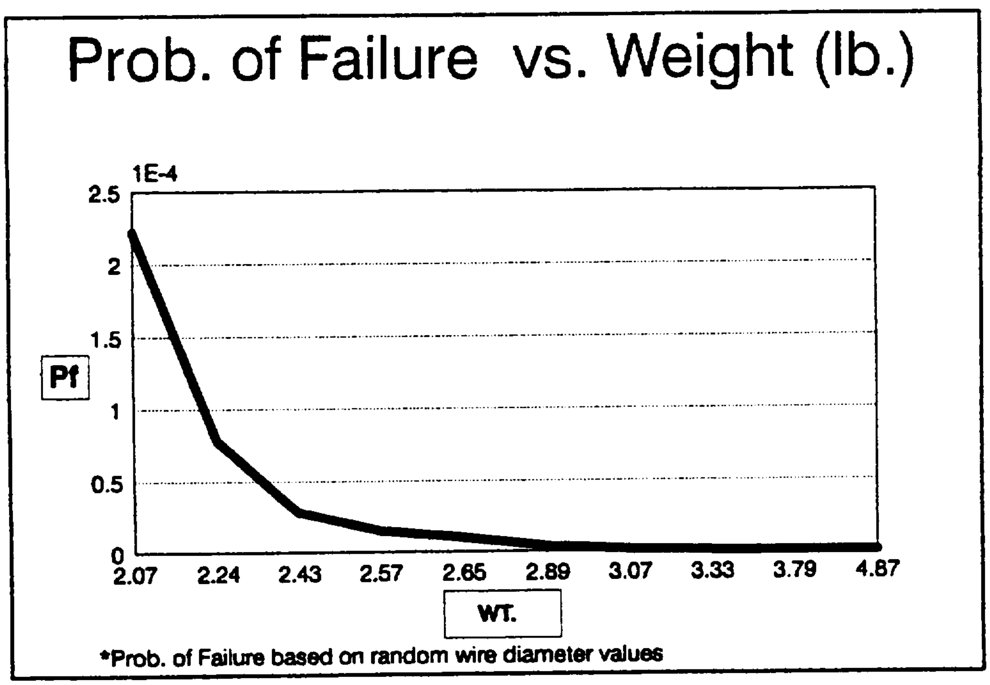

FIGURE 3-7: RELATIONSHIP BEWTEEN WEIGHT AND PROBABILITY OF FAILURE

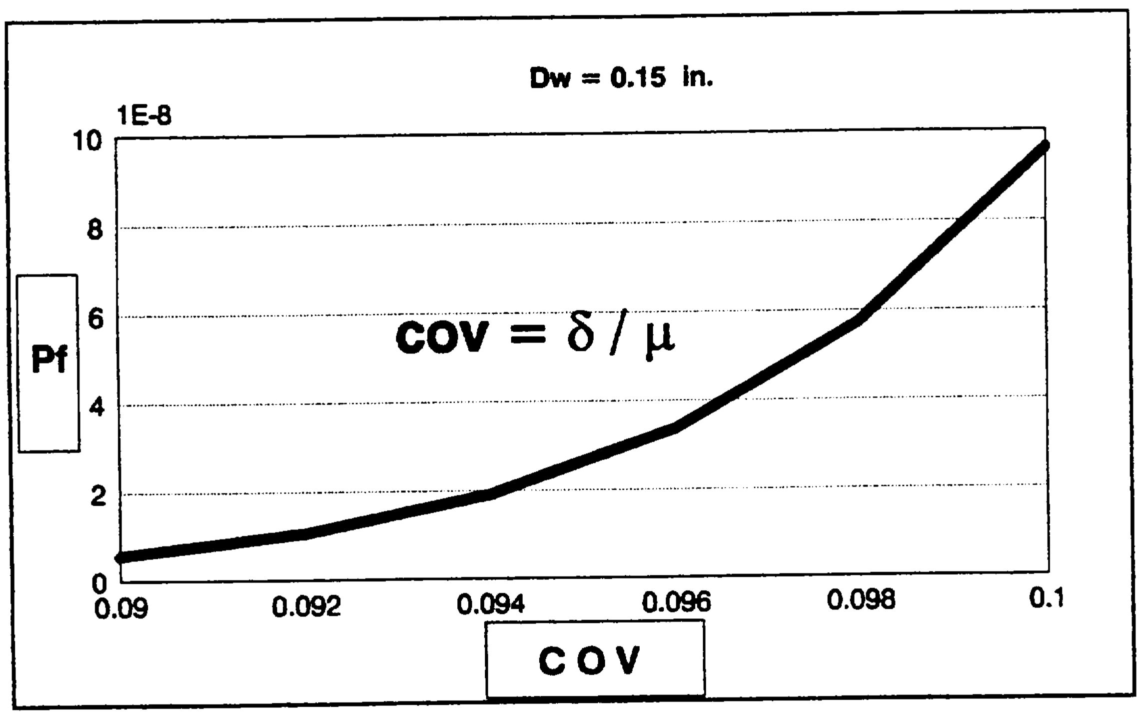

FIGURE 3-8: RELATIONSHIP BETWEEN COV AND PROBABILITY OF FAILURE

CHAPTER IV

CONCLUSION

Probabilistic Design Methodology is a valuable method in evaluating the reliability of structures and machine elements. This method was utilized to evaluate the reliability of a helical compression spring. A probabilistic analysis is performed on the spring using NESSUS, to model the spring's failure modes.

The methodology is effective in determining the reliability of irregularly shaped machine elements such as helical springs. Consideration for variations in the design parameters during the performance of the spring are taken into account. Variables like the spring modulus from the deflection failure mode, as seen in equation 3-5, allows for variability in the types of materials used. This means that a material, more conducive to increasing the reliability of the spring, can be selected. Deterministic methods do not take into account material selection because it does not consider the statistical nature of the spring modulus. No design parameters are taken to be variables in deterministic methods. Fluctuations of the design load are also taken into account, even though it is a specified condition of the system. For example, an elevator design has a maximum capacity limit, but the limit will be exceeded sometimes. Provisions must be made for these overloading conditions. However, designing to the worst possible condition can be very costly and unnecessary. The deterministic method assumes the worst case design, using the most unfavorable combination of design parameters to develop a safe and conservative design. The probabilistic design method quantifies the uncertainties in the design. This helps to eliminate the use of excessive material in order to achieve a safe design. Different characteristics of the spring such as weight can be determined as a function of reliability thus, developing a favorable probability of failure while satisfying various weight constraints. Other design constraints such as size can be taken into account as a function of reliability. It is also possible to develop a durable design by considering the reliability as a function of the total cost to achieve this reliability. Use of the worst case scenario by deterministic methods automatically discounts realistic economic considerations. This is because there is a very large margin of uncertainty of whether the design will fail or not.

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BIOGRAPHICAL SKETCH

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OBJECTIVE: I seek employment at a "cutting edge" company in the area of research and development, which offers realistic opportunities for employees to pursue higher education in engineering.

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WORK EXPERIENCE:

'94 - Present Research Assistant, conducted research on " The Design of Helical Springs using Probabilistic Design Methodology". Used NESSUS computer code for this study. **Tennessee State University.**

Summer '94 Summer Intern, designed GUI software (in C) to perform engineering economics and wrote technical paper on the subject. **NASA Lewis Research Center.**

Summer '93 Summer Intern, designed program to compute laser parameters (in FORTRAN), aided in writing technical paper for the research engineer. **NASA Lewis Research Center.**

Summer '93 Summer Intern, designed *laser to test cell* integration program (in QBASIC), presented work in NASA research symposium. **NASA Lewis Research Center.**

ACTIVITIES/ POSITIONS: '93-'94 Regional Rep. for NESCC; '93 NSBE, Publications Chair: *Carpe Diem Motivational Speaking*, President; T.A. (Physics, Circuits, Thermodynamics, Transport Phenomena, FORTRAN); ASME; ASHRAE; Honda All Star Academic Challenge Team, Captain; Intramural basketball and football, Captain; NAACP, Vice President ('93-'94)

AWARDS/ HONORS: Golden Key National Honor Society, Pi Tau Sigma Mechanical Engineering Honor Society, National Dean's List, NASA Scholar, 1st Place TSU Oratorical Contest, GEM Fellowship Award, Knights of Pythias Scholar, Who's Who Among American Colleges and Universities, Selected to present at National Undergraduate Research Conference in New York (Spring '95), keynote speaker for NASA SHARP Program, keynote speaker for Tri-C pre-college programs

REFERENCES: Dr. Decatur Rogers, Dean of Engineering, Tennessee State University
Mr. Thomas Benson, Senior Research Engineer, NASA Lewis Research Center

A P P E N D I X

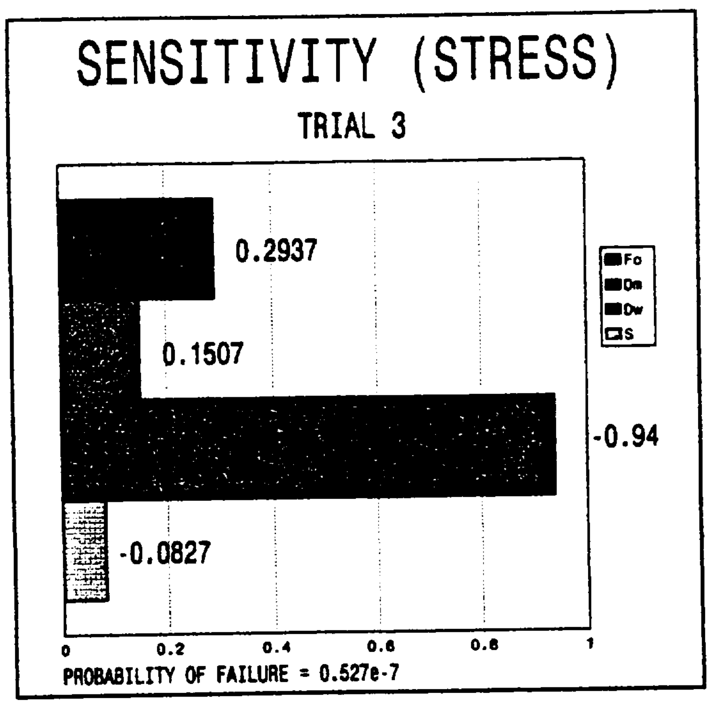

FIGURE A-1: TORSIONAL SHEAR STRESS SENSITIVITY ANALYSIS TRIAL 3

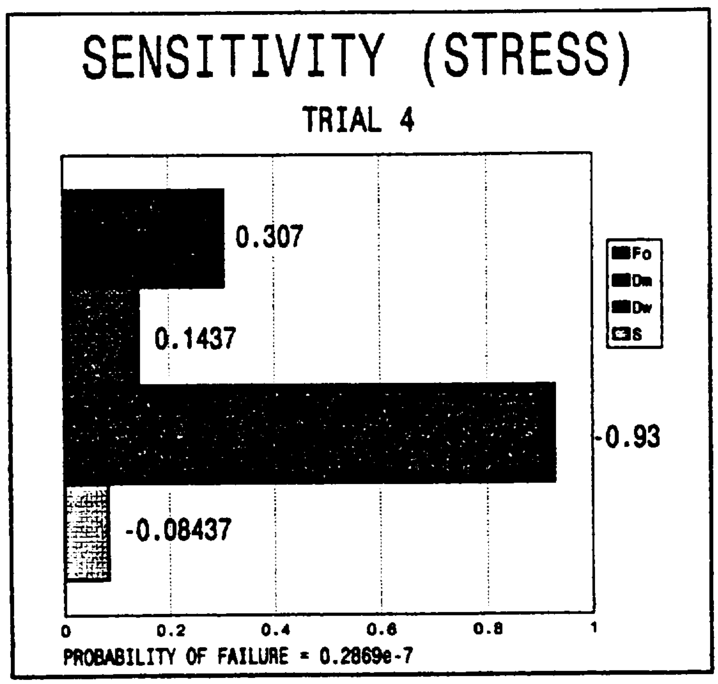

FIGURE A-2: TORSIONAL SHEAR STRESS SENSITIVITY ANALYSIS TRIAL 4

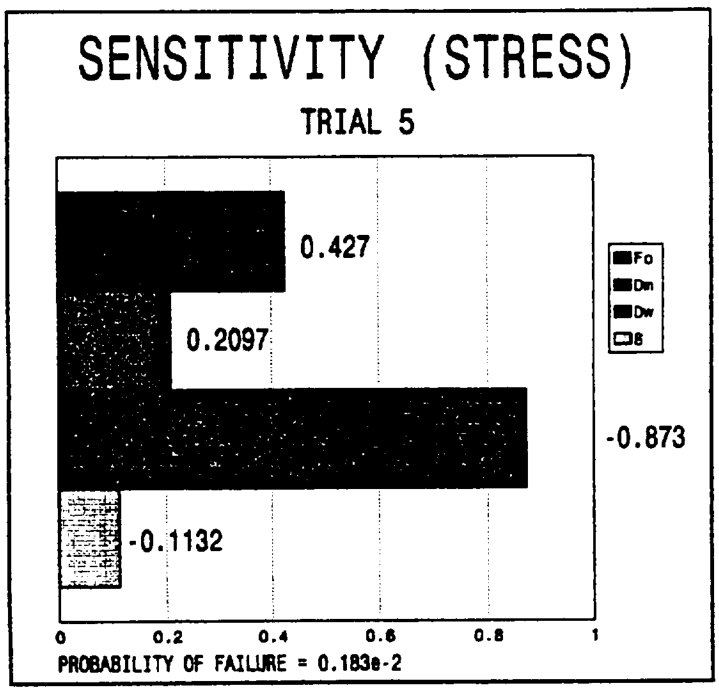

FIGURE A-3: TORSIONAL SHEAR STRESS SENSITIVITY ANALYSIS TRIAL 5

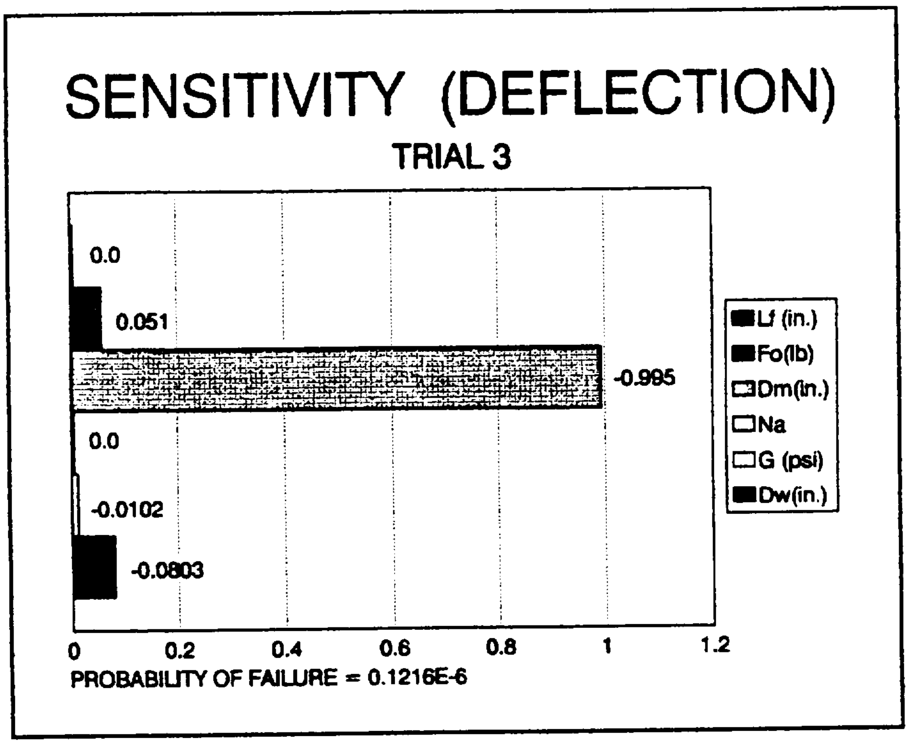

FIGURE A-4: DEFLECTION SENSITIVITY ANALYSIS TRIAL 3

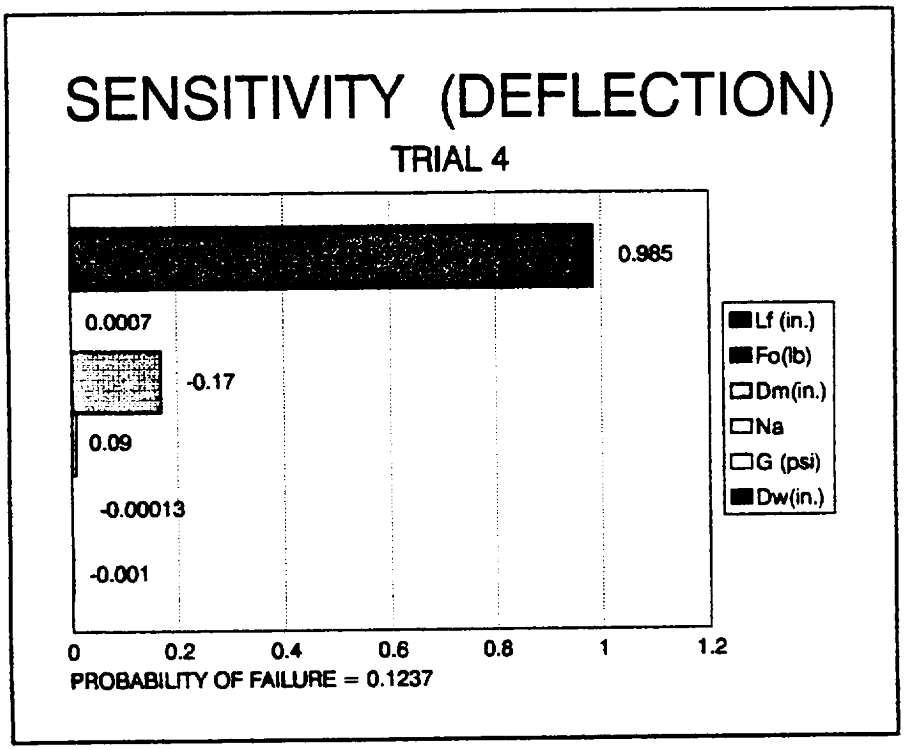

FIGURE A-5: DEFLECTION SENSITIVITY ANALYSIS TRIAL 4

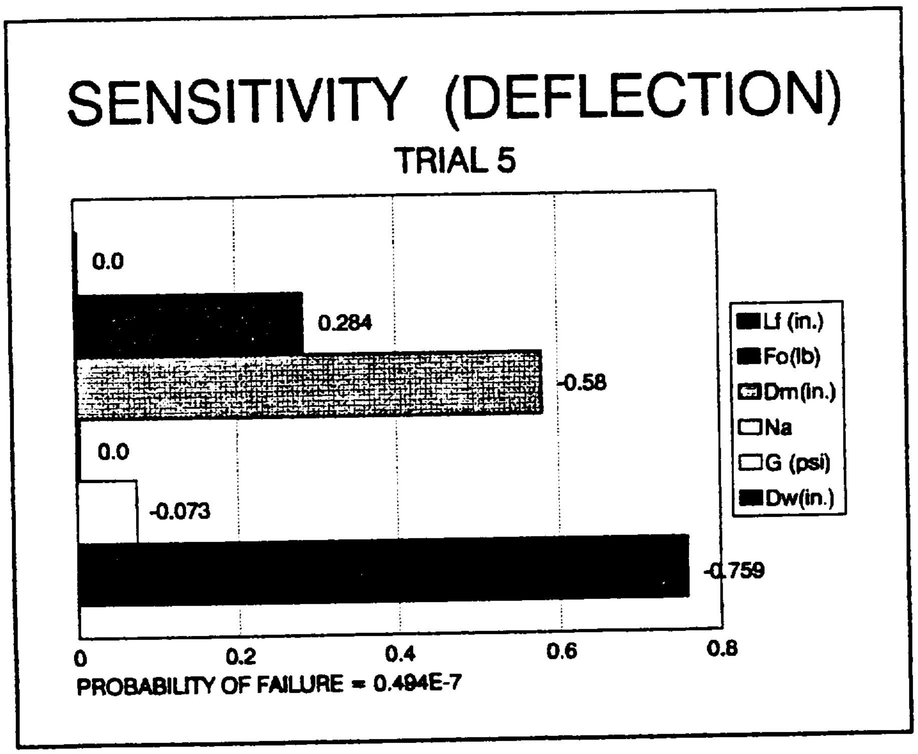

FIGURE A-6: DEFLECTION SENSITIVITY ANALYSIS TRIAL 5

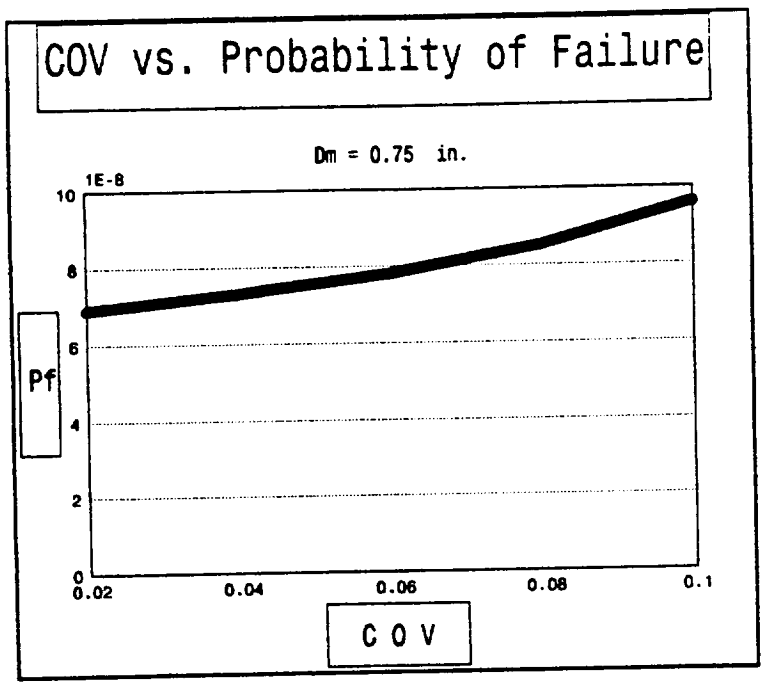

FIGURE A-7: RELATIONSHIP BETWEEN COV AND PROBABILITY OF FAILURE