TOOLS FOR NONLINEAR CONTROL SYSTEMS DESIGN

FINAL REPORT ON NAG2-243

by

S. S. Sastry

Memorandum No. UCB/ERL M97/83

23 November 1997

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S. S. Sastry

November 23, 1997

1 Introduction

This is a brief statement of the research progress made on Grant NAG2-243 titled "Tools for Nonlinear Control Systems Design", which ran from 1983 till December 1996. The initial set of PIs on the grant were C. A. Desoer, E. L. Polak and myself (for 1983). From 1984 till 1991 Desoer and I were the PIs and finally I was the sole PI from 1991 till the end of 1996.

The project has been an unusually longstanding and extremely fruitful partnership, with many technical exchanges, visits, workshops and new avenues of investigation begun on this grant. There were student visits, long term visitors on the grant and many interesting joint projects.

In this final report I will only give a cursory description of the technical work done on the grant, since there was a tradition of annual progress reports and a proposal for the succeeding year. These progress reports cum proposals are attached as Appendix A to this report. Appendix B consists of papers by me and my students as co-authors sorted chronologically. When there are multiple related versions of a paper, such as a conference version and journal version they are listed together. Appendix C consists of papers by Desoer and his students as well as "solo" publications by other researchers supported on this grant similarly chronologically sorted.

2 Synopsis of Research Topics Covered

The grant had very much of the character of a living object and it changed emphasis and focus over the years. The following were the main areas covered.

2.1 Adaptive Control

In the early years of the grant, a great deal of research on new algorithms for adaptive control, stability robustness, parameter convergence were covered. An extended research monograph based on the output of this research was published by Prentice Hall in 1989, entitled "Adaptive Systems: Stability, Convergence and Robustness" in addition to several papers on all aspects of adaptive control.

2.2 Nonlinear Control I/O Methods

Desoer worked on a number of topics involving the parameterization of all stabilizing nonlinear controllers, the use of dither, passivity in stabilization and nonlinear controller design methods from an Input Output point of view.

2.3 Geometric Nonlinear Control

Motivated by flight control applications for a VTOL Harrier at Ames, we began an investigation of "slightly non-minimum phase" nonlinear systems jointly with Dr. George Meyer. This work has actually continued till recently with recent collaboration between Dr. Meyer, and Prof. Hunt. We also studied nonlinear control of singular systems (with Petar Kokotovic of the University of California, Santa Barbara) (non-regular), adaptive control of nonlinear single input single output and multi input multi output systems (respectively with Alberto Isidori and Marika Di Benedetto from the University of Rome). Andy Teel and Raja Kadiyala worked on CAD tools for nonlinear control systems design. Raja Kadiyala developed a CAD toolbox called APLIN, which was to have been commercialized by Integrated Systems Inc.

2.4 Geometric Control of Robots and Multifingered Hands

Based on the geometric nonlinear control we developed adaptive and nonlinear control schemes for single and cooperative robots (multi-fingered hands). Partly based on this work, a text book entitled "A Mathematical Introduction to Robotic Manipulation" by R. Murray, Z. Li and S. Sastry was published by CRC Press in 1993.

2.5 Flight Management Systems and Air Traffic Control

Motivated by some work on nonholonomy in robots, we began a project with Richard Montgomery at the University of California, Santa Cruz in the area of optimal path planning for aircraft: a so-called "landing tower" problem. In response to this problem we developed tools for optimal control problems on Lie groups. Soon thereafter we realized that there were other deeper research issues in path planning, conflict resolution and hybrid control for aircraft inside TRACONS and we have begun a program of joint work with the NASA AATT program (in general) and the CTAS program in particular at Ames for path planning for aircraft. This work continues in the follow up grant NAG 2-1039 with Claire Tomlin, who spends a day a week at Ames and also represents the connections with Honeywell, Inc.

2.6 Hybrid Control

Motivated by hierarchical control of complex multi-agent systems (such as those in air traffic control) or multi-mode control systems (such as those encountered in flight control), we have begun a study of fundamental issues in the design of switched control systems and distributed decision making. This work continues in the follow up grant NAG 2-1039 entitled, "Design and Verification Tools for Hybrid Control Systems in Flight Vehicle Management". Claire Tomlin is the person primarily supported on this grant. Her role is to forge links between the basic hybrid systems research developed on the project and the Flight Systems Group of Dr. Meyer, with the ATM research under the leadership of Dr. Erzberger and Shridhar at NASA Ames.

3 Workshops Organized During Period of Grant

3.1 1989 NASA Ames Berkeley Workshop in Nonlinear Flight Control

This workshop held in August 1989 featured three days of presentations in areas of nonlinear and flight control. It was attended by about 80-100 people and is often cited as one of the premier workshops in nonlinear flight control.

3.2 1990 Second UC-NASA Workshop on Nonlinear Control

This workshop held at Santa Barbara in April 1990 just eight months after the 1989 workshop listed above, was a big success with about 80 speakers, a tributed to the vitality of the field.

3.3 1993 ARO NASA Workshop on "Formal Models for Intelligent Control"

This workshop co-sponsored by the Army Research Office was held at MIT at the beginning of October 1993. The total number of participants was over 150 and began an era of excitement about hybrid and intelligent control. The speakers covered hybrid systems, switched systems, neural and fuzzy control, flight control, discrete event systems, and hierarchical control.

3.4 1995 Bozeman Workshop on "Exterior Differential Systems and Hybrid Control"

This workshop also cosponsored by the Army Research Office was held at Bozeman (under the co-direction of Clyde Martin). Topics in exterior differential systems in nonlinear control and the new project on Air Traffic Management begun with NASA Ames were featured topics. It was attended by over fifty participants.

4 Personnel

4.1 Researchers supported on the grant

In addition to the PIs, and the students whose names are given in the next subsection, Profs. Michael Heymann, Alberto Isidori, Marica Di Benedetto, and Petar Kokotovic were partially supported for their work on the grant when they visited Berkeley and NASA Ames.

4.2 Completed Doctoral Dissertations Earned by Students Supported by the grant

The following doctoral students were either wholly or partialy supported by the grant. I have included their current whereabouts (if known) and date of completion. Some other students who finished with MS only were also supported on this grant: Joe Sifer(Desoer) and Shobana Venkataraman (Sastry).

1. Ramon Silva Madriz, graduated 1983,

2. Ching An Lin, graduated Dec. 1983, National Jiao Tung University, Taiwan (advised by Desoer)

3. Marc Bodson, graduated 1986, University of Utah, UT

4. Shahram Shahruz, graduated 1986, University of California, Berkeley, CA (advised by Desoer).

5. Li-Chen Fu, graduated 1987, National Taiwan University, Taiwan

6. Amit Bhaya, graduated 1987, Federal Institute of Technology, Rio de Janeiro, Brazil (advised by Desoer).

7. Er-Wai Bai, graduated 1987, University of Iowa, Iowa City, IA

8. Jeff Mason, graduated 1988, IBM Rochester, MN

9. John Hauser, graduated 1989, University of Colorado, CO

10. Richard Murray, graduated 1991, Caltech, Pasadena, CA

11. Guntekin Kabuli, graduated 1991, Univ of California, Davis, CA (advised by Desoer).
12. Andrew Teel, graduated 1992, University of California, Santa Barbara, CA
13. Raja Kadiyala, graduated 1992, Yasukawa Motors, San Francisco, CA.
14. A. K. Pradeep, graduated 1992, GE CR & D, Schenectady, NY.
15. Greg Walsh, graduated 1994, University of Maryland, MD.
16. Neil Getz, graduated 1995, private consultant, CA (co-advised with Marsden).
17. Datta Godbole, graduated 1994, UC Berkeley PATH program.
18. John Lygeros, graduated 1996, LCS, MIT, MA.

Of these students several spent substantial amounts of time at NASA Ames with Dr. George Meyer as student and post doctoral interns:

1. John Hauser
2. Joe Sifer (MS degree only, now at Aerospace Corp.)
3. Andrew Teel
4. Raja Kadiyala
5. Greg Walsh

5 Acknowledgement

The grant was a very valuable long range research tool. The consistent support of basic research by our program monitor, Dr. George Meyer was a big factor in being able to build a valuable community of students, researchers and scholars from all over the world, who have come to regard the collaborative arrangements between Berkeley and NASA Ames as a fantastic example of how basic research can be effectively transacted. I sincerely hope that, over time, the research supported on this project will be the best and most substantial testimonial to it.

Appendix A:
Annual Progress Reports and Proposed Research 1983-1996

ANNUAL REPORT 83-84 AND
RESEARCH PLANS 84-85
TO NASA-AMES RESEARCH CENTER

S. S. Sastry, C. A. Desoer, and E. Polak

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Two faculty members, Profs. S. S. Sastry and C. A. Desoer, and five research assistants were partially supported by the grant in this year:

1. Research Activities of S. S. Sastry and his students:

I. Adaptive control - we (Sastry and Marc Bodson) studied parameter convergence and robustness aspects of model reference adaptive control systems. We gave necessary and sufficient conditions for parameter convergence [1,2] and gave results showing the robustness of adaptive schemes when the reference signal is persistently exciting [3,4]. The following publications in this area were sponsored by this grant.

[1] S. Boyd and S. S. Sastry, "Parameter Convergence in Adaptive Control," Proceedings of the American Control Conference, San Diego, June 84.

[2] S. Boyd and S. S. Sastry, "Necessary and Sufficient Conditions for Parameter Convergence in Adaptive Control," Proceedings of the Ames-Berkeley Symposium on problems of nonlinear control a-d fluid mechanics.

ABSTRACT. Using *Generalized Harmonic Analysis*, we give a complete description of parameter convergence in Model Reference Adaptive Control (MRAC) in terms of the *spectrum* of the exogenous reference input signal. Roughly speaking, if the reference signal "contains enough frequencies" then the parameter vector converges to its correct value. If not, it converges to an easily characterizable subspace in parameter space.

[3] M. Bodson and S. S. Sastry, "Exponential Convergence and Robustness Margins in Adaptive Control," Proceedings of the Conference on Decision and Control, Las Vegas, December 84.

[4] M. Bodson and S. S. Sastry, "Small-Signal I/O Stability of Nonlinear Control Systems: Application to the Robustness of MRAC Schemes," ERL Memo No. M84/70, September 1984 - submitted to Automatica.

ABSTRACT. This paper presents general results relating the internal exponential stability of nonlinear time varying systems to their external input/output stability. Provided that the undriven system is exponentially stable, we give explicit bounds on the size of the input under which the driven system is stable. Moreover, the deviation from equilibrium of the driven system is at most proportional to the (L_∞) size of the input, and the L_∞ gain depends inversely on the rate of exponential convergence. These results are used to study the robustness properties of a model reference adaptive control scheme to various kinds of disturbances, and unmodelled dynamics. In most adaptive algorithms, the exponential convergence follows from a persistent excitation condition, so that this condition appears central to the robustness problem in adaptive control. The paper concludes with some remarks on the interpretation of these results for practical applications.

II. <u>Multiple Time Scale Systems</u> - We (Sastry and Ramon Silva) studied multiple time scales in input-output descriptions of linear and nonlinear systems arising from the dependence of their dynamics on a small parameter ε. The following publications [5,6,7] have appeared in this area so far, two more based on the Ph.D. dissertation [7] of Ramon Silva are in preparation.

[5] R. Silva-Madriz and S. S. Sastry, "Input-Output Description of Linear Systems with Multiple Time Scales," to appear in the International Journal of Control, October 84.

ABSTRACT. We study the multiple time scales structure of linear systems of the form

$$\dot{x} = A_0(\varepsilon)x + B_0(\varepsilon)u$$
$$y = C_0(\varepsilon)x$$

with a view to obtaining "approximate" lower order transfer functions valid at different time scales. Our development includes the classical two time scales as well. We use our results to study the positive realness of linear systems with multiple time scales in terms of the positive realness of the reduced order transfer functions.

[6] R. Silva-Madriz and S. S. Sastry, "Time Domain Models for Multiple Time Sclae Linear Systems," to appear in the Proceedings of the Conference on Decision and Control, Las Vegas, December 84.

[7] R. Silva-Madriz, "Reduced-Order Modelling in Linear and Nonlinear Multiple Time Scale Systems," Ph.D. dissertation, U.C. Berkeley, October 84.

III. State Space Solutions of Linear Matrix Equations Over Rings.

An important aspect of computer order linear control systems design is solving equations of the form

$$M_1 \, X \, M_2 = M_3$$

where M_1, M_2, M_3 are given matrices on rings of proper, stable or strictly proper transfer functions, for X proper, stable or both. As would be expected solutions in transfer function form are extremely numerically sensitive. We have developed techniques for solving these equations in the state space and are in the process of developing software to implement it. The reports (with Zexiang and Andy Packard) will appear in November.

2. Research Activities of C. A. Desoer and his students

From November 1, 1983 to May 15, 1984, Mr. C. A. Lin was an RA working with Professor C. A. Desoer on the NASA contract. During that period two research projects were brought to completion

a) A Comparative study of linear and nonlinear MIMO feedback configurations, by C. A. Desoer and C. A. Lin.

This paper has been presented at the American Control Conference, June 1984 and has been accepted for publication by the International Journal of Systems.

ABSTRACT. In this paper, we compare several feedback configurations which have appeared in the literature (e.g. unity-feedback, model-reference, etc.). We first consider the linear time-invariant multi-input multi-output case. For each configuration, we specify the stability conditions, the set of all achievable I/O maps and the set of all achievable disturbance-to-output maps, and study the effect of various subsystem perturbations on the system performance. In terms of these considerations, we demonstrate that one of the configurations considered is better than all the others. The results are then extended to the nonlinear multi-input multi-output case.

One of the interesting conclusions that the configuration used NASA helicopter project has the best features.

b) Nonlinear unity-feedback systems and Q-parametrization (improved version), by C. A. Desoer and C. A. Lin.

This is an improved version of a paper of similar title which will appear in the International Journal of Control. The improved version has been presented at the International Symposium on Systems and Optimization, Nice, June 1984 and published in the proceedings. The improvements consist of streamlined proofs and in more powerful statements of the theorem. We will submit this paper to the American Control Conference in June 1985.

ABSTRACT. This paper concerns nonlinear systems, defines a new concept of stability and extends to nonlinear unity-feedback systems the technique of Q-parametrization introduced by Zames and developed by Desoer, Chen and Gustafson. We specify 1) a global parametrization of all controllers that -stabilize a given -stable plant; 2) a parametrization of a class of controllers that stabilize an unstable plant; 3) necessary and sufficient conditions for a nonlinear controller to simultaneously stabilize two nonlinear plants.

In the first half of 1984, with C. A. Lin, we established an asymptotic result based on singular perturbations: namely, for a large class of nonlinear plants, a PID controller can always be used to achieve stability and to achieve the common asymptotic tracking and disturbance rejection property.

ABSTRACT. We study tracking and disturbance rejection of a class of MIMO nonlinear systems with linear proportional plus integral (PI) compensator. Roughly speaking, we show that if the given nonlinear plant is exponentially stable and has a strictly increasing dc steady-state I/O map, then a simple PI compensator can be used to yield a stable unity-feedback closed-loop system which asymptotically tracks reference inputs that tend to constant vectors and asymptotically rejects disturbances that tend to constant vectors.

Dr. C. A. Lin has graduated and is now working for Integrated Systems, Inc., in Palo Alto. Mr. S. Mohajadd-Shahruz is holding the RAship.

3. Research Plans for the Year 84-85

As suggested in the original proposal our work this year will follow the following three parallel streams:

(1) Practical Adaptive Control

We have within our grasp new techniques for studying the transient behavior of adaptive algorithms. Our results in robustness enhancement using persistent excitation will be simulated and further studied - along with new estimates of robustness margins. We are also in a position to propose some new adaptive schemes and also to study some MIMO adaptive schemes.

(2) State Space Solutions of Equations on Matrix Rings.

As part of the project to enhance our computer aided control systems design package, we will develop the theory and attendant numerical sensitivity issues to solve linear matrix equations over rings of (proper) stable transfer functions in state space form.

(3) Nonlinear Compensator Design Techniques using Dither.

We should be able to give a completely rigorous theory of the use of dither when there is a nonlinearity with hysteresis. In view of the complete lack of rigorous theory in this domain, this possibility is very tempting. So far the problem seems very tough: We are trying to appropriately adapt the methods of Zames and Schneydor (IEEE Trans. on Automatic Control, vol. AC-21, p. 660, 1976) to the hysteresis case. We note that Mossaheb (Int. J. of Control, vol. 38, no. 3, p. 557, 1983) did not do the job: his conclusions are only valid for a compact time interval.

Annual Report 1984 - 1985 and
Research Plans 1985 - 1986
to NASA-AMES RESEARCH CENTER
Grant Monitor: Dr. George Meyer

S.S. Sastry and C.A. Desoer

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Two faculty members, Professor S.S. Sastry and C.A. Desoer and four research assistants were partially supported by the grant in this year (M. Bodson, L-C. Fu, S. Shahran-Shahruz and A. Bhaya).

1. Research Activities of S.S. Sastry and his students

I. Adaptive control

We (Sastry, Marc Boson and Li-Chen Fu) studied the use of averaging methods in the analysis of adaptive systems. We derived new theorems on averaging and obtained convergence rate estimates of some adaptive systems. These are valuable tools for the determination of robustness margins and input signal design for adaptive systems. The following publications in this area were sponsored by this grant:

[1] L-C. Fu, M. Bodson, and S.S. Sastry, "New Stability Theorems for Averaging and Their Application to the Convergence Analysis of Adaptive Identification and Control Schemes; ERL Memo No. 85/21, March 1985. Also, Proceedings of the 24th IEEE Conference on Decision and Control, Fort Lauderdale, Florida, December 1985.

During the months of July - August 1985, S. Sastry and M. Bodson spent four weeks at the Australian National University, Canberra collaborating with Professors B.D.O. Anderson, and R.R. Bitmead analyzing the success and failure of the MIT rule for adaptation using averaging, as well as extensions to the results of [1]. The following paper is to appear based on this work and others will follow:

[2] S. Sastry, I. Mareels, B. Anderson, R. Bitmead and M. Bodson, "An averaging analysis of the MIT Rule for Adaptive Control," to be submitted to the IFAC workshop on Adaptive Identification and Control, July 1986, Lund, Sweden.

II. State Space Solutions of Linear Matrix Equations over various rings

An important aspect of computer aided linear control systems design is the solution of equations of the form

$$M_1 \times M_2 = M_3$$

where M_1, M_2, M_3 are given matrices on rings of proper stable or proper stable rational functions. We have developed techniques and software to implement solutions of these equations in the *state space*. The following publications in this area have been supported by this grant.

[3] A. Packard and S.S. Sastry, "Solution of Rational Matrix Equations in the State Space with Applications to Computer Aided Control Systems Design," presented at the 2nd IEEE Conference on Computer Aided Design, Santa Barbara, March 1985, to appear in *International Journal of Control*, 1986.

[4] Z. Li and S.S. Sastry "Conditioned Invariant Subspaces, Disturbance Decoupling and Solution of Rational Matrix Equations, ERL Memo No. 85/20, March 1985, to appear in *International Journal of Control*, 1986.

2. Research Activities of C.A. Desoer and his students

I. Stabilization by Dither

S.M Shahruz was supported for a while by this grant and later transferred to the JSEP program. The result is a paper accepted by the *International Journal of Control* whose title and abstract follow:

Stability of Dithered Nonlinear

Systems with Backlash or Hysteresis

We study the effect of dither on the nonlinear element of a single-input single-output feedback system. We consider nonlinearities with memory (backlash, hysteresis), in the feedforward loop; a dither of a given amplitude is injected at the input of the nonlinearity. The nonlinearity is followed by a linear element with low pass characteristic. The stability of the dithered system and an approximate equivalent system (in which the nonlinearity is a smooth function) are compared. Conditions on the input and on the dither frequency are obtained so that the approximate system stability guarantees that of the given hysteretic system.

II. Robust Linear Controller Design

Mr. A. Bhaya was supported from January 1, 1985 to the present. With Mr. Bhaya we investigated two basic robustness problems:

We propose a robust design method for a control structure which is perturbed by unmodeled dynamics. The unmodeled dynamics have the special feature that they are very lightly damped. As a specific example of such a control structure we consider a large flexible space structure (LFSS). For this example, when sensors (with suitable gains) and actuators are colocated, the plant is passive. Thus a strictly passive compensator results in an exponentially stable feedback system. An important robustness property of such compensators is that for any possible passive unmodeled dynamics, the resulting perturbed system is stable.

More generally, this design method applies to all MIMO linear time-invariant plants with passive transfer functions.

In the general case (for the LFSS example this means no colocation of actuators and sensors) we state necessary and sufficient conditions for the exponential stability of the control system. By means of a first-order perturbation calculation we find a condition that the Q-parameter must satisfy that ensures that feedback does not destabilize the unmodeled dynamics. The following publication is based on this research.

[1] A. Bhaya and C.A. Desoer, "On the design of Large Flexible Space Structures," to appear in *IEEE Transactions on Automatic Control*, also memorandum no UCB/ERL M84/106 of the Electronics Research Laboratory, U.C. Berkeley; to be presented at *24th IIIE Conference on Decision and Control*, December 1985, Florida.

Hitherto, results available on robust stability of feedback systems required one or the other restriction on the nature of the plant or the additive perturbation. For example, the extremely useful result of Doyle requires that the perturbation be normbounded on the $j\omega$-axis so that the perturbation may not have poles on the $j\omega$-axis and that the nominal plant and the perturbed plant have the same number of unstable poles.

We found a simple algebraic proof of a necessary and sufficient condition for stability of the unity-feedback system where the plant and the additive perturbation have independent inputs. This independent excitation of plant and perturbation occurs in many physical systems where the perturbation represents unmodeled dynamics rather than plant parameter changes. Another feature of our result is that it holds even if plant, compensator and perturbation are unstable.

[2] A. Bhaya and C.A. Desoer, "Robust Stability under Additive Perturbations," to appear in *IEEE Transactions on Automatic Control*, also Memorandum no UCB/ERL M84/110 of the Electronics Research Laboratory, U.C. Berkeley.

3. Research Plans for the Year 1985 - 86

Two students (M. Bodson and A. Bhaya) supported by the grant are expected to finish their Ph.D. dissertations this year respectively in the area of adaptive control and control of flexible structures.

(1) Stability, Convergence and Robustness of Adaptive Algorithms

We would like to synthesize the results of the part few years in aspects of stability, convergence and robustness of adaptive algorithms for continuous and discrete time systems into a research monograph whose primary audience is practicing flight control engineers, academics and other researchers. We have succeeded in unifying and organizing the literature as well as reformulated and solved adaptive control problems in a way we feel is at the same *time compact*, *rigorous*, and *easy to follow*. A first draft of the monograph has been prepared and new material that is under consideration for publication is being constantly added. We feel that such a well-written publication will greatly enhance the literatures. Also, we have efficient ways of producing the text using our UNIX facilities.

(2) Adaptive Control and Identification in the Presence of Prior Information

One of the greatest drawbacks of adaptive identification and control is the ability to incorporate reasonably prior information about the structure of the plant being identified or controlled—for instance, a few unknown gains in the interconnection of an otherwise well modelled linear system. Preliminary work has shown that considerable modification of present algorithms are needed in this context. Prior information in both a state space and transfer function context will be considered.

(3) Adaptive and Robust Control of Linearizable Non-linear Systems.

Two sets of problems need to be investigated for linearizable systems of the kind studied in the TAFCOS programs at times—

i) the on-line identification of some unknown parameters in the linearizing transformations, so as to iteratively improve the linearizations.

ii) the robustness to unmodelled dynamics of the linearized controller.

Research on part (ii) is rudimentary, but we have encountered some initial success in part (i) in the context of some linearizable systems encountered in the control of robot manipulators.

(4) Feasibility Study of Expert System Architectures for Computer Aided Control Systems Design (CACSD).

We will study the feasibility of providing a high level design environment to perform generic flight control CACSD such as:

modelling the plant, determining the characteristics of the plant model, modifying the configurations to make the plant more amenable to control (moving sensors/actuators), formulating line design specifications (reference model, trajectory generations, etc.), the actual design procedures (LQ, adaptive, linearization), design tradeoffs, and constant checkup, validation, documentation and *implementation*.

In the first year we will study the feasibility for future years we will expand this activity to a more constructive mode for prototype sets of control problems.

(5) Robust Designs for flexible structures

Our proposed method based on the Q-parameterization raises a number of implementation issues: in particular, to obtain an algorithm for designing the controller subject to the constraints imposed by the stabilization requirement.

We also suspect that for plants with slightly damped modes, there must be some fundamental limitations analogous to the ones derived by Zames and Doyle and Stein.

85-86
Progress

Two students Marc Bodson and Amit Bhaya supported by this grant finished their Ph.Ds this year. This brings to four the total number of Ph.Ds completed under this grant.

I. Research Activities of S. S. Sastry and His Students

A. *Stability, Convergence and Robustness of Adaptive Systems*

During the period September 1, 1985 to August 1986, Marc Bodson completed his Ph.D. dissertation. I am pleased to say that his *entire doctoral research* was supported by this NASA office – some of his work has appeared in print; but I have taken his doctoral dissertation as core to write an extended research monograph titled *Adaptive Control - Stability, Parameter Convergence and Robustness* under contract with Prentice Hall to be submitted by December 1986 and to appear by March 1987. The contents of this monograph are almost wholly on research supported by this grant and the writing is about three-fourths complete. A brief summary of some aspects of the book is given below:

[1] Marc Bodson, "Stability, Convergence and Robustness of Adaptive Systems," ERL Memorandum No. UCB/ERL M86/66, August 1986:

Abstract: The thesis addresses three issues of prime importance to adaptive systems: the stability under ideal conditions, the convergence of the adaptive parameters, and the robustness to modeling errors and to measurement noise. New results are presented, as well as simplified and unified proofs of existing results.

First, some identification algorithms are reviewed, and their stability and parameter convergence properties are established. Then, a new, input error, direct adaptive control scheme is presented. It is an alternate scheme to the output error scheme of Narendra, Lin, and Valavani, which does not require a strictly positive real condition on the reference model, or overparametrization when the high-frequency gain is unknown. Useful lemmas are presented and unified stability proofs are derived for the input and output error schemes, as well as for an indirect adaptive control properties. However, the input error and the indirect schemes have the advantage of leading to a linear error equation, and of allowing for a useful separation of identification and control.

The parameter convergence of the adaptive schemes is further analyzed using averaging techniques, assuming that the reference input possesses

some stationarity properties, and that the adaptation gain is sufficiently small. It is shown that the nonautonomous adaptive systems can be approximated by autonomous systems, thereby considerably simplifying the analysis. In particular, estimates of the rates of exponential convergence of the parameters are obtained for the linear identification scheme, as well for the nonlinear adaptive control scheme. The approach is particularly useful, as it leads to a frequency domain analysis, and has a vast potential for interesting extensions.

The Rohrs examples of instability in the presence of unmodeled dynamics are reviewed. A connection between exponential convergence and robustness is established in a general framework. The result is applied to a model reference adaptive control scheme, and stresses the importance of the persistency of excitation condition for robustness. Robustness margins of the adaptive control scheme are also obtained. The mechanisms of instability observed in the Rohrs examples are explained, and methods to improve robustness are briefly investigated.

Besides this summary the book will contain adaptive control of robots (joint work with J. Craig), as well as some work on adaptive control of partially known systems (see B below).

Other publications by Bodson and myself supported on this grant were:

[2] M. Bodson, S. Sastry, B. Anderson, R. Bitmead, and I Mareels, "Nonlinear Averaging theorems and the determination of parameter convergence rates in adaptive control," *Systems & Control Lettrs 7(1986) 145-157.*

Abstract: The paper presents nonlinear averaging theorems for two-time scale systems, where the dynamics of the fast system are allowed to vary with the slow system. The results are applied to the Narendra-Valavani adaptive control algorithm, and estimates of the parameter convergence rates are obtained which do not rely on a linearization of the system around the equilibrium, and therefore are valid in a larger region in the parameter space.

[3] S. Sastry, M. Bodson, B. Anderson, I. Mareels, and R. Bitmead, "The MIT Rule Revisited," Proceedings of the IFAC Workshop on Adaptive Identification and Control, Lund, Sweden July 1986, also submitted to Automatica.

Abstract: The MIT rule is a scalar parameter adjustment law which was proposed in 1961 for the model reference adaptive control of linear systems modeled as the cascade of a known stable plant and a single unknown gain. This adjustment law was derived by approximating a gradient descent procedure for an integral error squared performance criterion. For the early part of the 1960s this rule was the asis of many adaptive control schemes and a considerable wealth of practical experience and engineering folklore was amassed.

The MIT rule is in general not globally convergent nor stable but has a performance determined by several factors such as algorithm gain, reference input magnitude and frequency, and the particular transfer function appearing in the cascade. These restrictions on the MIT rule slowly came to be discerned through experimentation and simulation but effectively were without theoretical support until some novel algorithm modifications and stability analysis, so-called Lyapunov redesign, due to Parks. Our aim in this paper is to pursue a theoretical analysis of the original MIT rule to support the existing simulation evidence and to indicate mechanisms for treating questions of robustness of MIT-rule-based adaptive controllers with undermodelling effects.

The techniques that we apply to this problem centre on root locus methods, Nyquist methods and the application of the theory of averaging. Stability and instability results are presented and, using pertinent theories for different regimes of the gain-frequency plane, we approximate the experimentally derived stability margins, but for a broader signal class than simply periodic inputs. The mechanisms of instability and stability for these adaptive systems are highlighted and allow us to enunciate guidelines for the MIT rule to work. It is a pleasing by-product of this theoretical analysis that these guidelines coincide to a large degree with those advanced in earlier times on experimental and heuristic grounds.

B. *Averaging and Transient Analysis of Adaptive Systems*

An important aspect of the study of adaptive systems is their transient performance -- a frequent criticism of adaptive control has been its erratic transient performance. Given the nonlinear, time-varying nature of the adaptive control systems, a transient analysis is frequently complicated. However, the situation is tractable in the situation that the adaptation is slowed-down. We have studied several aspects of the transient behavior of adaptive systems: First we have extended to the discrete time case results which we obtained last year (on this grant with Fu and Bodson) for the continuous time case. It has been a dominant theme in our work to proceed with continuous and discrete time analysis in parallel and unify the often chaotic results in the literature:

[4] E.-W. Bai, L.-C. Fu, S. S. Sastry. "Averaging Analysis for Discrete Time and Sampled Data Adaptive Systems," ERL Memorandum No. UCB/ERL M86/67, September 1986, also to appear in the proceedings of the 28th IEEE

Conference on Decision and Control, Athens, Greece, December 1986, and submitted to the IEEE Trans. on Circuits and Systems:

Abstract: We extend our earlier continuous time averaging theorems to the nonlinear discrete time case. We use theorems for the study of the convergence analysis of discrete time adaptive identification and control systems. We also derive instability theorems and use them for the study of robust stability and instability of adaptive control schemes applied to sampled data systems. As a by product we also study the effects of sampling on unmodeled dynamics in continuous time systems.

Also in preparation is a paper discussing the choice of optimal input signal into an adaptive scheme for the fastest rate of convergence.

[5] L. C. Fu and S. S. Sastry, "Optimum Input Signal Design for Adaptive Identification and Control," invited to the American Control Conference, May 1987.

C. *Feasibility Study for the Use of Expert On-line Control Systems*

We spent a large amount of time during the year studying the literature and having numerous discussions on the integration of expert systems and control with a student (Richard Murray) and a visiting scientist on loan to us from E.T.C.A. Paris, Dr. Zavidovique. R. Murray has yet to synthesize a good survey of the literature, but some of my own thoughts on the subject of the integration of expert control are given in the following invited paper.

The paper is titled by S. V. Sastry and S. S. Sastry, "On the Integration of Robotics and Intelligent (Expert) Control with Power Systems Reliability," Proc. of the IFAC 86 Workshop on Power Systems Reliability, Bangalore, India, December 1986. Though power systems reliability and robotics are not within the framework of this grant, I feel that sections 0, 1, 2, and 3 in fact are not geared to that application. Further power systems and robotics are treated only as an example of the kind of complex, large-scale, multi-sensor, systems for which rule based control is relevant.

On the Integration of Robotics and Intelligent (Expert) Control into Power Systems Reliability

S. V. Sastry*   S. S. Sastry**

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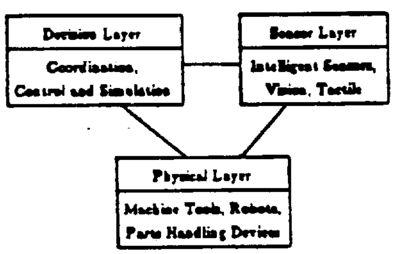
Introduction

In this talk we will present how we feel that the techniques of intelligent (expert) control can be integrated into power systems reliability. We will only mention in passing the use of robotics in enhancing power system reliability. Intelligent or expert or rule based control are a relatively new and unexplored area of control involving the interaction between an expert system and a real-time control system.

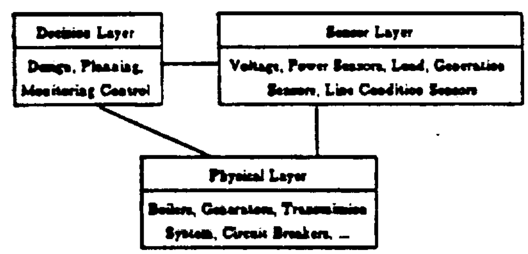
In recent years there have been (certainly in the U. S.) sharp declines in the rise of productivity and system effectiveness after a period of sharp rise (1945-1970). To identify the barriers to growth we regard complex engineering systems as compositions of three layers: the physical layer, sensor layer, and decision layer. In the context of power systems or flexible robotic manufacturing cells, the organization is as shown below.



Flexible Manufacturing Cell

Productivity increases of the pre-1970 era were due mainly to technological advances that made discrete operations in the physical layer more rapid, precise and automated. One consequence of the level of automation and reliability now available at the physical layer is that major increases in the overall efficiency of the systems is unlikely to be obtained by future innovation here. A second and unintended consequence of these technological advances is the tremendous burden placed on the decision layer in order to effectively design, plan and control the increasingly complex operations in the physical layer despite the massive amounts of detailed data about these operations provided by advanced sensor systems. *Current decision layers fail to cope with this complexity and data explosion and this is the basic barrier to the increase in productivity.*

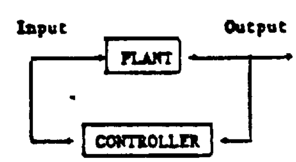
In this talk then we will present (somewhat abstractly and with only few specific allusions to power systems) on-going projects both at the Automotive Research Association of India, Pune and the Electronics Research Laboratory, Berkeley on *architectures and environments* for the decision layer. Control functions may be divided into "low level" continuous regulation of the physical layer and "high level" or "intelligent" functions mentioned above. Low level control is conveniently carried out by conventional feedback controllers which can be designed (though these too could be made intelligent - see Section 2), it is the higher level controller architecture we will primarily be interested in this talk: as we will explain in Sections 1 and 2. Section 3 contains a description of an expert system (intelligent control system) architecture for control and Section 4 some applications to power systems reliability.



Power System

Conventional Control and its Limitations

A basic framework for control is given in the following schematic



in which the plant to be controlled is viewed as an input-output system; with inputs time functions (belonging typically to $\mathbb{R}^{n_i}$) and outputs also have functions ($\in \mathbb{R}^{n_o}$) and the controller is a system which generates the requisite input in order to achieve a desirable output (regulation, for instance, demands that the outputs decay with time, etc). The control theoretic functions of:

(1) Modelling
(2) Identification
(3) Analysis, and
(4) Design

are implemented within this framework. The first step in this paradigm is a modelling step where the plant is modelled frequently as a state space dynamical system with input-output description given by a S triple: 3 spaces U- the input space, X-the state space, Y the output space, and two maps the state transition map $S: U \times X \times T \times T$ (T-the space which the time variable belongs to, typically $\mathbb{R}_+$ or Z_+) $\rightarrow Z$ and the output read out map $r = X \times T \times U \rightarrow Y$. For example, U could be $\mathbb{R}^{n_i}[0,\infty[$, $X \cong \mathbb{R}^n$ and Y could be $\mathbb{R}^{n_o}[0,\infty[$ and $J \cong \mathbb{R}_+$ in which case S a function of (t_1, t_0, x_0, u) with $t_1, t_0 \in T, x_0 \in \mathbb{R}^n$ and $u \in \mathbb{R}^{n_i}[0,\infty[$. and r is a function of t, x, u. The composition of these two maps is the input-output map.

Given this model of the dynamical system, the next step in the control paradigm is *identification*, namely the discovery of the state space X as well as the discovery of the state transition and output readout maps. This is performed on-line while the plant is in operation or off-line using scale models or replicas (simulations) of the plant.

The *analysis* phase consists in simplifying the model obtained through identification and making it tractable to available techniques of linear, non-linear control theory. Typically the output of this phase is a mathematical description of the plant, say of the form

$$
\begin{aligned}
\dot{x} &= f(x, u) \qquad x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^{n_i} \\
y &= s(x) \qquad y(t) \in \mathbb{R}^{n_o} \qquad (1.1)
\end{aligned}
$$

The *design* phase consists of applying state of the art methods of linear, nonlinear control to the control of the systems in the form (1.1) above accounting if necessary, for a certain amount of uncertainty, in some parameters of the model. Included as well are implementation issues such as sampling rates, microprocessors for performing the appropriate calculations, etc.

1. Limitations of Conventional Control

While the foregoing framework has proven to be extremely attractive conceptually and has had numerous successes in the fields of process control, flight control systems for aircraft, control of spacecraft, and the like; we have been confronted in recent years by systems which are too complex to fit into the current paradigm. The complexity can arise in many different ways:

(1) *Size and Location*: Several systems that one is called upon to control are geographically distributed and have very large numbers of semi-autonomous subsystems, coupled through communication networks with sizable transport delays, for example power systems, large computer communication systems, etc.

(2) *Complexity*: With the advent of new technologies the complexity of systems that need to be controlled has grown to such an extent that it is very difficult to obtain even the starting point for a good model in the paradigm described above. Consider the problem of controlling the flame of a gas-laser torch used for cutting metal. A detailed physical model from first principles is difficult to obtain and if obtained extremely cumbersome to manipulate because of its complexity. However, a great deal of expertise about the process and its dependence on certain parameters, such as the inlet pressure of oxygen can be gathered during its usage and control laws based on this experience.

(3) *Multiple Sensors*: Classical paradigms for control are motivated by considerations of very simple sensors for measuring well defined outputs for instance: in a process control, environment temperature, pressure sensors; in a flight control environment, gyro outputs, dynamic pressure sensors; in a robot manipulator control environment, velocity and acceleration sensors and so on. With the advent of new and more sophisticated and varied sensors, other kinds of sensor data are available, for example visual (outputs of cameras), both monochromatic and color, force or pressure profiles (outputs of tactile sensors). Frequently, too, multi-sensor data about the same object needs to be aggregated into a single coherent "state" of the process. Control of even relatively simple functions: such as on-line transient stability assessment is complicated by the fact that a variety of different kinds of sensor data from generators, buses, circuit breakers, and reactors need to be aggregated. In the years to come data fusion from even more and sophisticated vision sensors will need to be performed.

We feel that advances in expert or rule based control will be beneficial to the operations of power systems in three areas.

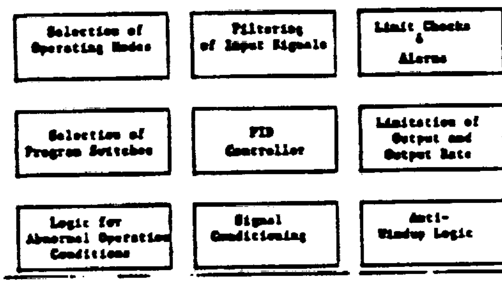
1) Monitoring: system diagnosis, fault analysis, stability assessment and stability margin enhancement.

2) Scheduling: of maintenance, of bringing up and shutting down both generation and load, unit commitment

3) Coordination: largely in a restorative phase. When a system needs to be reconnected after an islanding incident.

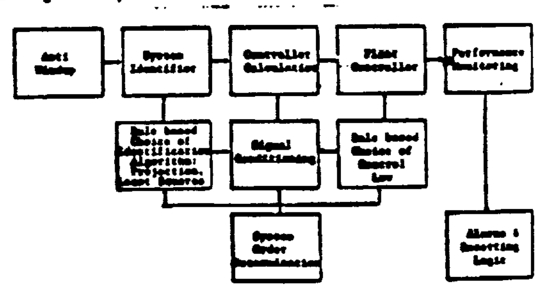
2. Introducing Intelligence into Control Systems

Almost unconsciously, good control engineers have always been using heuristics along with conventional control techniques. Astrom, *et al* [1] point out that a simple operational industrial PID (proportional integral derivative) regulator considers operator interfaces, operational issues like switching smoothly between manual and automatic operation, transients due to parameter changes, the effects of non-linear actuators, windup of the integral term, maximum and minimum selectors, etc. as shown in the figure below

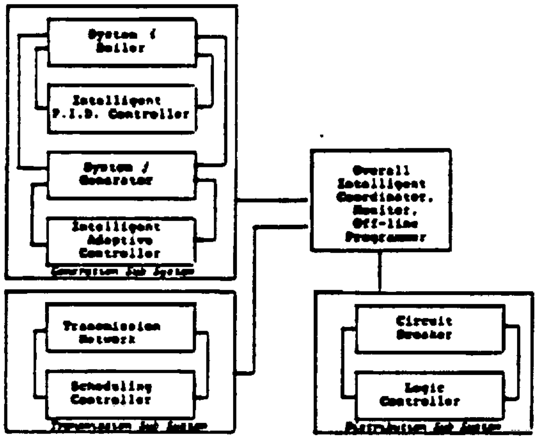


Block diagram of an industrial PID regulator with heuristics (after Astrom, *et al*)

It is therefore clear that heuristics based on the experience of experts is an essential part of a controller so as to enable the PID controller to function effectively. Similar such examples arise constantly in application of modern control - another example is an intelligent adaptive controller.



It is clear that complex industrial processes need several hierarchies of intelligence in control say as in the diagram below, drawn for a power system:



Another example is a complex flexible manufacturing cell involving the coordination of numerous robots, machine tools, intelligent sensors, etc. as shown in the figure below

SEE FIGURE AT END OF PAPER

The key characteristic of all these systems is a high level of distributed as well as centralized intelligence. Intelligent PID or adaptive controllers of the form discussed above supply local intelligence. More global rule based controllers deal with higher level problems of *monitoring, coordination, user interface, off-line programming, etc.*. At the current time the development, debugging, modification and testing of the rule-based part of the control logic is done in a *completely ad-hoc and insystematic fashion. This in part explains the relative lack of rapid increase in the performance of complex systems such as power systems, process control systems, or non-performance of certain other complex control systems - such as flexible manufacturing cells* in the face of rather impressive increases in technological progress in:

(1) Computation

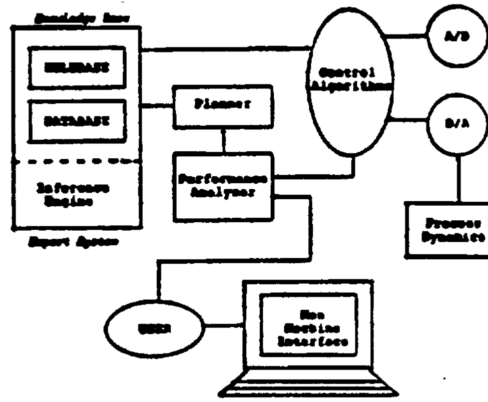
(2) High bandwidth communication

(3) High quality sensors

In the rest of this paper we will try to discuss a framework for developing intelligent (rule-based) or expert control systems and then potential applications in the field of power system reliability.

3. A Conceptual Architecture for Rule Based Control

A rule based, expert control system is somewhat different from a diagnostic or design oriented expert system in that it is a *piece of real-time software* interacting with a system whose dynamics are changed by the (feedback) rules of the expert control system. Thus, a block diagram of a rule based controller (see also Astrom, et al [1] and Waterman [2]) needs a few extra features:



The reader familiar with expert systems will see the addition of the new blocks: performance analyzer, control algorithms, A/D and D/A, as well as the dynamics of the process. We will discuss these briefly:

The System Database

This is a collection of facts, evidence, hypotheses and goals: for example, sensor measurement tolerances, operating thresholds, constants or operational sequencing, etc. Other examples indicate both static and dynamic goals: such as maintain stable operation or find optimal operating points (static) or reconfigure the system (dynamic).

The Rulebase

These typically contain production rules described as: if $<$ situation $>$ then $<$ action $>$. The situation represents facts, evidence, hypothesis and goals from the database, and the action can be either to add to/modify the database or to activate various identification/control algorithms. The rulebase is often structured in groups or knowledge sources that contain rules about the same subject to simplify the search.

Inference Engine

The purpose of the inference engine is to decide the context (database of facts, evidence, hypotheses and goals) which production rules to select next.

User Interface

This has two parts: the first gives development support such as a rule editor, and rule browser. The other part is a runtime interface which explains to the user how a certain rule was fixed or how a certain fact was concluded.

Performance Analyzer

This is a block which determines if a system is stable or whether the system is functioning appropriately at its current operating point or the security of the system at its current operating position.

Planner

A planner is needed to change production goals - for instance to change unit generation in response to changing loads, to reconfigure the network in accordance with some security enhancement routine or in response to some on-line faults. With many actions available and many sequences possible the development of a control plan may be viewed as a search through a large network for a path that reaches the currently established goals. This searching and planning based on the abstracted expertise of human experts is a fundamental part of the expert controller.

Such a planner is easy to visualize in problems such as those involving motion planning for robots in a flexible manufacturing cell.

Implementation issues involving the use of production systems (such as OPSG, OPSS), the kind of programming language (LISP, PROLOG), a mailbox type or blackboard type architecture, forward/backward chaining of rules, reasoning with uncertainty, etc. are too involved to be discussed further here.

4 Application Areas in Power Systems Reliability

Power Systems represent an extremely complex class of systems to be controlled reliably for the following reasons: They are:

1) large scale systems consisting of several hundreds of units in geographically widely distributed locations.

2) complex to model for a variety of different reasons: the dynamics of power systems happens on a variety of different time scales: time constants of circuit breakers, swing dynamics of the generators, boiler dynamics, scheduling daily, weekly, monthly, etc. Sometimes the dynamics are not either continuous time or discrete time but are *event driven* for instance when lightning strikes cause the landing of a network. Further good models are typically unavailable for customer load characteristics, probability of faults, etc.

3) They have multiple sensors whose measurements need to be aggregated for a coherent description of the state of the system.

We feel* that primary areas for the applicability of expert systems in power systems reliability are in

1 System Monitoring - diagnostics and fault analysis

2 Scheduling - maintenance, hydro-scheduling, unit commitment

3 Coordination and Restoration

1. Monitoring

System analysis and diagnosis are favorite domains for the application of expert systems. Monitoring means providing the dispatcher with a comprehensive analysis of the system and pointing out its vulnerabilities rather than displaying a large amount of unprocessed raw data.

This is a relatively easy task given that security assessment can be performed, the expertise of expert dispatchers codified and formulated into rules and implemented with a reasonably good separation of domain knowledge and the inference engine.

2. Scheduling

Mathematically speaking, scheduling is merely a very large scale constrainted optimization problem involving thousands of real and integer variables. Unfortunately, it is too large a system to easily implement. Consequently, it needs to be broken up into a strategic part and a sequence of smaller optimization problems. The strategic part including decomposition into smaller subproblems is what the codified knowledge of experts is good for

3. System Restoration

This is similar to scheduling, except that it is a multiobjective optimization problem involving the satisfaction of both static and dynamic constraints - power flow constraints require the generated power to match the load power, stability constants need to ensure that the system is able to endure salt change and synchronization. Finally, operating constraints involving generator load pick-up and line switching need to be considered.

Expert control systems are needed to plan and coordinate the use of several analysis packages as well as some heuristics involving the order of interconnections.

We feel that the three areas listed here are just the tip of the iceberg of possible further extensions of rule based control systems in power systems.

* This part of the paper is based largely on the M.S. report of A. Neyer.

Acknowledgements The research of S. S. Sastry was sponsored in part by NASA under grant NAG2-243. We would like to thank R. Murray, A. Neyer, and especially Professor B. Zavidovique of ETCA, Paris for several discussions.

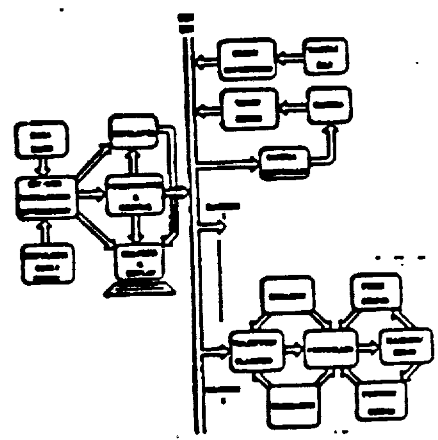
References

[1] Astrom, K. J., Anton, J. J., and Arzen, K. E.. "Expert Control" *Automatica*, Vol 22 (1986) pp. 277-286.

[2] Waterman, D. A., *Expert Systems*, Addison-Wesley, 1986.

[3] Neyer, A. F.. "Expert Systems for Power Engineering - A Tutorial", M.S. Plan II Report, University of California, Berkeley, December 1985.

FUNCTIONAL DIAGRAM OF [illegible]



II. Research Activities of C. A. Desoer and His Students

1. Dr. A. Bhaya was supported from September 1, 1985 to April 1986. Dr. Bhaya completed his thesis entitled

Issues in the Robust Control of Large Flexible Structures

Two new issues were considered:

1. Suppose we consider the general linear unity-feedback MIMO structure $S(P,C)$ where P is *unstable*. Even in that case, the stabilizing controller may be parameterized by Q: it is well known that if P is stable, then Q may be any proper stable transfer function. If P is unstable, the Q must satisfy certain structural constraints dictated by the instability structure of P. We found the necessary and sufficient condition on Q so that it gives a stabilizing controller for P. This is reported in the paper

   Necessary and Sufficient Conditions on Q $(=C(I+PC)^{-1})$
   for Stabilization of the Linear Feedback System $S(P,C)$
   A. Bhaya and C. A. Desoer.

   Abstract: We find the following necessary and sufficient conditions of $Q\ (:=C(I+PC)^{-1})$ to H-stabilize the standard linear time-invariant unity feedback system $S(P,C)$ where P has the l.c.f. (D_{pl}, N_{pl}) and the r.c.f (N_{pr}, D_{pr}): and H is a principal ideal domain. (i) Q must have elements in H, (ii) (resp. (iii)) Q must factorize in H with D_{pr} (resp. D_{pl}) as a left (resp. right) factor and (iv) $(I-QP)$ must factor in H with D_{pr} as a left factor.

2. In the usual large flexible structure model, it is known that in the colocated case, the plant has necessarily some jw-axis zeros. (Hence severe performance constraints follow). By reformulating the problem of the location of the zeros as an eigenvalue problem it is possible to show that as the number of actuators is increased the zeros of transmission have increased frequencies, without however breaking the interleaving property. Those results will be presented at the CDC in December 1986.

   Transmission Zeros of Large Flexible Space Structures (LFSS)
   Amit Bhaya and Charles A. Desoer

   Abstract We consider a general finite element (lumped) model for the LFSS. We assume small deformations, linear-elastic materials and neglect Coriolis forces, gyroscopic coupling and damping. We assume that the modal position is the output; colocated sensors and actuators with the sensor gains chosen so that it has the system representation $S_p = \{A, B, C\}$ where

$$A = \begin{bmatrix} 0 & \vdots & I_n \\ \cdots & \cdot & \cdots \\ -\Omega^2 & \vdots & 0 \end{bmatrix};\ B = \begin{bmatrix} 0 \\ \cdots \\ \hat{B} \end{bmatrix};\ C = \begin{bmatrix} \hat{B}^T & \vdots & 0 \end{bmatrix}$$

   with system matrix S, where $\hat{B} \in \mathbb{R}^{n\times m}$ and rk $(\hat{B}) = m \le n$, $\Omega^2 := (\omega_1^2, \cdots, \omega_n^2) \in \mathbb{R}^{n\times n}$ with $0 \le \omega_1 \le \omega_2 \le \cdots \le \omega_n$.

West-Vukovich *et al* characterized the transmission of zeros of S_p and showed that all $(n-m)$ nonpositive transmission zeros of the reduced system $\tilde{S}_p := \{-\Omega^2, \hat{B}, \hat{B}^T\}$ (with system matrix $\tilde{S}_p$) are the *squares* of the transmission zeros of S_p. Thus S_p was shown to have $2(n-m)$ jw-axis transmission zeros.

In this paper, we use the simple structures of the reduced system $\tilde{S}_p$ to reformulate the problem of finding the transmission zeros of $\tilde{S}_p$ (equivalently the zeros of det $[\tilde{S}_p(.)]$) as the numerically well-conditioned and standard problem of finding the $(n-m)$ real eigenvalues of a real, symmetric $(n-m)\times(n-m)$ matrix.

Earlier reported work, now published

A. Bhaya and C. A. Desoer, "Robust Stability under Additive Perturbations", *IEEE Trans. AC-30*, No. 12, pp. 1233-34, Dec. 1985.

2. Mr. G. Kabuli started to work this summer: he is familiarizing himself with the nonlinear literature and in particular is studying a proposal of J. Hammer to factorize nonlinear discrete-time plants in a manner similar to the linear case.

Related Work

3. Mr S. M. Shahruz has completed some of his work on nonlinear systems and has obtained a streamlined derivation of the singular perturbation problem with three time-scales. The result is a paper accepted for publication by *Circuits Systems and Signal Processing* entitled

Stability of Nonlinear Systems with Three Time Scales
C. A. Desoer and S. M. Shahruz

Abstract We study the asymptotic stability of a singularly perturbed nonlinear time-invariant system $S_{\xi\nu}$ which has three vastly different time scales. The system $S_{\xi\nu}$ is approximated by three simpler systems over different time intervals. We give a straight forward proof of the fact that the asymptotic stability of $S_{\xi\nu}$ is guaranteed when the equilibrium points of the three simpler systems are exponentially stable and when the parameters ξ and ν are sufficiently small.

The work on dithered systems reported earlier has been published:

C. A. Desoer and S. M. Shahruz "Stability of Ditherred Nonlinear Systems with Backlash or Hysteresis", *International Journal of Control*, 46, 4, pp. 1045 - 1060, 1986.

III. Research Plans for the Year 1986-1987

One student (Li-Chen Fu) supported by this grant is expected to finish his dissertation this year in the area of adaptive control.

A. *Frequency Domain Concepts for Robust, Adaptive Control*

A need has emerged for a synthesis of frequency domain techniques of analysis into the study of the transient behavior and design of adaptive systems. This is particularly difficult since adaptive systems are nonlinear and time-varying. We have encountered some success, however, in carrying out this program when the rate of parameter update is slowed down (consider our work on averaging and optimal input-design). We feel that this work can be extended on several directions:

(i) An understanding of the shift in the tuned value of the adaptive system in response to changes in the frequency content of the excitation

(ii) An understanding of mechanisms of instability including slow drift instability, high-gain instability

(iii) A design technique for the adaptive control of sampled-data systems

(iv) A systematic design procedure for general indirect-adaptive control laws. Our work so far has concentrated on direct adaptive (in fact, model reference adaptive control). We are now poised to use frequency domain technique for indirect schemes.

B. *Completion of our Monograph Project–Stability, Convergence and Robustness of Adaptive Algorithms*

This monograph (currently being classroom tested) is an attempt at providing the first *rigorous, compact* text on adaptive control expected to be finished by December 1986. S. S. Sastry and Marc Bodson propose to give a short course

1986-87

Progress Report of S.S. Sastry and his Students

At various times during the year four graduate students of S.S. Sastry were partially supported on the grant: E.W. Bai, L.C. Fu, J. Mason and R. Murray. E.W. Bai and L.C. Fu have graduated and are Assistant Professors at the University of Iowa and National Taiwan University, respectively. J. Mason is expected to complete his Ph.D. dissertation by January 1988. These three students worked in the area of linear adaptive control. Richard Murray has been studying high level languages for manipulation and trajectory planning for multi-fingered robot hands. In addition I have begun a systematic study of adaptive control of linearizable systems. Here are some details of the research on the grant and abstracts of the key publications:

1. Linear Adaptive Control

We have continued our ongoing work on the use of frequency domain techniques and averaging for the study of the transient behavior and robustness of adaptive control techniques.

1.1. Averaging Analysis for Discrete Time and Sampled Data Adaptive Systems (Bai, Fu and Sastry), to appear in the IEEE Transactions on Circuits and Systems, February 1988, also in the Proceedings of the Conference on Decision and Control, Dec. 86 (Athens).

Abstract: We extend our earlier continuous time averaging theorems to the nonlinear discrete time case. We use theorems for the study of the convergence analysis of discrete time adaptive identification and control systems. We also derive instability theorems and use them for the study of robust stability and instability of adaptive control schemes applied to sampled data systems. As a by product we also study the effects of sampling on unmodelled dynamics in continuous time systems.

1.2. Frequency Domain Synthesis of Optimal Inputs for Adaptive Identification and Control (Fu and Sastry), Proceedings of the American Control Conference, Minneapolis, June 87, also submitted to the IEEE Transactions on Automatic Control.

Abstract: In this paper, we precisely formulate the input design problem of choosing proper inputs for use in SISO Adaptive Identification and Model Reference Adaptive Control algorithms. Characterization of the optimal inputs is given in the frequency domain and is arrived at through the use of averaging theory. An expression for what we call the average information matrix is derived and its properties are studied. To solve the input design problem, we recast the design problem in the form of an optimization problem which maximizes the smallest eigenvalue of the average information matrix over power constrained signals. A convergent numerical algorithm is provided to obtain the global optimal solution. In the case where the plant has unmodelled dynamics, a careful study of the robustness of both Adaptive Identification and Model Reference Adaptive Control algorithms is performed using

averaging theory. With these results, we derive a bound on the frequency search range required in the design algorithm in terms of the desired performance.

1.3. Slowdrift Instability in Model Reference Adaptive Systems - an Averaging Analysis (Fu and Sastry), International Journal of Control, Vol. 45 (1987), pp. 503-527.

Abstract: This paper presents instability theorems for one and two-time scale time-varying non-linear systems. The theorems are applied to the model reference adaptive control system with unmodelled dynamics and output disturbances.

1.4. Frequency Domain Analysis and Synthesis Techniques (Ph.D. dissertation of Fu).

Abstract: In this thesis we use a mathematical technique, referred to as the method of averaging, to thoroughly analyze both adaptive identification and adaptive control schemes. In principle the results hold when the rate of parameter update in the adaptive loop is slow compared with the dynamics of the other state variables, but in practice they work for normal rates of parameter adaptation. Our analysis is not confined to the ideal case which consists of knowing the order of the unknown plant exactly and assuming there exists no external disturbances, but it also allows for unmodelled dynamics and additive output disturbances. We also make use of the method of averaging to solve the optimal input problem, i.e. the problem of choosing the input which produces the fastest rate of parameter convergence.
The results of this thesis are many. The first is a set of stability theorems which determine when a dynamical system possesses exponential stability, partial exponential stability or ultimate boundedness. Instability theorems for one- and two-time-scale systems are also given. Under the assumptions of a stationary reference input and slow adaptation these results are applied to adaptive systems. The next result is a calculable estimate of the rate of parameter convergence when various adaptation algorithms are used. When the plant contains unmodelled dynamics, we use the method of averaging to formally define the notion of a set of "tuned parameters". Under the assumptions of slow adaptation and persistency of excitation, we show that for the adaptive identifier, the actual identifier parameters converge to a ball which is centered at the tuned parameters and whose radius goes to zero as the adaptation gain goes to zero. Similar results, though slightly more complicated, are also obtained for the adaptive control case. To illustrate the importance of the choice of input signals, the phenomenon of slow-drigt instability is analyzed. Finally a frequency domain technique, for the synthesis of reference inputs which solve the optimal input problem, is given. An expression for what we call the average information matrix is derived and its properties are studied. The objective of the input synthesis technique is to specify the frequency content of a power constrained input signal, which maximizes the smallest eigenvalue of the average information matrix, and hence maximize the parameter convergence rate. A convergent numerical algorithm is given which obtains globally opimal solutions to the above problem. When the plant contains unmodelled dynamics, practical considerations of the range of support of the frequency content of the reference input is given.

1.5. Analysis of Adaptive Identifiers in the Presence of Unmodelled Dynamics: Averaging and Tuned Parameters (Mason, Bai, Fu, Bodson and Sastry), ERL Memo No. M87/25, submitted to IEEE Transactions on Automatic Control.

Abstract: Adaptive identifiers are designed with the assumption that the order of the plant is known. In this paper we analyze the behavior of a standard identifier when the plant contains additonal dynamics, called unmodelled dynamics, which invalidate the known order assumption. The first result of our analysis is an input richness condition which does not depend on the order of the unmodelleed dynamics to guarantee persistency of excitation of the regressor. Then we show that the PE condition leads to a BIBO stability property for the identifier. We use the method of averaging to formally define the notion of tuned parameters as the equilibrium of the identifier averaged system. It is shown that the tuned parameters always exist and that the actual parameters converge to some neighborhood of the tuned parameters. From the definition of the tuned parameters, we derive an explicit expression to calculate them and interpret them as the fixed parameter values which minimixe the mean squared output error.

We have continued to develop the results of (1.5) for the identification of the transfer function of a flexible space structure. The input-output data is obtained from a model of the structure at Caltech and is endowed with several modes which we choose not to model.

The monograph project started last year on this grant is nearing completion (draft enclosed).

2. Adaptive Control of Linearizable Systems (Sastry)

In recent years, starting with the pioneering work of Meyer and Cicolani at NASA-Ames an interesting methodology of non-linear control has evolved. A number of applications of these techniques have been made, however, their chief drawback appears to arise from the fact that they rely on an exact cancellation of non-linear terms in order to get linear input-output behavior. We suggest the use of adaptation as a technique to make asymptotically exact the exact cancellation of non-linear terms when the uncertainty in the non-linear terms is parametric. This work was carried out in collaboration with D. Normand-Cyrot of L.S.S. - E.S.E., Paris.

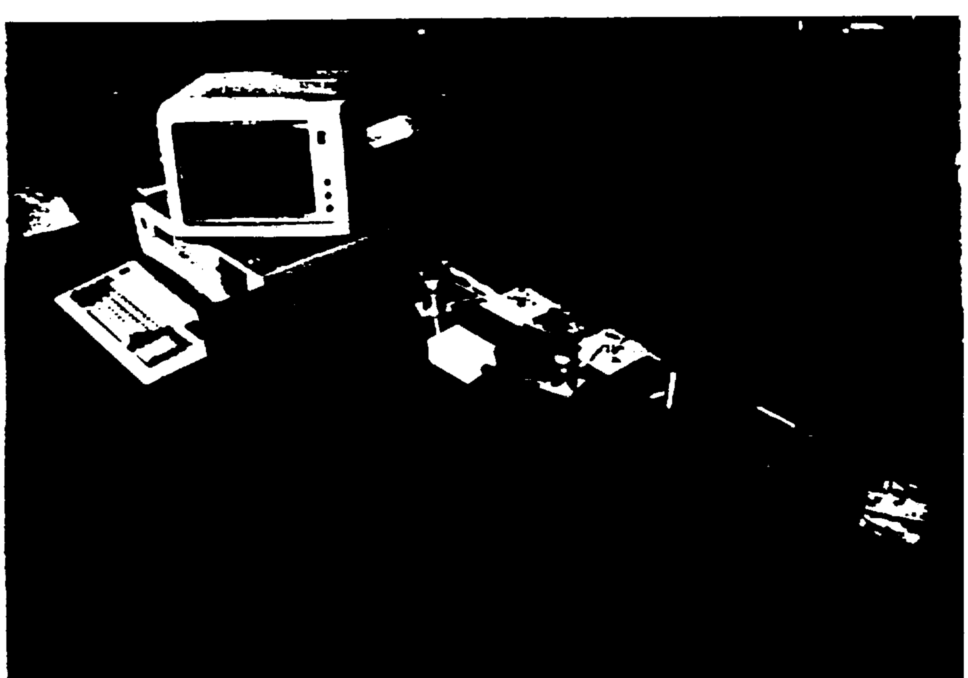
Abstract: In this paper we give some initial results on the adaptive control of "minimum-phase" non-linear systems which are exactly input-output linearizable by state feedback. Parameter adaptation is used as a technique to robustify the exact cancellation of non-linear terms which is called for in the linearization technique. We review the application of the techniques to the adaptive control of robot manipulators. Only the continuous time case is discussed in this paper--extensions to the discrete time and sampled data case are not obvious.

3. Trajectory Generation and Manipulation Primitives for a Multi-fingered hand (Murray and Sastry)

One of the most important areas of future research in control is the generation of command trajectories for a control system from a high level, qualitative description of a task. This problem is readily perceivable in a variety of applications: flight control (from the pilot's idea of a maneuver to a reference trajectory), robot path planning (from a pick and place description to a collision avoidance trajectory that respects a torque limits), etc. For our part we have been interested in understanding manipulation of objects by a multi-fingered hand, which of course can be thought of as being a collection of robots. Manipulation of objects, such as screwing parts and mating parts for assembly; reconfiguring can be achieved if the high level description could be specified in terms of a set of primitives. Candidate primitives include various types of grasps - power grasp, dextrous grasp, pinched grasp and some motions - twirl, push, apply contact force, compliance, etc. These in turn are to be complied to a set of control actions on the individual joints of the fingers taking into account their kinematics, collision avoidance, torque limits on the motors.

Such a program is undoubtedly ambitious and the project is in its infancy: we have built a two jointed, two-fingered hand (Styx II) controlled through a PC-AT currently instrumented with joint encoders and strain gauge finger tips are being designed to perform simple operations: such as peg in a hole insertion, grasping, etc. We hope to proceed inductively on this project - using our experimental experience to come up with notions of the correct theory to pursue in this regard.

The Berkeley Multi-Fingered Hand



Progress Report of C. A. Desoer and his student M. G. Kabuli

1. Nonlinear Systems, Factorizations and Stable Feedback Systems

The well established algebraic factorization theory for linear systems has been singularly successful in obtaining definitive results: class of all achievable input-output maps, the parametrization of all stabilizing compensators, the decoupling problem, the sensitivity minimization problem (H^∞ theory), the decentralized control problem, etc. Recently a few papers investigated some of the possibilities that factorization would offer in the *nonlinear* context. We are in the process of developing that line of research. So far we have completed the papers listed below:

1.1 C. A. Desoer, M. G. Kabuli, "*Stabilization and Robustness of the Nonlinear Unity-Feedback System: Factorization Approach*", University of California, Berkeley ERL Memo M87/28 1987, also accepted to International Journal of Control, Ref. No. IJC/87227.

> Abstract: This paper is a self-contained discussion of a right factorization approach in the stability analysis of the continuous-time or discrete-time, time-invariant or time-varying, well-posed unity feedback system $S(P, C)$. We show that a well-posed stable feedback system $S(P, C)$ implies that P and C have right factorizations. In the case where C is stable, P has a normalized right-coprime factorization. The factorization approach is used in stabilization and simultaneous stabilization results.

An abbreviated version of these results have been accepted for presentation at the 26th IEEE Conference on Decision and Control, Los Angeles, Dec. 1987, under the title "*Nonlinear Systems, Factorizations and Stable Feedback Systems*".

1.2 C. A. Desoer, M. G. Kabuli, "*Right Factorization of a Class of Time-Varying Nonlinear Systems*", University of California, Berkeley ERL Memo M87/55 1987, also submitted to IEEE Transactions on Automatic Control, Ref. No. CM-87298.

> Abstract: We consider a class of nonlinear continuous-time time-varying plants with a state-space description which has a uniformly completely controllable linear part. For this class, we obtain by calculation a right factorization. In the case where the state is available for feedback, we obtain a normalized right-coprime factorization.

1.3 C. A. Desoer, M. G. Kabuli, "*Linear Stable Unity-Feedback System: Necessary and Sufficient Conditions for Stability under Nonlinear Plant Perturbations*", University of California, Berkeley ERL Memo M87/64 1987, also submitted to IEEE Transactions on Automatic Control, Ref. No. CM-87378.

Abstract: We consider a linear (not necessarily time-invariant) stable unity-feedback system, where the plant and the compensator have normalized right-coprime factorizations; we study two cases of *nonlinear* plant perturbations (*additive* and *feedback*), with four subcases resulting from: *1)* allowing exogenous input to the perturbation or not, *2)* allowing the observation of the output of the perturbation or not. The plant perturbation is *not* required to be stable. Using the factorization approach we obtain *necessary* and *sufficient* conditions for all cases in terms of two pairs of nonlinear pseudo-state maps. Simple physical considerations explain the form of these necessary and sufficient conditions. Finally we obtain the characterization of all perturbations for which the perturbed system remains stable.

Some of these results in the paper above were presented at the 25th Annual Allerton Conference on Communication, Control and Computing, Urbana-Champaign, Sept. 1987, under the title "*Linear Stable Unity-Feedback System: Necessary and Sufficient Conditions for Stability Under Nonlinear Additive Plant Perturbations*".

1.4 C. A. Desoer, A. N. Gündeş, M. G. Kabuli, "*Coprime factorizations of individual subsystems in stable linear systems*", submitted to the 1988 American Control Conference, Atlanta, Georgia, June 1988.

Abstract: We consider a *stable* linear (not necessarily time-invariant) unity-feedback system and obtain the *necessary* and *sufficient* conditions under which the *individual* subsystems have right- and left-coprime factorizations. The linear time-invariant case is given as an example.

Research Plans for the Year 1987 - 1988

This year our research will be in three major areas. The focus will shift somewhat from problems of linear, adaptive control to non-linear flight control. The methodology of research will also shift somewhat from being purely theoretical to one which is more closely coupled to a few candidate applications:

(i) The control of VTOL aircraft, and in particular the Harrier at NASA - Ames.

(ii) The control of flexible robot manipulators, both for large slewing maneuvers and smaller dextrous motions.

(iii) The generation of command trajectories for the low-level control of multi-fingered hands and flight control systems.

In order to accomplish this shift in our research methodology, especially in the context of the first application listed above, we envision a much closer interaction with the Ames Flight Control group with the exchange of models, data and techniques. We will be significantly aided in this process by the presence in our research group of John Hauser, who has had considerable experience with flight control systems as a pilot with the Air- Force. At the outset Mr. Hauser will spend some time with the simulation facilities at Ames understanding the nature of the models and porting them in a form in which they can be understood and used by other people in the group. We then hope to embark on the following program of research:

Section 1. Identification of Non-linear Plants

Key problems arise in the development of non-linear models for flight control from simulation models and flight data. These problems center around coming up with analytical models of the non-linearities present in the system. The non-linearities need to be chosen so that they are a basis set for the system to be identified (they are rich enough to model the system completely), and in addition the resultant model needs to be robust. The tools which are used for linear model identification may be extended to linearizable systems with only a modest effort. Consequently, the choice of the non-linear functions chosen to represent the plant is critical. Some choices of functions may result in a linearizable system while others may not. The tools that we will bring to bear on this topic will be a combination of symbolic manipulation tools, simulation tools and identification tools. We will proceed in developing the theory in this regard inductively, i.e. we will start with some prototype identification problems encountered with the modelling of the dynamics of the Harrier from exisitng simulations and flight data and then proceed to propose a theory.

Section 2. Research in Linearizing Control of Non-linear Systems

While a great deal of progress has been made in applying the methodology of linearizing control to "minimum phase" non-linear systems very little progress has been made in the context of non-minimum phase non-linear systems. For such systems there are two approaches available: to explicitly include the non-minimum phase characteristic in the reference model or to modify the output of the system so as to make it minimum phase. The latter option has been used to get a system with no zeros for the longitudinal axis of the helicopter. We propose to build the theoretical backbone for such approaches.

Yet another area of active research in linearizing control has to do with when the linearizing conditions are not uniformly satisfied in regions of the state space. For example, in the instance of a relative degree 1 system the case when $L_g(h)$ is not bounded away from zero but changes sign in a certain region of the state space is not covered. We have encountered such a situation in the context of the control of a flexible robot manipulator. The situation is considerably more involved in the multivariable case. As in the previous section we hope to proceed inductively from our experience with the test bed flexible robot manipulator which we have built and instrumented. We will also commence the program of studying the linearizability of the Harrier dynamics.

A question related to the one raised in the preceeding paragraph is the question of near-linearizability of non-linear dynamics, i.e. when the linearizability conditions fail to hold, how does one determine by how much they fail to hold. In other words which systems are nearly linearizable and how does one perform approximate linearization. We propose to develop a systematic methodology for this task.

Section 3. Command Generation for Control Systems

A key problem in control systems is the development of a methodology for the generation of commands as inputs to a control system from a high level task description. Consider, for example the problem of trajectory generation for robot manipulators: from a description of the task and the configuration of obstacles in the environment one generates a collision avoidance trajectory in space. This needs to be indexed by time in such a manner as to include torque constraints on the motors, finite slew rates for the torques and so on. For a flight control system the problem is complicated by the fact that the constraints on actuators are more complicated and that certain trajectories are infeasible when the system is not minimum phase. Further, the process of compiling a low-level quantitative description of a task from a high-level qualitative one is not well understood. Infact, it is in the context of this problem that we have begun the work on multi-fingered hands described in the progress report. We propose to continue both that project as well as one involving command generation for the Harrier and the flexible arm this year.

Section 4. Adaptive Control Of Linearizable Systems

As pointed out in the progress report a key drawback of linearizing control is that it relies on exact cancellation of non-linear terms. Consequently, when these terms are not exactly known the cancellation needs to be robustified using adaptation. We have begun some work in this regard on this grant last year, but this is preliminary and will need to be extended to cover the situations described in Section 2 above and the three applications indicated above.

Progress Report of S.S. Sastry and his Students

1987 - 1988

During the past year three graduate students of S.S. Sastry were partially supported on this grant, Richard Murray, John Hauser, and Andrew Teel. Richard Murray continued the work he began last year in the control and trajectory generation of manipulation primitives for a multi-fingered hand. Both John Hauser and Andrew Teel have been working on the linearizing control of a class of non-linear systems, with specific emphasis on the dynamics of the Harrier, VTOL. In the area of linear adaptive control in work supported by this grant, I have completed a monograph with Marc Bodson (a student supported on this grant until January 1987) entitled "Adaptive Control: Stability, Convergence and Robustness," to be published by Prentice Hall in November 1988. This brings to a state of completion a long program of research in linear adaptive control, supported on this grant from October 1983 to the present, which also resulted in the completion of four Ph.D. dissertations either wholly or partially supported by this grant, M. Bodson (December 1986), E.W. Bai (August 1987), L.C. Fu (August 1987), J. Mason (June 1988). Here are some details of the research on the grant and abstracts of key publications:

1. Linear Adaptive Control

1.1. "Adaptive Control, Stability, Convergence and Robustness" (S. Sastry and M. Bodson), to be published by Prentice Hall, November 1988. This 378 page monograph is a synopsis of our work in the last four years in adaptive control with contributions from the dissertations of Fu, Bai and Mason.

1.2. "Analysis of Adaptive Identifiers in the Presence of Unmodelled Dynamics: Averaging and Tuned Parameters" (Mason, Bai, Fu, Bodson and Sastry), *IEEE Trans. on Automatic*

Control, Vol. 33, No. 10, 1988, pp. 969-975.

Abstract: Adaptive identifiers are designed with the assumption that the order of the plant is known. In this paper we analyze the behavior of a standard identifier when the plant contains additional dynamics, called unmodelled dynamics, which invalidate the known order assumption. The first result of our analysis is an input richness condition which does not depend on the order of the unmodelled dynamics to guarantee persistency of excitation of the regressor. Then we show that the PE condition leads to a BIBO stability property for the identifier. We use the method of averaging to formally define the notion of tuned parameters as the equilibrium of the identifier averaged system. It is shown that the tuned parameters always exist and that the actual parameters converge to some neighborhood of the tuned parameters. From the definition of the tuned parameters, we derive an explicit expression to calculate them and interpret them as the fixed parameter values which minimize the mean squared output error.

2. **"Trajectory Generation and Manipulation Primitives for a Multi-fingered Hand" (Murray and Sastry).**

We have completed the construction of a two jointed, two fingered hand (Styx II) controlled through a PC-AT to perform some simple operation. The two fingered hand has turned out to be an excellent test bed for trying out various kinds of low level control laws for position and force control. It has also helped us to formulate a comprehensive "universal" theory of Lagrangian control for single and multi-manipulators under a variety of contact conditions. As we had suggested in last year's progress report, we have studied questions of trajectory generation for a multi-fingered hand from a high level qualitative description of a task. Though we have not yet had some definitive results in the area, we have been studying the literature in the area of event driven control systems for additional tools.

2.1. "Control Experiments in Planar Manipulation and Grasping," (Murray and Sastry), submitted to the *1989 IEEE Conference on Robotics and Automation*, also to be a *UCB/ERL Memo*, October 1988.

Abstract: Many algorithms have been proposed in the literature for control of multi-fingered robot hands. This paper compares the performance of several of these algorithms, as well as some extensions of more conventional manipulator control laws, in the case of planar grasping. A brief introduction to the subject of robot hands and the notation used in this paper is included.

2.2. "Dynamics and Control of Constrained Robot Systems," (Murray, Hsu and Sastry). Submitted to the *IEEE Conference on Robotics and Automation/fR, 1989; also to be a UCB/ERL Memo*, October 1988.

Abstract: This paper presents a general framework for developing dynamics and control laws for a large class of robot systems. This class includes single robots in contact with the environment, multi-fingered hands, and multiple robots performing a coordinated task. Several well known trajectory control laws for robots are extended using this framework and proofs of exponential stability are given. A new control law which allows hybrid position/stiffness control is also presented along with proofs of stability. Experimental results of the control algorithms are given for the case of a two-fingered hand manipulating an object and a single robot applying a force to a surface.

3. Control of Linearizing Systems

We have continued the program of taking recent advances in non-linear control into a practical design methodology. In this work, we have been collaborating very closely with the flight control group at NASA-Ames. Two sets of projects are underway.

3.1. Flight Control of VTOL Aircraft (Hauser and Sastry in collaboration with G. Meyer of NASA-Ames). When the linearizing theory is blindly applied to the flight control of high performance aircraft, such as VTOL aircraft, the controller yields an internally unstable system. This is because linearizing control is based on cancellation of the zero dynamics, which are slightly non-minimum phase for the dynamics of the VTOL. We have developed methodology to modify the theory for these and other non-minimum phase systems.

"Linearization Techniques for Flight Control Systems," (Hauser, Sastry and Meyer).

Abstract: There has been a great deal of excitement recently over the development of a theory for explicitly linearizing the input-output response of a nonlinear system using state feedback. One shortcoming of this theory is the inability to deal with *non-minimum phase* nonlinear systems. Highly maneuverable jet aircraft, such as the V/STOL Harrier, belong to an important class of *slightly* non-minimum phase nonlinear systems. The non-minimum phase character of these aircraft is due in part to a slight coupling between rolling moments and lateral accelerations. In this paper, we show that while straightforward application of the linearization theory to a non-minimum phase system results in a system with a linear input-output response but unstable internal dynamics, designing a feedback control based on a minimum phase approximation to the true system results in a system with desirable properties such as bounded tracking and asymptotic stability.

3.2. "Computer Aided Non-linear Controller System," (Teel and Sastry).

To make the theory a practical design methodology we have developed a VAXIMA based design package for obtaining the "normal form" of non-linear systems starting from an analytical description of their dynamics. In future, we will also develop B-Spline software to take numerical data and convert it into an analytical form necessary for the follow up analysis and design.

Progress Report of C. A. Desoer and M. G. Kabuli

1987 - 1988

1. Nonlinear Systems, Factorizations and Stable Feedback Systems

Factorization theory for linear time-invariant systems has been the only approach that completely characterizes all stabilizing compensators, all achievable performance objectives in one framework. Recently, a few papers have investigated the extension of the factorization approach to a *nonlinear* setting, using bounded-input bounded-output stability arguments. This generalization is not an abstraction solely for theoretical purposes; it captures the essence of certain properties of the linear factorization theory often considered as limited to its constrained setting of the algebra of linear maps. When we restrict ourselves to additive feedback schemes, this generalization allows us to describe the set of feedback stabilizable nonlinear plants. In certain cases, one can find one, many or even all stabilizing compensators and describe the achievable closed-loop maps. As it is well-known not all linear systems admit certain factorizations required for the application of factorization theory; thus, it is not restrictive at all to consider special classes of plants in the generalization. The following papers emphasize these points.

1.1. C. A. Desoer, M. G. Kabuli, "Stabilization of a Class of Nonlinear Systems Using a Factorization Approach," *Proceedings of the IEEE 27th Conference on Decision and Control*, Austin, December 1988.

Abstract: Linear time-invariant proper plants with a minimal state-space description can *always* be stabilized in an observer-controller configuration where the feedback system consists of a two-input one-output compensator which is *stable;* hence, such plants *always* have right-coprime factorizations. Using observer based controllers, coprime factorizations of a class of *linear* plants have been obtained in the literature. In this paper, we generalize the right-coprimeness definition by requiring *only* that the corresponding plant's pseudo-state can be reconstructed by a two-input one-output *stable* observer. After pointing out the properties of this definition and the feedback system $\Sigma(P, Q)$, we obtain right-coprime factorizations for a class of nonlinear plants, where a two-input one-output pseudo-state observer is constructed. A feedback stabilization scheme is given using this observer.

1.2. C.A. Desoer, M.G. Kabuli, "Nonlinear Stable Unity-Feedback Systems with One Linear Subsystem," *UCB/ERL Memo* M87/91, December 1987.

Abstract: In this paper, we consider the *nonlinear* unity-feedback configuration where one of the two subsystems (either the plant or the compensator) is specified by a *linear* (not necessarily time-invariant) map. If the linear subsystem has a left-coprime factorization, the nonlinear subsystem is shown to have a specific normalized right-coprime factorization. If the linear subsystem also has a *normalized* right-coprime factorization, we obtain a parametrization of the set of all stabilizing nonlinear subsystems; this parametrization can be interpreted as: i) that of all stabilizing nonlinear compensators for a given *linear* plant or ii) that of all nonlinear fractional perturbations of a possibly nonlinear plant stabilized by a given *linear* compensator. It is interesting to note that the set of all stabilizing nonlinear compensators (for a given linear plant) (or interpreted differently, the set of all stable nonlinear fractional perturbations of a nonlinear plant, stabilized by a given linear compensator) is precisely of the form well-known for the linear case except that certain maps including the free stable parameter Q are nonlinear.

1.3. The following is a status update of the papers listed in the previous progress report.

- "Stabilization and Robustness of the Nonlinear Unity-Feedback System," *International Journal of Control*, Vol. 47, No. 4, pp. 1133-1148, April 1988.
- "Right-Factorization of a Class of Time-Varying Nonlinear Systems," *IEEE Transactions on Automatic Control*, Vol. 33, No. 8, pp. 755-757, August 1988.
- "Linear Stable Unity-Feedback System: Necessary and Sufficient Conditions for Stability under Nonlinear Plant Perturbations," *IEEE Transactions on Automatic Control*, Vol. 34, No. 1, January 1989.

Proposed Research 1988 - 1989

We will continue the new directions of research commenced last year with the shift in focus from linear, adaptive control to non-linear flight control with the methodology shifting to two candidate applications:

(i) The flight control of VTOL aircraft.

(ii) The generation of command trajectories for the low-level control of multi-fingered hands and flight control systems.

The past year has seen closer exchanges with the NASA Ames Flight Control group involving some students working with models, data and techniques such as John Hauser, Andrew Teel and Joseph Sifer. The following topics of research will be pursued:

1. Identification of Non-linear Systems

So far, we have developed generic control techniques for simplified models of VTOL aircraft. This year we will need to develop more sophisticated analytical models from simulation models and flight data. We plan to use B-splines as the basic set for modelling the uncertainties. They are certainly rich enough to model the system; whether or not they will provide robust models needs to be studied. We will bring to bear a combination of symbolic simulation tools and identification tools to model the dynamics of the Harrier from existing simulations and flight data.

2. A Robust Theory of Linearization

2.1. Slightly Non-Minimum Phase Systems

We have developed a theory for effectively dealing with the slightly non-minimum phase characteristics of planar VTOL models. We will follow up this work with the extension to more complicated three degree of freedom VTOL models. We have developed a joy stick based simulation package so that the resulting controller may be "flown" to determine its efficacy.

2.2. Linearization in the Presence of Uncertainty

Last year we inaugurated a program of adaptive control for linearizable non-linear systems. This was a technique for dealing with non-linear systems which have parametric uncertainty. We have now studied other techniques for robust linearization in the presence of uncertainty or poor modeling of the non-linearities such as sliding mode control and high gain; as well as the zero dynamics algorithm of Isidori. We will try some of these techniques on the Harrier dynamics.

2.3. Command Generation for Control Systems

Most control methodologies begin with a certain desired trajectory that the plant is required to track. The generation of these trajectories from a qualitative description of the control tasks is far from trivial or obvious - especially given the constraints on actuators, finite slow rates, etc. It is the context of this problem that we instigated the work on multi-fingered hands on this grant. This year we will use the expertise of a pilot to better understand the process of trajectory generation for the VTOL/Harrier. Also, proceeding apace will be high level control for the two-fingered hand.

3. Factorization Based Approaches

We propose to investigate the use of these factorization techniques to clarify two important problems:

(i) The robustness problem: that is, how to characterize the robust design, how to find the achievable robustness for a given specification. We must recall that an important advance in *linear* theory was Zames's elucidation of the trade-off between robustness and performance for the MIMO case.

(ii) Two of the most important properties of feedback systems are the disturbance rejection and tracking properties. We propose to study these problems using our factorization methods.

Progress Report of S. S. Sastry and his Students, 1988-1989

During the past year three graduate students of S.S.Sastry were supported by this grant: John Hauser, Andrew Teel and Raja Kadiyala. John Hauser finished his Ph. D. dissertation in August 1989 on *The Approximate Linearization of Nonlinear Systems* with emphasis on the dynamics of V/STOL aircraft. Both Andrew Teel and Raja Kadiyala have finished their M.S. project reports on the indirect adaptive control of nonlinear systems.

Also, using additional funding furnished by this grant we organized the 1989 NASA Ames - UC Berkeley workshop in Nonlinear Flight Control held between August 21st and August 24th this year. This workshop featured three days of presentations by specialists in the areas of both nonlinear control and flight control and was attended by about 80-100 researchers. The workshop also served to showcase some of the joint work in this area being done by NASA Ames and UC Berkeley. Here are some of the details of work done on the grant and the abstracts of key publications:

1. Approximate Linearization of Nonlinear Systems

While necessary and sufficient conditions for both full state and input-output linearization have been given in the literature a great deal remains to be done in making these conditions part of a comprehensive design methodology. In the work started on this project we have taken the first steps to identify those conditions in the linearization method which are not robust. We have then proceeded to examine how when these conditions fail to be satisfied but fail to do so by a small amount that conditions for the approximate linearization exist. The development of the theory has been guided by two key examples, the first being the model of a planar V/STOL aircraft and the second being a ball and beam modelling the dynamics of variable wing stores or fuel slosh in an aircraft wing. In the former case the nonlinear system is slightly nonminimum phase so that linearization results in a closed loop unstable system; while in the latter case the system fails to have relative degree so that the linearization technique cannot be directly applied. The following publications and their abstracts reflect this work:

Approximate Tracking for Nonlinear Systems
with Application to Flight Control

John Hauser

In this dissertation, we embark on a project to make recent theoretical advances in geometric nonlinear control into a *practicable control design methodology*.

The method of input-output linearization by state feedback provides a natural framework to design controllers for systems, such as aircraft, where output tracking rather than stabilization is the control objective. Central notions include relative degree and zero dynamics. Roughly speaking, the relative degree of a system is the dimension of the part of the system that can be input-output linearized and the zero dynamics are the remaining (unobservable) dynamics. Systems with exponentially stable zero dynamics are analogous trajectories with internal stability.

While investigating the use of these methods in the control of the V/STOL Harrier aircraft, we notice that the small forces produced when generating body moments caused the aircraft to have an unstable zero dynamics, i.e., to be nonminimum phase. However, if this coupling were zero, then the aircraft could be input-output linearized with no zero dynamics. In other words, a small change in a parameter resulted in a significant change in the system structure!

With this observation as the driving force, this dissertation studies the effects of system perturbations on the structure of the system and develops methods for tracking controller design based on approximate systems.

After reviewing the basics of geometric nonlinear control, we show that small regular perturbations in the system can result in singular perturbations in the zero dynamics. We give asymptotic formulas for the resulting fast dynamics.

Next we develop techniques for tracking control design for systems that do not have a well defined relative degree. Using an approximate tracking for the true system. This approach is shown to be superior to the usual Jacobian linearization method on a simple ball and beam system.

Returning to the aircraft control problem, we use a highly simplified planar VTOL aircraft model to illustrate the (slight) nonminimum phase characteristic of these systems and develop a controller to guarantee approximate tracking. We also develop a formal theory for this class of systems.

Zero Dynamics of Regularly Perturbed Systems are Singularly Perturbed

S. S. Sastry, J. E. Hauser and P. V. Kokotovic

In this paper we present new results on the structure of the zeros of linear and nonlinear systems under perturbation. In particular we show that when state space descriptions of linear or single-input single-output systems with relative degree ≥ 2 are regularly perturbed, then their zero dynamics are singularly perturbed and show a separation of time scales. In the SISO case, we give asymptotic formulas for the new *high frequency* zero dynamics arising form the regular perturbation.

Nonlinear Control via Approximate Input-Output Linearization: the Ball and Beam Example

J. E. Hauser, S. S. Sastry and P. V. Kokotovic

In this paper, we study approximate input-output linearization of SISO nonlinear systems which fail to have relative degree in the sense of Byrnes and Isidori. This work is in the same spirit as the earlier work of Krener on approximate full state linearization by state feedback. The general theory presented in this paper is motivated through it's application on a ball and beam system, a commonly used undergraduate control laboratory experiment; where it is shown to be superior to the standard Jacobian linearization for this example.

2. Adaptive Control of Nonlinear Systems

In previous work, we had studied the use of parameter adaptation to robustify the cancellation of nonlinear terms in the exact linearizing control laws. The work had been restricted to the use of direct adaptive control techniques, i.e. there was no explicit separation between identification and control. While the results were analytically quite pleasing and also practically implementable on a model of a robot manipulator we found that the amount of computation required in its implementation was rather large. Consequently we spent last year studying techniques for indirect adaptive control involving an explicit separation between identification and control. We began with a study of nonlinear identifiers both of the least squares and projection variety and then used them to give a few indirect adaptive schemes one based on a self tuning principle and the other on a reduced order nonlinear observer. The schemes were proven to be convergent and a systematic study of the relative merits of each of the schemes on several examples was completed in the following work:

Indirect Techniques for Adaptive Input Output Linearization of Nonlinear Systems

R. Kadiyala, A. Teel, P. Kokotovic and S. Sastry

A technique of indirect adaptive control based on certainty equivalence for input output linearization of nonlinear systems is proven convergent. It does not suffer from the overparameterization drawbacks of the direct adaptive control techniques on the same plant. This paper also contains a semi-indirect adaptive controller which has several attractive features of both the direct and indirect schemes.

Progress Report of C. A. Desoer and his Students, 1988-1989

Report on Research by C.A Desoer and M.G. Kabuli

(Kabuli, who was supported by this grant, completed his dissertation in August of 1989)

The results of our recent research on Factorization techniques for the study of feedback systems has been written up in a self consistent form in the following report. These new results were presented at the MTNS Symposium in Amsterdam, June, 1989.

Factorization Approach to Nonlinear Feedback Systems
by
C.A. Desoer and M.G. Kabuli
ERL Memorandum UCB/ERL M89/64

In its general algebraic framework, factorization theory has proven to be extremely useful in solving interesting control problems related to linear time-invariant systems that have transfer function representations. This work studies the extension of factorizations to nonlinear multiinput-multioutput maps.

The nonlinear maps considered are assumed to be causal (i.e., non-anticipatory) and are defined over input and output extended spaces; hence the setting is quite general and is suitable for analyzing unstable nonlinear maps. Due to the flexibility of choosing norms in input and output spaces, this input-output approach is suitable for generalized forms of bounded-input bounded-output stability analysis.

Factorization tools are applied to stability and robustness analysis of nonlinear additive feedback systems. These tools are also used to propose stabilizing feedback schemes.

Proper stable factorizations of linear time-invariant finite-dimensional systems and related key facts are reviewed for motivation; they lead to a compact self-contained formulation of stability and robustness properties.

Stabilizing feedback systems and existence of factorizations are studied based on a discussion of factorization tools for general linear maps. Using factorization tools, necessary and sufficient conditions are given for robust stability of the nominal linear unity-feedback system under nonlinear (possibly unstable) additive, feedback, pre-multiplicative and post-multiplicative plant perturbations.

Following a discussion for right-factorization tools for nonlinear causal maps, a stabilizing additive feedback configuration is proposed. Right-factorization and right-coprime factorization examples for some classes of nonlinear plants are explicitly worked out. After stating conditions on linear (not necessarily time-invariant) plants for parametrizing the set of all nonlinear stabilizing compensators in nonlinear unity-feedback systems the parametrization of all stabilizing nonlinear compensators is obtained. Stability and robustness of nonlinear unity-feedback system and conditions for simultaneous stabilization are studied using factorization tools.

RESEARCH PROPOSAL

1989-1990

Research Proposal 1989-1990

We will continue the work instigated last year on making a practicable design methodology for non-linear control systems with applications to flight control. The following topics will be pursued:

1. Linearization for Non-regular MIMO Systems

In work done on this grant last year, we studied the approximate linearization of nonlinear systems which were either slightly nonminimum phase or failed to have relative degree by a small amount. both of these investigations were primarily SISO for technical reasons having to do with the fact that the MIMO linearization results were not yet fully unified (except for the instance of full state linearization, which was solved by Hunt, Su and Meyer). Recently we have been made aware of the theory of differential algebra which is useful in defining the notions of regular MIMO systems and systems which are decouplable by dynamic state feedback. The MIMO counterpart, then, of systems which have relative degree is systems which are regular. Our program this year will be to study the approximate linearization of systems which fail to be regular or if they are regular are close to not being regular (like the V/STOL aircraft models). Some preliminary work in this regard has begun in joint work with DiBenedetto, Grizzle and Hauser.

2. Regulator Theory for Nonlinear Systems

In recent work by Byrnes and Isidori there have been new necessary and sufficient conditions for the existence of nonlinear regulators. There are two drawbacks to these results, even though they apply to non-minimum phase systems (indeed this is their chief virtue): they are too closely based on the Jacobian linearized system and they are somewhat non-constructive, in the sense that the compensator is described in terms of the existence of solutions to certain first order partial differential equations.

We will remedy both of these deficiencies: the former by incorporating into this theory recent work by Krener and other researchers on higher order (higher than 1, that is) observers and the latter by a systematic approximation technique for solution of the partial differential equation deriving from the fact that the regulator PDE is a center-manifold type of PDE for which a systematic approximation theory exists.

3. Command generation for Control Systems

Most control systems begin with a description of trajectories that need to be tracked or models of signals which need to be followed. The generation of these trajectories which respects actuator constraints as well as intrinsic dynamical constraints on the system is far from evident. While we have made very little progress on this topic last year we will continue to persevere in this line of investigation with inspiration from models derived from NASA.

4. From Numerical Models to Symbolic Computation

A key area of investigation is the ability to make analytically tractable models for design starting from numerical and wind tunnel data ... this is especially important when the analytical model is differentiated repeatedly to get the controller. During this year we hope to begin a program of research into systematic model identification in collaboration with Dr. G. Meyer of NASA Ames and Mr. Jim Smolka of NASA, Edwards.

89 - 90

Tools for Nonlinear Control Systems Design: Continuation Proposal

S. S. Sastry, C. A. Desoer

November 26, 1990

1 Introduction

In this year we have continued the program of developing theoretical and CAD tools for the rapid prototyping of nonlinear control systems. Highlights of the program this year were :

1. We sponsored a workshop on *Nonlinear Control* along with NSF and the UC Nonlinear Sciences Institute. This workshop was held in April 1990 at UC Santa Barbara and was billed as the second UC-NASA workshop in Nonlinear Control. Even though the workshop was only eight months after the workshop at Berkeley it was a big success, attended by almost 80 researchers, a tribute to the vitality of the field. We propose that a tradition be made of UC-NASA workshops held either every year or every other year with the location shifting between Berkeley and Santa Barbara.

2. Two Berkeley students spent the summer with Dr. George Meyer working with him on encoding the details of his flight control methodology with the eventual goal of making it the heart of a nonlinear controller design package for UNIX workstations under an X windows environment.

The rest of this renewal proposal is organized as follows: in Section 2 we give a brief progress report for the year 89 – 90 and in Section 3 we present the renewal proposal for 90 – 91.

2 Progress Report 89-90

During the past year three graduate students of S. S. Sastry were supported by this grant: Raja Kadiyala, A. K. Pradeep and Andrew Teel. Andrew Teel and Raja Kadiyala finished their M. S. project reports combining CAD tools for nonlinear control with indirect adaptive control of nonlinear systems. The topics can fit into two categories:

2.1 Robust and Adaptive Nonlinear Control

The first topic of research was to use adaptive and sliding mode control to robustify linearization by state feedback as a methodology for nonlinear control. Since linearization is based on exact cancellation it is prone to lack of robustness when the cancellation is not exact. In our past work we had produced new results in nonlinear adaptive control to make the cancellation of the nonlinear terms asymptotically exact. The present work is an extension of this work using techniques of indirect adaptive control, namely explicit identification followed by nonlinear control. We have also explored the use of sliding mode control as a methodology to reject model uncertainties of certain classes. The papers based on this research are the following:

1. **Indirect Techniques for Adaptive Input Output Linearization of Nonlinear Systems**, by Teel, Kadiyala, Kokotovic and Sastry, to appear in Intl. J. of Control, Dec 1990.

Abstract

A technique of indirect adaptive control based on certainty equivalence for input output linearization of nonlinear systems is proven convergent. It does not suffer from the overparameterization drawbacks of the direct adaptive control techniques on the same plant. This paper also contains a semi-indirect adaptive controller which has several attractive features of both the direct and indirect schemes.

2. **Generalized matching Conditions for Sliding Mode Control of Perturbed Nonlinear Systems**, by Pradeep, presented at the UC-NASA workshop at SB, April 90.

Abstract

There has been a great wealth of theoretical machinery built up for controlling and analyzing nonlinear systems culminating in a rather complete characterization of linearization by state feedback. Feedback linearization, however, has been somewhat slow to catch on in "real world" applications mostly due to the fact that there does not exist a good CAD tool which handles feedback linearization of nonlinear systems. In turn, the nonlinear CAD tool development has been hampered in the past since the calculations necessary to formulate the feedback law are symbolic in nature, but with the advent of new symbolic processing packages and the unleashing of new compute horsepower it has become feasible to carry out the necessary computations.

We will present the construct of a nonlinear control CAD tool which is in its preliminary stages and discuss issues such as model extraction and spline fitting to create the descriptional equations of a system. We will also propose possible architectures for real time controllers on which the generated controller my be implemented.

In yet another approach we have been exploring the use of the new results of Byrnes and Isidori on nonlinear regulation for systems which may not be minimum phase or even have well defined relative degree. A good practical system which has stimulated the theory in this direction has been the ball and beam system which is a model of fuel slosh in aircraft wings. The papers written in this area are:

1. **Applications of Nonlinear Output Regulation**, Teel and Sastry, UC-NASA workshop at SB, April 90.

Abstract

Recently, results have been established by Byrnes and Isidori [1] for output regulation of nonlinear systems. In this framework, deriving the appropriate control law involves solving a partial differential equation to find an invariant, error zeroing manifold. We have applied these results to the problem of tracking a reference trajectory for the ball and beam example. The output regulation theory is especially useful for this example because the ball and beam does not exhibit a well-defined relative degree. We have examined the effects of making polynomial approximations (of various orders) for the invariant manifold in this example which makes solving the P.D.E. more tractable.

The output regulation theory for nonlinear systems relies on exact knowledge of the exosystem which models the class of disturbance and reference signals. We have examined the tracking results when there is uncertainty in the model of the exosystem but its stability properties are retained. We plan to look at adaptively accounting for these uncertainties in this context. Further, we plan to look at tracking results when the exosystem varies slowly with time.

2. **Robust and Adaptive Nonlinear Output Regulation**, Teel, submitted to European Control Conference, Oct. 90

Abstract

The object of this paper is to prove the stability of an adaptive control scheme designed to asymptotically achieve output regulation for a class of nonlinear systems. Center manifold theory is the frame on which most of the analysis rests. The task is restricted to the tracking of signals that are generated by Poisson stable exosystems. No growth restrictions are made on the vector fields of the nonlinear system. This is attractive because the analysis can be extended to certainty equivalence of exact input-ouput linearizing control schemes for tracking of Poisson stable trajectories.

Keywords. Nonlinear Output Regulation, Adaptive Control, Center Manifold, Slowly-varying.

3. **Toward Larger Domains of Attraction for Local Nonlinear Control Schemes**, Teel, submitted to European Control Conference, Oct. 90

Abstract

This paper is motivated by the fact that the success of approaches to controlling nonlinear systems that cannot be exactly linearized is very sensitive to initial conditions. The focus of this paper will be extending the regions of attraction for recent control schemes when applied to SISO nonlinear systems that do not have a well-defined relative degree.

The basic conclusions that we have obtained so far show that the theory still has a fair distance to go before it is a practical design method, since the technique is very sensitive to initial conditions .As a consequence adaptation does not significantly enhance the domain of attraction.

2.2 CAD tools for nonlinear systems

We have proceeded steadily forward towards developing CAD tools for nonlinear control systems design. Two papers report the code written to date. For the most part the code is written using Mathematica as the basic computational engine. We are hoping to interface this code to a more standard simulation environment such as Matrix-X in future.

1. **CAD for Nonlinear Control Systems**, Teel, ERL Memo UCB/ERL M 89/135, December 1989.

Abstract

A computer software package has been developed to assist in the control design for a certain subclass of nonlinear systems. Much progress has been made in the area of nonlinear control recently. The design procedure that is a result of these advances has been automated in this software package. Both single-input single-output (SISO) and multi-input multi-output (MIMO) systems are handled.

2. CAD and Implementational Tools for Nonlinear Systems, Kadiyala and Sastry, UC-NASA workshop, SB, April 90.

There has been a great wealth of theoretical machinery built up for controlling and analyzing nonlinear systems culminating in a rather complete characterization of linearization by state feedback. Feedback linearization, however, has been somewhat slow to catch on in "real world" applications mostly due to the fact that there does not exist a good Computer Aided Design (CAD) tool which handles feedback linearization of nonlinear systems. In turn, the nonlinear CAD tool development has been hampered in the past since the calculations necessary to formulate the feedback law are symbolic in nature, but with the advent of new symbolic processing packages and the unleashing of new compute horsepower it has become feasible to carry out the necessary computations.

A package, which is specific to flight control, has been developed in conjunction with NASA Ames Research Center. Although this package is not symbolic in nature, it does carry through the necessary calculations numerically, which then renders an aircraft input-output linear. The controller created can than be readily implemented on the current generation of flight control computers.

We are currently constructing a nonlinear control CAD tool package which will implement the necessary operations for feedback linearization on a general class of systems and also handle the model extraction and spline fitting that is necessary to develop the descriptional equations of a system. The traditional nonlinear tools available in present packages will be incorporated as well. We will also look into possible architectures for real time controllers on which the generated controller may be implemented.

[1] George Meyer *Application of Brunowsky Froms in Multi-Mode Flight Control*, 1990 American Control Conference, Sheraton Harbor Island Hotel, San Diego, California, May 23-25, 1990

3 Proposal for 1990-91

We will continue the program of adaptive linearization: a new paper on direct adaptive linearization of MIMO nonlinear systems is under preparation (jointly with DiBenedetto). This work supported by this grant will give a comprehensive solution to the nonlinear MIMO model reference adaptive control problem. This work will bring to a finish the program of adaptive linearization begun three years ago. This work will be presented at the European Control Conference, 1991.

Further we will continue the program of developing the new nonlinear regulator theory (which is not based on cancellation of nonlinear terms) and study how to enhance it's domains of attraction and robustness. We feel that the area of nonlinear regulation is the best methodology available for controlling nonlinear systems which are either not linearizable or are non-minimum phase. In addition to making the theory more practical we will also study techniques for approximately solving the center manifold style partial differential equations.

In keeping with the long range goal of developing CAD tools for nonlinear control systems design we will continue to write code in an integrated framework both for nonlinear adaptive control, flight control and nonlinear regulators.

The key problem that we will address in this year however will be the following nonlinear flight control problem which combines aspects of approximate linearization for slightly nonminimum phase nonlinear systems and trajectory planning for nonlinear systems: Consider the following model for a rigid body model of an aircraft,

$$
\begin{aligned}
m\ddot{r} = & \; mg\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} + R_1(\begin{bmatrix} u_1 \\ \epsilon u_3 \\ \mu u_4 \end{bmatrix} + R_2|\dot{r}|^2\begin{bmatrix} c_D(1+c_1\alpha^2) \\ c_s\beta \\ c_L(1+c_2\alpha) \end{bmatrix}) \\
\dot{R}_1 = & \; \omega \times R_1 \\
J\dot{\omega} = & \; \omega \times J\omega + \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix}
\end{aligned}
$$

In the equation above r stands for the position of the CM of the aircraft in inertial coordinates, $R_1 \in SO(3)$ stands for the orientation of the aircraft body frame relative to the inertial frame and $R_2 \in SO(3)$ is the orientation of the aircraft wind frame relative to the aircraft body frame. The angle of attack angle α and the side slip angle β describe the orientation R_2 as

$$R_2 = e^{S(\hat{y})\alpha)} e^{S(\hat{z})\beta}$$

with $S(\hat{x})$ is the skew symmetric matrix associated with the cross product operation with the unit vector in the x direction, i.e. $(1,0,0)\times$. Similarly for $S(\hat{z})$. The coefficients c_D, c_S, c_L stand for the drag, side force and lift force coefficients. The coefficients ϵ, μ model the parasitic coupling between moment generation and force generation. The input u_1 is the thrust input and the inputs u_2, u_3, u_4 stand for the moment inputs to the aircraft. Thus the system has four inputs. The system can thus be used to independently control four outputs. A sample choice is the three position variables r and the roll angle θ. Of course θ is one of the variables in R_1 as

$$R_1 = e^{S(\hat{z})\phi} e^{-S(\hat{y})\theta} e^{S(\hat{x})\psi}$$

With these four outputs the system is weakly nonminimum phase, because of the cross coupling ϵ, μ. It is important to precisely formulate the different choices of outputs which can be decoupled approximately by state feedback. Such a program was laid out by Meyer in his paper at the American Control Conference , 1990. We will carry out the details of his program and describe the different modes which the aircraft can fly in.

In the second half of the program for this year we will study the problem of trajectory generation for aircraft on maneuvers involving a combination of different regimes of control and strategies for switching between them. We will discuss the implications of having a hybrid or event driven control system which has continuous system dynamics and discrete or logic dynamics. In this part of the program we will continue our close collaboration with Dr. Meyer.

Tools for Nonlinear Control Systems Design: Continuation Proposal

S. S. Sastry, C. A. Desoer

October 24, 1991

1 Introduction

In the year 1990-91, we worked on four areas of nonlinear control:

1. **Adaptive Control of MIMO Nonlinear Systems**
2. **Robust and Adaptive Regulators for Nonlinear Systems**
3. **Control of Nonlinear Systems in the Presence of Saturation**
4. **CAD tools for Approximate linearization**

2 Progress Report 90-91

During this year three graduate students of S. S. Sastry were supported on this grant: Raja Kadiyala, A. K. Pradeep and Andrew Teel. They all expect to finish their Ph. D. dissertations between January and June 1992. In addition we partially supported Prof. M. Di Benedetto during her visits here.

2.1 Robust and Adaptive Control of MIMO nonlinear systems

We developed starting from our earlier work on adaptive control of SISO nonlinear systems a theory of model reference adaptive control for MIMO nonlinear control systems. We also continued with our work on adaptive control of SISO nonlinear systems with a paper describing its application to the control of induction motors:

Indirect Adaptive Nonlinear Control of Induction Motors by Kadiyala, submitted to the IFAC Conference NOLCOS, June 1992 in Bordeaux, France.

An indirect adaptive control law based on certainty equivalence is designed for a model of the induction motor with the assumption that the magnetic subsystem is linear. This nonlinear control law renders the induction motor system input-output linear and also achieves input-output decoupling. In addition, we find for the specific case of the induction motor we are able to prove parameter convergence and asymptotic tracking of a reference trajectory using the indirect adaptive controller. This result differs from the generic case where we can only show asymptotic tracking. The indirect adaptive control methodology also does not suffer from the drawback of overparameterization as in the direct adaptive control technique. Simulations are also given comparing nonadaptive, direct adaptive, and indirect adaptive nonlinear controllers.

The work on adaptive control of MIMO nonlinear systems was as follows:

Adaptive Linearization and Model Reference adaptive Control for a Class of Nonlinear Systems, by Di Benedetto and Sastry, in proceedings of the European Control Conference, Grenoble, 1991 and to appear in the Journal of Mathematical Control, Estimation and Control, 1992.

This paper discusses two schemes for the adaptive control of MIMO nonlinear systems with parametric uncertainty in their dynamics. The first approach is an adaptive version of a dynamic precompensation scheme of Descusse and Moog for linearizing right invertible MIMO nonlinear systems. The second approach is Model Reference Adaptive Control. The scheme presented is an adaptive version of the scheme proposed by Di Benedetto for asymptotic model matching using static state feedback. We also show that the non-adaptive model matching control law can be specialized to yield a tracking control law by choice of a suitable model and give an adaptive version of this algorithm.

In this paper, the first topic that we cover is adaptive input-output linearization of general MIMO systems. We consider the class of systems that may be linearized by dynamic compensation. We then give an adaptive version of the algorithm of Descusse and Moog for dynamic linearization and decoupling. In the process, we give a precise characterization of the prior information needed to build the adaptive controller.

Next, we begin with the problem of general model reference adaptive control of nonlinear systems. We take advantage of some recent results on (non-adaptive) asymptotic model matching with stability for general MIMO nonlinear systems, to begin this program. These results are also interesting in that they allow for specialization to the problem of tracking for general MIMO systems by static state feedback. Our results on nonadaptive tracking using static feedback for general MIMO nonlinear systems is a by-product of the results of Di Benedetto but have not appeared in the literature, to our knowledge. Using these non-adaptive algorithms as basis we give an adaptive version of the algorithms. The prior information needed to implement the scheme and is also discussed.

2.2 Robust and Adaptive Regulators

In recent years there has been a great deal of interest in the use of nonlinear regulators for tracking signals being generated by exo-systems by plants which are not necessarily minimum phase or input-output linearizable. However, the solutions given by Byrnes and Isidori suffer from the drawback that they are extremely difficult to compute and that they have small domains of attractions. We have worked on ameliorating these difficulties and also in making the regulator adaptive:

Robust and Adaptive Nonlinear Output Regulation, by Teel, Proceedings of the European Control Conference, Grenoble, 1991.

Recently, Byrnes and Isidori have established results for output regulation of nonlinear systems. In this framework, deriving the appropriate control law involves solving a partial differential equation (PDE) to find an invariant. In this framework, deriving the appropriate control law involves solving a partial differential equation (PDE) to find an invariant, error zeroing manifold. In general, to achieve regulation, no uncertainty is permitted in the state equations of the plant or exo-system (which generates trajectories and disturbances.)

We have examined the output regulation problem in the case of parametric uncertainty in the plant and exo-system. We have shown that, when the states of the plant and exo-system are available for feedback, a slowly-varying adaptive scheme can be used to achieve regulation asymptotically. The basic structure of the adaptive scheme employs a standard "observer-based" identifier of our past work on indirect adaptive control and relies on the nonlinear slowly-varying result of Hoppensteadt.

Toward Larger Domains of Attraction for Local Nonlinear Control Schemes, by Teel, Proceedings of the European Control Conference, Grenoble, 1991.

This work is motivated by the observation that the success of some recent nonlinear control approaches is very sensitive to initial conditions. We especially focus on the recent results of Byrnes and Isidori for (local) output regulation of nonlinear systems. In this framework, achieving regulation involves solving a partial differential equation (PDE) to find an invariant, error zeroing manifold and then making this manifold attractive. Then, if the initial conditions of the system are very close to this manifold, the state trajectory decays to the manifold and regulation is achieved. However, it may not be realistic to expect the initial state to start very close to the manifold.

We have chosen to overcome this problem by augmenting the manifold in such a way that the initial conditions are necessarily close to the new manifold and the new manifold asymptotically becomes the original error zeroing manifold. We are able to augment the manifold by adding additional (decaying) states to the exo-system (which generates trajectories and disturbances.) In essence, we are changing the prescribed trajectory such that it is close to the initial states and asymptotically becomes the original trajectory. The results are still local. However, for some systems, this approach can greatly increase the range of initial conditions that can be handle with the nonlinear regulator.

2.3 Control of Systems with Actuator Saturation

In a set of two papers, Teel has developed a great deal of the machinery required to stabilize certain classes of nonlinear and linear systems with actuator limits. These papers have excited a great deal of interest in the community and have already been generalized.

Global Stabilization and Restricted Tracking for Multiple Integrators with Bounded Controls, by Teel, submitted to NOLCOS, Bordeaux, France, 1992, accepted in Systems and Control Letters.

Necessary and sufficient conditions for globally stabilizing linear systems with bounded controls are known. In essence, the system must be stabilizable and not have any eigenvalues in the open right half plane. Constructing such a control law, however, is not straightforward. In the work of Sussmann and Sonntag, a complicated recursive procedure was outlined for producing such a control which requires one to solve for a certain submanifold of the state space. Further, it was shown there that no saturation of a linear feedback can globally stabilize a simple chain of integrators of dimension 3 or more.

In our work, we have developed a *nonlinear* combination of saturation functions of *linear* feedbacks that globally stabilizes a chain of integrators of arbitrary order. The appealing feature of the proposed control is that it is easy to construct. Further, it is linear near the origin and can also be used to achieve trajectory tracking for a class of trajectories restricted by the absolute bound on the input.

Using Saturation to Stabilize a Class of Single-input partially linear Composite Systems, by Teel, submitted to NOLCOS, Bordeaux, France, 1992.

We are interested in globally (semi-globally) stabilizing single-input nonlinear systems that cannot be globally full-state linearized. We focus on partially linear composite systems where the dynamics of the nonlinear subsystem are *not* zero-input asymptotically stable. We specify a class of such systems where a linear plus (small) bounded control can be used to stabilize the composite system. A subset of systems in this class can be globally (semi-globally) stabilized using only a bounded control.

We propose algorithms that use our recent result for stabilizing a (linear) chain of integrators with bounded controls. In the nonlinear setting, the success of our algorithms depend only on the general properties of the nonlinear terms and not on their explicit form. Consequently the stability property is robust to unmodeled nonlinear terms that satisfy the general properties as well as unknown (possibly time-varying) bounded parameters.

2.4 CAD Tools for Nonlinear Systems

In work supported by this grant Raja Kadiyala has been developing a tool kit for approximate linearization called AP-LIN. This is based upon his experience in working with Dr. Meyer at NASA Ames in the summer of 1989 and is an attempt towards developing a real-time nonlinear control systems design toolkit: the software has been partially written and tested on some classroom style examples. More will be done in the Ph. D. dissertation work of Kadiyala. The following paper summarizes his work to date:

AP-LIN: A Tool Box for Approximate Linearization of Nonlinear Systems. by Kadiyala, submitted to the IEEE Conference on Computer Aided Control Systems Design.

A toolbox for nonlinear control system design is presented. This package contains modules to approximate systems to polynomials systems of arbitrary order and then render them input-output linear or input-state linear with arbitrary order error terms. We also discuss possibilities for real-time control.

2.5 Structure of the zero dynamics of nonlinear control systems

In addition we continued with our other work on structure of MIMO zero dynamics and how they change under perturbations. A paper based on this co-authored with Isidori, Kokotovic and Byrnes is to appear in the IEEE Transactions on Automatic Control in 1991.

Singularly perturbed zero dynamics of nonlinear systems by Isidori, Sastry, Kokotovic and Byrnes, to appear in the IEEE Transactions on Automatic Control, 1991.

Stability properties of zero dynamics are among the crucial input-output properties of both linear and nonlinear systems. Unstable, or "non-minimum phase", zero dynamics are a major obstacle to input-output linearization and high gain designs. An analysis of the effects of regular perturbations in system equations on zero dynamics shows that, whenever a perturbation decreases the system's relative degree, it manifests itself as a singular perturbation of zero dynamics. Conditions are given under which the zero dynamics evolve in two timescales characteristic of a standard singular perturbation form that allows a separate analysis of slow and fast parts of the zero dynamics. The slow part is shown to be identical to the zero dynamics of the unperturbed system, while the fast part, represented by the so called *boundary layer system*, describes the effects of perturbations.

3 Proposal for 1991-92

3.1 Approximate Linearization and Inversion for MIMO nonlinear systems

In past work on this grant, we have studied approximate linearization for systems without relative degree (for example, the ball and beam system) and for systems which are slightly non-minimum phase (the V/STOL Harrier example). In the first example, the system did not have relative degree and had ot be approximated by a system which was *regular*, that is, it had relative degree. In the second example, certain terms in the decoupling matrix representing the parasitic generation of small forces by moment generating control surfaces needed to be neglected in order to:

- improve the numerical conditioning of the inverse of the decoupling matrix.
- avoid cancellation of far off right half plane zeros.

In this work we will attempt to generalize this problem to the study of approximate inverses of nonlinear systems. The general observation appears to be that if there are small terms in the decoupling matrix, not only do they contribute to large numbers in the inverse, but they are also symptomatic of the appearance of far off zeros which would be cancelled by input-output linearization schemes unless they were neglected in the algorithm. Which terms need to be neglected is rather obvious in the instance of flight control, but is less so in other applications. With Di Benedetto and Grizzle we have begun to give methods for inversion of MIMO nonlinear systems which are both numerically robust and also do not cancel far-off zeros. So far, we have succeeded in obtaining robust versions of the Descusse Moog algorithm for input-output decoupling by dynamic extension. We have not yet obtained robust versions of the Singh algorithm or the Di Benedetto, Grizzle, Moog algorithm.

3.2 CAD Tools for Approximate Linearization

The basic package AP-LIN written this year needs a user interface, also it needs to be tired out on a number of design examples and extended from SISO to MIMO systems. The Ph. D. dissertation of Kadiyala will contain the user's manual and is expected between January and June 1992.

3.3 Control of Systems with Saturation

The Ph. D. dissertation of Teel will deal with the design of nonlinear control laws in the presence of actuator saturation. The linear situation has been addressed already and extended somewhat to the nonlinear context, but the algorithms could be improved further. Connections with sliding mode control laws will also be made.

3.4 Flight Control and Trajectory Generation

Though the questions of trajectory generation and switches in the operating regimes of an aircraft under the assumption of simplified models for the dynamics was proposed last year, we have not as yet made substantial project in this regard. We have however been studying the work of Singh using models of a slightly different nature. As soon as we have a firm handle in this regard. we hope to begin with a more detailed investigation of mode switching in flight control laws. Such an effort would complement one already under way at Ames under Dr. Meyer in collaboration with Drs. Hunt and Su.

3.5 NASA Ames — UC Berkeley Nonlinear Control Workshop

In October 1992, we will host at Berkeley the second NASA - Berkeley workshop in nonlinear control bringing together researchers from the US and overseas to participate in a workshop on new advances in nonlinear control, with emphasis on how these advances may impact control of aircraft and spacecraft. We will have active participation from European researchers who are involved in the European Space Agency (ESA) programs. The workshop will be about 3 days long and we will (as in 1989) provide some travel support to students to attend the workshop.

Tools for Nonlinear Control Systems Design: Progress Report for 1991-92 and Proposal for 1992-93

S. S. Sastry

October 19, 1992

1 Introduction

In the year 1991-92, we worked on the following areas of nonlinear control:

1. **Adaptive Control of MIMO Nonlinear Systems: Applications**
2. **Stabilization of Nonlinear Systems in the Presence of Saturation in the Actuators**
3. **CAD tools for Approximate linearization**

2 Progress Report 91-92

During this year four graduate students of S. S. Sastry were supported on this grant: Raja Kadiyala, A. K. Pradeep and Andrew Teel and Datta Godbole. Pradeep finished his Ph. D. in March 1992, Kadiyala finished his Ph. D. in April 1992 and Teel in May 1992. They are currently respectively at GE Corporate Research Center, Schenectady, Teknekron Control and Automation (a small start up in the area of control in Emeryville, CA) and at the Ecole des Mines (though Teel will take up a position at the University of Minnesota in January 1993).

2.1 Abstracts of Ph. D. dissertations completed in 1991-92

In this section, we have collected (verbatim) the abstracts of the three Ph. D. students, who finished this year and were supported on the grant, from 1989-1992.

2.1.1 Feedback Stabilization: Nonlinear Solutions to Inherently Nonlinear Problems — A. R. Teel

Control strategies are developed for nonlinear systems that fail to satisfy differential geometric conditions for input-to-state linearizability under state feedback and change of coordinates.

The central part of this work is motivated primarily by a popular "ball and beam" laboratory experiment. For this example, the differential geometric conditions for input-to-state linearizability are not satisfied. Strategies have been developed previously to overcome this limitation in a neighborhood of an equilibrium manifold in order to achieve (approximate) tracking and local stabilization. However, the domains of attraction for these methods are very small.

Control strategies are presented for a general class of nonlinear systems, of which the "ball and beam" is an example, which result in arbitrarily large domains of attraction for both the small signal tracking problem and the stabilization problem. The main component of the approach is the use of saturation functions to limit the destabilizing effects that cannot be removed by geometric linearization techniques. One of the new elements of this work is the nesting of saturation functions to systematically isolate and diminish these destabilizing effects.

One can think of a linear chain of integrator systems that are subject to "actuator constraints" as nonlinear systems that cannot be made to appear linear globally. The methodology of nested saturation functions provides new, simple globally stabilizing control laws for such systems.

In addition to developing methodologies for systems like the "ball and beam" and linear systems subject to "actuator constraints", asymptotically stabilizing control strategies are developed for a class of nonholonomic control systems. These systems generically do not satisfy geometric conditions for input-to-state linearization. New, smooth time-varying and locally stabilizing control laws are developed based on previous work in the literature on steering nonholonomic systems with sinusoids. Globally stabilizing strategies are then achieved by again introducing saturation functions.

Finally, results are presented that improve regions of feasibility for a recently developed nonlinear adaptive control scheme.

These different settings are used to argue for the desirability of tackling inherently nonlinear control problems with new, inherently nonlinear solutions. The case is made for continued research to develop powerful, specialized tools to add to the nonlinear control toolbox.

2.1.2 CAD Tools for Nonlinear Control — R. Kadiyala

In this dissertation I present a toolbox for nonlinear control system design. This toolbox (AP_LIN) contains modules to approximate systems by polynomials systems of arbitrary order and then render them input-output linear or input-state linear with error terms of arbitrarily high order. The approximation of the full nonlinear system to a polynomial nonlinear system allows us to compute the control law numerically as opposed to symbolically. Hence the computations can be made extremely fast. Furthermore, since the AP_LIN package is a stand alone package written in C, we have the ability to repeat the computations in real-time along the trajectory of the controlled system.

The task of approximating a system and updating the control law accordingly is very similar to adaptive control and we present a technique of indirect adaptive control based on certainty equivalence for input output linearization of nonlinear systems. This adaptive control scheme does not suffer from the overparameterization drawbacks of the direct adaptive control techniques on the same plant.

We give an example of the indirect control scheme by designing a controller for a model of the induction motor with the assumption that the magnetic subsystem is linear. We find that the adaptive nonlinear control law asymptotically renders the induction motor system input-output linear and also achieves input-output decoupling. In addition, we find that for the specific case of the induction motor we are able to prove parameter convergence and asymptotic tracking of an open set of reference trajectories using the indirect adaptive controller. This differs from the general indirect controller structure, where we cannot guarantee parameter convergence.

We also present a visualization tool which allows one to view the stability regions of nonlinear ordinary differential equations in three dimensions. We find that these computations may be carried out in parallel and present an algorithm for multiple networked workstations. We also discuss various viewing alternatives for the visualization of these dynamics.

An outline of the dissertation is as follows. We start with a review of the nonlinear control techniques implemented in the package and continue with a development of the indirect adaptive control scheme followed by the induction motor example. The AP_LIN package is then presented along with Sys_View, a tool for the visualization of stability domains for dynamical systems.

2.1.3 Sliding Mode Control of Perturbed Nonlinear Systems — A. K. Pradeep

In this dissertation, we present techniques and conditions for the robust control of perturbed nonlinear systems.

First, we develop matching conditions i.e., conditions to be satisfied by perturbations such that the control objective, namely asymptotic regulation, is achieved by the perturbed system, utilizing control laws for the unperturbed system. In the first three chapters of this dissertation we present statements and proofs of matching conditions for:

1. Perturbed SISO systems.
2. Perturbed, MIMO systems that possess vector relative degree.
3. Perturbed MIMO systems that are invertible but do not possess vector relative degrees.

We consider control laws for such systems developed in the framework of the zero dynamics algorithm and the dynamic extension method.

In chapter 4, we review in our notation some basic results on existence and uniqueness of systems with discontinuous right hand sides. Finally in chapter 5 we develop techniques that utilize sliding mode control theory to identify unknown parameters for a class of nonlinear systems. We then develop robust control laws using a Lyapunov control method that ensure stabilization in the presence of mismatched perturbations for a class of nonlinear systems.

We utilize sliding mode control theory for the purpose of synchronous regulation utilizing multiple sliding surfaces, and conclude this dissertation with a conjecture on fractional control.

3 List of Manuscripts Submitted or Published

1. R. R. Kadiyala, CAD Tools for Nonlinear Control, Proceedings of the IEEE Conference on Computer Aided Design, Napa, April 1992.
2. R. R. Kadiyala, Sys_View: A Visualization Tool for viewing the regions of validity and attraction of nonlinear systems, ERL Memo NO. M 92/21, March 1992.
3. A. Teel, Global Stabilization and Restricted Tracking for Multiple Integrators with Bounded Controls, to appear in Systems and Control Letters, also ERL Memo M 91/58, revised October 1991.
4. A. Teel, Error Based Adaptive Nonlinear control and regions of feasibility, submitted to Systems and Control Letters, also ERL Memo M 91/96.
5. A. Teel, Using Saturation to Stabilize a class of partially linear composite systems, Proceedings of the IFAC conference on Nonlinear Control Systems Design (NOLCOS), pp. 224-229.
6. G. Burgio, M. Di Benedetto and S. Sastry, Adaptive Linearization by dynamic state feedback: a case study, Proceedings of the IFAC conference on Nonlinear Control Systems Design (NOLCOS), pp. 594-600.
7. R. Kadiyala, Indirect Adaptive Control of Induction Motors, Proceedings of the IFAC conference on Nonlinear Control Systems Design (NOLCOS), pp. 144-150.
8. A. Teel, Semi-global stabilization of minimum phase nonlinear systems in special normal forms, to appear in International Journal of Control, also ERL Memo M 91/101, revised February 1992.

9. A. Teel, R. Murray and G. Walsh, Nonholonomic Control Systems: from Steering to stabilization with sinusoids, ERL Memo No. M 92/28, March 1992.

10. A. Teel, Beyond Linear Feedback for the nonlinear regulator, ERL Memo M 92/29, March 1992.

11. R. Kadiyala, AP_LIN: a tool box for approximate linearization of nonlinear systems, to appear in the IEEE Control Systems Magazine, also ERL Memo M 92/22, March 1992.

12. M. Di Benedetto and S. Sastry, "Adaptive Control for MIMO Nonlinear Systems. to appear in Journal of Mathematical Systems and Estimation, 1992.

13. A. R. Teel, R. M. Murray and G. Walsh, "Nonholonomic Control Systems: from Steering to Stabilization with Sinusoids", ERL Memo M 92/28, March 1992 and submitted to the IEEE Conference on Decision and Control, 1992.

4 Brief Outline of Research Findings

In this funding period we have begun a large activity in developing software for nonlinear control systems design. The software is being evaluated and tested on numerous practical control systems design problems. We have also continued our activity in adaptive control of nonlinear systems, both adaptive regulation and adaptive tracking and also sliding mode control of nonlinear systems. We also began a project on the use of saturation functions for stabilizing systems which either had finite escape time or for which the presence of actuator constraints actually made the linearizing control laws difficult to implement.

1. **CAD and Implementational Tools for Nonlinear Systems**

   The toolbox (AP_LIN) contains modules to approximate systems by polynomial systems of arbitrary order and then to render them input-output or input-state linear with error terms of arbitrarily high order. The approximation of the full nonlinear system by a polynomial nonlinear system allows us to compute the control law numerically as opposed to symbolically. Furthermore, since the package is a stand alone package written in C, we have the ability to repeat computations in real time along the trajectory of the controlled system. The task of approximating a system and updating the control law accordingly is very similar to adaptive control.

Finally, we have a visualization tool called SYS_VIEW which allows one to view the stability regions of the control laws. The calculations for this purpose are carried out in parallel on multiple networked workstations and the results presented on a Silicon Graphics workstation.

2. Adaptive Tracking for nonlinear systems

   In this research, we (joint work with M. Di Benedetto from the Università di Roma) studied two schemes for the adaptive tracking control of MIMO systems with parametric uncertainty in their dynamics. The first approach is an adaptive version of a static feedback law for tracking control based on some results on asymptotic model matching recently proposed by Di Benedetto. This scheme is based on some new techniques for extending the so-called zero dynamics algorithm of Isidori and Byrnes to problems of stable model matching followed by their specialization to tracking. The second scheme is an adaptive version of a dynamic precompensation law of Descusse and Moog for linearization using dynamic state feedback.

3. Sliding Mode Control of MIMO nonlinear systems

   The problem of developing precise matching conditions for nonlinear systems which are not linearizable by static state feedback has proved to be a surprisingly hard nut to crack. In early work on the grant we encountered success in developing matching conditions for MIMO systems linearizable by static state feedback. The extension of these results to either dynamically decouplable MIMO systems or other more general systems is not yet complete.

   However, our earlier experiments with sliding mode control laws have enabled us to understand solutions to stabilization problems where it may be shown that the underlying control system cannot be stabilized by continuous, state feedback.

4. **Using Saturation to Stabilize Nonlinear Systems**

   For a large class of systems with polynomial nonlinearities, there is a problem of finite escape time and also the problem that the systems cannot be globally full-state linearized. In the dissertation work of Teel, he used saturation functions to stabilize such systems and also to stabilize systems which could not be linearized because of actuator constraints. These results represent solutions to several classical problems of stabilizing systems in the presence of a very common nonlinearity: saturation. Among the surprises, one can show that it is not completely obvious how to stabilize a *linear* system consisting of a chain of integrators when saturation is presented. Teel presented a methodology using nested sequences of saturation functions for this purpose.

5 Proposal for 1992-93

5.1 Development of Rapid Prototyping Controller Design Tools

One of the most important goals of this grant is to develop at least at a conceptual level user friendly tools for nonlinear control, which contain on the one hand recent advances in the theory, but on the other hand also take advantage of recent advances in workstations to provide graphical and symbolic visualization of simulations. Our software has incorporated graphical depiction of our control laws on Sun workstations. This, we believe, is essential to allow for rapid prototyping of new nonlinear and adaptive control laws. The systematic development of the software in C with a good user interface are current topics of research. What has begun as an off-line CAD tool design effort has, owing to the development of computer hardware, become an attractive option for real time control: consequently the real time aspects of the computations are our future priorities.

5.2 Intelligent Control of Hybrid Systems

One of the major challenges in the design and prototyping of controller designs is the problem of aggregating continuous time dynamics and control with discrete event dynamics, arising from logical operations. This problem is particularly acute in the context of complex hierarchically organized control systems where at all but the lowest levels of the hierarchy the control is on models of a coarser granularity then differential equations. Such problems arise, for example, in highway automation problems, problems of command and control and also more generically in flight control (with many modes of operation) and process control. This represents a new direction of research on the grant and a transition towards tools for intelligent control systems combining features of low-level nonlinear control and control of discrete systems. A preliminary position paper explaining our current directions of investigation are contained in the next section.

5.3 Flight Control and Trajectory Generation

Though the questions of trajectory generation and switches in the operating regimes of an aircraft under the assumption of simplified models for the dynamics was proposed last year, we have not as yet made substantial progress in this regard. We have, however, been studying the work of Singh using models of a slightly different nature. As soon as we have a firm handle in this regard, we hope to begin with a more detailed investigation of mode switching in flight control laws. Such an effort would complement one already under way at Ames under Dr. Meyer in collaboration with Drs. Hunt and Su.

5.4 NASA Ames — UC Berkeley Nonlinear Control Workshop

In Summer 1993, we will host at Berkeley the third NASA - Berkeley workshop in nonlinear and intelligent control bringing together researchers from the US and overseas to participate in a workshop on new advances in nonlinear control and intelligent control of hybrid systems, with emphasis on how these advances may impact control of aircraft and spacecraft. We will have active participation from European researchers who are involved in the European Space Agency (ESA) programs and the Center for Intelligent Control Systems located at Harvard and MIT. The workshop will be about 3 days long and we will (as in 1989) provide some travel support to students to attend the workshop. We are hoping that the European Space Agency will be able to fund the travel of European researchers to attend the meeting as well.

6 Intelligent Control Systems: a position paper

We (myself and S. K. Mitter) have been teaching a 3 hour a week seminar class on intelligent control for the past 7 weeks this term at MIT, to try to organize our thinking about the research agenda in Intelligent control. Here are some notes that I have put down about my thoughts thus far. It is to be thought of as a preliminary position paper. The position paper is incomplete in the sense that it does not have a complete discussion of the literature and opportunities in Discrete Event Dynamical Systems (DEDS) and is somewhat superficial in its review of the literature on grammars, automata (especially Būchi automata) and on verification of program correctness, all of which are essential for a comprehensive research program in Intelligent Control Systems.

In the course, we have been emphasizing a pragmatic view of *Intelligent Control* as the *structure of complex, hierarchically organized control systems with several levels of sensory-motor control loops.* This keeps us from going off at tangents about the precise definition of the term "intelligent control". Some examples of such systems occur in biological motor control ([1], [2]), in Intelligent Vehicle Highway Systems (IVHS) ([10], [13]), in flight control, automotive control and in signal processing ([5]), to mention a few examples.

These examples are characterized by the following features:

1. Hybrid Dynamics consisting of continuous time dynamics and logic.
2. Hierarchically organized control, with the low level control being nonlinear control at the level of differential equations and the higher levels being analogous to the control of Markov chains (namely stochastic control of a finite state non-deterministic process).
3. Distributed intelligence and adaptation at all levels of the hierarchy.

Let us consider some of the examples a little further:

- **Biological Motor Control**
  Control of our musculo-skeletal system involves control at the muscular level (perhaps of the PD kind), at the spinal level, the pre-cortical and the cortical level. Some sequences of motor actions appear to be "hard-wired" into the spinal level, such as walking sequences, yet others seem to need cerebellar intervention (for example, motions that involve balancing or shifting weight) and still others appear to need full cortical intervention (especially those which involve some hand-eye coordination). It appears that the dynamical models used to plan one's movements are progressively less quantitative and more symbolic the higher one ascends in the hierarchy. Further, it appears that high level commands issued at the cortical level appear to generate progressively larger and larger groups (schemas) of commands (rather like a computer interpreter or compiler) down the hierarchy. Dual to this, there is a fan in of sensory information from the individual muscle fibers (with their muscle spindles and Golgi tendon organs) and the proprioceptors and tactile sensors up the same hierarchy with a view to presenting the relevant kind of signal for feedback at each level of the hierarchically organized control.

- **Intelligent Vehicle Highway Systems**
  One of the methods that is being suggested for easing of congestion on the highways is the idea of platooning cars so as to be able to steer them down highways at a high speed. Such an operation has been characterized as "making trains out of cars", but the situation is far more subtle: in particular, unlike a centralized controller for railroad cars or a completely decentralized controller for autonomous mobile robots, one has the need for decentralized intelligence in every car making for allowances in the needs and desires of the driver. Thus, the control of such an Intelligent Vehicle Highway System involves control at the *regulation layer* (nonlinear steering control in the lateral and longitudinal directions), *platoon layer* (control of a finite state process whose states are the cars in each platoon with the velocity and acceleration of the platoon leader), *link layer* involving segments of the freeway where platoons need to be monitored for optimal size and velocity and *network layer* assigning optima; travel time routes to each car entering the IVHS compatible with maintaining a good through-put of cars through

the system.

- **Flight Control, Automotive Control**
In the design of control systems for systems as diverse as aircraft and cars one builds controller subsystems for controlling different parts of the composite: thus, for example, in a car, one has anti-lock brakes, fuel injection, suspension control, intelligent cruise control (with collision monitoring and avoidance), etc. However, these systems do in fact interact dynamically with each other so that a logic level controller (involving a finite state process) is superimposed on these low-level controllers. It is seldom clear as to how to design the logic in a manner which is not ad-hoc and does not produce deadlocks or other incompatibilities. A similar situation exists for aircraft where control systems such as autopilots, engine control systems, flutter suppression, environment controllers all need to interact in order to not deadlock or otherwise confuse the composite system.

The chief features of these systems are hierarchies of models of increasing granularity with concomitant compression (or fan-in) of sensor data up the hierarchy and demodulation or decompression (fan-out) of control commands down the hierarchy. The hierarchy of models could for instance range from partial differential equation models to ordinary differential equation models to finite state machines (also, some times called De Marco diagrams) to formal languages and grammars. An example of a language for addressing these issues and also the issues of *uncertainty* at several scales is contained in ([6], [12], [7]).

Related to the question of hierarchies of models is the question of *signal to symbol* or *symbol to signal* transduction which seems to be the fundamental endeavor in the neural networks literature ([11]). Indeed the goals of neural networks are to train a circuit using signals so as to get them to encode patterns as symbols, which they recall at later times or use as a basis for generalization. This also appears to be the point to *fuzzy control* ([11], [14]) where the mechanism of fuzzy reasoning is used as an interface between the signal world and the symbol world (which is linguistic in a literal sense). Another such instantiation of a signal to symbol transducer is found in the so-called H dot equation of Brockett ([8], [9]) which uses a differential equation

$$\dot{H} = [H, [H, N]]$$

to perform several combinatorial optimization problems and discrete searches.

Questions of model uncertainty, adaptation and learning certainly permeate our notions of hierarchy with update of models to be performed at every level of the hierarchy in decentralized fashion.

I may add in closing that this view of intelligence is diametrically opposed to that espoused in say ([3]) where the point of view taken is not hierarchical but really integrative of a large number of elementary sensory motor loops acting on an individual level. This point of view has been developed quite thoroughly by Brooks. We are at the current time unable to make a more scholarly comparison of the pluses and minuses of the approaches.

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Tools for Nonlinear Control Systems Design: Progress Report for 1992-93 and Proposal for 1993-94

S. S. Sastry

January 11, 1994

1 Introduction

One of the major challenges in the design and prototyping of controller designs is the problem of aggregating continuous time dynamics and control with discrete event dynamics, arising from logical operations. This problem is particularly acute in the context of complex hierarchically organized control systems where, at all but the lowest levels of the hierarchy the control is on models of a coarser granularity then differential equations. Such problems arise in flight control (with many modes of operation) and process control. In this proposal, we discuss the issues in the control of hybrid systems arising from intelligent flight control. The research proposed here is complimentary to an ongoing activity at NASA Ames with potential payoff in the design of integrated flight control systems for high speed civilian transport aircraft (SST).

2 Progress Report 1992-93

In the year 1992-93, we worked on three projects on the grant NAG 2-243:

1. **Path planning for aircraft**

   In the area of path-planning for aircraft, we set ourselves the so-called "landing tower problem". In this problem, we are given a sequence of via-points and times by the control tower for the position and orientation of the aircraft and asked to plan a trajectory through those points. We attacked this problem using a kinematic model for the motion of the aircraft. More specifically if $g \in SE(3)$ models the position and orientation of the aircraft and the inputs u_1, u_2, u_3 stand for the rates of rotation about the principal (body) axes of the aircraft and v the velocity of the (CTOL) aircraft, then the kinematic equations of motion are given by

$$\dot{g} = g \begin{bmatrix} 0 & -\omega_1 & \omega_2 & 0 \\ \omega_1 & 0 & -\omega_3 & 0 \\ -\omega_2 & \omega_3 & 0 & v \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{1}$$

Here g is represented in homogeneous coordinates and the velocity v is normalized to 1, representing the fact that the velocity of the aircraft is roughly constant. We solve the problem of steering the aircraft between g_o and g_f in "optimal fashion", that is to minimize

$$\int_0^T \sum_{i=1}^{3} u_i^2 \, dt.$$

We have solved this problem and shown that the optimal inputs are elliptic functions (so-called Weierstrass P-functions) and have devised computational procedures for steering the system. The problem is undergoing further refinement in terms of reducing the use of the yawing input and also to treat the velocity of the aircraft as input. A notable drawback of the approach is the use of the *kinematic* model for the planning. This is something that we plan to address in this year's research. The work done in this project has not yet been published but is in preparation. G. Walsh will file his M. A. dissertation in Mathematics on this topic in January 1994.

2. **Steering and Control of a Nonholonomic system: a Bicycle**

   The project on building and controlling a bicycle was undertaken on this grant in an attempt to understand mode switching and nonlinear control laws for a complex dynamical system whose equations of motion possess non-holonomic constraints. Mode-switching was between starting the bicycle, riding in a straight line and turning. A children's bicycle was instrumented with motors and a 486-based computer. A model of the bicycle was derived and simulated. Control laws for turning were derived and are to be presented at the Control and Decision Conference this year. Mechanical problems with measuring the attitude of the bicycle and academic difficulties faced by the student (N. Getz) who was working on this project have caused us to rethink the future of this project. It will in all likelihood be revived with a different student.

3. **ARO-NASA workshop on "Formal Models for Intelligent Control"**

   Also in this funding period one of our activities was the planning and organization of a workshop on "Formal Models for Intelligent Control", co-sponsored by the Army Research Office and held at MIT, between September 30th and October 2nd, 1993. This workshop featured over twenty speakers representing different points of view on formal models for intelligent control. Some areas represented were

   (a) Hybrid systems: Antsaklis, Brockett, Grossman, Ho, Kohn, Nerode and Ramadge.
   (b) Switched systems: Guckenheimer and Morse.
   (c) Neural and fuzzy systems: Sontag, Tomizuka and Wang.
   (d) Flight control: Hauser, Meyer and Stengel.
   (e) Discrete event systems: Caines and Lafortune.
   (f) Hierarchical control systems: Mitter, Sastry and Varaiya.

   A large number of students also attended the workshop and the headcount of the total attendance at the workshop was about a hundred and fifty. The workshop was also attended by industrial participants from General Electric, Siemens, Lincoln Laboratories and Draper Laboratories. Program managers from the Army Research Office, AFOSR, NIST and NSF also showed their enthusiasm for the proceedings.

   It is planned to invite the speakers to submit some of their published papers and to compile a volume with three or four long survey papers written with the aid of the students Branicky, Deshpande, Godbole and Lygeros who were at the meeting. Also the IEEE Transactions on Automatic Control is considering at its Editorial Board Meeting in December a special issue on "Formal Models for Intelligent Control" inspired by the workshop.

3 List of Manuscripts Submitted or Published

1. M. Di Benedetto and S. Sastry, "Adaptive Control for MIMO Nonlinear Systems, Journal of Mathematical Systems and Estimation, Vol. 3, No. 1, 1993 pp. 73-105.
2. A. R. Teel, R. M. Murray and G. Walsh, "Nonholonomic Control Systems: from Steering to Stabilization with Sinusoids", ERL Memo M 92/28, March 1992 Proceedings of the IEEE Conference on Decision and Control, Scottsdale, 1992, submitted to the IEEE Transactions on Automatic Control.
3. N. Getz, "Control of modified Caplygin nonholonomic systems," Proceedings of the IEEE Conference on Decision and Control, San Antonio, 1993.
4. G. Walsh, S. Sastry and R. Montgomery, "The Landing Tower Problem: Optimal Control on Lie groups", (in preparation).

4 Proposal for 1993-94

Control of Hybrid Systems in Flight Control

One of the major challenges in the design and prototyping of controller designs is the problem of aggregating continuous time dynamics and control with discrete event dynamics, arising from logical operations. This problem is particularly acute in the context of complex hierarchically organized control systems where at all but the lowest levels of the hierarchy the control is on models of a coarser granularity then differential equations. Such problems arise, for example, in highway automation problems, problems of command and control and also more generically in flight control (with many modes of operation) and process control. This represents a new direction of research on the grant and a transition towards tools for intelligent control systems combining features of low-level nonlinear control and control of discrete systems. Here we discuss the issues in control of hybrid systems arising from intelligent flight control (such as those arising in the development of high speed civilian transport or the SST).

In this context, the control of the overall system has the following features:

1. Pilot plans a path from source to destination, using criteria such as weather, time of flight and fuel consumptions. This is referred to as the *strategic level.*

2. The Air Traffic Control system modifies the flight plan to be compatible with traffic considerations and possible other criteria such as anticipated changes in weather and safety considerations.

3. The flight path from the Air Traffic Control is then converted into a *tactical level* plan consisting of steering the aircraft through different flight regimes, such as

   (a) Conventional or CTOL mode
   (b) Helicopter or HELI mode
   (c) Attitude control or ATT mode
   (d) Vertical Take Off and Landing or VTOL mode

4. Each of the regimes of operation come equipped with a nonlinear control law, consisting of approximate full-state linearization. The reason for having approximate control is to prevent the cancellation of fast unstable zero dynamics arising from parasitic coupling effects between the mechanism for moment and force generation in the aircraft. These different nonlinear controllers need to be spliced together in a smooth fashion.

5. The performance of each "linearizing" control in each regime needs to be improved by using either anticipatory control of the non-minimum phase dynamics or a non-linear regulator approach. Further the implementation of the controller in each regime involves switching brought about by changes in the coordinate charts describing the orientation of different axes (body, wind) on the aircraft.

From looking at these features, the following hierarchical control architecture has been proposed by Meyer at NASA Ames:

1. **Strategic Level Planner** This combines both the pilot's planning of her route from one place to another along with the air traffic controller's allocation of check points on the trajectory. The outcome of this planner is a sequence of discrete events or commands.

2. **Tactical Level Planner** The coarse and incomplete plan arising from the strategic level is converted into a detailed tactical plan consisting of chunks of trajectory labeled with the appropriate modal conditions: such as HELI (for helicopter mode), VTOL (for vertical take off and landing mode) or CTOL (for conventional take off and landing mode), to pick a few examples. The tactical level planner tries to take into account performance limitations of the aircraft as well as comfort considerations of the passengers.

3. **Arithmetic or Low-level controller** Using detailed dynamic models of the aircraft linearizing controllers using different outputs are selected with different controllers being in effect for each mode. Further, switches in the controllers occur when singularities of the coordinate charts parameterizing orientation are encountered.

Our research will be complimentary to the intelligent flight control project already under way at Ames. The goal of our research will be to develop tools for the analysis of hierarchical hybrid dynamical systems motivated from specific problems encountered in solving the integrated flight management system described above. Challenges abound at various levels in this sort of a hierarchical system and we will list a few sample problems that we will attack:

1. Path planning for aircraft including the classification of trajectories into the different modes of operation. This is planning at two levels: at the level of the air traffic control and at the level of the on-board controller.

2. Switching between different modes of control. This is involved in several different ways:

(a) switching between modes of operation, such as VTOL, HELI, CTOL, etc.

(b) switching between approximate inverses or anticipatory tracking control laws in each mode

(c) switching between different coordinate charts in the same mode of operation.

3. Specification and verification tools for hybrid control systems.

More specifically, our efforts will be in:

4.1 Trajectory Planning at the tactical level

Trajectory planning is a critical issue both from the standpoint of the control tower as well as the aircraft. Several approaches including optimal control and searching through a data base of stored trajectories have been suggested. Our own approach thus far has been kinematic. As a consequence, the trajectories may run into difficulties when implemented on dynamic models of aircraft. Also the path planner should generate mode types for the different segments of the flight so as to be of use for the lower levels of control and also to enable transitions in the event of sudden changes necessitated by emergency conditions. Results on classification of bundles of trajectories will be valuable in this context.

Unlike many hierarchical systems where models become increasingly coarse and unrefined the higher one travels in the hierarchy, the flight control application calls for a fairly detailed dynamical model and a sense of actuator limits and state variable constraints in order to develop a trajectory planner. The question of how few details will be needed to plan the trajectory and tag the discrete components of the state will be the first area of research that we will address. While it is clear as a general principle of modeling that the sophistication of the model of a control system should reflect the difficulty of the control task at hand, the implementation of this principle for hierarchical designs appears to be a problem that has yet to receive attention. It is the problem of *functionally based model reduction.* Our approach to making a dent on this problem will be inductive from a solution of the specific problem of trajectory planning models.

4.2 Switching Controllers

While certain classes of switched dynamical systems such as those arising from relaxed control or sliding mode control have been well investigated, it is fair to say that a systematic investigation of switching between different nonlinear control laws is in its infancy. The literature in gain scheduling does little from a design point of view in establishing rules of transition to preserve continuity of the controller inputs. We will begin a systematic study of performance specifications and design guidelines for control systems with switches. We will be aided in this regard by some related work in the project on highway automation, PATH.

While the underlying dynamics of the aircraft are smooth functions, the control laws based on full state approximate linearization are different in different modes of operation. This has to do with changes in the independent and dependent outputs of the aircraft depending on the flight regime of the aircraft. Thus, the *arithmetic level* controller is switched at each instant of mode switching. Mode switching needs to be accompanied by smooth transitions in the control inputs. This in turn needs the design of so-called *transitional or interface level* controllers for mode switching.

4.3 Specification and verification tools for hybrid control systems

In systems which have the level of functional and hierarchical complexity of flight control systems systematic design tools for verification of whether the control schemes meet the specifications need to be developed. The level of sophistication of current hybrid systems methodologies is severely limited to analysis of systems with clocks or systems whose dynamic performance can be abstracted by clocks. New frameworks and a fresh new approach to these problems is warranted.

Tools for Nonlinear Control Systems Design: Progress Report for 1994 and Proposal for 1995

S. S. Sastry

January 19, 1995

1 Introduction

One of the major challenges in the design and prototyping of nonlinear control systems is the problem of aggregating continuous time dynamics and control with the discrete event dynamics which arise , arising from logical operations. This problem is particularly acute in the context of complex hierarchically organized control systems where, at all but the lowest levels of the hierarchy, the control is on models of a coarser granularity than differential equations. Such problems arise in flight control (with many modes of operation) and process control. In this proposal, we discuss the issues in the control of hybrid systems arising from intelligent flight control.

In 1994, we began a project aimed to be coordinated with Dr. Meyer of the Ames research center on the use of control theoretic methods for air traffic control. In this research, we aimed to understand aspects of the planning of trajectories for an aircraft auto-pilot given via points for the aircraft to follow, and to formulate control laws for achieving and modifying these planned trajectories. Actuator limitations and non-minimum phase characteristics of the aircraft need to be taken into account to modify the planned trajectories. In continuing work, we will combine this work with planning of trajectories at the level of the air traffic control with the auto-pilot.

2 Progress Report 1994

In the year 1994, we worked on three projects on the grant NAG 2-243:

1. **Path planning for aircraft**

   In the area of path-planning for aircraft, we set ourselves the so-called "landing tower problem". In this problem, we are given a sequence of via-points and times by the control tower, specifying the position and orientation of the aircraft, and we are asked to plan a trajectory through those points. We approached this problem using a kinematic model for the motion of the aircraft. More specifically, if $g \in SE(3)$ models the position and orientation of the aircraft and the inputs u_1, u_2, u_3 stand for the rates of rotation about the principal (body) axes of the aircraft and v the velocity of the (CTOL) aircraft, then the kinematic equations of motion are given by

$$\dot{g} = g \begin{bmatrix} 0 & -u_1 & u_2 & 0 \\ u_1 & 0 & -u_3 & 0 \\ -u_2 & u_3 & 0 & v \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{1}$$

   Here g is represented in homogeneous coordinates and the velocity v is normalized to 1, representing the fact that the velocity of the aircraft is roughly constant. We solve the problem of steering the aircraft between the initial and final configurations, g_o and g_f in "optimal fashion", that is to minimize

$$\int_0^T \sum_{i=1}^{3} u_i^2 \, dt.$$

   We have solved this problem and shown that the optimal inputs are elliptic functions (so-called Weierstrass P-functions) and have devised computational procedures for steering the system. One very interesting aspect of the explicit solution is that it includes *helices, circles and straight line* segments. This is especially pleasing, since such solutions are currently used frequently by the ATC. The problem is undergoing further refinement in terms of reducing the use of the yawing input and also to treat the velocity of the aircraft as input. A notable drawback of the approach is the use of the *kinematic* model for the planning.

Computational approaches to the efficient generation of trajectories are also being pursued.

2. **Stabilization of Satellite using fewer than 3 actuators**

   The dynamics and control of satellites has received a tremendous amount of attention in the control literature. When there are fewer than 3 actuators the steering of the satellite from one configuration to another can be achieved, if the satellite is not symmetric. These open loop steering control laws can be stabilized using a "time-varying" linearization about the trajectory. However, for set point regulation of the satellite, stabilizing control laws need to be derived. Early work of Meyer solved this problem for the case of 3 actuators and this work has been extended by other researchers recently as well. We revisited the problem of stabilizing a satellite with two or fewer actuators and gave time varying smooth control laws, for this purpose. This is perhaps the best one can do, since a theorem of Brockett guarantees that no smooth time-invariant stabilizing control law can be found for this class of systems.

3. Modeling of the dynamics and control of a model helicopter

   In work on the grant this year, we have set up a test rig consisting of a model helicopter mounted on a six degree of freedom robot in the laboratory. The purpose of the robot is to measure the position and orientation of the helicopter in a reasonably cheap fashion. The helicopter is equipped with motors for operating its rotors and swash plate. The goal of the project is to interface this to a Unix workstation as a test bed for trying out different approaches to its control: at the current time we are contemplating a nonlinear control approach, which needs detailed identification of the helicopter, and a fuzzy control approach, which needs only a rudimentary model. These two approaches will be compared.

3 List of Manuscripts Submitted or Published

1. G. Walsh, S. Sastry and R. Montgomery, "The Landing Tower Problem: Optimal Control on Lie groups", Proceedings of the 1994 Conference on Decision and Control, Orlando, Florida, December 1994, to be submitted to Journal of Optimization Theory and its Applications.

2. G. Walsh, R. Montgomery and S. Sastry, " Stabilization of a satellite with two control inputs", Proceedings of the 1994 American Control Conference, Baltimore, Md, June 1994, to be submitted to the IEEE Transactions on Automatic Control.

4 Proposal for 1995: Control of Hybrid Systems in Flight Control

In this project we are interested in pursuing the study of hierarchical, hybrid control systems in the context of air traffic control. A representative scenario could be as follows, in the vicinity of a large urban airport:

1. The Air Traffic Control (ATC) gives the pilot a sequence of via points and approach vectors corresponding to sequencing the aircraft for landing.

2. The (auto-)pilot plans a path to interpolate between these via points based on other data such as time of flight and fuel consumption. If the path is not feasible or is unsatisfactory, the pilot re-negotiates with the ATC.

3. The planned flight path is then converted into a *tactical level* plan consisting of steering the aircraft through different flight regimes, such as

   (a) Conventional or CTOL mode

   (b) Helicopter or HELI mode

   (c) Attitude control or ATT mode

(d) Vertical Take Off and Landing or VTOL mode

4. Each of the regimes of operation comes equipped with a nonlinear control law, consisting of approximate full-state linearization. The reason for having approximate control is to prevent the cancellation of fast unstable zero dynamics arising from parasitic coupling effects between the mechanism for moment and force generation in the aircraft. These different nonlinear controllers need to be spliced together in a smooth fashion.

5. The performance of each "linearizing" control in each regime needs to be improved by using either anticipatory control of the non-minimum phase dynamics or a non-linear regulator approach. Further, the implementation of the controller in each regime involves switching brought about by changes in the coordinate charts describing the orientation of different axes (body, wind) with respect to the aircraft.

From looking at these features, the following hierarchical control architecture has been proposed by Meyer at NASA Ames:

1. **Strategic Level Planner** This combines both the pilot's planning of his route from one place to another along with the air traffic controller's allocation of check points on the trajectory. The outcome of this planner is a sequence of discrete events or commands.

2. **Tactical Level Planner** The coarse and incomplete plan arising from the strategic level is converted into a detailed tactical plan consisting of chunks of trajectory labeled with the appropriate modal conditions, such as HELI (for helicopter mode), VTOL (for vertical take off and landing mode) or CTOL (for conventional take off and landing mode), to pick a few examples. The tactical level planner tries to take into account performance limitations of the aircraft as well as comfort considerations of the passengers.

3. **Arithmetic or Low-level controller** Using detailed dynamic models of the aircraft, linearizing controllers using different choices of output functions are selected. Different controllers are in effect for each mode. Further, switches in the controllers occur when singularities of the coordinate charts parameterizing orientation are encountered.

Our research will be complementary to the intelligent flight control project already under way at Ames. Both the PI and at least one of his students will interact closely with the groups of Drs. Meyer and Tobias (ATC) at NASA, Ames to help get a realistic formulation of the research issues. The student (identified as Ms. Claire Tomlin) will spend the summer of 1995 at Ames, to help this dialog. The goal of our research will be to develop tools for the analysis of hierarchical hybrid dynamical systems motivated from specific problems encountered in solving the integrated flight management system described above. Challenges abound at various levels in this sort of a hierarchical system and we will list a few sample problems that we will attack:

1. Path planning for aircraft including the classification of trajectories into the different modes of operation. This is planning at two levels: at the level of the air traffic control and at the level of the on-board controller.

2. Switching between different modes of control. This is involved in several different ways:

   (a) switching between modes of operation, such as VTOL, HELI, CTOL, etc.

   (b) switching between approximate inverses or anticipatory tracking control laws in each mode

   (c) switching between different coordinate charts in the same mode of operation.

3. Specification and verification tools for hybrid control systems.

4.1 Trajectory Planning at the tactical level

Trajectory planning is a critical issue both from the standpoint of the control tower as well as from the aircraft. Several approaches including optimal control and searching through a database of stored trajectories have been suggested. Our own approach thus far has been kinematic. As a consequence, the trajectories may run into difficulties when implemented on dynamic models of aircraft. Also the path planner should generate mode types for the different segments of the flight so as to be of use for the lower levels of control and also to enable transitions in the event of sudden changes necessitated by emergency conditions. Results on classification of bundles of trajectories will be valuable in this context.

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4.3 Specification and verification tools for hybrid control systems

In systems which have comparable functional and hierarchical complexity of flight control, systematic design tools for verification that the control schemes meet the specifications need to be developed. The level of sophistication of current hybrid systems methodologies is severely limited to analysis of systems with clocks or systems whose dynamic performance can be abstracted by clocks. New frameworks and a fresh new approach to these problems is warranted, since the origin of hybrid control laws in the context flight control arises not only from the interaction of planning and continuous control, but also the interaction between different kinds of nonlinear control laws (on the same nonlinear control system).

Tools for Nonlinear Control Systems Design: Progress Report for 1995 and Proposal for 1996

S. S. Sastry

1 Introduction

One of the major challenges in the design of control systems is the problem of aggregating continuous time dynamics and control with the discrete event dynamics arising from logical operations. This problem is particularly acute in the context of complex hierarchically organized control systems where, at all but the lowest levels of the hierarchy, the control is on models of a coarser granularity than differential equations. Such problems arise in flight control (with many modes of operation) and process control. In addition, the problem of integrating discrete planning with continuous dynamics arises when there is a need for coordination among multiple agents. In this proposal, we discuss our background work in air traffic management systems. We then discuss our plans for research on this grant to design and implement *Flight Vehicle Management Systems* for multi-agent coordination maneuvers, which are guaranteed to be safe.

2 Progress Report 1995

In the year 1995, we worked on three projects on the grant NAG 2-243. These are discussed below. As other activity on the grant, we helped Clyde Martin organize a workshop at Bozeman, Montana from July 24-28th, 1995, on the topic of "Exterior Differential Systems and Hybrid Control". Our role was to increase the scope of the workshop to include recent work on Exterior Differential Systems and bring in some supplementary funding from the Army Research Office to allow a large number of students to attend the meeting.

2.1 Stabilization and Tracking of Feedback Linearizable Systems under Input Constraints

The problem of feedback linearization of nonlinear systems in the presence of input saturation was considered in this work. Although different approaches for stabilization of linear and nonlinear systems under bounded controls have been suggested in the literature, the need to perform output tracking for nonlinear systems dictates the choice of feedback linearization in our control design. This method results in linear error dynamics allowing the estimation of the convergence rate. Explicit characterization of the space of feasible trajectories that can be tracked using bounded inputs has been made, followed by simulation results. In later work, we have considered the problem of avoiding saturation by trajectory reparameterization for feedback linearizable systems. The desired trajectory is reparameterized on a slower time scale in order to avoid input saturation. Necessary conditions that the reparameterizing function must satisfy have been derived. The deviation from the nominal trajectory is minimized by formulating the problem as an optimal control problem.

2.2 Hybrid Control in Air Traffic Management Systems

In a collaborative project involving the University of California, Berkeley, NASA Ames Research Center, and Honeywell Systems Research Center, we have begun the study of hierarchical, hybrid control systems in the framework of air traffic management systems (ATMS). The need for a new ATMS arises from the overcrowding of large urban airports and the need to more efficiently land and take off larger numbers of aircraft, without building new runways. Technological advances that make a more advanced air traffic control system a reality include the availability of relatively inexpensive and fast real time computers (both on board the aircraft and in the control tower) and global positioning systems. The usefulness of these technological advances is currently limited by today's air traffic control system, which involves the use of "freeways" in the Terminal Radar Approach Control (TRACON) region around urban airports. These freeways are set approach patterns to runways which do not allow for the possibility of so-called "free flight" by an aircraft to its destination. Limiting the aircraft trajectories in this manner results in the addition of both planned and unplanned delays to air travel. Research has focused on hierarchical, hybrid control systems and their application to air traffic management. One proposed decision support hierarchy describes an automated, decentralized ATMS: much of the control functionality exists in the flight vehicle management system on board each aircraft rather than at the ground-based air traffic control. Aircraft trajectories are restricted in the airport region; outside this region the architecture allows for so-called "free flight". A simulation and visualization package for hybrid systems in the framework of ATMS, called SmartPlanes, is being developed to aid the design process.

2.3 Output Tracking for a Non-Minimum Phase Dynamic CTOL Aircraft Model

In this work, a dynamic model of a conventional takeoff and landing (CTOL) aircraft was used to study output tracking in the presence of non-minimum phase dynamics. Non-minimum phase characteristics in this model result from the fact that the process of generating an upward pitch moment produces a small downward force, causing the aircraft to lose altitude. The model is not full state linearizable and the internal (zero) dynamics which remain after input-output linearization using the coordinates of the center of mass as outputs are unstable. The CTOL model is not "flat" with respect to fixed points on the aircraft body. The nonlinear inversion technique of Devasia and Paden produces stable trajectories for the states of the internal dynamics, but the corresponding feed-forward force inputs required to track these trajectories are large. *Approximate* linearization techniques which ignore the coupling between the pitch moment and the vertical and horizontal aircraft dynamics, have been used by us to calculate inputs of smaller magnitude.

3 List of Manuscripts Submitted or Published

1. S. Sastry, G. Meyer, C. Tomlin, J. Lygeros, D. Godbole and G. Pappas, "Hybrid Control in Air Traffic Management Systems", Proceedings of the 1995 IEEE Conference on Decision and Control, New Orleans, LA, December 1995.
2. C. Tomlin, J. Lygeros, L. Benvenuti and S. Sastry, "Output Tracking for a Non-minimum phase dynamic CTOL aircraft model", Proceedings of the 1995 IEEE Conference on Decision and Control, New Orleans, LA, December 1995.
3. C. Tomlin, G. Pappas, J. Lygeros, D. Godbole, S. Sastry, and G. Meyer, "Hybrid Control in Air Traffic Management Systems", to appear in the Proceedings of the 1996 IFAC, San Francisco, July 1996.
4. G. Pappas, J. Lygeros and D. Godbole, "Stabilization and tracking of feedback linearizable systems under input saturation constraints", Proceedings of the 1995 IEEE Conference on Decision and Control, New Orleans, LA, December 1995.

4 Proposal for 1996: Hybrid Systems in Flight Control

4.1 Background: A Decentralized Decision Making System

We propose a partially decentralized air traffic management decision support system which transfers much of the current air traffic control (ATC) functionality to each individual aircraft. We expect our proposed system to be more robust and reliable, reduce ATC workload, and be more suitable for free flight.

- A distributed system is more fault tolerant. If a single aircraft's computer system fails, most of the ATMS system is still intact, and the aircraft may be guided by voice to the nearest airport. If in a centralized system the central computer fails, the results could be catastrophic.
- Congestion is increasing monthly in large urban airports. A distributed system is more suited to handling increasing numbers of aircraft than is a centralized system, since

a new aircraft and its own computer system may be added easily to the system. A centralized system would require upgrades to the ATC computer regularly.

- Free flight, one of the features of a distributed control system, minimizes fuel consumption since each aircraft may optimally plan its own trajectory, which in turn results in reduced delays outside TRACONs. Free flight is also an advantage in avoiding turbulence, since the aircraft would not have to wait for clearance from ATC to avoid rough weather patches.

- A distributed system reduces the ATC workload, allowing ATC to spend more time in resolving safety critical situations.

- In a centralized system, because of the large number of aircraft it has to deal with, ATC can only use a rough approximation of each aircraft's dynamics to calculate feasible trajectories. In our ATMS model, each aircraft's autopilot contains a detailed model of its own dynamics, and thus the aircraft itself is well equipped to plan its own trajectory in a conflict resolution maneuver.

The functionality of the proposed air traffic management architecture is slightly different for the region inside the TRACON (take off and landing) than it is for the region outside the TRACON (en-route airspace). In the region outside the TRACON, the density of aircraft is relatively small, and we propose that each aircraft be allowed to completely plan its own trajectory (to be called the "free flight" region). In this region, conflicts will be resolved by coordination between aircraft, the role of the ground based ATC will simply be to provide information about the route, such as upcoming weather conditions. Inside the TRACON, the ATC will determine the approach trajectory for each aircraft. These trajectories will depend on local weather conditions and traffic. A block diagram of our proposed architecture for ATMS inside the TRACON region is presented in Figure 1. The levels of architecture below ATC reside on the individual aircraft and comprise what is known as the aircraft's *Flight Vehicle Management System*, or FVMS. The FVMS consists of four layers, the strategic, tactical, and trajectory planners, and the regulation layer.

4.1.1 Air Traffic Control

When an aircraft enters the TRACON region of an airport, ATC passes a sequence of *waypoints* to the strategic planner on board the aircraft. These waypoints are a discretization of a kinematic trajectory, accessed from a database of stored kinematic trajectories, which have been calculated offline for different combinations of:

- aircraft kinematics;

- wind magnitude and direction;

- runway configurations.

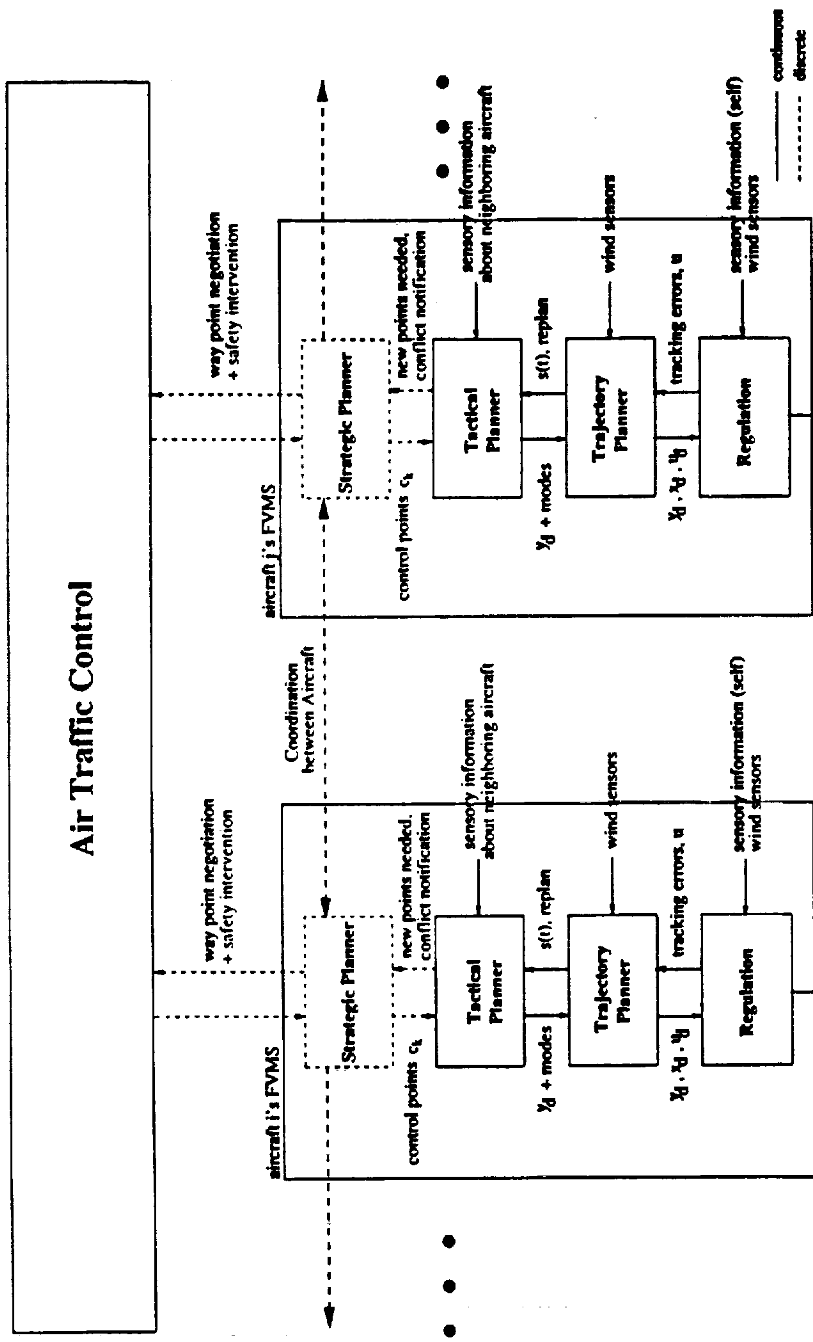

Figure 1: Proposed ATMS Architecture: Inside TRACON

These pre-computed trajectories have been optimized to provide a minimum-time path for the given aircraft kinematics. The waypoints from ATC are time-stamped to provide a suggested schedule of landings, which is designed to meet the announced arrival times and reflects conflict resolution and compromises between airline schedules. Once ATC has calculated these waypoints and passed them to the strategic planner, all of the planning and control tasks are taken over by the FVMS on board the individual aircraft. If the FVMS changes the waypoints for some reason (such as in a safety critical situation), then the new set of waypoints is passed to ATC. Outside the TRACON region, free flight is under effect and the role of ATC is minimal. ATC passes only TRACON exit and entry information to the tactical planner of the aircraft, and then the tactical planner takes over the role of calculating an initial kinematic trajectory for the aircraft.

4.1.2 Strategic Planner

The main role of the strategic planner is to resolve conflicts between aircraft. Inside the TRACON, the strategic planner has the additional role of designing a coarse trajectory for the aircraft in the form of a sequence of control points, c_k, which interpolate the waypoints from ATC. If the tactical planner on board the aircraft predicts that a conflict will occur between its aircraft and other aircraft, it notifies the strategic planner. Conflict resolution is to be achieved by communication between aircraft at the strategic level. The strategic planners of all aircraft involved in the potential conflict determine a sequence of maneuvers which will result in conflict-free trajectories, and then each strategic planner commands its own tactical planner to follow these maneuvers. Inside the TRACON, the commands are passed down to the tactical planner in the form of a modified sequence of control points. ATC is notified if this modified sequence deviates from the original set of waypoints. Outside the TRACON, where the upper levels of the ATMS architecture have looser control over trajectory planning, the commands themselves are passed.

4.1.3 Tactical Planner

Inside the TRACON, the tactical planner refines the strategic plan by interpolating the control points with a smooth output trajectory, denoted y_d in Figure 1. Outside the TRACON, it calculates this trajectory from scratch, using only the initial and final conditions of free flight (the TRACON exit and entry information). In addition to the output trajectory, the tactical planner determines the sequence of *flight modes* necessary to execute the kinematic plan. In both regions, the tactical planner is responsible for predicting conflicts.

The tactical planner uses a simple kinematic model of the aircraft for all trajectory calculation. For conflict prediction, it uses information about the positions and velocities of neighboring aircraft (available through radar) and kinematic models to predict their movement. If more information, such as neighboring aircraft type and capabilities, is available through communication, the models can be refined. Simple models are used at this stage since very detailed models may unnecessarily complicate the calculations, which are assumed to be approximate and have large safety margins.

4.1.4 Trajectory Planner and Regulation

The trajectory planner uses a detailed dynamic model of the aircraft, sensory input which measures the wind's magnitude and direction, and the tactical plan consisting of an output trajectory and sequence of modes, to design a full state and input trajectory for the aircraft. This trajectory is passed to the regulation layer which directly controls the aircraft.

4.2 A Hybrid System Methodology for Multi-Agent Conflict Resolution

A method for conflict prediction and resolution in a multi-agent system is to be developed in the context of our decision support system for ATMS.

Conflicts arise due to the different objectives of the individual agents, and must be resolved to avoid collision between agents. The method will use optimal control in a game theoretic setting in a way being developed by us for hybrid systems verification ([1]) to calculate the maneuver which resolves a potential collision. If the problem cannot be solved, the agents resort to a sequence of sub-optimal protocols to avoid the conflict.

The algorithm for predicting and resolving conflicts is presented in Figure 2. Each agent in the system performs the algorithm simultaneously. For example, in the case of air traffic management systems (ATMS), the algorithm is run on board each aircraft within the strategic and tactical planners of the flight vehicle management system.

At the first step of the algorithm (level (1)), long range conflict detection is performed. This process determines whether or not a conflict is likely to occur between agents, and identifies the set of agents involved in the potential conflict. Methods such as forward extrapolation the current motion of the agents (such as in [4]), or simply choosing a sphere around each aircraft (whose radius depends on the velocity of the aircraft) as a possible conflict region, could be used to determine the agents involved in the potential conflict. These agents are then treated as players in a zero sum dynamic game (level (2) of the algorithm), in which each agent tries its best to avoid a collision in an environment in which all of the other agents are its opponents, and are trying their best to cause a collision. A solution to the dynamic game is a sequence of control strategies for each agent, which will guarantee that the agent avoids collision assuming that the actions of all of the other agents are unknown, but lie within a known disturbance set.

In our research on the grant this year, we will refine our game theoretic techniques for hybrid systems design, first introduced in [1] and adapted for ATMS in [3] to develop the set of initial conditions in which conflict resolution needs to be undertaken. This research is already quite interesting and challenging in two dimensions, where it extends the basic state of the art in pursuit evasion games ([2]). Of course, our primary interest will be in three dimensions.

When collisions seem unavoidable, we will develop protocols for collision avoidance and verify their safeness. Then, we will develop the dynamic control laws (including potential mode changes in the FVMS) to implement them. Verification of the composite hybrid system

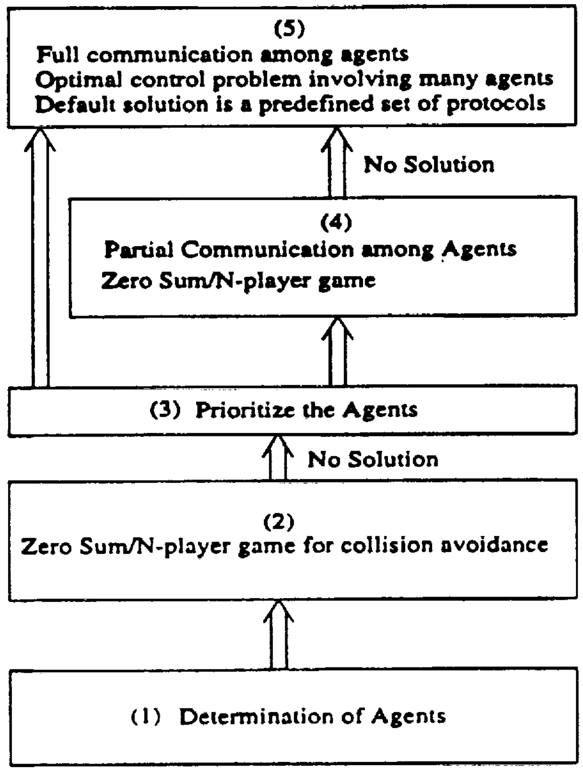

Figure 2: Strategy for conflict prediction and resolution in a multi-agent setting.

may or may not fit into the game theoretic framework alluded to earlier, and will have to be studied. We expect to make progress for at least two kinds of conflicts, which we have begun studying, namely *overtake* and *collision*. The 2-agent overtake conflict describes the situation in which two agents are traveling in the same direction, and the agent who is behind has the greater speed. The 2-agent collision conflict is one in which the agents are traveling towards each other.

In our research this year, we will concentrate on generalizing these ideas and developing them in a way so as to allow for the interaction between the tactical layer and the conflict avoidance module, possibly including some dynamic replanning and mode setting of the maneuvers.

4.3 Implementation, Verification and Validation of Hybrid Control Laws

In addition to the design of the architecture for a complex system such as ATMS, an important issue is the reliable implementation of hybrid system algorithms as well as their verification and validation.

4.3.1 HOPTs: Hierarchy of Operational Procedure Tables

One methodology which seems to address these issues is HOPTs (Hierarchy of Operational Procedure Tables), developed by Honeywell Technology Center in collaboration with NASA Ames.

HOPTs is a model-based formal specification language, which automates the translation of designer specifications to software code, documentation, and test plans, through the use of *operational procedure tables.* Operational procedures define a set of high level tasks which may be sequenced together to complete a *mission.* The operational procedure tables consist of *mission scenarios*, which are sets of external circumstances (such as windshear and collision avoidance), and *mission behaviors*, which are the responses of the system to the given scenarios. The system is considered to be *complete* and *consistent* if it responds to all possible mission scenarios with acceptable mission behaviors.

The tabular structure of HOPTs is amenable to correctness verification. The model is validated, a table at a time, in order to prove correctness of mode switching logic and to ensure completeness and consistency. However, verification and validation of the hybrid systems is not yet resolved. In work on the grant this year, we will study the problems associated with the HOPTs implementation of our FVMS conflict resolution protocols.

4.3.2 Simulation and Visualization Issues

The complexity of large scale projects renders simulation a valuable tool both in the design of various control laws and coordination protocols as well as in the evaluation of overall system performance. Furthermore, a good simulation package may also be used as a debugging tool in the design process. This requires the development of a simulation package for hybrid systems that will be able to simulate both the low level differential equation models as well as the high level finite state machine models. The complexity of the system also emphasizes the need for efficient computational schemes, such as parallel and distributed computation algorithms. Each simulation run results in a tremendous amount of data that needs to be analyzed and interpreted. Visualization techniques, such as animation, can be used to present the simulation results in a manner which is much easier to analyze by the designer. In this direction, the development of *SmartPlanes*, a simulation and visualization facility for ATMS, has begun. At the current stage, SmartPlanes is a visualization tool which allows the user to view the trajectory of a single aircraft from various perspectives. In future versions, multiple aircraft will be shown as well as a local radar, and the user will have the ability to configure the airport so as to meet the needs of different cities.

4.4 Visitors

In lieu of having the traditional NASA-Berkeley workshop this year, we have elected to have 3-5 day long presentations by eminent researchers at Moffett Field, so as to enable us to understand their point of view and question them in depth. Examples of such visitors could include W. M. Wonham (University of Toronto), Wolf Kohn (Sagent Corporation) and Anil Nerode (Cornell University), and Sanjoy Mitter (MIT). The last named has some very interesting new insights into the use of infinite linear programming in logic verification. Budgets for these speakers are included in the proposed budget.

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[1] J. Lygeros, D. Godbole and S. Sastry, "A Game Theoretic Approach to Hybrid System Design," *UCB ERL Memo* M 95/77, Univ of Calif., Berkeley, October 1995.

[2] T. Basar and J. Olsder, *Dynamic Non-cooperative Game Theory*, Academic Press, 2nd Edition, 1995.

[3] G. Pappas, C. Tomlin, S. Sastry, "Conflict Resolution in Multi-Agent Path Planning," to be submitted to *1996 IEEE Conference on Decision and Control.*

[4] R. Slattery and S. Green, "Conflict Free Trajectory Planning for Air Traffic Control Automation," NASA Technical Memorandum 108790, 1994.

[4] G. Meyer, L. R. Hunt and R. Su "Nonlinear System Guidance in the presence of transmission zero dynamics", NASA Technical Memorandum 4661, 1995, also in the *Proceedings of the 1996 IEEE Conference on Decision and Control.*

[5] J. Lygeros, C. Tomlin and S. Sastry, "Multiobjective Controller Synthesis," in the *Proceedings of the Workshop on Hybrid Systems, Grenoble, France, March 1997.*

[6] R. Slattery and S. Green, "Conflict Free Trajectory Planning for Air Traffic Control Automation," NASA Technical Memorandum 108790, 1994.

[7] C. Tomlin and S. Sastry, "Bounded Tracking for Nonminimum Phase Nonlinear Systems with Fast Zero Dynamics", *Proceedings of the 35th Conference on Decision and Control, 1996.* (also submitted to the *International Journal of Control*, August 1996)

[8] C. Tomlin and S. Sastry, "Switching through Singularities," presented at the *Fourth International Conference on Hybrid Systems, Ithaca*, October 1996. (To be submitted to Systems and Control Letters, October 1996)

[9] L. Sherry, D. Youssefi, and C. Hynes, "A Formalism for the Specification of Operationally Embedded Reactive Avionic Systems", Honeywell Publication C69-5370-001, Phoenix, Az, October 1995.

[10] D. Cofer, "Reliable Implementation of Hybrid Control Systems for Advanced Avionics," to appear in "Hybrid Systems IV", Springer Verlag, 1997.

[11] M. Heymann, F. Lin and G. Meyer, "Control Synthesis for a class of hybrid systems subject to configuration based safety constraint," to appear in "Hybrid Systems IV", Springer Verlag, 1997.

3 Progress Report for 1996

In the year 1996, we worked on five projects on the grant NAG 2-1039. These are discussed here. I may add that the past year has been an unusually productive year in terms of the output of the researcher supported on this grant, Ms. Claire Tomlin, who also spends a day a week at NASA Ames.

3.1 Bounded Tracking for Nonminimum Phase Nonlinear Systems with Fast Zero Dynamics

In this work, we derive tracking control laws for nonminimum phase nonlinear systems with both fast and slow, possibly unstable, zero dynamics. The fast zero dynamics arise from a perturbation of a *nominal* system. These fast zeros can be problematic in that they may be in the right half plane and may cause large magnitude tracking control inputs. We combine the ideas from some recent work of Hunt, Meyer, and Su with that of Devasia, Paden, and

Chen on an asymptotic tracking procedure for nonminimum phase nonlinear systems. We give (somewhat subtle) conditions under which the tracking control input is bounded as the magnitude of the perturbation of the nominal system becomes zero. Explicit bounds on the control inputs are calculated for both SISO and MIMO systems using some interesting non-standard singular perturbation techniques. The method is applied to a suite of examples, including the simplified planar dynamics of VTOL and CTOL aircraft.

3.2 Switching through Singularities

The nonlinear control toolbox has built up a fair level of sophistication with the use of techniques for input-output and full state linearization, approximate linearization, and bounded tracking for nonminimum phase systems. One area in which results have been hard to come by is the tracking of *singular* or *non regular* nonlinear control systems, i.e. those that fail to have relative degree. While the problem of trying to track trajectories that go through singularities was begun by Hirschorn and Davis who limited the class of outputs that could be tracked, the first set of practical schemes for approximate asymptotic tracking through singularities was given by Hauser, Sastry and Kokotović, using an approximation technique. This in turn spurred the development of a result by Grizzle, Di Benedetto and Lamnabhi-Lagarrigue which proved the necessity of "regularity" for asymptotically exact tracking.

In parallel with this activity has been the interest in using switching control laws for adaptive control, for steering and stabilization, as well as activity in *hybrid control systems*. We combine ideas from switching along with our results on exact tracking of slightly non-minimum phase systems to describe our results in *approximate tracking for singular or non-regular nonlinear systems*.

3.3 Multiobjective Hybrid Controller Synthesis

The problem of systematically synthesizing hybrid controllers that satisfy multiple control objectives is considered. We present a technique, based on the principles of optimal control, for determining the class of least restrictive controllers that satisfy the most important requirement (which we refer to as safety). The system performance with respect to the lower priority requirement (which we refer to as efficiency) can then be optimized within this class. We motivate our approach by three examples, one purely discrete (the problem of reachability in finite automata) one hybrid (the steam boiler problem) and one primarily continuous (a flight vehicle management system).

3.4 Conflict Resolution for Air Traffic Management: A Case Study in Multi-Agent Hybrid Systems

A conflict resolution architecture for multi-agent hybrid systems with emphasis on Air Traffic Management Systems (ATMS) is presented. In such systems, conflicts arise in the form of potential collisions which are resolved locally by inter-agent coordination. This results in a decentralized architecture in which safety issues are resolved locally and central agencies, such as Air Traffic Controllers, focus on global issues such as efficiency and optimal throughput. In order to allow optimization of agents' objectives, inter-agent coordination is minimized by noncooperative conflict resolution methods based on game theory. If noncooperative methods are unsuccessful, then cooperative methods in the form of coordinated maneuvers are used to resolve conflicts. The merging of inter-agent coordination, which is modeled by discrete event systems, and agent dynamics, which are modeled by differential equations, results in *hybrid systems.*

3.5 Hybrid Control in Air Traffic Management Systems

The congestion of aircraft in the airspace around large urban airports strongly suggests the need to increase the efficiency of the current air traffic management system (ATMS). In a research collaboration which includes UC Berkeley, NASA Ames Research Center, and Honeywell Systems Research Center, an architecture for a new ATMS is proposed. Research is focused on hierarchical, hybrid control systems and their application to air traffic management. The proposed architecture describes an automated, decentralized ATMS: much of the control functionality exists in the flight vehicle management system on board each aircraft rather than at the ground-based air traffic control. Aircraft trajectories are restricted in the airport region; outside this region the architecture allows for so-called "free flight". A simulation and visualization package for hybrid systems in the framework of ATMS is being developed to aid the design process.

4 List of Manuscripts Submitted or Published

1. C. Tomlin, J. Lygeros, L. Benvenuti and S. Sastry, "Output Tracking for a Non-Minimum Phase Dynamic CTOL Aircraft Model", *Proceedings of the 34th IEEE Conference on Decision and Control*, pp. 1867-1872, 1995.

2. C. Tomlin and S. Sastry, "Bounded Tracking for Nonminimum Phase Nonlinear Systems with Fast Zero Dynamics", *Proceedings of the 35th Conference on Decision and Control, 1996.* (also submitted to the *International Journal of Control*, August 1996)

3. C. Tomlin, G. Pappas, and S. Sastry, "Conflict Resolution for Air Traffic Management: A Case Study in Multi-Agent Hybrid Systems", *Submitted to the IEEE Transactions on Automatic Control, Special Issue on Hybrid Systems, July 1996.* (also appears in Proceedings of the 35th Conference on Decision and Control, 1996).

4. S. Sastry, G. Meyer, C. Tomlin, J. Lygeros, D. Godbole and G. Pappas, "Hybrid Control in Air Traffic Management Systems", *Proceedings of the 34th IEEE Conference in Decision and Control*, pp. 1478-1483, New Orleans LA, 1995.

5. C. Tomlin, G. Pappas, J. Lygeros, D. Godbole, S. Sastry, and G. Meyer, "Hybrid Control in Air Traffic Management Systems", *Proceedings of the 13th Congress of the International Federation of Automatic Control*, July 1996.

6. J. Lygeros, C. Tomlin, and S. Sastry, "Multiobjective Hybrid Controller Synthesis", *To appear in 1997 International Workshop on Hybrid and Real-Time Systems, Grenoble, FRANCE.*

7. C. Tomlin and S. Sastry, Switching through Singularities, presented at the *Fourth International Conference on Hybrid Systems, Ithaca*, October 1996. (To be submitted to Systems and Control Letters, October 1996)

Appendix B: Publications of S. Sastry and coworkers on NAG 2-243

Professor S. Shankara Sastry Publications - Grant 23223

1. S. Sastry and S. Boyd, "On Parameter Convergence in Adaptive Control," *Systems and Control Letters,* Vol. 3, pp. 311-319, 1983.

2. S. Sastry and S. Boyd, "Necessary and Sufficient Conditions For Parameter Convergence in Adaptive Control," *Proceedings of American Control Conference,* San Diego, CA, June 1984; *Proceedings of Berkeley-Ames Symposium on Nonlinear Problems in Fluid Mechanics and Control,* pp. 81-103, Math. Sci. Press, Brookline, MA, August 1984; *Automatica,* Vol. 22, No. 6, pp. 629-639, September 1986.

3. S. Sastry and M. Bodson, "Small-Signal I/O Stability of Nonlinear Control Systems: Application to the Robustness of MRAC Schemes," *ERL Memo M84/70,* September 1984.

4. S.S. Sastry and R.S. Madriz, "Input-Output Description of Linear Systems with Multiple Time Scales," *International Journal of Control,* Vol. 40, No. 4, pp. 699-721, October 1984.

5. S. Sastry and M. Bodson, "Robustness For Model Reference Adaptive Control Schemes," *Proceedings of IEEE International Symposium on Decision and Control,* Las Vegas, NV, December 12-15, 1984.

6. S.S. Sastry and R.S. Madriz, "Multiple Time Scales in Nonlinear Systems," *Proceedings of IEEE Conference on Decision and Control,* Las Vegas, NV, December 12-15, 1984; *Circuits, Systems and Signal Processing,* Vol. 5, No. 1, pp. 153-169, March 1986.

7. S. Sastry and A. Packard, "Solution of Rational Matrix Equations in the State Space and its Application to Computer Aided Control Systems Design," *Proceedings of Conference on Computer Aided Control Systems Design,* Santa Barbara, CA, March 29-30, 1985, also *International Journal of Control,* Vol. 3, pp. 65-90, 1986.

8. S. Sastry, M. Bodson, B. Anderson, I. Mareels, and R. Bitmead, "The MIT Rule Revisited," *Proceedings of the IFAC Workshop on Adaptive Identification and Control,* Lund, Sweden, July 1986.

9. S. Sastry, M. Bodson, B.D.O. Anderson, I. Mareels and R. Bitmead, "Nonlinear Averaging Theorems and the Determination of Parameter Convergence Rates in Adaptive Control," *Systems and Control Letters,* September 1986.

10. S.S. Sastry and S.V. Sastry, "On the Integration of Robotics and Intelligent (Expert) Control with Power Systems Reliability," *Proceedings of the IFAC '86 Workshop on Power Systems Reliability,* Bangalore, India, December 1986.

11. S. Sastry and Z. Li, "Conditioned Invariant Subspaces, Disturbance Decoupling and Solution of Rational Matrix Equations," *International Journal of Control,* Vol. 43, No. 1, pp. 91-108, 1986.

12. S. Sastry, L.C. Fu, and M. Bodson, "New Stability Theorems For Averaging and their Application to the Convergence Analysis of Adaptive Identification and Control Schemes," *Singular Perturbations and Asymptotic Analysis in Control Systems,* Springer Verlag Lecture Notes in Control and Information Sciences, New York, Vol. 90, pp. 374-417, 1986.

13. S. Sastry, J. Mason, E. Bai, L.C. Fu, and M. Bodson, "Analysis of Adaptive Identifiers in the Presence of Unmodelled Dynamics," *Proceedings of IEEE Conference on Decision and Control,* Los Angeles, CA, December 1987, also *IEEE Transactions AC.,* Vol. 33, pp. 969-976, 1986.

14. S.S. Sastry and L.C. Fu, "Slow Drift Instability in Model Reference Adaptive Systems — An Averaging Analysis," *International Journal of Control*, Vol. 45, No. 2, pp. 503-527, 1987.

15. S.S. Sastry, E. Bai, and L.C. Fu, "Averaging Analysis For Discrete Time and Sampled Data Adaptive Systems," *IEEE Transactions on Circuits and Systems,* Vol. 35, pp. 137-149, February 1988.

16. S.S. Sastry and R.M. Murray, "Control Experiments in Grasping," *Proceedings of IEEE Conference on Robotics and Automation,* Phoenix, AZ, May 1989.

17. S. Sastry, J. Hauser and P. Kokotovic, "Zero Dynamics of Regularly Perturbed Systems May Be Singularly Perturbed," *Systems and Control Letters,* Vol. 13, pp. 299-314, 1989.

18. S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness,* Prentice Hall, 1989.

19. S.S. Sastry and L.C. Fu, "Frequency Domain Synthesis of Optimal Inputs For On-Line Identification and Control," *IEEE Transactions on Auto Control,* Vol. 36, No. 3, pp. 354-358, March 1991.

20. S. Sastry, A. Teel, R. Kadiyala and P. Kokotovic, "Indirect Techniques For Adaptive Input-Output Linearization," *International Journal of Control,* Vol. 52, pp. 193-222, 1991.

21. S. Sastry, J. Hauser and G. Meyer, "Nonlinear Control Design For Slightly Nonminimum Phase Systems: Application to V/STOL Aircraft," *Automatica,* 28, pp. 665-679, February 1992.

22. S. Sastry, J. Hauser and P. Kokotovic, "Approximate Linearization For Systems Without Relative Degree: The Ball and Beam Example," *IEEE Transactions on Automatic Control,* 37, pp. 392-398, 1992.

23. S. Sastry, G. Burgio and M. Di Benedetto, "Adaptive Linearization by Dynamic State Feedback: A Case Study," *Proceedings of the IFAC Conference on Nonlinear Control Systems Design (NOLCOS),* Bordeaux, France, pp. 594-600, June 1992.

24. S. Sastry and M. Di Benedetto, "Adaptive Control For MIMO Nonlinear Systems," *Journal of Mathematical Systems, Estimation and Control,* 3(1), 1993.

25. S. Sastry, G. Walsh and R. Montgomery, "Orientation Control of the Dynamic Satellite," *Proceedings of 1994 American Control Conference*, Baltimore, MD, pp. 135-142, June 1994.

26. S. Sastry, G. Walsh and R. Montgomery, "Optimal Path Planning on Matrix Lie Groups," *Proceedings of 1994 33rd IEEE Conference on Decision and Control*, Lake Buena Vista, FL, Vol. 2, pp. 1258-63, December 1994.

27. S. Sastry, G. Meyer, C. Tomlin, J. Lygeros, D. Godbole and G. Pappas, "Hybrid Control in Air Traffic Management Systems," *Proceedings of 1995 IEEE Conference on Decision and Control*, New Orleans, LA, December 1995.

28. S. Sastry, C. Tomlin, J. Lygeros and L. Benvenuti, "Output Tracking For a Non-Minimum Phase Dynamic CTOL Aircraft Model," *Proceedings of 1995 IEEE Conference on Decision and Control*, New Orleans, LA, December 1995.

29. S. Sastry, C. Tomlin, G. Pappas, J. Lygeros, D. Godbole and G. Meyer, "Hybrid Control in Air Traffic Management Systems," *Proceedings of 1996 IFAC*, San Francisco, CA, July 1996.

Appendix C: Publications of C. Desoer and coworkers, and other researchers on NAG 2-243

"Other" Publications - Grant 23223

1. Silva-Madriz R., "Reduced-Order Modelling in Linear and Nonlinear Multiple Time Scale Systems," Ph.D. dissertation, U.C. Berkeley, October 1984.

2. Desoer, C.A. and C.A. Lin, "Nonlinear Unity-Feedback Systems and Q-Parametrization," *Proc. 6e International Conference Analyse et Optomization des Systemes,* Nice, France, June 1984. Also *International Journal Control,* Vol. 40, No. 1, pp. 37-51, 1984.

3. Desoer, C.A. and C.A. Lin, "A Comparative Study of Linear and Nonlinear MIMO Feedback Configurations," *International Journal Systems Science*, Vol. 16, No. 7, pp. 789-813, 1985.

4. Desoer, C.A. and A. Bhaya, "On the Design of Large, Flexible Space Structures," *IEEE Transactions,* AC-30, No. 11, pp. 1118-1120, November 1985.

5. Desoer, C.A. and A. Bhaya, "Robust Stability Under Additive Perturbations," *IEEE Transactions*, AC-30, No. 12, pp. 1233-1234, December 1985.

6. Desoer, C.A. and A. Bhaya, "Necessary and Sufficient Conditions for $Q = C(I + PC)^{-1}$ for Stabilization of the Linear Feedback System $S(P,C)$," *Systems and Control Letters*, 7, 1, pp. 35-39, February 1986.

7. Bhaya, A., "Issues in the Robust Control of Large Flexible Structures," Ph.D. dissertation, U.C. Berkeley, April 1986.

8. Desoer, C.A. and S.M. Shahruz, "Stability of Dithered Nonlinear Systems with Backlash or Hysteresis," *International Journal Control*, 43, 4, pp. 1045-1060, April 1986.

9. Bodson, M., "Stability, Convergence and Robustness of Adaptive Systems," Ph.D. dissertation, U.C. Berkeley, August 1986.

10. Desoer, C.A. and S.M. Shahruz, "Stability of Nonlinear Systems with Three Time Scales," *Circuits Systems and Signal Processing*, Vol. 5, No. 4, pp. 450-464, 1986.

11. Fu, L.C., "Frequency Domain Analysis and Synthesis Techniques," Ph.D. dissertation, U.C. Berkeley, 1987.

12. Desoer, C.A. and M.G. Kabuli, "Linear Stable Unity-Feedback Systems: Necessary and Sufficient Conditions for Stability Under Nonlinear Additive Plant Perturbation," *Proceedings of 25th Allerton Conference,* Illinois, pp. 522-524, 1987.

13. Desoer, C.A. and M.G. Kabuli, "Stabilization and Robustness of Nonlinear Unity-Feedback System: Factorization Approach," *International Journal Control*, Vol. 47, 4, pp. 1133-1149, April 1988.

14. Desoer, C.A. and M.G. Kabuli, "Right Factorization of a Class of Time-Varying Nonlinear Systems," *IEEE Transactions*, AC-33, pp. 755-757, August 1988.

15. Desoer, C.A., A.N. Gundes and M.G. Kabuli, "Coprime Factorizations in Stable Linear Systems," *Proceedings of American Control Conference*, Atlanta, GA, pp. 1252-1254, 1988.

16. Desoer, C.A., and M.G. Kabuli, "Feedback Stabilization of a Class of Nonlinear Plants Using a Factorization Approach," *Proceedings of 27th Conference on Decision and Control*" Austin, TX, Vol. 1, pp. 455-456, December 1988.

17. Desoer, C.A. and M.G. Kabuli, "Factorization Approach to Nonlinear Feedback Systems," *UCB/ERL Memo M89/64*, April 1989.

18. Hauser, J., "Approximate Tracking for Nonlinear Systems with Application to Flight Control," Ph.D. dissertation, U.C. Berkeley, August 1989.

19. Kabuli, M.G., "Factorization Approach to Nonlinear Feedback Systems," Ph.D. dissertation, U.C. Berkeley, August 1989.

20. Teel, A., "CAD for Nonlinear Control Systems," *ERL Memo UCB/ERL M 89/135,*, December 1989.

21. Teel, A., "Toward Larger Domains of Attraction for Local Nonlinear Control Schemes," *European Control Conference*, October 1990.

22. Teel, A., "Robust and Adaptive Nonlinear Output Regulation," *European Control Conference*, 1991.

23. Teel, A., "Feedback Stabilization: Nonlinear Solutions to Inherently Nonlinear Problems," Ph.D. dissertation, U.C. Berkeley, 1992.

24. Kadiyala, R., "CAD Tools for Nonlinear Control," Ph.D. dissertation, U.C. Berkeley, 1992.

25. Pradeep, A.K., "Sliding Mode Control of Perturbed Nonlinear Systems," Ph.D. dissertation, U.C. Berkeley, 1992.

26. Kadiyala, R., "CAD Tools for Nonlinear Control," *Proceedings of IEEE Conference on Computer Aided Design*, Napa, CA, April 1992.

27. Kadiyala, R., "Sys_View: A Visualization Tool For Viewing the Regions of Validity and Attraction of Nonlinear Systems," *ERL Memo No. M 92/21*, March 1992.

28. Teel, A., "Global Stabilization and Restricted Tracking For Multiple Integrators With Bounded Controls," *Systems and Control Letters*, 1992.

29. Teel, A., "Error Based Adaptive Nonlinear Control and Regions of Feasibility," *Systems and Control Letters,* 1992.

30. Teel, A., "Using Saturation to Stabilize a Class of Partially Linear Composite Systems," *Proceedings of IFAC Conference in Nonlinear Control Systems Design (NOLCOS)*, pp. 224-229, 1992.

31. Kadiyala, R., "Indirect Adaptaive Control of Induction Motors," *Proceedings of IFAC Conference on Nonlinear Control Systems Design (NOLCOS)*, pp. 144-150, 1992.

32. Teel, A., "Semi-Global Stabilization of Minimum Phase Nonlinear Systems in Special Normal Forms," *International Journal of Control*, 1992.

33. Teel, A., R. Murray and G. Walsh, "Nonholonomic Control Systems: From Steering to Stabilization With Sinusoids," *ERL Memo, No. M 92/28*, March 1992, also *IEEE Conference on Decision and Control*, 1992.

34. Teel, A., "Beyond Linear Feedback For the Nonlinear Regulator," *ERL Memo, No. M 92/29*, March, 1992.

35. Kadiyala, R., "AP_LIN: A Tool Box For Approximate Linearization of Nonlinear Systems," *IEEE Control Systems Magazine*, March 1992.

36. Getz, N., "Control of Modified Caplygin Nonholonomic Systems," *Proceedings of IEEE Conference on Decision and Control*, San Antonio, TX, 1993.

37. Pappas, G., J. Lygeros and D. Godbole, "Stabilization and Tracking of Feedback Linearizable Systems Under Input Saturation Constraints," *Proceedings of 1995 IEEE Conference on Decision and Control*, New Orleans, LA, December 1995.