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Solar-driven neutral density waves

P. Blum[1], **P. Gangopadhyay**[2], **H.S. Ogawa**[2], **and D.L. Judge**[2]

[1] Institut für Astrophysik, University of Bonn, Auf dem Hügel 71, W-5300 Bonn 1, Germany
[2] Space Sciences Center and Department of Physics, University of Southern California, Los Angeles, CA 90089-1341, USA

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Abstract. Interstellar neutral hydrogen atoms flowing into the solar system are attracted by the solar gravitational force, repelled by solar hydrogen Ly-α radiation pressure, and are ionized, primarily, through charge exchange with the solar wind protons. The solar cycle variation of the radiation pressure causes the net central solar force to fluctuate between attraction and repulsion resulting in the modulation of the neutral hydrogen density about the usual time independent model. The calculation presented here shows that the time dependent downstream density is strongly modulated by a large number of travelling neutral density waves. The waves possess a continuous range of wavelengths as is to be expected for a Maxwellian gas subjected to several eleven year solar cycle variations during its journey through the solar system. The amplitudes of the density modulation were found to be quite large. The backscattered glow was found to depend on the position of the detector and the phase of the solar cycle. At the most favorable condition a deviation of the order of 25% from the time independent glow might be observed.

Key words: interplanetary medium – interstellar hydrogen

1. Introduction

Neutral hydrogen atoms entering the solar system experience a time varying central force resulting from the variation of the solar Ly-α radiation pressure. Since the solar gravitational force and the radiation force are nearly equal, but opposite in direction, the solar cycle variation of radiation pressure will make the net central force attractive or repulsive depending on the phase of the solar cycle. The neutral atom density along the downstream axis is particularly sensitive to the solar cycle effect. For example, in the zero temperature limit only those hydrogen atoms which experience a net attractive central force while passing close to the Sun will reach the downstream axis. Thus, depending on the hydrogen temperature, the downstream time averaged density derived from models not taking into account solar activity variations can differ considerably from the instantaneous density.

A straightforward analytic approach to this time dependent problem would require that the time dependent Boltzmann equation be solved. Fahr et al. (1987) derived a formal solution to the Boltzmann equation. It was solved, however, only for the time independent central force problem, which is valid for helium but not for hydrogen. Recently, Fahr & Scherer (1990) have derived an explicit expression for the Fourier transformed neutral gas density for a time dependent central force. However, this density was not back-transformed into real space and time. We have taken an alternate and less sophisticated approach by following neutral hydrogen atoms through space over many solar cycles for a period of ~ 50 yr. The density is obtained by simply counting the number of atoms crossing the downstream axis at given time intervals and assigning appropriate weights to them. This, however, is only the first step. Since we do not perform a full scale simulation and have restricted ourselves to following particles with discrete sets of velocities in the x and y directions to generate a provisional density, we have used a Fourier transform technique (described later) to compute a complete density distribution. Our goal in this paper is to study the nominal size of the effect for a time varying radiation force without going through an expensive and time consuming full scale simulation.

2. Numerical calculation

Figure 1 shows the geometry of the simulation. The x-axis is taken parallel to the neutral hydrogen flow at large distances from the Sun. The simulation is simplified by assuming azimuthal symmetry about the x-axis and is carried out in the x–y plane. The neutral hydrogen atoms are launched from a preselected launch space. The launch space is bounded by $-220 \leq x \leq -20$ AU and $0 < y < y_{max}$. It may be thought that launching particles from $x \sim 20$ AU is too close to the Sun for the Maxwellian distribution to hold, however, our result (Fig. 4) suggests that this approximation is adequate. As a further test of our launch space we obtain the correct density in the $T = 0$ K limit, as shown

Send offprint requests to: P. Gangopadhyay later. Y_{max} is determined empirically, i.e. y is increased till no particle reaches the downstream axis within 200 AU of the Sun. All the particles are launched at the same time, in 1940. The neutral atoms follow trajectories that are non-Keplerian because the central force acting on them is time dependent. The total radial acceleration of an interplanetary hydrogen atom is given by

$$\ddot{r} = -(1-\mu)\frac{GM}{r^2} + r\dot{\phi}^2 \quad (1)$$

where μ is the ratio of the Lyman-alpha radiation pressure to the solar gravitational attraction, G is the gravitational constant and M is the solar mass. The distance of the hydrogen atom from the Sun is r and its azimuthal angle relative to the upwind axis is ϕ.

The solar cycle variation of the Lyman-alpha radiation is described by the time dependence of μ. μ is known to vary from about 0.75 at solar minimum to 1.35 at solar maximum. Based on observations of the solar Lyman-alpha radiation we have approximated the variation of μ by

$$\mu = 1.06 + 0.29\cos\left[\frac{2\pi(\text{time}-1980)}{11}\right], \quad (2)$$

where time is given in years AD.

The density of downwind neutral hydrogen atoms near the axis of symmetry is a function of distance to the sun r and time t, i.e. den = den(r,t). In order to obtain den(r,t) we have divided the launch area (Fig. 1) into grid points spaced by 1/4 AU in the x and y directions. On this space grid we have superimposed a velocity grid, three values of V_x and V_y covering the range appropriate to a Maxwellian velocity distribution of an 8000 K gas. From each point of the combined space–velocity grid one particle is launched. The orbits of all particles are followed and those which cross the downwind region are put into various bins according to the position r and the time t of their downwind axis crossing. Each bin covers a spatial region of dimension $\Delta x\,\Delta y$ centered about the position $r = x$ and $y = 0$. The particles are collected over a time interval Δt at time t. Δx and Δy were both taken to be 1 AU and Δt was taken to be one year. After all the atoms passed through the downwind region, the density den(r,t) was found by adding all properly weighted particles which passed through each bin. The weighting factor takes into account all of the following points:

(a) The probability that the atom at the launch position has the velocity components V_x and V_y.

(b) The volume of the launch space, which is proportional to $2\pi y_{in}$, where y_{in} is the y-coordinate at the launch point. This factor arises from our assumption of azimuthal symmetry.

(c) The survival probability of the atom against ionization.

(d) The time span during which a particle remains in its particular cell in the downwind region.

We will briefly discuss the various terms of the weighting factor.

We assume the neutrals in our launch space to have a temperature T and therefore a Maxwellian velocity distribution centered on $V_x = V_b$ and $V_y = 0$, where V_b is the bulk velocity of the neutrals taken to be 20 km s^{-1}. The density of the neutrals in the launch region is taken to be uniform and the velocity distribution of the neutrals in the launch region is proportional to

$$\exp\left[-\frac{m}{2kT}(V_x - V_b)^2 + V_y^2\right],$$

where m is the hydrogen atom mass, T the temperature and k the Boltzmann constant.

The downwind neutral density is proportional to the survival probability surv(r,t) of neutrals against interplanetary ionization processes. The predominant ionization process is charge exchange with solar wind protons, and the probability of survival against this process is given by $\exp(-\int dt/\tau) = \exp(-\int n_p \sigma V_{rel}\, dt)$. Here τ is the lifetime of a H atom against ionization by charge exchange. n_p is the density of protons at (r, ϕ), σ is the charge exchange cross section, and V_{rel} is the relative velocity between the neutral atoms and protons. The angular momentum of a particle at any point along a trajectory is constant and given by $l = mr^2\, d\phi/dt$. Hence, we can substitute $dt = (mr^2/l)\, d\phi$ into the expression for surv. The final expression for surv is $\exp(-\int n_p \sigma V_{rel} mr^2\, d\phi/l)$.

The time interval during which a particular particle remains in a given bin on the downstream axis is proportional to $\Delta y/v_y$. Particles which traverse more than one bin along the axis are in this approximation placed in the first bin which they enter.

Based on these considerations we obtain den(r,t) by summing over all the weighted particles in the bin defined by r and t:

$$\begin{aligned}\text{den}(r,t) \sim \sum_{\text{particle}} & \exp(-\textstyle\int n_p \sigma V_{rel} mr^2\, d\phi/l) \\ & \times \exp\left\{-\left[\frac{m}{2kT}(V_x - V_b)^2 + V_y^2\right]\right\} \\ & \times 2\pi y_{in}\left[\frac{|\Delta y|}{V_y(r,\phi,t)}\right]. \end{aligned} \quad (3)$$

3. Results and discussion

The zero order density calculated by this method uses a limited set of velocities. So we use Fourier transform method to compute the complete density as discussed below. We computed the zero order neutral hydrogen atom density for the downstream region for various years between 1972 to 1992 by counting the particles collected in the various bins. The statistical accuracy of the provisional density varied with the distance from the Sun. The accur-

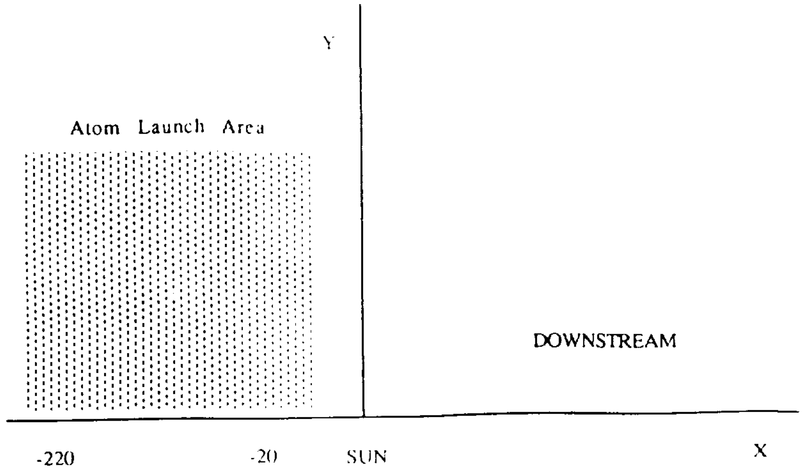

Fig. 1. The launch geometry is shown. The shaded area is the region from which the atoms are launched. The black dots schematically represent the discrete launch points

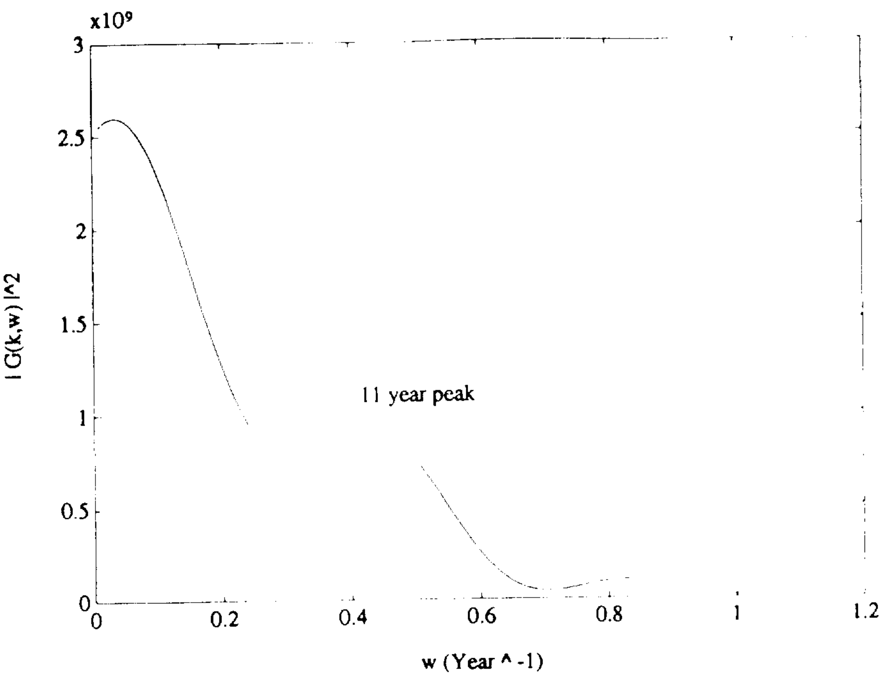

Fig. 2. The power spectrum of the full density $|G(k,\omega)|^2$ for $k = 0.1$ AU^{-1} is plotted against ω(yr^{-1}). This calculation was performed for a 20 yr period from 1972 to 1992

acy was of high order $<5\%$ at heliocentric distance >100 AU. Near the Sun (<50 AU) the accuracy declines to 10% or more. Since it is clear that the neutral hydrogen atom density will carry information about the solar cycle variation of the radiation pressure we have calculated the power spectrum of the density fluctuations as a function of the wave vector k and frequency ω. The Fourier integral transform of den(r,t) is given by

$$G(k,\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{den}(r,t)\, e^{-i(kr-\omega t)}\, \mathrm{d}r\, \mathrm{d}t. \qquad (4)$$

The power spectrum in ω, $|G(k,\omega)|^2$, for a particular value of k is shown in Fig. 2. The first peak at $\omega \approx 0$ is the time independent part of the density while the peak at $\omega \approx 2\pi/11$ yr^{-1} corresponds to the 11 yr period arising naturally from the solar activity cycle. Since we did not carry out an exact simulation it is not surprising that the expected $\omega = 0$ peak is not exactly at $\omega = 0$. The third peak may be an artifact of the approximation method used here. This result has enabled us to write the density den(r,t) as den$(r,t) = \mathrm{den}_0(r)[1 + d_1(r)\exp(-i\omega t)]$ where den$_0(r)$ is the time independent density. The absence of neutral hydrogen atoms close to the Sun ($r = 0$) imposes the condition that den$_0(0) = 0$. The time independent density den$_0(r)$ is of the form $a(1 - e^{-br})$. The time independent density is of the exponential form if the solar gravitational force and radiation force cancel each other (Axford 1972; Isenberg 1986). The constants a and b were obtained by a least squares fit to the simulated time dependent data. The fluctuating part $d_1(r)\exp(-i\omega t)$ was obtained from the ratio $[\mathrm{den}(r,t) - \mathrm{den}_0(r)]/\mathrm{den}_0(r)$.

The Fourier transform of the oscillating density is given by

$$
\begin{aligned}
g(k,\omega) &= \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d_1(r)e^{i\omega_0 t}e^{-i(kr+\omega t)}\,dr\,dt \\
&= \frac{1}{(2\pi)}\int_{-\infty}^{\infty} d_1(r)e^{-ikr}\,dr\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(\omega_0-\omega)t}\,dt\right] \\
&= \frac{1}{(2\pi)}\int_{-\infty}^{\infty} d_1(r)e^{-ikr}\,dr\,\delta(\omega_0-\omega) = g(k)\,\delta(\omega_0-\omega).
\end{aligned}
\tag{5}
$$

The power spectrum in k space for the oscillating density, $|g(k)|^2$, computed for $t = 1988$ is shown in Fig. 3. The expression for $g(k)$ was obtained from Eq. (5). For the year 1988 there are three clear peaks in k space. These three peaks correspond approximately to the three (V_x) velocities of launch. An exact simulation using a finer velocity grid would reveal a continuous spectrum in k. This is a reflection of the Maxwellian distribution of velocities. There is also an interesting feature in the power spectrum in ω. The density power spectrum is in principle a δ-function in ω-space because all the atoms were subjected to the same

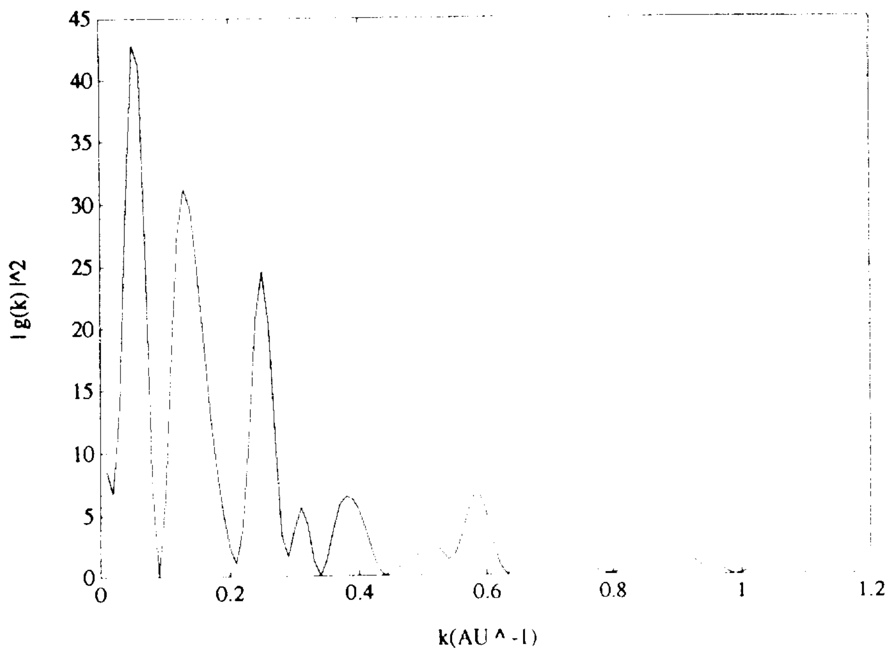

Fig. 3. The power spectrum of the residual neutral density $|g(k)|^2$ is plotted against wave vector k(AU^{-1}), where $g(k) = \int e^{ikr} d_1(r)\,dr$. This calculation was performed over a distance between 5 to 140 AU for the year 1988

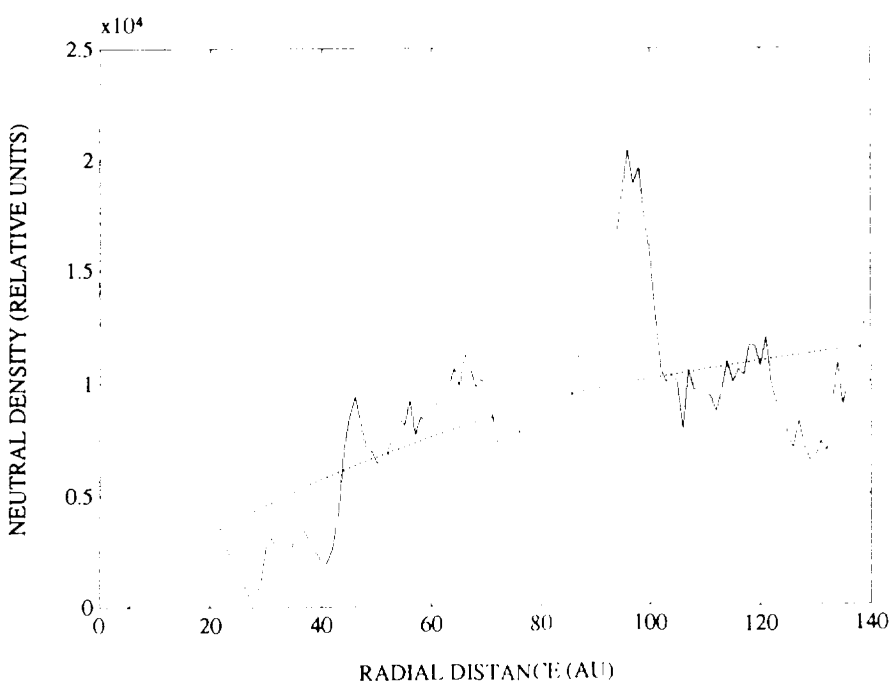

Fig. 4. The last square fitted neutral densities for 1988 (dots) and the modulated density (solid line) are plotted against the radial distance from the Sun in the downwind direction

11 yr variation of the net central force. The finite spatial and temporal simulation of the neutral atom flow through the solar system, however, "broadens" the δ-function which shows up as the broad peak in Fig. 2.

Since there is a continuous spectrum in k the density should be written in a Fourier integral from:

$$\mathrm{den}(r,t)=\mathrm{den}_0(r)\left(1+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(k,\omega)\mathrm{e}^{\mathrm{i}(kr-\omega t)}\mathrm{d}k\,\mathrm{d}\omega\right)$$

$$=\mathrm{den}_0(r)\left(1+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(k) \times \delta(\omega-\omega_0)\mathrm{e}^{\mathrm{i}(kr-\omega t)}\,\mathrm{d}k\,\mathrm{d}\omega\right)$$

$$=\mathrm{den}_0(r)\left(1+\mathrm{e}^{-\mathrm{i}\omega_0 t}\int_{-\infty}^{\infty} g(k)\,\mathrm{e}^{\mathrm{i}kr}\,\mathrm{d}k\right). \tag{6}$$

We have accordingly used Eq. (6) to calculate the total density in 1988 and the results are shown in Fig. 4. Of course, the physical density is the real part of $\mathrm{den}(r,t)$. It is interesting to note that the size of the modulation is quite large in the downstream direction. This is consistent with the limiting case of a cold gas, $T=0$ K (Fig. 5), where the

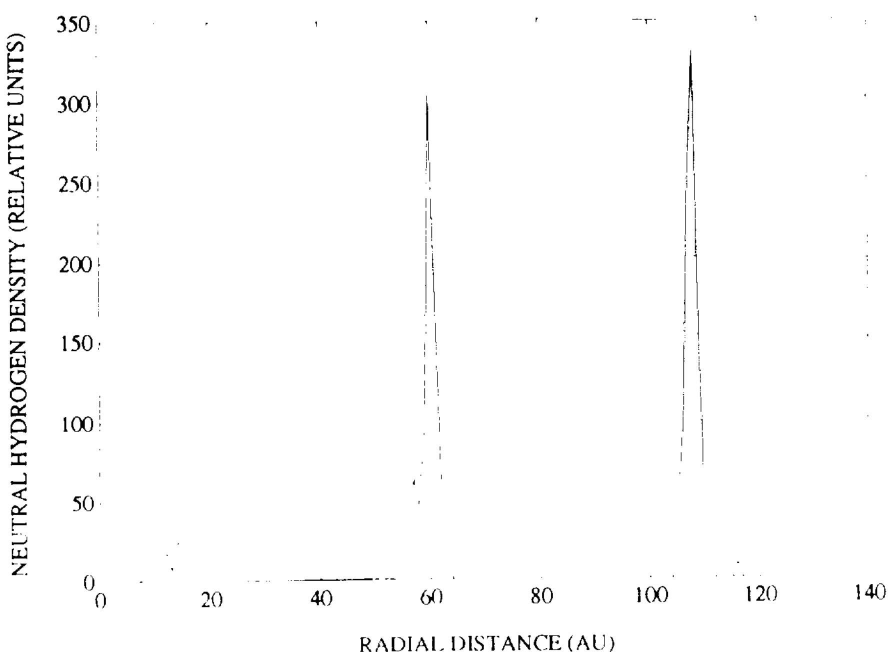

Fig. 5. The downstream neutral hydrogen density for a 0 K temperature gas as obtained from the simulation is plotted against the heliocentric distance

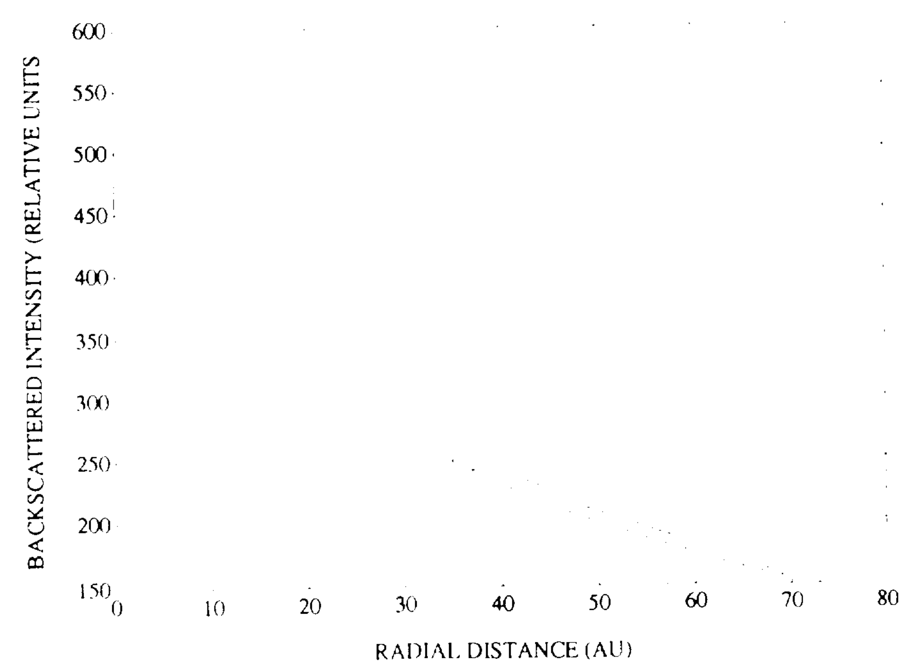

Fig. 6. The quantity $\int(n\,r^2)\,\mathrm{d}r$, which is proportional to the backscattered intensity, is plotted for the year 1988 against heliocentric distance for cases (i) the time independent density (dots) and (ii) modulated density (solid line)

modulation is 100% on the downwind axis giving rise to travelling hydrogen rich and hydrogen empty regions.

Would it be possible to see these travelling waves by using the UV photodetectors aboard the deep space probes Pioneers 10/11 and Voyagers 1/2? We have carried out a demonstrative calculation to explore the effect of the solar cycle driven wave on the backscattered intensity. We calculated $\int (n/r^2)\,dr$ (a quantity proportional to intensity in the single scattering approximation) where n is the hydrogen density and r is the heliocentric distance to the scattering volume. For simplicity we assumed an isotropic phase function and a radial field of view. We calculated the backscattered hydrogen Ly-α intensity for both time dependent and time independent density for the year 1988, i.e.

(i) $n = \mathrm{den}_0(r)$ and

(ii) $n = \mathrm{den}_0(r,t)[1 + d_1(r)\exp(-i\omega t)]$. (7)

In Fig. 6 we have plotted the backscattered intensities in the downwind direction for cases (i) and (ii). Any deviation in the backscattered glow due to the travelling wave is seen to depend on the spacecraft position and the time of observation and can be as large as $\sim 25\%$. The Pioneer 10 glow data may contain evidence of this effect since it is moving in the downstream direction where the density variations are significant and in principle observable. The density modulations might also be large enough to be directly detected by a neutral particle in-situ detector.

4. Summary

The effect of solar cycle radiation pressure variations on the downstream neutral hydrogen density has been explored. The Fourier transform of the density in k and ω space was found to depend on a spectrum of k values but only on a single $\omega(\simeq 2\pi/11\ \mathrm{yr\,s^{-1}})$. There is clear evidence for a series of travelling waves with a continuous spread of wave vectors. Thus, a spacecraft going downstream should be able to detect a deviation in the backscattered glow intensity, relative to that expected on the basis of a time independent density, for selected detector positions and observation times.

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