

Reduction of Model Complexity by Active Control

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Abstract

The mathematical models of many dynamic systems of interest in the aerospace industry are inherently complex and of high order. Rather than grapple with the full-complexity models for such systems, the control designers often elect to derive low-order, reduced-complexity models for which a control system can be designed. The subject of model-order reduction has received a significant amount of attention in control engineering literature. This paper describes and demonstrates a novel active-control methodology that can be used to design a control system that automatically forces the original system to behave like a chosen reduced-complexity model by treating the effects of the original complexity-related terms as disturbances acting on the reduced-complexity system. The method is demonstrated by several worked examples involving both linear and nonlinear systems, supported by simulation results.

Introduction

The difficulty in developing high-order models for complex systems in the aerospace industry has been facilitated by the emergence of high-speed digital computers and automated modeling software. The realistic behavior of even extremely complicated systems can now be effectively modeled using finite element analyses and/or complex system simulations embodying relevant characteristics such as nonlinearities, coupling, time-varying parameters, large numbers of inputs and outputs, time delays, and input-derivative terms. Most models of complex systems are inherently high-order, even when the other complexities identified above are eliminated via linearization, estimation, or prudent disregarding of the offending terms. Consequently, the control system designer must often cope with unwieldy, high-order models which lead to the design of inordinately high-

order control systems. Although, in theory, the controller developed provides the desired performance, implementation of the full-order controller is sometimes not practical.

In recent years, a considerable amount of research effort has been directed towards the problem of developing reliable order-reduction techniques for both mathematical system models and their associated controllers. Reduction in model order generally leads to a simpler, lower-order controller design, thereby increasing the feasibility and practicality of implementation and potentially reducing cost. The subsequent reduction in the degrees-of-freedom associated with the reduced-order model aids in the designer's visualization and understanding of the system and may lead to further insight into a control system design. Typically, the complexity of a model is reduced by first eliminating or simplifying nonlinearities, coupling effects, time-varying parameters, and other complicating factors. The model is then further simplified by reducing the order of the less complex model via frequency domain or state-space techniques. Control systems designed in this manner can provide acceptable performance in principle. However, in some instances the controller may require further tuning to improve performance, or may necessitate the addition of filters to eliminate the effects of unmodeled dynamic modes that interact with the control system in an undesirable fashion.

The technique presented and illustrated in this paper constitutes a new and innovative way to achieve effective reduction of model-complexity by active control. The examples presented are simple engineering examples, however they demonstrate the potential for application of this method to more complicated systems. The traditional approach of first reducing the model-complexity and then designing the control system provides no mechanism whereby the credibility of the

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reduced-complexity model (RCM) is guaranteed. The term "reduced-complexity" is used in place of "reduced-order" because the Model Complexity Reduction by Active Control (MCRAC) method produces a controller that is inherently capable of eliminating or minimizing the undesirable effects of phenomena such as coupling and nonlinear terms, as will be demonstrated via example problems. The method links the model-complexity reduction with the controller design method such that upon implementation of the controller, the credibility of the assumed RCM is automatically enforced by the controller actions, thereby achieving a new level of effectiveness in applying the classical linearization techniques.

The proposed method employs "disturbance observer" theory to generate real-time estimates of the time-domain effects of the unwanted terms associated with model complexity. The total control effort is divided in such a way that part of the controller action annihilates, or minimizes, the disturbing effects of the model-complexity terms on the time response of the system. The other part of the control effort provides the desired closed-loop system performance. The control system designer is thus free to tailor the desired RCM and develop a control strategy such that the closed-loop RCM meets the desired performance goals and rejects disturbances acting on the system. The following sections present details of the method followed by three illustrative examples. Included in the Summary and Conclusions section are plans to extend the method to more complicated problems.

Model Complexity Reduction

Model complexity reduction is more generally referred to as model order reduction and is considered to be the reduction of the dynamic order of the model. Given a system of order n , the goal of the model order reduction process is to derive a model of order k , where $k < n$, such that for a given input, the reduced-order model output(s) closely tracks that of the full order system model. An error norm is examined to determine the accuracy of the reduced-order model. A plethora of methods exists by which this may be accomplished using time- and frequency-domain techniques as indicated in the extensive bibliographies in References 1 and 2. Indeed this area of model simplification has garnered the majority of the attention of researchers in this area. Techniques including balanced realization, component mode synthesis, aggregation methods, perturbation methods, and continued

fraction methods have been developed to accomplish the task.

Less frequently addressed are other aspects of model complexity reduction. These include approximation procedures for dealing with nonlinearities, time-varying parameters, coupling, and the number of system inputs. System nonlinearities are typically addressed by conventional linearization about a nominal operating point (or points) or a nominal trajectory. Functional linearizations such as describing functions may also be employed. Time-varying parameters are typically eliminated by using averaging methods or frozen-time eigenvalue methods. Coupling effects are sometimes ignored based on the assumption that they manifest fast transients. Coupling is also eliminated from high-order models using transformations to alternate coordinate systems, such as modal coordinates. The transformation masks the existence of coupling and the transformed model is then more easily reduced for controller design. For systems with a "large" number of inputs, often the number of inputs is reduced for simplification of the design via a fixed coordination of multiple control inputs or by ignoring certain disturbance inputs. Multiple-input multiple-output techniques are currently enjoying a much wider application to this modeling aspect.

The MCRAC design philosophy is that a linear controller for the original full-complexity system model, M , can be designed using a judiciously chosen reduced complexity model, m , such that the resulting controller, $u(m)$, provides performance which meets a given set of closed-loop specifications when implemented with the original system model, M . Furthermore, the action of $u(m)$ enforces the RCM selected and guarantees the desired performance of M . The reduced complexity model is of lower order than the original and is linear. The resulting control design is, in general, of lower order than one designed using the original model.

Assume that M has the standard state space form given in equation (1) below, where the matrix A may be time-varying, nonlinear, highly coupled, or embody other complexities.

$$\begin{aligned}\dot{x} &= F(x,t,u,w) \\ \dot{x} &= Ax + Bu + Fw + \text{h.o.t.'s} \\ y &= Cx\end{aligned}\tag{1}$$

In the equation above, x is the state variable, u is the control variable, w is an external disturbance, y is the output, and "h.o.t.'s" stands for higher order terms which arise from classical linearization. A reduced complexity model may be developed from (1) in the following manner. A linear RCM system matrix A_r and the state vector $\dot{x}_r$ are selected by the designer to embody the primary dynamics to be controlled. If $\dot{x}_s$ is defined as the state vector which embodies the secondary dynamics, then the full system model can be written as given in equation (2) which, for now, ignores the h.o.t.'s derived from linearization.

$$\dot{x} = \begin{bmatrix} A_r & A_{rs} \\ A_{sr} & A_s \end{bmatrix} \begin{bmatrix} x_r \\ x_s \end{bmatrix} + Bu(m) + Fw \quad (2)$$

$$y = C_r x_r$$

The reduced complexity model is now given by

$$\dot{x}_r = A_r x_r + A_{rs} x_s + B_r u(m) + F_r w$$

$$y = C_r x_r \quad (3)$$

with the dynamics of x_s determined by equation (4) below.

$$\dot{x}_s = A_{sr} x_r + A_s x_s \quad (4)$$

Before explaining how the effects of the secondary dynamics are handled in the design, the structure of the controller $u(m)$, given in equation (5) below, is examined.

$$u(m) = u_p(m) + u_d(m) + u_c(m) \quad (5)$$

where

$u_p(m) \equiv$ control providing desired response
 $u_d(m) \equiv$ control for external disturbances
 $u_c(m) \equiv$ control for system complexities.

Note that the total control effort $u(m)$ is divided into three parts, the primary part $u_p(m)$ that provides the desired closed-loop system response, the disturbance accommodation part, $u_d(m)$ that is designed to cancel or minimize external disturbances, and the RCM part $u_c(m)$ that is designed to cancel or minimize the effects of the unmodeled modes, nonlinearities (or h.o.t.'s resulting from linearization), and other complexities on the behavior of the reduced-complexity system. Those effects are viewed as time-varying disturbances, and thus $u_c(m)$ can be designed using Disturbance Accommodating

Control (DAC) methods.[3] Consider the external disturbance term and recall that for cancellation

$$-B_r u_d(m) = Fw \quad (6)$$

The disturbance w is modeled in state-space form as

$$\dot{z} = Dz + \sigma$$

$$w = Hz \quad (7)$$

In equation (7), z is the state of the disturbance w and D is the system matrix governing the dynamics of z . The matrix D is developed directly from the time-domain or differential equations defining the waveform structure of w . The σ are Dirac impulses which arrive at random intervals with random intensities. The derivation of the matrix D is explained thoroughly in the reference. From equation (6), disturbance cancellation occurs when

$$u_d(m) = -B_r^{-1} F H z \quad (8)$$

which requires that

$$\text{rank}[B] = \text{rank}[B F H] \quad (9)$$

If the disturbance dynamics described in equation (4) manifest a waveform structure, which is typically the case for physical systems, then $u_c(m)$ can be designed using the same principles as described above for the external disturbances. A disturbance state model is developed based on the waveform structure of x_s leading to the development of a state disturbance model as in equation (10).

$$\dot{z}_s = D_s z_s + \sigma$$

$$x_s = H z_s \quad (10)$$

Therefore, for cancellation,

$$u_c(m) = -B_r^{-1} A_{rs} H z_s \quad (11)$$

with the rank condition as indicated in equation (9) satisfied. When full cancellation occurs, the dynamic order of the system is reduced. The control $u_c(m)$ must eliminate the effects of the coupling term, or nonlinear terms (demonstrated in the third example in the following section), without destabilizing the system.

The total control action $u(m)$ can be divided into additional parts, as required, to control any other disturbances that may act on the plant. The

$u_p(m)$ part of the controller is designed using the engineer's method of choice so long as the controller meets the performance requirements. A composite observer is designed using standard observer theory to estimate the system states required for controller implementation as well as the effects of the unmodeled dynamics and disturbances. The observer order generally determines the order of the controller, therefore, care must be taken when selecting the disturbance model for the full-order model complexities. The choice of the disturbance model will be discussed in the following section. The outputs of the observer are used with the appropriate gains, calculated using the RCM, to implement $u(m)$. The method will be demonstrated and elaborated upon using three simple examples in the following section.

Illustrative Examples

Three design examples are presented here to demonstrate the concept of the MCRAC method for controller design. It is emphasized that the results shown here are preliminary, but show a high degree of promise for the method. All examples are single-input, single-output systems with the assumption of co-location of the sensor and actuator. It is further assumed that the complexities removed from the full-order model may be modeled as an input disturbance. The first example comprises an eight-order spring-mass system. The second example is a highly-coupled sixth order system which simulates a flexible beam. The third example is a nonlinear second-order system.

Shown in Figure 1 is a diagram representing an eighth-order dynamic spring-mass system, which is the focus of the first example. The values of the masses and spring constants used in this example are given below the diagram in Table 1. The control objective is to move m_1 to a given set point position and hold it there, regardless of the motion of the other three masses. Note that

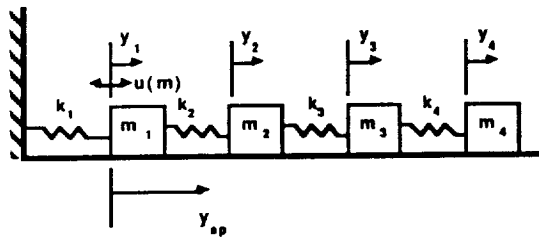


Figure 1. Spring-mass system for Example 1.

$m_1 = 10$	$k_1 = 5$
$m_2 = 5$	$k_2 = 10$
$m_3 = 2.5$	$k_3 = 5$
$m_4 = 5$	$k_4 = 2.5$

Table 1. Parameter values used in Example 1.

there is no damping in the system so that the masses m_2 , m_3 , and m_4 will continually oscillate. It is assumed that the position of m_1 can be sensed and that the control action is applied at m_1 . The equations of motion were derived and are given below in equations (12).

$$\begin{aligned}\ddot{y}_1 &= -\left(\frac{k_1 + k_2}{m_1}\right)y_1 + \frac{k_2}{m_1}y_2 + \frac{1}{m_1}u \\ \ddot{y}_2 &= \frac{k_2}{m_2}y_1 - \left(\frac{k_2 + k_3}{m_2}\right)y_2 + \frac{k_3}{m_2}y_3 \\ \ddot{y}_3 &= \frac{k_3}{m_3}y_2 - \left(\frac{k_3 + k_4}{m_3}\right)y_3 + \frac{k_4}{m_3}y_4 \\ \ddot{y}_4 &= \frac{k_4}{m_4}y_3 - \frac{k_4}{m_4}y_4\end{aligned}\quad (12)$$

The eight-order model was cast in state space form for complexity reduction and control design. Because the goal of the closed-loop controller is to move m_1 to a particular position and hold it there regardless of the motion of the other masses, the reduced-complexity second order RCM, m , given in equations (13) below was selected. In this equation, y_r represents the variable y_1 , therefore, the primary states for the system are those associated with y_1 . The motion of the other masses and the resulting forces imparted on m_1 are secondary and are modeled as a disturbance input w_s , which equates to x_s in equation (10).

$$\ddot{y}_r + \frac{k_1 + k_2}{m_1}y_r = \frac{1}{m_1}u(m) + \frac{k_2}{m_1}w_s \quad (13)$$

The control input $u(m)$ is divided as in equation (5) except that there is no $u_d(m)$ term as no external disturbances are considered. The primary control $u_p(m)$ is designed such that the closed-loop response of m manifests the integral time absolute error (i.t.a.e.) response characteristics. The ideal design model is given in equation (14) below.

$$\ddot{y}_m + 1.4\omega_0\dot{y}_m + \omega_0^2y_m = y_{sp} \quad (14)$$

To design $u_p(m)$, the standard set point error dynamics are examined for the ideal model and the RCM. When the error dynamics are equated, the gains for $u_p(m)$ are generated. The resulting controller providing i.t.a.e. response characteristics for the RCM is given below (for the parameter values given in Table 1).

$$u_p(m) = 10(\omega_0^2 - 1.5)(y_{sp} - y_1) - 14\omega_0\dot{y}_1 + 15y_{sp} \quad (15)$$

For $\omega_0=10$,

$$u_p(m) = 1000y_{sp} - 985y_1 - 140\dot{y}_1 \quad (16)$$

The character of the disturbance acting on m_1 is of waveform structure because of the sinusoidal motions exhibited by spring-mass systems. Therefore, the principles discussed in the previous section may be applied to develop the model of the disturbance for incorporation into the observer design. The motion of the system is sinusoidal, and certainly the disturbance may be modeled as a linear combination of sinusoidal functions of proper frequency. However, to be more general and to allow for unforeseen parameter variations, the spline disturbance model given in equation (17) is selected. The variable t is time and the c_i 's are arbitrary constants.

$$w_s = c_1 t^2 + c_2 t + c_3 \quad (17)$$

The resulting "disturbance" state model is given in (18) where D_s and H are defined shortly.

$$\begin{aligned} \dot{z}_s &= D_s z_s + \sigma \\ w_s &= H z_s \end{aligned} \quad (18)$$

For cancellation of the effects of the model complexities, the rank condition given in equation (11) must be satisfied, which is the case for this example. Therefore, the expression for $u_c(m)$ is derived immediately in equation (19).

$$\begin{aligned} u_c(m) &= -B_r^{-1} A_{rs} H z_s \\ u_c(m) &= \begin{pmatrix} 0 & m_1 \end{pmatrix} \begin{bmatrix} 0 & 0 \\ \frac{k_2}{m_1} & 0 \end{bmatrix} z_s = -k_2 z_1 \end{aligned} \quad (19)$$

An observer is designed using the composite dynamics involving m and the disturbance model for controller implementation.

The composite dynamics state model is given in equations (20).

$$\begin{aligned} \begin{pmatrix} \dot{x}_r \\ \dot{z}_s \end{pmatrix} &= \begin{bmatrix} A_r & A_{rs}H \\ 0 & D_s \end{bmatrix} \begin{pmatrix} x_r \\ z_s \end{pmatrix} + \begin{bmatrix} B_r \\ 0 \end{bmatrix} u + \begin{pmatrix} 0 \\ \sigma \end{pmatrix} \\ &= \tilde{A}\tilde{x} + \tilde{B}u + \begin{pmatrix} 0 \\ \sigma \end{pmatrix} \\ \tilde{y} &= \tilde{C}\tilde{x} \end{aligned} \quad (20)$$

where

$$A_r = \begin{bmatrix} 0 & 1 \\ \frac{k_1 + k_2}{m_1} & 0 \end{bmatrix}, \quad B_r = \begin{bmatrix} 0 \\ \frac{1}{m_1} \end{bmatrix}$$

$$\begin{aligned} A_{rs}H &= \begin{bmatrix} 0 & 0 & 0 \\ \frac{k_2}{m_1} & 0 & 0 \end{bmatrix}, \quad D_s = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ \tilde{C} &= [1 \ 0 \ 0 \ 0 \ 0] \end{aligned}$$

The observer design equation is given in (21), where λ_{oi} are the observer poles, placed for this design at $\lambda_{oi} = -20$, and K_o are the observer gains to be calculated.

$$\det[(\tilde{A} + K_o \tilde{C}) - \lambda_{oi} I] = 0 \quad (21)$$

The design procedure for such observers is detailed in the DAC reference cited above. The resulting estimator, and thus controller, is 5th order. The numerical values of the gains for the state and disturbance estimates, indicated by the carets, for the closed-loop system are given in equation (22). Note that the disturbance derivatives are not required in the controller.

$$u(m) = 1000y_{sp} - 985\hat{x}_1 - 140\hat{x}_2 - 10\hat{z}_1 \quad (22)$$

The plot in Figure 2 is a plot of the response of the full-order system model when controlled by $u(m)$ as designed above. In the figure, the system output is the solid line, the set point is the dashed line, and the estimate of the output is another dashed line which tracks the true output exactly. Figure 3 is a plot of the disturbance (solid line), its estimate (tracks exactly the disturbance signal), and the control signal (dashed line). The control signal has been scaled in the plot to that it is evident that the controller is tracking the disturbance dynamics,

caused by the oscillations of the masses m_2 , m_3 , and m_4 and providing a signal which eliminates the effects of these oscillations on the position of m_1 . To test the robustness of the design to full-order model parameter variations, a series of simulations was executed while varying the m_i 's and k_i 's for the system. The plot in Figure 4 shows that the system continues to track the set point input even with the system matrix parameters varied by 100%. No change to the control gains or the observer were made to achieve this result.

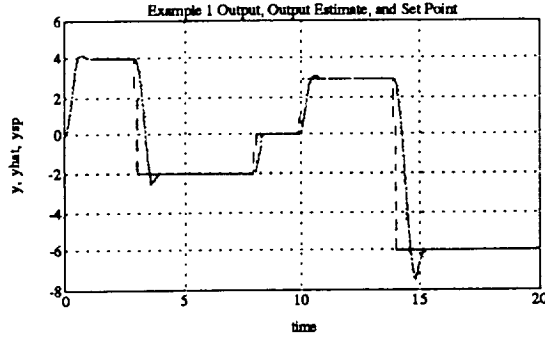


Figure 2. Set Point response with MCRAC controller.

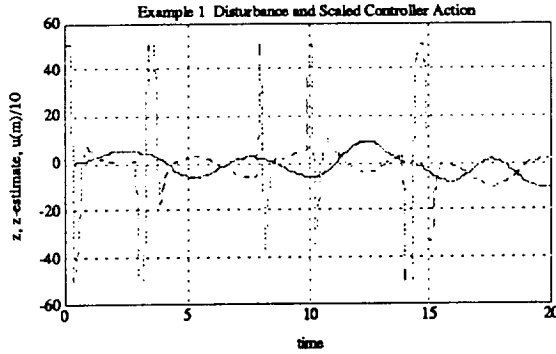


Figure 3. Control Signal and Disturbance.

The second example, diagramed in Figure 5, focused on a simple model of a vertically suspended flexible beam. For simplicity, the beam is modeled as three rigid beams with torsional springs at the joints. Small angles are assumed in deriving the equations of motion for the sixth order system. The controller goal is to maintain θ_3 at zero degrees regardless of the motion of the other two rigid members. The system is highly coupled. The equations of motion for the system were derived using the Lagrange method and are presented in equations (23), with the definition of

terms as indicated in figure 5 and the r_i equal to the midpoint of each beam as measured from each joint.

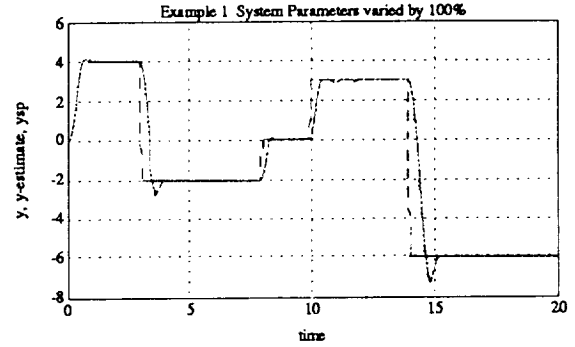


Figure 4. Response with full-order model parameters varied by 100%.

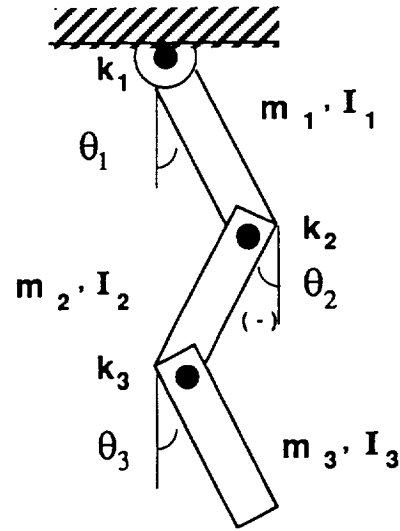


Figure 5. Rigid link "beam" model.

$$\begin{aligned}
 & \ddot{\theta}_1(I_1 + m_1 r_1^2 + (m_2 + m_3)l_1^2) + \ddot{\theta}_2(m_2 l_1 r_2 + m_3 l_1 l_2) \\
 & + \ddot{\theta}_3 m_3 l_2 r_3 + (m_1 g r_1 + m_2 g l_1 + m_3 g l_1)\theta_1 \\
 & + K_1 \theta_1 + K_2 \theta_1 - K_2 \theta_2 = 0 \\
 & \ddot{\theta}_2(I_2 + m_2 r_2^2 + m_3 l_2^2) + \ddot{\theta}_1(m_2 l_1 r_2 + m_3 l_1 l_2) \\
 & + \ddot{\theta}_3 m_3 l_2 r_3 + (m_2 g r_2 + m_2 g l_1 + m_3 g l_2)\theta_2 \\
 & + K_2 \theta_2 - K_2 \theta_1 + K_3 \theta_2 - K_3 \theta_3 = 0 \\
 & \ddot{\theta}_3(I_3 + m_3 r_3^2) + \ddot{\theta}_1 m_3 l_1 r_3 \\
 & + \ddot{\theta}_2 m_3 l_2 r_3 + m_2 g r_3 \theta_3 \\
 & - K_3 \theta_2 + K_3 \theta_3 = 0
 \end{aligned} \quad (23)$$

After obtaining a general expression for the mass and stiffness matrix, numerical values were

substituted and the full-order state space model M derived. For the results shown below, $m_i=12$, $l_i=1$, $k_i=10$, and the length of each beam was set to 1. The RCM selected for this example was a simple second-order system involving the states for θ_3 . The model took the same form as that for Example 1. The same disturbance model was selected and the observer designed in a similar fashion to that in the previous example. The controller $u(m)$, designed using m , was then implemented with the full-order model. In the simulation, the controller is activated at $t=2$ seconds. The plot shown in Figure 6 is the θ_3 closed-loop response and the estimate, which tracks exactly. Figure 7 is a plot of the motion of θ_1 and θ_2 .

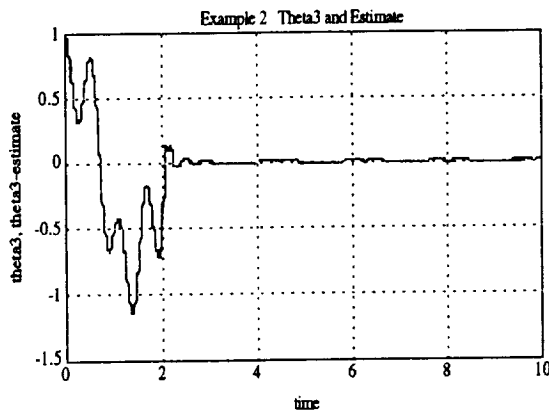


Figure 6. Closed-loop response for Example 2.

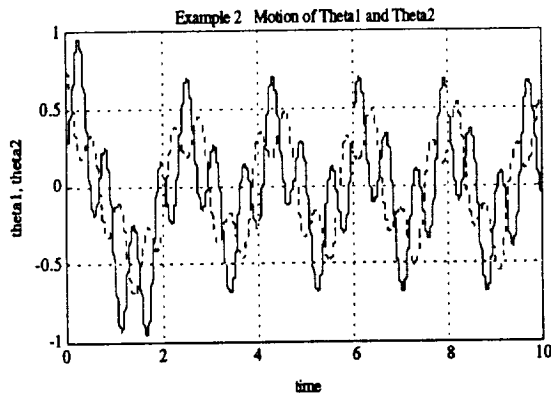


Figure 7. Motion of θ_1 (solid line) and θ_2 (dashed line).

Shown in Figure 8 is the control signal. Note that the controller produces a signal which cancels the effect of the motion of the upper two beam elements on the third. The controller is designed to accommodate all coupling effects and higher order terms as described in the previous

section. These are estimated and eliminated through the MCRAC action.

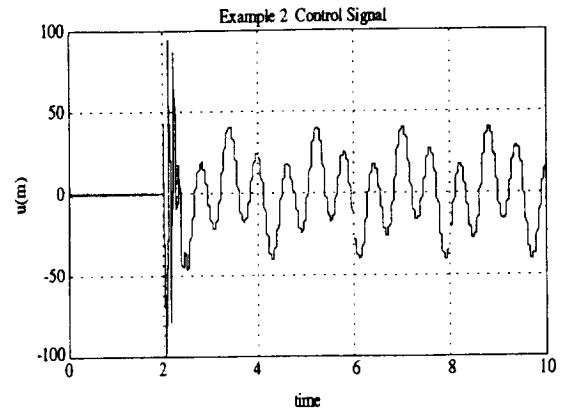


Figure 8. Control signal for Example 2.

The third and final example involves the nonlinear second-order system given in equation (12).

$$\ddot{y} + (1 + \dot{y}^2) \sin \omega y = 0 \quad (12)$$

The nonlinearity is the product of a sine function and the square of the derivative of the output y . The RCM chosen for this example was a simple double-integrator. An i.t.a.e. controller was designed for the RCM. The disturbance model selected to eliminate the effects of the nonlinear term is again a second-order spline implying a third-order disturbance model. The resulting controller is fifth order. The curves plotted in Figure 9 below indicate the system closed-loop response with $u(m)$ in the loop with the full-complexity model. The initial conditions on the system are $(y, \dot{y}) = (1, 0)$, the extremes on ω are $-100 \leq \omega \leq 100$, and selected curves are plotted. The controller action is initiated at $t=2$ seconds. Similar results are obtained for varying the initial conditions on y and its derivative.

A controller was designed using a linearized plant model for equation (12) and a similar set of simulation runs executed. The domain in which the controller, designed using the linearized model, produced acceptable results was $-5 \leq \omega \leq 5.484$. The controller designed using MCRAC ideas provides a substantially improved operating range over parameter variations.

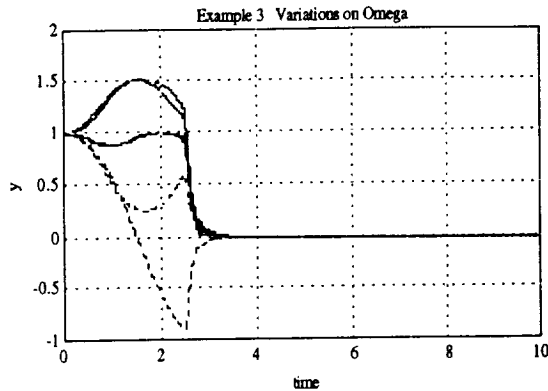


Figure 9. Closed-loop response of nonlinear plant as ω is varied from -100 to 100.

Summary and Conclusions

The Model Complexity-Reduction by Active Control methodology has been introduced in this paper. The method shows potential for application to systems in which the assumptions described in the previous examples hold. That is, single-input, single-output systems with a co-located sensor and actuator pair. It is also assumed that the complexities removed from the full-order model may be modeled as an input disturbance. The controllers derived are of lower order than those derived using the full-complexity models for the two dynamic system examples and much higher order for the nonlinear system. However, the improvement in performance of the higher order controller over the second order controller designed using the linearized model for example 3 is significant. In all example cases, parameter variations of significant magnitude may be imparted on the full-order system matrix with virtually no effect on system performance.

Future research will be directed toward further development of the theory and to the application of MCRAC principles to more complex systems for which the assumptions listed above do not necessarily hold. Based on the results presented in this paper, the method holds promise for application to the class of problems discussed.

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