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Approximations for Quantitative Feedback Theory Designs

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Introduction

QUANTITATIVE feedback theory (QFT) is a powerful technique for the design of multi-input/multi-output (MIMO) flight control systems, which guarantees performance and stability robustness in the presence of significant parametric uncertainty in the vehicle model.[1] The performance specifications can be stated in either the time or the frequency domain, with the latter being more common. In employing QFT in the frequency domain, the designer must specify bounds on the amplitude ratios of on-axis and off-axis response-to-command transfer functions (desired tracking performance and desired cross-coupling minimization). Whereas specifying tracking bounds is fairly straightforward, especially in flight control problems where handling qualities specifications can provide some guidance, the specification of cross-coupling bounds

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can be problematic. This is not a minor concern as these cross-coupling bounds can drive the entire QFT design. Finally, MIMO QFT designs are usually approached using a sequential loop closure technique to minimize conservatism.[2] Until now, no method other than trial and error could be employed to determine the loop closure sequence. As will be seen, both the problem of determining cross-coupling bounds and loop closure sequence can be solved using an approximate predesign technique (PDT).

Flight Control Example

Consider Fig. 1, which shows a feedback topology for the design of a lateral-directional flight control system for a supermaneuverable fighter. A single diagonal compensation matrix $G_{\mathrm{QFT}}(s)$ is to provide performance and stability robustness across a variety of flight conditions. Here, $P(s)$ represents a 2×5 plant matrix with outputs of sideslip β and roll rate p. There are five control effectors: differential horizontal stabilator, yaw thrust vectoring, differential pitch thrust vectoring, aileron, and rudder. The matrix S is a 5×2 control distribution matrix. Also shown is an inner, yaw-rate r feedback loop, which has been closed with a fixed compensator $G_r(s)$. The matrix $G_c(s)$ is a constant 2×2 precompensation matrix, which approximately decouples the plant across the flight conditions being studied.[3] Finally, the matrix $F(s)$ is a diagonal 2×2 prefilter matrix. The goal of the QFT design is to specify the diagonal elements of $G_{\mathrm{QFT}}(s)$ and $F(s)$ to meet performance and stability requirements in the presence of the uncertainty in $P(s)$ introduced by considering 15 different flight conditions.

PDT

Approximately Decoupled Controls

The PDT has its basis in an assumption regarding the diagonal compensation elements of $G_{\mathrm{QFT}}(s)$. Referring to the example, if the pseudocontrols u_β and u_p are approximately decoupled, then the following relationship can be employed:

$$\begin{aligned} G_{\mathrm{QFT}_{\beta\beta}} &\approx \left(\omega_{c_\beta}/s\right) \cdot [1/(\beta/u_\beta)] \\ G_{\mathrm{QFT}_{pp}} &\approx \left(\omega_{c_p}/s\right) \cdot [1/(p/u_p)] \end{aligned} \tag{1}$$

where the double subscripts on the left-hand sides of the equations represent diagonal elements and the $\omega_{c_{(-)}}$ represent crossover frequencies. Equation (1) exploits the well-known fact that the loop transmission $L(s)$ of a well-designed single-input/single-output system, or the loop transmissions of a decoupled MIMO system, each resemble $\omega_{c_{(-)}}/s$ near the region of crossover. Equation (1) extends this approximation to all frequencies. In terms of approximating the elements of $G_{\mathrm{QFT}}(s)$, low-frequency characteristics ($\omega \ll \omega_c$) are relatively unimportant provided $|L(j\omega)| \gg 1.0$, and the high-frequency characteristics ($\omega \gg \omega_c$) are relatively unimportant provided $|L(j\omega)| \ll 1.0$. These conditions are guaranteed by Eq. (1). For QFT designs, a nominal plant is selected to define a nominal loop transmission on the Nichols chart. For the PDT, this simply means choosing one of the possible plants out of the uncertain set to define the denominator of the right-hand sides of Eq. (1).

The PDT is limited to minimum phase systems, or at least those possessing no low-frequency zeros in the right-half plane. Thus, the diagonal elements of $P \cdot S \cdot G_c(s)$ should have no right-half plane zeros at or below the loop crossover frequency.

Coupled Controls

If a precompensator $G_c(s)$ is not being employed, and if the plant itself is not adequately decoupled, the relation of Eq. (1) may have to be modified to account for the control cross coupling that may exist. In such a case, Eq. (1) becomes

$$G_{\mathrm{QFT}_{\beta\beta}} \approx \frac{\omega_{c_\beta}}{s} \cdot \frac{1}{(\beta/u_\beta)_{p \to u_p}}, \qquad G_{\mathrm{QFT}_{pp}} \approx \frac{\omega_{c_p}}{s} \cdot \frac{1}{(p/u_p)_{\beta \to u_\beta}} \tag{2}$$

The subscripts on the parenthetic terms in the denominators on the right-hand sides of Eqs. (2), e.g., $p \to u_p$, mean that, in calculating the transfer function in parentheses, the remaining control loop is closed. In the analysis, one of the compensators will have to be determined first, and the compensation in the remaining loop will be unknown. The most expedient course is simply to approximate the unknown compensator by the appropriate relation in Eq. (1).

Implementation

In employing the PDT as part of a QFT design, a relatively simple implementation procedure can be followed, here couched in terms of the flight control example of Fig. 1.

1) Tracking performance bounds are selected. Initial estimates for the crossover frequencies ω_{c_β} and ω_{c_p} are chosen. Approximations for the diagonal compensation elements in $G_{\mathrm{QFT}}(s)$ are created from either Eqs. (1) or (2). With approximations for the compensators thus obtained, closed-loop relations for the tracking and cross-coupling transfer functions can be obtained for each flight condition. If the variations in the amplitude of the tracking transfer functions across all of the flight conditions exceed the bounds at some frequency or frequencies within the range of interest, then the crossover frequencies are increased and the procedure repeated. When the variations are acceptable, elements in the prefilter matrix $F(s)$ can be determined so that the actual tracking transfer function amplitudes lie within the prescribed bounds.

2) Next is cross-coupling minimization. After completing step 1, the amplitudes of the closed-loop cross-coupling transfer functions can be obtained for each flight condition. Least-upper bounds on these amplitudes will provide realistic bounds for cross-coupling minimization for the formal QFT design. Of course, if cross-coupling bounds are available ab initio (typically an unlikely event), then crossover frequency selection in step 1 is predicated on meeting these as well as the tracking bounds.

3) Last is cost of feedback. An examination of the predicted compensation from Eqs. (1) or (2) in the region beyond crossover provides an estimate of the sensor noise propagation to the plant inputs. This noise propagation has been called cost of feedback by Horowitz.[1]

Results

The results of the implementation of the PDT just described will be 1) estimates of the loop crossover frequencies, 2) estimates of

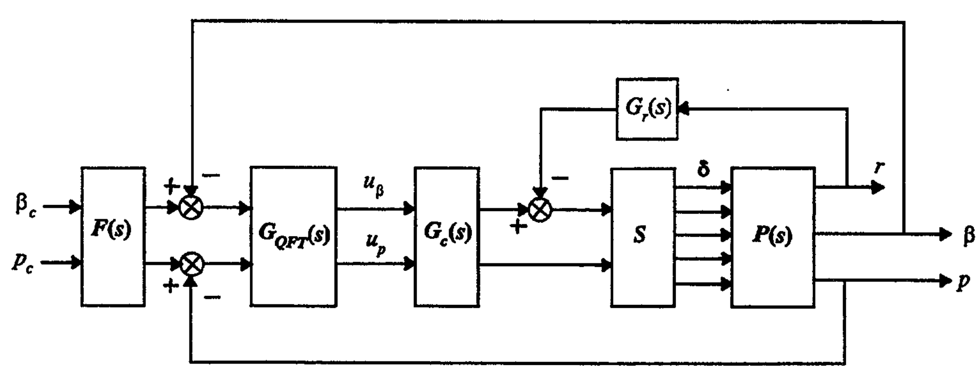

Fig. 1 MIMO lateral-directional flight control system.

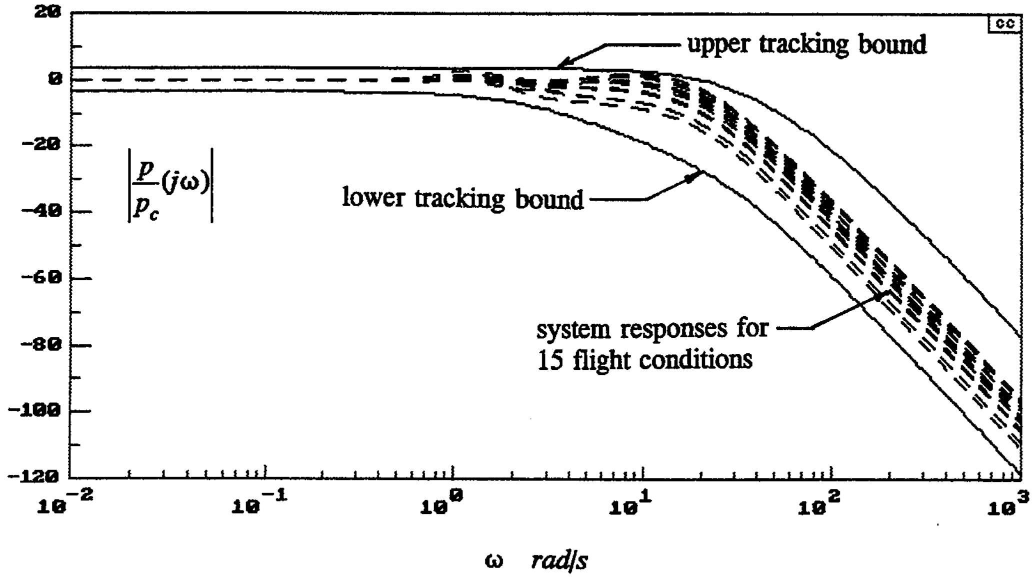

Fig. 2 Tracking performance results for roll-rate to roll-rate command with PDT for 15 flight conditions.

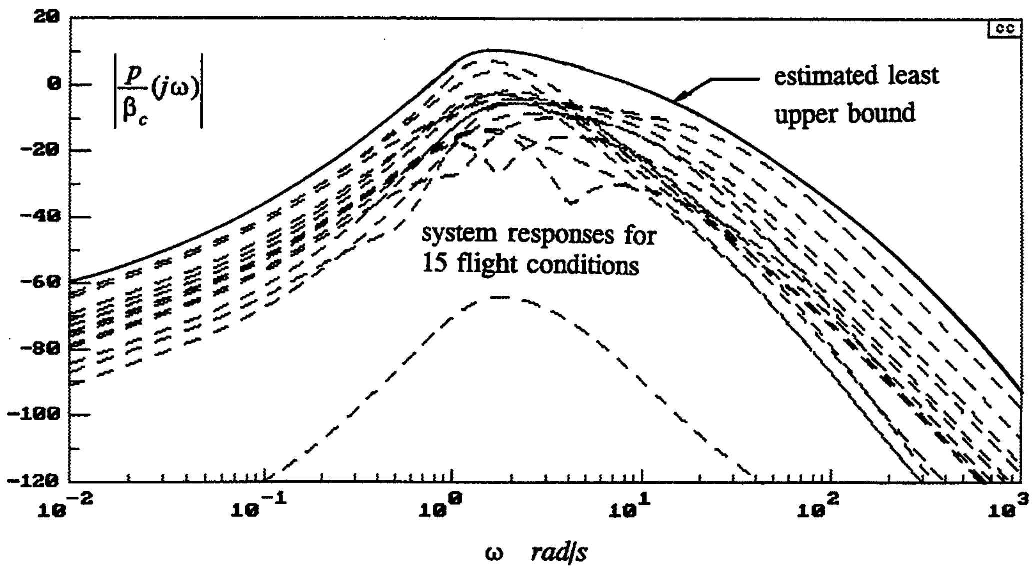

Fig. 3 Cross-coupling minimization for roll-rate to sideslip command with PDT for 15 flight conditions.

the required compensation and prefilters (with cost of feedback), 3) estimates of realistic cross-coupling bounds to be used in the formal QFT procedure, and 4) a suggested loop closure sequence for the formal QFT procedure. Because the formal QFT procedure will likely involve sequential loop closure, and because this first loop closure involves the most conservatism, the loop with the lower estimated crossover frequency should be the first to be closed in the formal QFT design.

Application to the Flight Control Example

Tracking performance bounds were selected for the flight control example outlined earlier, and the steps just outlined were performed. Figure 2 shows the estimated roll-rate tracking performance that resulted, i.e., $|(p/p_c)(j\omega)|$ for the 15 flight conditions considered. Figure 3 shows the estimated cross coupling between roll-rate and sideslip command, i.e., $|(p/\beta_c)(j\omega)|$. A suggested bound to be used in the formal QFT design is also shown. The estimated crossover frequencies that yielded the performance shown were $\omega_{c_\beta} = 3.0$ rad/s and $\omega_{c_p} = 6.0$ rad/s. These values suggest that the β loop should be closed first in the formal QFT design. An examination of the elements of the total compensation matrix $G_c \cdot G_{\mathrm{QFT}}(s)$ indicates that the β loop will involve a considerably higher cost of feedback than the p loop.

The results of predesign exercises as described have compared quite favorably with those obtained in formal QFT designs that entailed none of the approximations used in the PDT.[3]

Conclusions

The computational requirements for obtaining the results summarized in the preceding section were very modest and were easily accomplished using computer-aided control system design software. Of special significance is the ability of the PDT to indicate a loop closure sequence for MIMO QFT designs that employ sequential loop closure. Although discussed as part of a 2 × 2 design, the PDT is obviously applicable to designs with a greater number of inputs and system responses.

Acknowledgments

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