## NASA/ASEE SUMMER FACULTY FELLOWSHIP PROGRAM

# MARSHALL SPACE FLIGHT CENTER THE UNIVERSITY OF ALABAMA 

## SCATTERING AND THE POINT SPREAD FUNCTION OF THE NEW GENERATION SPACE TELESCOPE

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## INTRODUCTION

Preliminary design work on the New Generation Space Telescope (NGST) is currently under way. This telescope is envisioned as a lightweight, deployable Cassegrain reflector with an aperture of 8 meters, and an effective focal length of 80 meters. It is to be folded into a small-diameter package for launch by an Atlas booster, and unfolded in orbit, as shown in Figure 1. The primary is to consist of an octagon with a hole at the center, and with eight segments arranged in a flower petal configuration about the octagon. The corners of the petal-shaped segments are to be trimmed so that the package will fit atop the Atlas booster. This mirror, along with its secondary will focus the light from a point source into an image which is spread from a point by diffraction effects, figure errors, and scattering of light from the surface. The distribution of light in the image of a point source is called a point spread function (PSF).


Figure 1. The folded and deployed configuration of the NGST.

The obstruction of the incident light by the secondary mirror and its support structure, the trimmed corners of the petals, and the grooves between the segments all cause the diffraction pattern characterizing an ideal point spread function to be changed, with the trimmed corners causing the rings of the Airy pattern to become broken up, and the linear grooves causing diffraction spikes running radially away from the central spot, or Airy disk. Any figure errors the mirror segments may have, or any errors in aligning the petals with the central octagon will also spread the light out from the ideal point spread function.

A point spread function for a mirror the size of the NGST and having an incident wavelength of 900 nm in Figure 2. Most of the light is confined in a circle with a diameter of 0.05 arc seconds. The ring pattern ranges in intensity from $10^{-2}$ near the center to $10^{-6}$ near the edge of the plotted field, and can be clearly discerned in a log plot of the intensity.

The total fraction of the light scattered from this point spread function is called the total integrated scattering (TIS), and the fraction remaining is called the Strehl ratio. The angular distribution of the scattered light is called the angle resolved scattering (ARS), and it shows a strong spike centered on a scattering angle of zero, and a broad, less intense distribution at larger angles. ${ }^{1}$ It is this scattered light, and its


Figure 2. The point spread function for the NGST at 900 nm wavelength. The area covered is approximately 0.9 arc seconds on a side.
effect on the point spread function which is the focus of this study. The effects of the peculiar mirror shape, and of figure and alignment errors are left for a later date.

## CALCULATING THE PSF

A Microsof ${ }^{\star}$ FORTRAN program, PFSARRAY, was written to compute the angle resolved scattering and point spread function of the NGST for a range of rms roughness values. This program was based on the program, ZERTAPER written by Mike Jones of Lockheed Martin Co. In Fort Worth, Texas. ZERTAPER computes the point spread function of a circular mirror with a central obstruction, and having known coefficients for the Zernike polynomials describing the figure errors. This program was converted from code for a Sun workstation to code for a PC. It was also modified to compute the angle resolved scattering, and total integrated scattering, and to use them to compute a final point spread function. PFSARRAY also stores the arrays representing the ARS and PSF on a floppy disk in a format which can be read by the software package MATLAB. MATLAB was used to plot this data, and to store the images in PCX files which can be imported into documents on the PC.

PSFARRAY begins by building a complex array representing a pupil function for the telescope. The primary mirror is covered by a square array 80 elements on a side. Each element falling on the mirror is given a magnitude of 1.0, and each element outside the mirror edge, or inside the central obstruction, is given a magnitude of zero. The coefficients of the Zernike polynomials are then used to find the wave front errors, and then the phase angles of the array elements, and from them the real and imaginary parts of the complex array elements.

The complex pupil function is then transformed by a fast Fourier transform subroutine into an original PSF array having a size of 256 by 256 . The center 128 by 128 elements of this array are then taken as a complex image array. The square of the magnitudes of the image array produce the elements of an intensity array which is the actual original PSF for the telescope.

A profile of the surface roughness of a mirror can be used to produce an autocovariance function for the surface. A fast Fourier transform of the autocovariance yields the power-spectraldensity (PSD) function of the of the surface microstructure. A default, normalized PSD function is stored in PSFARRAY, with an option to change it. This default function was taken from Bennett and Mattsson ${ }^{1}$, and came from a profile of a nickel mirror with an rms surface roughness of 5A. The general form of this normalized function represents surfaces with roughly Gaussian distributions of surface height, regardless of what the rms roughness is measured to be. If the Fourier transform of the PSF is multiplied point for point by the PSD, the inverse Fourier transform of the product is the ARS for the telescope. PSF array uses its complex FFT subroutine to perform these operations.

The PSD function must be converted into an array for use by the FFT routine. The first task is to determine the size of an array element in this (spatial) frequency space. This array element size corresponds to the lowest frequency, or longest wavelength in the spatial plot of the PSF. The longest wavelength would be the array size, N , times the width of one array element, $\Delta \mathrm{x}$. The inverse of this longest length is the element size in the frequency domain of the PSD function, $\triangle$ PSD. The size of $\Delta x$ is determined from the fact that the first dark ring in the Airy pattern occurs 4 array
elements from the center, and that distance is given by

$$
r_{o}=\frac{\lambda D}{d}
$$

where $D$ is the diameter of the telescope objective, and $d$ is the effective focal length. The result is that

$$
\triangle P S D=\frac{1}{N}\left(\frac{8 \pi d}{\lambda D}\right)
$$

The PSD array is filled in using the calculated element size by interpolation from the linear PSD function stored in the program.

A second fitting problem arises out of the fact that the PSD function is normalized, and produces an ARS distribution with an undetermined intensity level. The general form of the ARS is shown in Figure 3, and was taken from Maradudin and Mendez ${ }^{2}$. It looks like an inverted parabola, with a spike at the center. The light is scattered into a full hemisphere, but with decreasing intensity at increasing scattering angles, $\theta_{s}$. The same amount of scattered light, if uniformly intense would fill a solid angle of 3.7 steradians. The area under the ARS curve is simply the total integrated scattering (TIS), which is calculate from the rms surface roughness, $\delta$, and the incident wavelength, $\lambda$, by the relation given by Bennett and Mattsson ${ }^{1}$,


Figure 3. ARS for a silver surface with a TIS of $1.9 \%$.

$$
T I S=\left(\frac{4 \pi \delta}{\lambda}\right)^{2}
$$

From the TIS, the intensity at the top of the inverted parabola which would be scattered into a solid angle the size of one array element in the PSF array may be calculated. Since the intensity of the ARS was still decreasing at the edge of the ARS array, it was assumed it would continue to decrease by a factor of ten before it reached the nearly horizontal portion of the general ARS function. This method of fitting the calculated ARS to the TIS is somewhat inexact, but should not cause substantial errors in the intensities of the ARS array.

$$
\text { Strehl }=e^{-\left(\frac{4 \pi \delta}{\lambda}\right)^{2}}
$$

The Strehl ratio is given by the expression ${ }^{3}$, It is the fraction of the incident light that is not scattered. If the original PSF of the telescope is diminished by the Strehl ratio, and the ARS, calculated as described above, are added together, the result is the final PSF of the telescope ${ }^{4}$.

## RESULTS

The ARS and PSF arrays for a telescope the size of the NGST were calculated for a number of rms roughness values, ranging from 5 A to 500 A . Figure 4 shows the intensity of the scattered light for an rms roughness of 500 A . As can be seen, the intensity is $10^{-9}$ of the intensity in the PSF. This has a negligible effect when added to the PSF. On the other hand, this large a surface roughness produces a Strehl ratio of only 0.51 , meaning that the intensity of the light in the PSF has been cut nearly in half by the light which was scattered out of it. One would not like to lose this much light from the image.

Figure 5 shows a cross section through the center of the PSF array for a roughness of 100A. Here the Strehl ratio is 0.98 , and at an ms roughness of 50 A it ${ }^{\frac{1}{10}}$ would be 0.995 . In Figure 5 the central disk of the Airy pattern, and the surrounding bright rings can easily be seen, and there is essentially no contribution by scattered light. At longer wavelengths the scattered light would have even less effect.


Figure 4. ARS for a surface roughness of 500A.


Figure 5. The PSF with 100 A rms surface roughness

## CONCLUSION

Scattered light will not degrade the image quality of the NGST. A Strehl ratio, or TIS value, which will not appreciably diminish the intensity of the original PSF is sufficient to specify an acceptable finish of the mirror. A value of 50 A for the rms roughness would have a negligible effect on the image.

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