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**ELECTRON FLOW TO A SATELLITE AT HIGH POSITIVE POTENTIAL**

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## INTRODUCTION

The Tethered Satellite System (TSS) is designed to deploy a 1.6 m diameter spherical satellite a distance of 20 km above the space shuttle orbiter on an insulated conducting tether. Because of the passage of the conducting tether through the earth's magnetic field, an emf is generated producing a positive satellite potential of about 5000 V. Electron flow under the influence of this high positive potential is the focus of the present analysis.

The ionospheric parameters at TSS orbit altitude are; thermal velocity of electrons,  $1.9 \times 10^5$  m/s, thermal velocity of the ions,  $1.1 \times 10^3$  m/s, velocity of the satellite  $8 \times 10^3$  m/s. The electrons, with a Debye length,  $\lambda_D = 0.49$  cm, spiral about the earth's magnetic field lines (0.4 Gauss) with a radius of about 3 cm and the ions spiral with a radius of 5 m. Under these conditions, the electron thermal energy,  $kT$  is 0.17 eV. The TSS satellite radius,  $r_p$  is 163 Debye lengths.

There is an extensive literature on the interaction of satellites with the near-earth ionospheric plasma. The space charge limitation to the electron current collected by a sphere at positive electrical potential was calculated by Langmuir and Blodgett (1924). Parker and Murphy (1967) recognized the importance of the influence of the earth's magnetic field and used the guiding center approximation to calculate the electron current collected by a positive charged satellite. More recently Ma and Schunk (1989) have calculated the time dependent flow of electrons to a spherical satellite at positive potential utilizing numerical methods and Sheldon (1994) used similar methods to solve this problem for the steady state.

In order to analyze some of the phenomena that occurred in the ionosphere during the TSS flights, it would be useful to have analytic expressions for these electron flows. The governing equations are very complex and an exact analytical solution is not likely. An approximate analytical solution is feasible however, and the results of one attempt are presented herein.

## MATHEMATICAL MODEL

Electron flow to the spherical satellite is modelled here by the use of the cold plasma one-fluid momentum and continuity equations with the Poisson equation used previously [Sheldon (1994)]. In the present calculation, it is assumed that there will be a sheath region around the satellite devoid of ions due to positive satellite potential being much higher than the ram ion energy. Electrons drift toward this sheath along the earth's magnetic field lines with an average velocity  $(kT/m)^{1/2}$ , neglecting the average initial velocity due to their ambient spirals. At the outer boundary of this sheath there will be a negative space charge potential barrier approximately equal in magnitude to the electron ambient thermal energy. The governing

equations are written in non-dimensional spherical coordinates  $(r, \theta, \phi)$  with the magnetic field aligned with the polar axis. The steady-state momentum equations for  $u_r$ ,  $u_\theta$  and  $u_\phi$ , the  $r$ ,  $\theta$ , and  $\phi$  components of the average velocity of the electron flow, are then:

$$u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} - \frac{u_\phi^2}{r} - \frac{\partial V}{\partial r} + Bu_\phi \sin \theta = 0 \quad (1)$$

$$u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_\theta u_r}{r} - \frac{u_\phi^2}{r} \cot \theta - \frac{1}{r} \frac{\partial V}{\partial \theta} + u_\phi B \cos \theta = 0 \quad (2)$$

and

$$u_r \frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{u_r u_\phi}{r} + \frac{u_\phi u_\theta}{r} \cot \theta - Bu_r \sin \theta - Bu_\theta \cos \theta = 0 \quad (3)$$

where  $r$  is in units of  $\lambda_D$  and the velocities are in units of  $u_A = (kT/m)^{1/2}$ , where  $m$  is the electron mass. The electric potential energy,  $V(r, \theta)$  is in units of  $kT$  and  $B = \omega_c/\omega_{ep} = 0.2$ , where  $\omega_c$  is the electron cyclotron frequency in the earth's magnetic field and  $\omega_{ep}$  is the electron plasma frequency. The continuity equation is

$$\frac{1}{r} \frac{\partial}{\partial r} (r^2 n u_r) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta n u_\theta) = 0 \quad (4)$$

where  $n(r, \theta)$  is the electron density in units of the ambient electron density,  $n_A$  and Poisson's equation is

$$\frac{\partial}{\partial r} (r^2 \frac{\partial V}{\partial r}) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial V}{\partial \theta}) = n r^2 \quad (5)$$

## SOLUTION

In order to get an approximate solution to Eqs(1)-(5) the magnetic field parameter,  $B$  is considered a perturbation parameter. Using the perturbation expansion in (1)-(5), the zero order equations (no magnetic field) are,

$$u_r^0(r) \frac{\partial u_r^0(r)}{\partial r} - \frac{\partial V^0(r)}{\partial r} = 0 \quad (6)$$

$$\frac{1}{r} \frac{\partial}{\partial r} [r^2 n^0(r) u_r^0(r)] = 0 \quad (7)$$

$$\frac{\partial}{\partial r} (r^2 \frac{\partial V^0(r)}{\partial r}) = n^0(r) r^2 \quad (8)$$

The equations to first order in B are

$$\frac{\partial}{\partial r} [u_r^1(r, \theta) u_r^0(r) - V^1(r, \theta)] = 0 \quad (9)$$

$$u_r^0(r) \frac{\partial}{\partial r} [r u_\theta^1(r, \theta)] - \frac{\partial V^1(r, \theta)}{\partial \theta} = 0 \quad (10)$$

$$u_r^0(r) \left[ \frac{\partial u_\phi^1(r, \theta)}{\partial r} + \frac{u_\phi^1(r, \theta)}{r} - B \sin \theta \right] = 0 \quad (11)$$

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} [r^2 (n^1(r, \theta) u_r^0(r) + n^0(r) u_r^1(r, \theta))] \\ + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} [n^0(r) u_\theta^1(r, \theta) \sin \theta] = 0 \end{aligned} \quad (12)$$

$$\frac{\partial}{\partial r} [r^2 \frac{\partial V^1(r, \theta)}{\partial r}] + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} [\frac{\partial V^1(r, \theta)}{\partial \theta} \sin \theta] = n^1(r, \theta) r^2 \quad (13)$$

The solution to the zero order Eqs. (11)-(13) can be obtained from Langmuir and Blodgett (1924). They considered a spherical electron emitter of radius  $r_m$  outside of a collector with negligible initial velocity of the electrons leaving the emitter. The boundary conditions were  $V^0(r_m) = 0$  and  $(dV^0/dr)|_{r_m} = 0$ . In the present case the outer boundary condition is taken at a virtual cathode of radius  $r_0$ , where  $u_r^0(r_0) = 1$  and  $n^0(r_0) = 1$ .

With the boundary condition,  $u_\phi^1(r_0, \theta) = 0$ , Eq. (11) gives,

$$u_\phi^1(r, \theta) = B \frac{\sin \theta}{2r} (r_0^2 - r^2) \quad (14)$$

The remaining first order perturbation equations are linear and allow the separation of variables. Defining  $u_r^1 = g_r(r) g_\theta(\theta)$ ,  $u_\theta^1 = h_r(r) h_\theta(\theta)$ ,  $n^1 = k_r(r) k_\theta(\theta)$ ,  $V^1 = f_r(r) f_\theta(\theta)$ , the separated  $\theta$ -dependent Eqs. yield the following results

$$h_\theta(\theta) = - \sum_l A_l P_l'(\cos \theta) \sin \theta \quad (15)$$

$$f_\theta(\theta) = g_\theta(\theta) = \sum_l A_l P_l(\cos \theta) \quad (16)$$

$$A_\ell = \frac{2\ell+1}{2} \int_{-1}^{+1} (1-x) P_\ell(x) dx \quad (17)$$

$P_\ell(\cos\theta)$  is the Legendre polynomial of the first kind.

The  $r$ -dependent equations can be combined into a coupled set, of linear Eqs. which do not reduce to a standard form. An approximate solution is obtained by expanding the dependent variables in a Taylor's series about  $r_0$ . Only the leading terms are used here. Combining with the  $\theta$ -dependent functions, the final results largest order are

$$u_r(r, \theta) \approx u_r^0(r) = [(9/2) r_m^{-2} \alpha^2]^{1/3} \quad (18)$$

$$u_\theta(r, \theta) \approx \left(\frac{r_0}{r}\right) \sin\theta \quad (19)$$

$$u_\phi(r, \theta) \approx \frac{\sin\theta}{2r} (r_0^{-2} - r^{-2}) \quad (20)$$

$$n(r, \theta) \approx n^0(r) = i \left(\frac{r_m}{r}\right)^2 \left[\frac{2\alpha^2}{9r_m^2}\right]^{1/3} \quad (21)$$

$$V(r, \theta) \approx V^0(r) = (2^{-1/3}) [(3/2) r_m \alpha]^{4/3} \quad (22)$$

$$\alpha(\gamma) = \gamma + 0.3\gamma^2 + 0.075\gamma^3 + 0.01432\gamma^4 + 0.00216\gamma^5 \quad (23)$$

where  $\gamma = \ln(r_m/r)$  and  $i = 4\pi r_0^2 n_A u_A$ .

The results for the azimuthal velocity, Eq.(14), are compared with the numerical results of earlier work [Sheldon(1994)] in Fig. 1. While the present method introduces considerable error in the actual values  $u_\phi$ , the radial profiles have the correct shape. Similar results were obtained for

$u_r(r, \theta)$ ,  $v(r, \theta)$  and  $n(r, \theta)$ .

## CONCLUSIONS

The perturbation method has allowed approximate determination of the electron flow in the proposed model. The previous numerical solution was more accurate, however the intention here was not accuracy, but a better understanding of the influence of the controlling parameters. This is available in Eqs(18)-(23).

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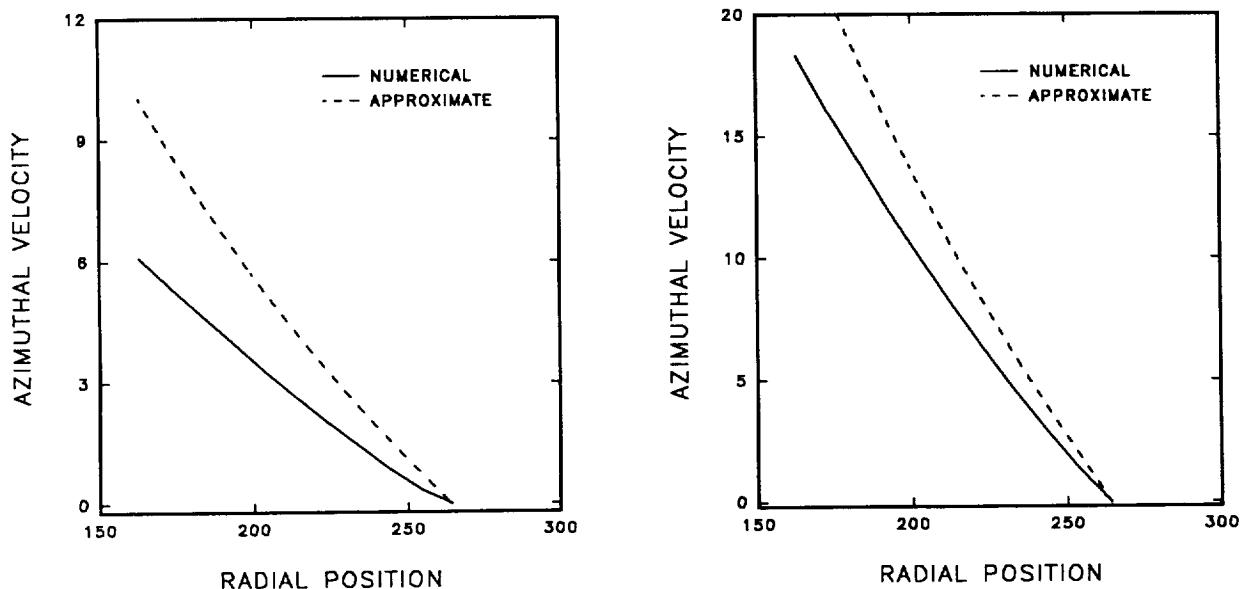


Figure 1. Radial profiles of the azimuthal electron velocity.  
 $V_p = 1000$ . Nondimensional units  $\lambda_b$  and  $u_A$  defined in text.  
(a)  $\theta = 22.5^\circ$ , (b)  $\theta = 67.5^\circ$ .

