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**Modeling of Radiative Heating in Base Region of Jovian
Entry Probe**

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MODELING OF RADIATIVE HEATING IN BASE REGION OF JOVIAN ENTRY PROBE

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Abstract

A theoretical model is derived to determine the average thermodynamic properties in the expanding region, recirculating region, recompression region, and neck region through application of one-dimensional conservation equations. Radiative transfer is calculated using spectrally detailed computer codes accounting for nonequilibrium. The results show that the most severe heating occurs immediately behind the frustum, and that the recompression and neck regions are the major sources of radiation that heats the base stagnation point. The radiation flux to the base point is slightly stronger with ablation than without. Its value is $0.11(43p_b/p_s)^2$ times that to the front stagnation point where the base pressure p_b is defined as the average pressure in the recirculating region and p_s is the front stagnation-point pressure. The time-integrated heat load to the base point is $18(43p_b/p_s)^2$ kJ/cm².

Nomenclature

e = radiative power emission from unit volume
 H = total enthalpy
 h = static enthalpy
 \dot{M} = mass flow rate through neck
 N_e = electron density
 p_b = average pressure in base region
 p_s = front stagnation-point pressure
 q = radiative heat-transfer rate to a base point
 q_s = radiative heat-transfer rate to front stagnation point
 R_f = frustum radius
 R_n = neck radius

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MODELING OF RADI

r = radius
 T_e = electron temperature
 t = time
 u = tangential velocity
 u_e = velocity at outer edge
 u_e' = velocity at outer edge
 x = distance along centerline
 x_n = distance between reattachment points
 Y_n = coordinate normal to surface
 Δ = shock standoff distance
 δ = thickness of layer (boundary layer) by free-shear layer
 λ = wavelength of radiation
 ρ = density
 θ = viewing angle measured from normal
($\bar{}$) = volume average

In a future space mission, the probe will be flown into the atmosphere. The heat transfer to the probe vehicle is dominated by radiation. Up to the present, the heat-transfer problem focuses on the front stagnation point where the heat-transfer rate is most important. Little effort has been made to study the heat-transfer rate to the base region. Existing reports on the heat-transfer rate to the base region are limited. The front stagnation point heat-transfer to the base region is the dominant body region, one should expect the heat-transfer rate to the base region. The present work is a preliminary study of the radiative heat-transfer rate to the base region.

The problem of radiative heat-transfer to the base region was studied by Stephenson.^{2,3} He measured the heat-transfer rate behind a model flying through air consisting of sensors located outside the model. The sensors consisted of air, and the heat-transfer was by molecular dissociation. The heat-transfer rate of a molecular nature was measured at the base region of the sensor location from the heat-transfer rate carrying out the associated heat-transfer. The radiative heat-transfer rate was about 90 W/cm² under his test conditions. The heat-transfer rate is calculated to be about 100 W/cm².

r = radius
 T_e = electron temperature
 t = time
 u = tangential velocity
 u_e = velocity at outer edge of free-shear layer
 $u_{e'}$ = velocity at outer edge of neck
 x = distance along centerline axis
 x_n = distance between reattachment point and frustum edge
 Y = coordinate normal to flow direction
 Δ = shock standoff distance
 δ = thickness of layer (in shock layer) eventually entrained by free-shear layer
 λ = wavelength of radiation
 ρ = density
 θ = viewing angle measured at base stagnation point
 $(\bar{})$ = volume average

Introduction

In a future space mission, a probe vehicle is planned to be flown into the atmosphere of the planet Jupiter. Heat transfer to the probe vehicle is expected to be due mostly to radiation. Up to the present, most studies of the radiative heat-transfer problem focused on the forebody region of the vehicle where the heat-transfer problem is undoubtedly more important. Little effort has been expended to determine the heat-transfer rate to the afterbody, that is, to the base region. Existing reports indicate that the convective heat-transfer rate to the base region is typically 1% of that to the front stagnation point.¹ Unknown, however, is the rate of heat-transfer to the base region by radiation. Since radiation is the dominant mode of energy transport in the forebody region, one should expect the same for the afterbody region. The present work is an attempt to determine theoretically the radiative heat-transfer rate to the base region.

The problem of radiative base heating was studied first by Stephenson.^{2,3} He measured the luminosity of the trail behind a model flying through a free-flight range with radiation sensors located outside the model. The ambient gas consisted of air, and the flow around the model was dominated by molecular dissociation. The radiation observed was therefore of a molecular nature. He deduced the radiative heating rate of the base region of the model by hypothetically moving the sensor location from outside to inside the model and by carrying out the associated geometrical-optics calculations. The radiative heat-transfer rate was determined to be up to about 90 W/cm² under his test conditions.² This heat-transfer rate is calculated to be equivalent to about 1/3 that to the

front stagnation point. The experiment showed that the neck region (i.e., the region around and immediately downstream of the reattachment point) and the expanding region behind the shoulder (hereafter referred to as shoulder-expansion region) are highly luminous. In a later unpublished preliminary calculation communicated internally within NASA, Stephenson recognized the recirculating region adjacent to the base wall to be a major contributor to base heating because, even though it is less luminous than the other two regions, it is closer to the base wall. Thus, three important sources of radiation are identified: the shoulder-expansion region, the recirculating region, and the neck region.

Recently, Shirai and Park⁴ measured the radiative base heating rate with radiation sensors imbedded in the model in a shock-tube flow. The flow was in an ionizing regime, and the radiative heat-transfer rate to the base stagnation point was found to be on the order of 1 kW/cm^2 , or 10% of that to the front stagnation point. The luminosity photographs of the flow field obtained by Shirai and Park showed the same features as seen by Stephenson. A theoretical analysis is required, however, to extrapolate their experimental results to the full-scale case since the tests did not simulate the correct flight Mach numbers or the size.

An attempt was made recently by Nestler and Brant⁵ to determine theoretically the heat-transfer rate to the base region of the Jovian probe. Through a detailed analysis of the flow field in the recirculating region, they calculated the convective heat-transfer rate for the base region to a relatively high precision. Only preliminary results are presented in their work, however, on the radiative contribution, which is not adequate for the purpose of spacecraft design.

The present analysis complements the work of Nestler and Brant.⁵ It makes use of simple one-dimensional conservation equations to determine the average thermodynamic properties in the luminous regions. Radiative-transfer calculations are carried out using spectrally detailed computer codes accounting for the effect of nonequilibrium in the shoulder-expansion region. The theoretical model qualitatively reproduces the features found experimentally by Stephenson^{2,3} and Shirai and Park.⁴ The calculation is carried out for the full-scale Jovian probe configuration, which consists of a 45° sphere cone with a nose radius of 31.12 cm and a frustum radius of 62.24 cm. The base wall of the vehicle also is spherical (radius of 62.24 cm), with the base stagnation point 34.9 cm downstream of the plane of the frustum edge. The flow environments considered are those encountered in a -9° entry

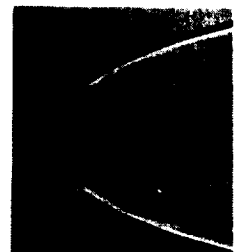
calculated by Moss et al. heating rates of the base immediately behind the fr

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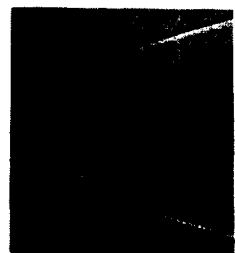
Shoulder-Expansion Region

Analysis of base flow region. Provided the initial conditions (over the frustum) and the boundary conditions between the expansion region and the base are known, it would seem possible to solve the flow numerically by using a finite difference method. It is very difficult to do this, however, because of radiation, and chemical n

To provide an alternate flow field, a free flow field, one important parameter is the boundary-layer thickness. Theoretical and experimental data dictate the dynamics of nonablating cases.^{7,8} The Mach number and body geom



a) Velocity =



b) Velocity =
Fig. 1 Free-

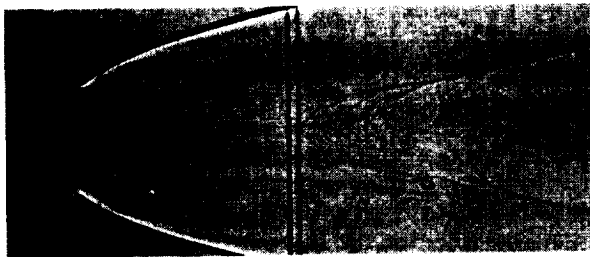
calculated by Moss et al.⁶ The analysis derives the radiative heating rates of the base stagnation point and of the region immediately behind the frustum.

Method of Solution

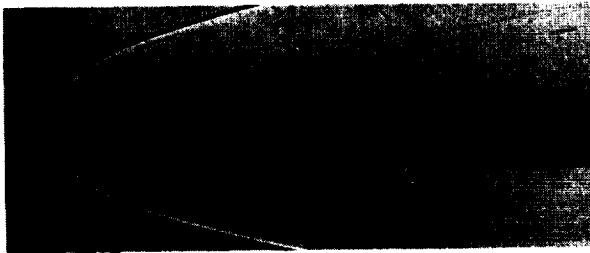
Shoulder-Expansion Region

Analysis of base flow starts from the shoulder-expansion region. Provided the initial conditions (i.e., the flow field over the frustum) and the geometry of the free-shear layer between the expansion region and the recirculating region are known, it would seem possible in principle to solve for the flow numerically by using existing techniques.⁷ In practice it is very difficult to do so because of ablation,⁸ radiation, and chemical nonequilibrium.

To provide an alternative way to determine the base region flow field, a free-flight test was performed at Ames Research Center. In experimentally simulating the base flow field, one important parameter is believed to be the ratio of boundary-layer thickness to shock-layer thickness. Existing theoretical and experimental evidence indicates that this ratio dictates the dynamics of base flow field in both ablating and nonablating cases.^{7,8} This ratio depends, in turn, for a given Mach number and body geometry, mainly on the effective



a) Velocity = 5.34 km/s, model cold.



b) Velocity = 5.22 km/s, model ablating.

Fig. 1 Free-flight shadowgraph.

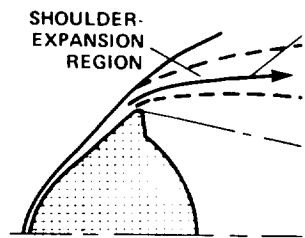
specific heat ratio γ prevailing in the forebody shock layer and the characteristic Reynolds number. In the Ames experiment, therefore, an attempt was made to reproduce these two parameters. An iron scale model of the Jovian probe, 2.03 cm diam, was flown in an ambient atmosphere consisting of 39% oxygen and 61% argon by volume at 122 Torr, at a velocity of 5.3 km/sec. The mixture produced a specific heat ratio γ between 1.2 and 1.4 in the forebody region, a fair simulation of the Jovian flight conditions. The solutions by Moss et al.⁶ show γ to vary from 1.1 at the wall to about 1.3 at the shock. The test also produced a Reynolds number that agrees closely with that in the Jovian flight case. Shadowgraph pictures were taken at several stations along the flight path. Two such photographs, one taken at an upstream station where the model is cold and the other at a downstream station where the model is ablating, are shown in Figs. 1 a and 1 b. The shadowgraphs show the following features:

- (1) For the nonablating case, the angle of the free-shear layer, as judged by the line of demarcation caused by the temperature gradient along the layer, is -11.5° on the average.
- (2) For the nonablating case, the neck is located about 1.5 diam downstream of the frustum.
- (3) The diameter of the neck is about 2/3 that of the body diameter for both ablating and nonablating cases. Here the neck radius is defined as the extreme outer edge of the domain containing the ablation product.

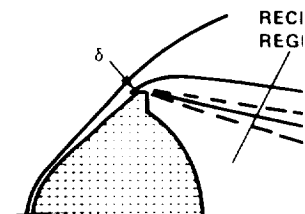
In addition to these features, the present analysis makes use of the results of Stephenson² which showed that, for the ablating case, the location of the neck is at 1.5 body diam downstream of the frustum. One notices here that some of Stephenson's luminosity photographs show the neck radius to be about half the body radius rather than 2/3 of the radius. However, this can be attributed to photographic deception: the weak luminosity of the edge of the neck could be below the lower cutoff of the photographic sensitivity.

A method-of-characteristics calculation was carried out by the present author for the shoulder-expansion region, assuming the gas to be perfect with $\gamma = 1.2$. The γ value was chosen because (1) it is an average between the values at the wall and the shock wave and (2) it is approximately the largest γ value for which the method of characteristics is practical. (At higher γ values, the flow over the frustum becomes subsonic, thereby rendering the method invalid.) The shear layer was replaced by a straight backward-facing cone of -11.5° half-angle for this calculation (see Fig. 2 a). From

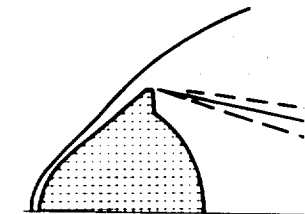
MODELING OF RADIATION



a) Stream tube for luminous region.



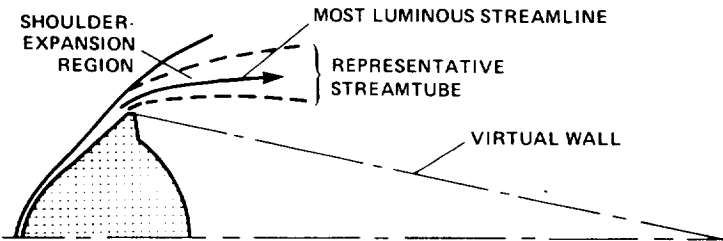
b) Velocity region.



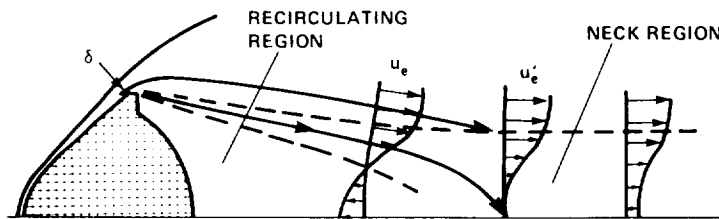
c) Static pressure region.

Fig. 2 Schematic diagrams of the stream tube for the luminous region.

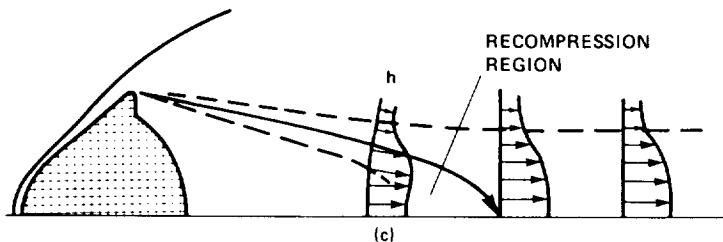
the solution, a stream tube of maximum pressures and radiation intensity increases. It will be the strongest at the thickness of the stream tube. The shock-layer thickness at the neck is to be a mixture only of hydrogen and ablation product gases are present. The present choice of representative values may overestimate pressure and radiation power emitted by the shock layer. This is partly, by the fact that the portion that came past the neck is neglected in the radiation



a) Stream tube for luminosity calculation in shoulder-expansion region.



b) Velocity profile.



c) Static enthalpy profile.

Fig. 2 Schematic of base flow field.

the solution, a stream tube was chosen which is along the path of maximum pressures and temperatures (see Fig. 2 a). Because radiation intensity increases with pressure and temperature, it will be the strongest along this streamline. The initial thickness of the stream tube is taken to be (outer) 80% of the shock-layer thickness at the frustum edge. The gas is assumed to be a mixture only of hydrogen and helium; concentration of ablation product gases are most likely to be negligibly small. The present choice of representative stream tube tends to overestimate pressure and temperature on the average and hence radiation power emitted by the gas mass that passed through the shock layer. This tendency is compensated for, at least partly, by the fact that the flow region above, that is, the portion that came past the extended bow shock wave, is neglected in the radiation calculation.

Flow properties are then calculated along this representative stream tube using a computer code for hydrogen-helium mixture developed by the present author which accounts for finite-rate ionic reactions of hydrogen ions and electrons and nonequipartition of energies between electrons and heavy particles.⁹ The calculation uses the pressure variation specified by the method-of-characteristics solution. Radiation power emitted by the stream tube is then calculated using the method to be described later (see the section on Radiation).

Recirculating and Recompression Regions

To calculate radiative properties in the recirculating region, one must first determine the static pressure in the region. It is known that pressure varies considerably within the region between the body surface and the neck reattachment point. It is generally low near the body surface, the lowest point being at a point a short distance away from the base stagnation point.⁸ Existing experimental evidence,⁸ the theory of Weng and Chow,⁷ and the recent unpublished calculations made at Ames Research Center indicate that pressure rises almost linearly with axial distance, starting from a point about one body radius away from the stagnation point. Traditionally, base pressure is defined as the pressure at the base stagnation point.^{5,10} This pressure value is appropriate in calculating convective heat-transfer rates. Since radiative emission occurs mainly in the high-pressure region away from the surface, however, this traditional base pressure value is of little consequence in the present problem.

In the present work, the region between the body surface and the neck point is divided arbitrarily into a "recirculating region" and a "recompression region." The recirculating region is considered to extend from the body surface to 2/3 of the distance from the surface to the neck point. For the purpose of simplifying radiation calculations, pressure was assumed to be constant in each of the two regions. These pressure values, which are the bulk average values in the regions, are estimated from the following procedure:

From the turning angle around the frustum edge obtained from the free-flight tests, one can calculate the pressure behind the frustum edge using the Prandtl-Meyer expansion method. The flow-turning angle is first determined by subtracting the sonic-point angle, which is the angle of the body surface where the flow reaches sonic velocity, from the angle of the free-shear layer. Pressure change is then determined assuming that the flow process follows Prandtl-Meyer expansion over the turning angle. For the nonablating case

with $\gamma = 1.3$, the sonic angle is determined, and hence the turning angle to a base-to-front stagnation point. The average pressure in the recirculating region is then calculated using the method described above.

It is to be noted that the pressure value adopted in the present work is 0.8% of the front stagnation pressure. If a traditional base pressure is adopted, the Mach number becomes close to 5. Using the method described above, one finds the neck pressure value.⁸ Since static pressure increases linearly with distance from the surface, the average pressure in the recirculating region becomes the average of the front stagnation pressure and the neck pressure.

The second important aspect of radiative properties is enthalpy. In the recirculating region, one examines the average enthalpy. For the dividing streamline, the edge velocity u_e (velocity at the edge of the layer, see Ref. 8). The average enthalpy along the centerline is then calculated. For the purpose of calculating the average velocity in the recirculating region, the algebraic average velocity in the region is calculated as

$$\bar{h} = \bar{h}_e - \frac{1}{2} \Delta h$$

This approximation introduces an error in the radiation calculation. Since the region is a small fraction of the total enthalpy.

The average total enthalpy in the recompression regions is then calculated. The layer allows rapid transfer of enthalpy across it, therefore, the enthalpy will be almost uniform of the flow mass enthalpy. The recompression reduces, therefore, to

with $\gamma = 1.3$, the sonic speed is reached at the frustum edge and hence the turning angle is $45^\circ + 11.5^\circ = 56.5^\circ$. This leads to a base-to-front stagnation-point pressure ratio of 0.023. The average pressure in the recompression region is taken to be the algebraic average between the average pressure in the recirculating region and the neck pressure which is determined using the method described later.

It is to be noted here that the average base pressure value adopted in the present work is consistent with the traditional base pressure value which is known to be about 0.8% of the front stagnation-point value.⁵ If the 0.8% value is adopted, the Mach number in the shoulder expansion region becomes close to 5. Using the Chapman-Korst theory, one then finds the neck pressure to be at least 10 times the 0.8% value.⁸ Since static pressure increases approximately linearly with distance (starting from one body radius away from the surface), the volume-averaged pressure in the recirculating region becomes nearly equal to or greater than 2% of the front stagnation-point value.

The second important parameter needed in the determination of radiative properties is static enthalpy. To determine the enthalpy, one examines velocity distribution within the recirculating and recompression regions (see Fig. 2 b). Along the dividing streamline, velocity is approximately half the edge velocity u_e (velocity at the outer edge of the free-shear layer, see Ref. 8). The maximum velocity of the reverse flow along the centerline axis is also approximately $u_e/2$ (Ref. 8). For the purpose of calculating radiative properties, the average velocity in the region u is assumed to be $u_e/4$, that is, the algebraic average between the maximum and minimum velocity in the region. The average static enthalpy is calculated as

$$\bar{h} = \bar{H} - (1/2)\bar{u}^2 = \bar{H} - (1/2)(u_e/4)^2$$

This approximation introduces only a small uncertainty into the radiation calculation because the average kinetic energy in the region is a small fraction (about 3%) of the total enthalpy.

The average total enthalpy \bar{H} within the recirculating and recompression regions must now be determined. Since the shear layer allows rapid transport of mass, momentum, and total enthalpy across it, the total enthalpy H within the regions will be almost uniform and equal to the average total enthalpy of the flow mass entrained into the shear layer. The problem reduces, therefore, to that of determining the average total

enthalpy of the flow entrained by the shear layer. The mass entrained by the shear layer originates in the lower portion of the shock layer, indicated in Fig. 2a by $0 < Y < \delta$. If δ is known, the average total enthalpy \bar{H} is determined as

$$\bar{H} = \int_0^\delta \rho u H dY / \int_0^\delta \rho u dY$$

The point δ is determined in the present work by imposing the requirement that the calculated neck radius equals the value observed experimentally. The details of the method are explained in the following section.

The average enthalpy \bar{H} is used also in determining the shear-layer edge velocity u_e . The process of expansion from the frustum edge to the edge of shear layer is represented by a stream tube. The pressure variation along this stream tube is assumed to be such that pressure reaches the average base pressure p_b within about 15 cm from the frustum edge. For the hypothetical case of no ablation calculated by Moss,⁶ the non-equilibrium one-dimensional flow code for hydrogen-helium mixture used for the shoulder-expansion region is used here also to determine the edge velocity u_e . The resulting u_e value is found to correspond to about 50% of the total enthalpy, which is substantially smaller than that obtained under the assumption of equilibrium flow. For the case with ablation, similar calculation is presently impractical because the rate coefficients associated with the reactions involving carbonaceous species are unknown. The velocity u_e is determined for this case, therefore, assuming that 50% of the total enthalpy is converted to kinetic energy, as is found to be the case for the nonablating case.

Since the flow velocity is relatively slow in the recirculating and recompression regions, and since the length over which the gas therein must travel is long, it is likely that thermodynamic equilibrium prevails within the region. From the pressure and static enthalpy values determined previously, therefore, temperature and concentrations of each chemical species are determined uniquely via an equilibrium thermochemistry calculation. Radiative properties of the region are then determined by the method described later. For the purpose of radiative-transfer calculations, the shape of the recirculating and decompression regions is taken to be a truncated cone of 11.5° half-angle.

Neck Region

The dividing streamline converges into a reattachment point where velocity is zero. Since the dividing streamline

meets the centerline at the axial velocity profile to radius, as shown schematically in Fig. 2b, the low velocity constitutes mass entrained in the cross-sectional area of the shear layer because of the neck. The velocity determined in the present work

The flow velocity at the neck is determined from the assumption that the pressure corresponds to the Prandtl-Meyer function of $0.92 u_e$.

From the velocity profile is constructed the static enthalpy in the neck is and recompression region. The same entrainment principle is used for the layer. As was done for the shoulder and neck regions, the radiative properties are determined by assuming uniform at the average static enthalpy and average kinetic energy. The average kinetic velocity is assumed velocity profile

$$\pi R_n^2 \bar{u}^2 = 2$$

Static pressure is determined at the free-shear layer. The flow is assumed steady. As the flow approaches the reattachment point, pressure rises gradually and, to a lesser extent, the flow field, that is, the pressure rise due to the function mostly of the distance from the reattachment point about 3 in the present case. Theoretical data indicate that the pressure at the reattachment point is about 3 times that outside the flow.^{7,8,12} (At a high Mach number, correspondingly larger. an isentropic compression, Prandtl-Meyer function)

meets the centerline axis at a right angle at this point, the axial velocity profile has zero first derivative with respect to radius, as shown schematically in Fig. 2 b. The region of low velocity constitutes a "neck" through which the entire flow mass entrained in the shear layer must pass.¹¹ The overall cross-sectional area of the neck is much larger than that of the shear layer because the average flow velocity is slow in the neck. The velocity profile across the neck is approximated in the present work by the quadratic expression:

$$u = u_e'(r/R_n)^2$$

The flow velocity at the outer edge of the neck, u_e' , is determined from the assumption of isentropic compression corresponding to the Prandtl-Meyer turning angle of 11.5° to be $0.92 u_e$.

From the velocity profile obtained, the static enthalpy profile is constructed (Fig. 2 c). The average total enthalpy in the neck is the same as that in the recirculating and recompression regions because all three regions are fed by the same entrainment process occurring in the free shear layer. As was done for the recirculating and recompression regions, the radiative properties in the neck region are determined by assuming that the static enthalpy therein is uniform at the average value determined by subtracting the average kinetic energy from the average total enthalpy. The average kinetic velocity \bar{u} is calculated in turn from the assumed velocity profile as

$$\pi R_n^2 \bar{u}^2 = 2\pi \int_0^{R_n} r u^2 dr = (\pi/16) R_n^2 u_e'^2$$

Static pressure is almost constant in the early stage of the free-shear layer. The edge velocity u_e is correspondingly steady. As the flow approaches the reattachment point, pressure rises gradually due mostly to recovery of dynamic pressure and, to a lesser extent, by virtue of axisymmetry of the flow field, that is, narrowing of cross-sectional area. The pressure rise due to recovery of dynamic pressure is a function mostly of the Mach number of the edge flow, which is about 3 in the present case. Existing experimental and theoretical data indicate that, at a Mach number of 3, the pressure at the reattachment point is slightly under three times that outside the shear layer for the two-dimensional flow.^{7,8,12} (At a higher Mach number, the ratio is correspondingly larger.) A similar value is obtained through an isentropic compression calculation — an inverse two-dimensional, Prandtl-Meyer calculation corresponding to the

turning angle of 11.5° gives a pressure rise factor of 2.8. Existing literature indicates that axisymmetry adds slightly to this pressure rise.⁷ The present work assumes therefore that the pressure in the neck region is three times the average base pressure.

Since the average velocity is relatively slow and the pressure is high, thermodynamic equilibrium is expected to be established within the neck region. From the average enthalpy and pressure values obtained, therefore, one can determine the rest of the state properties through an equilibrium thermochemistry calculation. Using the average density $\bar{\rho}$ thus obtained, the mass flow rate through the neck \dot{M} is calculated as

$$\dot{M} = 2\pi\bar{\rho} \int_0^{R_n} ru \, dr = (1/2) \pi R_n^2 \bar{\rho} u_e$$

By requiring that the calculated neck radius R_n equals the value observed experimentally, that is,

$$R_n = (2/3)R_f$$

\dot{M} is uniquely determined. From the \dot{M} value, the thickness of the inner shock layer swallowed by the shear layer, δ (Fig. 2 b) is determined from the relation

$$\dot{M} = 2\pi R_f \int_0^\delta \rho u \, dY$$

From the value of δ determined in this way, one can calculate the average total enthalpy \bar{H} needed to determine the static enthalpy values for both the recirculating/recompression region and the neck region using the method described earlier. The calculation is carried out in a trial-and-error manner. An arbitrary value of δ is first assumed, and the corresponding value of R_n is calculated from it. The R_n value is then compared with the experimental value, and the parameter δ is adjusted until the calculated R_n agrees with the observed value.

Downstream of the neck, the flow accelerates gradually because of the shear force exerted on it by the outside flow. In return, the outer flow is accelerated and is entrained in the inner region. The flow thus evolves into a wake. Typical velocity profiles and enthalpy profiles in the early stage of the wake are shown in Figs. 2 b and 2 c.

Experimental data of Stephenson^{2,3} show that the high luminosity of the neck region extends over approximately 1

body diameter. In the re luminosity attenuates gra calculation, the neck-plu a cylinder of uniform lum neck radius and a length

Radiation

The foregoing calcul carried out for the Jovian Moss et al.⁶ The base-fl six entry trajectory poin of zero ablation and one l benchmark point is approx (for the front stagnation with finite ablation is of shield, assuming the ablat shock-layer profiles gener determining the average to concentrations in the base re pressure is assumed to be pressure which is derived

The radiative transfe both the nonequilibrium re expansion region) and the recompression, and neck re region, the species contri hydrogen (H) and ionized h ablation products and radi weak. For the equilibrium calculations are carried o the hypothetical zero abla containing ablation produc oxygen, and their molecula

For the nonequilibrium properties can be calculate computer code NEQRAP.¹³ Th equilibrium population of e date techniques and account broadening of hydrogen line calculation with a slight n excited hydrogen atoms is a the free (i.e., ionized) st represents a variation from dynamic equilibrium) approx pseudo-LTE approximation,¹³ rigorous solutions yield ex

body diameter. In the region downstream of the bright region, luminosity attenuates gradually. For the purpose of radiation calculation, the neck-plus-wake region is represented here by a cylinder of uniform luminosity having a radius equal to the neck radius and a length equal to 1.5 body diameters.

Radiation

The foregoing calculation of base-flow parameters is carried out for the Jovian entry environments calculated by Moss et al.⁶ The base-flow calculations are carried out for six entry trajectory points under the hypothetical assumption of zero ablation and one benchmark point with ablation. The benchmark point is approximately at the peak heating point (for the front stagnation point). The benchmark calculation with finite ablation is obtained for a carbon-phenolic heat shield, assuming the ablation layer to be turbulent. The shock-layer profiles generated by Moss's solutions are used in determining the average total enthalpies and species concentrations in the base region. The volume-averaged base pressure is assumed to be 2.3% of the front stagnation-point pressure which is derived for the nonablating case for γ 1.3.

The radiative transfer calculation is carried out for both the nonequilibrium region behind the frustum (shoulder-expansion region) and the equilibrium region (recirculating, recompression, and neck regions). For the nonequilibrium region, the species contributing to radiation are neutral hydrogen (H) and ionized hydrogen (H^+). The flow contains no ablation products and radiation from helium is negligibly weak. For the equilibrium regions along the axis, radiation calculations are carried out for both the $H+He$ mixture (for the hypothetical zero ablation condition) and for the mixture containing ablation products (hydrogen, helium, carbon, oxygen, and their molecular compounds and ions).

For the nonequilibrium ionized hydrogen, radiative properties can be calculated to a high precision using the computer code NEQRAP.¹³ The code calculates the nonequilibrium population of excited states using the most up-to-date techniques and accounts rigorously for the Stark broadening of hydrogen lines. The code is used in the present calculation with a slight modification: the population of excited hydrogen atoms is assumed to be in equilibrium with the free (i.e., ionized) state. This assumption, which represents a variation from the popular LTE (local thermodynamic equilibrium) approximation and hence can be termed a pseudo-LTE approximation,¹³ is based on the finding that the rigorous solutions yield excited state populations that are

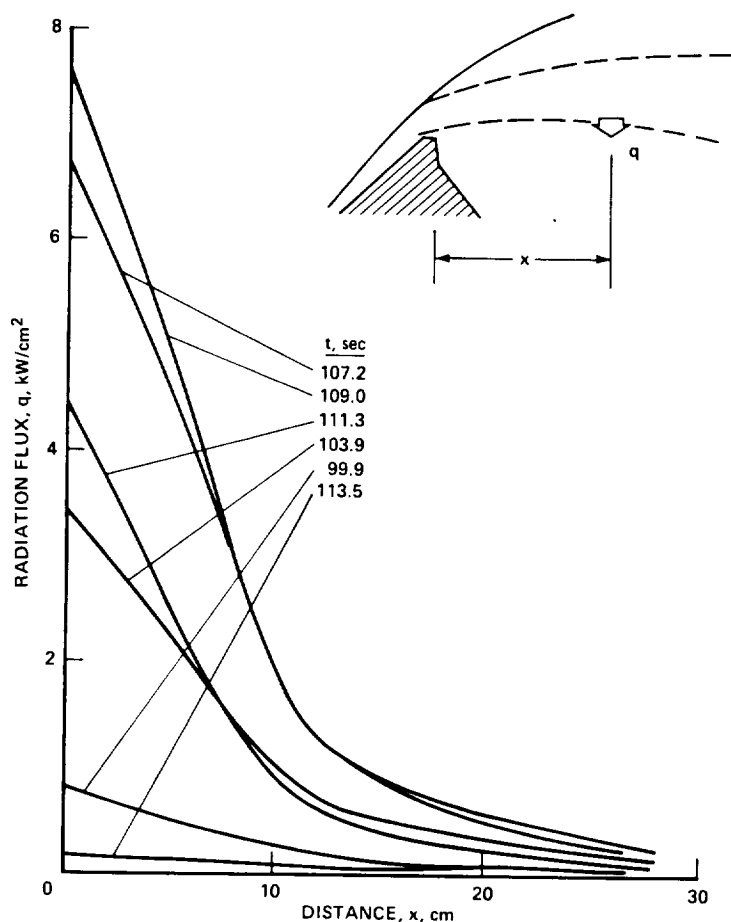


Fig. 3 Calculated radiation flux emerging from shoulder-expansion region.

very close to those obtained from the pseudo-LTE approximation. The largest deviation of the pseudo-LTE value from the rigorous population value, which is still under 10%, occurs for the first excited state from which the Lyman-alpha line emanates. But since the Lyman-alpha line is mostly self-absorbed, inaccuracy of its population value does not lead to significant error. For the second excited state and higher states, which produce Balmer and Paschen lines and hence contribute significantly, deviation of the pseudo-LTE values from the rigorous solutions is less than 2%. The calculation procedure adopted in the present work is believed to be accurate, therefore, to within about 2% in the radiation properties. The photon energy range of 0.2 to 16.2 eV is divided into 3000 equally spaced intervals for the calculation.

A one-dimensional adopted for the calculation shoulder-expansion region is calculated only in the centerline axis, and multiplying the intensity procedure implicitly assumes thinner than the body diameter optically thin for the. The assumptions are below of 10%, which is acceptable.

For the three equilibrium central axis (recirculation) an axisymmetric radiation for the base stagnation falling on the stagnation function of the viewing formed along 16 different are chosen at especially resolve the rapidly varying the neck region. Radiation are obtained using

For the case with the code ARCRAP indicates negligible for the benchmark is above 10,000 K in all molecules are very small oxygen species are seen species contributing significantly therefore, are hydrogen properties of hydrogen and neutral carbon included listed in the NBS tables used for these lines) are upper state of principal missing from the NBS tables obtained from the corresponding

Altogether 374 lines the carbon lines is taken Gaussian component of the from temperature. The line profile consists of three natural broadening, Stark broadening. The width of exactly. All Stark lines

A one-dimensional radiative transfer approximation is adopted for the calculation of radiative transfer for the shoulder-expansion region. That is, intensity of radiation is calculated only in the radially inward direction normal to the centerline axis, and the radiation flux is obtained by multiplying the intensity of the normal ray by 2π . The procedure implicitly assumes that the emitting layer is much thinner than the body diameter, and that the medium is optically thin for the Balmer-Paschen lines and continua. The assumptions are believed to lead to an error of the order of 10%, which is acceptable in the present work.

For the three equilibrium regions located along the central axis (recirculating, recompression, and neck regions), an axisymmetric radiative-transport calculation is performed for the base stagnation point. To obtain radiation intensity falling on the stagnation point as a sufficiently fine function of the viewing polar angle, the calculation is performed along 16 different polar angle directions. The first 6 are chosen at especially small angular intervals in order to resolve the rapidly varying contribution of different parts of the neck region. Radiative properties needed in the calculation are obtained using the computer code ARCRAP.¹⁴

For the case with ablation, preliminary calculation with the code ARCRAP indicates that all molecular radiation is negligible for the benchmark case. This is because temperature is above 10,000 K in all three regions; concentrations of molecules are very small at such high temperatures. Also, oxygen species are seen to contribute negligibly. The only species contributing significantly to radiative transfer, therefore, are hydrogen and carbon species. The radiative properties of hydrogen species are well known. The lines of neutral carbon included in the calculation are: (1) all lines listed in the NBS tables¹⁵ (f numbers given in the tables are used for these lines) and (2) all lines that emanate from an upper state of principal quantum numbers of 5 or less but are missing from the NBS tables (f numbers for these lines are obtained from the corresponding transitions in N^+ , O^{++} , or H_e).¹⁶

Altogether 374 lines of carbon are included. The shape of the carbon lines is taken to be the Voigt profile.¹⁷ The Gaussian component of the Voigt profile is uniquely determined from temperature. The Lorentzian component of the Voigt profile consists of three parts, those corresponding to natural broadening, Stark broadening, and Van der Waals broadening. The width of naturally broadened line is known exactly. All Stark line widths obtained for carbon by Griem¹⁸

are accommodated. For the lines with unknown Stark line widths, the half-half-width $\Delta\lambda$ is estimated with the formula:

$$\Delta\lambda = 0.025(\lambda/10,000)(N_e/10^{16})^0, \text{ \AA}$$

where N_e is electron density (in cm^{-3}). The formula approximately represents the mean value of all known Stark widths of carbon.¹⁸ The Van der Waals line width is subdivided into two parts, the broadening due to hydrogen atoms and that due to all other atoms and molecules. The reason for this subdivision is that hydrogen atoms induce exceptionally large broadening. The broadening due to hydrogen atom is approximated by

$$\Delta\lambda = 0.02(\lambda/2000)^2([H]/10^{16}), \text{ \AA}$$

where $[H]$ is the concentration of hydrogen atoms (in cm^{-3}). The formula approximately represents the mean values of the line widths for silicon atoms produced by the same mechanism.¹⁴ The approximation is made here because carbon is similar to silicon in spectroscopic properties. The Lorentzian line widths due to broadening by collisions between carbon and all atoms and molecules other than hydrogen are calculated as in the work of Park and Arnold.¹⁹

The bound-free continua of C and free-free continua of C^+ are calculated using the cross-sectional data of Peach.²⁰ The photon energies in the range of 0.2 to 16.2 eV are divided into 18,500 equally spaced intervals for this calculation.

No attempt is made to calculate the absorption of radiation from the hot gases by the cold gases adjacent to the wall. Since pressure is low in the base region, absorption by the cold gases is not likely to be significant. Moreover, any reduction in radiative heat-transfer rate due to absorption is likely to be partially compensated for by the rise in convective heat-transfer rate caused by the absorption.

Results

The results of the foregoing calculations are shown in Figs. 3-7. They are given in greater detail in Ref. 21. Figure 3 shows the intensity of the inward radiation flux emerging from the shoulder-expansion region as a function of distance x . At $x = 0$, the calculated flux value represents approximately 80% of the heat flux in the frustum edge on the forebody side. This is because the initial thickness of the expansion region is taken to be 80% of that of the shock-layer thickness (see section on Shoulder-Expansion Region).

A close examination that the radiation heat flux is the same as those from an optical source by

$$e = (2.7 \pm 0.68) \times 10^8 \text{ W/m}^2$$

The upper error limit occurs when the radiation limit pertains to high pressure. As evident from Fig. 3, the radiation flux is a function mostly of electron density and is independent of atom density. The well-known effect of nonequilibrium properties were calculated. The densities would be considered here, leading to corresponding values.¹³

Figure 3 also shows that this region is the strongest than the peak heating point. This early arrival of the shoulder-expansion region flight velocity and chemical density. The radiation from cylindrical surfaces surrounding the total inward heat flux value at $x = 0$ are given

In Figs. 4 and 5, the radiation reaching the base stagnation point is abating and the abating is many more lines than Fig. 3. The carbon lines included in these figures, radiation at photon energies is weak. This is attributed to the high-energy photons are not present. The function value is small for the radiation. This partly justifies neglecting the radiation adjacent to the wall since primarily those photons are

Figure 6 shows the radiation flux received by the base. In this figure, radiation received is slightly higher (27%) than the radiation from the ablation. For both cases, the radiation received is in the angular range of small angular range is irradiated

A close examination of the calculated results revealed that the radiation heat flux values are approximately the same as those from an optically thin gas with emissivity given by

$$e = (2.7 \pm 0.68) \times 10^{-32} (10,000/T_e)^{1.64} N_e^2, \text{ W/cm}^3$$

The upper error limit occurs at low pressures, and the lower limit pertains to high pressure within the pressure range of concern. As evident from the expression, radiation intensity is a function mostly of electron density and is virtually independent of atom density. The expression underscores the well-known effect of nonequilibrium.^{9,13,22} If the flow properties were calculated assuming equilibrium, electron densities would be considerably smaller than those obtained here, leading to correspondingly small heat-transfer rate values.¹³

Figure 3 also shows that the radiative heat flux q in this region is the strongest at $t = 109$ sec, which is earlier than the peak heating point for the front stagnation point. This early arrival of the peak heating point for the shoulder-expansion region is caused by the combination of high flight velocity and chemical nonequilibrium caused by low density. The radiation flux q is integrated over the cylindrical surfaces surrounding the frustum edge to obtain the total inward heat flux. The results and the maximum q value at $x = 0$ are given in Table 1.

In Figs. 4 and 5, the spectra of the radiation fluxes reaching the base stagnation point are shown for the non-ablating and the ablating cases, respectively. Figure 5 shows many more lines than Fig. 4. The difference is due to atomic carbon lines included in the calculation for Fig. 5. In both figures, radiation at photon energies greater than 10 eV are weak. This is attributed to the combination of two facts: the high-energy photons are self-absorbed and the blackbody-function value is small for the high-energy range. This fact partly justifies neglecting absorption by the cold gases adjacent to the wall since absorption would attenuate primarily those photons in the high-energy range.

Figure 6 shows the angular distribution of radiation flux received by the base stagnation point. As seen in the figure, radiation received in the case with ablation is slightly higher (27%) than that received in the case without ablation. For both cases, over half the total radiation received is in the angular range of $0 < \theta < 0.6$ rad. This small angular range is irradiated primarily by the radiation

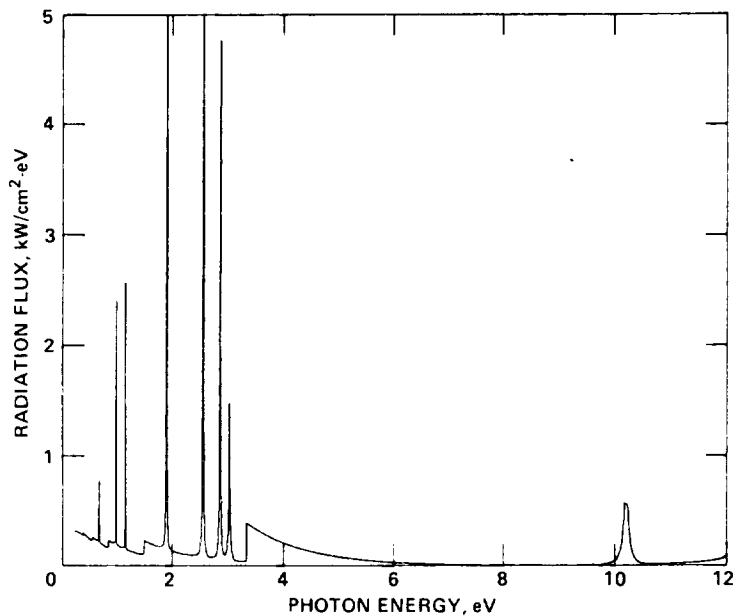


Fig. 4 Calculated spectra of radiation flux at base stagnation-point at benchmark point, without ablation.

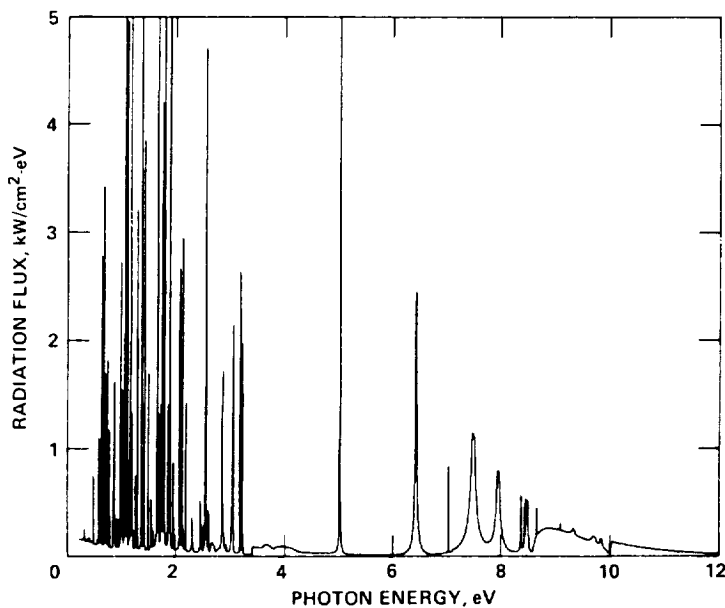


Fig. 5 Calculated spectra of radiation flux at base stagnation-point at benchmark point, with ablation.

emitted by the neck and re the figure. By integrati obtains the total radiatio point given in Table 1.

Figure 7 a shows the $\oint q \, dA$ from the shoulder-ex along the entry trajectory over the surface of the ir schematically in Fig. 7 a because it was thought to the heat-shield design. transfer rate for the base function of time. For th occurs only slightly ahead front stagnation point. 7 b leads to $\int q \, dt = 1.4$

The results just show radiative heating of the l behind the frustum. This the same radiative heating luminosity persists behind region, partly because of

The radiative heating is substantially greater i Its absolute value is only and without ablation (the larger) even though the so circulating, recompression colder when ablation produ difference between the two that carbon radiates more hydrogen.

Several assumptions i the present theory, the mo be the value for base pres based on an average base p pressure or, equivalently pressure. In order to inv pressure, calculations we pressure values for the b The results show that the rate q is approximately p to-front stagnation-point

emitted by the neck and recompression regions as indicated in the figure. By integrating under the curves in Fig. 6, one obtains the total radiation flux q for the base stagnation point given in Table 1.

Figure 7 a shows the area-integrated radiation power $\oint q \, dA$ from the shoulder-expansion region as a function of time along the entry trajectory. The areal integration here is over the surface of the imaginary semi-infinite cylinder shown schematically in Fig. 7 a. The quantity is shown here because it was thought to be more useful than the q values in the heat-shield design. In Fig. 7 b, the radiative heat-transfer rate for the base stagnation point is shown as a function of time. For this case, the peak heating point occurs only slightly ahead of the peak heating point for the front stagnation point. Integrating under the curve in Fig. 7 b leads to $\int q \, dt = 1.4 \times 10^4 \, \text{J/cm}^2$.

Discussion

The results just shown indicate that the most severe radiative heating of the base region would occur immediately behind the frustum. This region is subject approximately to the same radiative heating as the forebody. Relatively high luminosity persists behind the frustum in the expansion region, partly because of the nonequilibrium effect.

The radiative heating rate of the base stagnation point is substantially greater than the convective heating rate.⁵ Its absolute value is only slightly different in cases with and without ablation (the ablating case value is 1.27 times larger) even though the sources of radiation (i.e., the recirculating, recompression, and neck regions) are appreciably colder when ablation products are present. Such a small difference between the two cases can be attributed to the fact that carbon radiates more strongly at low temperatures than hydrogen.

Several assumptions have been made in the development of the present theory, the most important of which is believed to be the value for base pressure. The present calculations are based on an average base pressure that is 4.5% of the frustum pressure or, equivalently, 2.3% of the front stagnation-point pressure. In order to investigate the possible effect of base pressure, calculations were made with two different base-pressure values for the benchmark condition ($t = 111.3 \, \text{sec}$). The results show that the base stagnation-point heat-transfer rate q is approximately proportional to the square of the base-to-front stagnation-point pressure ratio p_b/p_s . For the

Table 1 Summary of heat-transfer rate q calculations

Forebody data (Moss ⁶ solutions)									
Trajectory point	612	652	685	703	726	748			
Time, sec	99.5	103.9	107.2	109.0	111.3	113.5			
Altitude, km	195.6	169.6	149.1	138.6	126.1	115.3			
Freestream density, $g/cm^3 \times 10^8$	2.65	7.19	16.4	24.5	43.6	70.2			
Flight velocity, km/sec	48.07	46.96	44.83	42.88	39.29	34.67			
Front stagnation-point pressure, atm	0.56	1.44	2.97	4.20	6.01	7.45			
Front stagnation point q , without ablation									
Convective, kw/cm^2	7.90	11.4	14.0	14.8	14.7	12.8			
Radiative, kw/cm^2	2.70	10.2	25.0	35.3	42.3	32.0			
Front stagnation point q , with ablation									
Convective, kw/cm^2	0.02	...			
Radiative, kw/cm^2	19.4	...			
Afterbody data (radiative only)									
Shoulder-expansion region									
Max q , kw/cm^2	0.82	3.42	6.66	7.48	4.47	1.98			
ϕq dA, MW	3.22	13.5	24.8	25.5	13.5	1.04			
Base stagnation point q									
Without ablation, kw/cm^2	0.091	0.391	1.02	1.45	1.72	1.27			
With ablation, kw/cm^2	2.19	...			

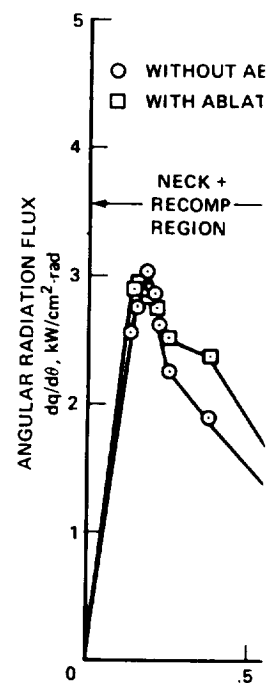
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Fig. 6 Angular dist stagnation-point fo

benchmark case with abl
front stagnation-point

$$q/q_s$$

Assuming that the ratio
without ablation applie
estimate of the heat lo
calculated trajectory b

$$\int q \, dt = 1.8$$

The present analys
concerns only the two e.
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treme outer edge. The
core regions (i.e., rec
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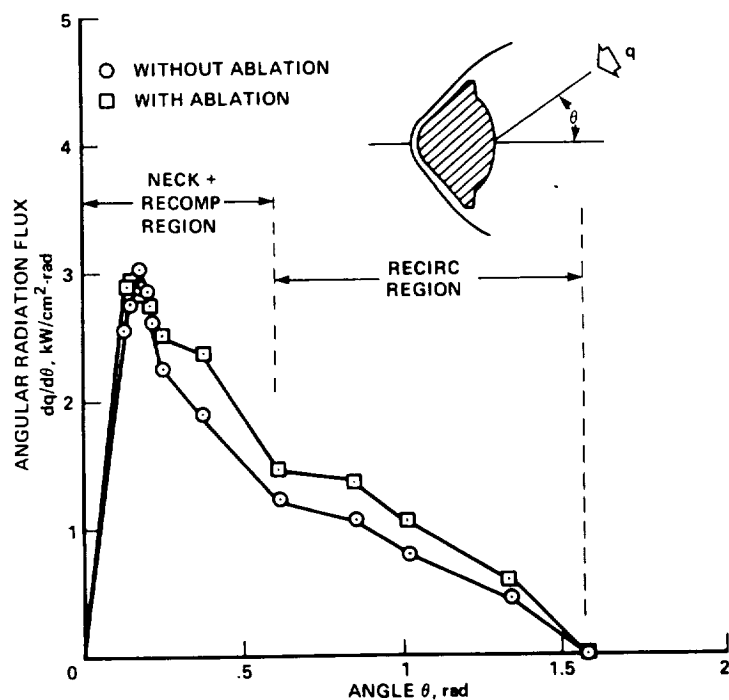


Fig. 6 Angular distribution of radiation flux at base stagnation-point for benchmark point.

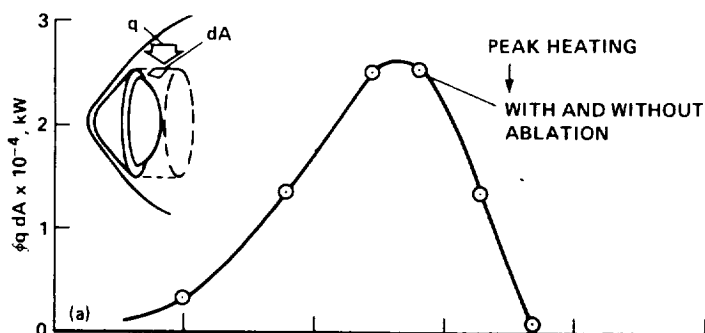
benchmark case with ablation, therefore, the ratio of base-to-front stagnation-point heat-transfer rate becomes

$$q/q_s = 0.113(43.4p_b/p_s)^2$$

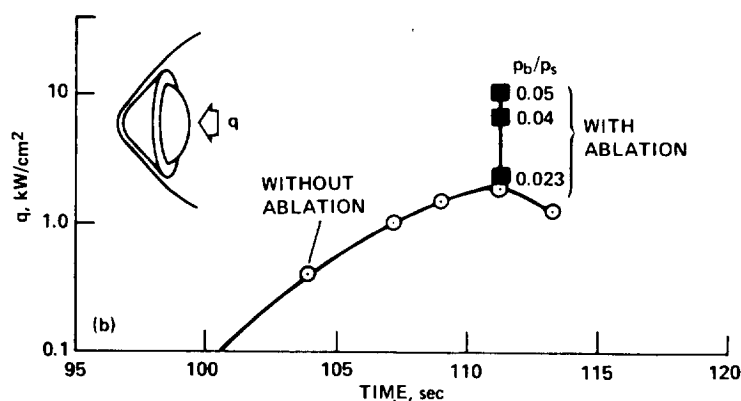
Assuming that the ratio of 1.27 between the cases with and without ablation applies at all trajectory points, the best estimate of the heat load to the stagnation point for the calculated trajectory becomes

$$\int q \, dt = 1.8 \times 10^4 (43.4p_b/p_s)^2, \text{ J/cm}^2$$

The present analysis of radiative heat-transfer rate concerns only the two extreme positions on the base heat shield — the base stagnation point (axis-point) and the extreme outer edge. The base stagnation point is heated by the core regions (i.e., recirculating, recompression, and neck regions) and the outer edge is irradiated by the shoulder-expansion region. An intermediate, off-axis point between the two extreme positions will be subject to heating from both sources of radiation. The intensity of radiation cast by the



a) Integrated heat load from shoulder-expansion region.



b) Base stagnation-point heat-transfer rate.

Fig. 7 Heat load in base region.

three core regions on the off-axis point will be approximately the same as that on the axis point. The average distance from the recirculating region to the off-axis point is slightly farther than to the axis point, but its effect is offset by the fact that the viewing angle of the neck region is larger. Since the off-axis point is at an oblique angle to the normal plane, the heating rate will be lower by the cosine of the angle. This slight reduction will be offset by the added heating from the shoulder-expansion region. As a result, the off-axis point will be subject essentially to the same or slightly greater radiative heating than the base stagnation point. The experimental data of Shirai and Park⁴ taken at an off-axis point confirms this trend.

The present analysis entails a scale effect for the shoulder-expansion region. The flow in the region will tend to freeze chemically when the body size becomes small, thereby

producing large electrical radiation power. But rate is virtually independent of sources of radiation (regions) are in equilibrium the luminous regions a contradiction to the argument base stagnation-point body size. Stephenson observations made at low Reynolds number predict a size-dependent average total enthalpy layer will depend on Reynolds number is greater than average enthalpy becomes number since the volume shear layer will be determined parameter alone, which the Jovian flight case Shirai and Park,⁴ the is believed to be in the is unaffected by the Reynolds

Radiative heating frustum on a Jovian equilibrium expansion significant. The base recirculating, recompression and neck. The base stagnation-point slightly greater in the absence. Near the peak point heating rate is volume-averaged pressure ratio to the front stagnation and the time-integrated

Sincere thanks are group at Ames Research C. E. De Rose, and the flight experiment.

producing large electron density values and hence large radiation power. But the base stagnation-point heat-transfer rate is virtually independent of size because the three sources of radiation (recirculating, recompression, and neck regions) are in equilibrium and because the viewing angles of the luminous regions are independent of size. This conclusion contradicts the argument of Stephenson² which predicts the base stagnation-point heat-transfer rate to be dependent on body size. Stephenson's argument is based on experimental observations made at relatively low Reynolds numbers. In a low Reynolds number regime, the present theory will also predict a size-dependent heat-transfer rate because the average total enthalpy of the flow entrained by the free-shear layer will depend on Reynolds number. When the Reynolds number is greater than a certain critical value, however, the average enthalpy becomes virtually independent of Reynolds number since the volume of the mass entrained by the free-shear layer will be determined by the shear-spreading parameter alone, which is approximately constant.⁵ In both the Jovian flight case and the experimental conditions of Shirai and Park,⁴ the Reynolds number is relatively high and is believed to be in the regime where the heat-transfer rate is unaffected by the Reynolds number.

Conclusions

Radiative heating is most severe immediately behind the frustum on a Jovian entry probe. Radiation from the non-equilibrium expansion region downstream of the frustum is also significant. The base stagnation point is irradiated by the recirculating, recompression, and neck regions, with the recompression and neck regions contributing the major portion. The base stagnation-point radiative heat-transfer rate is slightly greater in the presence of ablation than in its absence. Near the peak heating point, the base stagnation-point heating rate is $2.2(43 p_b/p_s)^2$ W/cm² where p_b is the volume-averaged pressure in the recirculating region. Its ratio to the front stagnation-point value is $0.11(43 p_b/p_s)^2$, and the time-integrated heat load is $18(43 p_b/p_s)^2$ kJ/cm².

Acknowledgments

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