On-Orbit Model Refinement for Controller Redesign

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Abstract – High performance control design for a flexible space structure is challenging since high fidelity plant models are difficult to obtain *a priori*. Uncertainty in the control design models typically require a very robust, low performance control design which must be tuned on-orbit to achieve the required performance. A new procedure for refining a multivariable open loop plant model based on closed-loop response data is presented. Using a minimal representation of the state space dynamics, a least squares prediction error method is employed to estimate the plant parameters. This control-relevant system identification procedure stresses the joint nature of the system identification and control design problem by seeking to obtain a model that minimizes the difference between the predicted and actual closed-loop performance. This paper presents an algorithm for iterative closed-loop system identification and controller redesign along with illustrative examples.

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1. Introduction

Implicit in the design of high performance control systems is the availability of an accurate model of the system to be controlled. Although system identification and control design are both critical aspects of high performance model based control design, the theoretical foundations of these two disciplines have developed distinctly. Developments in system identification have been directed toward obtaining accurate nominal models with bounds on the associated uncertainty. Recognizing this dependence on accurate nominal design models, robust control theory has been developed to accommodate modeling errors. Recently, attention has been drawn to the fact that the issues of system identification and control design must be treated as *mutually dependent*. In Ref. [1], Skelton points out that since the magnitude and spectrum of excitation forces are controller dependent, an appropriate model for control design cannot be determined independent of the controller. The point is made that the validity of the model is dictated by the controller instead of the opposite, as is usually assumed. Since the model that is most appropriate depends on the control design, the open-loop response of a model is not sufficient to indicate the fidelity of the model for control design. Additionally, robust performance requires an accurate model of the plant in the controller crossover frequency range [2], indicating that the amount of model error that can be tolerated is frequency and controller dependent. Hence, the issues of model identification and model based control design must be treated as a joint problem suggesting an iterative solution [1],[3].

Closed-loop system identification, (i.e. identification of the open-loop plant given closed-loop response data and knowledge of the compensator dynamics), is currently a field of active research. Most of the methods that identify state space models of the open-loop plant are based on identifying the closed-loop Markov parameters from frequency response functions and then extracting the open-loop Markov parameters. A discrete time state space realization is then obtained from these identified Markov parameters. An approach to iterative closed-loop system identification and controller redesign is given by Liu and Skelton where a state space model is estimated using the q-Markov Cover algorithm and a controller is designed using the Output Variance Constraint algorithm [4]. q-Markov Cover theory describes all realizations of a linear system which match the first q Markov parameters and covariance parameters of the true system generated by pulse responses. First the entire closed-loop system is estimated and then using knowledge of the compensator dynamics, the plant is extracted. However, the identified open-loop plant has dimension equal to the controller dimension plus the dimension of the identified closed-loop plant. Model order reduction is used to remove the superfluous states. A similar approach is taken in Ref. [5] where Phan *et al.* formulate a method which first obtains the closed-loop Markov parameters and then the open-loop Markov parameters are recovered. A discrete time state space realization is then obtained from the Markov parameters. Other related approaches are given in Refs. [6] and [7].

To provide synergism, an iterative process should match the system identification objectives with the control design objectives. Ref. [3] presents an iterative algorithm for frequency-response identification from closed-loop data and robust control design. The identification phase is control oriented with the objective of providing robust performance by closely approximating the achieved closed-loop performance. The interplay between identification and control design is formalized by specifying a performance metric (norm) for model based optimal control design and an identification cost function that minimizes the difference in the achieved performance and nominal design performance as defined by the same metric. Many algorithms have been developed based on this general framework which utilize different performance norms and identification algorithms. An excellent survey of this topic is given in Ref. [8]. Most of the work cited therein has been done in the context of single-input, single-output (SISO) systems.

A classical approach to parameter estimation which has been extended to closed-loop system identification is the prediction error approach [9]. This optimization method estimates the parameters of a linear system by minimizing the squared sum of the errors between the actual measurements and the predicted measurements. Zang, Bitmead, and Gevers present an iterative prediction error identification and control design algorithm based on the H_2 norm [10]. The control objective is used to frequency weight the identification cost functional and the resulting prediction error spectrum is used to frequency weight the control design. In this manner the control is penalized heavily where the SISO transfer function model fit is poor and the model is weighted to fit best in the regions most critical to performance. Another approach utilizes the dual Youla parameterization of all plants stabilized by a given controller. This approach was introduced by Hansen and Franklin [11] and further elaborated by Hansen, *et al.* in Ref. [12] as applied to closed-loop experiment design. Schrama applied the dual Youla parameterization to closed-loop system identification in Refs. [13] and [14] as did Anderson and Kosut in Ref. [15]. A related method directed toward identification of the coprime factors of the plant was introduced by Schrama in Ref. [13] and was further elaborated in Refs. [14] and [16].

In this paper the prediction error method is extended to closed-loop identification for multivariable systems. Building on a method developed for estimation of the parameters of an open-loop system in canonical form from open-loop data, a new procedure for closed-loop system identification is developed and demonstrated.

2. Background

The traditional approach to control design is to obtain a nominal model of the plant, $\hat{P}$, which is the basis for control design. Since $\hat{P}$ is an approximation of the true plant, P, the model based compensator, $C_{\hat{P}}$, must provide a certain level of robust stability (i.e., $C_{\hat{P}}$ must internally stabilize $\hat{P}$ and P). In the system identification process an attempt is made to bound the model error $\| P - \hat{P} \|$, which determines the amount of robustness required by the control design. It is also desirable that the controller provide some level of performance robustness; that is the achievable performance should not differ significantly from the nominal design performance. High performance control design for a flexible space structure is especially challenging since high fidelity nominal plant models are difficult to obtain. The large error bounds that result typically require a very robust, low performance control design which must be tuned on orbit to achieve the required performance.

An upper bound on achievable performance is [3], [14]

$$\| J(P, C_{\hat{P}}) \| \leq \| J(\hat{P}, C_{\hat{P}}) \| + \| J(P, C_{\hat{P}}) - J(\hat{P}, C_{\hat{P}}) \| \quad (1)$$

where

$\| J(P, C_{\hat{P}}) \|$: the achieved performance
$\| J(\hat{P}, C_{\hat{P}}) \|$: the nominal performance, and
$\| J(P, C_{\hat{P}}) - J(\hat{P}, C_{\hat{P}}) \|$: the performance differential.

The choice of a specific performance metric J and norm $\| * \|$ is determined by the control design methodology. To achieve high performance requires

- high nominal performance ($\Leftrightarrow \| J(\hat{P}, C_{\hat{P}}) \|$ small)
- robust performance ($\Leftrightarrow \| J(P, C_{\hat{P}}) - J(\hat{P}, C_{\hat{P}}) \| \ll \| J(\hat{P}, C_{\hat{P}}) \|$)

The second condition is actually a requirement on model fidelity and indicates that the nominal closed-loop system model must closely approximate the actual closed-loop system performance when the compensator $C_{\hat{P}}$ is used. Therefore, the "fitness" of the nominal model is a function of the compensator and must be judged from a closed-loop perspective. This fitness is not guaranteed by good open-loop model matching nor is it precluded by poor open-loop model matching [1].

When the model error $\| P - \hat{P} \|$ is large, stability and performance robustness necessitates a low authority controller. Since a low authority controller is not as sensitive to model errors, the performance differential will be small as compared to high authority controllers. However, the performance may not be satisfactory. To achieve high performance, the issues of modeling and control design must be treated as a joint problem. The fitness of the design model $\hat{P}$ is a function of $C_{\hat{P}}$, which is itself a function of the design plant. In some cases, high performance control design requires an iterative closed-loop system identification and control design procedure.

A new closed-loop system identification method is presented in this chapter which is one step of an iterative closed-loop system identification and control design procedure. It is assumed that a moderately accurate dynamic model of the system to be controlled is available for the initial low authority controller design. However, this initial design model is not of sufficient fidelity to permit high authority control design. The objective of the closed-loop system identification procedure is to refine the initial control design model based on closed-loop response data.

In the development of a closed-loop system identification method, consideration must be given to the non-uniqueness of the triple (A, B_2, C_2) in the identified realization. Although there are an infinite number of equivalent state space realizations for a system, a system with n states, nu inputs, and ny outputs can be uniquely expressed with a minimum of $n(nu + ny)$ parameters. Having as the objective of the closed-loop system identification process the ability to refine an existing design model, one approach which circumvents the non-uniqueness problem is to realize the open-loop system matrices in a unique, minimal form and directly identify the canonical parameters from closed-loop response data. Denery has developed a method of parameter estimation for multivariable state space systems from open-loop test data using canonical forms [17]. By utilizing the structure of the closed-loop system matrices, an extension of Denery's algorithm is developed herein to estimate the plant parameters based on closed-loop response data.

Proper selection of the objectives of system identification and control design further stresses the joint nature of the identification and control problem. Based on the prediction error method, the objective of the new closed-loop identification procedure developed in this chapter is to obtain a model $\hat{P}$ that minimizes the performance differential $\| T-\hat{T} \|_2$ where T is the actual closed-loop system and $\hat{T}$ is the identified closed-loop system. This system norm cannot be evaluated since T is not known. However the actual and predicted closed-loop measurements are known and an equivalent objective is to minimize the prediction error of the closed-loop system, $\| y-\hat{y} \|_2$. The actual and predicted closed-loop system outputs, y and $\hat{y}$, respectively, are determined for the same set of inputs. The least squares cost functional for control dependent closed-loop system identification then is

$$J = \frac{1}{2}\int_0^{t_f} (y-\hat{y})^T W (y-\hat{y}) dt \tag{2}$$

where W is a constant matrix chosen to weight the relative importance of different measurement outputs. The control objective is matched with the identification objective by designing the controller to minimize the H_2 criterion $\| \hat{T} \|_2$.

The rest of the paper is structured as follows. First, the algorithm developed by Denery for open-loop system identification procedure is presented in detail. Next, the extension of the canonical system identification algorithm to closed-loop system identification is presented. Finally, an algorithm for iterative closed-loop system identification and control redesign is presented along with illustrative examples.

3. System Identification Algorithm

In Ref. [17], a two-step procedure is given which generates parameter estimates based on noisy measurement data. The algorithm begins with an equation error procedure, which is similar to a linear observer, to generate an initial estimate of the parameters. Noisy measurement data may cause the equation error estimates to be biased, but they are sufficiently accurate to initialize the second step, an iterative quasi-linearization output error procedure. Since the structure of the two procedures are identical with one exception that will pointed out in the following, these two procedures are combined to form an iterative algorithm that is robust to initial parameter estimates and relatively insensitive to measurement noise. First, the details of the equation error procedure will be presented, followed by the output error procedure.

Equation Error Procedure

Consider a model of the state space system to be identified:

$$\begin{aligned} \dot{z} &= \mathcal{F}z + \mathcal{G}u, \quad z(0) = z_0 && (3)\\ \hat{y} &= \mathcal{H}z && (4) \end{aligned}$$

The objective is to identify some $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and z_0 which represents the dynamics of the unknown system based on knowledge of the inputs, $u(t)$, and noisy measurements, $y(t)$. An estimate of the unknown parameters may be obtained by minimizing the cost functional given in Eq. 2. Directly minimizing J results in a nonlinear optimization process, but in the absence of measurement noise, a linear formulation may be obtained. Recognizing that for a perfect model the output in Eq. 4 will exactly equal the measurements and $y - \hat{y} = 0$, this difference can be fed back to the model with arbitrary gains $\mathcal{K}$ and $\mathcal{M}$ according to

$$\begin{aligned} \dot{z} &= \mathcal{F}z + \mathcal{G}u + \mathcal{K}(y - \mathcal{H}z) && (5)\\ \hat{y} &= \mathcal{H}z + \mathcal{M}(y - \mathcal{H}z) && (6) \end{aligned}$$

which can also be written

$$\dot{z} = \mathcal{F}_N z + \mathcal{G}_N u + \delta\mathcal{G}u + \mathcal{K}y \tag{7}$$

$$z(0) = z_{N0} + \delta z_0 \tag{8}$$
$$\hat{y} = \mathcal{H}_N z + \mathcal{M} y \tag{9}$$

by use of the definitions

$$\mathcal{F}_N = \mathcal{F} - \mathcal{K}\mathcal{H} \tag{10}$$
$$\mathcal{G}_N = \mathcal{G} - \delta\mathcal{G} \tag{11}$$
$$\mathcal{H}_N = (I - \mathcal{M})\mathcal{H} \tag{12}$$
$$z_{N0} = z_0 - \delta z_0 \tag{13}$$

The elements of $(\mathcal{F} - \mathcal{K}\mathcal{H})$ and $(I - \mathcal{M}\mathcal{H})$ may be chosen independently of the unknown elements of $\mathcal{F}$ and $\mathcal{H}$ by using a maximum of $n * ny$ parameters in $\mathcal{K}$ and $\mathcal{M}$ when the structure of the system is in a specific form. As a consequence, $\mathcal{F}_N$, $\mathcal{G}_N$, $\mathcal{H}_N$, and z_{N0} are chosen and the unknown parameters are contained in $\mathcal{K}$, $\mathcal{M}$, $\delta\mathcal{G}$, and δz_0.

The structure of the system must be such that the unknown parameters in $\mathcal{F}$ are coefficients of measured states. To obtain this structure, Denery developed a canonical form for multivariable systems which is analogous to a canonical form in Ref. [18] for multi-input systems. Denery's canonical form is called the observer canonical form in Ref. [19], which is dual to the controller canonical form presented in Chapter III. It can be shown that if the plant dimension is an even multiple of the number of outputs and the first n rows of the observability matrix are linearly independent, then the realization is canonical and $\mathcal{H}$ will consist only of ones and zeros. Otherwise, some elements of $\mathcal{H}$ will be included as unknown parameters in the estimation procedure.

Eq. 9 can now be rewritten as

$$\hat{y} = y_N + f(t)\gamma \tag{14}$$

where y_N is the output of the linearized trajectory

$$\dot{z}_N = \mathcal{F}_N z_N + \mathcal{G}_N u, \quad z_N(0) = z_{N0} \tag{15}$$
$$y_N = \mathcal{H}_N z_N \tag{16}$$

The sensitivity matrix, $f(t)$, is given by

$$f(t) = \frac{\partial \hat{y}}{\partial \gamma} \tag{17}$$

where the vector of parameters to be estimated is

$$\gamma = \begin{bmatrix} vec(\mathcal{K}) \\ vec(\delta\mathcal{G}) \\ vec(\mathcal{M}) \\ vec(\delta z_0) \end{bmatrix} \tag{18}$$

Using the expression for $\hat{y}$ from Eq. 14 in the cost functional, Eq. 2, results in J becoming quadratic in the unknown parameters. By differentiating J with respect to the unknown parameters and equating the result to zero, the estimate of γ is given by

$$\hat{\gamma} = \left[\int_0^{t_f} f(t)^T W f(t) dt\right]^{-1} \tag{19}$$
$$\text{x} \left[\int_0^{t_f} f(t)^T W (y - \hat{y}) dt\right] \tag{20}$$

or for discrete measurements

$$\hat{\gamma} = \left[\sum_{i=1}^{N} f(t_i)^T W f(t_i)\right]^{-1} \tag{21}$$
$$\text{x} \left[\sum_{i=1}^{N} f(t_i)^T W (y(t_i) - \hat{y}(t_i))\right] \tag{22}$$

The i^{th} column of $f(t)$ is $\hat{y}_{\gamma_i}(t)$, which is the output of the i^{th} sensitivity equation

$$\dot{z}_{\gamma_i} = \mathcal{F}_N z_{\gamma_i} + \frac{\partial \mathcal{K}}{\partial \gamma_i} y + \frac{\partial(\delta\mathcal{G})}{\partial \gamma_i} u \tag{23}$$
$$z_{\gamma_i}(0) = \frac{\partial(\delta z_0)}{\partial \gamma_i} \tag{24}$$
$$\hat{y}_{\gamma_i} = \mathcal{H}_N z_{\gamma_i} + \frac{\partial \mathcal{M}}{\partial \gamma_i} y \tag{25}$$

From $\hat{\gamma}$, the system matrices are obtained by solving Eqs. 10 - 13 for $\mathcal{H}$, $\mathcal{G}$, $\mathcal{F}$, and z_0. These values are used in $\mathcal{F}_N$, $\mathcal{G}_N$, $\mathcal{H}_N$, and z_{N0} as the initial values for the next iteration.

Output Error Procedure

If the measurement data used in the equation error procedure is corrupted with unbiased measurement noise, a bias error will result in the parameter estimates. This can be circumvented by using an output error procedure which yields unbiased parameter estimates based on unbiased noisy measurements, provided the initial estimates are sufficiently accurate. Typically the biased estimates obtained from the equation error procedure are sufficiently accurate to initialize the output error procedure. Hence the two procedures form a combined algorithm for unbiased parameter estimates based on unbiased noisy measurements.

The output error procedure implements the method of quasi-linearization, which is a well-known approach to minimize Eq. 2 subject to Eqs. 3 - 4. The method of quasi-linearization approximates the response of the system model by a nominal trajectory y_N, based on the initial parameter estimates, plus a linearized correction about the nominal trajectory. By defining the initial estimates $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and z_0 to be $\mathcal{F}_N$, $\mathcal{G}_N$, $\mathcal{H}_N$, and z_{N0}, respectively, $\hat{y}$ may be approximated by $\hat{y}_A$ from

$$\dot{\hat{z}}_A = \mathcal{F}_N \hat{z}_A + [\mathcal{F} - \mathcal{F}_N] z_N + [\mathcal{G}_N + \delta\mathcal{G}] u \tag{26}$$
$$\hat{z}_{A0} = z_{N0} + \delta z_0 \tag{27}$$
$$\hat{y}_A = \mathcal{H}_N \hat{z}_A + [\mathcal{H} - \mathcal{H}_N] z_N \tag{28}$$

where z_N is obtained from

$$\dot{z}_N = \mathcal{F}_N z_N + \mathcal{G}_N u \tag{29}$$
$$z_N(0) = z_{N0}, \quad y_N = \mathcal{H}_N z_N \tag{30}$$

Using the definitions in Eqs. 10 - 11, and recognizing that the initial estimates are sufficiently accurate, implies

$$\begin{aligned} \mathcal{K}\mathcal{H} &= \mathcal{K}[\mathcal{H}_N + \mathcal{M}\mathcal{H}] \approx \mathcal{K}\mathcal{H}_N \quad (31) \\ \mathcal{M}\mathcal{H} &= \mathcal{M}[\mathcal{H}_N + \mathcal{M}\mathcal{H}] \approx \mathcal{M}\mathcal{H}_N \quad (32) \end{aligned}$$

Substituting these expressions in Eqs. 26 - 28 yields

$$\begin{aligned} \dot{\hat{z}}_A &= \mathcal{F}_N \hat{z}_A + \mathcal{G}_N u + \delta\mathcal{G} u + \mathcal{K} y_N, \quad (33) \\ z_{A0} &= z_{N0} + \delta z_0 \quad (34) \\ \hat{y}_A &= \mathcal{H}_N \hat{z}_A + \mathcal{M} y_n \quad (35) \end{aligned}$$

which is identical to Eqs. 7 - 9 except y_N replaces y. Using $\hat{y}_A$ in Eq. 2 instead of $\hat{y}$ reduces the minimization problem to a form identical to the equation error procedure, except y_N is used in the place of y when computing f in the sensitivity equations, Eqs. 23 - 25. In the absence of noise in the measurements, the equation error estimate is the same as the quasi-linearization estimate.

Closed-Loop System Identification Algorithm

Denery's algorithm is extended to closed-loop system identification by expressing the plant in observer canonical form and exploiting the structure of the closed-loop matrices. For the plant given by

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \quad (36) \\ y &= C_2 x \quad (37) \end{aligned}$$

and a dynamic compensator

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c y \quad (38) \\ u &= -C_c x_c \quad (39) \end{aligned}$$

the resulting closed-loop dynamics are

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} &= \begin{bmatrix} A & -B_2 C_c \\ B_c C_2 & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} \\ &\quad + \begin{bmatrix} B_1 & B_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \quad (40) \\ y &= \begin{bmatrix} C_2 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} \quad (41) \end{aligned}$$

As with the open loop algorithm, noisy measurements are accounted for by averaging over multiple data sets.

If the plant (A, B_2, C_2) matrices are expressed in observer form, then the C_2 matrix consists of ones and zeros and the unknown parameters in A are coefficients of the measured transformed states. (As stated earlier, in some cases the transformation may not be canonical resulting in the C_2 matrix having additional nonzero elements, but these parameters can be estimated as well.) It is assumed that the compensator state vector time history is recorded. Comparison of Eqs. 3 - 4 with Eqs. 40 - 41 indicates that

$$\mathcal{F} = \begin{bmatrix} A & -B_2 C_c \\ B_c C_2 & A_c \end{bmatrix}, \quad \mathcal{G} = \begin{bmatrix} B_1 & B_2 \\ 0 & 0 \end{bmatrix} \quad (42)$$

$$\mathcal{H} = \begin{bmatrix} C_2 & 0 \\ 0 & I \end{bmatrix} \quad (43)$$

Since the only unknowns in $\mathcal{F}$ and $\mathcal{G}$ are the plant matrices A, B_1, and B_2, the unknown parameter matrices are defined as

$$\begin{aligned} \mathcal{K} &= \begin{bmatrix} \mathcal{K}_{11} & -\mathcal{K}_{12} C_c \\ 0 & 0 \end{bmatrix} \quad (44) \\ \delta\mathcal{G} &= \begin{bmatrix} \delta\mathcal{G}_{11} & \delta\mathcal{G}_{12} \\ 0 & 0 \end{bmatrix} \quad (45) \end{aligned}$$

In the case of a true canonical form for the plant, $\mathcal{H}$ is completely known. Note that $\mathcal{K}_{11}$ corresponds to the unknown parameters in A, $\mathcal{K}_{12}$ corresponds to the unknown parameters in B_2, $\delta\mathcal{G}_{11}$ corresponds to the unknown parameters in B_1, and $\delta\mathcal{G}_{12} = \mathcal{K}_{12}$.

For closed-loop system identification, the partial derivative expressions in the sensitivity equations, Eqs. 23 - 25, are modified as follows and the unknown parameter vector is defined as

$$\gamma = \begin{bmatrix} vec(\mathcal{K}_{11}) \\ vec(\mathcal{K}_{12}) \\ vec(\delta\mathcal{G}_{11}) \\ vec(\delta z_0) \end{bmatrix} \quad (46)$$

For γ_i corresponding to the (j,k) element of $\mathcal{K}_{11}$,

$$\frac{\partial\mathcal{K}}{\partial\gamma_i} = e_j e_k^T \quad (47)$$

and

$$\frac{\partial\delta\mathcal{G}}{\partial\gamma_i} = 0, \ \frac{\partial\mathcal{M}}{\partial\gamma_i} = 0, \ \frac{\partial\delta z_0}{\partial\gamma_i} = 0 \quad (48)$$

where $e_j e_k^T$ is a matrix of zeros except for a 1 in the (j,k) element. For γ_i corresponding to the (j,k) element of $\mathcal{K}_{12}$,

$$\frac{\partial\mathcal{K}}{\partial\gamma_i} = \begin{bmatrix} 0 & \frac{\partial(-\mathcal{K}_{12}C_c)}{\partial\gamma_i} \\ 0 & 0 \end{bmatrix} \quad (49)$$

where

$$\frac{\partial\mathcal{K}_{12}C_c}{\partial\gamma_i} = -e_j e_k^T C_c \quad (50)$$

which is an $nxnc$ matrix of zeros except for the j^{th} row which is comprised of the k^{th} row of C_c. Since $\delta\mathcal{G}_{12} = \mathcal{K}_{12}$,

$$\frac{\partial\delta\mathcal{G}}{\partial\gamma_i} = \begin{bmatrix} 0 & \frac{\partial\delta\mathcal{G}_{12}}{\partial\gamma_i} \\ 0 & 0 \end{bmatrix} \quad (51)$$

where $\frac{\partial\delta\mathcal{G}_{12}}{\partial\gamma_i} = \frac{\partial\mathcal{K}_{12}}{\partial\gamma_i}$ is zero except for a one in the (j,k) element. The terms $\frac{\partial\mathcal{M}}{\partial\gamma_i}$ and $\frac{\partial\delta z_0}{\partial\gamma_i}$ are identically zero. For γ_i corresponding to the (j,k) element of $\delta\mathcal{G}_{11}$,

$$\frac{\partial\delta\mathcal{G}}{\partial\gamma_i} = \begin{bmatrix} \frac{\partial\delta\mathcal{G}_{11}}{\partial\gamma_i} & 0 \\ 0 & 0 \end{bmatrix} \quad (52)$$

where $\frac{\partial\delta\mathcal{G}_{11}}{\partial\gamma_i}$ is zero except for a one in the (j,k) element and all others are identically zero. Similarly, for γ_i

corresponding to the j^{th} element of δz_0, $\frac{\partial \delta z_0}{\partial \gamma_i}$ is a zero vector with a one in the j^{th} element and all others are identically zero.

Note that this algorithm is not guaranteed to converge. Since the estimates are determined by minimizing the error in the closed loop time response and not the error in the open loop plant parameter estimates, the plant parameter estimates may not converge to the "true" plant parameters but still provide a good control design model.

4. Iterative Control Redesign Examples

The iterative closed-loop system identification and control design procedure implemented herein is patterned after the approach of Ref. [3] with one notable exception to be pointed out below.

Iterative Closed-Loop Identification and Control Redesign Algorithm:

1. Beginning with model $\hat{P}_i$, design a set of H_2 controllers of varying control authority.
2. Evaluate actual and estimated output and control costs and performance differential.
3. Determine highest performance control design point which satisfies performance differential constraint threshold, denoted $C_{\hat{P},i}$.
4. Using closed-loop data from $T(P, C_{\hat{P},i})$ and $\hat{P}_i$, do closed-loop system identification to determine $\hat{P}_{i+1}$.
5. Repeat until desired performance is attained.

This algorithm differs from the framework presented in Ref. [3] in that the amount of control authority increase between iterations is more formally quantified. Recognizing that small changes in controller authority tend to result in small changes in performance, a constant scaling factor was used in the control design step in Ref. [3] which was slightly increased each identification/control design iteration. Thus the control authority was gradually increased each iteration until an appropriate model and high authority control design was achieved. In the procedure introduced above, the performance is evaluated for a set of controllers with varying authority to ascertain the onset of performance differential due to model mismatch. The output and control costs for performance assessment are evaluated from the mean-square closed-loop output response to white noise inputs and the mean-square control signal, respectively.

Instead of numerous iterations of identification and control design, the emphasis is placed on evaluating a set of controllers designed for a common model. By explicitly evaluating the performance differential for each controller, larger steps in control authority may be taken with each iteration resulting in fewer identification/control design iterations. Although the iterative procedure is not guaranteed to converge, the convergence may be checked at each iteration by evaluating the performance at each iteration.

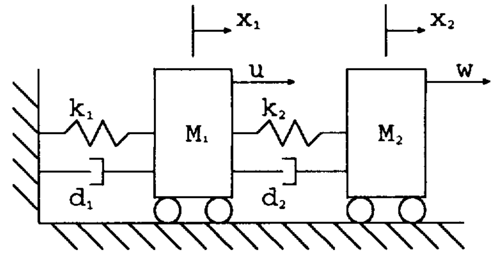

Figure 1: Coupled Mass Benchmark Problem

Coupled Mass Example

As an example of the iterative identification and control procedure, the coupled two mass problem illustrated in Fig. 1 is used. This example problem highlights robust control issues as related to flexible space structures and was used as a benchmark problem in Ref. [20] (with $k_1 = 0$ and $d_1 = 0$). A disturbance acts on mass two and the control force is applied to the first mass. The coefficient matrices are:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{d_1+d_2}{m_1} & \frac{d_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{d_2}{m_2} & -\frac{d_2}{m_2} \end{bmatrix} \quad (53)$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ 0 \end{bmatrix} \quad (54)$$

The state vector is $\bar{x} = [x_1,\ x_2,\ \dot{x}_1,\ \dot{x}_2]^T$ and the measurements are $y = [x_1,\ x_2]^T$, so

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (55)$$

Two cases will be considered, the first being open-loop stable and the second case having a rigid body mode. The stable system is described by $k_1 = k_2 = 1.2$, $m_1 = m_2 = 1.5$, $\zeta_1 = \zeta_2 = 0.1$ and the damping constant is computed by $d_i = 2\zeta_i\sqrt{k_i/m_i}$.

The procedure begins with an initial plant for control design. As an extreme case, the initial plant is obtained by adding 50% error to k_1, k_2, ζ_1, and ζ_2 and 5% error for each of the two masses. After transforming the true $(A, [B_1\ B_2], C_2)$ triple to observer canonical form, the resulting realization is

$$At = \begin{bmatrix} 0 & -0.8000 & 0 & 0.8000 \\ 1.0000 & -0.1789 & 0 & 0.1789 \\ 0 & 0.8000 & 0 & -1.6000 \\ 0 & 0.1789 & 1.0000 & -0.3578 \end{bmatrix} \quad (56)$$

$$[B1t \ \ B2t] = \begin{bmatrix} 0.6667 & 0 \\ 0 & 0 \\ 0 & 0.6667 \\ 0 & 0 \end{bmatrix}, \quad (57)$$

$$C20 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (58)$$

and the corresponding initial triple in observer canonical form is

$$A0 = \begin{bmatrix} 0 & -1.1429 & 0 & 1.1429 \\ 1.0000 & -0.3207 & 0 & 0.3207 \\ 0 & 1.1429 & 0 & -2.2857 \\ 0 & 0.3207 & 1.0000 & -0.6414 \end{bmatrix} \quad (59)$$

$$[B10 \ \ B20] = \begin{bmatrix} 0.6349 & 0 \\ 0 & 0 \\ 0 & 0.6349 \\ 0 & 0 \end{bmatrix} \quad (60)$$

In observer canonical form, the C2 matrix is fixed for a given set of observability indices and the columns of the A matrix with free elements corresponds to the columns of C2 that have an element equal to one. Note that the resulting initial design plant elements varied by 79.28% and 42.86% in the A matrix, and 4.76% in the B_1 and B_2 matrices from the truth model.

A set of LQG controllers of varying authority were designed for the initial design plant using the weighting matrices

$$W = \begin{bmatrix} I_n & 0 \\ 0 & \rho I_{nu} \end{bmatrix}, \quad V = \begin{bmatrix} I_n & 0 \\ 0 & I_{ny} \end{bmatrix}, \quad (61)$$

where ρ is used to vary the control authority. The performance of this set of controllers was then evaluated with both the design model and the truth model to assess the performance differential that results from the initial erroneous model. Recall from the beginning of this chapter that the performance differential is a measure of performance robustness. Fig. 2 indicates a large performance difference at all control authority levels, so a controller with a moderate authority level ($\rho = 5$) is chosen for initial implementation.

Using the LQG controller designed for $\rho = 5$ with the initial design model, the closed-loop is excited with unit intensity, zero mean random noise low pass filtered at 25 Hz. The closed-loop measurements are corrupted with a low intensity random measurement noise (the standard deviation of the noise was equal to 20% of the standard deviation of the measurements). In Ref. [17], measurement noise is accounted for by averaging over multiple experiment sets. In this example, five sets of measurements are used and the five resulting sets of estimated system matrices are averaged.

Table 1 gives the initial, actual, and estimated parameters of the system matrices in observer canonical form. The significant error is clear as well as the convergence of the parameter estimates after 50 iterations. As with

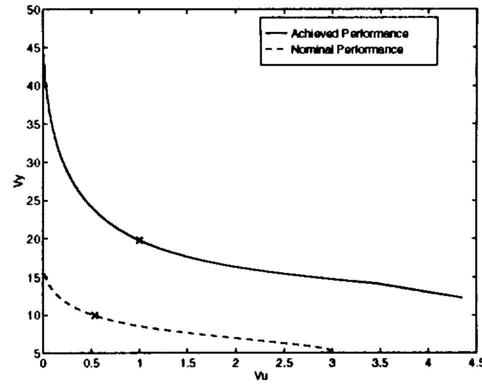

Figure 2: Performance Differential Using Initial Plant: Coupled Mass Problem

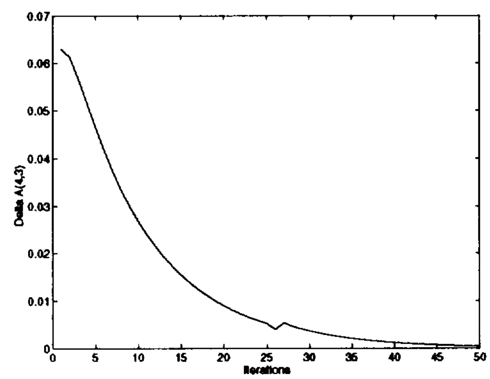

Figure 3: Convergence of Δ A(4,3)

the combined open-loop algorithm, the first 25 iterations used the equation error method and the second 25 iterations used the output error method. The convergence over 50 iterations for the correction to the $A(4,3)$ parameter is shown in Fig. 3. This parameter corresponds to the largest element of γ (required the largest correction) at the first correction iteration. A slight discontinuity is evident at the 25^{th} iteration when the algorithm switched from the equation error method to the output error method. However, this is removed after one iteration.

Having refined the initial design model to obtain a more accurate model, a second set of LQG controllers is designed and the performance differential evaluated. Fig. 4 shows that the gap between design performance and achieved performance is considerably decreased at all authority levels when compared to Fig. 2. The identified model results in robust performance (as defined at the beginning of this chapter in regard to performance differential) and good nominal performance. It bears pointing out that when the identification experiment was conducted without measurement noise, the achieved performance and design performance curves were indistinguishable, indicating that the difference in Fig. 4 is due to measurement noise. More averages and more iterations could possibly further diminish the per-

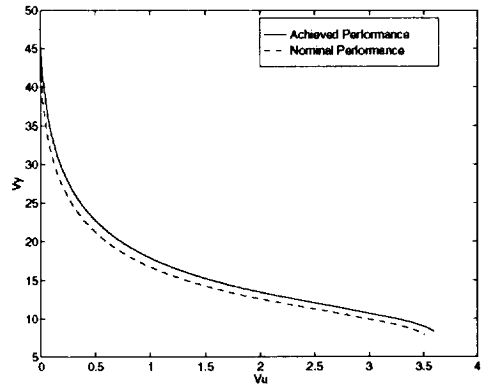

Figure 4: Performance Differential Using Estimated Plant: Coupled Mass Problem

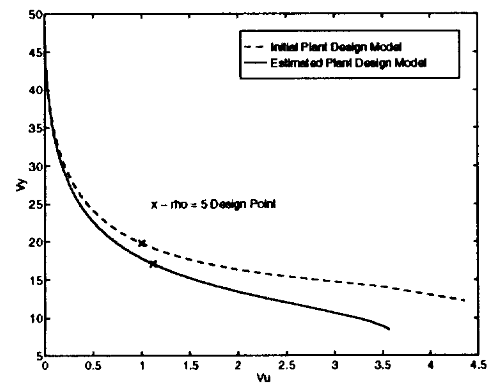

Figure 5: Comparison of Achieved Performance: Coupled Mass Problem

formance differential. Another perspective is to compare the achieved performance for a set of LQG designs based on the initial plant model with the achieved performance for the designs based on the estimated model, shown in Fig. 5. The difference between the dashed line and the solid line indicates the performance increase attained by performing closed-loop system identification and controller redesign. Although the amount of performance sacrificed by designing the controller based on an initial open-loop design model depends on the specific plant and control design, Fig. 5 illustrates the fact that model error limits achievable performance and better performance can be obtained by refining the model to reduce the error. The design point corresponding to $\rho = 5$ which was used in the closed-loop system identification is indicated by 'x' on Fig. 5.

A more difficult identification problem is obtained by removing the spring and damper denoted k_1 and d_1 on Fig. 1 resulting in an unstable rigid body mode in the open-loop plant. Identification of open-loop unstable systems (such as spacecraft) is a difficult task which is a further motivation for closed-loop system identification. Using the same initial stiffness and damping error, Table 2 indicates the convergence of the estimated system parameters after 50 iterations. Again, the measurements were noise corrupted and the estimates were averaged after five identification experiments. Similar convergence of the parameter estimates was observed for this case as for the open-loop stable example.

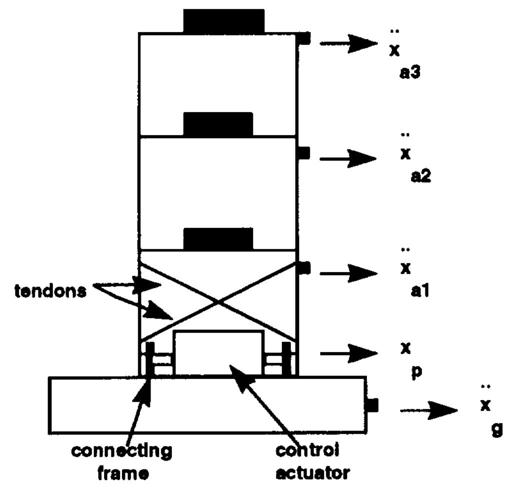

Figure 6: Structural Control Experiment

Building Control Example

A second example is derived from a benchmark problem in vibration control of a building subject to an earthquake excitation [21]. The problem is based on an experimental model of a three-story tendon controlled structure at the National Center for Earthquake Engineering Research [22] is depicted in Fig. 6.

A 20 state model is provided in Ref. [22] that was obtained by system identification experiments on the laboratory structure. A six state nominal control design model was obtained by balancing and residualizing the 20 state model, retaining modes at 2.268, 7.332, and 12.240 Hz. Inputs to the six state nominal design model are the ground acceleration and control force and the outputs are the relative displacements of the three floors, which results in 30 parameters to be estimated. An initial erroneous design model is obtained by adding 5% error to the natural frequency square terms and the B_1 and B_2 matrices. Although only 5% error is introduced, the maximum errors in the elements of the A, B_1, and B_2 matrices (in observer canonical form) are 72.2%, 9.9%, and 39.6%, respectively.

Using the initial model, a set of LQG controllers is designed using ρ to vary the control authority. For this example, filtered noise is used as input excitation and perfect measurements are assumed. Fig. 7 indicates the performance differential resulting from controllers designed for the initial model, which is relatively constant at all

Table 1: Comparison of Initial, Actual, and Estimated Parameters For Open-Loop Stable Coupled Mass Problem

Initial Parameters	True Parameters	Estimated Parameters
-1.1429	-0.8000	-0.8240
-0.3207	-0.1789	-0.1832
1.1429	0.8000	0.8252
0.3207	0.1789	0.1993
1.1429	0.8000	0.8219
0.3207	0.1789	0.1720
-2.2857	-1.6000	-1.6531
-0.6414	-0.3578	-0.3815
0.6349	0.6667	0.6672
0	0	-0.0064
0	0	-0.0027
0	0	0.0016
0	0	-0.0004
0	0	0.0113
0.6349	0.6667	0.6733
0	0	0.0039

Table 2: Comparison of Initial, Actual, and Estimated Parameters For Open-Loop Unstable Example

Initial Parameters	True Parameters	Estimated Parameters
-1.1429	-0.8000	-0.8228
-0.3207	-0.1789	-0.1830
1.1429	0.8000	0.8253
0.3207	0.1789	0.1937
1.1429	0.8000	0.8228
0.3207	0.1789	0.1835
-1.1429	-0.8000	-0.8231
-0.3207	-0.1789	-0.1860
0.6349	0.6667	0.6637
0	0	0.0009
0	0	-0.0011
0	0	-0.0016
0	0	0.0016
0	0	0.0034
0.6349	0.6667	0.6678
0	0	0.0045

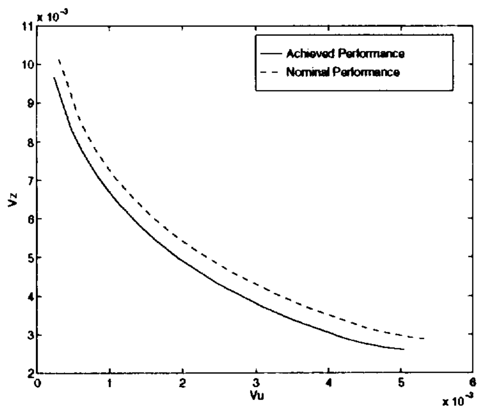

Figure 7: Performance Differential Using Initial Plant: Building Problem

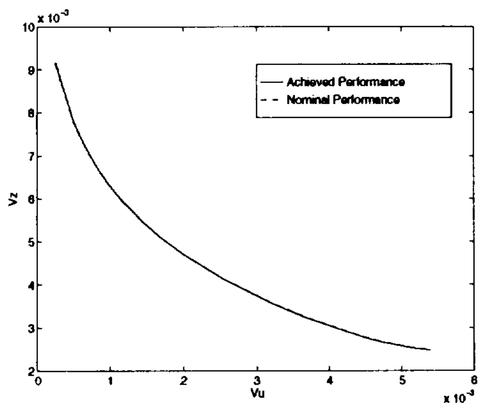

Figure 8: Performance Differential Using Estimated Plant: Building Problem

control authority levels. A low authority controller is used for closed-loop system identification which results in an estimated plant with the performance differential shown in Fig. 8. For this estimated model there is virtually no performance differential.

Discussion

Since the end objective is an iterative system identification and model-based control design procedure, additional constraints are placed on the input and output processes. The system identification is based on closed-loop test data which mandates that the generalized plant for control design consist of actuated inputs and measured outputs only. Fig. 9 illustrates this requirement where the disturbance inputs, w and w_p, act through the control input and sensor channels and the performance variables, z and z_p, must be linear combinations of the sensed variables, y, and the control inputs, u. Hence a constraint is placed on the generalized plant formulation by the system identification such that the columns of B_1 and B_p lie in the column space of B_2 and the rows of C_1 and C_p lie in the row space of C_2. With regard to the H_∞ subproblem, this requires the uncertainty representation to consist only of (input or output) multiplicative and additive uncertainty models, which implies that it is not possible, for example, to include parametric uncertainty such as mass or stiffness uncertainty in the generalized plant formulation. The matrices D_{21}, D_{2p}, D_{1p}, and D_{12} are obtained from the input/output uncertainty models and disturbances, and do not contain parameters to be estimated. Consequently, the plant matrices A, B_2, and C_2 are the only matrices to be estimated by closed-loop system identification. To relax this constraint would require introducing additional actuators and sensors for the sole purpose of system identification.

A brief discussion on the closed-loop system identification algorithm from a numerical implementation perspective is warranted. The stability of the algorithm is sensitive to several factors. The primary factor influencing convergence of the parameter estimation is the matrix inversion in the computation of $\hat{\gamma}$ (Eqs. 20 or 22). For this inverse to exist, the $(ny + nc)\mathrm{x}N_p$ sensitivity matrix f must have full row rank where $N_p = n * (nu + ny + nw)$ is the number of unknown parameters. Recall that the columns of f are time histories of the sensitivity equations, Eqs. 23 - 25, which include the compensator state time histories. If the input is not sufficiently rich to excite the measurement and compensator states, then f will not have full rank. This difficulty is compounded as the number of parameters increases. If the matrix tends to singularity, the magnitude of the elements of $\hat{\gamma}$ diverge. In order to alleviate the divergence of $\hat{\gamma}$ in the examples above, a relaxation factor was introduced that scaled $\hat{\gamma}$. Scaling $\hat{\gamma}$ by a relaxation factor of 0.5 typically was sufficient to produce smooth convergence as seen in Fig. 3. Without the relaxation factor, the estimates would overshoot and overcorrect, resulting in divergence of the parameter estimates. The relaxation factor in essence damped the overshoot of the correction steps at each iteration. This could possibly have been accomplished by using a pseudo-inverse of the matrix to zero the small singular values, but that would have introduced error. Using the relaxation factor did not introduce error but only slowed the convergence.

5. Conclusions

This paper has shown that to achieve high performance control often requires reducing model error through system identification. In many cases open-loop testing is not possible and even when it is, often the most appropriate model for control design is obtained from closed-loop response data. A new method for refining a control design model from closed-loop response data is presented herein. Based on a prediction error method, the open-loop plant parameters are estimated in a canonical form. Examples have shown that higher performance can be obtained when the controller is redesigned based on the refined model.

A major shortcoming of the closed-loop system identifi-

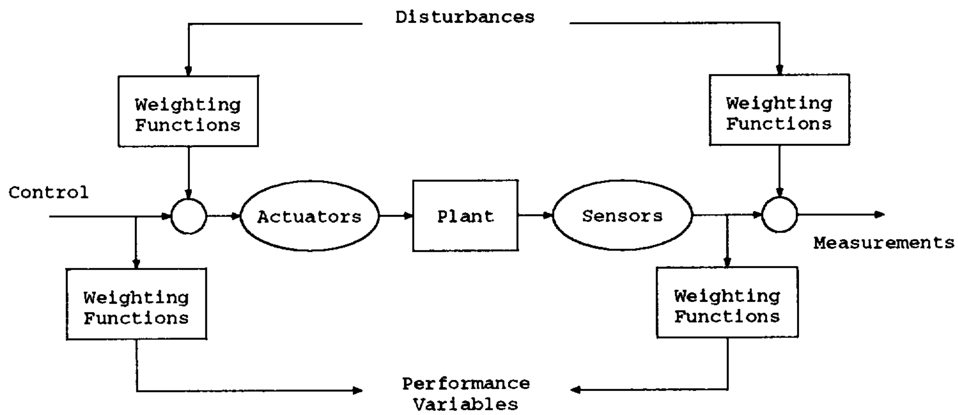

Figure 9: Input/Output Constraint Relationships

cation procedure is numerical sensitivity. The solution procedure presented herein requires the inversion of a large data matrix which tends to be ill-conditioned. Ensuring full-rank of the data matrix requires sufficiently rich excitation in all of the modes to be identified which is often quite difficult in practice due to limitations such as sensor and actuator dynamics. Methods which do not involve matrix inversion such as genetic algorithms have potential for the closed-loop parameter estimation. An additional benefit of genetic algorithms is that the system identification can be operating in the background as part of an autonomous identification/controller tuning process. Future research should be conducted to that end. Finally, the measurement noise properties (intensity, frequency spectrum, etc.) should be explicitly accounted for in the parameter estimation instead of the ad hoc use of ensemble averaging.

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