NASA/CR–1998–206877

# The Development of a Degree 360 Expansion of the Dynamic Ocean Topography of the POCM_4B Global Circulation Model

*Richard H. Rapp*
*Department of Civil and Environmental Engineering and Geodetic Science*
*The Ohio State University, Columbus, Ohio*

NAS5–32352

National Aeronautics and
Space Administration

**Goddard Space Flight Center**
Greenbelt, Maryland 20771

September 1998

**Acknowledgments**

This report was prepared to help provide documentation of some results described in a presentation by Rapp and Zhang at the 1996 Spring AGU meeting. Additional results are also described in this report.

The material presented here is based in part on material prepared by Changyou Zhang. This material was part of the documentation for the research reported at the AGU meeting. Without this material this report could not have been prepared. In addition, Dr. Zhang provided information on calculation and programs in response to questions posed by R. Rapp.

The preparation of this document was supported by NASA (Contract NAS5-32352) through Raytheon STX.

Available from:

NASA Center for AeroSpace Information
7121 Standard Drive
Hanover, MD 21076–1320
Price Code: A17

National Technical Information Service
5285 Port Royal Road
Springfield, VA 22161
Price Code: A10

# TABLE OF CONTENTS



# LIST OF TABLES



# LIST OF FIGURES

## LIST OF FIGURES (continued)

## Abstract

The paper documents the development of a degree 360 expansion of the dynamic ocean topography (DOT) of the POCM_4B ocean circulation model. Most computations were originally carried out in 1996 for a presentation by Rapp and Zhang at the Spring Meeting of the American Geophysical Union. The paper describes the principles and software used leading to the final 360 model. A key principle was the development of interpolated DOT values into land areas to avoid discontinuities at or near the land/ocean interface. The power spectrum of the POCM_4B is also presented with comparisons made between orthonormal (ON) and spherical harmonic (SH) magnitudes to degree 24. A merged file of ON and SH computed degree variances is proposed for applications where the DOT power spectrum from low to high (360) degrees is needed.

## Introduction

A component of the POCM_4B global circulation model (Stammer et al, 1996) is the dynamic ocean topography (DOT). The DOT estimates have been used (e.g. Rapp, Zhang, Yi, 1996) in comparison with DOT determination from TOPEX altimeter data and geoid undulation values implied by various geopotential models. The primary mode of comparison has been through orthonormal (ON) expansions valid for a specific ocean domain of interest. The determination of the ON coefficients of the DOT is done by first estimating, through a least squares adjustment, a low degree (e.g. 24) spherical harmonic coefficient set, which is then transformed to the ON coefficient sets. For applications requiring greater DOT resolutions a higher degree expansion is desirable. Such expansions can be created by quadrature procedures assuming DOT values are defined in a global sense if spherical harmonic coefficients are to be determined. Techniques used for the determination of high degree (e.g. 360) expansions of the geopotential can be also be applied to the DOT values with special consideration being given to the definition of DOT values in areas (e.g. land) in which the DOT values are not defined. Rapp and Zhang (1996) have presented a degree 360 expansion of the POCM_4B model. The purpose of this note is to document aspects of the development based on notes prepared by C. Zhang in February–May 1996, and by R. H. Rapp in February 1996, and to add some additional comments and results.

## The POCM_4B DOT Values

The DOT values were originally given on a grid with a longitude spacing of 0.4° and a latitude ($\phi$) spacing of 0.4° cos $\phi$. There were 902 longitude values and 505 latitude values with latitude extremes of 74.97° S to 64.85° N. The DOT values represented a mean set based on the averaging of 3 day instantaneous files for the 2 year time period of January 1, 1993 to December 31, 1994.

## The Low Degree Spherical Harmonic Coefficient Estimations

A description of the determination of the spherical harmonic coefficient values to degree 24 is given in Rapp, Zhang, Yi (1996, p. 22, 580). Specific items to note are:

1. The zero degree term of the DOT was forced to be zero through a priori weighting of this coefficient.

2. Priori A coefficient weights in the adjustment were based on DOT degree variances computed from TOPEX altimeter data (cycles 9 to 82) and the geoid evaluation of the JGM-3 geopotential model. The degree variance values are given in Table 2 of the Rapp, Zhang, Yi (1996) paper.

3. The DOT values of POCM_4B were defined to be zero in locations where another DOT model (POP 96) was not defined between 60° S and 75° N. These values were down weighted in the least squares adjustment by a factor of 4 with respect to the given values of the POCM_4B DOT. This procedure helped constrain the DOT values in the areas where such values did not exist without creating significant DOT gradients near the land/ocean interface.

The spherical harmonic coefficients and their standard deviations were estimated on the CRAY using the program (ZHANG account) /a/osu1615/SC/sstadj.var2.cosw. This program is also found on the mainframe as RHRAPP.ZHANGC.SSTADJ.VAR2.COSW. The input data was placed on a temporary CRAY file /osu1615/SC/sc.sst.std.dot. This file contains the latitude, longitude, DOT, and DOT standard deviation as formatted data. This file may have been created by the CRAY program stored at: /a/osu1615/SC/read_two year.f (main frame: RHRAPP.ZHAN GC.READ.TWOYEAR.F). Alternately, the file may have been created by a program on the Silicon Graphs (czhang account, in directory SC) designated read.f. The coefficients and their standard deviations were initially placed on a CRAY file and then ftped to the main frame: ZHANGC.SC.TWOYEAR.VAR2.COSW2.ZEROLA. The spherical harmonic coefficients were converted to ON coefficients using software described elsewhere. The mainframe file containing the spherical harmonic and ON coefficients is: ZHANGC.ORTH.SC.TWOYEAR.VAR2. COSW2.ZEROLA, an ascii file.

**The High Degree Spherical Harmonic Coefficient Estimation**

Considerable information is available in the POCM_4B gridded data set that is not represented by a degree 24 expansion. To represent this information, spherical harmonic expansion software used for degree 360 expansions of the geopotential could be used provided a global set of DOT values could be defined. For the 360 expansion a global set of 30'x30' mean values were to be determined. The complication in doing this is that the DOT values are only defined in (most) ocean areas. Values for other areas (primarily land) were needed. Although one might set land and other undefined areas to be zero, the consequences of such a procedure are undesirable because of the discontinuity that would be introduced at the land/ocean interface. The degree 360 expansion of such a data set would imply DOT values with high (and unrealistic) gradients at the interface. In addition, the power spectrum of the DOT representation could be in error because of the artificial high frequency information introduced when values on land are simply set to zero. Consequently, an alternate procedure was sought for the definition of land values of DOT.

The initial step in the definition of a global set of 30' mean DOT values was the calculation of such values based on non zero values of the original POCM_4B grid. This non zero value POCM4B data set is called ZHANGC.POCM4B. It contains 316,512 values. An average value of the non-zero DOT values was computed to define a 30' value using program ZHANGC. GRID.POCM4B. As an initial test all undefined 30' value were set to zero and the resultant data set expanded into a degree 360 representation using mainframe program ZHANGC.GCM (F419) (also see ZHANGC.F419.POCM4B or ZHANGC.AGU.F419). When a plot of the DOT in the Gulf Stream region (25° N to 50° N, 275° to 310°) was made, significant DOT gradients were seen at the coastlines as would be expected.

The next step was the creation of a global (30') set of DOT values from a low (10) degree expansion of the DOT from the Los Alamos National Laboratory. The LA95 model was a precursor to the model (POP, 96) described in Rapp, Zhang and Yi (1996, p.22, 586). The spherical harmonic coefficients of the LA95 model were estimated to degree 24 using a least squares adjustment procedure. The procedure used was essentially the same as that described for the POPCM_4B model. The LA95 point data set was: ZHANGC.POPCM5.

This data set was ftped to the CRAY and used to determine the spherical harmonic coefficients to degree 24 using the following program: /a/osu1615/SC/popcm5.f (RHRAPP.ZHANGC.POPCM5.F). The output file from the CRAY run (stored in /a/osu1615/SC/popcm5.o32001) was ftped to the mainframe and stored as ZHANGC.POPCM5.VAR2.COSW2.ZERO1. This SH coefficient file was then used in program ZHANGC.SST.F388.LA, with NMAX=10, to generate a global set of DOT values on a 0.5°x0.5° grid. The data set name of this binary file was ZHANG. LA.LAND.T010.

The next test was made by calculating 30' mean values in areas previously set to zero using the DOT values of the LA95 (or POPCM5) model to degree 10. The hope here was that the introduction of a long wavelength DOT model, for zero values of the POCM_4B model, would reduce the land/ocean discontinuity. A degree 360 expansion of the new file was made and DOT values plotted (Figure 1). Clearly, the land/ocean large gradients remained. Experiments were then started to find a way to reduce the large gradients in the DOT near the shoreline that were implied by a degree 360 model.

One experiment was tried interpolating "real" DOT values from POCM_4B into 30' cells that had no given value. The interpolation procedure used known cells within a specified latitude and longitude range of the cell whose value was being estimated. The weight used was 1/distance. (Program RHRAPP.ZHANGC.CRAY.MODEL.SST.GRID.DIS.F which came from the CRAY account of Zhang (/a/osu1615/model.sst/grid/dis.f)). The "search radius" or delta latitude or delta longitude value was set, in this test, to 3°. This new data set of 30' values would still have zero values that were filled in with values from a degree 10 expansion (DOT data set ZHANGC.LA.LAND.T010). The full set of DOT values was then expanded into a degree 360 expansion using ZHANGC.F419.POCM4B (This basic program is also found in ZHANGC.GCM (F419)). Figure 2 shows the DOT in the Gulf Stream region based on this expansion (ZHANGC.POCM 4B.COEF.TODEG360.DIS3D). One notes that the DOT gradient has been moved inland leaving no gradient near the shoreline—a desirable factor. However, the significant gradients were not desirable so another weighting procedure was considered in the next experiment.

In the next procedure an interpolation was carried out using an exponential decay function which was dependent on distance in the following form:

$$z(\phi',\lambda') = \frac{\sum_{i=1}^{n} z_i(\phi',\lambda') e^{-d_i/D}}{n} \tag{1}$$

where:

| | |
|---|---|
| z | is the interpolated 30' x 30' DOT value: |
| $\varphi',\lambda'$ | are coordinates of the 30' cell whose DOT value is being estimated; |
| $z_i(\varphi,\lambda)$ | are the non-zero DOT values within a specified $\Delta\lambda$ from the cell whose DOT is being determined. |
| n | is the number of non-zero 30' cells being used; |
| D | is a scaling factor that determines the rate of decay |

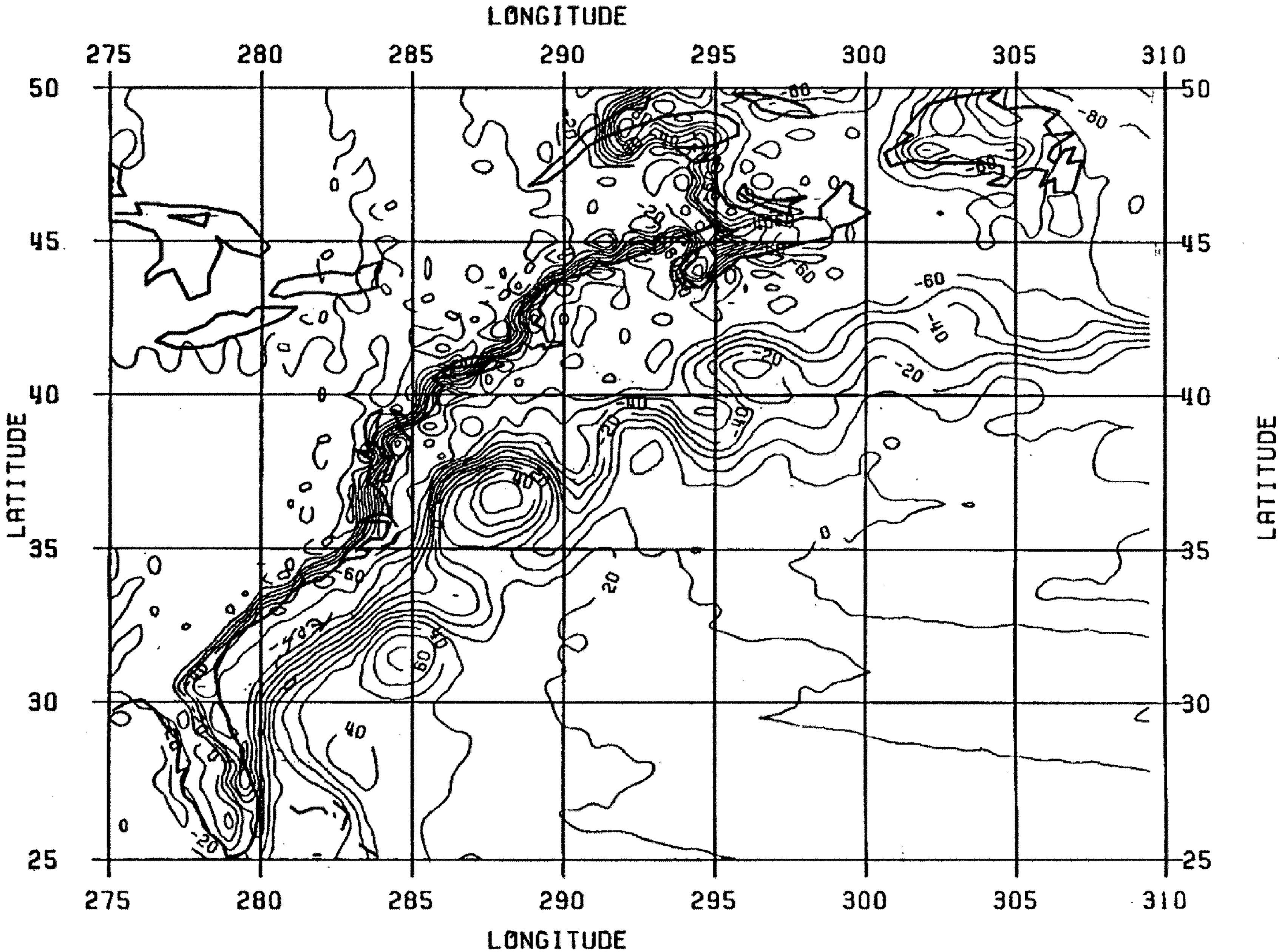


Figure 1. Dynamic Ocean Topography in the Gulf Stream Region Based on a Degree 360 Expansion of POCM_4B Using a Degree 10 Model in Undefined 30' Cells (coefficients: ZHANGC.POCM4B.COEF10.TODEG360).

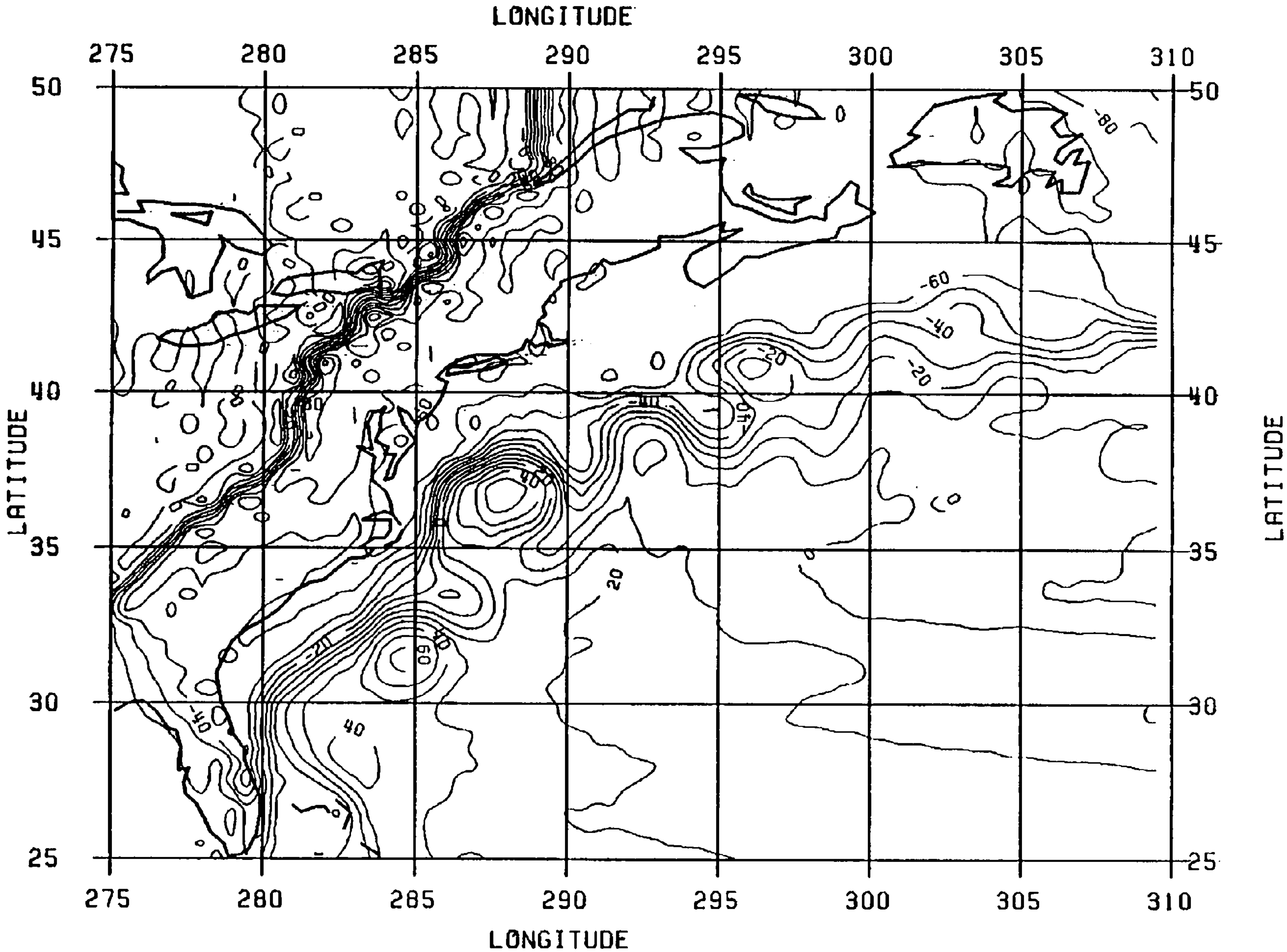


Figure 2. Dynamic Ocean Topography in the Gulf Stream Region Based on a Degree 360 Expansion of POCM_4B Using an Inverse Distance Interpolation with Search Increments of 3° and a Degree 10 Model for Remaining 30' Cells (coefficients: ZHANGC.POCM4B.COEFF.TODEG360.DIS3D).

The idea behind this equation was to have an interpolator that "slowly" let the interpolated values decay from the known values to avoid the steep gradients created when going from the known value to near zero. The larger the D value, the slower would be the decay. Tests were carried out with the following D values: 350 km, 450 km and 750 km.

In one test the exponential interpolator, with D=750 km was applied to the data set creat-ed by the 1/d weighting ($\Delta\varphi$, $\Delta\lambda$=3°). The $\Delta\varphi$, $\Delta\lambda$ increments in the application of the exponential interpolator were also set to 3°. (See RHRAPP.ZHANGC.CRAY.MODEL.SST.GRID.EX P.F). The new 30' values were supplemented by the degree 10 solution where 30' cells remained zero. The global data set was then expanded into a degree 360 model (ZHANGC.POCM4B.CO EF.TODEG360.DIS3D.EXP750). Figure 3 shows the DOT values generated form this expansion. Comparing Figure 3 to Figure 2, one sees a shift and a decrease of the gradient regions in the land area. For example, the gradient pattern at $\varphi$=45°, $\lambda$=286° in Figure 2 has shifted to $\lambda$=280°. In addition the gradient has decreased.

In the final test described here the exponential function with D=350 km and latitude/longitude limits of 1.5° was applied to the previous data set calculated with D=750 km. The program used for this calculation was found on the CRAY as /a/osu1615/model.sst/grid.exp.h.f The program was retained on the mainframe as RHRAPP.ZHANGC.CRAY.MODEL. SST.GRID.EXP.H.F.

A version of F419 (ZHANGC.AGU.F419 or ZHANGC.F419.POCM4B) was then used to calculate the degree 360 expansion, creating the data set called ZHANGC.POCM4B.COEF. TODEG360.FINAL123. DOT values still zero were replaced by the values from the degree 10 expansion (ZHANG.LA.LAND.T010). The expansion was created on or near 2/27/96. The DOT in the Gulf Stream region from this model is given in Figure 4. Comparing the land gradient pattern in the Great Lakes area, one sees that it has been shifted west by about 70 km. The location of features to the east of the land gradient pattern in Figure 3 is visibly unchanged in Figure 4. In addition, the land gradient in Figure 4 is not as great as it was in Figure 3, which is a desirable attribute of this interpolation process.

Another region of significant gradient in DOT is in the Kuroshio Current region. Figure 5 shows the DOT implied by the "FINAL123" model in a large (60° N to 10° N, 80° to 140°) area. The gradient associated with the current is clearly seen. No large gradient appears near the coastline. Considering the land regions, one sees the DOT slowly decaying to zero in regions about 700 km from the coastline (30° N, 90° E). (Note: The generation of a DOT grid is carried out by program RHRAPP.F388.SST.FROM.POCM4B.) The creation of a Postscript plot file is done by program RHRAPP.PLT.SST. The PS file has been ftped to the LAN (OSU, geodetic science) from OHSTMVSA for printing.

**The Power Spectrum of POCM_4B**

For numerous purposes it is helpful to know the power spectrum or the degree variances of a representation of dynamic ocean topography. The need for such information includes its use as a priori weighting information in least squares solutions of DOT from satellite altimeter data. In addition, knowledge of the power of DOT at high frequencies is helpful in the assessment of the value of gravity field mapping missions considering the inferred geoid undulation accuracy.

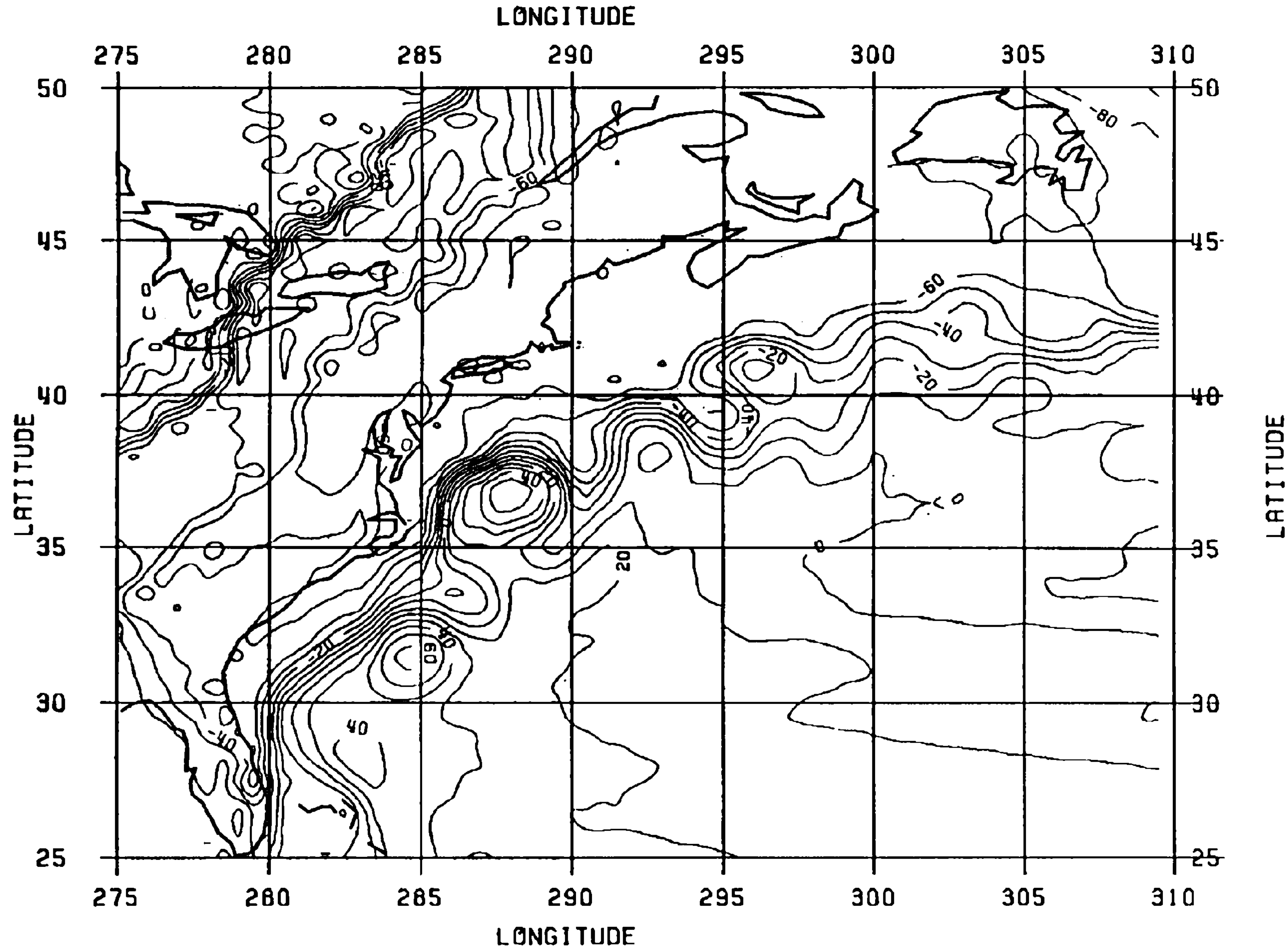


Figure 3. Dynamic Ocean Topography in the Gulf Stream Region Based on a Degree 360 Expansion of POCM_4B Using an Exponential Decay (D=750 km) Interpolation with Search Increments of 3° and a Degree 10 Model for Remaining 30' Cells (coefficients: ZHANGC.POCM4B.COEF.TODEG360.DIS3D.EXP750).

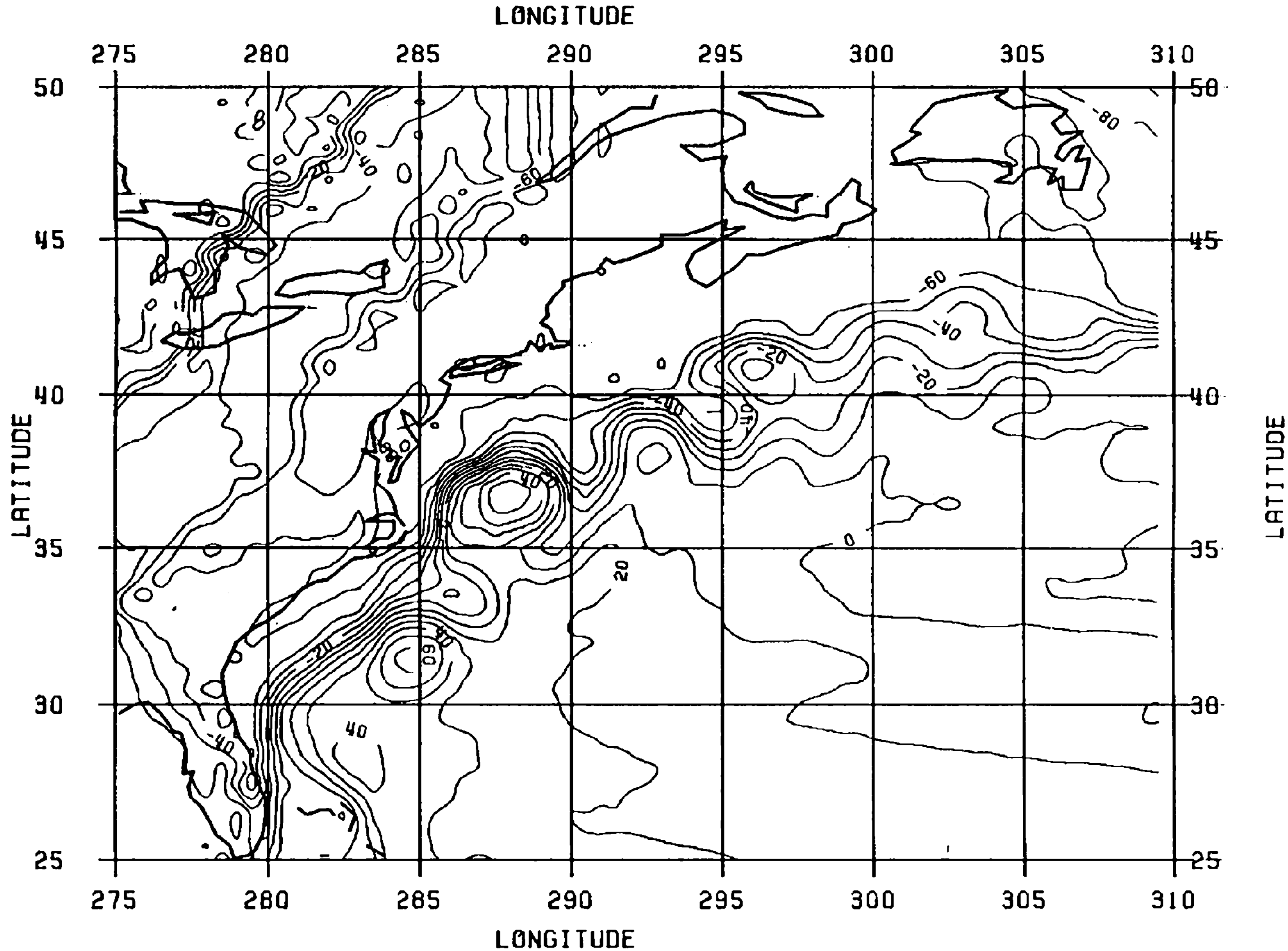


Figure 4. Dynamic Ocean Topography in the Gulf Stream Region Based on a Degree 360 Expansion of POCM_4B Using an Exponential Decay (D=350 km) Interpolation with Search Increments of 1.5° and a Degree 10 Model for Remaining 30'Cells (coefficients: ZHANGC.POCM4B.TODEG360.FINAL123).

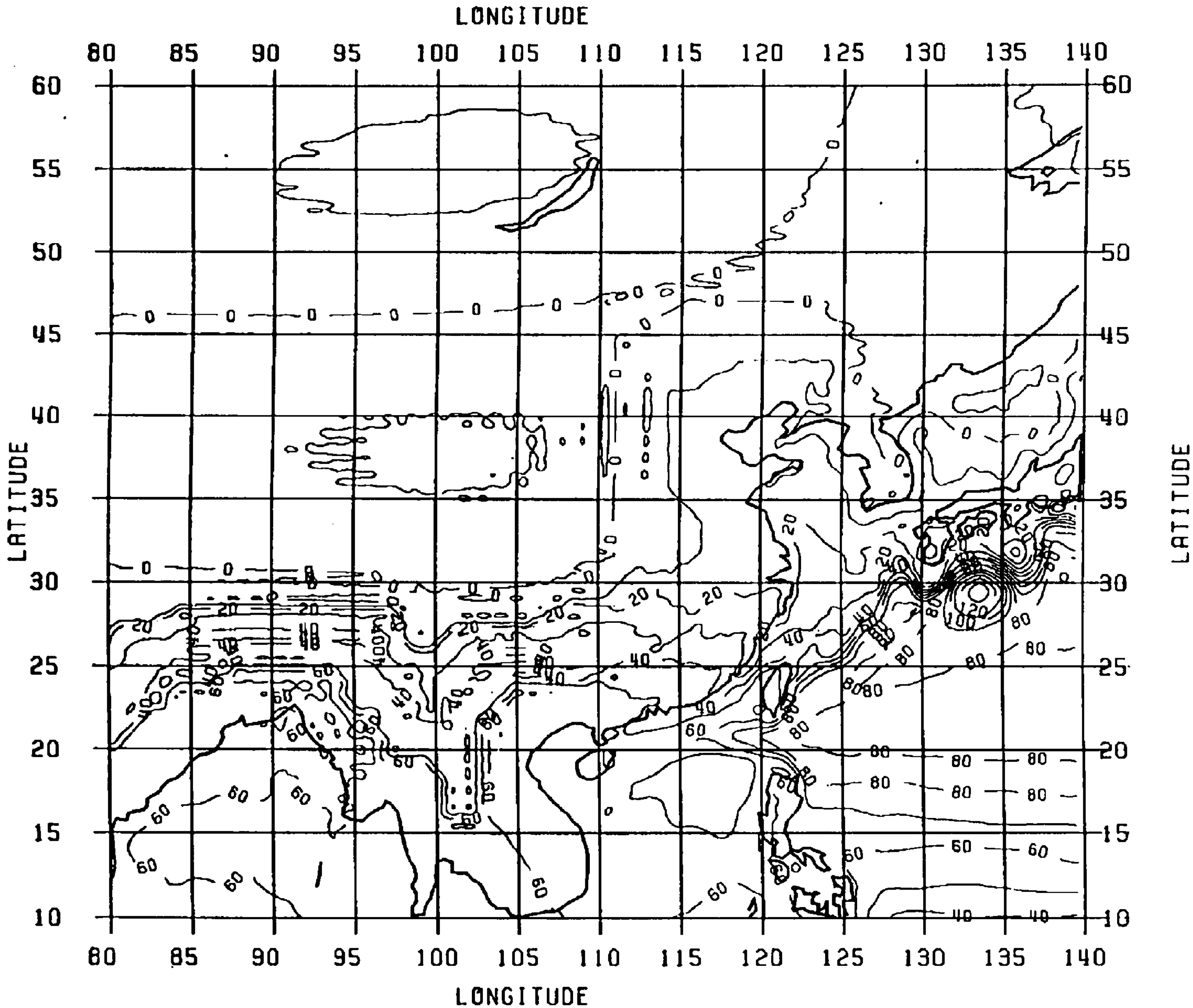


Figure 5. Dynamic Ocean Topography in the Kuroshio Current Region Based on a Degree 360 Expansion of POCM_4B Using an Exponential Decay (D=350 km) Interpolation With Search Increments of 1.5°, and a Degree 10 Model for  Remaining 30' Cells (coefficients: ZHANGC.POCM4B.TODEG360.FINAL123).

The difficulty in describing the spectrum occurs because DOT is not a global phenomenon; rather a quantity that is defined in a specific domain. Thus, considering the power spectrum from a spherical harmonic expansion is problematic since such an expansion is built on a globally defined function. Unless care is taken in appropriately defining this function, an incorrect spectrum could be inferred. For example, a global set of DOT with zero's on land creates, we saw, significant gradients in some coastal regions. Such gradients imply high frequency power that is an artifact of the way in which the DOT function is defined. It is for this reason that some care was taken to create a global DOT data set that avoids shoreline discontinuities and tapers the DOT to near zero away from the coast.

Let the spherical harmonic expansion of the DOT be given by the following expression:

$$\zeta(\theta,\lambda)=\sum_{n=0}^{k}\sum_{m=0}^{n}[c_{nm}R_{nm}(\theta,\lambda)+s_{nm}S_{nm}(\theta,\lambda)] \tag{2}$$

where $R_{nm}$, and $S_{nm}$ are the fully normalized spherical harmonics; $c_{nm}$ and $s_{nm}$ are the spherical harmonic coefficients with $\theta$ equal to co-latitude; and k is the maximum degree of the expansion. The DOT degree variances would be:

$$\zeta_n^2=\sum_{m=0}^{m}\left(c_{nm}^2+s_{nm}^2\right) \tag{3}$$

The square root of the degree variances for the POCM_4B model (based on the FINAL123 degree 360 model) are given for degree 1 to 24 and selected degrees to 360 in Table 1. The complete list is included in Appendix A. (The file is RHRAPP.POCM4B.SRDEGVAR).

As noted early in this documentation, an alternative representation of DOT is through orthonormal functions that are defined for an ocean domain. We let the orthonormal expansion be:

$$\zeta(\theta,\lambda)=\sum_{n=0}^{k}\sum_{m=0}^{n}[a_{nm}O_{nm}(\theta,\lambda)+b_{nm}Q_{nm}(\theta,\lambda)] \tag{4}$$

where $O_{nm}$ and $Q_{nm}$ are the orthonormal functions and $a_{nm}$, $b_{nm}$ are the orthonormal coefficients. In our studies the orthonormal coefficients are determined from spherical harmonic coefficients using transformation procedures as described by Hwang (1991) and Wang and Rapp (1994) with the most current procedures described in Rapp, Zhang, and Yi (1996). The ON degree variances are computed from (3) replacing the SH coefficients with the ON coefficient.

Using the SH coefficients to degree 24 of the degree 360 DOT solution (FINAL123), the corresponding ON coefficients (ZHANGC.ORTH.POCM4B.FINAL123) were computed. The ocean definition for the SH to ON conversions is defined as domain 7 in OSU software. The degree variances from these ON coefficients are given as the third column in Table 1. The magnitude of the power is similar to the SH results for most degrees, although significant changes are noted at degrees 4 to 10. Above degree 10 the magnitude are comparable but clearly not identical.

**Table 1.**

Square Root of Dynamic Ocean Topography Degree Variances (Units are cm)

| Degree | POCM4B* | | POCM4B** | TOPEX/geoid | |
|---|---|---|---|---|---|
| | SH | ON | ON | JGM-3 (ON) | EGM96 |
| 1 | 25.58 | 29.78 | 30.12 | 29.04 | 30.10 |
| 2 | 40.30 | 44.32 | 43.80 | 39.28 | 39.93 |
| 3 | 17.00 | 17.58 | 18.07 | 17.39 | 17.87 |
| 4 | 5.41 | 15.98 | 15.96 | 18.43 | 16.95 |
| 5 | 6.70 | 7.63 | 7.65 | 8.50 | 7.88 |
| 6 | 16.61 | 5.26 | 5.46 | 7.07 | 6.47 |
| 7 | 12.60 | 4.36 | 4.04 | 4.65 | 3.99 |
| 8 | 14.02 | 7.01 | 7.05 | 8.01 | 7.74 |
| 9 | 10.23 | 4.96 | 4.87 | 4.48 | 4.84 |
| 10 | 8.90 | 3.49 | 3.82 | 3.74 | 3.68 |
| 11 | 3.87 | 2.77 | 2.97 | 3.76 | 2.90 |
| 12 | 3.67 | 2.62 | 2.71 | 2.74 | 2.77 |
| 13 | 2.81 | 2.78 | 2.50 | 2.80 | 2.56 |
| 14 | 2.60 | 3.41 | 3.27 | 3.41 | 3.07 |
| 15 | 2.63 | 2.50 | 2.55 | 1.98 | 1.73 |
| 16 | 3.25 | 2.30 | 2.21 | 1.90 | 1.71 |
| 17 | 2.60 | 1.43 | 1.62 | 2.15 | 1.92 |
| 18 | 2.11 | 1.72 | 1.83 | 1.86 | 1.70 |
| 19 | 1.92 | 1.65 | 1.86 | 1.86 | 1.51 |
| 20 | 1.89 | 1.74 | 1.87 | 2.02 | 1.22 |
| 21 | 1.78 | 1.51 | 1.56 | 1.99 | 1.01 |
| 22 | 1.68 | 1.17 | 1.12 | 1.40 | 0.84 |
| 30 | 1.25 | | | | |
| 50 | 0.65 | | | | |
| 100 | 0.41 | | | | |
| 150 | 0.17 | | | | |
| 180 | 0.13 | | | | |
| 200 | 0.11 | | | | |
| 250 | 0.08 | | | | |
| 300 | 0.09 | | | | |
| 360 | 0.07 | | | | |

*quadrature solution
**SH coefficients by least squares adjustment

The POCM_4B model data (point DOT values) were also used to determine a spherical harmonic expansion to degree 24 using a least squares procedure as described on page 22,586 of Rapp, Zhang and Yi (1996). The spherical harmonic coefficients were converted to orthonormal coefficients (ZHANGC.ORTH.POPCM4B. TWOYEAR.VAR2.COSW2.ZEROLA) and the degree variances computed. The results are given in the fourth column in Table 1. Note the generally good agreement with the ON degree variances in the second column. This is encouraging since it shows the degree 360 model has not significantly distorted the DOT power spectrum in the ocean domain of the ON functions.

It is also of interest to consider the degree variances implied by the analysis of TOPEX satellite altimeter data with geoid undulations computed from a potential coefficient model. Such calculations have been described for the JGM-3 geopotential model in the Rapp, Zhang, Yi (1996) paper (see Figure 3 on page 22,590). The square root of the degree variances using the JGM-3/OSU91A model are given in the fifth column of Table 1. The results when the EGM96 geopotential model (Lemoine et al., 1997, 1998) are used are given as the last column in the table. (The SH and ON coefficients to degree 24 of the DOT representation using two years of TOPEX data and the EGM96 geopotential model are found in the following ASCII file: ZHANGC.OR TH.C12.TO84.PGS7337B.HDM190.TRACK). The magnitude of the DOT is similar when using the two geopotential models. However the results presented in Lemoine et al. (1997) demonstrate that the EGM96 model gives better agreement with the POCM_4B model than when the JGM-3 model are used.

As noted earlier there are some applications that require the power spectrum of DOT. The use of the results from the complete degree 360 SH expansion is possible but it was noted in the discussion of Table 1 that some of the lower degrees (4 to 10) differ significantly from the corresponding ON representation. In addition, the power spectrum computed from the ON representation from the SH least squares fit to the POCM_4B ocean data should be a more correct measure of the DOT power in the oceans than the one based on the quadrature solution based on the global set of 30' mean DOT values. Consequently, a single file of DOT power (see Table 1) (square root of degree variance) by degree has been created by merging the values from degrees 1 to 20 from column 4 ON coefficients by least squares fit to ocean POCM_4B data points (in a defined domain) with the power from degree 360 quadrature expansion of POCM_ 4B and land values computed as described previously. This merged ASCII file is RHRAPP.POCM4B.DEG VAR. The decision to start the SH power from degree 21 was somewhat arbitrary as degrees 21 or 22 could also have been used. In the case of the degree 20 transition, the power at degree 20 is 1.87 cm (ON, least squares) and 1.78 cm with the SH quadrature expansion.

One needs to recognize the merged file is just an approximation. The merger of ON power and SH power spectrums could be argued inappropriate because of the different nature of the functions. However, recognizing the limitations of the process, the results appear to be useful for studies needing a set of values representing the power the POCM_4B DOT to high degrees.

Another quantity of interest is the cumulative power of the DOT to a specified degree. Results for several cases are given in Table 2. The ON estimate of the total power depends on the domain in which the ON functions are defined. This domain is described Rapp, Zhang, and Yi (1996, p. 22,584). The magnitude implied by the spherical harmonic (quadrature) expansion will be low because of the way in which values of DOT were computed in the land areas. In a test solution a quadrature expansion of POCM_4B was carried out using zero values for all 30' cells for which a POCM_4B estimate could not be determined. (The data set name of this file is ZHANGC.POCM4B.COEF.TODEG360.ZERO). For this solution the cumulative value of DOT to degree 22 was ±53.8 cm, and to degree 360, ±55.3 cm. Both values are less than that reported in Table 2 for the SH solution involving special treatment of land DOT values.

**Table 2**

Cumulative Value of DOT (POCM_4B) to Specified
Degree Based on SH and ON Expansions

| Degree | ON (quadrature) | ON (least squares) | SH |
|---|---|---|---|
| 22 | ±60.7 cm | ±60.6 cm | ±59.5 cm |
| 360 | | | ±60.0 cm |

The standard deviation of the original point POCM_4B values was calculated using $\cos^2\phi$ weighting (Rapp, Zhang, Yi, 1996, p.22,586). The calculation was first done using all values with the result being ±66.7 cm. For comparison with the results given in Table 2 for the ON results, the calculation was restricted to points between 65° N and 66° S that approximates the ocean domain used for the ON expansions. In this case the standard deviation was found to be ±60.85 cm which is in excellent agreement with the Table 2 values.

## Discussion

This document has been prepared to describe the determination of a degree 360 expansion of dynamic ocean topography estimate of the POCM_4B model. A key element in the discussion is the way in which land and undefined values of DOT were interpolated in the calculation of a global 30' x 30' mean DOT data set. In the development of the interpolation process a goal was to avoid DOT gradients near the shoreline in the high degree expansion. In addition significant gradients in land areas were considered undesirable.

A degree 360 expansion was created from the 30' DOT values using the standard program (F419) used in the calculation of high degree expansions of the geopotential from mean gravity anomalies or expansions of the topography from 30' mean elevation data.

Orthonormal (ON) expansions were also described that are valid for a specific ocean domain. Degree variances of the ON and SH expansions were compared. The low degree (4 to 10) spectrum of the SH expansion differed from the ON expansion discussed. (Actually, two ON expands were described—one based on the SH quadrature expansion and one on a least squares SH fit to the POCM4B data points). At degrees higher than 10 the ON and SH power was similar. This led to the formation of a power spectrum file based on the merger of ON power, from the least squares solution (degrees 1 to 20), with the SH power from degree 21 to 360. This combined data set is RHRAPP.POCM4B.DEGVAR with some text lines explaining how the merger was done.

Improvement in the procedures described here can be made. Highly desirable are higher degree ON expansions. If such functions could be obtained, the concern on the determination of DOT for non-defined areas would be reduced or eliminated.

**References**


Hwang C (1991) Orthogonal functions over the oceans and applications to the determination of orbit error, geoid and sea surface topography from satellite altimetry, Rep. 414, Dep of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio.

Lemoine FG et al. (1997) The development of the NASA GSFC and NIMA Joint Geopotential Model, in Proc of the Int Symp on Gravity, Geoid, and Marine Geodesy, J Segawa, H Fujimoto, and S Okuba (eds), 461-469, IAG Symposium Series, Springer, New York, Berlin.

Lemoine FG et al. (1998) The Development of the Joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96, NASA/TP—1998–206861, NASA Goddard Space Flight Center, Greenbelt, Maryland.

Rapp RH, C Zhang, Y Yi (1996) Analysis of dynamic ocean topography using TOPEX data and orthonormal functions, J. Geophys Res., 101: 22583-22598.

Rapp RH, C Zhang (1996) Comparison and blending of dynamic ocean topography estimates from TOPEX Data and Global Circulation Models, presented at 1996 Spring AGU Meeting, (abstract), EOS, April 23, p. 576.

Stammer DR, Tokmakian A, Semtner and Wunsch C (1996) How closely does a 1/4° degree global circulation model simulate large scale observations? J. Geophys Res., 101: 25779-25812.

Wang YM, Rapp RH (1994) Estimation of sea surface dynamic topography ocean tides, and secular changes from Topex altimeter data, Rpt. 430, Dept of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio.

# Appendix A

Square Root of DOT Degree Variances of the Degree 360 POCM4B Expansion
(based on ZHANGC.POCM4B.COEF.TO DEG 360.FINAL 123)
(Units are cm)

| n | Value | n | Value | n | Value | n | Value |
|---|---|---|---|---|---|---|---|
| 0 | 1.1762 | | | | | | |
| 1 | 25.579 | 46 | 0.6945 | 91 | 0.4144 | 136 | 0.2238 |
| 2 | 40.3011 | 47 | 0.7213 | 92 | 0.4267 | 137 | 0.2018 |
| 3 | 16.9966 | 48 | 0.7716 | 93 | 0.4144 | 138 | 0.2099 |
| 4 | 5.4077 | 49 | 0.6618 | 94 | 0.4168 | 139 | 0.2064 |
| 5 | 6.7048 | 50 | 0.6538 | 95 | 0.4289 | 140 | 0.1976 |
| 6 | 16.6092 | 51 | 0.6022 | 96 | 0.3985 | 141 | 0.2138 |
| 7 | 12.604 | 52 | 0.5642 | 97 | 0.3682 | 142 | 0.1901 |
| 8 | 14.0152 | 53 | 0.6457 | 98 | 0.4003 | 143 | 0.1840 |
| 9 | 10.2299 | 54 | 0.6568 | 99 | 0.3705 | 144 | 0.1828 |
| 10 | 8.9008 | 55 | 0.6522 | 100 | 0.4091 | 145 | 0.1849 |
| 11 | 3.8709 | 56 | 0.5887 | 101 | 0.3388 | 146 | 0.1806 |
| 12 | 3.6655 | 57 | 0.5636 | 102 | 0.3797 | 147 | 0.1799 |
| 13 | 2.8064 | 58 | 0.5815 | 103 | 0.3506 | 148 | 0.2022 |
| 14 | 2.6016 | 59 | 0.5619 | 104 | 0.3531 | 149 | 0.1781 |
| 15 | 2.6323 | 60 | 0.5167 | 105 | 0.3485 | 150 | 0.1734 |
| 16 | 3.2452 | 61 | 0.4928 | 106 | 0.3817 | 151 | 0.1646 |
| 17 | 2.6024 | 62 | 0.5258 | 107 | 0.3429 | 152 | 0.1647 |
| 18 | 2.1138 | 63 | 0.4656 | 108 | 0.3352 | 153 | 0.1567 |
| 19 | 1.9211 | 64 | 0.4898 | 109 | 0.3286 | 154 | 0.1558 |
| 20 | 1.8896 | 65 | 0.4848 | 110 | 0.3068 | 155 | 0.1419 |
| 21 | 1.7842 | 66 | 0.5845 | 111 | 0.3345 | 156 | 0.1593 |
| 22 | 1.6889 | 67 | 0.5483 | 112 | 0.3378 | 157 | 0.1614 |
| 23 | 1.8545 | 68 | 0.4789 | 113 | 0.3242 | 158 | 0.1591 |
| 24 | 1.8643 | 69 | 0.4681 | 114 | 0.3379 | 159 | 0.1654 |
| 25 | 1.9106 | 70 | 0.4781 | 115 | 0.2999 | 160 | 0.1557 |
| 26 | 1.5320 | 71 | 0.4533 | 116 | 0.3021 | 161 | 0.1539 |
| 27 | 1.7303 | 72 | 0.4363 | 117 | 0.2914 | 162 | 0.1626 |
| 28 | 1.7518 | 73 | 0.4899 | 118 | 0.2939 | 163 | 0.1495 |
| 29 | 1.5157 | 74 | 0.5027 | 119 | 0.3219 | 164 | 0.1442 |
| 30 | 1.2469 | 75 | 0.4819 | 120 | 0.2858 | 165 | 0.1589 |
| 31 | 1.2471 | 76 | 0.4569 | 121 | 0.2702 | 166 | 0.1355 |
| 32 | 1.3058 | 77 | 0.4758 | 122 | 0.2526 | 167 | 0.1353 |
| 33 | 1.1054 | 78 | 0.4652 | 123 | 0.2649 | 168 | 0.1333 |
| 34 | 1.0186 | 79 | 0.5288 | 124 | 0.2562 | 169 | 0.1285 |
| 35 | 1.1806 | 80 | 0.4425 | 125 | 0.2767 | 170 | 0.1288 |
| 36 | 0.9944 | 81 | 0.4424 | 126 | 0.2609 | 171 | 0.1368 |
| 37 | 1.2291 | 82 | 0.4666 | 127 | 0.2497 | 172 | 0.1536 |
| 38 | 1.0890 | 83 | 0.4908 | 128 | 0.2405 | 173 | 0.1407 |
| 39 | 0.8434 | 84 | 0.4330 | 129 | 0.2703 | 174 | 0.1481 |
| 40 | 0.8154 | 85 | 0.4463 | 130 | 0.2395 | 175 | 0.1261 |
| 41 | 0.7834 | 86 | 0.4466 | 131 | 0.2521 | 176 | 0.1327 |
| 42 | 0.8318 | 87 | 0.4696 | 132 | 0.2239 | 177 | 0.1391 |
| 43 | 0.7843 | 88 | 0.4385 | 133 | 0.2161 | 178 | 0.1255 |
| 44 | 0.7752 | 89 | 0.4553 | 134 | 0.2184 | 179 | 0.1379 |
| 45 | 0.8614 | 90 | 0.4191 | 135 | 0.2021 | 180 | 0.1263 |

## Appendix A (Continued)

Square Root of DOT Degree Variances of the Degree 360 POCM4B Expansion
(based on ZHANGC.POCM4B.COEF.TO DEG 360.FINAL 123)
(Units are cm)

| n | Value | n | Value | n | Value | n | Value |
|---|---|---|---|---|---|---|---|
| 181 | 0.1308 | 226 | 0.0941 | 271 | 0.0865 | 316 | 0.0875 |
| 182 | 0.1266 | 227 | 0.1050 | 272 | 0.0821 | 317 | 0.0831 |
| 183 | 0.1117 | 228 | 0.0978 | 273 | 0.0780 | 318 | 0.0859 |
| 184 | 0.1364 | 229 | 0.1028 | 274 | 0.0788 | 319 | 0.0805 |
| 185 | 0.1064 | 230 | 0.1038 | 275 | 0.0879 | 320 | 0.0804 |
| 186 | 0.1235 | 231 | 0.0982 | 276 | 0.0828 | 321 | 0.0747 |
| 187 | 0.1319 | 232 | 0.1155 | 277 | 0.0792 | 322 | 0.0847 |
| 188 | 0.1247 | 233 | 0.0924 | 278 | 0.0869 | 323 | 0.0810 |
| 189 | 0.1247 | 234 | 0.1088 | 279 | 0.0802 | 324 | 0.0768 |
| 190 | 0.1149 | 235 | 0.0958 | 280 | 0.0897 | 325 | 0.0788 |
| 191 | 0.1158 | 236 | 0.1029 | 281 | 0.0852 | 326 | 0.0847 |
| 192 | 0.1131 | 237 | 0.0945 | 282 | 0.0936 | 327 | 0.0783 |
| 193 | 0.1151 | 238 | 0.0970 | 283 | 0.0925 | 328 | 0.0767 |
| 194 | 0.1229 | 239 | 0.1055 | 284 | 0.0913 | 329 | 0.0799 |
| 195 | 0.1179 | 240 | 0.1037 | 285 | 0.0946 | 330 | 0.0759 |
| 196 | 0.1267 | 241 | 0.1011 | 286 | 0.0901 | 331 | 0.0756 |
| 197 | 0.1149 | 242 | 0.0966 | 287 | 0.0864 | 332 | 0.0761 |
| 198 | 0.1108 | 243 | 0.0919 | 288 | 0.0881 | 333 | 0.0724 |
| 199 | 0.1173 | 244 | 0.0941 | 289 | 0.0890 | 334 | 0.0721 |
| 200 | 0.1121 | 245 | 0.0907 | 290 | 0.0819 | 335 | 0.0738 |
| 201 | 0.1157 | 246 | 0.0901 | 291 | 0.0784 | 336 | 0.0725 |
| 202 | 0.1115 | 247 | 0.0940 | 292 | 0.0854 | 337 | 0.0753 |
| 203 | 0.1229 | 248 | 0.0815 | 293 | 0.0825 | 338 | 0.0693 |
| 204 | 0.1144 | 249 | 0.0898 | 294 | 0.0845 | 339 | 0.0769 |
| 205 | 0.1203 | 250 | 0.0788 | 295 | 0.0862 | 340 | 0.0693 |
| 206 | 0.1170 | 251 | 0.1060 | 296 | 0.0884 | 341 | 0.0707 |
| 207 | 0.1108 | 252 | 0.0925 | 297 | 0.0921 | 342 | 0.0712 |
| 208 | 0.1085 | 253 | 0.1103 | 298 | 0.0893 | 343 | 0.0706 |
| 209 | 0.1115 | 254 | 0.0973 | 299 | 0.0988 | 344 | 0.0683 |
| 210 | 0.1083 | 255 | 0.0891 | 300 | 0.0934 | 345 | 0.0695 |
| 211 | 0.1063 | 256 | 0.0953 | 301 | 0.0920 | 346 | 0.0679 |
| 212 | 0.0988 | 257 | 0.0828 | 302 | 0.0943 | 347 | 0.0716 |
| 213 | 0.1029 | 258 | 0.0846 | 303 | 0.0875 | 348 | 0.0675 |
| 214 | 0.1092 | 259 | 0.0780 | 304 | 0.0897 | 349 | 0.0734 |
| 215 | 0.1191 | 260 | 0.0728 | 305 | 0.0859 | 350 | 0.0701 |
| 216 | 0.1073 | 261 | 0.0798 | 306 | 0.0822 | 351 | 0.0719 |
| 217 | 0.1115 | 262 | 0.0761 | 307 | 0.0863 | 352 | 0.0708 |
| 218 | 0.1101 | 263 | 0.0850 | 308 | 0.0844 | 353 | 0.0666 |
| 219 | 0.1209 | 264 | 0.0888 | 309 | 0.0870 | 354 | 0.0725 |
| 220 | 0.1098 | 265 | 0.0967 | 310 | 0.0838 | 355 | 0.0639 |
| 221 | 0.1057 | 266 | 0.0963 | 311 | 0.0837 | 356 | 0.0722 |
| 222 | 0.1181 | 267 | 0.0825 | 312 | 0.0940 | 357 | 0.0679 |
| 223 | 0.1101 | 268 | 0.0980 | 313 | 0.0892 | 358 | 0.0707 |
| 224 | 0.1117 | 269 | 0.0812 | 314 | 0.0874 | 359 | 0.0666 |
| 225 | 0.0971 | 270 | 0.0921 | 315 | 0.0821 | 360 | 0.0697 |

## Appendix B

Program and Data File Names

In this appendix we list data set and program names that are related to the computations described in this paper. The programs have primarily obtained form accounts maintained by Changou Zhang on the CRAY computer at The Ohio Supercomputer Center (OSC), the Silicon Graphics Computer (Helmert) in the Geodetic Science and Surveying sections, and in the mainframe OSU Computer (OHSTMVSA). The files were ftped to a site at GSFC as part of the documentation of the work. Programs where the first part of the filename is zhangc were files stored on the mainframe with the high level index of rhapp after saving from a zhang based account. Files without zhangc were files transformed (for the most part) from zhang files stored under a zhang account. All files were set up as ascii files, some of which were compressed on the Silicon Graphics computer because of the file size. The files are:

agu.f419
agu96.f159
convert.la.land.to10a
f419.pocm 4b
grid.pocm 4b
la.land.to10
orth.c12.to84.pgs7337b.hdm190.track
orth.sc.twoyear.var2.cosw2.zerola
pocm4b
pocm4b.coef.todeg360.dis3d
pocm4b.coef.todeg360.final123
pocm4b.coef.todeg360.zero
pocm4b.coef.todeg360.dis3d.exp750
pocm4b.degvar
pocm4b.g30x30
pocm4b.g30x30.set2
pocm4b.srdegvar
popcm5.var2.cosw2.zero1
popcm5.var2.cosw2.zero1.new
sc.twoyear.var2.cosw2.zerola
zhangc.gcm.grid.d.f
zhangc.gcm.grid.f1
zhangc.cray.model.sst.grid.dis.f
zhangc.cray.model.sst.grid.exp.f
zhangc.cray.model.sst.grid.exp.h.f
zhangc.geoid.f159.on
zhangc.read.twoyear.f
zhangc.sst.f388.la
zhangc.sstadj.var2.cosw

# REPORT DOCUMENTATION PAGE

*Form Approved*
*OMB No. 0704-0188*



**1. AGENCY USE ONLY** *(Leave blank)*

**2. REPORT DATE**
September 1998

**3. REPORT TYPE AND DATES COVERED**
Contractor Report

**4. TITLE AND SUBTITLE**
The Development of a Degree 360 Expansion of the Dynamic Ocean Topography of the POCM_4B Global Circulation Model

**5. FUNDING NUMBERS**
Code 926
NAS5-32352

**6. AUTHOR(S)**
Richard H. Rapp

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS (ES)**
Goddard Space Flight Center
Greenbelt, Maryland 20771

**8. PEFORMING ORGANIZATION REPORT NUMBER**
98A01485

**9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS (ES)**
National Aeronautics and Space Administration
Washington, DC 20546-0001

**10. SPONSORING / MONITORING AGENCY REPORT NUMBER**
CR—1998–206877

**11. SUPPLEMENTARY NOTES**
R. Rapp: Dept. of Civil and Environmental Engineering and Geodetic Science, The Ohio State University, Columbus, Ohio 43210–1275. E-mail address: rapp.1@osu.edu

**12a. DISTRIBUTION / AVAILABILITY STATEMENT**
Unclassified - Unlimited
Subject Category: 48
Report available from the NASA Center for AeroSpace Information,
800 Elkridge Landing Road, Linthicum Heights, MD 21090; (301) 621-0390.

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** *(Maximum 200 words)*

This paper documents the development of a degree 360 expansion of the dynamic ocean topography (DOT) of the POCM_4B ocean circulation model. The principles and software used that led to the final model are described. A key principle was the development of interpolated DOT values into land areas to avoid discontinuities at or near the land/ocean interface. The power spectrum of the POCM_4B is also presented with comparisons made between orthonormal (ON) and spherical harmonic magnitudes to degree 24. A merged file of ON and SH computed degree variances is proposed for applications where the DOT power spectrum from low to high (360) degrees is needed.

**14. SUBJECT TERMS**
Geosciences, Oceanography, Mathematical and Computer Sciences

**15. NUMBER OF PAGES**
17

**16. PRICE CODE**

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| Unclassified | Unclassified | Unclassified | UL |