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TECHNICAL NOTE

D-324

THE DESIGN OF FEEDBACK CONTROL SYSTEMS CONTAINING
A SATURATION TYPE NONLINEARITY

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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SUMMARY

This report is an extension of NASA TN D-20. The extension includes a derivation of the optimum response for a step input for plant transfer functions which have an unstable pole and further data on plants with a single zero in the left half of the s plane. The calculated data are presented tabulated in normalized form.

Optimum control systems are considered. The optimum system is defined as one which keeps the error as small as possible regardless of the input, under the constraint that the input to the plant (or controlled system) is limited. Intuitive arguments show that in the case where only the error can be sensed directly, the optimum system is obtained from the optimum relay or on-off solution. References to known solutions are presented. For the case when the system is of the sampled-data type, arguments are presented which indicate the optimum sampled-data system may be extremely difficult if not impossible to realize practically except for very simple plant transfer functions.

Two examples of aircraft attitude autopilots are presented, one for a statically stable and the other for a statically unstable airframe. The rate of change of elevator motion is assumed limited for these examples. It is shown that by use of nonlinear design techniques described in NASA TN D-20 one can obtain near optimum response for step inputs and reasonable response to sine wave inputs for either case. Also, the nonlinear design prevents inputs from driving the system unstable for either case.

INTRODUCTION

Saturation (or limiting) in a feedback control system can be analyzed and compensated for (if required) by an appropriate application of the root-locus and switch-time methods described in reference 1. The principles of these two methods are reviewed here for the reader's convenience.

To apply the root-locus method one draws the loci of the roots of the characteristic equation as a function of the equivalent¹ gain of the saturating device (equivalent limiter gain). One then views the closed-loop poles as moving on these loci as a function of the input magnitude. If the poles stay in well-damped regions of the s plane for all expected values of equivalent gain, one is assured that saturation causes no instability of the closed-loop system. Conversely, if the closed-loop poles move to lightly damped or unstable regions of the s plane for low values of equivalent limiter gain (large input magnitudes), the system response will be oscillatory or unstable when it is subjected to very large transients. In order to obtain qualitative ideas of how to compensate for the instability one studies how various changes in feedback parameters shift the zeros of the root-locus diagram. Shifts which are suitable are those that allow the equivalent limiter gain to decrease while the closed-loop poles stay in well-damped regions of the s plane. The feedback parameters (or zeros) are made to shift with one or more functions which are similar to the limiter input (such as the error).

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The switch-time method is based on the fact that the existence of saturation allows one to define an optimum response. The optimum response uses the saturated variable in such a manner as to restore the error and its derivatives to zero in a minimum time. It has been shown (ref. 1) that the first reversal time (after the application of a step) of the saturated variable can be used to obtain a quantitative measure for comparing the particular system to the optimum. If the actual first reversal time is longer than the optimum, then overshoot exists; conversely, if it is smaller, the actual response time is longer than the optimum. By designing nonlinear functions to make the actual first reversal time equal to the optimum for large step inputs, one obtains a design which has a stable nonoscillatory response for all input magnitudes.

As can be seen, the application of the switch-time method requires a knowledge of the optimum response for the particular "plant"² under consideration. A method for obtaining this type data was described in reference 1 where it was shown that if the plant transfer function has a zero in the left half of the s plane, then a considerable reduction in the response time to a step input can be achieved. The solution derived, however, requires some of the variables of the plant to change in a certain manner while the error and its derivatives remain zero.

¹Describing function analysis shows that saturation (or limiting) can be treated as an equivalent gain whose value decreases as the input magnitude increases (see, e.g., ref. 2).

²The word "plant" refers to the controlled system. The plant transfer function could, for example, be the mathematical relationship between the output of an amplifier (which saturates for large inputs) and a motor shaft position in a servo position controller.

One limitation of reference 1 was that no systems were derived to demonstrate that the above solution, obtained for a plant with a zero in its transfer function, was practically realizable.

A second limitation was that the limiter input equations were not derived for the optimum system. The optimum system is defined here as one which maintains the error as small as possible regardless of the input or initial condition under the constraint of finite limits on the plant input.

A third limitation was that no optimum curves or examples were derived which considered the problem of controlling an unstable plant (i.e., the transfer function has a pole in the right half of the s plane). This problem has considerable practical importance in certain autopilot problems where the aircraft is statically unstable in certain areas of its flight envelope.

It is the purpose of this investigation to remove these limitations. In particular, the following subjects are investigated and some practical solutions studied or suggested.

1. The optimum control system for plants with a saturation-type nonlinearity. The plant in this case is stable and has no zeros.

2. The influence of a zero in the plant transfer function on the optimum control system. The examples considered are for a type 1³ second-order plant and a type 2 third-order plant.

3. The influence of an unstable pole in the plant transfer function on the optimum system for a type 1 second-order plant.

4. The determination of the optimum response of plant transfer functions of the type encountered in pitch-rate and pitch-attitude autopilots where the control surface rate is limited. This amounts to considering plant transfer functions of the form

$$(a) \quad \frac{K(\tau_1 s + 1)}{s \left(\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1 \right)}$$

$$(b) \quad \frac{K(\tau_1 s + 1)}{s^2 \left(\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1 \right)}$$

³Type in this usage was defined in reference 1 as the number of poles at the origin in the s plane. A type 1 second-order plant (with one zero) has a transfer function, $G(s) = \frac{K(\tau_1 s + 1)}{s(\tau_2 s + 1)}$; similarly a type 2 third-

order plant (with zero) has a transfer function, $G(s) = \frac{K(\tau_1 s + 1)}{s^2(\tau_2 s + 1)}$.

where ξ can take on all positive values and

$$(c) \quad \frac{K(\tau_1 s + 1)}{s(\tau_2 s + 1)(\tau_3 s - 1)} \quad (d) \quad \frac{K(\tau_1 s + 1)}{s^2(\tau_2 s + 1)(\tau_3 s - 1)}$$

The (c) and (d) cases are for statically unstable airframes.

5. Practical examples of an attitude autopilot. For this, two airframes are considered to demonstrate the difference in control systems for stable and unstable airframes.

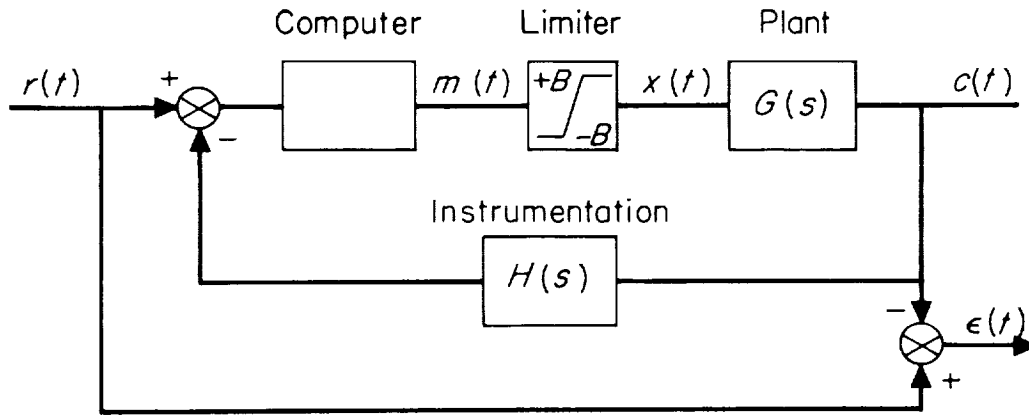
NOTATION

B	limit level
c(t)	controlled quantity
C(s)	Laplace transform of controlled quantity
D(s)	controller transfer function
D(z)	digital controller transfer function
e	2.7183
E(s)	Laplace transform of error
G(s)	plant transfer function
h_p	density altitude, ft
H(s)	feedback transfer function
K	gain
$L[f(t)]$	Laplace transform of $f(t)$
m(t)	controller output
M	Mach number
r(t)	input reference quantity
R_0	input step magnitude
R(s)	Laplace transform of reference quantity

s	Laplace operator
t	time, sec
T	sampling period, sec
T_1	first reversal time, sec
T_m	minimum response time, sec
$u(t)$	unit step function
$U(s)$	Laplace transform of unit step function
$x(t)$	limiter output (saturated variable)
$X(s)$	Laplace transform of saturated variable
z	$e^{-sT} = Z$ transform operator
δ	control surface deflection, radians
$e(t)$	error function, $r(t) - c(t)$
ζ	damping ratio
θ	pitch angle, radians
τ	time constant, sec
ω_n	natural frequency, radians/sec

DESCRIPTION OF THE PROBLEM

The problem is best described with the help of the block diagram of sketch (a). The computer in this diagram will be assumed capable of providing any linear or nonlinear relationship desired between its input and output. The output of the computer, $m(t)$, will be assumed to be continuous if an analog computer is used or a staircase signal (a zero-order hold circuit on the output of a digital computer) if a digital computer is used.



Sketch (a)

The output of the computer, $m(t)$, is fed to a limiter. The output of the limiter, $x(t)$, obeys the following equations

for

$$|m(t)| \leq B \quad (1)$$

$$x(t) = m(t)$$

for

$$|m(t)| > B \quad (2)$$

$$x(t) = B \operatorname{sgn}[m(t)]$$

The plant is assumed to be linear and representable by a transfer function, $G(s)$. The plant transfer functions in table I are considered here. The first seven cases were also treated in reference 1.

The feedback transfer function is $H(s)$. In general, one desires the error, $e(t) = r(t) - c(t)$, to be as small as possible for all the inputs, $r(t)$, the system receives. As will be shown later, this implies that $H(s) = 1$ and all compensation (linear or nonlinear) be accomplished by the computer.

The general problem is how to design the computer operations in order to maintain the error as small as possible under the constraint that $|x(t)| \leq B$. This, as will be shown, generally implies very complicated computer equations for the plant transfer functions for which solutions have been found. At the present time, the solution is not known for most

plants whose characteristic equation is of third or higher order. Thus, in this investigation we are principally interested in finding approximate solutions.

The problem will be investigated in the following order:

1. Determination of the optimum switching times for the plants given in table I. This will be done in the section entitled "Optimum Responses."

2. Derivation (based on intuitive argument) of the optimum system. This subject is covered in the section entitled "Optimum Systems." Reference data to known previous work as well as new work which includes simple plants with an unstable pole or stable zero are contained in this section.

3. Approximate optimum systems for

a. A pitch attitude autopilot for a stable airframe.

b. A pitch attitude autopilot for a statically unstable airframe.

These systems are derived in the section entitled "Approximate Optimum Systems."

OPTIMUM RESPONSES

A general theorem which was partially proved in reference 1 is as follows: If a Laplace transformable function of time, $f(t)$, is truncated (i.e., $f_T(t) = 0$ for $a > t > b$ where $f_T(t)$ is the truncated time signal), then the Laplace transform of $f_T(t)$, $L[f_T(t)]$, is an entire function.

This theorem can be used to advantage in deriving the optimum response if further information regarding the shape of $x(t)$, the bounded variable, is attainable. Consider sketch (a) for $r(t) = R_0 u(t)$ and zero initial conditions. Then

$$L[\epsilon(t)] = E(s) = \frac{R_0}{s} - X(s) G(s) \quad (3)$$

A necessary but not a sufficient condition if we are to attain the optimum is that $E(s)$ must be an entire function because $\epsilon(t) = 0$ for $t < 0$ and we desire $\epsilon(t) = 0$ for $t > T_m$. In other words, the desired $\epsilon(t)$ is a truncated time signal. In order to meet the sufficiency condition one must know the optimum shape of $x(t)$ in general terms of unknown reversal times. One then derives $X(s)$ and determines the relationship between these unknown reversal times and R_0 by forcing equation (3) to be an entire function.

The general shape of $x(t)$ for the minimum response time can be found by either inspection, or by use of a theorem proved by Bellman, Glicksberg, and Gross (ref. 3) for the conditions for which it applies. The theorem applied to this problem states that, in order to have a minimum response time to an input step, $x(t)$ should be at its maximum value plus or minus throughout the response, the maximum number of reversals being equal to $n-1$ where n is the order of the system. The proof, however, is only for $G(s)$ having real, distinct, and negative roots. It is shown later that if one confines his interest to plants having only poles in the left half plane or at $s = 0$ and is interested only in responses to step inputs, the theorem gives a sufficient number of reversals of $x(t)$ in order to restore the error and its derivatives to zero in a finite time, that is, to allow $E(s)$ given by equation (3) to be an entire function. As will be demonstrated, the response time obtained by using this number of reversals may not be the minimum for plants with lightly damped complex poles. It will also be shown here that if zeros exist in $G(s)$ then one does not want the bounded variable, $x(t)$, to be at its maximum throughout the entire response, that is, until the time when all the states of the system are restored to zero or a constant.

The steps involved in obtaining the optimum responses are summarized below.

(1) Determine $x(t)$ in terms of the unknown switch times for the optimum response. This function will be the limit level times a sum of delayed steps for $G(s)$ having only poles. The delay times will be written as undetermined coefficients.

(2) Determine $X(s)$ from $x(t)$ with the switching times as parameters.

(3) Use the fact that the error transform must be an entire function to obtain algebraic or transcendental equations relating R_0 and the unknown switching times and exponential decays (if any) of $x(t)$.

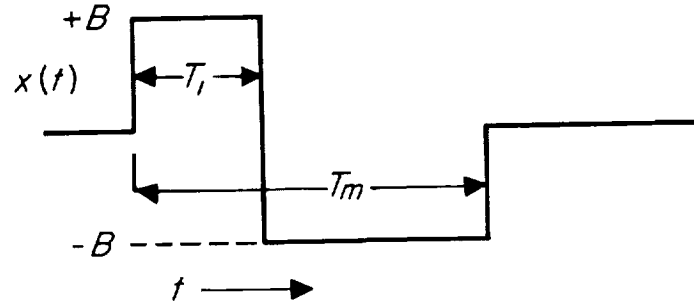
(4) Solve the equations of step (3) to obtain the optimum first reversal time and the minimum response time as functions of the input step magnitude, R_0 .

In reference 1 these steps were followed in deriving the optimum responses for the first 7 cases of table I. The optimum responses for cases 8 to 12 will be derived here.

A Type 1 Second-Order Unstable Plant

The plant transfer function for this example (case 8, table I) is $G(s) = K/s(\tau_s - 1)$. Arguments similar to those presented in reference 1

show that for $x(t) = R_0 u(t)$ where $u(t)$ is the unit step function, $x(t)$ must be of the form shown in sketch (b).



Sketch (b)

The Laplace transform of $x(t)$ is

$$X(s) = B \frac{1 - 2e^{-T_1 s} + e^{-T_m s}}{s} \quad (4)$$

The Laplace transform of the error is (see eq. (3))

$$E(s) = \frac{R_0}{s} - BK \frac{1 - 2e^{-T_1 s} + e^{-T_m s}}{s^2(s\tau - 1)} \quad (5)$$

The relationships which must be satisfied in order that equation (5) be an entire function are

$$1 - 2e^{-T_1/\tau} + e^{-T_m/\tau} = 0 \quad (6)$$

$$\frac{R_0}{BK} = T_m - 2T_1 \quad (7)$$

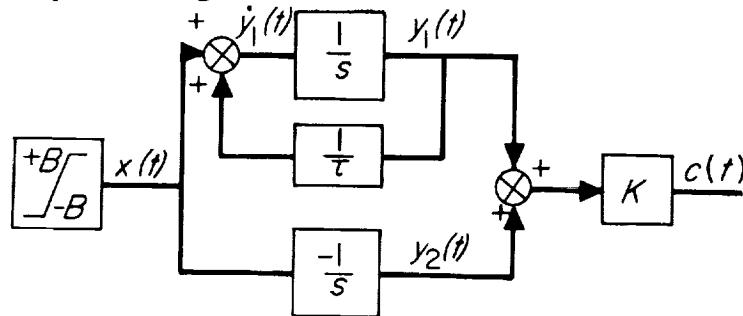
from which one can find

$$\frac{R_0}{BK} = \tau \ln \frac{1}{2e^{-T_1/\tau} - 1} - 2T_1 \quad (8)$$

$$\frac{R_0}{BK} = T_m - 2\tau \ln \frac{2}{1 + e^{-T_m/\tau}} \quad (9)$$

Equations (8) and (9) are normalized by dividing by τ and the curves showing T_1/τ and T_m/τ versus $R_0/BK\tau$ are presented in figure 1.

As can be seen from figure 1(a), for the unstable plant the optimum first reversal time approaches a constant for R_0 approaching infinity. Since this condition will exist for all the unstable plants considered here, the reason for its existence will be derived. The derivation is accomplished by expanding the plant transfer function by means of partial fractions and by defining state variables as shown in sketch (c).



Sketch (c)

As can be seen from sketch (c), the transfer function, $Y_1(s)/X(s)$, contains the unstable mode (pole in the right half plane). Thus, if $x(t)$ is a step function and $y_1(0) = 0$, then $y_1(t)$ is a growing exponential.

From sketch (c) it is seen that the input to the integrator, $\dot{y}_1(t)$, is $\dot{y}_1(t) = x(t) + (1/\tau)y_1(t)$. If $y_1(0) = 0$ and $x(t) = Bu(t)$, then the input to the integrator grows with time. If control is to be maintained of $y_1(t)$ by $x(t)$, then $x(t)$ must be able to reverse the sign of $\dot{y}_1(t)$. This is impossible if $y_1(t) > \tau B$ when $x(t)$ is limited to a magnitude of B . Thus if we use the optimum motion of $x(t)$ shown in sketch (b), T_1 must never exceed a certain value. This value is, in general, (from eq. (8)) $T_1 = \tau \ln 2$.

This simple derivation is easily shown for the second-order example by phase plane analysis. It amounts to the fact that $y_1(t)$ must be held within certain limits. If $y_1(t)$ is ever allowed out of these limits, then $c(t)$ grows indefinitely, regardless of $x(t)$. This is the principal difference between stable and unstable plants; that is, with an unstable plant the phase space must be limited if one is to control the plant with a limited input.

A Type 1 Third-Order Stable Plant (With Zero)

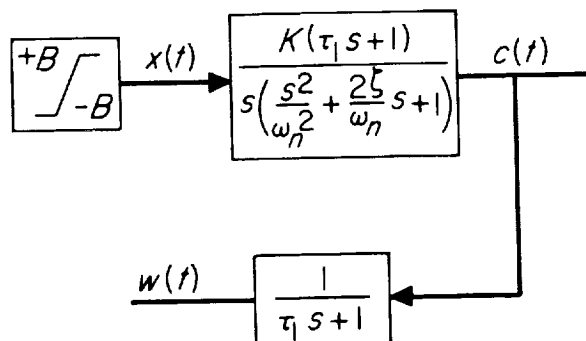
For this example (case 9 of table I)

$$G(s) = \frac{K(\tau_1 s + 1)}{s[(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1]}$$

The problem is to determine the optimum motion of $x(t)$. There are several possibilities. One scheme is to use a pole to cancel the zero,

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as illustrated in sketch (d). If this solution is used, $w(t)$ will be a zero overshoot response for a step input to the system. The output, $c(t)$, will overshoot, however. The amount of overshoot can be extremely large if τ_1 is large. It therefore would seem desirable to find a solution which has zero overshoot.

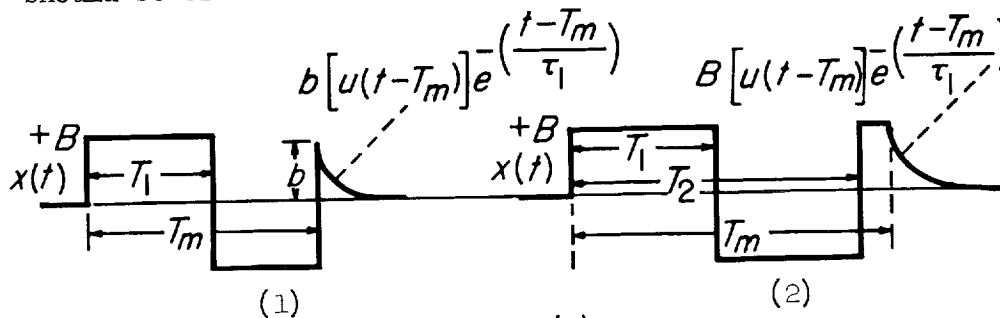


Sketch (d)

It was shown in reference 1 for a simple plant transfer function that if $x(t)$ is a bang-bang motion followed by a tailing off exponential of time constant τ_1 , then $c(t)$ has no overshoot. With this solution all of the states of the plant are not zero when the error and its derivatives are zero and there are practical questions as to how such a system can be implemented. The questions will be resolved later in this report.

For the more complicated plant considered here one questions whether a solution of the type desired can be found by simply taking more switching times for the bang-bang part of $x(t)$. This actually turns out to be the case. The solution which is derived here was actually used in an example in reference 4 and thus one has experimental evidence of its existence.

From these considerations one concludes that the optimum motion of $x(t)$ should be of the form shown in sketch (e). A partial proof that these



Sketch (e)

motions are optimum is obtained if we are successful in finding solutions to the equations which force $E(s)$ to be an entire function; these solutions must have a shorter minimum response time than the one obtained by the scheme shown in sketch (d).

If $x(t)$ is of the form shown in sketch (e 1), the equations which force $E(s)$ to be an entire function are

$$\frac{R_0 \omega_n}{BK} = 2\omega_n T_1 - \omega_n T_m + \frac{b}{B} \omega_n \tau_1 \quad (10)$$

$$(1 - 2e^{\alpha \omega_n T_1} + e^{\alpha \omega_n T_m})(1 - \alpha \omega_n \tau_1) - \frac{b}{B} \alpha \omega_n \tau_1 e^{\alpha \omega_n T_m} = 0 \quad (11)$$

$$(1 - 2e^{\bar{\alpha} \omega_n T_1} + e^{\bar{\alpha} \omega_n T_m})(1 - \bar{\alpha} \omega_n \tau_1) - \frac{b}{B} \bar{\alpha} \omega_n \tau_1 e^{\bar{\alpha} \omega_n T_m} = 0 \quad (12)$$

where

$$\alpha = \zeta + \sqrt{\zeta^2 - 1}$$

$$\bar{\alpha} = \zeta - \sqrt{\zeta^2 - 1}$$

If $|b/B|$ from this solution is greater than unity, one must use the $x(t)$ motion given in sketch (e 2). If this motion is used, the equations which force $E(s)$ to be an entire function are

$$\frac{R_0 \omega_n}{BK} = 2\omega_n T_1 - 2\omega_n T_2 + \omega_n T_m + \omega_n \tau_1 \quad (13)$$

$$(1 - 2e^{\alpha \omega_n T_1} + 2e^{\alpha \omega_n T_2} - e^{\alpha \omega_n T_m})(1 - \alpha \omega_n \tau_1) - \alpha \omega_n \tau_1 e^{\alpha \omega_n T_m} = 0 \quad (14)$$

$$(1 - 2e^{\bar{\alpha} \omega_n T_1} + 2e^{\bar{\alpha} \omega_n T_2} - e^{\bar{\alpha} \omega_n T_m})(1 - \bar{\alpha} \omega_n \tau_1) - \bar{\alpha} \omega_n \tau_1 e^{\bar{\alpha} \omega_n T_m} = 0 \quad (15)$$

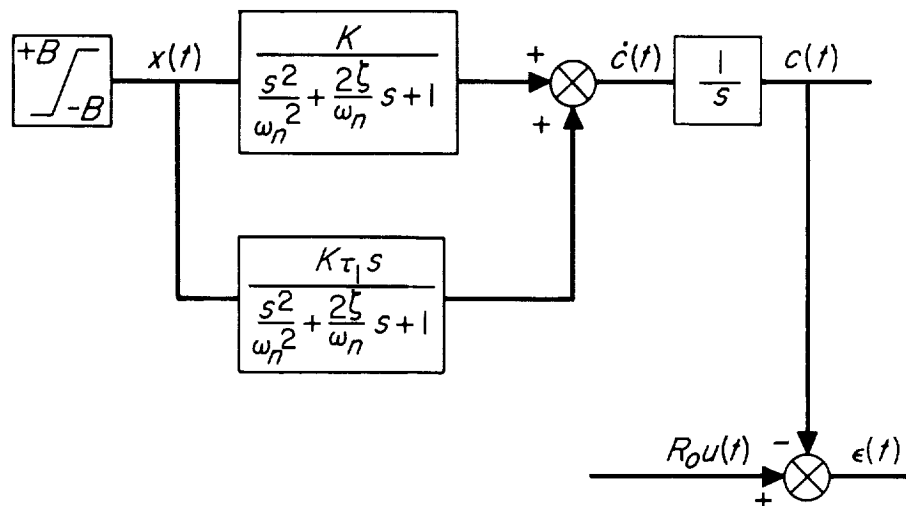
where

$$\alpha = \zeta + \sqrt{\zeta^2 - 1}$$

$$\bar{\alpha} = \zeta - \sqrt{\zeta^2 - 1}$$

As was the case for the type 1 third-order example considered in reference 1, these equations cannot be solved analytically. It is therefore necessary to resort to an iterative procedure and use a digital computer. The digital computer was programmed to solve equations (10), (11), and (12). If the solution obtained has a value of $|b/B| > 1$, this solution is discarded and equations (13), (14), and (15) are used.

As it turns out, for $\zeta > 1$, a sensible solution to these equations always exists. For $\zeta < 1$, however, a sensible solution does not exist for all values of the variables. One can give a physical argument which substantiates this fact. Consider the block diagram of the plant shown in sketch (f).



Sketch (f)

In order for $c(t)$ to reach R_0 (for R_0+) in the minimum time, $\dot{c}(t)$ should have the highest value permissible in the interval. The quantity, $x(t)$, during a large portion of this interval is equal to the positive limit level, B . The motion of $\dot{c}(t)$, until T_1 , is seen from sketch (f) to be the sum of two components. Since $x(t) = Bu(t)$ for $t < T_1$, $\dot{c}(t)$ can be expressed as (for $t < T_1$)

$$\dot{c}(t) = KB \left[\left(\text{unit step response of a second-order system} \right) + \tau_1 \left(\text{unit impulse response of a second-order system} \right) \right] \quad (16)$$

The unit step and impulse response of a second-order system for various values of ζ is given in figure 2.

As can be seen from figure 2 (or proved mathematically) for $\zeta > 1$, the impulse response is positive for all time. Since the unit step response is positive (for $\zeta > 0$), we can say that $\dot{c}(t)$ will be positive until T_1 if $\zeta > 1$, regardless of the value of τ_1 .

If $\zeta < 1$, however, the unit impulse is negative part of the time. This means that $\dot{c}(t)$, given by equation (16), can reverse sign for certain values of t , ζ , and τ_1 . For small values of T_1 , however, $\dot{c}(t)$ will be positive so a sensible solution using the $x(t)$ motion given in sketch (e) exists. These facts generally imply that the $x(t)$ motion chosen is non-optimum for all values of the variables. Rather than attempting to find other motions of $x(t)$, we have chosen to present the data where it is a sensible solution. Since the possible size step input or transient to most physical systems is bounded and since sensible solutions exist for a certain range of the step input (starting at zero), this restriction of scope is perhaps not as serious as it could be.

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The data summarizing the switch times, b/B , and input step magnitude for various values of ζ and τ_1 are presented in table II. It is believed that the range of the parameters chosen, ζ and τ_1 , should be sufficient for a large number of control systems.

A typical plot of $\omega_n T_1$ and $\omega_n T_m$ versus $R_0 \omega_n / KB$ for $\zeta = 0.5$ which shows the effects of $\omega_n \tau_1$ is presented in figure 3. As can be seen from this figure a sensible solution exists for all the plotted values of $\omega_n \tau_1$ except $\omega_n \tau_1 = 16$. Here, the solution does not exist for $R_0 \omega_n / KB \gtrsim 18$.

As can be seen also, the response time is much shorter for large values of $\omega_n \tau_1$ than for $\omega_n \tau_1 = 0$. Thus a large benefit in speed of response of the system is possible in cases where the plant has a zero close to the origin in comparison with other poles.

A Type 1 Third-Order Unstable Plant (With Zero)

The plant transfer function for this example (case 10, table I) is

$$G(s) = \frac{K(\tau_1 s + 1)}{s(\tau_2 s + 1)(\tau_3 s - 1)}$$

Arguments identical to the ones given for the previous example indicate the $x(t)$ motion should be one of the forms shown in sketch (e). Note also that the results of the second-order unstable plant presented earlier indicate T_1 should approach a finite limit as T_m and R_0 approach infinity.

If the motion of $x(t)$ shown in sketch (e) is assumed, the equations which must be satisfied in order that $E(s)$ be an entire function are

$$\frac{R_0}{BK\tau_2} = \frac{T_m}{\tau_2} - \frac{2T_1}{\tau_2} - \frac{b}{B} \frac{\tau_1}{\tau_2} \quad (17)$$

$$\left(1 - 2e^{T_1/\tau_2} + e^{T_m/\tau_2}\right) \left(1 - \frac{\tau_1}{\tau_2}\right) - \frac{b}{B} \frac{\tau_1}{\tau_2} e^{T_m/\tau_2} = 0 \quad (18)$$

$$\left[1 - 2e^{-\frac{T_1}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right)} + e^{-\frac{T_m}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right)}\right] \left[1 + \frac{\tau_1}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right)\right] + \frac{b}{B} \frac{\tau_1}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right) e^{-\frac{T_m}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right)} = 0 \quad (19)$$

Equations (17) to (19) are normalized in a manner such that solutions for $R_0/BK\tau_2$, T_m/τ_2 , T_1/τ_2 , and b/B are desired for various values of the ratios τ_1/τ_2 and τ_2/τ_3 . The ratio τ_1/τ_2 can be expressed as

$$\frac{\tau_1}{\tau_2} = \frac{\text{distance of stable pole from the origin of the } s \text{ plane}}{\text{distance of stable zero from the origin of the } s \text{ plane}}$$

similarly

$$\frac{\tau_2}{\tau_3} = \frac{\text{distance of unstable pole from the origin of the } s \text{ plane}}{\text{distance of stable pole from the origin of the } s \text{ plane}}$$

Expressing the ratios in this form gives an understanding of their meaning in terms of a pole-zero plot of the plant transfer function. One can also see that if $\tau_1/\tau_2 \gg 1$ that one might eliminate τ_2 as an approximation and use data for a simpler plant. Also note that for $\tau_1/\tau_2 = 1$ the pole and zero cancel and the data for the second-order unstable plant presented earlier is the solution.

If the solution of these equations gives a value of $|b/B|$ greater than unity, one must discard the answer and derive the solution using the motion of $x(t)$ given in sketch (e 2). If $x(t)$ is of this form, the equations which must be satisfied in order that $E(s)$ be an entire function are

$$\frac{R_0}{BK\tau_2} = \frac{2T_2}{\tau_2} - \frac{2T_1}{\tau_2} - \frac{T_m}{\tau_2} - \frac{\tau_1}{\tau_2} \quad (20)$$

$$\left(1 - 2e^{-\frac{T_1}{\tau_2}} + 2e^{-\frac{T_2}{\tau_2}} - e^{-\frac{T_m}{\tau_2}}\right)\left(1 - \frac{\tau_1}{\tau_2}\right) - \frac{\tau_1}{\tau_2} e^{-\frac{T_m}{\tau_2}} = 0 \quad (21)$$

$$\left[1 - 2e^{-\frac{T_1}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right)} + 2e^{-\frac{T_2}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right)} - e^{-\frac{T_m}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right)}\right]\left(1 + \frac{\tau_1}{\tau_3}\right) + \frac{\tau_1}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right) e^{-\frac{T_m}{\tau_2} \left(\frac{\tau_2}{\tau_3}\right)} = 0 \quad (22)$$

As was the case for the previous example, these two sets of equations, (17) to (19) and (20) to (22) must be solved by an iterative procedure. However, this example is somewhat simpler than the previous one since one can derive solutions which are valid for $T_m \rightarrow \infty$. In equation (19) for example, if $T_m \rightarrow \infty$ we find

$$T_1 = \tau_3 \ln 2 \quad (23)$$

Similarly, one can divide equation (18) by e^{T_m/τ_2} and then let $T_m \rightarrow \infty$ and find

$$\frac{b}{B} = \frac{\tau_2}{\tau_1} - 1 \quad (24)$$

Equation (23) shows that T_1 approaches the same limit for $T_m \rightarrow \infty$ as for the second-order example considered earlier. For the reasons given earlier, this is the maximum value of T_1 .

Equation (24) shows that if $\tau_2/\tau_1 \leq 2$ (or $\tau_1/\tau_2 \geq 0.5$) then as $T_m \rightarrow \infty$, $|b/B| \leq 1$. As it turns out, b/B is very small for T_m small and the value given by equation (24) is the maximum regardless of the value of T_m .

Equations (17) to (19) were solved by means of a digital computer for the quantities $R_0/BK\tau_2$, T_1/τ_2 , T_m/τ_2 , and b/B for various values of the ratios τ_1/τ_2 and τ_2/τ_3 . Only one value of $\tau_2/\tau_1 > 2$ (i.e., $\tau_1/\tau_2 = 0$) was computed; therefore equations (20) to (22) were needed for this case only.

The results of the computations are presented in table III. A typical plot of the tabulated results showing the effects of the ratio, τ_1/τ_2 , for one value of the ratio, τ_2/τ_3 , is presented in figure 4. As can be seen from figure 4(b), a zero in the left half plane much closer

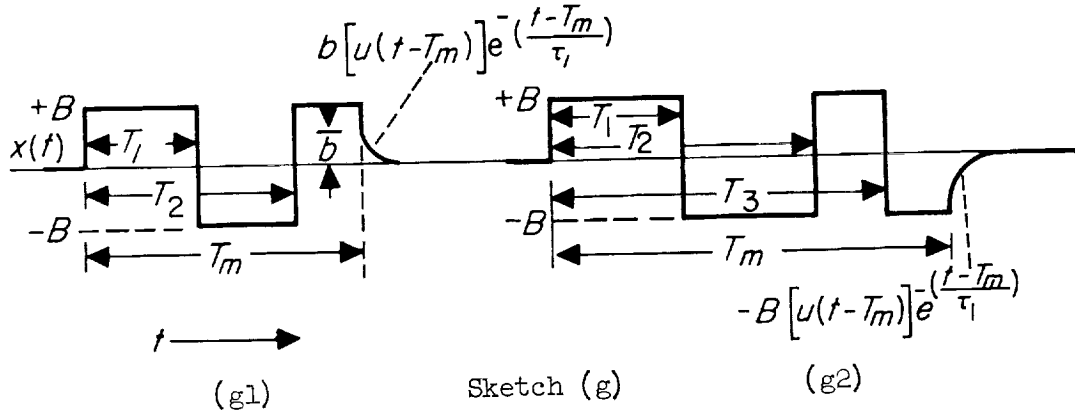
to the origin than a pole (i.e., $\tau_1/\tau_2 \gg 1$) provides a considerably smaller response time than if $\tau_1/\tau_2 = 0$. Thus, as was the case for the previous example, a left half plane zero of the plant can be used to advantage if a control system can be designed which provides the bang-bang followed by a tailing off exponential motion required of the saturated variable, $x(t)$.

A Type 2 Fourth-Order Plant (With Zero)

The plant transfer function for this example (case 11, table I) is

$$G(s) = \frac{K(\tau_1 s + 1)}{s^2[(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1]} \quad (25)$$

Arguments similar to the ones given previously show the optimum motion of $x(t)$ should be one of the two forms shown in sketch (g).



Note that these two motions are identical to sketch (e) except that an additional switch time has been added to the bang-bang part of the motion.

If the $x(t)$ motion is as shown in sketch (g 1), the equations which must be satisfied in order that $E(s)$ be an entire function are

$$2\omega_n T_1 - 2\omega_n T_2 + \omega_n T_m + \frac{b}{B} \omega_n \tau_1 = 0 \quad (26)$$

$$\left(1 - 2e^{\omega_n T_1} + 2e^{\omega_n T_2} - e^{\omega_n T_m}\right) \left(1 - \alpha \omega_n \tau_1\right) - \frac{b}{B} \alpha \omega_n \tau_1 e^{\alpha \omega_n T_m} = 0 \quad (27)$$

$$\left(1 - 2e^{\bar{\alpha}\omega_n T_1} + 2e^{\bar{\alpha}\omega_n T_2} - e^{\bar{\alpha}\omega_n T_m}\right) \left(1 - \bar{\alpha}\omega_n \tau_1\right) - \frac{b}{B} \bar{\alpha}\omega_n \tau_1 e^{\bar{\alpha}\omega_n T_m} = 0 \quad (28)$$

$$\begin{aligned} \frac{2R_0\omega_n^2}{BK} &= 2\omega_n \tau_1 (2\omega_n T_1 - 2\omega_n T_2 + \omega_n T_m) - 2(\omega_n T_1)^2 + 2(\omega_n T_2)^2 \\ &\quad - (\omega_n T_m)^2 - 2 \cdot \frac{b}{B} (\omega_n \tau_1) (\omega_n T_m) \end{aligned} \quad (29)$$

where

$$\left. \begin{aligned} \alpha &= \zeta + \sqrt{\zeta^2 - 1} \\ \bar{\alpha} &= \zeta - \sqrt{\zeta^2 - 1} \end{aligned} \right\} \quad (30)$$

If the motion of $x(t)$ is as shown in sketch (g 2), the equations which must be satisfied in order that $E(s)$ be an entire function are

$$2\omega_n T_1 - 2\omega_n T_2 + 2\omega_n T_3 - \omega_n T_m - \omega_n \tau_1 = 0 \quad (31)$$

$$\left(1 - 2e^{\alpha\omega_n T_1} + 2e^{\alpha\omega_n T_2} - 2e^{\alpha\omega_n T_3} + e^{\alpha\omega_n T_m}\right) \left(1 - \alpha\omega_n \tau_1\right) + \alpha\omega_n \tau_1 e^{\alpha\omega_n T_m} = 0 \quad (32)$$

$$\left(1 - 2e^{\bar{\alpha}\omega_n T_1} + 2e^{\bar{\alpha}\omega_n T_2} - 2e^{\bar{\alpha}\omega_n T_3} + e^{\bar{\alpha}\omega_n T_m}\right) \left(1 - \bar{\alpha}\omega_n \tau_1\right) + \bar{\alpha}\omega_n \tau_1 e^{\bar{\alpha}\omega_n T_m} = 0 \quad (33)$$

$$\begin{aligned} \frac{2R_0\omega_n^2}{BK} &= 2(\omega_n \tau_1)^2 - 2(\omega_n T_1)^2 + 2(\omega_n T_2)^2 - 2(\omega_n T_3)^2 \\ &\quad + (\omega_n T_m)^2 + 2(\omega_n \tau_1) (\omega_n T_m) \end{aligned} \quad (34)$$

Equations (26), (27), and (28) are solved for $\omega_n T_1$, $\omega_n T_2$, and b/B for assumed values of $\omega_n T_m$, ξ , and $\omega_n \tau_1$. If the value of $|b/B|$ obtained is less than unity, equation (29) is used to obtain $R\omega_n^2/BK$. If the value of $|b/B| > 1$, the solution is discarded and equations (31), (32), and (33) are solved for $\omega_n T_1$, $\omega_n T_2$, and $\omega_n T_3$ for the same values of $\omega_n T_m$, ξ , and $\omega_n \tau_1$. Equation (34) is used for the correct value of $R\omega_n^2/BK$. The equations were solved by an iterative method on a digital computer. The results are presented in table IV. A typical plot of the results is given in figure 5. As was the case for the previous examples, it can be seen from figure 5(b) that the minimum response time is reduced as τ_1 is increased.

As was the case for the type 1 third-order stable plant (with zero) considered earlier, a sensible solution to the equations does not exist for $\xi < 1$ for certain values of $\omega_n \tau_1$ and $\omega_n T_m$. The reason for this is similar to the one presented earlier except, in this case, the output acceleration reverses sign for certain values of $\omega_n \tau_1$ and $\omega_n T_m$ in the interval $t < T_1$ rather than the output velocity as for the previous case. The $x(t)$ motion of sketch (g) is nonoptimum for these ranges of parameters. Rather than attempting to find the optimum motion of $x(t)$ for all ranges of parameters we have chosen to present the data where it is a sensible solution.

A Type 2 Fourth-Order Unstable Plant (With Zero)

The plant transfer function for this example (case 12, table I) is

$$G(s) = \frac{K(\tau_1 s + 1)}{s^2(\tau_2 s + 1)(\tau_3 s - 1)} \quad (35)$$

The optimum motion of $x(t)$ is one of the two forms shown in sketch (g). As was the case for the previous unstable example T_1 must approach a limit as $T_m \rightarrow \infty$.

If the motion of $x(t)$ is as shown in sketch (g 1), the equations which must be satisfied in order that $E(s)$ be entire are

$$2 \frac{T_1}{\tau_2} - 2 \frac{T_2}{\tau_2} + \frac{T_m}{\tau_2} + \frac{b}{B} \frac{\tau_1}{\tau_2} = 0 \quad (36)$$

$$\left(1 - 2e^{\frac{T_1}{\tau_2}} + 2e^{\frac{T_2}{\tau_2}} - e^{\frac{T_m}{\tau_2}}\right) \left(1 - \frac{\tau_1}{\tau_2}\right) - \frac{b}{B} \frac{\tau_1}{\tau_2} e^{\frac{T_m}{\tau_2}} = 0 \quad (37)$$

$$\left[1 - 2e^{-\frac{T_1}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right)} + 2e^{-\frac{T_2}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right)} - e^{-\frac{T_m}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right)} \right] \left[1 + \frac{\tau_1}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right) \right] + \frac{b}{B} \left(\frac{\tau_1}{\tau_2} \right) \left(\frac{\tau_2}{\tau_3} \right) e^{-\frac{T_m}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right)} = 0 \quad (38)$$

$$\frac{2R_0}{BK\tau_2^2} = -2 \frac{\tau_1}{\tau_2} \left(2 \frac{T_1}{\tau_2} - 2 \frac{T_2}{\tau_2} + \frac{T_m}{\tau_2} \right) + 2 \left(\frac{T_1}{\tau_2} \right)^2 - 2 \left(\frac{T_2}{\tau_2} \right)^2 + \left(\frac{T_m}{\tau_2} \right)^2 + 2 \frac{b}{B} \left(\frac{\tau_1}{\tau_2} \right) \left(\frac{T_m}{\tau_2} \right) \quad (39)$$

If the motion of $x(t)$ is as shown in sketch (g 2), the equations which must be satisfied in order that $E(s)$ be entire are

$$2 \frac{T_1}{\tau_2} - 2 \frac{T_2}{\tau_2} + 2 \frac{T_3}{\tau_2} - \frac{T_m}{\tau_2} - \frac{\tau_1}{\tau_2} = 0 \quad (40)$$

$$\left(1 - 2e^{\frac{T_1}{\tau_2}} + 2e^{\frac{T_2}{\tau_2}} - 2e^{\frac{T_3}{\tau_2}} + e^{\frac{T_m}{\tau_2}} \right) \left(1 - \frac{\tau_1}{\tau_2} \right) + \frac{\tau_1}{\tau_2} e^{\frac{T_m}{\tau_2}} = 0 \quad (41)$$

$$\left[1 - 2e^{-\frac{T_1}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right)} + 2e^{-\frac{T_2}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right)} - 2e^{-\frac{T_3}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right)} + e^{-\frac{T_m}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right)} \right] \left[1 + \left(\frac{\tau_1}{\tau_2} \right) \left(\frac{\tau_2}{\tau_3} \right) \right] - \frac{\tau_1}{\tau_2} \left(\frac{\tau_2}{\tau_3} \right) e^{-\frac{T_m}{\tau_2} \frac{\tau_2}{\tau_3}} = 0 \quad (42)$$

$$\frac{2R_0}{BK\tau_2^2} = -\frac{2\tau_1^2}{\tau_2^2} + 2 \left(\frac{T_1}{\tau_2} \right)^2 - 2 \left(\frac{T_2}{\tau_2} \right)^2 + 2 \left(\frac{T_3}{\tau_2} \right)^2 - \left(\frac{T_m}{\tau_2} \right)^2 - 2 \left(\frac{\tau_1}{\tau_2} \right) \left(\frac{T_m}{\tau_2} \right) \quad (43)$$

The solution of these equations is by an iterative procedure so similar to previous examples that no further explanation is considered necessary. The solutions are presented in table V for the same values of τ_2/τ_3 and τ_1/τ_2 as were given for the type 1 third-order plant presented earlier.

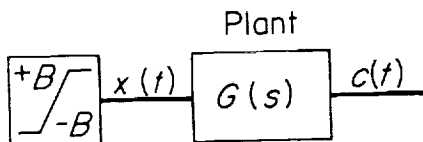
Typical plots of the data are shown in figure 6. As can be seen from figure 6(a), T_1/τ_2 approaches the limit $T_1/\tau_2 = (\tau_3/\tau_2)\ln 2$ as $R_0/BK\tau_2^2 \rightarrow \infty$. This is the same limit value as for the previous unstable case. As can be seen from figure 6(b), the minimum response time for a particular input decreases as τ_1 increases which has been true for all the plants considered.

A Method for Determining Approximate Switch Time Data

The data presented in this report and reference 1 cover a large number of plant transfer functions. The examples given here (in later sections) and those of reference 1 serve to show the switch time method has fairly general application if one designs to (a) force the first reversal time to be optimum and (b) force the zeros to shift in the s plane in such a manner that as the equivalent gain of the limiter reduces, the closed-loop poles remain in reasonably damped regions. In the examples considered it is only necessary to change the zero positions as a function of the error. Other examples may require further complications or additions; however, it is believed the general concepts are still applicable.

One of the limitations in applying the concept is the limited optimum switching time data available. The plant may be nonlinear or have a transfer function for which data are not available. It is the purpose of this section to show an approximate method for obtaining these data. The method is well suited to analog computer computation or tests on the actual hardware.

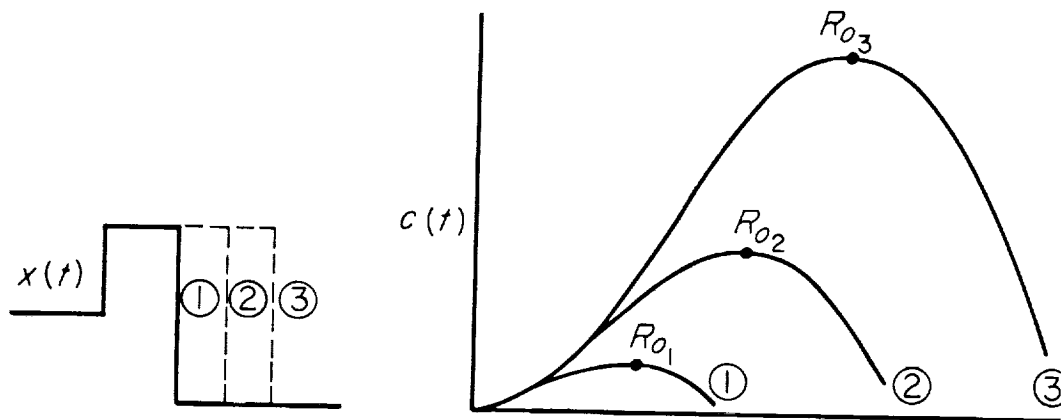
Consider the system of sketch (h). It is desired to know the optimum



Sketch (h)

first reversal time, T_1 , as a function of input step magnitude, R_0 . It has generally been shown that the optimum motion of $x(t)$ is to remain at the maximum in one direction (+B) until T_1 and then reverses to (-B), the maximum in the opposite direction. As the order of the system is increased, further switchings take place at later times.

The $c(t)$ motions for several $x(t)$ motions which are maximum one way and reverse at T_1 will generally be as shown in sketch (i).



Sketch (i)

To obtain an approximate curve one could plot the peak value of $c(t)$, (R_{01} , R_{02} , R_{03}), versus the switching times (1, 2, 3). If the plant is simulated on an analog computer or if actual hardware is tested such plots are easily accomplished since the $x(t)$ motion of sketch (i) can be readily generated by electronic devices such as interval timers. This simple scheme was originally suggested in reference 4. It can be seen to be exact for second-order plants since the output velocity is zero at $c(t)$ maximum and returning $x(t)$ to zero at this time (or using a tailing off exponential for plants with a zero in the left half plane) will cause no further change in $c(t)$.

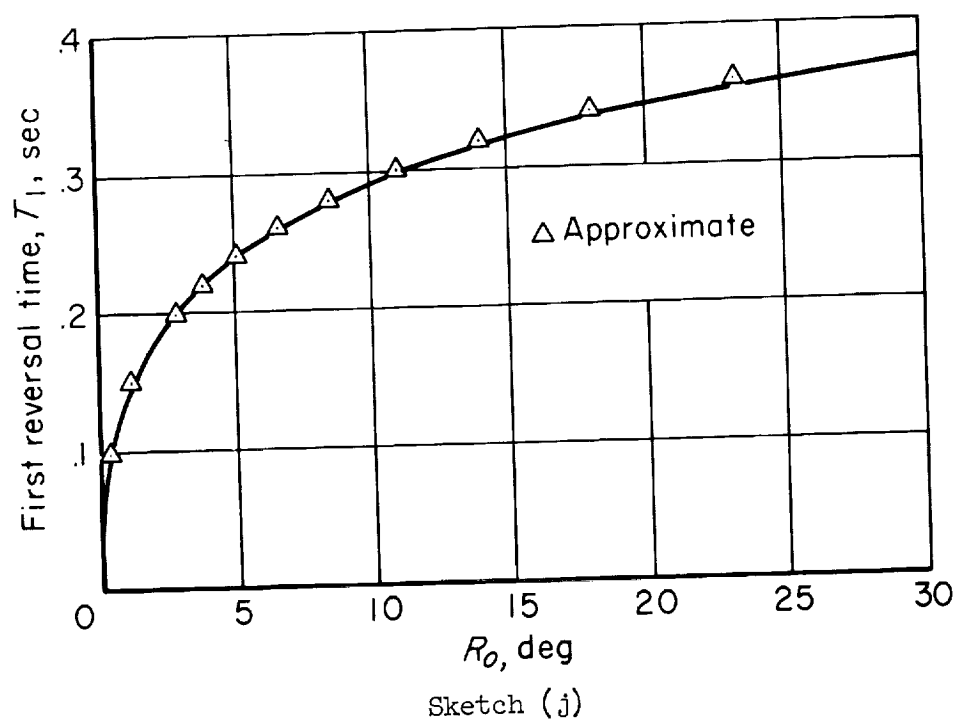
For higher order plants one questions the accuracy of such a simple technique. To obtain an idea of the relative accuracy the relationships between R_0 and T_1 used in this scheme can be compared to the exact relationship. This has been done for type 3 third-order and type 4 fourth-order plants. The results are

<u>Plant transfer function</u>	<u>Exact</u>	<u>Approximate</u>
$\frac{K}{s^3}$	$T_1^3 = \frac{R_0}{2KB}$	$T_1^3 = \frac{R_0}{1.91 KB}$
$\frac{K}{s^4}$	$T_1^4 = \frac{R_0(0.1766)}{KB}$	$T_1^4 = \frac{R_0(0.2107)}{KB}$

As can be seen, the error is about 5 percent for the third-order plant and 15 percent for the fourth-order plant. If some of the roots of the plant denominator are in the left half plane, it is believed the error

will be less than that given by the above cases. The reason for this belief is that some of the output derivatives are bounded (by the fact that $x(t)$ is bounded) when there are poles in the left half plane. The result is that the ensuing motions required to force all derivatives of $c(t)$ to be zero have less effect than in the cases shown above. To obtain an idea of the relative accuracy for unstable plants approximate data are compared in sketch (j) with exact data (from table V) for a plant transfer function

$$G(s) = \frac{3.125(s + 1)}{s^2(0.183 s + 1)(0.683 s - 1)}$$



As can be seen, the approximate data, obtained from an analog computer simulation, are in close agreement with the exact curve.

One point should be mentioned with regard to the use of approximate first reversal time data. That is, the approximate T_1 is slightly larger than the exact T_1 . The result is that one will obtain a design which has some overshoot. This overshoot is usually readily removed by slight adjustments in the nonlinear function.

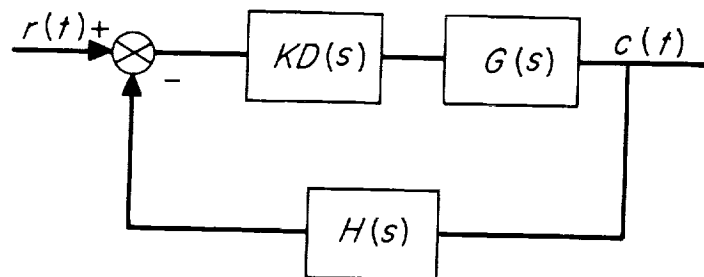
OPTIMUM SYSTEMS

It is the purpose of this section to consider the problem of designing an optimum feedback control system. This type system⁴ will be defined as one which maintains the error as small as possible under the constraint that the plant input variable is bounded. Unfortunately, very little is known about such systems at the present time. If the system is continuous of the type shown in sketch (a), and it is subjected to initial conditions only ($r(t) = 0$), it is generally accepted that the relay or bang-bang system is the optimum.⁵ If the system is of the sampled-data type ($m(t)$ of sketch (a) is a staircase signal), little is known about the design of the optimum system.

In this section the discussion will be restricted to the plant transfer functions given in table I.

The Optimum Continuous Linear System

Prior to discussing cases where saturation is present, it is pertinent to discuss what the previous definition of an optimum system means in terms of a linear system. To have the optimum (i.e., zero error to any input) it is quite obvious that the closed-loop transfer function relating $C(s)/R(s)$ of sketch (k) must be unity.



Sketch (k)

⁴Another definition of optimum is made when the input is contaminated with noise. The optimum system is then defined as one which minimizes the quantity $H_d r(t) - c(t)$. The input (without noise) is $r(t)$; $c(t)$ is the output; and H_d is the mathematical representation of the desired response which may or may not be physically realizable (see, e.g., ref. 5).

⁵Optimum in the sense that the error is restored to zero in the minimum time.

Since the closed-loop transfer function must be unity then

$$\frac{C(s)}{R(s)} = 1 = \frac{KD(s)G(s)}{1 + KD(s)G(s)H(s)} \quad (44)$$

Thus, one obtains the following two conditions if equation (44) is to be satisfied:

$$H(s) = 1 \quad (45)$$

$$K = \infty \quad (46)$$

A third condition is obtained from stability considerations:

$$D(s)G(s) = \frac{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_0}{b_ns^n + b_{n-1}s^{n-1} + \dots} \quad (47)$$

The roots of the numerator of equation (47) must have real parts which are negative and the order of the numerator can only be 1 lower than the order of the denominator. The following arguments based on root-locus rules verify this.

a. For $K \rightarrow \infty$ all poles of the root locus equation move to zeros; therefore, all zeros must be in the left half of the s plane or the system will be unstable. The roots of the numerator must, therefore, have negative real parts.

b. If the order of the numerator is 3 (or higher) less than the denominator, some of the closed-loop poles for $K \rightarrow \infty$ are in the right half plane; therefore, the system will be unstable.

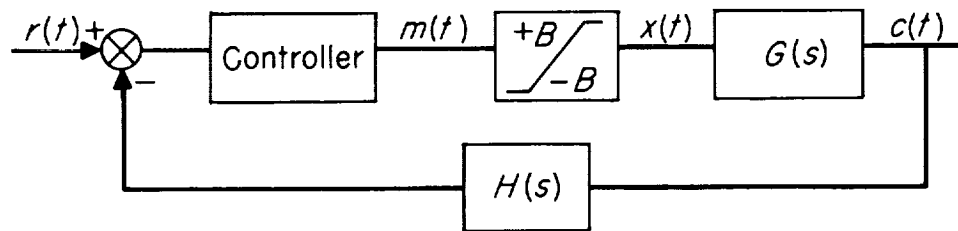
c. If the order of the numerator is 2 lower than the denominator, two of the poles move to infinity (as $K \rightarrow \infty$) with asymptotes which have a finite real part. This real part can be negative; however, since it is finite, the impulse response time is finite. Therefore, the closed-loop transfer function cannot be unity.

d. If the order of the numerator is 1 less than the denominator, one pole moves to infinity (as $K \rightarrow \infty$) along the negative real axis. With $K = \infty$ this provides a closed-loop transfer function of first order with a time constant equal to zero which is essentially a transfer function of unity.

The important point gained from this consideration of linear systems is that if $G(s)$ has n poles and no zeros, $n-1$ derivatives of the error must be available in order to design the optimum linear system.⁶

The Optimum Continuous Control System With Saturation

Consider the system shown in sketch (1).



Sketch (1)

If the controller's characteristics are linearized about zero input, it can be seen that the same conditions given for the optimum linear system must be true for this system; that is,

$$H(s) = 1$$

$$\frac{M(s)}{E(s)}_{K \rightarrow \infty} = K(1 + a_1 s + a_2 s^2 + a_3 s^3 + \dots)$$

The roots of s of the equation determined by setting

$$\frac{M(s)}{E(s)} = 0$$

must be in the left half of the s plane. Furthermore, since $x(t)$ is bounded and $X(s)G(s) = C(s)$, certain output derivatives or sums of output derivatives are bounded. For example if

⁶Practically speaking, the optimum linear system cannot exist, since it is impossible to differentiate. Any practical differentiating circuits also introduce poles which, when included in the system, cause the system to show an instability at some finite value of gain, K . If the poles introduced by the differentiating circuits are sufficiently far from the origin in the s plane compared to the plant poles and to the zeros introduced by the differentiating circuits, then one can obtain a system design which approximates the optimum (with a finite gain) over a low-frequency band width.

$$G(s) = \frac{K}{s(\tau_2 s + 1)} \quad (48)$$

then (see sketch (1))

$$|x| = \left| \frac{\tau_2 \ddot{c} + \dot{c}}{K} \right| \leq B \quad (49)$$

If these same bounds are placed on the input which for this example gives

$$\left| \frac{\tau_2 \ddot{r} + \dot{r}}{K} \right| \leq B \quad (50)$$

then the plant has the capability of following the input exactly. Thus, bounding $x(t)$ forces one to put certain bounds on derivatives of the input (depending on $G(s)$) if zero error is to be obtained.

The question therefore is what can be said about the optimum system when equation (50) is not satisfied. One answer could be that, regardless of the present error and its derivatives, we would like $x(t)$ to take the optimum motion to reduce the error to zero in a minimum time.⁷

If this answer is correct, then the characteristics of the controller⁸ would be (with $H(s) = 1$)

$$m(t) = Kf(\epsilon, \dot{\epsilon}, \ddot{\epsilon}, \dots) \quad (51)$$

where

$$K = \infty$$

The function of error and its derivatives, $f(\epsilon, \dot{\epsilon}, \ddot{\epsilon}, \dots)$, is the equation for the optimum switching surface in the phase space $\epsilon, \dot{\epsilon}, \ddot{\epsilon}, \dots$.

A second answer to the question has been pursued by Hopkin and Wang (ref. 6). Their answer amounts to making the error zero at some predicted time in the future. For this answer $m(t)$ becomes

$$m(t) = Kf(\epsilon, \dot{\epsilon}, \ddot{\epsilon}, \dots, \dot{r}, \ddot{r}, \dots) \quad (52)$$

where

$$K = \infty$$

⁷The input could be such that it was impossible for the error to ever be zero. For example, the plant given by equation (48) could not follow a constant acceleration input in a system of the type given in sketch (1), since the limit on $x(t)$ forces the output velocity to be bounded.

⁸The input to the plant $x(t)$ can (for $K = \infty$) be written as $B \operatorname{sgn} f(\epsilon, \dot{\epsilon}, \ddot{\epsilon}, \dots)$. There is a perfect equivalence between considering the controller to be an infinite gain device and the more customary way of lumping the controller and limiter together and calling the combination an ideal relay.

The derivatives of r occurring in equation (52) are a result of prediction of the future error. For the second-order example in which they had run simulation tests their system worked considerably better than a linear function of e and $\dot{e}$ alone; however, no data were presented for the system of equation (52) where nonlinear functions are required.

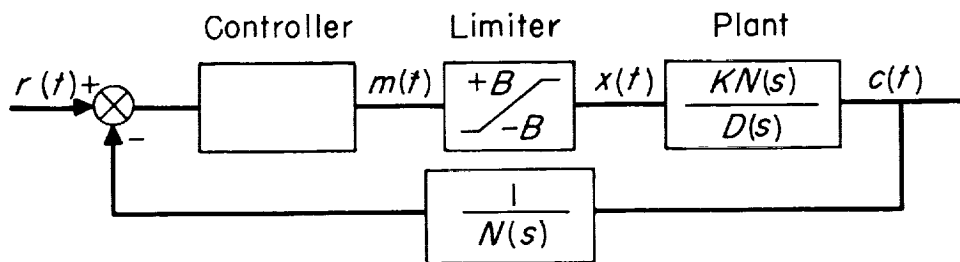
In many cases it is not possible to measure the input; that is, the error is sensed directly (for example radar or infrared seekers). For these cases the only choice for an optimum would be given by equation (51).

There is an apparent need for more work on the problem of the optimum system where saturation is present; however, we will assume the system of the type given by equation (51) is the optimum for many situations where the input is not sensed directly. If this assumption is valid, there are previous results available. Table VI lists previous work in which the optimum controllers of some of the plants (given by case number) can be found. The expressions are written out for the examples where this is possible. As can be seen from the table, to the authors' knowledge, only the simplest plants have been studied sufficiently by previous investigators to define the optimum switching surface. For these cases, a study of the reference for the particular plant involved allows one to determine the complexity of the control equation required if one desires the optimum system. The minimum response data of reference 1 and the previous section demonstrate what can be gained (compared to some linear controller) by the use of this usually complicated control equation.

A
3
9
3

The Influence of Plant Zeros on the Optimum System

It was shown in sections 3.5 and 3.8 of reference 1 and the previous section of this report that zeros in the plant transfer function make the minimum response time considerably lower than the value obtained when the zeros are neglected. A system which gives the relay, or bang-bang, solution can always be obtained with zeros in the plant transfer function (if they are in the left half plane) by the simple artifice of adding a lag network in the feedback path as was shown in sketch (d) and repeated in sketch (m) for the convenience of the reader.



Sketch (m)

The terms $N(s)$ and $D(s)$ represent the numerator and denominator of $G(s)$, respectively, written in polynomial form. The controller equation is then that given or referred to in table VI for the particular plant (if such data are available).

It has already been shown that this system is nonoptimum for step inputs and it is quite easy to show that this system is not the optimum for slow changing inputs. To show this, one linearizes the controller equations around zero input. For a very high gain controller then

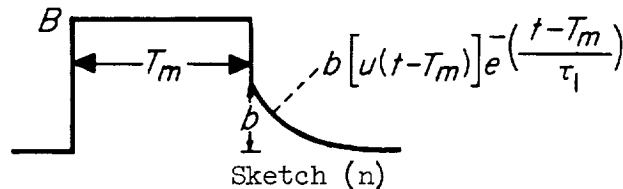
$$\frac{C(s)}{R(s)} \approx N(s) \quad (53)$$

or, the output "leads" the input. Two cases will be investigated in this section to show how one can use the knowledge gained thus far in deriving an optimum system.

A type 1 second-order plant.— For this case

$$G(s) = \frac{K(\tau_1 s + 1)}{s(\tau_2 s + 1)}$$

It was shown in chapter III of reference 1 that if one restricts his interest to plants where $\tau_1/\tau_2 > 1/2$, then the optimum motion for $x(t)$ for step inputs is as shown in sketch (n). The error to step inputs is



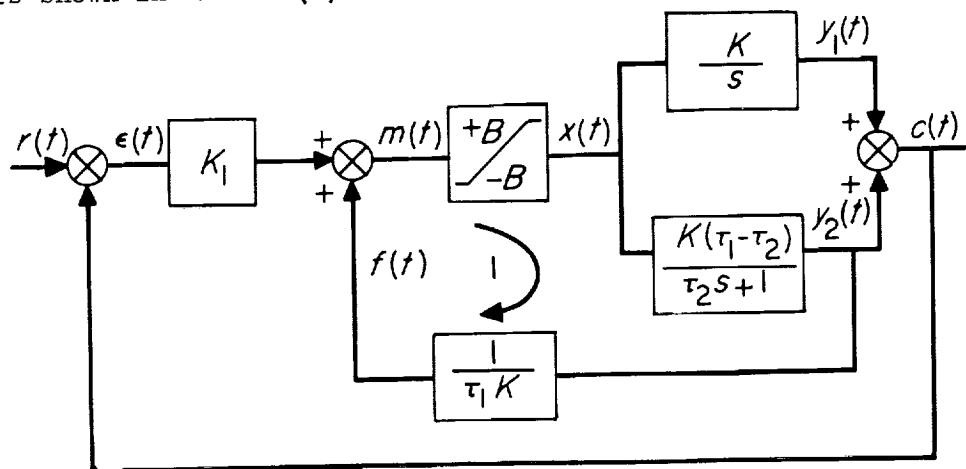
zero when $t = T_m$. Thus, the control system desired is one that holds $x(t)$ on its limits until the error is zero and then causes the motion of $x(t)$ to have an exponential decay of time constant, τ_1 .

The amplitude of the tailing off exponential, $x(T_m) = b$, is

$$b = B \left(1 - e^{-\frac{T_m}{\tau_2}} \right) \left(1 - \frac{\tau_2}{\tau_1} \right) \quad (54)$$

(See ref. 1, pp. 44-50 for the derivation of this equation.)

A control system which provides the optimum motion of $x(t)$ of sketch (n) is shown in sketch (o).



Sketch (o)

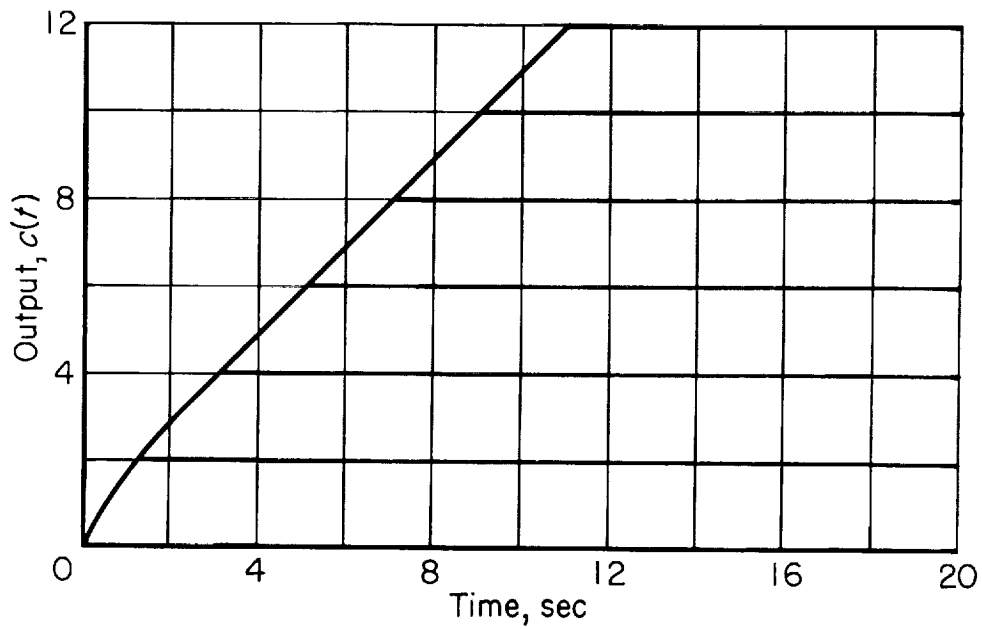
The quantity K_1 of sketch (o) approaches infinity with the restriction that $\epsilon K_1 = \infty$ if ϵ has any value and $\epsilon K_1 = 0$ if $\epsilon = 0$.

A study of sketch (o) shows that the main control of $x(t)$ comes from the error signal $e(t)$. In other words, if the error has any value whatsoever, $c(t)$ will be moving in such a direction to reduce the error as fast as possible; that is, $\dot{c}(t)$ will be at its maximum. Only when the error is zero does the signal $f(t)$ have any effect. The time constant of loop number 1 when $x(t)$ is not at its limit can be readily calculated to be equal to τ_1 . The value of $x(T_m)$ can be seen from sketch (o) to be equal to $y_2(T_m)/\tau_1 K$. Since $y_2(0) = 0$, the value of $y_2(T_m)$ is readily calculated by the Laplace transform to be

$$y_2(T_m) = L^{-1} \left\{ \left(\frac{B}{s} \right) \left[\frac{K(\tau_1 - \tau_2)}{(\tau_2 s + 1)} \right] \right\} \Big|_{T_m} = BK\tau_1 \left(1 - \frac{\tau_2}{\tau_1} \right) \left(1 - e^{-\frac{T_m}{\tau_2}} \right) \quad (55)$$

It can be seen that $x(T_m) = y_2(T_m)/K\tau_1$ agrees with equation (54).

No system has been derived for the case of $\tau_1/\tau_2 < 1/2$; however, since one reversal of $x(t)$ is required before the error is zero, it is probable that a more complicated system is required than that shown in sketch (o). Sketch (p) shows some typical time histories of the output response, $c(t)$, for various step input magnitudes. For this example $\tau_1 = 2$, $K = 1$, $\tau_2 = 1$, and $|B| = 1$.

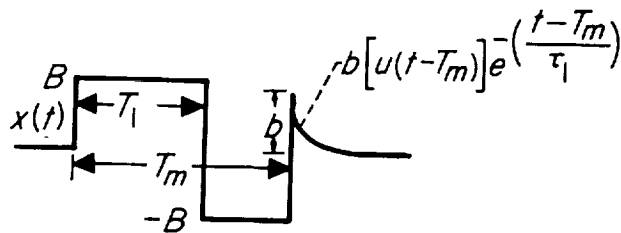


Sketch (p)

A type 2 third-order plant.— For this example

$$G(s) = \frac{K(\tau_1 s + 1)}{s^2(\tau_2 s + 1)}$$

If one restricts his interest to examples where $\tau_1/\tau_2 > 1/2$, then the optimum motion of $x(t)$ is as given in sketch (q).

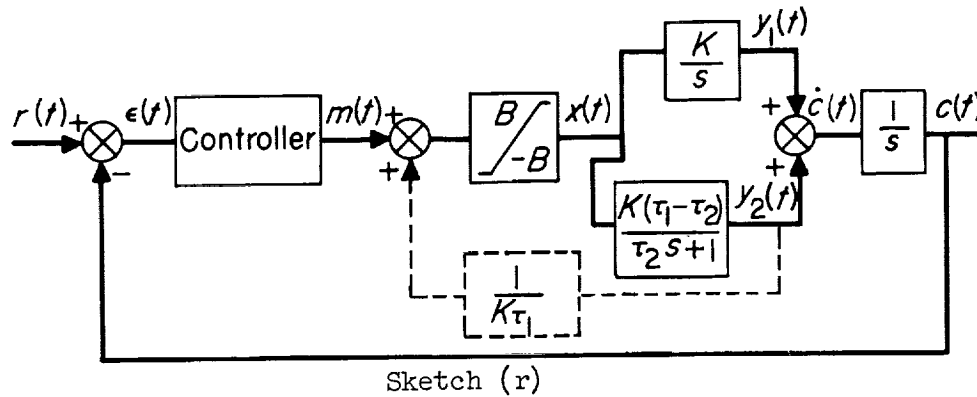


Sketch (q)

One can expand $G(s)$ and define state variables as follows:

$$G(s) = \frac{K(\tau_1 s + 1)}{s^2(\tau_2 s + 1)} = \frac{1}{s} \left[\frac{K}{s} + \frac{K(\tau_1 - \tau_2)}{\tau_2 s + 1} \right] = \frac{1}{s} \left[\frac{Y_1(s)}{X(s)} + \frac{Y_2(s)}{X(s)} \right] \quad (56)$$

This system is shown in the following block diagram.



The dotted feedback in sketch (r) has been added here for the same reason as the previous example; that is, if we get the error and error rate to zero, this feedback allows $y_1(t)$ and $y_2(t)$ to change while $\dot{c}(t)$ remains zero and $c(t)$ remains a constant (or zero). The fact that the solution following T_m of sketch (r) will be correct can be seen by checking the time constant of the dotted loop as well as reviewing section 3.8 of reference 1.

The completion of the design requires that the controller equations be determined and realized. In order to overcome the dotted feedback the gain of the controller must be infinite. Consider step inputs only; the motion shown in sketch (q) shows that only one reversal need take place since, when $t = T_m$, both error and error rate are zero. This suggests that an equation which is a function of ϵ and $\dot{\epsilon}$ of form, for example

$$m(t) = [\epsilon + f(\dot{\epsilon})]K_1 \quad (57)$$

$$K_1 \rightarrow \infty$$

should be sufficient.

The nonlinear function, $f(\dot{\epsilon})$, given in equation (57) was calculated using the method presented in chapter II of reference 1 to make T_1 equal to the optimum, and it was found that this scheme will not work since

$m(t)$ given by equation (57) does not reverse sign at T_1 if $x(t)$ is the shape of sketch (q).⁹ The reason for this is best explained if the switching curve in the phase plane ($\epsilon, \dot{\epsilon}$) is determined and an optimum trajectory for one input magnitude is shown. We know all the necessary information to compute both the trajectory and the optimum switching curve. For example

$$\epsilon(t) = R_0 u(t) - L^{-1} \left[X(s) G(s) \right] \quad (58)$$

$X(s)$ is the Laplace transform of $x(t)$ shown in sketch (q). The relationship between R_0 , T_1 , and T_m is determined from the optimum curves of chapter III of reference 1. The error velocity is

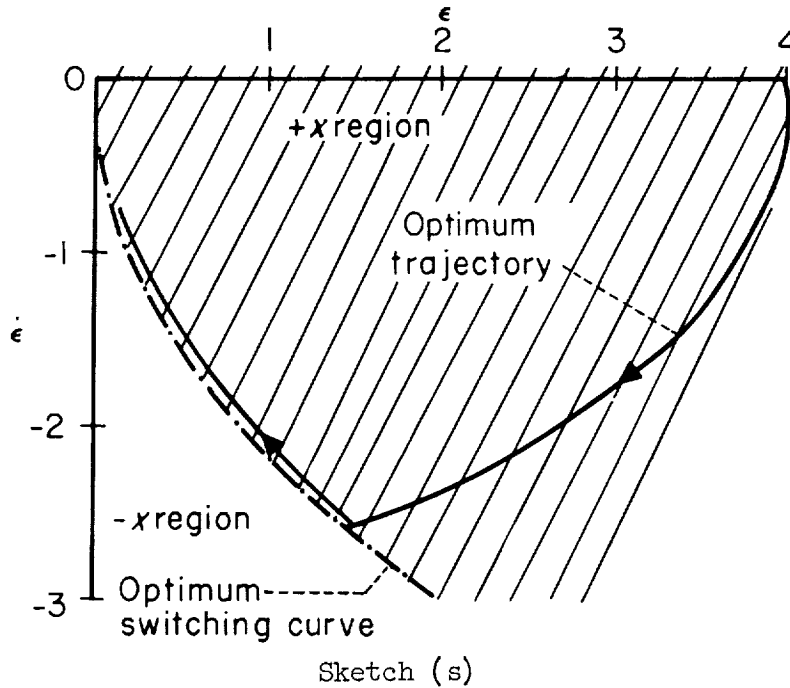
$$\dot{\epsilon}(t) = R_0 \delta(t) - L^{-1} \left[sX(s) G(s) \right] \quad (59)$$

The δ function may be omitted since it is only an impulse at the beginning of the trajectory. For any assumed value of R_0 , one can therefore compute and plot ϵ versus $\dot{\epsilon}$.

The optimum switching curve is found by plotting $\epsilon(T_1)$ versus $\dot{\epsilon}(T_1)$ as obtained from equations (58) and (59); R_0 in equation (58) is obtained from the curves of chapter III (ref. 1) for each value of T_1 .

The results of these computations for the example where $B = 1$ and $G(s) = (2s + 1)/s^2(s + 1)$ are shown in sketch (s). An examination of the optimum trajectory after it meets the optimum switching curve shows the reason why the switching function of equation (57) is unsatisfactory. The optimum trajectory stays in the $+x$ region; therefore, a system which uses this scheme cannot be expected to be optimum. The simulation results showed the system to have a reasonably large first overshoot, but no undershoot to the response. The amount of overshoot was smaller than that which would be given by a system of the type shown in sketch (m); however, any overshoot at all means we do not have the optimum system.

⁹A note should be added that this is not a limitation of the switch-time method. This method is based on the fact that we have a linear region and the switch-time method simply provides techniques for determining non-linear functions which provide near-optimum response for large step inputs. For the case at hand, we are attempting to use the switch-time method for determining an optimum switching surface in the phase space, since this is a third-order problem. We therefore cannot use the switch-time method directly and expect to have an optimum system.



What is really needed to obtain the optimum system is the equation for the switching surface in the phase space. This equation is not known at this time, so we shall, instead, design the system which will have optimum response to steps but nonoptimum response to other inputs. We must introduce a new variable in the system so that when the trajectory hits the switching surface, x will change sign. The particular system derived here will, in reality, be an approximate optimum system and should perhaps have been introduced in the next section. This was not done since a somewhat different design viewpoint will be introduced then.

It may be noted from sketch (s) that if a straight line is drawn from the origin to the desired switching point on the optimum switching curve, then the optimum trajectory will always be to the left of this line. This suggests that a switching equation of the type

$$m(t) = K_1[\epsilon + \dot{\epsilon}f(y_3)] \quad (60)$$

$$K_1 \rightarrow \infty$$

will be satisfactory if the variable y_3 is "slow" compared with $\dot{\epsilon}$. There are probably a number of choices which could be made; however, for this example, the choice was that

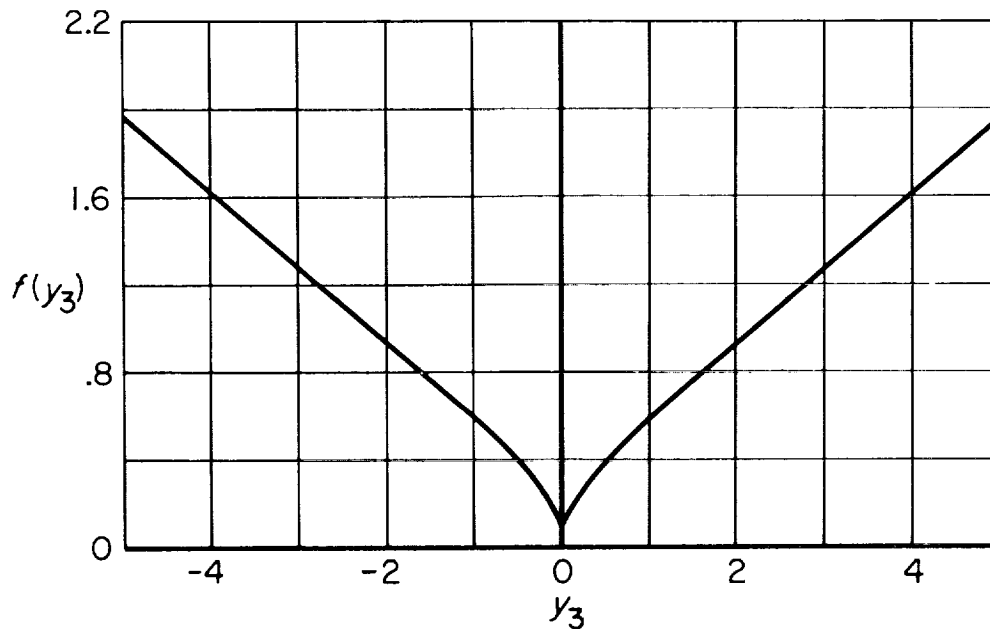
$$y_3(t) = y_1(t) - y_2(t) \quad (61)$$

For the particular values of τ_1 and τ_2 chosen for the example,

$$Y_3(s) = \frac{sC(s)}{2s + 1}$$

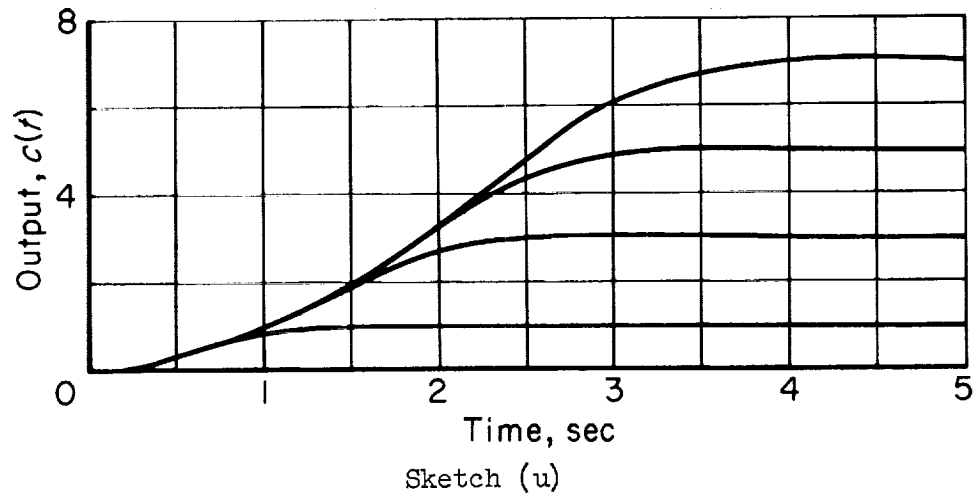
Thus $y_3(t)$ is nothing more than the output velocity fed through a time lag. Since the choice was not based on phase space considerations, it, therefore, is not surprising that the results of tests conducted on the analog computer showed optimum response to steps and certain initial conditions but showed nonoptimum response to other initial conditions. In all cases where nonoptimum response was found, however, it was due to excessive damping rather than instabilities or low damping.

The function $f(y_3)$ can be obtained for any example by forcing the first reversal time to be correct by the switch time method given in chapter II of reference 1. For the example case, the function $f(y_3)$ is shown in sketch (t). The step responses obtained from the analog computer



Sketch (t)

are shown in sketch (u). As was mentioned, good response was obtained for a number of initial conditions; however, it was not the optimum for all conditions. The question remains then as to what the optimum controller is for this example. The need for further research for the case where the plant has zeros is certainly indicated.



The Influence of an Unstable Pole on the Optimum System

As has been shown earlier, if the plant has an unstable pole, the optimum first reversal time of the saturated variable must approach a finite limit as $R_0 \rightarrow \infty$; otherwise, control over the plant is impossible. To demonstrate what this implies in terms of the optimum system it is convenient to use a type 1 second-order plant. The optimum switching times for this example are presented in figure 1.

The plant transfer function is

$$G(s) = \frac{K}{s(\tau s - 1)}$$

Although one can obtain the optimum switching curve by either the switch-time method or phase plane for second-order problems (as was shown in ref. 1), it is believed the phase plane derivation is the simplest.

By use of the fact that $x(t)$ is either $\pm B$ one can derive the trajectory equations in the phase plane $c(t)$, $\dot{c}(t)$. For initial conditions (or steps) one can substitute ϵ for $-c$ and $\dot{\epsilon}$ for $-\dot{c}$. For this example

$$\tau \ddot{c}(t) - \dot{c}(t) = \pm KB \quad (62)$$

Let

$$\left. \begin{aligned} c(t) &= x & \dot{c}(t) &= y \\ \text{then} & & \ddot{c}(t) &= \frac{dy}{dt} = \frac{dy}{dt} \left(\frac{dt}{dx} \right) y = y \frac{dy}{dx} \end{aligned} \right\} \quad (63)$$

Substituting the relationships of equation (63) in (62) one obtains

$$\frac{dy}{dx} = \frac{1}{\tau} \left(\frac{y \pm KB}{y} \right) \quad (64)$$

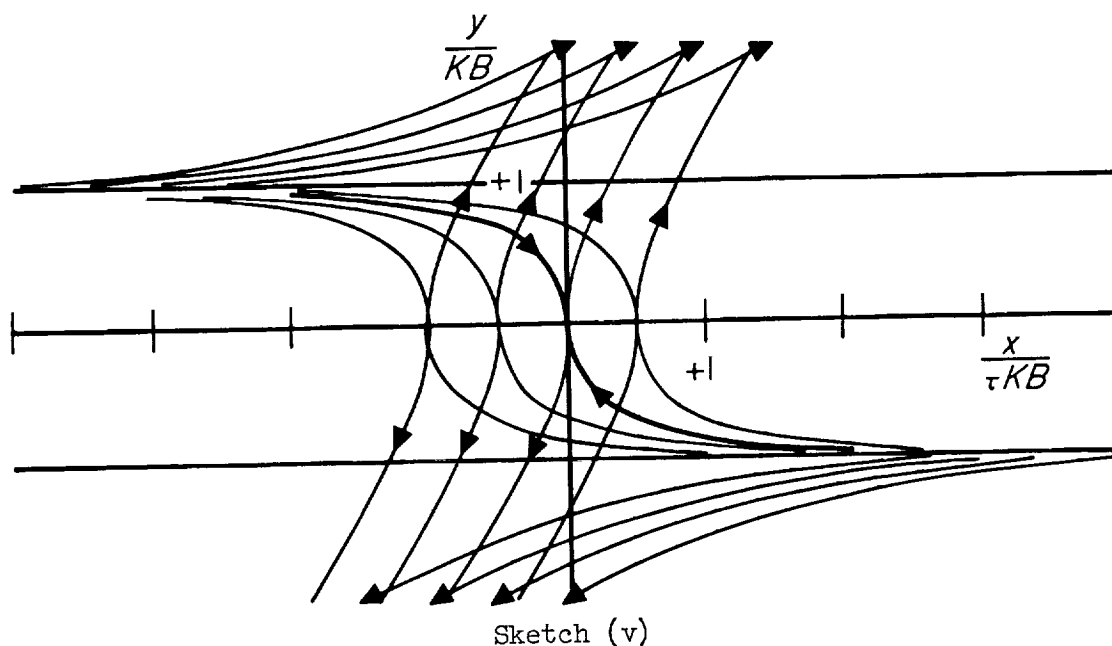
The solution of equation (64) is

$$\frac{x}{\tau} = y \mp KB \ln(y \pm KB) + C \quad (65)$$

If equation (65) is divided by KB one obtains

$$\frac{x}{\tau KB} = \frac{y}{KB} \mp \ln(\pm KB) \mp \ln \left(1 + \frac{y}{\pm KB} \right) + C_1 \quad (66)$$

If C_1 is chosen to be $\pm \ln(\pm KB)$ then x will be zero when y is zero. With this choice of constant, equation (66) represents the two branches of the optimum switching curve. Typical trajectories along with the optimum switching curve are shown in sketch (v). As is seen in sketch (v),



if $|y/KB| < 1$, there are trajectories which lead to the origin. If $|y/KB| > 1$, there are no trajectories which lead to the origin. Thus, $|y/KB|$ must be kept less than unity if one is to maintain control over the unstable plant.

It is apparent that the optimum system for step inputs or initial conditions will use an ideal relay whose switching equation could be of the form

$$f(\epsilon) + \dot{\epsilon} = 0 \quad (67)$$

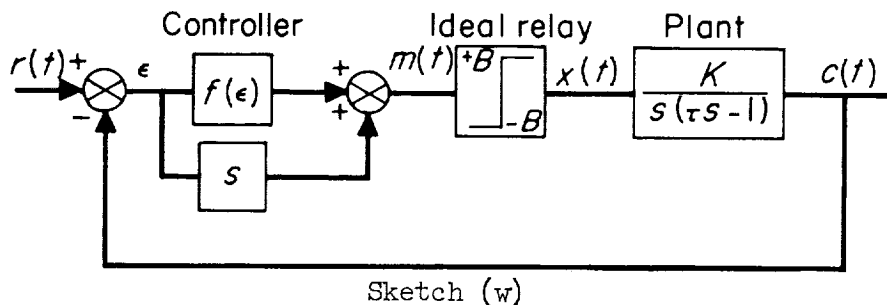
or

$$\epsilon + f(\dot{\epsilon}) = 0 \quad (68)$$

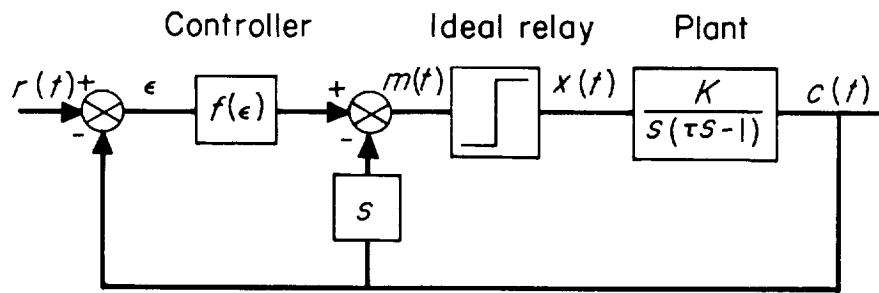
Equations (67) and (68) are the only possibilities if only functions of a single variable are considered.

The function $f(\epsilon)$ in normalized form is the optimum curve shown in sketch (v) if one views $x/KB\tau$ as ϵ , the input to the function generator. Similarly, $f(\dot{\epsilon})$ is the optimum switching curve shown in normalized form in sketch (v) if one views y/KB as $\dot{\epsilon}$, the input to the function generator. From practical standpoints it is believed that equation (67) is easier to fabricate since the function has the characteristic of a saturating device.

The optimum system for steps and initial conditions will therefore be of the form shown in sketch (w).



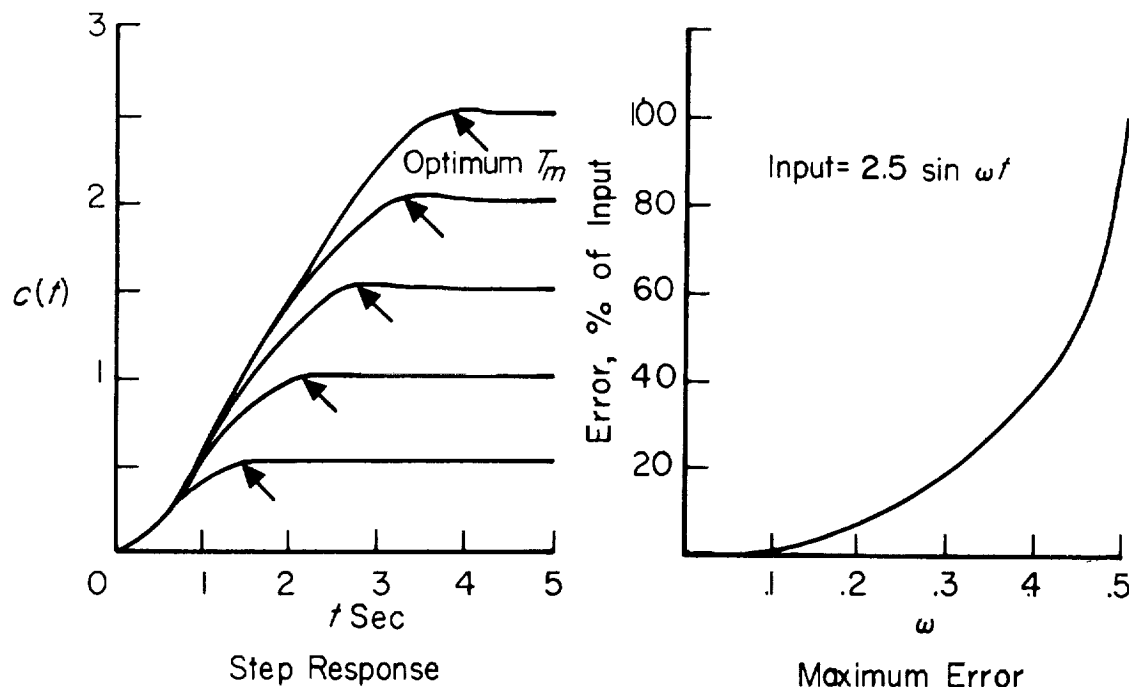
This system is probably near optimum for slowly changing inputs (i.e., $\dot{r}(t) \ll KB$); however, it certainly is nonoptimum for $\dot{r}(t) > KB$ since this condition (if held for any length of time) could drive the system into instability. One way of preventing the input from being able to drive the system into instability is to use output rate feedback rather than error rate feedback. This alternative is shown in sketch (x).



Sketch (x)

The problem with this alternative is that zero error is not possible for changing inputs. The $f(\epsilon)$ function shown in sketch (w) is very high gain for small errors, however, so the error for slowly changing inputs will be small. It shall be assumed that the system shown in sketch (x) is a near optimum system. The optimum system probably requires sensing of input derivatives and will not be determined here.

Some simulation tests were made of the system shown in sketch (x) for $B = K = \tau = 1$. The step response for several input magnitudes and the maximum error versus frequency for a constant amplitude sine wave input are shown in sketch (y). The step responses show the characteristic



Sketch (y)

of velocity limiting for the large inputs. This velocity limiting is a result of the nonlinear function of sketch (v). It must exist in order to prevent the system from becoming unstable. The maximum error versus

frequency curve shows the characteristic one would expect from sketch (v). That is, for $\dot{r}(\max) = 2.5\omega < 1/4$, the error is quite small. As the maximum rate of the input approaches unity and above, however, the maximum error increases rapidly.

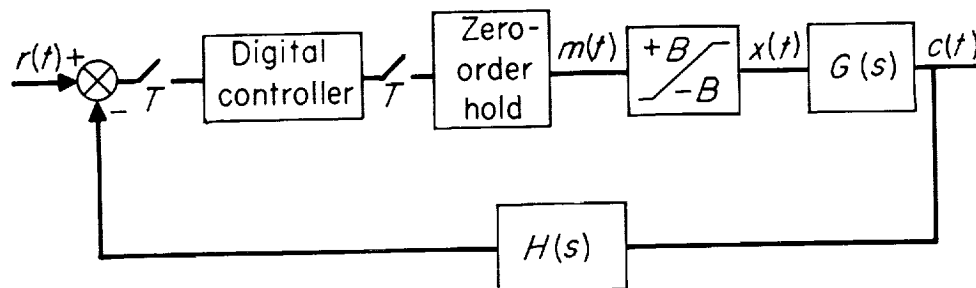
Optimum Sampled-Data Systems

As has been shown, the optimum, continuous, saturated control system is directly obtained from the relay, or bang-bang, solution (at least, for stable plants). There still remain questions as to whether these systems are optimum for all inputs and certainly practical questions as to how one differentiates the error a number of times. The problem, at least, is very well understood in comparison to saturated sampled-data control systems. Perhaps because of the newness of sampled-data systems and the additional complexity caused by the introduction of sampling, very little is known about optimum, saturated, sampled-data systems.

It is the purpose of this section to consider the questions:
 (1) What is a definition of the optimum, saturated, sampled-data system and what are its qualitative characteristics? (2) What is the optimum sampled-data system for a type 1 first-order plant? and (3) What are some of the pitfalls of trying to derive optimum systems for higher order plants? These three questions are considered in sequence.

An optimum, saturated, sampled-data system shall be defined as one which, regardless of the input or initial conditions, will maintain the error as small as possible with the operating sampling period, T , and with a limited input to the plant. The addition of the T is what makes the problem difficult. As $T \rightarrow 0$ the sampled-data system approaches the continuous system. As T becomes very large, the sampled-data system performance with respect to varying inputs and response time to step inputs becomes very poor.

Consider the system shown in sketch (z).



Sketch (z)

Although there are many other sampled-data configurations possible, we shall restrict ourselves to this system where there is one sampler and a digital controller in the loop. The following information regarding the systems can be obtained from linear considerations. To have an optimum, one needs a transfer function of unity. Therefore

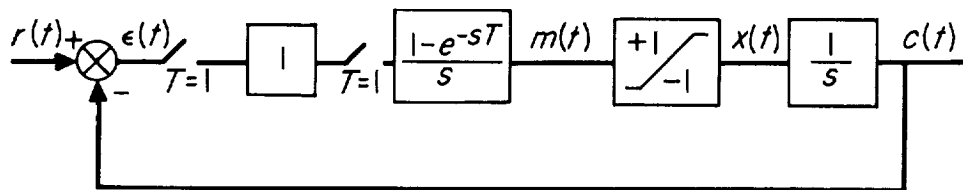
$$\left. \begin{aligned} H(s) &= 1 \\ T &\rightarrow 0 \\ \text{Gain of digital controller} &\rightarrow \infty \end{aligned} \right\} \quad (69)$$

If T is any finite value, the gain can never be infinite, so perhaps the optimum sampled-data system becomes a "finite settling time design" (refs. 7, 8, 9) with $H(s) = 1$. We must for relatively obvious reasons introduce the constraint of "zero ripple" (refs. 7, 9). The finite settling time design provides:

(1) The minimum number of sampling instants to restore the error to zero from an input transient. (Steps, ramps, etc., are permitted, depending upon the number of poles of $G(s)$ at $s = 0$.) See, for example, reference 7 or 9.

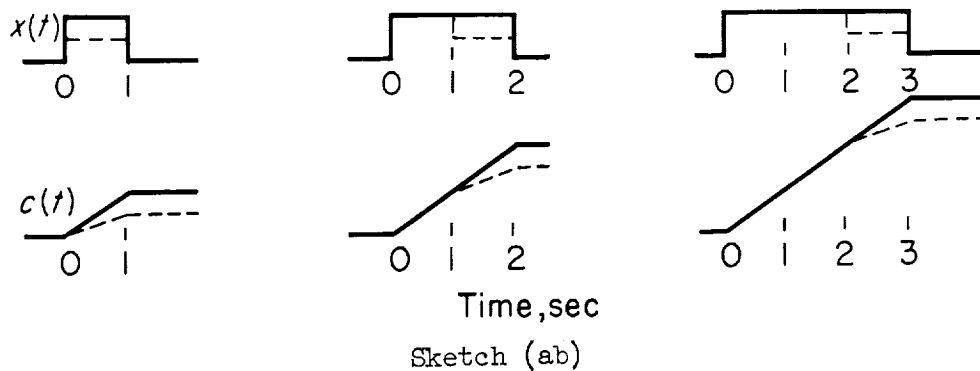
(2) The highest gain permissible at the prescribed sampling rate under the constraints of (1). Thus, for slowly varying inputs, the error will be very small. (Band width of the input should be small compared to the sampling frequency.)

Let us now examine a type 1, first-order plant with a finite settling time design. The block diagram is shown in sketch (aa) with values for $T = 1.0$, $B = 1$, $K = 1$.



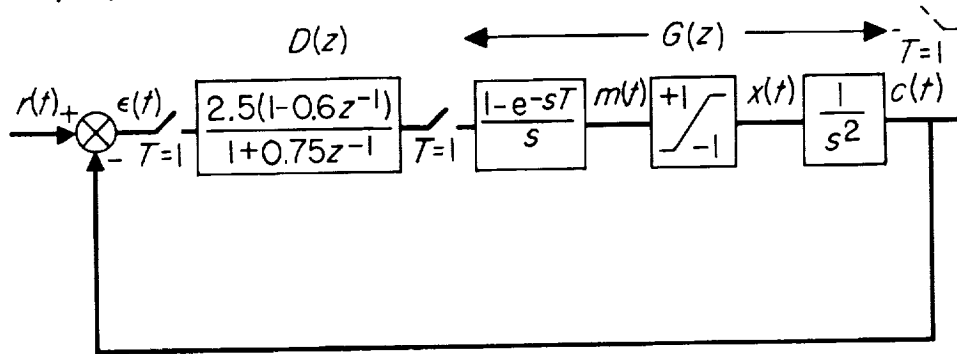
Sketch (aa)

Now consider a step input in the ranges shown in sketch (ab).



Quite obviously, from a study of the step responses, the finite settling time design is optimum with only a gain in the digital controller. A little further thought will show that this system should be optimum for arbitrary inputs, since the output will always move in a direction to reduce the error to zero (with no overshoot) in a minimum time based on the error information available at the present sampling instant. If $T \rightarrow 0$ in this design, the gain of the digital controller approaches infinity and the over-all system approaches the continuous system optimum given or referred to in table VI.

Let us examine the design for the simple second-order plant where $G(s) = 1/s^2$, $T = 1$, $B = 1$. This design is given in sketch (ac).



The pulse transfer function

$$\frac{M(z)}{R(z)} = \frac{D(z)}{1 + G(z)D(z)} = 2.5(1 - 0.6z^{-1})(1 - z^{-1})^2 \quad (70)$$

and for a step of $r(t)$ of magnitude R_0

$$M(z) = R_0(2.5)(1 - 1.6z^{-1} + 0.6z^{-2}) \quad (71)$$

If $R_0 > 1/(2.5)(1.6)$, the value of $m(t)$ exceeds unity (the limit level) and the pulse transfer function given in equation (70) is no longer valid. What happens for a large step of $r(t)$? Certainly the input to the plant is saturated during the first sampling instant (see eq. (71)); therefore, one can compute what $m(t)$ will be at the second sampling instant. This is most conveniently found by subtracting the value of the feedback quantity from the step response of $D(z)$.

$$m(1.0) = \left[\frac{R_0 D(z)}{(1 - z^{-1})} - 2.5 \sum_{n=1}^{\infty} \frac{(n)^2 z^{-n}}{2} \right] \bigg|_{n=1} \quad (72)$$

where $m(1.0)$ means the value of m between the first and second sampling instant.

The first term in the result must be expanded in a power series expression and the value of the coefficient of z^{-1} obtained. The second term is simply the step response of the plant and is valid so long as $m > 1$.

The result in carrying through the calculation is

$$m(1.0) = -0.875 R_0 - 1.25 \quad (73)$$

In other words, for large inputs, the input to the plant will always reverse at the first sampling instant. This is a result of having a pole of $D(z)$ which is lightly damped and is the reason such systems were considered undesirable in chapter II of reference 1.

If the plant input reverses at the first sampling instant, it should be quite obvious from the arguments presented in chapter II of reference 1 that one cannot obtain anywhere close to optimum response with this linear digital controller. Thus, one recognizes that a nonlinear controller of some type must be used. Exactly what means one uses to design this controller is not recognized at present.

In summary, one can state the following about optimum saturated sampled-data systems:

1. They should be finite settling time designs in the linear region.
2. They will never be as good as the continuous system but will approach it as $T \rightarrow 0$.
3. Although the first-order case is very simple, extension even to second order probably requires a relatively complex nonlinear digital controller.

4. The determination of the nonlinear digital controller's mathematical operations does not appear to be a simple problem.

The fact that finite settling time designs are very sensitive to parameter variations (see refs. 8 and 9) and to the other difficulties listed probably means that optimum, saturated, sampled-data systems can never exist. Thus probably other designs are required for both nonlinear and linear operation; two such examples were given in chapter IV of reference 1.

APPROXIMATE OPTIMUM SYSTEMS

This section is devoted to the problem of designing an approximate optimum attitude autopilot for an aircraft. By approximate optimum is meant that for small or slowly varying inputs, if the aircraft has the capability of following a desired input (pitch attitude), the error will be essentially zero. For step inputs that may come about as a result of engaging the system, the response will be near optimum, that is, the error will be reduced to zero in a near minimum time. Disturbance inputs which may come about in actual practice as a result of gusts, will be neglected in this study; however, the resulting design which is a very high gain closed-loop system should have a good response to disturbances.

It will be assumed that the limiting in the system is a result of hydraulic valve bottoming in the control-surface actuator. For simplicity purposes it is further assumed that mechanical limits on the hydraulic valve are equivalent to control-surface rate limits, that is, the actuator is a perfect integrator. The control-surface rate limit will be assumed to be 10^0 per second. Control-surface position limiting will be neglected in this study.

Two transfer functions relating control-surface rate to pitch attitude are considered:

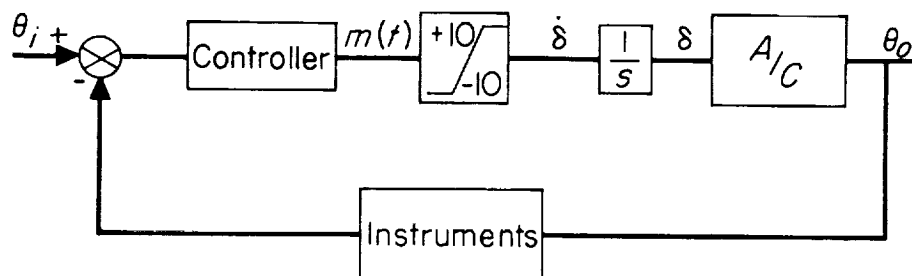
$$\frac{\theta}{\delta_e} = \frac{1.5625(s + 1)}{s^2 \left[\left(\frac{s}{4} \right)^2 + 2 \left(\frac{0.1875}{4} \right) s + 1 \right]} \quad (74)$$

$$\frac{\theta}{\delta_e} = \frac{3.125(s + 1)}{s^2(0.183 s + 1)(0.683 s - 1)} \quad (75)$$

Equation (74) is the approximate linear relationship for the F-86 in normal configuration flying at $M = 0.8$, $h_p = 30,000$ feet. Equation (75)

is the approximate linear relationship for the F-86 with the c.g. shifted aft to give a statically unstable airframe. The flight condition is the same for both examples.

With the assumptions made, the problem reduces to one of designing the controller and instrumentation shown in the block diagram of sketch (ad).



Sketch (ad)

From considerations given in the previous section it is known that the desired instrument transfer function is unity and for very small errors, the linearized controller transfer function should be of the form

$$m(t) = K(1 + as + bs^2)\epsilon \quad (76)$$

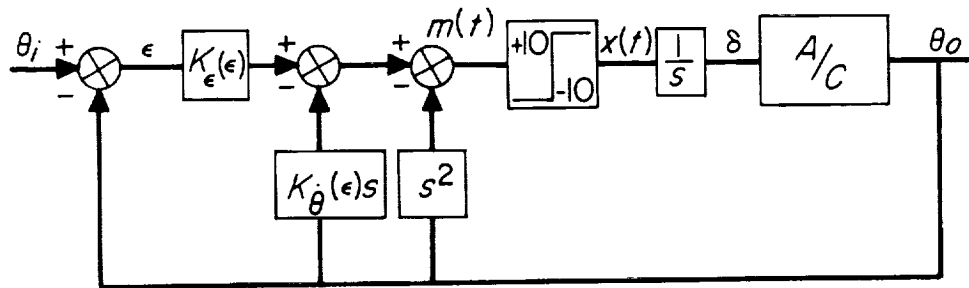
The open loop transfer function system given by sketch (ad) contains three zeros and four poles. Two of the zeros are a result of the controller; the other is from the aircraft transfer function. So long as the zeros are in the left half plane, in theory infinite gain is possible and the closed-loop transfer function θ_o/θ_i will be unity.

The problems associated with differentiation make it impossible to obtain the desired controller operations of equation (76). In general, one must add at least one pole for each differentiation operation. Furthermore, instruments must have dynamics associated with their measurements. The net result is that one cannot in practice obtain an open-loop transfer function with three zeros and only four poles. As a result, the high gain saturating control system will in practice have a limit cycle or chatter. It has been shown (ref. 10) that if this limit cycle is very high in frequency compared with the plant dynamics, one can closely approximate the infinite gain system.

For the unstable dynamics given by equation (75) it was shown in a previous example (for a simpler case) that if one uses error rate, the input is able to drive the system into instability. As a result of this system being higher order, we must neither use error rate nor error acceleration as stabilizing quantities. For our purposes we will use

output rate and output acceleration ($\dot{\theta}_o, \ddot{\theta}_o$) for both aircraft transfer functions (eqs. (74), (75)). The result will be near optimum step response and a finite error to all inputs which are nonconstant.

These assumptions reduce the problem to the design of the system of sketch (ae) for the two plants of equations (74) and (75).



Sketch (ae)

The characteristic equations for the two systems are (under the assumption that the relay may be replaced by an equivalent gain, K):

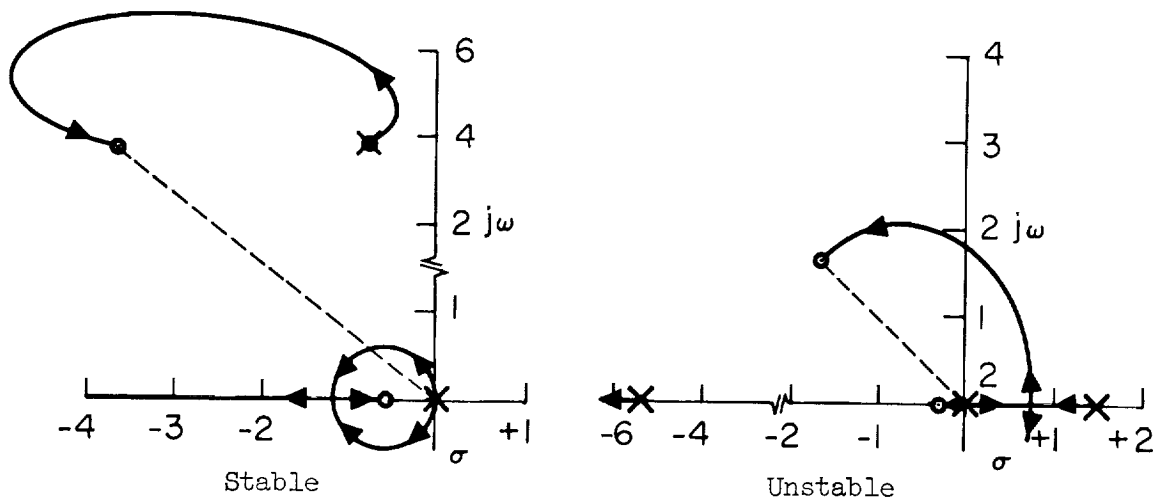
Stable airframe

$$1 + K[s^2 + K_{\dot{\theta}}(\epsilon)s + K_{\epsilon}(\epsilon)] \overbrace{\left\{ \frac{1.5625(s+1)}{s^2 \left[\left(\frac{s}{4}\right)^2 + 2 \left(\frac{0.1875}{4}\right)s + 1 \right]} \right\}}^{G(s)} = 0 \quad (77)$$

Unstable airframe

$$1 + K[s^2 + K_{\dot{\theta}}(\epsilon)s + K_{\epsilon}(\epsilon)] \overbrace{\left[\frac{3.125(s+1)}{s^2(0.183s+1)(0.683s-1)} \right]}^{G(s)} = 0 \quad (78)$$

The root locus as a function of equivalent limiter gain could be as shown in sketch (af) (for one value of $K_{\dot{\theta}}(\epsilon)$ and $K_{\epsilon}(\epsilon)$).

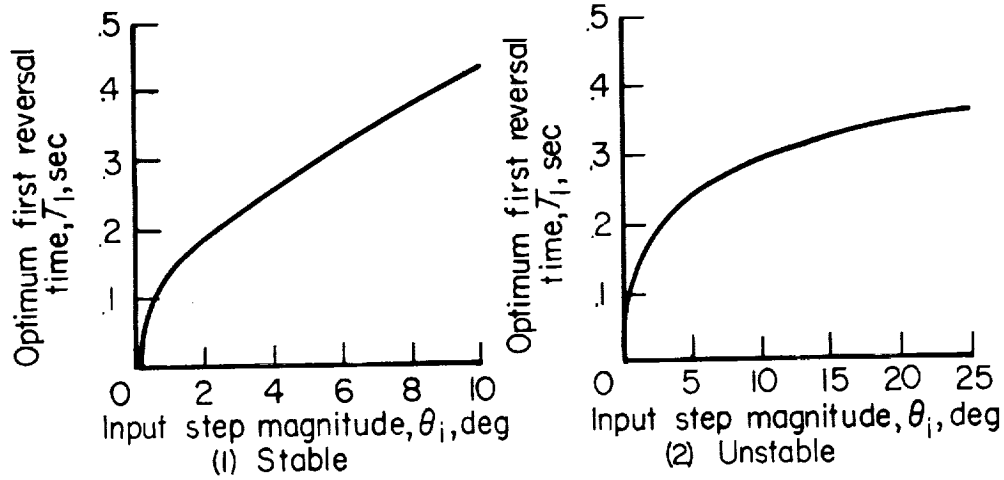


Sketch (af)

As can be seen from sketch (af), the stable system will be lightly damped and the unstable system will diverge for low values of equivalent limiter gain (or large step inputs). To have a good response for large inputs and a very fast system for small inputs the following two conditions will be imposed on the gains $K_e(\epsilon)$ and $K_\theta(\epsilon)$.

1. They shall be such that the first reversal time for step inputs is the optimum.
2. They shall only be allowed to vary in a manner such that the damping ratio of the zeros determined by these quantities is a constant value of 0.7.

These two conditions are identical to those used for the third example of reference 1. To impose condition (1) it is necessary to have the optimum first reversal time as a function of the input step magnitude for the two example $G(s)$ transfer functions. This information is available in normalized form in table IV for the stable airframe and in table V for the unstable airframe. Since tabulated data are not directly obtainable for the particular dynamics given by equations (74) and (75), one must interpolate. Nonlinear interpolation was used for these examples to increase the accuracy. The resultant optimum first reversal time curves are plotted in sketch (ag).



Sketch (ag)

To insure that the first reversal time of the system be optimum, it is necessary that $m(t)$, the input to the ideal relay, reverse sign at the optimum first reversal time given in sketch (ag) above. The equation for $m(t)$, the input to the ideal relay, derived from sketch (ae) is

$$m(t) = K_\epsilon(\epsilon)\epsilon + K_{\dot{\theta}}(\epsilon)\dot{\theta}_o + \ddot{\theta}_o \quad (79)$$

For step inputs and for a time of T_1 , the first reversal time,

$$m(T_1) = 0 = K_\epsilon(\epsilon)[\theta_i(T_1) - \theta_o(T_1)] + K_{\dot{\theta}}(\epsilon)\dot{\theta}_o(T_1) + \ddot{\theta}_o(T_1) \quad (80)$$

The quantity $\theta_i(T_1)$ is determined from sketch (ae) for the particular plant being studied. The quantities $\theta_o(T_1)$, $\dot{\theta}_o(T_1)$, $\ddot{\theta}_o(T_1)$ are the responses of the output, output velocity, and output acceleration, respectively, for a step input $x(t) = Bu(t)$ evaluated at $t = T_1$. Thus, to satisfy the first condition one simply finds an equation of form

$$K_\epsilon(\epsilon)[\epsilon(T_1)] + K_{\dot{\theta}}(\epsilon)\dot{\theta}_o(T_1) + \ddot{\theta}_o(T_1) = 0 \quad (81)$$

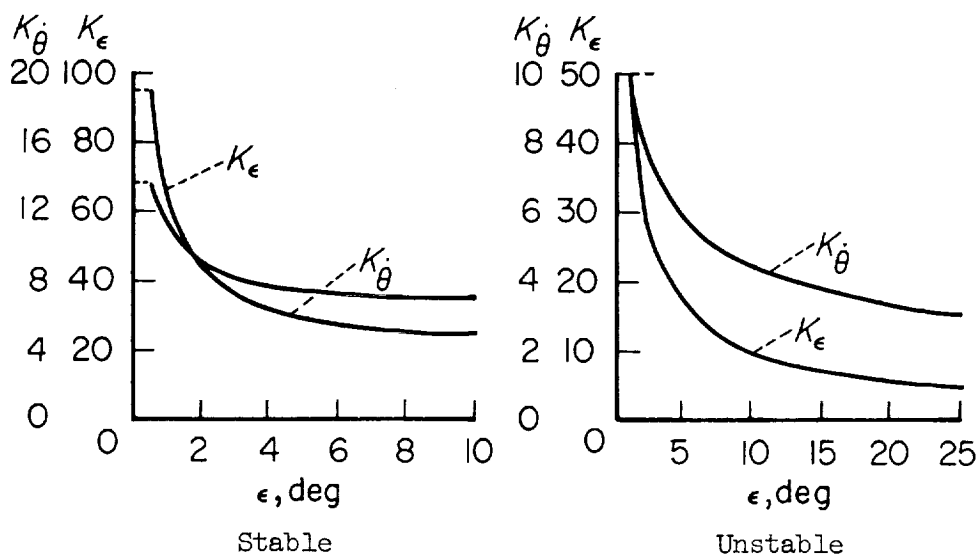
where $\epsilon(T_1)$, $\dot{\theta}_o(T_1)$, and $\ddot{\theta}_o(T_1)$ are constant for any particular value of T_1 .

To satisfy the second condition one evaluates the damping ratios of the zeros determined by the quantities $K_\epsilon(\epsilon)$ and $K_{\dot{\theta}}(\epsilon)$; namely,

$$\zeta_o = \frac{1}{2} \frac{K_{\dot{\theta}}(\epsilon)}{\sqrt{K_\epsilon(\epsilon)}} \quad (82)$$

Setting $\zeta_0 = 0.7$ in equation (82) one can solve for $K_\theta(\epsilon)$ and substitute into equation (81) to obtain a solution for $K_\epsilon(\epsilon)$ for each T_1 . Alternatively one can plot $K_\theta(\epsilon)$ versus $K_\epsilon(\epsilon)$ from equation (82). Equation (81) is a straight line (for each T_1) on this same plot. The intersection of the two curves is the desired value of $K_\epsilon(\epsilon)$ and $K_\theta(\epsilon)$ for the particular choice of T_1 .

The functions of error were computed using this second alternative. The results are shown in sketch (ah).



Sketch (ah)

As can be seen, both $K_\epsilon(\epsilon)$ and $K_\theta(\epsilon)$ approach infinity as $\epsilon \rightarrow 0$. This means the zeros (and two of the closed-loop poles for the infinite gain system) go to infinity along the dotted line of sketch (af). For our purposes the gains were arbitrarily made constant for errors less than $1/2^\circ$ for the stable case and less than 1° for the unstable case, as indicated by the dotted lines of sketch (ah). Thus, in the chatter mode of operation (see ref. 10), the system's responses with the stable airframe will be closely approximated by a second-order system with a natural

frequency of $\sqrt{K_\epsilon(\epsilon)} = \sqrt{95} = 9.75$ radians per second, and a damping ratio of 0.7.

For the unstable airframe the system's response (in the chatter mode of operation) will be closely approximated by a second-order system of natural frequency $\sqrt{K_\epsilon(\epsilon)} = 6.86$ radians per second and damping ratio of 0.7. In the chatter mode of operation both these systems are very high gain representing very "tight" attitude autopilots. The system will therefore closely follow any normal inputs and should have good response to gusts. In addition, the response of the system to step inputs over the range of inputs considered (10° max in these cases) should be near optimum.

The two systems were simulated on an analog computer and the results of the simulation are presented in figures 7 and 8. Figure 7(a) shows the step responses of the system with stable aerodynamics. Figure 7(b) shows the control-surface deflection belonging to the step responses.

As can be seen from the control-surface deflection, the end of the response has a long tailing off exponential. The motion is a result of the fact that the system is an essentially infinite gain and one closed-loop pole of the system has a time constant of 1 second. As a result of the 1 second time constant zero in the aircraft transfer function, however, this motion does not occur on the output. Aerodynamically speaking, one could say that the pitch attitude is held constant while angle of attack and flight-path angle are slowly changing to take on new trim conditions.

The step responses are near optimum as can be seen by the arrows on the response representing the minimum response time for the particular input.

Figure 8(a) shows the step responses of the attitude control system for the unstable airframe. In this case also the response time is near the minimum. Figure 8(b) shows the control-surface motion for the various steps of figure 8(a). A comparison of figure 7(b) with 8(b) shows a significant difference in the character of the response. The difference is the first reversal time. As was mentioned earlier and shown in reference 1, the first reversal time for stable systems continues to increase with the input step magnitude. As the input goes to infinity, $T_m - T_1$ approaches a constant. For unstable systems, however, T_1 approaches a constant as $T_m \rightarrow \infty$. Since we have a near optimum system for both these examples, this fact is quite readily recognized from figures 7(b) and 8(b).

Figure 9 shows the maximum error as a function of frequency for the two systems. The input for both cases was $10 \sin \omega t$. As was hypothesized earlier, there is practically no error for low-frequency inputs. The error does increase with frequency, however, as a result of the use of output derivative feedbacks instead of error derivatives (for the lower frequencies) and a combination of this and limiting at the higher frequencies.

CONCLUSIONS

It has been shown that the switch-time method for synthesis of nonlinear controllers to compensate for the effects of saturation on the input to the plant has very general application. It appears that only two restrictions must be imposed on the nonlinear controller design in order that one obtain a system which has a near optimum step response and well-damped response to other inputs. These restrictions are:

- (1) The first reversal time of the bounded variable shall be the optimum first reversal time of the optimum bang-bang control system.
- (2) The zeros (in the open loop transfer function) introduced by the controller should change (as a function of error) in such a manner that as the limiter equivalent gain is decreased, the poles of the closed-loop system remain in well-damped regions of the s plane.

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To meet restriction (1) it is necessary to have data on the optimum first reversal time as a function of input step magnitude. This report has presented such data for several additional plant transfer functions over those considered in reference 1. In addition, an approximate method for determining such data has been presented. The approximate method has been shown to be reasonably accurate for plants of at least fourth order.

To meet restriction (2) it has been found that if only one zero is introduced by the controller, then as the error increases, this zero should move toward the origin on the negative real axis. For controllers with two zeros, it has been found that if the zeros move toward the origin along a line of constant damping ratio (as the error increases) good characteristics are obtained. Systems with three or more zeros introduced by the controller have not been studied.

It has been shown that if the plant has a zero in the left half plane, the system's response can be made faster than if the zero is neglected. This solution requires some of the plant variables to change as the error and error derivatives remain constant; however, examples have shown that it is a practical solution.

The effects of an unstable pole are to force the controller to keep certain output derivatives from exceeding certain limits; otherwise, control over the plant is lost and the system response diverges. It is interesting to note that if the plant has stable poles, some of the output derivatives are bounded because the plant input is bounded. Thus there are not significant differences between stable and unstable plants in this respect.

With regard to optimum continuous systems it has been shown that with certain restrictions they are derivable from the optimum relay solution. Optimum sampled-data systems, however, need considerably more research before one can derive the optimum controller except for extremely simple plants.

One can generally conclude that saturation in single variable control systems is one type of nonlinearity that lends itself to analytical treatment such that its effects in any particular system can be well understood. Furthermore, it can be concluded that if the effects are undesirable, such

as to make the system unstable or oscillatory for large inputs, the switch time method can be used to compute compensating nonlinear functions in the controller which remove these undesirable characteristics.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., April 8, 1960

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TABLE I.- TRANSFER FUNCTIONS OF PLANTS CONSIDERED

Case	$G(s)$	Type
1	$\frac{K}{s}$	1 - first order
2	$\frac{K(\tau_1 s + 1)}{s(\tau_2 s + 1)}$	1 - second order
3	$\frac{K}{s[(s^2/\omega_n^2) + (2\zeta s/\omega_n) + 1]}$	1 - third order
4	$\frac{K}{s^2}$	2 - second order
5	$\frac{K(\tau_1 s + 1)}{s^2(\tau_2 s + 1)}$	2 - third order
6	$\frac{K}{s^2[(s^2/\omega_n^2) + (2\zeta s/\omega_n) + 1]}$	2 - fourth order
7	$\frac{K}{s^3}$	3 - third order
8	$\frac{K}{s(\tau s - 1)}$	1 - second order, unstable
9	$\frac{K(\tau_1 s + 1)}{s[(s^2/\omega_n^2) + (2\zeta s/\omega_n) + 1]}$	1 - third order (with zero)
10	$\frac{K(\tau_1 s + 1)}{s(\tau_2 s + 1)(\tau_3 s - 1)}$	1 - third order (with zero) unstable
11	$\frac{K(\tau_1 s + 1)}{s^2[(s^2/\omega_n^2) + (2\zeta s/\omega_n) + 1]}$	2 - fourth order (with zero)
12	$\frac{K(\tau_1 s + 1)}{s^2(\tau_2 s + 1)(\tau_3 s - 1)}$	2 - fourth order (with zero) unstable

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TABLE II.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 1 THIRD-ORDER PLANT WITH ZERO

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	b/B	$R_0 \omega_n / BK$
$\xi = 0.10; \omega_{n1} = 0$					$\xi = 0.10; \omega_{n1} = 1.0$					$\xi = 0.25; \omega_{n1} = 0$				
0.5	1.2852	1.7423		0.1719	8.0	8.6151	9.1176		8.8875	9.0	9.7561	10.8091		9.2970
1.0	2.0402	2.8951		.8148	9.0	9.4434	10.3296		10.4429	10.0	10.7226	11.7792		10.3540
1.5	2.5065	3.7169		1.7038	10.0	10.2668	11.1636		11.5901	11.0	11.6898	12.7270		11.3473
2.0	2.8570	4.3717		2.6577	11.0	11.2339	11.7699		12.3021	12.0	12.6954	13.6971		12.3062
2.5	3.1743	4.9161		3.5674	12.0	12.2628	12.4510		12.8853	13.0	13.7153	14.7038		13.2732
3.0	3.5069	5.3616		4.3479	13.0	13.4320	13.5429		13.6790	14.0	14.7294	15.7280		14.2691
3.5	3.8828	5.7119		4.9463	14.0	14.5226	14.8861		14.6430	15.0	15.7295	16.7447		15.2858
4.0	4.3080	5.9868		5.3708	15.0	15.4667	16.0901		16.1567	16.0	16.7212	17.7450		16.3026
5.0	5.2790	6.4610		5.9029	16.0	16.3667	17.0602		17.3228	17.0	17.7139	18.7353		17.3074
6.0	6.4358	7.1970		6.3254	17.0	17.3146	17.8648		18.2355	18.0	18.7124	19.7264		18.3015
7.0	7.7642	8.6216		7.0932	18.0	18.3275	18.6692		19.0142	19.0	19.7155	20.7244		19.2933
8.0	8.8011	10.0313		8.4292	19.0	19.3969	19.6503		19.8565	20.0	20.7192	21.7281		20.2897
9.0	9.6275	11.1206		9.8656	20.0	20.4594	20.8075		20.8874	$\xi = 0.25; \omega_{n1} = 0.5$				
10.0	10.4707	11.9576		11.0161	21.0	21.4524	21.9513		22.0464	.1		.2129	0.3721 -1	.5741 -2
11.0	11.4190	12.6723		11.8343	22.0	22.4017	22.9765		23.1730	.2		.4413	.1339	.2564 -1
12.0	12.4878	13.4908		12.5151	23.0	23.3603	23.8862		24.1686	.3		.6707	.2665	.6253 -1
13.0	13.6423	14.6078		13.3232	24.0	24.3599	24.7737		25.0619	.4		.8913	.4173	.1174
14.0	14.7141	15.8501		14.4219	25.0	25.3870	25.7315		25.9574	.5		1.0983	.5755	.1894
15.0	15.6500	16.9637		15.6636	26.0	26.4246	26.7910		26.9419	.6		1.2905	.7353	.2772
16.0	16.5515	17.9120		16.8091	27.0	27.4330	27.8792		28.0131	.7		1.4682	.8933	.3785
17.0	17.4994	18.7624		17.7637	28.0	28.4110	28.9166		29.0946	.8	1.6324	1.6560		.4912
18.0	18.5203	19.6399		18.5994	29.0	29.3844	29.8849		30.1162	.9	1.7812	1.8761		.6136
19.0	19.5952	20.6550		19.4646	30.0	30.3749	30.8213		31.0725	1.0	1.9166	2.0773		.7441
20.0	20.6516	21.7745		20.4713	$\xi = 0.10; \omega_{n1} = 2.0$					1.5	2.4625	2.8971		1.4721
$\xi = 0.10; \omega_{n1} = 0.5$					.1		.2019	0.1120 -1	.2048 -1	2.0	2.8950	3.5335		2.2434
.1		.2161	0.4401 -1	.5925 -2	.2		.4034	.4311 -1	.4283 -1	2.5	3.2899	4.0648		2.9870
.2		.4537	.1621	.2732 -1	.3		.5987	.9247 -1	.1862	3.0	3.6811	4.5250		3.6628
.3		.6958	.3289	.6860 -1	.4		.7840	.1557	.3273	3.5	4.0908	4.9345		4.2528
.4		.9298	.5233	.1319	.5		.9573	.2294	.5014	4.0	4.5266	5.3121		4.7589
.5		1.1489	.7321	.2171	.6		1.1182	.3106	.7030	4.5	4.9908	5.6792		5.1976
.6		1.3511	.9470	.3224	.7		1.2672	.3970	.9269	5.0	5.4834	6.0600		5.5931
.7	1.5335	1.6125		.4455	.8		1.4054	.4868	1.1682	5.5	5.9855	6.9548		6.3638
.8	1.6929	1.8698		.5840	.9		1.5340	.5783	1.4226	6.0	6.5495	7.6962		7.2576
.9	1.8337	2.1033		.7359	1.0		1.6543	.6705	1.6867	6.5	7.1493	8.0962		8.3097
1.0	1.9594	2.3178		.8990	1.5	2.1611	2.3814		3.0592	7.0	7.6493	8.0962		10.4721
1.5	2.4397	3.2012		1.8217	2.0	2.4953	3.4878		4.4973	8.0	8.6921	9.1939		12.4170
2.0	2.7921	3.8955		2.8112	$\xi = 0.10; \omega_{n1} = 4.0$					10.0	10.6233	11.2187		14.3787
2.5	3.1063	4.4687		3.7560	.1		.1995	.9868 -2	.3997 -1	12.0	12.6040	13.1249		16.4159
3.0	3.4356	4.9298		4.5585	.2		.3944	.3808 -1	.1580	14.0	14.6378	15.1553		18.4137
3.5	3.8124	5.2764		5.1516	.3		.5801	.8198 -1	.3478	16.0	16.6305	17.1769		20.4013
4.0	4.2413	5.5312		5.5487	.4		.7543	.1386	.6001	18.0	18.6214	19.1565		22.4074
5.0	5.2116	5.9412		6.0179	.5		.9160	.2051	.9044	20.0	20.6280	21.1573		24.4093
6.0	6.3475	6.5963		6.4012	.6		1.0654	.2789	1.2503	22.0	22.6285	23.1645		26.4062
7.0	7.6836	8.0448		7.1776	.7		1.2036	.3580	1.6283	24.0	24.6299	25.1611		28.4069
8.0	8.7346	9.5183		8.5491	.8		1.3318	.4405	2.0303	26.0	26.6268	27.1599		30.4077
9.0	9.5627	10.6417		10.0163	.9		1.4513	.5251	2.4492	28.0	28.6274	29.1617		
10.0	10.4060	11.4779		11.1660	1.0		1.5635	.6107	2.8792	30.0	30.6268	31.1614		
11.0	11.3529	12.1625		11.9567	1.5	2.0460	2.1422		5.0502	$\xi = 0.25; \omega_{n1} = 1.0$				
12.0	12.4150	12.9427		12.6128	$\xi = 0.10; \omega_{n1} = 8.0$					.1		.2037	.1415 -1	.1043 -1
13.0	13.5678	14.0530		13.4173	.1		.1983	.9657 -2	.7896 -1	.2		.4097	.5262 -1	.4290 -1
14.0	14.6456	15.3235		14.5323	.2		.3898	.3729 -1	.3085	.3		.6113	.1091	.9781 -1
15.0	15.5847	16.4620		15.7926	.3		.5708	.8035 -1	.6720	.4		.8039	.1760	.1740
16.0	16.4865	17.4164		16.9434	.4		.7392	.1360	1.1487	.5		.9853	.2548	.2695
17.0	17.4334	18.2539		17.8872	.5		.8949	.2015	1.7168	.6	1.1547	1.3362		.3815
18.0	18.4514	19.1109		18.7081	.6		1.0383	.2743	2.3559	.7	1.3127	1.4201		.5074
19.0	19.5245	20.1165		19.5676	.7		1.1708	.3524	3.0483	.8	1.4601	.5049		.6448
20.0	20.5828	21.2459		20.5804	.8		1.2938	.4341	3.7768	.9	1.5981	.5893		.7912
21.0	21.5720	22.3528		21.7088	.9		1.4085	.5179	4.5349	1.0	1.7278	.6727		.9449
22.0	22.5202	23.3598		22.8194	1.0		1.5163	.6028	5.3060	1.5	2.2855	2.3408		1.7697
23.0	23.4794	24.2835		23.8247	1.5	1.9831	2.0946		9.1284	2.0	2.7299	3.0687		2.6090
24.0	24.4774	25.1955		24.7407	$\xi = 0.10; \omega_{n1} = 16.0$					2.5	3.1254	3.6515		3.4008
25.0	25.5108	26.1725		25.6508	.1		.1977	.9665 -2	.1569	3.0	3.5168	4.1355		4.1019
26.0	26.5476	27.2261		26.6309	.2		.3876	.3733 -1	.6097	3.5	3.9265	4.5449		4.6919
27.0	27.5538	28.2938		27.6863	.3		.5661	.8045 -1	1.3211	4.0	4.3630	4.9021		5.1762
28.0	28.5305	29.3165		28.7555	.4		.7317	.1362	2.2474	4.5	4.8268	5.2338		5.5803
29.0	29.5039	30.2861		29.7782	.5		.8843	.2018	3.3447	5.0	5.3160	5.5700		5.9379
30.0	30.4956	31.2358		30.7446	.6		1.0247	.2748	4.5728	5.5	5.8360	6.3848		6.2528
$\xi = 0.10; \omega_{n1} = 1.0$					.7		1.1544	.3532	5.8971	6.0	6.4696	7.4850		7.5499
.1		.2067	.1746 -1	.1075 -1	.8		1.2746	.4352	7.2885	6.5	7.1493	8.0962		8.6297
.2		.4211	.6698 -1	.4551 -1	.9		1.3870	.5194	8.7231	7.0	7.6493	8.0962		10.8233
.3		.6344	.1409	.1065	1.0		1.4925	.6046	10.1811	7.5	8.2055	8.6707		12.7434
.4		.8399	.2337	.1939	$\xi = 0.25; \omega_{n1} = 0$					8.0	8.5205	9.1736		14.7050
.5		1.0335	.3392	.3058	.5		1.2567	1.6609	.1475	8.5	9.0855	9.6464		16.7507
.6		1.2135	.4529	.4394	1.0		2.0301	2.7264	.6662	9.0	9.6505	10.2106		18.7447
.7		1.3801	.5715	.5914	1.5		2.5573	3.4842	1.3695	9.5	10.1212	10.6806		20.7304
.8		1.5340	.6926	.7586	2.0		2.9834	4.0876	2.1208	10.0	10.5966	11.1506		22.7388
.9		1.6766	.8145	.9379	2.5		3.3754	4.6002	2.8495	10.5	11.0722	11.6206		24.7404
1.0		1.8093	.9358	1.1265	3.0		3.7672	5.0517	3.5174	11.0	11.5478	12.1506		26.7437
1.5	2.3196	2.7934		2.1542	3.5		4.1769	5.4612	4.1074	11.5	12.0234	12.6806		28.7464
2.0	2.6722	3.5907		3.2463	4.0		4.6129	5.8462	4.6203	12.0	12.4991	13.2106		30.7386
2.5	2.9728	4.2482												

TABLE II.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 1 THIRD-ORDER PLANT
 WITH ZERO - Continued

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_m$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_m$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_m$	b/B	$R_0 \omega_n / BK$
$\xi = 0.25; \omega_{n1} = 2.0$					$\xi = 0.50; \omega_{n1} = 0$					$\xi = 0.50; \omega_{n1} = 4.0$				
0.8		1.3359	0.3691	1.0023	16.0	16.9505	17.7223		15.8214	0.1		0.1923	0.7352	-2
0.9		1.4594	0.4354	1.2115	17.0	17.9505	18.7224		16.3215	0.2		0.3693	0.6655	-1
1.0		1.5761	0.5018	1.4276	18.0	18.9504	19.7224		17.8216	0.3		0.5318	0.5450	-1
1.5		2.0868	0.8157	2.5446	19.0	19.9504	20.7223		18.8216	0.4		0.6815	0.8834	-1
2.0	2.5260	2.6653		3.6133	20.0	20.9503	21.7223		19.8216	0.5		0.8200	0.1263	
2.5	2.9136	3.3940		4.5667	$\xi = 0.50; \omega_{n1} = 0.5$					0.6		0.9492	0.1670	
3.0	3.2958	3.9519		5.3602	-1		0.2079	0.2668	-1	0.7		1.0705	0.2094	1.1672
3.5	3.7056	4.3609		5.9496	-2		0.4231	0.9272	-1	0.8		1.1854	0.2528	1.4258
4.0	4.1502	4.6548		6.3544	-3		0.6354	1.797	-1	0.9		1.2948	0.2965	1.6912
4.5	4.6216	4.8841		6.6410	-4		0.8387	2.754	-1	1.0		1.3997	0.3401	1.9605
5.0		5.1115	0.9924	6.8732	-5		1.0307	3.729	-1	1.5		1.6806	0.5451	3.2999
5.5		5.1115	0.9924	6.8732	-6		1.2108	4.686	-1	2.0		2.3251	0.7122	4.5237
6.0		6.1705	0.7848	7.3992	-7		1.3796	5.609	-1	2.5		2.7607	0.8310	5.5632
6.5		7.2745	0.7820	8.2896	-8		1.5383	6.491	-1	3.0		3.2007	0.9006	6.4018
7.0		8.3166	0.8986	9.4807	-9		1.6879	7.328	-1	3.5		3.6522	0.9271	7.0560
8.0		10.2848		11.7797	-10		1.8295	8.121	-1	4.0		4.1183	0.9207	7.5645
10.0	10.2526			13.6161	0.1		1.6879	7.328	-1	4.5		4.5993	0.8941	7.9772
12.0		12.2291	0.9226	15.5738	0.2		1.8295	8.121	-1	5.0		5.0934	0.8996	8.3450
14.0		14.2627	0.9183	17.6521	1.5	2.4475	2.5179		1.1229	5.5		5.5971	0.8268	8.7100
16.0		16.2568	0.9545	19.6334	2.0	2.9663	3.1456		1.7130	6.0		6.1060	0.8014	9.0995
18.0		18.2471	0.9402	21.6126	2.5	3.4412	3.6896		2.3073	6.5		6.6129	0.7788	9.4991
20.0		20.2335	0.9330	23.6285	3.0	3.9003	4.1871		2.8866	7.0		7.1259	0.7825	10.9933
22.0		22.2543	0.9414	25.6292	4.0	4.8228	5.1205		3.9748	7.5		7.6365	0.7935	12.0364
24.0		24.2516	0.9404	27.6230	5.0	5.7820	6.0465		4.9824	8.0		8.1365	0.8008	13.0687
26.0		26.2524	0.9377	29.6255	6.0	6.7776	7.0093		5.9542	9.0		9.1375	0.7935	14.0630
28.0		28.2531	0.9393	31.6267	8.0	8.8020	9.0229		7.9188	12.0		12.1306	0.8011	15.0738
30.0		30.2525	0.9386		10.0	10.8057	11.0371		9.9256	14.0		14.1313	0.7986	16.0630
$\xi = 0.25; \omega_{n1} = 4.0$					12.0	12.8020	13.0336		11.9295	16.0		16.1317	0.7988	17.0630
0.1		0.1967	0.8880	-2	14.0	14.8020	15.0321		13.9282	18.0		18.1316	0.7991	18.0630
0.2		0.3842	0.3341	-1	16.0	16.8025	17.0327		15.9278	20.0		20.1316	0.7991	19.0630
0.3		0.5600	0.7041	-1	18.0	18.8024	19.0329		17.9280	22.0		22.1316	0.7990	20.0630
0.4		0.7233	0.1169	-1	20.0	20.8024	21.0327		19.9280	24.0		24.1316	0.7990	21.0630
0.5		0.8745	0.1705	-1	22.0	22.8024	23.0327		21.9280	$\xi = 0.50; \omega_{n1} = 8.0$				
0.6		1.0147	0.2290	-1	24.0	24.8024	25.0328		23.9280	0.1		0.1912	0.8061	-2
0.7		1.1450	0.2910	-1	26.0	26.8024	27.0328		25.9280	0.2		0.3653	0.2924	-1
0.8		1.2670	0.3551	-1	$\xi = 0.50; \omega_{n1} = 1.0$					0.3		0.5239	0.5982	-1
0.9		1.3817	0.4203	-1	0.1		0.1991	0.9011	-2	0.4		0.6690	0.9706	-1
1.0		1.4903	0.4858	-1	0.2		0.3931	0.3232	-1	0.5		0.9273	1.838	-1
1.5		1.9708	0.7986	-1	0.3		0.5790	0.6513	-1	0.6		1.0440	2.307	-1
2.0	2.3914			7.3172	0.4		0.7552	1.039	-1	0.7		1.1545	2.787	-1
2.5	2.7307				0.5		0.9213	1.460	-1	0.8		1.2599	3.271	-1
$\xi = 0.25; \omega_{n1} = 8.0$					0.6		1.0778	1.898	-1	0.9		1.3610	3.755	-1
0.1		0.1955	0.9030	-2	0.7		1.2254	2.342	-1	1.0		1.4629	4.239	-1
0.2		0.3799	0.3400	-1	0.8		1.3652	2.785	-1	1.5		1.8269	6.036	-1
0.3		0.5512	0.7170	-1	0.9		1.4979	3.220	-1	2.0		2.2620	7.900	-1
0.4		0.7093	0.1192	-1	1.0		1.6245	3.645	-1	2.5		2.6920	9.227	-1
0.5		0.8551	0.1739	-1	1.5		2.1906	5.548	-1	3.0	3.1291	3.1341		10.8759
0.6		0.9898	0.2339	-1	2.0		2.6896	7.015	-1	3.5	3.5775	3.8195		11.6645
0.7		1.1149	0.2975	-1	2.5		3.1580	8.024	-1	4.0	4.0447	4.2290		12.1397
0.8		1.2320	0.3634	-1	3.0		3.6155	9.006	-1	4.5		4.5279	0.9933	12.4188
0.9		1.3422	0.4305	-1	3.5		4.0738	9.954	-1	5.0		5.0235	0.9547	12.6144
1.0		1.4467	0.4979	-1	4.0		4.5378	10.872	-1	5.5		5.5295	0.9180	12.8153
1.5		1.9116	0.8210	-1	4.5		5.0000	11.775	-1	6.0		6.0387	0.8896	13.0783
2.0	2.3079			10.4393	5.0		5.4580	12.654	-1	6.5		6.5499	0.8720	13.4261
$\xi = 0.25; \omega_{n1} = 16.0$					5.5		5.9180	13.518	-1	7.0		7.0596	0.8643	13.8551
0.1		0.1949	0.9215	-2	6.0		6.3780	14.366	-1	7.5		7.5696	0.8685	14.2778
0.2		0.3777	0.3470	-1	6.5		6.8380	15.198	-1	8.0		8.0700	0.8685	14.6959
0.3		0.5468	0.7320	-1	7.0		7.2980	16.018	-1	8.5		8.5700	0.8685	15.1095
0.4		0.7023	0.1217	-1	7.5		7.7580	16.828	-1	9.0		9.0700	0.8685	15.5180
0.5		0.8454	0.1777	-1	8.0		8.2180	17.628	-1	9.5		9.5700	0.8685	15.9215
0.6		0.9774	0.2390	-1	8.5		8.6780	18.418	-1	10.0		10.0700	0.8685	16.3215
0.7		1.0999	0.3040	-1	9.0		9.1380	19.198	-1	10.5		10.5700	0.8685	16.7180
0.8		1.2145	0.3714	-1	9.5		9.5980	19.968	-1	11.0		11.0700	0.8685	17.1115
0.9		1.3225	0.4401	-1	10.0		10.0580	20.728	-1	11.5		11.5700	0.8685	17.5015
1.0		1.4249	0.5092	-1	10.5		10.5180	21.478	-1	12.0		12.0700	0.8685	17.8880
1.5		1.8821	0.8402	-1	11.0		10.9780	22.218	-1	12.5		12.5700	0.8685	18.2715
$\xi = 0.50; \omega_{n1} = 0$					11.5		11.4380	22.958	-1	13.0		13.0700	0.8685	18.6515
0.5	1.2188	1.5561		1.1186	12.0		11.8980	23.698	-1	13.5		13.5700	0.8685	19.0280
1.0	2.0141	2.5362		1.5080	12.5		12.3580	24.438	-1	14.0		14.0700	0.8685	19.4015
1.5	2.6105	3.2559		1.9350	13.0		12.8180	25.178	-1	14.5		14.5700	0.8685	19.7715
2.0	3.1183	3.8491		2.4126	13.5		13.2780	25.918	-1	15.0		15.0700	0.8685	20.1380
2.5	3.5879	4.3745		2.9198	14.0		13.7380	26.658	-1	15.5		15.5700	0.8685	20.5015
3.0	4.0446	4.8627		3.4270	14.5		14.1980	27.398	-1	16.0		16.0700	0.8685	20.8615
3.5	4.5019	5.3316		3.9342	15.0		14.6580	28.138	-1	16.5		16.5700	0.8685	21.2180
4.0	4.9666	5.7935		4.4414	15.5		15.1180	28.878	-1	17.0		17.0700	0.8685	21.5715
5.0	5.9277	6.7274		5.4486	16.0		15.5780	29.618	-1	17.5		17.5700	0.8685	21.9215
6.0	6.9255	7.6985		6.4558	16.5		16.0380	30.358	-1	18.0		18.0700	0.8685	22.2680
7.0	7.9391	8.7015		7.4630	17.0		16.4980	31.098	-1	18.5		18.5700	0.8685	22.6115
8.0	8.9507	9.7149		8.4702	17.5		16.9580	31.838	-1	19.0		19.0700	0.8685	22.9515
9.0	9.9548	10.7241		9.4774	18.0		17.4180	32.578	-1	19.5		19.5700	0.8685	23.2880
10.0	10.9537	11.7264		10.4846	18.5		17.8780	33.318	-1	20.0		20.0700	0.8685	23.6215
11.0	11.9513	12.7249		11.4918	19.0		18.3380	34.058	-1	$\xi = 0$				

TABLE II.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 1 THIRD-ORDER PLANT
WITH ZERO - Continued

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	b/B	$R_0 \omega_n / BK$
$\xi = 0.75; \omega_{nT_1} = 0$					$\xi = 0.75; \omega_{nT_1} = 4.0$					$\xi = 1.0013; \omega_{nT_1} = 0$				
4.0	5.2305	5.8146		3.3536	4	0.6486	0.6629	-1	0.4166	4.1952	5.6501	6.0789		3.1691
5.0	6.2153	6.8013		4.3707	5	.7783	.9334	-1	.5951	5.2440	6.7162	7.1487		4.2042
6.0	7.2066	7.7907		5.3774	6	.9000	1.219		.7877	6.2929	7.7720	8.2060		5.2476
7.0	8.2035	8.7856		6.3785	7	1.0154	1.514		.9902	7.3417	8.8236	9.2582		6.2943
8.0	9.2032	9.7843		7.3779	8	1.1257	1.812		1.1992	8.3905	9.8735	10.3083		7.3423
9.0	10.2036	10.7844		8.3771	9	1.2319	2.111		1.4125	9.4393	10.9227	11.3576		8.3908
10.0	11.2040	11.7848		9.3767	1.0	1.3349	2.407		1.6280	10.4881	11.9717	12.4067		9.4394
11.0	12.2042	12.7850		10.3766	1.5	1.4389	2.709		1.8468	11.5369	13.0206	13.4556		10.4881
12.0	13.2043	13.7851		11.3766	2.0	1.5429	3.012		2.0683	12.5857	14.0694	14.5044		11.5370
13.0	14.2043	14.7852		12.3766	2.5	1.6469	3.315		2.2900	13.6345	15.1182	15.5532		12.5858
$\xi = 0.75; \omega_{nT_1} = 0.5$					3.0	3.2029	.6341		5.3336	14.6833	16.1670	16.6020		13.6346
-1	.2033	0.1704	-1	.5204	3.5	3.6748	.6690		6.0014	15.7322	17.2158	17.6508		14.6834
-2	.4075	.5746	-1	.2128	4.0	4.1550	.6871		6.5934	16.7810	18.2647	18.6997		15.7322
-3	.6063	.8090	-1	.4823	4.5	4.6354	.6940		7.6408	17.8298	19.3135	19.7485		16.7810
-4	.7967	1.0641	-1	.8541	5.0	5.1133	.6868		8.6160	$\xi = 1.02; \omega_{nT_1} = 0.5$				
-5	.9777	1.2191	-1	1.3119	5.5	5.5934	.6762		10.5703	1.0	.1988	0.7502	-2	.4956
-6	1.1494	1.3719	-1	1.8655	6.0	6.0729	.6751		12.5641	2.0	.3928	.2458	-1	.1949
-7	1.3125	1.5217	-1	2.4883	6.5	6.5529	.6755		14.5657	3.0	.5800	.4569	-1	.4289
-8	1.4678	1.6683	-1	3.1664	7.0	7.0328	.6756		16.5661	4.0	.7596	.6782	-1	.7430
-9	1.6161	1.8117	-1	3.8997	7.5	7.5129	.6756		18.5661	5.0	.9319	.8942	-1	1.1128
1.0	1.7582	1.9519	-1	4.678	8.0	8.0000	.6756		20.5661	6.0	1.0972	1.098	-1	1.5777
1.5	2.3991	2.6131	-1	9.075	8.5	8.4791	.6756		22.5661	7.0	1.2562	1.286	-1	2.081
2.0	2.9658	3.226	-1	1.3955	9.0	8.9582	.6756		24.5661	8.0	1.4094	1.458	-1	2.635
3.0	3.9996	4.8415	-1	2.4211	9.5	9.4373	.6756		26.5660	9.0	1.5575	1.615	-1	3.233
4.0	4.9888	6.4819	-1	3.4522	10.0	9.9164	.6756		28.5660	1.0	1.7010	1.758	-1	3.869
5.0	5.9649	8.117	-1	4.4760	$\xi = 0.75; \omega_{nT_1} = 8.0$					1.5	2.3651	2.298	-1	7.498
6.0	6.9611	9.738	-1	5.4758	1.0	1.3002	2.919		3.0348	2.0	2.9682	2.638	-1	1.1637
7.0	7.9619	11.359	-1	6.4746	1.5	1.7700	4.606		4.9150	3.0	4.0779	3.002	-1	2.0722
8.0	8.9622	12.980	-1	7.4734	2.0	2.2220	5.994		6.5729	4.0	5.1254	3.163	-1	3.0327
9.0	9.9622	14.601	-1	8.4722	2.5	2.6745	7.031		7.9503	5.0	6.1558	3.320	-1	4.0077
10.0	10.9622	16.222	-1	9.4710	3.0	3.1346	7.737		9.0590	6.0	7.1558	3.477	-1	5.0077
11.0	11.9622	17.843	-1	10.4698	3.5	3.6044	8.166		9.9288	7.0	8.1618	3.632	-1	6.0027
12.0	12.9622	19.464	-1	11.4686	4.0	4.0835	8.389		10.6276	8.0	9.1618	3.787	-1	7.0027
13.0	13.9622	21.085	-1	12.4674	4.5	4.5626	8.612		11.3264	9.0	10.1618	3.942	-1	8.0027
14.0	14.9622	22.706	-1	13.4662	5.0	5.0417	8.835		12.0252	10.0	11.1618	4.097	-1	9.0027
15.0	15.9622	24.327	-1	14.4650	5.5	5.5208	9.058		12.7240	11.0	12.1618	4.252	-1	10.0027
16.0	16.9622	25.948	-1	15.4638	6.0	6.0000	9.281		13.4228	12.0	13.1618	4.407	-1	11.0027
$\xi = 0.75; \omega_{nT_1} = 1.0$					6.5	6.4791	9.504		14.1216	13.0	14.1618	4.562	-1	12.0027
1.0	.1948	.4315	-2	.9501	7.0	6.9582	9.727		14.8204	14.0	15.1618	4.717	-1	13.0027
2.0	.3789	.8630	-1	1.9002	7.5	7.4373	9.950		15.5192	15.0	16.1618	4.872	-1	14.0027
3.0	.5630	1.2945	-1	2.8503	8.0	7.9164	10.173		16.2180	16.0	17.1618	5.027	-1	15.0027
4.0	.7471	1.7260	-1	3.8004	8.5	8.3955	10.396		16.9168	$\xi = 1.02; \omega_{nT_1} = 1.0$				
5.0	.9312	2.1575	-1	4.7505	9.0	8.8746	10.619		17.6156	1.0	.1906	.3302	-2	.9066
6.0	1.1153	2.5890	-1	5.7006	9.5	9.3537	10.842		18.3144	2.0	.3657	.6606	-1	.3602
7.0	1.2994	3.0205	-1	6.6507	10.0	9.8328	11.065		19.0132	3.0	.5408	1.0114	-1	.6901
8.0	1.4835	3.4520	-1	7.6008	10.5	10.3119	11.288		19.7120	4.0	.7159	1.3622	-1	1.1140
9.0	1.6676	3.8835	-1	8.5509	11.0	10.7910	11.511		20.4108	5.0	.8910	1.7136	-1	1.5380
1.0	1.8517	4.3150	-1	9.5010	11.5	11.2701	11.734		21.1096	6.0	.9661	2.0650	-1	1.9620
1.5	2.0358	4.7465	-1	1.0451	12.0	11.7492	11.957		21.8084	7.0	1.1412	2.4164	-1	2.3860
2.0	2.2199	5.1780	-1	1.1392	12.5	12.2283	12.180		22.5072	8.0	1.3163	2.7678	-1	2.8100
2.5	2.4040	5.6095	-1	1.2333	13.0	12.7074	12.403		23.2060	9.0	1.4914	3.1192	-1	3.2340
3.0	2.5881	6.0410	-1	1.3274	13.5	13.1865	12.626		23.9048	1.0	1.6665	3.4706	-1	3.6580
3.5	2.7722	6.4725	-1	1.4215	14.0	13.6656	12.849		24.6036	1.5	1.8416	3.8220	-1	4.0820
4.0	2.9563	6.9040	-1	1.5156	14.5	14.1447	13.072		25.3024	2.0	2.0167	4.1734	-1	4.5060
4.5	3.1404	7.3355	-1	1.6097	15.0	14.6238	13.295		26.0012	2.5	2.1918	4.5248	-1	4.9300
5.0	3.3245	7.7670	-1	1.7038	15.5	15.1029	13.518		26.7000	3.0	2.3669	4.8762	-1	5.3540
5.5	3.5086	8.1985	-1	1.7979	16.0	15.5820	13.741		27.3988	3.5	2.5420	5.2276	-1	5.7780
6.0	3.6927	8.6300	-1	1.8920	$\xi = 0.75; \omega_{nT_1} = 16.0$					4.0	2.7171	5.5790	-1	6.2020
6.5	3.8768	9.0615	-1	1.9861	1.0	.1867	.7888	-2	.1395	4.5	2.8922	5.9304	-1	6.6260
7.0	4.0609	9.4930	-1	2.0802	1.5	.3510	1.577	-1	.4921	5.0	3.0673	6.2818	-1	7.0500
7.5	4.2450	9.9245	-1	2.1743	2.0	.5153	2.366	-1	.9845	5.5	3.2424	6.6332	-1	7.4740
8.0	4.4291	10.3560	-1	2.2684	2.5	.6796	3.155	-1	1.4769	6.0	3.4175	6.9846	-1	7.8980
8.5	4.6132	10.7875	-1	2.3625	3.0	.8439	3.944	-1	1.9693	6.5	3.5926	7.3360	-1	8.3220
9.0	4.7973	11.2190	-1	2.4566	3.5	1.0082	4.733	-1	2.4617	7.0	3.7677	7.6874	-1	8.7460
9.5	4.9814	11.6505	-1	2.5507	4.0	1.1725	5.522	-1	2.9541	7.5	3.9428	8.0388	-1	9.1700
10.0	5.1655	12.0820	-1	2.6448	4.5	1.3368	6.311	-1	3.4465	8.0	4.1179	8.3902	-1	9.5940
10.5	5.3496	12.5135	-1	2.7389	5.0	1.5011	7.100	-1	3.9409	8.5	4.2930	8.7416	-1	10.0180
11.0	5.5337	12.9450	-1	2.8330	5.5	1.6654	7.889	-1	4.4353	9.0	4.4681	9.0930	-1	10.4420
11.5	5.7178	13.3765	-1	2.9271	6.0	1.8297	8.678	-1	4.9297	9.5	4.6432	9.4444	-1	10.8660
12.0	5.9019	13.8080	-1	3.0212	6.5	1.9940	9.467	-1	5.4241	10.0	4.8183	9.7958	-1	11.2900
12.5	6.0860	14.2395	-1	3.1153	7.0	2.1583	10.256	-1	5.9185	10.5	4.9934	10.1472	-1	11.7140
13.0	6.2701	14.6710	-1	3.2094	7.5	2.3226	11.045	-1	6.4129	11.0	5.1685	10.4986	-1	12.1380
13.5	6.4542	15.1025	-1	3.3035	8.0	2.4869	11.834	-1	6.9073	11.5	5.3436	10.8500	-1	12.5620
14.0	6.6383	15.5340	-1	3.3976	8.5	2.6512	12.623	-1	7.4017	12.0	5.5187	11.2014	-1	12.9860
14.5	6.8224	15.9655	-1	3.4917	9.0	2.8155	13.412	-1	7.8961	12.5	5.6938	11.5528	-1	13.4100
15.0	7.0065	16.3970	-1	3.5858	9.5	2.9798	14.201	-1	8.3905	13.0	5.8689	11.9042	-1	13.8340

TABLE II.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 1 THIRD-ORDER PLANT
WITH ZERO - Continued

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	b/B	$R_0 \omega_n / BK$
$\zeta = 1.02; \omega_{n1} = 4.0$					$\zeta = 1.50; \omega_{n1} = 0$					$\zeta = 1.50; \omega_{n1} = 2.0$				
0.8	1.0774	0.1249		1.0220	7.5	9.5329	9.7958		5.7300	22.0	22.3884	-0.2238		21.1641
.9	1.1816	.1446		1.1968	8.0	10.0458	10.3089		6.2172	24.0	24.3885	-.2238		23.1640
1.0	1.2834	.1641		1.3730	8.5	10.5565	10.8194		6.7064	26.0	26.3885	-.2238		25.1640
1.5	1.7718	.2547		2.2469	9.0	11.0636	11.3266		7.1994					
2.0	2.2470	.3299		3.0727	9.5	11.5708	11.8338		7.6922					
3.0	3.2007	.4337		4.5340	10.0	12.0769	12.3399		8.1862					
4.0	4.1715	.4898		5.7878										
6.0	6.1484	.5312		7.9763										
8.0	8.1432	.5401		10.0174										
10.0	10.1421	.5420		12.0258										
12.0	12.1419	.5423		14.0274										
14.0	14.1418	.5424		16.0277										
16.0	16.1418	.5424		18.0278										
18.0	18.1418	.5424		20.0278										
$\zeta = 1.02; \omega_{n1} = 8.0$					$\zeta = 1.50; \omega_{n1} = 0.5$					$\zeta = 1.50; \omega_{n1} = 4.0$				
.1	.1834	.0635	-2	.6706	-1	.1916	-0.7579	-2	.4570	.1	.1762	.2404	-2	.3145
.2	.3413	.2145	-1	.2304	.2	.3711	-.2378	-1	.1697	.2	.3273	.7805	-2	.1039
.3	.4818	.4186	-1	.4532	.3	.5427	-.4298	-1	.3581	.3	.4598	.1474	-1	.1992
.4	.6099	.6549	-1	.7140	.4	.7066	-.6251	-1	.6019	.4	.5817	.2251	-1	.3083
.5	.7291	.9109	-1	.9996	.5	.8700	-.8106	-1	.8944	.5	.6966	.3073	-1	.4263
.6	.8416	.1178		1.3011	.6	1.0279	-.9806	-1	1.231	.6	.8066	.3917	-1	.5500
.7	.9491	.1452		1.6123	.7	1.1827	-.1133		1.607	.7	.9133	.4768	-1	.6774
.8	1.0528	.1727		1.9288	.8	1.3347	-.1368		2.019	.8	1.0175	.5618	-1	.8072
.9	1.1536	.2001		2.2474	.9	1.4842	-.1599		2.431	.9	1.1199	.6458	-1	.9384
1.0	1.2522	.2272		2.5657	1.0	1.6314	-.1837		2.842	1.0	1.2210	.7284	-1	1.0704
1.5	1.7273	.3535		4.1010	1.5	2.3365	-.2333		5.717	1.5	1.7167	.1113		1.7284
2.0	2.1931	.4588		5.4777	2.0	2.9988	-.2806		9.004	2.0	2.2072	.1442		2.3696
2.5	2.6614	.5417		6.6725	2.5	4.2305	-.2172		1.6609	2.5	3.1901	.1949		3.5896
3.0	3.1355	.6044		7.6997	3.0	5.3795	-.2233		2.5088	3.0	4.1782	.2300		4.7417
3.5	3.6156	.6504		8.5875	3.5	7.5418	-.2277		4.3443	4.0	6.1648	.2705		6.9173
4.0	4.1010	.6834		9.3662	4.0	9.6144	-.2292		6.2710	6.0	8.1587	.2895		8.9994
5.0	5.0830	.7228		10.0998	4.5	11.6475	-.2297		8.2376	8.0	10.1559	.2984		11.0378
6.0	6.0742	.7416		11.8587	5.0	13.6629	-.2300		10.2221	10.0	12.1545	.3026		13.0557
8.0	8.0682	.7542		13.9655	6.0	15.6700	-.2301		12.2150	12.0	14.1539	.3044		15.0638
10.0	10.0669	.7568		15.9872	8.0	17.6733	-.2302		14.2117	14.0	16.1536	.3054		17.0679
12.0	12.0667	.7573		17.9914	10.0	19.6748	-.2302		16.2101	16.0	18.1535	.3058		19.0697
14.0	14.0666	.7574		19.9923	12.0	21.6755	-.2302		18.2094	18.0	20.1535	.3060		21.0705
16.0	16.0666	.7574		21.9925	14.0	23.6759	-.2302		20.2090	20.0	22.1534	.3061		23.0709
18.0	18.0666	.7574		23.9925	16.0	25.6760	-.2302		22.2088	22.0	24.1534	.3061		25.0710
20.0	20.0666	.7574		25.9923	18.0	27.6761	-.2302		24.2088	24.0	26.1534	.3061		27.0710
$\zeta = 1.02; \omega_{n1} = 16.0$					$\zeta = 1.50; \omega_{n1} = 1.0$					$\zeta = 1.50; \omega_{n1} = 8.0$				
.1	.1829	.7264	-2	.1334	.1	.1839	-.7667	-2	.8388	.1	.1772	.4930	-2	.6223
.2	.3396	.2472	-1	.4560	.2	.3462	-.2468	-1	.2915	.2	.3242	.1601	-1	.2038
.3	.4765	.4825	-1	.0935	.3	.4956	-.4612	-1	.5833	.3	.4540	.3025	-1	.3880
.4	.6049	.7550	-1	1.4031	.4	.6367	-.6958	-1	.9374	.4	.5730	.4622	-1	.5968
.5	.7222	.1050		1.9580	.5	.7722	-.9375	-1	1.341	.5	.6848	.6313	-1	.8202
.6	.8330	.1359		2.5410	.6	.9037	-.1179		1.784	.6	.7918	.8052	-1	1.0523
.7	.9387	.1674		3.1400	.7	1.0322	-.1415		2.262	.7	.8955	.9808	-1	1.2892
.8	1.0408	.1992		3.7454	.8	1.1586	-.1644		2.770	.8	.9967	.1156		1.5282
.9	1.1400	.2309		4.3540	.9	1.2832	-.1863		3.305	.9	1.0963	.1330		1.7675
1.0	1.2372	.2622		4.9580	1.0	1.4065	-.2071		3.864	1.0	1.1947	.1501		2.0059
1.5	1.7063	.4082		7.8243	1.5	2.0090	-.2951		6.959	1.5	1.6784	.2298		3.1597
2.0	2.1676	.5300		10.3114	2.0	2.5999	-.3590		1.0451	2.0	2.1597	.2983		4.2268
2.5	2.6332	.6258		12.3804	2.5	3.1707	-.4047		1.4246	2.5	2.6295	.3674		5.2942
3.0	3.1053	.6984		14.0687	3.0	3.7348	-.4375		1.8276	3.0	3.1295	.4042		6.3604
3.5	3.5841	.7516		15.4417	3.5	4.2950	-.4691		2.2659	3.5	3.6095	.4476		7.4150
4.0	4.0686	.7898		16.5686	4.0	4.8552	-.5005		2.6893	4.0	4.0874	.4868		8.4646
5.0	5.0495	.8355		18.3185	5.0	5.4154	-.5302		3.1095	5.0	4.5774	.5268		9.5150
6.0	6.0402	.8752		19.6754	6.0	5.9756	-.5591		3.5297	6.0	5.0774	.5668		10.5650
7.0	7.0359	.9073		20.8403	7.0	6.5358	-.5875		3.9499	7.0	5.5774	.6068		11.6150
8.0	8.0339	.9378		21.9152	8.0	7.0960	-.6154		4.3699	8.0	6.0774	.6468		12.6650
9.0	9.0330	.9673		22.9489	9.0	7.6562	-.6429		4.7899	9.0	6.5774	.6868		13.7150
10.0	10.0326	.9948		23.9638	10.0	8.2164	-.6700		5.2099	10.0	7.0774	.7268		14.7650
11.0	11.0324	.9792		24.9705	11.0	8.7766	-.6967		5.6299	11.0	7.5774	.7668		15.8150
12.0	12.0323	.9754		25.9735	12.0	9.3368	-.7230		6.0499	12.0	8.0774	.8068		16.8650
13.0	13.0323	.9794		26.9746	13.0	9.8970	-.7487		6.4699	13.0	8.5774	.8468		17.9150
14.0	14.0323	.9755		27.9755	14.0	1.0459	-.7740		6.8899	14.0	9.0774	.8868		18.9650
15.0	15.0323	.9755		28.9755	15.0	1.1061	-.7993		7.3099	15.0	9.5774	.9268		19.9650
$\zeta = 1.50; \omega_{n1} = 0$					$\zeta = 1.50; \omega_{n1} = 2.0$					$\zeta = 1.50; \omega_{n1} = 16.0$				
.5	1.1321	1.3279		.6379	.1	.1801	-.1922	-2	.1607	.1	.1767	.6283	-2	.1238
1.0	1.9812	2.2168		.2944	.2	.3335	-.6230	-2	.5399	.2	.3227	.2041	-1	.4038
1.5	2.7239	2.9731		.8509	.3	.4715	-.1174	-1	.1050	.3	.4511	.3856	-1	.7658
2.0	3.4041	3.6992		1.2152	.4	.5996	-.1789	-1	.1646	.4	.5687	.5092	-1	1.1740
2.5	4.0426	4.3008		1.6088	.5	.7210	-.2436	-1	.2303	.5	.6791	.6050	-1	1.6088
3.0	4.6507	4.9103		2.0237	.6	.8378	-.3097	-1	.3003	.6	.7846	.7027	-1	2.0583
3.5	5.2377	5.4979		2.4556	.7	.9511	-.3761	-1	.3737	.7	.8869	.8000	-1	2.5147
4.0	5.8058	6.0672		2.9008	.8	1.0619	-.4420	-1	.4497	.8	.9867	.8967	-1	2.9729
4.5	6.3611	6.6229		3.3587	.9	1.1709	-.5068	-1	.5277	.9	1.0850	.9867	-1	3.4292
5.0	6.9034	7.1656		3.8225	1.0	1.2785	-.5703	-1	.6075	1.0	1.1822	.1915		3.8814
5.5	7.4398	7.7022		4.2907	1.5	1.8040	-.8613	-1	1.4590	1.5	1.6606	.2933		6.0321
6.0	7.9718	8.2342		4.7660	2.0	2.3199	-.1105		1.9068	2.0	2.1379	.3809		7.9570
6.5	8.4967	8.7594		5.2464	2.5	2.8318	-.1307		2.3641	2.5	2.6184	.4548		9.6578
7.0	9.0163	9.2791			3.0	3.3414	-.1472		2.8114	3.0	3.1025	.5165		11.1608
					3.5	3.8510	-.1686		3.2587	3.5	3.5896	.5678		12.4949
					4.0	4.3560	-.1719		3.7060	4.0	4.0792	.6104		13.6875
					4.5	4.8610	-.1886		4.1533	4.5	4.5640	.6514		14.8800
					5.0	5.3661	-.1998		4.6.27					

TABLE II.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 1 THIRD-ORDER PLANT
WITH ZERO - Concluded

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_m$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_m$	b/B	$R_0 \omega_n / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_m$	b/B	$R_0 \omega_n / BK$
$\zeta = 2.00; \omega_n T_1 = 0$					$\zeta = 2.00; \omega_n T_1 = 1.0$					$\zeta = 2.00; \omega_n T_1 = 4.0$				
0.5	1.1032	1.2615		0.5509 -1	6.0		7.3185	-0.7214	3.9602	20.0		20.1688	0.6072 -1	20.0740
1.0	1.9773	2.1560		.2014	8.0		9.4817	-.7262	5.7921	22.0		22.1688	.6085 -1	22.0746
1.5	2.7624	2.9456		.4209	10.0		11.5750	-.7280	7.6970	24.0		24.1688	.6094 -1	24.0750
2.0	3.4921	3.6769		.6927	12.0		13.6288	-.7287	9.6425	26.0		26.1688	.6094 -1	26.0750
2.5	4.1809	4.3661		1.0042	14.0		15.6600	-.7291	11.6109	28.0		28.1688	.6094 -1	28.0750
3.0	4.8382	5.0238		1.3473	16.0		17.6781	-.7293	13.5926	30.0		30.1687	.6103 -1	30.0754
3.5	5.4708	5.6562		1.7147	18.0		19.6887	-.7294	15.5819	$\zeta = 2.00; \omega_n T_1 = 8.0$				
4.0	6.0842	6.2698		2.1014	20.0		21.6949	-.7294	17.5757	.1		.1716	.3695 -2	.5792 -1
4.5	6.6787	6.8643		2.5068	22.0		23.6985	-.7295	19.5720	.2		.3101	.1152 -1	.1821
5.0	7.2567	7.4423		2.9288	24.0		25.7006	-.7295	21.5699	.3		.4322	.2121 -1	.3375
5.5	7.8263	8.0119		3.3593	26.0		27.7019	-.7295	23.5686	.4		.5451	.3181 -1	.5094
6.0	8.3854	8.5710		3.8003	28.0		29.7025	-.7295	25.5679	.5		.6525	.4285 -1	.6903
6.5	8.9373	9.1230		4.2485	30.0		31.7030	-.7295	27.5675	.6		.7565	.5407 -1	.8761
7.0	9.4822	9.6679		4.7036	$\zeta = 2.00; \omega_n T_1 = 2.0$					.7		.8582	.6531 -1	1.0643
7.5	10.0213	10.2071		5.1645	.1		.1743	-.5370 -2	.1494 -1	.8		.9585	.7647 -1	1.2533
8.0	10.5540	10.7397		5.6316	.2		.3185	-.1620 -1	.4810 -1	.9		1.0578	.8749 -1	1.4421
8.5	11.0834	11.2691		6.1023	.3		.4477	-.3070 -1	.9094 -1	1.0		1.1565	.9833 -1	1.6301
9.0	11.6086	11.7943		6.5770	.4		.5684	-.4593 -1	.1398	1.5		1.6468	.1492	2.5467
9.5	12.1305	12.3161		7.0551	.5		.6840	-.6171 -1	.1926	2.0		2.1366	.1942	3.4170
10.0	12.6493	12.8350		7.5364	.6		.7962	-.7767 -1	.2484	3.0		3.1200	.2685	5.0278
$\zeta = 2.00; \omega_n T_1 = 0.5$					.7		.9063	-.9356 -1	.3065	4.0		4.1079	.3256	6.4970
.1		.1852	-0.2119 -1	.4229 -2	.8		1.0150	-.1093	.3665	6.0		6.0923	.4031	9.1328
.2		.3530	-.6405	.1495 -1	.9		1.1227	-.1247	.4280	8.0		8.0836	.4487	11.5059
.3		.5129	-.1132	.3052 -1	1.0		1.2296	-.1397	.4909	10.0		10.0785	.4754	13.7249
.4		.6689	-.1620	.5013 -1	1.5		1.7593	-.2093	.8222	12.0		12.0756	.4911	15.8530
.5		.8228	-.2075	.7331 -1	2.0		2.2854	-.2691	1.1765	14.0		14.0740	.5002	17.9280
.6		.9761	-.2482	.9975 -1	3.0		3.3318	-.3640	1.9401	16.0		16.0730	.5056	19.9718
.7		1.1289	-.2837	.1292	4.0		4.3717	-.4334	2.7614	18.0		18.0724	.5088	21.9976
.8		1.2814	-.3140	.1616	6.0		6.4330	-.5218	4.5234	20.0		20.0721	.5106	24.0125
.9		1.4335	-.3396	.1967	8.0		8.4735	-.5702	6.3860	22.0		22.0719	.5116	26.0213
1.0		1.5852	-.3608	.2344	10.0		10.4992	-.5972	8.3064	24.0		24.0718	.5123	28.0268
1.5		2.3308	-.4224	.4580	12.0		12.5149	-.6125	10.2600	26.0		26.0717	.5127	30.0296
2.0		3.0464	-.4454	.7309	14.0		14.5244	-.6213	12.2330	28.0		28.0717	.5128	32.0307
3.0		4.3854	-.4588	1.3852	16.0		16.5300	-.6264	14.2172	30.0		30.0717	.5129	34.0318
4.0		5.6285	-.4620	2.1405	18.0		18.5333	-.6294	16.2080	$\zeta = 2.00; \omega_n T_1 = 16.0$				
6.0		7.9345	-.4634	3.8338	20.0		20.5353	-.6311	18.2026	.1		.1712	.5402 -2	.1153
8.0		10.1029	-.4637	5.6653	22.0		22.5364	-.6321	20.1994	.2		.3087	.1165 -1	.3610
10.0		12.1980	-.4638	7.5701	24.0		24.5371	-.6327	22.1976	.3		.4297	.3102 -1	.6666
12.0		14.2525	-.4639	9.5155	26.0		26.5375	-.6330	24.1965	.4		.5414	.4652 -1	1.0030
14.0		16.2841	-.4639	11.4839	28.0		28.5377	-.6332	26.1959	.5		.6476	.6268 -1	1.3553
16.0		18.3025	-.4639	13.4656	30.0		30.5379	-.6333	28.1955	.6		.7503	.7910 -1	1.7153
18.0		20.3132	-.4639	15.4549	$\zeta = 2.00; \omega_n T_1 = 4.0$					.7		.8509	.9556 -1	2.0781
20.0		22.3194	-.4639	17.4486	.1		.1725	.4478 -2	.2926 -1	.8		.9500	.1119	2.4404
22.0		24.3230	-.4639	19.4450	.2		.3128	-.1396 -2	.9275 -1	.9		1.0482	.1280	2.8005
24.0		26.3252	-.4639	21.4428	.3		.4373	-.2567 -2	1.730	1.0		1.1459	.1439	3.1568
26.0		28.3264	-.4639	23.4416	.4		.5527	-.3850 -2	.2627	1.5		1.6315	.2184	4.8635
28.0		30.3272	-.4639	25.4409	.5		.6627	-.5183 -2	.3581	2.0		2.1176	.2844	6.4336
30.0		32.3276	-.4639	27.4404	.6		.7692	-.6537 -2	.4570	2.5		2.6056	.3425	7.8743
$\zeta = 2.00; \omega_n T_1 = 1.0$					.7		.8735	-.7892 -2	.5581	3.0		3.0955	.3935	9.1998
.1		.1779	-.1428 -1	.7786 -2	.8		.9762	-.9236 -2	.6607	4.0		4.0797	.4774	11.5582
.2		.3299	-.4420 -1	.2587 -1	.9		1.0780	-.1056 -1	.7642	5.0		5.0681	.5418	13.6009
.3		.4692	-.8055 -1	.5022 -1	1.0		1.1791	-.1187 -1	.8684	6.0		6.0596	.5913	15.4007
.4		.6015	-.1195	.7901 -1	1.5		1.6801	-.1797 -1	1.3917	8.0		8.0484	.6582	18.4833
.5		.7298	-.1590	.1113	2.0		2.1790	-.2335 -1	1.9144	10.0		10.0421	.6975	21.1187
.6		.8556	-.1980	.1464	3.0		3.1766	-.3219 -1	2.9521	12.0		12.0384	.7206	23.4905
.7		.9800	-.2360	.1840	4.0		4.1748	-.3897 -1	3.9811	14.0		14.0363	.7340	25.7077
.8		1.1036	-.2725	.2239	6.0		6.1723	-.4812 -1	5.0202	16.0		16.0351	.7419	27.8357
.9		1.2268	-.3072	.2659	8.0		8.1708	-.5348 -1	6.0431	18.0		18.0344	.7466	29.9107
1.0		1.3499	-.3402	.3099	10.0		10.1699	-.5663 -1	7.0566	20.0		20.0339	.7492	31.9540
1.5		1.9671	-.4764	.5565	12.0		12.1694	-.5846 -1	8.0644	22.0		22.0337	.7508	33.9795
2.0		2.5887	-.5694	.8420	14.0		14.1692	-.5954 -1	9.0690	24.0		24.0336	.7518	35.9948
3.0		3.8279	-.6654	1.5067	16.0		16.1690	-.6017 -1	10.0717	26.0		26.0335	.7523	38.0038
4.0		5.0336	-.7011	2.2653	18.0		18.1689	-.6054 -1	11.0733	28.0		28.0334	.7526	40.0089
										30.0		30.0334	.7528	42.0114

TABLE III.- NORMALIZED OPTIMUM RESPONSE FOR A TYPE 1 THIRD-ORDER
UNSTABLE PLANT WITH ZERO

T_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$	T_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$	T_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$
$\tau_2/\tau_3 = 0.10; \tau_1/\tau_2 = 0$					$\tau_2/\tau_3 = 0.10; \tau_1/\tau_2 = 2.0$					$\tau_2/\tau_3 = 0.10; \tau_1/\tau_2 = 5.0$				
0.2528	-1	0.7528	-1	0.0281	-1	0.5048	-1	0.1	-0.7314	-3	0.5002	-3	0.5086	-1
.5112	-1	.1511	.2	.2529	-4	.1019	.2	.2	-.2850	-2	.1997	-2	.1034	.2
.7751	-1	.2275	.3	.8445	-4	.1540	.3	.3	-.6247	-2	.4485	-2	.1575	.3
.1044	.3045	.4	.1996	-3	.2068	.4	.1681	.4	-.1681	-1	.7956	-2	.2130	.4
.1319	.3821	.5	.3905	-3	.2602	.5	-.1644	.5	-.1644	-1	.1240	-1	.2699	.5
.1599	.4602	.6	.6730	-3	.3141	.6	-.2304	.6	-.2304	-1	.1779	-1	.3280	.6
.1884	.5390	.7	.1067	-2	.3684	.7	-.3050	.7	-.3050	-1	.2413	-1	.3873	.7
.2175	.6183	.8	.1590	-2	.4230	.8	-.3873	.8	-.3873	-1	.3139	-1	.4476	.8
.2470	.6981	.9	.2261	-2	.4779	.9	-.4764	.9	-.4764	-1	.3955	-1	.5089	.9
.2770	.7785	1.0	.3095	-2	.5329	1.0	-.5715	1.0	-.5715	-1	.4899	-1	.5710	1.0
.3070	.8589	1.1	.4032	-1	.5879	1.1	-.6809	1.1	-.6809	-1	.5907	-1	.6361	1.1
.3370	.9393	1.2	.5070	-1	.6429	1.2	-.7910	1.2	-.7910	-1	.7018	-1	.7018	1.2
.3670	1.0197	1.3	.6208	-1	.6979	1.3	-.9011	1.3	-.9011	-1	.8119	-1	.8119	1.3
.3970	1.0997	1.4	.7346	-1	.7529	1.4	-1.0112	1.4	-1.0112	-1	.9220	-1	.9220	1.4
.4270	1.1797	1.5	.8484	-1	.8079	1.5	-1.1213	1.5	-1.1213	-1	1.0321	-1	1.0321	1.5
.4570	1.2597	1.6	.9622	-1	.8629	1.6	-1.2314	1.6	-1.2314	-1	1.1422	-1	1.1422	1.6
.4870	1.3397	1.7	1.0760	-1	.9179	1.7	-1.3415	1.7	-1.3415	-1	1.2523	-1	1.2523	1.7
.5170	1.4197	1.8	1.1898	-1	.9729	1.8	-1.4516	1.8	-1.4516	-1	1.3624	-1	1.3624	1.8
.5470	1.4997	1.9	1.3036	-1	1.0279	1.9	-1.5617	1.9	-1.5617	-1	1.4725	-1	1.4725	1.9
.5770	1.5797	2.0	1.4174	-1	1.0829	2.0	-1.6718	2.0	-1.6718	-1	1.5826	-1	1.5826	2.0
.6070	1.6597	2.1	1.5312	-1	1.1379	2.1	-1.7819	2.1	-1.7819	-1	1.6927	-1	1.6927	2.1
.6370	1.7397	2.2	1.6450	-1	1.1929	2.2	-1.8920	2.2	-1.8920	-1	1.8028	-1	1.8028	2.2
.6670	1.8197	2.3	1.7588	-1	1.2479	2.3	-2.0021	2.3	-2.0021	-1	1.9129	-1	1.9129	2.3
.6970	1.8997	2.4	1.8726	-1	1.3029	2.4	-2.1122	2.4	-2.1122	-1	2.0230	-1	2.0230	2.4
.7270	1.9797	2.5	1.9864	-1	1.3579	2.5	-2.2223	2.5	-2.2223	-1	2.1331	-1	2.1331	2.5
.7570	2.0597	2.6	2.0998	-1	1.4129	2.6	-2.3324	2.6	-2.3324	-1	2.2432	-1	2.2432	2.6
.7870	2.1397	2.7	2.2136	-1	1.4679	2.7	-2.4425	2.7	-2.4425	-1	2.3533	-1	2.3533	2.7
.8170	2.2197	2.8	2.3274	-1	1.5229	2.8	-2.5526	2.8	-2.5526	-1	2.4634	-1	2.4634	2.8
.8470	2.2997	2.9	2.4412	-1	1.5779	2.9	-2.6627	2.9	-2.6627	-1	2.5735	-1	2.5735	2.9
.8770	2.3797	3.0	2.5550	-1	1.6329	3.0	-2.7728	3.0	-2.7728	-1	2.6836	-1	2.6836	3.0
.9070	2.4597	3.1	2.6688	-1	1.6879	3.1	-2.8829	3.1	-2.8829	-1	2.7937	-1	2.7937	3.1
.9370	2.5397	3.2	2.7826	-1	1.7429	3.2	-2.9930	3.2	-2.9930	-1	2.9038	-1	2.9038	3.2
.9670	2.6197	3.3	2.8964	-1	1.7979	3.3	-3.1031	3.3	-3.1031	-1	3.0139	-1	3.0139	3.3
.9970	2.6997	3.4	3.0102	-1	1.8529	3.4	-3.2132	3.4	-3.2132	-1	3.1240	-1	3.1240	3.4
1.0270	2.7797	3.5	3.1240	-1	1.9079	3.5	-3.3233	3.5	-3.3233	-1	3.2341	-1	3.2341	3.5
1.0570	2.8597	3.6	3.2378	-1	1.9629	3.6	-3.4334	3.6	-3.4334	-1	3.3442	-1	3.3442	3.6
1.0870	2.9397	3.7	3.3516	-1	2.0179	3.7	-3.5435	3.7	-3.5435	-1	3.4543	-1	3.4543	3.7
1.1170	3.0197	3.8	3.4654	-1	2.0729	3.8	-3.6536	3.8	-3.6536	-1	3.5644	-1	3.5644	3.8
1.1470	3.0997	3.9	3.5792	-1	2.1279	3.9	-3.7637	3.9	-3.7637	-1	3.6745	-1	3.6745	3.9
1.1770	3.1797	4.0	3.6930	-1	2.1829	4.0	-3.8738	4.0	-3.8738	-1	3.7846	-1	3.7846	4.0
1.2070	3.2597	4.1	3.8068	-1	2.2379	4.1	-3.9839	4.1	-3.9839	-1	3.8947	-1	3.8947	4.1
1.2370	3.3397	4.2	3.9206	-1	2.2929	4.2	-4.0940	4.2	-4.0940	-1	4.0048	-1	4.0048	4.2
1.2670	3.4197	4.3	4.0344	-1	2.3479	4.3	-4.2041	4.3	-4.2041	-1	4.1149	-1	4.1149	4.3
1.2970	3.4997	4.4	4.1482	-1	2.4029	4.4	-4.3142	4.4	-4.3142	-1	4.2250	-1	4.2250	4.4
1.3270	3.5797	4.5	4.2620	-1	2.4579	4.5	-4.4243	4.5	-4.4243	-1	4.3351	-1	4.3351	4.5
1.3570	3.6597	4.6	4.3758	-1	2.5129	4.6	-4.5344	4.6	-4.5344	-1	4.4452	-1	4.4452	4.6
1.3870	3.7397	4.7	4.4896	-1	2.5679	4.7	-4.6445	4.7	-4.6445	-1	4.5553	-1	4.5553	4.7
1.4170	3.8197	4.8	4.6034	-1	2.6229	4.8	-4.7546	4.8	-4.7546	-1	4.6654	-1	4.6654	4.8
1.4470	3.8997	4.9	4.7172	-1	2.6779	4.9	-4.8647	4.9	-4.8647	-1	4.7755	-1	4.7755	4.9
1.4770	3.9797	5.0	4.8310	-1	2.7329	5.0	-4.9748	5.0	-4.9748	-1	4.8856	-1	4.8856	5.0
1.5070	4.0597	5.1	4.9448	-1	2.7879	5.1	-5.0849	5.1	-5.0849	-1	4.9957	-1	4.9957	5.1
1.5370	4.1397	5.2	5.0586	-1	2.8429	5.2	-5.1950	5.2	-5.1950	-1	5.1058	-1	5.1058	5.2
1.5670	4.2197	5.3	5.1724	-1	2.8979	5.3	-5.3051	5.3	-5.3051	-1	5.2159	-1	5.2159	5.3
1.5970	4.2997	5.4	5.2862	-1	2.9529	5.4	-5.4152	5.4	-5.4152	-1	5.3260	-1	5.3260	5.4
1.6270	4.3797	5.5	5.4000	-1	3.0079	5.5	-5.5253	5.5	-5.5253	-1	5.4361	-1	5.4361	5.5
1.6570	4.4597	5.6	5.5138	-1	3.0629	5.6	-5.6354	5.6	-5.6354	-1	5.5462	-1	5.5462	5.6
1.6870	4.5397	5.7	5.6276	-1	3.1179	5.7	-5.7455	5.7	-5.7455	-1	5.6563	-1	5.6563	5.7
1.7170	4.6197	5.8	5.7414	-1	3.1729	5.8	-5.8556	5.8	-5.8556	-1	5.7664	-1	5.7664	5.8
1.7470	4.6997	5.9	5.8552	-1	3.2279	5.9	-5.9657	5.9	-5.9657	-1	5.8765	-1	5.8765	5.9
1.7770	4.7797	6.0	5.9690	-1	3.2829	6.0	-6.0758	6.0	-6.0758	-1	5.9866	-1	5.9866	6.0
1.8070	4.8597	6.1	6.0828	-1	3.3379	6.1	-6.1859	6.1	-6.1859	-1	6.0967	-1	6.0967	6.1
1.8370	4.9397	6.2	6.1966	-1	3.3929	6.2	-6.2960	6.2	-6.2960	-1	6.2068	-1	6.2068	6.2
1.8670	5.0197	6.3	6.3104	-1	3.4479	6.3	-6.4061	6.3	-6.4061	-1	6.3169	-1	6.3169	6.3
1.8970	5.0997	6.4	6.4242	-1	3.5029	6.4	-6.5162	6.4	-6.5162	-1	6.4270	-1	6.4270	6.4
1.9270	5.1797	6.5	6.5380	-1	3.5579	6.5	-6.6263	6.5	-6.6263	-1	6.5371	-1	6.5371	6.5
1.9570	5.2597	6.6	6.6518	-1	3.6129	6.6	-6.7364	6.6	-6.7364	-1	6.6472	-1	6.6472	6.6
1.9870	5.3397	6.7	6.7656	-1	3.6679	6.7	-6.8465	6.7	-6.8465	-1	6.7573	-1	6.7573	6.7
2.0170	5.4197	6.8	6.8794	-1	3.7229	6.8	-6.9566	6.8	-6.9566	-1	6.8674	-1	6.8674	6.8
2.0470	5.4997	6.9	6.9932	-1	3.7779	6.9	-7.0667	6.9	-7.0667	-1	6.9775	-1	6.9775	6.9
2.0770	5.5797	7.0	7.1070	-1	3.8329	7.0	-7.1768	7.0	-7.1768	-1	7.0876	-1	7.0876	7.0
2.1070	5.6597	7.1	7.2208	-1	3.8879	7.1	-7.2869	7.1	-7.2869	-1	7.1977	-1	7.1977	7.1
2.1370	5.7397	7.2	7.3346	-1	3.9429	7.2	-7.3970	7.2	-7.3970	-1	7.3078	-1	7.3078	7.2
2.1670	5.8197	7.3	7.4484	-1	4.0079	7.3	-7.5071	7.3	-7.5071	-1	7.4179	-1	7.4179	7.3
2.1970	5.8997	7.4	7.5622	-1	4.0629	7.4	-7.6172	7.4	-7.6172	-1	7.5280	-1	7.5280	7.4
2.2270	5.9797	7.5	7.6760	-1	4.1179	7.5	-7.7273	7.5	-7.7273	-1	7.6381	-1	7.6381	7.5
2.2570	6.0597	7.6	7.7898	-1	4.1729	7.6	-7.8374	7.6	-7.8374	-1	7.7482	-1	7.7482	7.6
2.2870	6.1397	7.7	7.9036	-1	4.2279	7.7	-7.9475	7.7	-7.9475	-1	7.8583	-1	7.8583	7.7
2.3170	6.2197	7.8	8.0174	-1	4.2829	7.8	-8.0576	7.8	-8.0576	-1	7.9684	-1	7.9684	7.8
2.3470	6.2997	7.9	8.1312	-1	4.3379	7.9	-8.1677	7.9	-8.1677	-1	8.0785	-1	8.0785	7.9
2.3770	6.3797	8.0	8.2450	-1	4.3929	8.0	-8.2778	8.0	-8.2778	-1	8.1886	-1	8.1886	8.0
2.4070	6.4597	8.1	8.3588	-1	4.4479	8.1	-8.3879	8.1	-8.3879	-1	8.2987	-1	8.2987	8.1
2.4370	6.5397	8.2	8.4726	-1	4.5029	8.2	-8.4980	8.2	-8.4980	-1	8.4088	-1	8.4088	8.2
2.4670	6.6197	8.3	8.5864	-1	4.5579	8.3	-8.6081	8.3	-8.6081	-1	8.5189	-1	8.5189	8.3
2.4970	6.6997	8.4	8.7002	-1	4.6129	8.4	-8.7182	8.4	-8.7182	-1	8.6290	-1	8.6290	8.4
2.5270	6.7													

TABLE III.- NORMALIZED OPTIMUM RESPONSE FOR A TYPE 1 THIRD-ORDER
UNSTABLE PLANT WITH ZERO - Continued

T_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$	T_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$	T_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$
$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 0.5$					$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 3.0$					$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 0$				
0.4857 -1	0.1	0.5099 -2	0.3128 -3	0.5050 -1	0.1	0.9556 -3	0.1873 -2	0.2515 -1	0.7516 -1	0.1	0.5099 -1	1.507	0.2	0.1562 -4
0.9473 -1	0.2	0.1857 -1	0.1258 -2	0.1019	0.2	0.3755 -2	0.7486 -2	0.5059 -1	1.507	0.2	0.5099 -1	1.507	0.2	0.1249 -3
1.1390	0.3	0.3818 -1	0.2849 -2	0.1540	0.3	0.8293 -2	1.681 -1	0.7630 -1	2.265	0.3	0.7630 -1	2.265	0.3	0.4215 -3
1.1819	0.4	0.6226 -1	0.5111 -2	0.2068	0.4	0.1446 -1	0.2979 -1	1.022	3.027	0.4	1.022	3.027	0.4	0.9983 -3
2.236	0.5	0.8954 -1	0.8069 -2	0.2600	0.5	0.2215 -1	0.4638 -1	1.284	3.794	0.5	1.284	3.794	0.5	0.1948 -2
2.644	0.6	1.119	0.1175 -1	0.3136	0.6	0.3124 -1	0.6651 -1	1.547	4.564	0.6	1.547	4.564	0.6	0.3362 -2
3.044	0.7	1.500	0.1620 -1	0.3674	0.7	0.4164 -1	0.9008 -1	1.812	5.339	0.7	1.812	5.339	0.7	0.5332 -2
3.438	0.8	1.820	0.2143 -1	0.4213	0.8	0.5321 -1	1.170	2.079	6.118	0.8	2.079	6.118	0.8	0.7946 -2
3.827	0.9	2.144	0.2749 -1	0.4752	0.9	0.6586 -1	1.472	2.346	6.902	0.9	2.346	6.902	0.9	0.1129 -1
4.211	1.0	2.469	0.3440 -1	0.5289	1.0	0.7947 -1	1.806	2.614	7.691	1.0	2.614	7.691	1.0	0.1546 -1
4.607	1.5	4.037	0.8280 -1	0.7923	1.5	0.1584	3.907	3.955	1.1712	1.5	3.955	1.1712	1.5	0.5148 -1
4.785	2.0	5.410	0.1565	1.0393	2.0	0.2467	6.617	5.270	1.5869	2.0	5.270	1.5869	2.0	0.1197
4.977	2.5	6.543	0.2575	1.2628	2.5	0.3344	9.776	6.525	2.0165	2.5	6.525	2.0165	2.5	0.2276
1.1203	3.0	7.445	0.3871	1.4596	3.0	0.4141	1.3232	7.690	2.4596	3.0	7.690	2.4596	3.0	0.3812
1.2737	3.5	8.143	0.5454	1.6293	3.5	0.4814	1.6857	8.742	2.9153	3.5	8.742	2.9153	3.5	0.5822
1.4172	4.0	8.672	0.7320	1.7741	4.0	0.5349	2.0566	9.668	3.3820	4.0	9.668	3.3820	4.0	0.8303
1.5505	4.5	9.064	0.9459	1.8973	4.5	0.5773	2.4313	1.0464	3.8577	4.5	1.0464	3.8577	4.5	1.1226
1.6734	5.0	9.349	1.1858	2.0026	5.0	0.6046	2.8086	1.1134	4.3406	5.0	1.1134	4.3406	5.0	1.4544
1.7862	5.5	9.553	1.4500	2.0933	5.5	0.6252	3.1890	1.1687	4.8288	5.5	1.1687	4.8288	5.5	1.8202
1.8891	6.0	9.697	1.7370	2.1720	6.0	0.6393	3.5739	1.2139	5.3210	6.0	1.2139	5.3210	6.0	2.2142
1.9826	6.5	9.797	2.0449	2.2409	6.5	0.6488	3.9646	1.2502	5.8158	6.5	1.2502	5.8158	6.5	2.6312
2.0673	7.0	9.865	2.3721	2.3015	7.0	0.6551	4.3624	1.2793	6.3125	7.0	1.2793	6.3125	7.0	3.0664
2.1437	7.5	9.911	2.7170	2.3551	7.5	0.6593	4.7677	1.3023	6.8104	7.5	1.3023	6.8104	7.5	3.5161
2.2125	8.0	9.942	3.0778	2.4025	8.0	0.6619	5.1809	1.3205	7.3090	8.0	1.3205	7.3090	8.0	3.9770
2.2743	8.5	9.962	3.4532	2.4446	8.5	0.6637	5.6019	1.3349	7.8082	8.5	1.3349	7.8082	8.5	4.4467
2.3297	9.0	9.976	3.8418	2.4819	9.0	0.6648	6.0305	1.3461	8.3077	9.0	1.3461	8.3077	9.0	4.9231
2.3792	9.5	9.985	4.2423	2.5151	9.5	0.6655	6.4662	1.3549	8.8074	9.5	1.3549	8.8074	9.5	5.4049
2.4235	10.0	9.990	4.6535	2.5446	10.0	0.6659	6.9086	1.3618	9.3072	10.0	1.3618	9.3072	10.0	5.8907
2.4681	11.0	9.996	5.5040	2.5940	11.0	0.6664	7.8112	1.3714	10.3070	11.0	1.3714	10.3070	11.0	6.3711
2.5172	12.0	9.998	6.3856	2.6328	12.0	0.6666	8.7341	1.3773	11.3070	12.0	1.3773	11.3070	12.0	7.8593
2.5646	14.0	1.0000	8.2188	2.6872	14.0	0.6667	10.6255	1.3808	12.3070	13.0	1.3808	12.3070	13.0	8.8521
2.6200	16.0	1.0000	10.1160	2.7206	16.0	0.6667	12.5588	1.3830	13.3069	14.0	1.3830	13.3069	14.0	9.8478
2.7235	18.0	1.0000	12.0530	2.7410	18.0	0.6667	14.5180	1.3843	14.3069	15.0	1.3843	14.3069	15.0	10.8452
2.7428	20.0	1.0000	14.0145	2.7534	20.0	0.6667	16.4932	1.3851	15.3069	16.0	1.3851	15.3069	16.0	11.8436
2.7545	22.0	1.0000	15.9911	2.7609	22.0	0.6667	18.4781	1.3856	16.3069	17.0	1.3856	16.3069	17.0	12.8426
2.7616	24.0	1.0000	17.9768	2.7655	24.0	0.6667	20.4690	1.3858	17.3069	18.0	1.3858	17.3069	18.0	13.8420
2.7659	26.0	1.0000	19.9682	2.7683	26.0	0.6667	22.4634	1.3860	18.3069	19.0	1.3860	18.3069	19.0	14.8417
2.7685	28.0	1.0000	21.9629	2.7700	28.0	0.6667	24.4600	1.3861	19.3069	20.0	1.3861	19.3069	20.0	15.8414
2.7701	30.0	1.0000	23.9597	2.7710	30.0	0.6667	26.4580	1.3862	20.3069	21.0	1.3862	20.3069	21.0	16.8413
2.7711	32.0	1.0000	25.9578	2.7716	32.0	0.6667	28.4567			$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 0.5$				
2.7717	34.0	1.0000	27.9566	2.7720	34.0	0.6667	30.4560	0.4827 -1	0.1	0.5666 -2	0.6259 -3			
2.7720	36.0	1.0000	29.9559	2.7722	36.0	0.6667	32.4555	0.9359 -1	0.2	0.2063 -1	0.2515 -2			
2.7723	38.0	1.0000	31.9555	2.7724	38.0	0.6667	34.4552	0.1366	0.3	0.4240 -1	0.5699 -2			
2.7724	40.0	1.0000	33.9552	2.7725	40.0	0.6667	36.4551	0.1776	0.4	0.6911 -1	1.022 -1			
2.7725	42.0	1.0000	35.9551					0.2171	0.5	0.9935 -1	1.614 -1			
$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 2.0$					$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 5.0$					$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 2.0$				
0.5029 -1	0.1	0.9140 -3	0.1249 -2	0.5066 -1	0.1	0.8905 -3	0.3123 -2	0.2552	0.7516 -1	0.1	0.5666 -2	0.6259 -3		
0.1011	0.2	0.3562 -2	0.4992 -2	0.1026	0.2	0.3522 -2	1.248 -1	0.2922	1.507	0.2	0.2063 -1	0.2515 -2		
0.1522	0.3	0.7805 -2	1.1121 -1	0.1556	0.3	0.7829 -2	2.801 -1	0.3282	2.265	0.3	0.4240 -1	0.5699 -2		
0.2036	0.4	0.1351 -1	0.1968 -1	0.2053	0.4	0.1374 -1	0.4963 -1	0.3632	3.027	0.4	0.6911 -1	1.022 -1		
0.2550	0.5	0.2053 -1	0.3097 -1	0.2643	0.5	0.2118 -1	0.7725 -1	0.3973	3.794	0.5	0.9935 -1	1.614 -1		
0.3065	0.6	0.2874 -1	0.4443 -1	0.3198	0.6	0.3006 -1	1.1107	0.5563	4.564	0.6	1.320	0.2351 -1		
0.3579	0.7	0.3801 -1	0.6022 -1	0.3722	0.7	0.4031 -1	1.499	0.6971	5.339	0.7	1.663	0.3240 -1		
0.4091	0.8	0.4823 -1	0.7828 -1	0.4222	0.8	0.5183 -1	1.947	0.8202	6.118	0.8	2.016	0.4287 -1		
0.4600	0.9	0.5926 -1	0.9857 -1	0.4758	0.9	0.6454 -1	2.449	0.9260	6.902	0.9	2.374	0.5498 -1		
0.5105	1.0	0.7101 -1	1.210	0.5285	1.0	0.7836 -1	3.002	1.0154	7.691	1.0	2.732	0.6879 -1		
0.5611	1.5	1.1368	0.2634	1.0991	1.5	0.1610	6.478	1.0895	8.407	1.5	3.1320	0.8326		
0.6116	2.0	0.2065	0.4498	1.3473	2.0	0.2583	1.0934	1.1503	9.222	2.0	3.5448	1.0375		
1.1864	2.5	0.2722	0.6717	1.5661	2.5	0.3604	1.6076	1.1995	10.037	2.5	3.9409	1.2389		
1.3685	3.0	0.3293	0.9215	1.7524	3.0	0.4584	2.1600	1.2390	10.852	3.0	4.3406	1.4448		
1.5290	3.5	0.3758	1.1935	1.9067	3.5	0.5455	2.7226	1.2705	11.677	3.5	4.7481	1.651		
1.6700	4.0	0.4119	1.4837	2.0325	4.0	0.6174	3.2737	1.2955	12.502	4.0	5.1499	1.859		
1.7940	4.5	0.4388	1.7895	2.1349	4.5	0.6732	3.8009	1.3152	13.327	4.5	5.5563	2.066		
1.9033	5.0	0.4582	2.1098	2.2189	5.0	0.7142	4.3012	1.3307	14.152	5.0	5.9620	2.273		
2.0000	5.5	0.4718	2.4438	2.2888	5.5	0.7431	4.7776	1.3428	14.977	5.5	6.3694	2.480		
2.0857	6.0	0.4813	2.7911	2.3479	6.0	0.7629	5.2367	1.3524	15.802	6.0	6.7769	2.687		
2.1620	6.5	0.4877	3.1513	2.3986	6.5	0.7760	5.6844	1.3598	16.627	6.5	7.1847	2.894		
2.2300	7.0	0.4920	3.5239	2.4425	7.0	0.7847	6.1262	1.3656	17.452	7.0	7.5925	3.101		
2.2906	7.5	0.4948	3.9084	2.480480										

TABLE III.- NORMALIZED OPTIMUM RESPONSE FOR A TYPE 1 THIRD-ORDER
UNSTABLE PLANT WITH ZERO - Continued

τ_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$	τ_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$	τ_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$
$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 2.0$					$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 5.0$					$\tau_2/\tau_3 = 1.0; \tau_1/\tau_2 = 2.0$				
0.8302	2.0	-0.2572	0.8541	1.3661	7.5	-0.7970	8.7526	0.6582	3.0	-0.4287	2.5411			
0.9567	2.5	-0.3274	1.2413	1.3706	8.0	-0.7982	9.2496	0.6723	3.5	-0.4559	3.0672			
1.0544	3.0	-0.3820	1.6552	1.3741	8.5	-0.7989	9.7462	0.6807	4.0	-0.4730	3.5846			
1.1293	3.5	-0.4217	2.0848	1.3768	9.0	-0.7993	10.2430	0.6856	4.5	-0.4835	4.0957			
1.1867	4.0	-0.4492	2.5249	1.3789	9.5	-0.7996	10.7401	0.6886	5.0	-0.4900	4.6027			
1.2310	4.5	-0.4675	2.9731	1.3805	10.0	-0.7997	11.2377	0.6904	5.5	-0.4939	5.1070			
1.2652	5.0	-0.4795	3.4285	1.3828	11.0	-0.7999	12.2339	0.6915	6.0	-0.4963	5.6096			
1.2918	5.5	-0.4872	3.8906	1.3842	12.0	-0.8000	13.2315	0.6921	6.5	-0.4977	6.1112			
1.3126	6.0	-0.4920	4.3589	1.3850	13.0	-0.8000	14.2299	0.6925	7.0	-0.4986	6.6122			
1.3288	6.5	-0.4951	4.8326	1.3855	14.0	-0.8000	15.2289	0.6928	7.5	-0.4992	7.1128			
1.3414	7.0	-0.4970	5.3111	1.3858	15.0	-0.8000	16.2283	0.6929	8.0	-0.4995	7.6131			
1.3513	7.5	-0.4981	5.7937	1.3860	16.0	-0.8000	17.2280	0.6930	8.5	-0.4997	8.1134			
1.3590	8.0	-0.4989	6.2797	1.3861	17.0	-0.8000	18.2278	0.6931	9.0	-0.4998	8.6135			
1.3650	8.5	-0.4993	6.7686	1.3862	18.0	-0.8000	19.2276							
1.3697	9.0	-0.4996	7.2598											
1.3734	9.5	-0.4997	7.7528											
1.3762	10.0	-0.4998	8.2473											
1.3802	11.0	-0.4999	9.2395											
1.3826	12.0	-0.5000	10.2348											
1.3840	13.0	-0.5000	11.2319											
1.3849	14.0	-0.5000	12.2301											
1.3855	15.0	-0.5000	13.2291											
1.3858	16.0	-0.5000	14.2284											
1.3860	17.0	-0.5000	15.2280											
1.3861	18.0	-0.5000	16.2278											
1.3862	19.0	-0.5000	17.2276											
$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 3.0$					$\tau_2/\tau_3 = 1.0; \tau_1/\tau_2 = 0$					$\tau_2/\tau_3 = 1.0; \tau_1/\tau_2 = 3.0$				
0.5017	-1	-1.1365	-2	0.3748	-2	0.7501	-1	0.3121	-4	0.4953	-1	0.1	-0.2183	-2
0.1006	-2	-0.5362	-2	0.1497	-1	0.4994	-1	0.1501	2	0.9790	-1	0.2	-0.8564	-2
0.1510	-3	-0.1183	-1	0.3358	-1	0.7479	-1	0.2252	3	0.1448	0.3	0.3	-0.1885	-1
0.2012	-4	-0.2061	-1	0.5947	-1	0.9950	-1	0.3005	4	0.1900	0.4	0.4	-0.3273	-1
0.2510	-5	-0.3153	-1	0.9249	-1	1.2400	-1	0.3760	5	0.2333	0.5	0.5	-0.4984	-1
0.3004	-6	-0.4440	-1	1.324	-1	1.483	-1	0.4517	6	0.2744	0.6	0.6	-0.6977	-1
0.3490	-7	-0.5904	-1	1.791	-1	1.723	-1	0.5277	7	0.3132	0.7	0.7	-0.9213	-1
0.3968	-8	-0.7527	-1	2.321	-1	1.960	-1	0.6040	8	0.3495	0.8	0.8	-1.1165	-1
0.4437	-9	-0.9290	-1	2.913	-1	2.194	-1	0.6806	9	0.3833	0.9	0.9	-1.424	-1
0.4894	-10	-1.1117	-1	3.564	-1	2.423	-1	0.7577	10	0.4146	1.0	1.0	-1.694	-1
0.5361	-11	-1.2175	-1	4.275	-1	2.646	-1	0.8337	11	0.4377	1.1	1.1	-2.0691	-1
0.5822	-12	-1.3421	-1	5.045	-1	2.864	-1	0.9054	12	0.4639	1.2	1.2	-2.5130	-1
0.6281	-13	-1.4841	-1	5.874	-1	3.075	-1	1.0408	13	0.4905	1.3	1.3	-3.0708	-1
0.6734	-14	-1.6440	-1	6.761	-1	3.283	-1	1.1428	14	0.5180	1.4	1.4	-3.6676	-1
0.7181	-15	-1.8215	-1	7.707	-1	3.488	-1	1.2532	15	0.5463	1.5	1.5	-4.3055	-1
0.7622	-16	-2.0165	-1	8.714	-1	3.690	-1	1.3732	16	0.5753	1.6	1.6	-5.0590	-1
0.8057	-17	-2.2290	-1	9.781	-1	3.888	-1	1.5032	17	0.6050	1.7	1.7	-5.9803	-1
0.8486	-18	-2.4585	-1	10.908	-1	4.083	-1	1.6432	18	0.6354	1.8	1.8	-6.9934	-1
0.8909	-19	-2.7050	-1	12.095	-1	4.275	-1	1.7932	19	0.6664	1.9	1.9	-8.1127	-1
0.9326	-20	-2.9685	-1	13.342	-1	4.464	-1	1.9532	20	0.6979	2.0	2.0	-9.3411	-1
0.9737	-21	-3.2490	-1	14.649	-1	4.650	-1	2.1232	21	0.7300	2.1	2.1	-10.6795	-1
1.0142	-22	-3.5465	-1	16.016	-1	4.833	-1	2.3032	22	0.7626	2.2	2.2	-12.1279	-1
1.0542	-23	-3.8610	-1	17.443	-1	5.014	-1	2.4932	23	0.7957	2.3	2.3	-13.6863	-1
1.0937	-24	-4.1925	-1	18.930	-1	5.193	-1	2.6932	24	0.8292	2.4	2.4	-15.3547	-1
1.1327	-25	-4.5410	-1	20.477	-1	5.370	-1	2.9032	25	0.8632	2.5	2.5	-17.1331	-1
1.1712	-26	-4.9065	-1	22.084	-1	5.545	-1	3.1232	26	0.8977	2.6	2.6	-19.0215	-1
1.2092	-27	-5.2890	-1	23.751	-1	5.718	-1	3.3532	27	0.9327	2.7	2.7	-21.0200	-1
1.2467	-28	-5.6885	-1	25.478	-1	5.889	-1	3.5932	28	0.9682	2.8	2.8	-23.1285	-1
1.2837	-29	-6.1050	-1	27.265	-1	6.058	-1	3.8432	29	1.0042	2.9	2.9	-25.3470	-1
1.3202	-30	-6.5385	-1	29.112	-1	6.225	-1	4.1032	30	1.0407	3.0	3.0	-27.6655	-1
1.3562	-31	-7.0000	-1	31.019	-1	6.390	-1	4.3732	31	1.0777	3.1	3.1	-30.0840	-1
1.3917	-32	-7.4885	-1	32.986	-1	6.553	-1	4.6532	32	1.1152	3.2	3.2	-32.6025	-1
1.4267	-33	-8.0040	-1	34.913	-1	6.714	-1	4.9432	33	1.1532	3.3	3.3	-35.2210	-1
1.4612	-34	-8.5465	-1	36.900	-1	6.873	-1	5.2432	34	1.1917	3.4	3.4	-37.9395	-1
1.4952	-35	-9.1160	-1	38.947	-1	7.030	-1	5.5532	35	1.2307	3.5	3.5	-40.7580	-1
1.5287	-36	-9.7135	-1	41.054	-1	7.185	-1	5.8732	36	1.2702	3.6	3.6	-43.6765	-1
1.5617	-37	-10.3390	-1	43.221	-1	7.338	-1	6.2032	37	1.3102	3.7	3.7	-46.6950	-1
1.5942	-38	-10.9825	-1	45.448	-1	7.489	-1	6.5432	38	1.3507	3.8	3.8	-49.8135	-1
1.6262	-39	-11.6475	-1	47.735	-1	7.638	-1	6.8932	39	1.3917	3.9	3.9	-53.0320	-1
1.6577	-40	-12.3300	-1	50.082	-1	7.785	-1	7.2532	40	1.4332	4.0	4.0	-56.3505	-1
1.6887	-41	-13.0425	-1	52.499	-1	7.930	-1	7.6232	41	1.4752	4.1	4.1	-59.7690	-1
1.7192	-42	-13.7850	-1	54.976	-1	8.073	-1	8.0032	42	1.5177	4.2	4.2	-63.2875	-1
1.7492	-43	-14.5575	-1	57.513	-1	8.214	-1	8.3932	43	1.5607	4.3	4.3	-66.9060	-1
1.7787	-44	-15.3600	-1	60.110	-1	8.353	-1	8.7932	44	1.6042	4.4	4.4	-70.6245	-1
1.8077	-45	-16.1925	-1	62.767	-1	8.490	-1	9.2032	45	1.6482	4.5	4.5	-74.4430	-1
1.8362	-46	-17.0550	-1	65.484	-1	8.625	-1	9.6232	46	1.6927	4.6	4.6	-78.3615	-1
1.8642	-47	-17.9475	-1	68.261	-1	8.758	-1	10.0532	47	1.7377	4.7	4.7	-82.3800	-1
1.8917	-48	-18.8700	-1	71.098	-1	8.889	-1	10.5032	48	1.7832	4.8	4.8	-86.5085	-1
1.9187	-49	-19.8225	-1	73.995	-1	9.018	-1	10.9732	49	1.8292	4.9	4.9	-90.7470	-1
1.9452	-50	-20.8050	-1	76.952	-1	9.145	-1	11.4632	50	1.8757	5.0	5.0	-95.0855	-1
1.9712	-51	-21.8175	-1	80.000	-1	9.270	-1	11.9732	51	1.9227	5.1	5.1	-99.5240	-1
2.0000	-52	-22.8600	-1	83.125	-1	9.393	-1	12.5032	52	1.9702	5.2	5.2	-104.0625	-1
2.0217	-53	-23.9425	-1	86.330	-1	9.514	-1	13.0532	53	2.0182	5.3	5.3	-108.7010	-1
2.0462	-54	-25.0650	-1	89.605	-1	9.633	-1	13.6232	54	2.0667	5.4	5.4	-113.4395	-1
2.0702	-55	-26.2275	-1											

TABLE III.- NORMALIZED OPTIMUM RESPONSE FOR A TYPE 1 THIRD-ORDER
UNSTABLE PLANT WITH ZERO - Concluded

T_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$	T_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$	T_1/τ_2	T_2/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2$
$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 0.5$					$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 3.0$					$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 0.5$				
0.4649 -1		0.1	0.9051 -2	0.2501 -2	0.2962		1.0	-0.2523	1.1647	0.1269		0.6	0.3027	0.1949
.8678 -1		.2	.3281 -1	.1002 -1	.3305		1.5	-.4014	2.0432	.1312		.7	.3642	.2556
.1219		.3	.6706 -1	.2262 -1	.3414		2.0	-.5030	2.8263	.1339		.8	.4219	.3212
.1527		.4	.1085	.4037 -1	.3448		2.5	-.5669	3.5109	.1357		.9	.4753	.3910
.1796		.5	.1547	.6334 -1	.3460		3.0	-.6060	4.1261	.1369		1.0	.5243	.4643
.2033		.6	.2038	.9156 -1	.3464		3.5	-.6299	4.6969	.1385		1.5	.7106	.8678
.2240		.7	.2541	.1250	.3465		4.0	-.6443	5.2400	.1386		2.0	.8244	1.3106
.2420		.8	.3046	.1637	$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 5.0$					$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 2.0$				
.2577		.9	.3544	.2074	.4839 -1		.1	-.4345 -2	.2494 -1	.4427 -1		.1	-.6625 -2	.2471 -1
.2712		1.0	.4029	.2561	.9319 -1		.2	-.1707 -1	.9897 -1	.7720 -1		.2	-.2493 -1	.9545 -1
.3151		1.5	.6116	.5640	.1339		.3	-.3751 -1	.2197	.1001		.3	-.5162 -1	.2031
.3341		2.0	.7973	.9531	.1702		.4	-.6471 -1	.3831	.1151		.4	-.8311 -1	.3361
.3418		2.5	1.3909	1.8558	.2020		.5	-.9753 -1	.5836	.1245		.5	-.1163	.4838
.3448		3.0	.9092	1.8558	.2293		.6	-.1346	.8146	.1302		.6	-.1302	.6381
.3459		3.5	.9448	2.3358	.2523		.7	-.1747	1.0688	.1336		.7	-.1807	.7941
.3463		4.0	.9665	2.8241	.2714		.8	-.2164	1.3389	.1357		.8	-.2100	.9487
.3465		4.5	.9797	3.3172	.2870		.9	-.2585	1.6183	.1369		.9	-.2371	1.1003
$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 2.0$					.2896		1.0	-.3001	1.9010	.1376		1.0	-.2618	1.2484
.4805 -1		.1	-.3040 -2	.9977 -2	.3327		1.5	-.4806	3.2373	.1386		1.5	-.3553	1.9334
.9196 -1		.2	-.1177 -1	.3963 -1	.3425		2.0	-.6033	4.3316	$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 3.0$				
.1315		.3	-.2551 -1	.8811 -1	.3453		2.5	-.6802	5.2103	.4443 -1		.1	-.8635 -2	.3705 -1
.1664		.4	-.4345 -1	.1540	.3462		3.0	-.7272	5.9438	.7763 -1		.2	-.3272 -1	.1429
.1970		.5	-.6470 -1	.2355	.3464		3.5	-.7558	6.5863	.1007		.3	-.6815 -1	.3031
.2232		.6	-.8836 -1	.3304	.3465		4.0	-.7732	7.1730	.1157		.4	-.1101	.4990
.2454		.7	-.1136	.4363	$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 0$					.1251		.5	-.1546	.7136
.2641		.8	-.1395	.5509	.2372 -1		.1	0.7379 -1	.1558 -3	.1307		.6	-.1986	.9345
.2796		.9	-.1656	.6720	.4476 -1		.2	.1454	.1238 -2	.1340		.7	-.2407	1.1540
.2923		1.0	-.1912	.7977	.6309 -1		.3	.2151	.4128 -2	.1359		.8	-.2799	1.3677
.3280		1.5	-.3018	1.4475	.7873 -1		.4	.2835	.9625 -2	.1371		.9	-.3160	1.5738
.3402		2.0	-.3775	2.0745	.9182 -1		.5	.3510	.1842 -1	.1377		1.0	-.3490	1.7715
.3444		2.5	-.4252	2.6617	.1026		.6	.4181	.3105 -1	.1386		1.5	-.4737	2.6440
.3458		3.0	-.4545	3.2175	.1113		.7	.4853	.4794 -1	$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 5.0$				
.3463		3.5	-.4724	3.7522	.1182		.8	.5529	.6937 -1	.4455 -1		.1	-.1017 -1	.6174 -1
.3465		4.0	-.4833	4.2736	.1235		.9	.6213	.9547 -1	.7799 -1		.2	-.3876 -1	.2378
.3465		4.5	-.4898	4.7866	.1276		1.0	.6908	.1263	.1012		.3	-.8110 -1	.5031
$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 3.0$					.1368		1.5	1.0604	.3472	.1163		.4	-.1315	.8249
.4824 -1		.1	-.3815 -2	.1496 -1	.1384		2.0	1.4686	.6603	.1256		.5	-.1849	1.1736
.9263 -1		.2	-.1489 -1	.5941 -1	.1386		2.5	1.9080	1.0389	.1311		.6	-.2380	1.5276
.1328		.3	-.3251 -1	.1320	$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 0.5$					.1343		.7	-.2885	1.8740
.1685		.4	-.5577 -1	.2303	.4297 -1		.1	.1571 -1	.6208 -2	.1362		.8	-.3357	2.2060
.1997		.5	-.8359 -1	.3514	.7385 -1		.2	.5577 -1	.2442 -1	.1372		.9	-.3791	2.5209
.2265		.6	-.1148	.4916	.9548 -1		.3	.1107	.5367 -1	.1378		1.0	-.4187	2.8179
.2491		.7	-.1483	.6468	.1103		.4	.1733	.9271 -1	.1386		1.5	-.5684	4.0650
.2680		.8	-.1830	.8131	.1203		.5	.2384	.1402					
.2835		.9	-.2179	.9868										

TABLE IV.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER PLANT WITH ZERO

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$
$\zeta = 0.10; \omega_n T_1 = 0$						$\zeta = 0.10; \omega_n T_1 = 4.0$					
0.5	1.5061	2.4715	2.9308		0.2050	0.5000 -1	0.1499		0.2	-0.6176 -4	0.1001 -2
1.0	2.4965	3.8584	4.7240		1.5036	.1003	.2993		.4	-.4829 -3	.8021 -2
1.5	3.1995	4.6451	5.8914		3.7642	.1511	.4479		.6	-.1595 -2	.2716 -1
2.0	3.7771	5.1630	6.7719		6.5393	.2030	.5956		.8	-.3705 -2	.6467 -1
2.5	4.3012	5.5559	7.5092		9.5765	.2563	.7421		1.0	-.7103 -2	.1271
3.0	4.8091	5.8930	8.1675		12.7548	.3083	1.1022		1.5	-.2302 -1	.4377
3.5	5.3273	6.2178	8.7810		16.0227	.3586	1.4529		2.0	-.5282 -1	1.0666
4.0	5.8847	6.5713	9.3733		19.3772	.7457	1.7945		2.5	-.1006	2.1543
4.5	6.5280	7.0118	9.9677		22.8769	.9699	2.1292		3.0	-.1704	3.8633
5.0	7.3584	7.6569	10.5972		26.6670						
5.5	8.6154	8.7735	11.3163		31.0297						
6.0	10.8398	10.9577	12.2357		36.2864						
6.5	13.0501	13.8887	14.0775		42.4894						
7.0	14.1977	15.0782	15.7614		49.4334						
7.5	15.1214	16.1535	17.0642		57.0636						
8.0	15.9189	16.9835	18.1293		65.3068						
8.5	16.6414	17.6578	19.0328		74.0104						
9.0	17.3310	18.2503	19.8386		83.0742						
9.5	18.0204	18.8143	20.5881		92.4397						
10.0	18.7402	19.3962	21.3118		102.0811						
10.5	19.5319	20.0508	22.0377		112.0403						
11.0	20.4650	20.8632	22.7966		122.3928						
11.5	21.6020	21.9130	23.6220		133.2165						
12.0	23.0171	23.3084	24.5827		144.6568						
12.5	24.5443	24.9449	25.8014		156.7788						
13.0	26.3433	26.4370	27.2056		169.5661						
13.5	26.9012	27.6595	28.5166		182.9750						
14.0	27.8381	28.6714	29.6669		196.9730						
14.5	28.7013	29.5481	30.6934		211.4666						
15.0	29.5228	30.3358	31.6259		226.4358						
$\zeta = 0.10; \omega_n T_1 = 0.5$						$\zeta = 0.10; \omega_n T_1 = 8.0$					
.4802 -1	.1475		.2	-0.1890 -2	.1253 -3	.5016 -1	.1500		.2	-.3059 -4	.2001 -2
.9299 -1	.2899		.4	-.1253 -1	.1008 -2	.1009	.2999		.4	-.2422 -3	.1604 -1
.1360	.4272		.6	-.3536 -1	.3443 -2	.1525	.4493		.6	-.8097 -3	.5430 -1
.1780	.5603		.8	-.7084 -1	.8299 -2	.2055	.5979		.8	-.1904 -2	.1293
.2196	.6901		1.0	-.1181	.1657 -1	.2602	.7455		1.0	-.3693 -2	.2539
.3252	1.0049		1.5	-.2810	.6016 -1	.4075	1.1083		1.5	-.1231 -1	.8727
.4383	1.3138		2.0	-.4979	.1564	.5759	1.4600		2.0	-.2897 -1	2.1176
.7018	1.9204	2.9686	3.0		.6328	.7741	1.7956		2.5	-.5637 -1	4.2458
.9802	2.4272	3.6970	4.0		1.5125	1.0122	2.1242		3.0	-.9702 -1	7.5252
$\zeta = 0.10; \omega_n T_1 = 1.0$						$\zeta = 0.10; \omega_n T_1 = 16.0$					
.4911 -1	.1489		.2	-.4078 -3	.2503 -3	.5023 -1	.1501		.2	-.1540 -4	.4003 -2
.9684 -1	.2954		.4	-.2964 -2	.2008 -2	.1012	.3002		.4	-.1227 -3	.3208 -1
.1437	.4392		.6	-.9123 -2	.6812 -2	.1532	.4499		.6	-.4128 -3	.1806
.1903	.5804		.8	-.1979 -1	.1627 -1	.2068	.5990		.8	-.9764 -3	.2585
.2370	.7192		1.0	-.3550 -1	.3211 -1	.2623	.7471		1.0	-.1905 -2	.5075
.3568	1.0575		1.5	-.9853 -1	.1124	.4125	1.1109		1.5	-.6442 -2	1.7427
.4862	1.3877		2.0	-.1971	.2804	.5852	1.4624		2.0	-.1536 -1	4.2186
.8023	2.0467		3.0	-.5111	1.0897	.7894	1.7979		2.5	-.3018 -1	8.4184
1.2517	2.7405	3.9887	4.0		3.0332	1.0344	2.1160		3.0	-.5229 -1	14.8052
$\zeta = 0.10; \omega_n T_1 = 2.0$						$\zeta = 0.25; \omega_n T_1 = 0$					
.4970 -1	.1496		.2	-.1369 -3	.5004 -3	.5	1.4949	2.3961	2.8024		.1702
.9906 -1	.2980		.4	-.1044 -2	.4012 -2	1.0	2.5448	3.8009	4.5122		1.2094
.1485	.4451		.6	-.3367 -2	.1359 -1	1.5	3.3615	4.6917	5.6602		3.0567
.1984	.5907		.8	-.7639 -2	.3238 -1	2.0	4.0807	5.3559	6.5502		5.4192
.2491	.7348		1.0	-.1431 -1	.6369 -1	2.5	4.7676	5.9242	7.3131		8.1249
.3820	1.0881		1.5	-.4390 -1	.2204	3.0	5.4624	6.4699	8.0149		11.0983
.5291	1.4332		2.0	-.9584 -1	.5409	3.5	6.2011	7.0496	8.6971		14.3261
.8981	2.1124		3.0	-.2857	2.0128	4.0	7.0135	7.7072	9.3872		17.8490
1.4432	2.8184		4.0	-.6248	5.3583	4.5	7.9460	8.5061	10.1201		21.7439
$\zeta = 0.25; \omega_n T_1 = 0.5$						5.0	9.0425	9.5100	10.9349		26.1118
.4836 -1	.1479		.2	-.1644 -2	.1254 -3	5.5	10.3104	10.7539	11.8871		31.0588
.9425 -1	.2915		.4	-.1090 -1	.1008 -2	6.0	11.6378	12.1510	13.0263		36.6344
.1387	.4310		.6	-.3076 -1	.3440 -2	6.5	12.8284	13.4541	14.2514		42.8564
.1824	.5670		.8	-.6159 -1	.8285 -2	7.0	13.9272	14.6581	15.4617		49.6402
.2261	.7005		1.0	-.1026	.1652 -1	7.5	14.9338	15.7335	16.5994		56.9950
.3385	1.0277		1.5	-.2430	.5964 -1	8.0	15.8895	16.7188	17.6987		64.8703
						8.5	16.8127	17.6405	18.6553		73.2419
						9.0	17.7023	18.4919	19.5792		82.0943
						9.5	18.6491	19.3934	20.4885		91.3236
						10.0	19.6565	20.3853	21.4574		101.0258

TABLE IV.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER PLANT
WITH ZERO - Continued

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$
$\zeta = 0.25; \omega_n T_1 = 0.5$						$\zeta = 0.50; \omega_n T_1 = 0$					
0.4602	1.3535		2.0	-0.4269	0.1538	0.5	1.4860	2.3159	2.6598		0.1319
.5972	1.6853		2.5	-.6477	.3301	1.0	2.6095	3.7520	4.2847		.9122
.9124	2.3380	3.4256	3.5		1.0240	1.5	3.5710	4.8085	5.4752		2.3589
1.2745	2.9615	4.1870	4.5		2.2400	2.0	4.4574	5.6844	6.4538		4.3825
1.7534	3.6757	4.9223	5.5		4.3324	2.5	5.3274	6.4958	7.3356		6.8496
2.3530	4.4955	5.6425	6.5		7.4602	3.0	6.2052	7.2951	8.1798		9.7408
3.0456	5.4343	6.3887	7.5		11.5655	3.5	7.1083	8.1201	9.0234		13.0538
$\zeta = 0.25; \omega_n T_1 = 1.0$						4.0	8.1246	9.0985	9.9477		16.7050
.4947 -1	.1493		.2	-.3399 -3	.2503 -3	4.5	9.1587	10.1119	10.9063		20.8961
.9521 -1	.2970		.4	-.2471 -2	.2008 -2	5.0	10.1779	11.1221	11.8889		25.5598
.1467	.4429		.6	-.7603 -2	.6809 -2	5.5	11.1779	12.1221	12.8889		30.8107
.1954	.5872		.8	-.1649 -1	.1625 -1	$\zeta = 0.50; \omega_n T_1 = 0.5$					
.2447	.7299		1.0	-.2957 -1	.3204 -1	.4893 -1	.1486		.2	-0.1233 -2	.1253 -3
.3732	1.0823		1.5	-.8190 -1	.1118	.9634 -1	.2943		.4	-.8171 -2	.1007 -2
.5141	1.4326		2.0	-.1631	.2771	.1430	.4373		.6	-.2305 -1	.3432 -2
.8598	2.1526		3.0	-.4144	1.0519	.1897	.5782		.8	-.4608 -1	.8244 -2
1.3414	2.9452		4.0	-.7925	2.8370	.2368	.7176		1.0	-.7658 -1	.1638 -1
$\zeta = 0.25; \omega_n T_1 = 2.0$						.3594	1.0646		1.5	-.1792	.5836 -1
.5006 -1	.1499		.2	-.1190 -3	.5005 -3	.4931	1.4158		2.0	-.3090	.1477
.1005	.2996		.4	-.9078 -2	.4011 -2	.6424	1.7782		2.5	-.4568	.3094
.1517	.4487		.6	-.2927 -2	.1358 -1	1.0016	2.5552		3.5	-.7860	.9725
.2039	.5973		.8	-.6639 -2	.3235 -1	1.4287	3.3730	4.4443	4.5		2.2092
.2577	.7452		1.0	-.1243 -1	.6358 -1	1.9007	4.2134	5.3127	5.5		4.0405
.4010	1.1130		1.5	-.3806 -1	.2193	2.4417	5.1499	6.2082	6.5		6.6428
.5626	1.4798		2.0	-.8281 -1	.5357	3.0215	6.1639	7.1424	7.5		9.9752
.9723	2.2292		3.0	-.2431	1.9547	$\zeta = 0.5; \omega_n T_1 = 1.0$					
1.5593	3.0458		4.0	-.5136	5.0079	.5006 -1	.1499		.2	-.2255 -3	.2502 -3
2.3682	3.9920		5.0	-.8762	10.0944	.1005	.2997		.4	-.1646 -2	.2006 -2
$\zeta = 0.25; \omega_n T_1 = 4.0$						.1516	.4491		.6	-.5061 -2	.6793 -2
.5037 -1	.1503		.2	-.5720 -3	.1001 -2	.2038	.5983		.8	-.1096 -1	.1618 -1
.1017	.3008		.4	-.4470 -3	.8019 -2	.2573	.7475		1.0	-.1962 -1	.3182 -1
.1544	.4515		.6	-.1476 -2	.2714 -1	.3992	1.1222		1.5	-.5389 -1	.1098
.2089	.6020		.8	-.3428 -2	.6460 -1	.5561	1.5032		2.0	-.1058	.2678
.2654	.7523		1.0	-.6567 -2	.1268	.9323	2.3044		3.0	-.2559	.9645
.4190	1.1265		1.5	-.2123 -1	.4356	1.4110	3.1853		4.0	-.4514	2.4123
.5961	1.4990		2.0	-.4855 -1	1.0568	1.9889	4.1582		5.0	-.6612	4.8025
.8058	1.8719		2.5	-.9193 -1	2.1200	2.6256	5.2043		6.0	-.8426	8.0590
1.3646	2.6385		3.5	-.2380	6.1157	3.2678	6.2877		7.0	-.9604	12.0388
2.1689	3.4909		4.5	-.4640	13.1341	4.4440	8.4505		9.0	-.9871	21.0322
$\zeta = 0.25; \omega_n T_1 = 8.0$						5.4623	10.5277		11.0	-.8691	30.9252
.5053 -1	.1504		.2	-.2944 -4	.2002 -2	6.4124	12.5255		13.0	-.7717	42.0335
.1024	.3014		.4	-.2329 -3	.1604 -1	7.3684	14.4982		15.0	-.7406	55.2318
.1559	.4528		.6	-.7785 -3	.5426 -1	8.3518	16.4769		17.0	-.7498	70.7319
.2115	.6042		.8	-.1830 -2	.1291	9.3546	18.4705		19.0	-.7680	88.5147
.2597	.7555		1.0	-.3546 -2	.2534	10.3630	20.4734		21.0	-.7792	108.4113
.4292	1.1321		1.5	-.1179 -1	.8684	$\zeta = 0.5; \omega_n T_1 = 2.0$					
.6158	1.5052		2.0	-.2765 -1	2.0982	.5057 -1	.1506		.2	-.8924 -4	.5003 -3
.8392	1.8752		2.5	-.5348 -1	4.1800	.1029	.3022		.4	-.6803 -3	.4006 -2
1.4386	2.6195		3.5	-.1423	11.7570	.1569	.4547		.6	-.2191 -2	.1355 -1
$\zeta = 0.25; \omega_n T_1 = 16.0$						.2131	.6081		.8	-.4961 -2	.3220 -1
.5061 -1	.1505		.2	-.1511 -4	.4003 -2	.2717	.7625		1.0	-.9268 -2	.6312 -1
.1027	.3017		.4	-.1203 -3	.3207 -1	.4312	1.1530		1.5	-.2814 -1	.2156
.1566	.4534		.6	-.4047 -3	.1085	.6133	1.5529		2.0	-.6042 -1	.5186
.2129	.6052		.8	-.9567 -3	.2581	.8232	1.9658		2.5	-.1074	1.0286
.2719	.7569		1.0	-.1865 -2	.5064	1.3429	2.8475		3.5	-.2454	2.8793
.4347	1.1344		1.5	-.6290 -2	1.7339	1.9958	3.8137		4.5	-.4272	5.0272
.6265	1.5070		2.0	-.1493 -1	4.1791	2.7330	4.8672		5.5	-.6158	10.3320
.8572	1.8738		2.5	-.2918 -1	8.2885	4.1811	7.0561		7.5	-.8750	20.8069
1.1373	2.2359		3.0	-.5017 -1	14.4569	5.3879	9.2198		9.5	-.9181	31.9569
						6.3529	11.2329		11.5	-.8101	42.9159
						7.2309	13.2791		13.5	-.7018	54.6799
						8.1589	15.2434		15.5	-.6655	68.9510
						9.1455	17.2230		17.5	-.6735	86.1143
						10.1617	19.2198		19.5	-.6919	105.7668

TABLE IV.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER PLANT
 WITH ZERO - Continued

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$
$\zeta = 0.5; \omega_n T_1 = 4.0$						$\zeta = 0.75; \omega_n T_1 = 1.0$					
0.5099 -1	0.1509		0.2	-0.4956 -4	0.1001 -2	0.5066 -1	0.1506		0.2	-0.1133 -3	0.2501 -3
.1042	.3034		.4	-.3870 -3	.8009 -2	.1027	.3023		.4	-.8220 -3	.2002 -2
.1599	.4573		.6	-.1276 -2	.2707 -1	.1565	.4552		.6	-.2522 -2	.6765 -2
.2185	.6126		.8	-.2958 -2	.6429 -1	.2120	.6093		.8	-.5448 -2	.1606 -1
.2803	.7690		1.0	-.5653 -2	.1259	.2695	.7646		1.0	-.9714 -2	.3144 -1
.4519	1.1657		1.5	-.1811 -1	.4281	.4228	1.1596		1.5	-.2633 -1	.1068
.6532	1.5715		2.0	-.4085 -1	1.0233	.5915	1.5661		2.0	-.5067 -1	.2549
.8908	1.9890		2.5	-.7590 -1	2.0110	.7768	1.9863		2.5	-.8110 -1	.5007
1.4959	2.8738		3.5	-.1860	5.4771	1.1982	2.8720		3.5	-.1525	1.3738
2.2707	3.8416		4.5	-.3395	11.0210	1.6785	3.8156		4.5	-.2258	2.8581
4.0268	5.9609		6.5	-.6580	25.8268	2.1976	4.8030		5.5	-.2891	4.9938
5.6545	8.1275		8.5	-.8885	42.3835	2.7339	5.8158		6.5	-.3361	7.7454
$\zeta = 0.5; \omega_n T_1 = 8.0$						$\zeta = 0.75; \omega_n T_1 = 2.0$					
.5115 -1	.1510		.2	-.2750 -4	.2001 -2	.5128 -1	.1512		.2	-.5949 -4	.5001 -3
.1048	.3040		.4	-.2174 -3	.1602 -1	.1053	.3048		.4	-.4527 -3	.3999 -2
.1614	.4585		.6	-.7256 -3	.5412 -1	.1621	.4607		.6	-.1455 -2	.1349 -1
.2214	.6146		.8	-.1702 -2	.1285	.2220	.6188		.8	-.3284 -2	.3196 -1
.2851	.7719		1.0	-.3290 -2	.2514	.2893	.7791		1.0	-.6111 -2	.6236 -1
.4638	1.1705		1.5	-.1083 -1	.8532	.4587	1.1904		1.5	-.1830 -1	.4937
.6768	1.5766		2.0	-.2505 -1	2.0313	.6599	1.6174		2.0	-.3849 -1	.1617
.9320	1.9918		2.5	-.4755 -1	3.9678	1.1243	2.5223		3.0	-.1020	1.6177
1.5923	2.8614		3.5	-.1202	10.5876	1.6790	3.4920		4.0	-.1870	3.6186
3.4139	4.8213		5.5	-.3356	32.7151	2.7857	4.5088		5.0	-.2770	6.4820
5.3321	6.9395		7.5	-.5356	58.0204	2.9062	5.5494		6.0	-.3568	10.0589
7.1241	9.0394		9.5	-.7087	85.0474	4.0957	7.6376		8.0	-.4580	18.7198
8.8832	11.1305		11.5	-.8757	115.4568	5.1814	9.6930		10.0	-.4883	28.8283
$\zeta = 0.5; \omega_n T_1 = 16.0$						$\zeta = 0.75; \omega_n T_1 = 4.0$					
.5123 -1	.1511		.2	-.1462 -4	.4002 -2	.5161 -1	.1515		.2	-.4191 -4	.9999 -3
.1052	.3042		.4	-.1163 -3	.3203 -1	.1066	.3059		.4	-.3269 -3	.7993 -2
.1622	.4591		.6	-.3906 -3	.1082	.1653	.4631		.6	-.1075 -2	.2695 -1
.2229	.6155		.8	-.9215 -3	.2569	.2279	.6229		.8	-.2484 -2	.6378 -1
.2875	.7732		1.0	-.1792 -2	.5025	.2947	.7853		1.0	-.4472 -2	.1243
.4701	1.1723		1.5	-.5982 -2	1.7031	.4821	1.2023		1.5	-.1492 -1	.4163
.6896	1.5776		2.0	-.1400 -1	4.0453	.7013	1.6354		2.0	-.3296 -1	.9737
.9544	1.9896		2.5	-.2689 -1	7.8704	1.2418	2.5519		3.0	-.9496 -1	3.1289
1.2716	2.4106		3.0	-.4512 -1	13.4113	1.9039	3.5313		4.0	-.1863	6.8073
3.0381	4.2541		5.0	-.1605	50.2963	2.6406	4.5560		5.0	-.2923	11.8074
5.0827	6.3265		7.0	-.2820	93.4738	4.1436	6.6586		7.0	-.4925	24.3369
7.0018	8.3965		9.0	-.3882	136.2389	5.5287	8.7604		9.0	-.6341	38.6541
$\zeta = 0.75; \omega_n T_1 = 0$						$\zeta = 0.75; \omega_n T_1 = 8.0$					
.5	1.4863	2.2708	2.5690		.1024	.5177 -1	.1517		.2	-.2556 -4	.2000 -2
1.0	2.6800	3.7856	4.2111		.7183	.1073	.3065		.4	-.2017 -3	.1598 -1
1.5	3.6975	4.8829	5.3709		2.0022	.1669	.4642		.6	-.6717 -3	.5387 -1
2.0	4.6743	5.8647	6.3807		3.8115	.2310	.6247		.8	-.1570 -2	.1274
2.5	5.6071	6.8049	7.3957		6.2306	.2999	.7878		1.0	-.3021 -2	.2483
3.0	6.6048	7.8037	8.3973		8.9842	.4955	1.2063		1.5	-.9798 -2	.8293
3.5	7.6033	8.8026	9.3983		12.2387	.7285	1.6397		2.0	-.2218 -1	1.9328
4.0	8.6001	9.7999	10.4000		16.0025	1.3173	2.5509		3.0	-.6662 -1	6.1338
4.5	9.6000	10.8000	11.4000		20.2513	2.0575	3.5178		4.0	-.1349	13.0935
5.0	10.6000	11.8000	12.4000		25.0000	3.7730	5.5592		6.0	-.3034	32.6550
5.5	11.6000	12.8000	13.4000		30.2500	5.5142	7.6431		8.0	-.4678	55.8863
$\zeta = 0.75; \omega_n T_1 = 0.5$						$\zeta = 0.75; \omega_n T_1 = 0.5$					
.4949 -1	.1493		.2	-.8219 -3	.1251 -3	.4949 -1	.1493		.2	-.8219 -3	.1251 -3
.9842 -1	.2971		.4	-.5442 -2	.1005 -2	.9842 -1	.2971		.4	-.5442 -2	.1005 -2
.1473	.4435		.6	-.1533 -1	.3417 -2	.1473	.4435		.6	-.1533 -1	.3417 -2
.1967	.5891		.8	-.3055 -1	.8179 -2	.1967	.5891		.8	-.3055 -1	.8179 -2
.2470	.7343		1.0	-.5058 -1	.1617 -1	.2470	.7343		1.0	-.5058 -1	.1617 -1
.3783	1.0992		1.5	-.1165	.5665 -1	.3783	1.0992		1.5	-.1165	.5665 -1
.5208	1.4717		2.0	-.1964	.1402	.5208	1.4717		2.0	-.1964	.1402
.8479	2.2559		3.0	-.3682	.5144	.8479	2.2559		3.0	-.3682	.5144
1.2366	3.1041		4.0	-.5302	1.2990	1.2366	3.1041		4.0	-.5302	1.2990
1.6814	4.0148		5.0	-.6664	2.6244	1.6814	4.0148		5.0	-.6664	2.6244
2.1676	4.9753		6.0	-.7695	4.5555	2.1676	4.9753		6.0	-.7695	4.5555
2.6777	5.9680		7.0	-.8388	7.0923	2.6777	5.9680		7.0	-.8388	7.0923
3.1972	6.9775		8.0	-.8788	10.1983	3.1972	6.9775		8.0	-.8788	10.1983
4.2337	9.0083		10.0	-.9018	17.9589	4.2337	9.0083		10.0	-.9018	17.9589
5.2528	11.0295		12.0	-.8931	27.6400	5.2528	11.0295		12.0	-.8931	27.6400
6.2586	13.0382		14.0	-.8816	39.2163	6.2586	13.0382		14.0	-.8816	39.2163
7.2587	15.0400		16.0	-.8751	52.7308	7.2587	15.0400		16.0	-.8751	52.7308
8.2576	17.0394		18.0	-.8728	68.2257	8.2576	17.0394		18.0	-.8728	68.2257
9.2566	19.0385		20.0	-.8725	85.7226	9.2566	19.0385		20.0	-.8725	85.7226
10.2562	21.0380		22.0	-.8728	105.2271	10.2562	21.0380		22.0	-.8728	105.2271

TABLE IV.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER PLANT
WITH ZERO - Continued

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$
$\zeta = 0.75; \omega_n T_1 = 16.0$						$\zeta = 1.02; \omega_n T_1 = 1.0$					
0.5185 ⁻¹	0.1517		0.2	-0.1413 ⁻⁴	0.3999 ⁻²	4.2954	9.3083		10.0	0.2577 ⁻¹	17.9106
.1077	.3068		.4	-.1122 ⁻³	.3196 ⁻¹	4.7913	10.3045		11.0	.2634 ⁻¹	22.4094
.1678	.4648		.6	-.3759 ⁻³	.1077	5.2886	11.3019		12.0	.2673 ⁻¹	27.4173
.2326	.6256		.8	-.8837 ⁻³	.2548	5.7868	12.3003		13.0	.2700 ⁻¹	32.9322
.3026	.7890		1.0	-.1700 ⁻²	.4961	6.2856	13.2992		14.0	.2718 ⁻¹	38.9521
.5027	1.2077		1.5	-.5623 ⁻²	1.6552	6.7848	14.2985		15.0	.2730 ⁻¹	45.4759
.7433	1.6402		2.0	-.1289 ⁻¹	3.8491	7.2843	15.2980		16.0	.2738 ⁻¹	52.5024
1.0285	2.0857		2.5	-.2411 ⁻¹	7.3027	7.7840	16.2977		17.0	.2744 ⁻¹	60.0308
1.3595	2.5439		3.0	-.3945 ⁻¹	12.1162	8.2838	17.2975		18.0	.2747 ⁻¹	68.0607
3.0397	4.4883		5.0	-.1314	42.5620	8.7836	18.2974		19.0	.2750 ⁻¹	76.5916
4.9321	6.5275		7.0	-.2381	81.3945	9.2835	19.2973		20.0	.2751 ⁻¹	85.6232
6.8108	8.5801		9.0	-.3413	123.2668	10.2834	20.2972		22.0	.2753 ⁻¹	105.1879
$\zeta = 1.01; \omega_n T_1 = 0$						11.2834	21.2971		24.0	.2754 ⁻¹	126.7535
.50	1.4669	2.1794	2.4250		.9230 ⁻¹	12.2833	22.2970		26.0	.2754 ⁻¹	150.3197
1.0	2.7049	3.7414	4.0729		.6130	13.2833	23.2969		28.0	.2754 ⁻¹	175.8861
1.5	3.8405	5.0533	5.4256		1.6816	14.2833	24.2968		30.0	.2755 ⁻¹	203.4526
2.0	4.9245	6.2440	6.6391		3.3028	$\zeta = 1.02; \omega_n T_1 = 2.0$					
2.5	5.9769	7.3633	7.7727		5.4634	.5194 ⁻¹	.1519		.2	-.2734 ⁻⁴	.4996 ⁻³
3.0	7.0020	8.4186	8.8332		8.1672	.1078	.3076		.4	-.2077 ⁻³	.3987 ⁻²
3.5	8.0289	9.4802	9.9025		11.3698	.1676	.4669		.6	-.6650 ⁻³	.1340 ⁻¹
4.0	9.0365	10.4962	10.9195		15.1056	.2314	.6299		.8	-.1494 ⁻²	.3158 ⁻¹
4.5	10.0448	11.5146	11.9399		19.3418	.2991	.7963		1.0	-.2764 ⁻²	.6123 ⁻¹
5.0	11.0507	12.5290	12.9564		24.0757	.4849	1.2267		1.5	-.8107 ⁻²	.2016
5.5	12.0585	13.5461	13.9754		29.3154	.6925	1.6759		2.0	-.1662 ⁻¹	.4620
6.0	13.0587	14.5463	14.9753		35.0660	1.1605	2.6191		3.0	-.4142 ⁻¹	1.4270
6.5	14.0588	15.5463	15.9752		41.3152	1.6742	3.6024		4.0	-.7176 ⁻¹	3.0356
7.0	15.0587	16.5464	16.9753		48.0634	2.2087	4.6065		5.0	-.1022	5.2722
7.5	16.0589	17.5469	17.9760		55.3118	2.7481	5.6189		6.0	-.1292	8.0870
8.0	17.0658	18.5632	18.9949		63.0523	3.2842	6.6329		7.0	-.1513	11.4330
$\zeta = 1.02; \omega_n T_1 = 0.5$						3.8140	7.6455		8.0	-.1685	15.2770
.5011 ⁻¹	.1500		.2	-.3775 ⁻³	.1252 ⁻³	4.3370	8.6558		9.0	-.1813	19.6004
.1006	.3000		.4	-.2499 ⁻²	.1003 ⁻²	4.8541	9.6636		10.0	-.1905	24.3953
.1519	.4501		.6	-.7016 ⁻²	.3396 ⁻²	5.3665	10.6694		11.0	-.1971	29.6608
.2040	.6006		.8	-.1393 ⁻¹	.8085 ⁻²	5.8752	11.6736		12.0	-.2016	35.3994
.2573	.7516		1.0	-.2293 ⁻¹	.1588 ⁻¹	6.3812	12.6765		13.0	-.2047	41.6154
.3965	1.1335		1.5	-.5179 ⁻¹	.5446 ⁻¹	6.8854	13.6785		14.0	-.2069	48.3132
.5452	1.5239		2.0	-.8497 ⁻¹	.1313	7.3882	14.6799		15.0	-.2083	55.4968
.7874	2.3360		3.0	-.1495	.4558	7.8902	15.6809		16.0	-.2093	63.1697
1.2414	3.1910		4.0	-.2020	1.0954	8.3915	16.6815		17.0	-.2100	71.3345
1.6439	4.0836		5.0	-.2410	2.1365	8.8923	17.6819		18.0	-.2104	79.9934
2.0738	5.0066		6.0	-.2687	3.6391	9.3929	18.6822		19.0	-.2107	89.1480
2.5246	5.9526		7.0	-.2881	5.6399	9.8933	19.6824		20.0	-.2109	98.7996
2.9905	6.9152		8.0	-.3015	8.1575	10.3938	20.6826		22.0	-.2111	119.5967
3.4672	7.8896		9.0	-.3106	11.1992	11.8940	23.6827		24.0	-.2112	142.3893
3.9514	8.8722		10.0	-.3168	14.7658	12.8940	25.6828		26.0	-.2113	167.1796
4.4407	9.8605		11.0	-.3210	18.8549	13.8941	27.6828		28.0	-.2113	193.9688
4.9336	10.8526		12.0	-.3239	23.4631	14.8941	29.6828		30.0	-.2113	222.7575
5.4288	11.8473		13.0	-.3258	28.5865	$\zeta = 1.02; \omega_n T_1 = 4.0$					
5.9256	12.8438		14.0	-.3271	34.2219	.5227 ⁻¹	.1522		.2	-.3364 ⁻⁴	.9988 ⁻³
6.4234	13.8414		15.0	-.3279	40.3663	.1092	.3087		.4	-.2619 ⁻³	.7969 ⁻²
6.9220	14.8398		16.0	-.3285	47.0176	.1710	.4693		.6	-.8584 ⁻³	.2676 ⁻¹
7.4210	15.8388		17.0	-.3289	54.1738	.2377	.6337		.8	-.1973 ⁻²	.6301 ⁻¹
7.9204	16.8381		18.0	-.3292	61.8337	.3094	.8020		1.0	-.3731 ⁻²	.1220
8.4200	17.8376		19.0	-.3293	69.9963	.5109	1.2378		1.5	-.1153 ⁻¹	.3999
8.9197	18.8373		20.0	-.3295	78.6608	.7428	1.6932		2.0	-.2481 ⁻¹	.9106
9.4194	19.8370		22.0	-.3296	97.4934	1.2855	2.6507		3.0	-.6738 ⁻¹	2.7605
10.9192	22.8368		24.0	-.3296	118.3289	1.9016	3.6502		4.0	-.1257	5.7302
11.9192	24.8368		26.0	-.3297	141.1659	2.5534	4.6713		5.0	-.1910	9.6788
12.9191	26.8367		28.0	-.3297	166.0035	3.2130	5.7008		6.0	-.2561	14.4199
13.9191	28.8367		30.0	-.3297	192.8414	3.8636	6.7314		7.0	-.3161	19.7931
$\zeta = 1.02; \omega_n T_1 = 1.0$						4.4964	7.7595		8.0	-.3685	25.6781
.5130 ⁻¹	.1513		.2	.9033 ⁻⁵	.2500 ⁻³	5.1077	8.7835		9.0	-.4121	31.9904
.1052	.3052		.4	.6559 ⁻⁴	.1996 ⁻²	5.6971	9.8031		10.0	-.4470	38.6765
.1616	.4617		.6	.2007 ⁻³	.6721 ⁻²	6.2664	10.8184		11.0	-.4740	45.7100
.2205	.6207		.8	.4315 ⁻³	.1588 ⁻¹	6.8183	11.8299		12.0	-.4942	53.0865
.2818	.7822		1.0	.7651 ⁻³	.3088 ⁻¹	7.3563	12.8384		13.0	-.5089	60.8167
.4453	1.1963		1.5	.2035 ⁻²	.1028	7.8833	13.8446		14.0	-.5194	68.9205
.6221	1.6240		2.0	.3819 ⁻²	.2390	8.4023	14.8489		15.0	-.5267	77.4207
1.0100	2.5141		3.0	.8216 ⁻²	.7679	8.9154	15.8519		16.0	-.5318	86.3393
1.4335	3.4399		4.0	.1271 ⁻¹	1.7143	9.4244	16.8539		17.0	-.5353	95.6959
1.8823	4.3906		5.0	.1662 ⁻¹	3.1348	9.9305	17.8553		18.0	-.5376	105.5061
2.3484	5.3582		6.0	.1972 ⁻¹	5.0578	10.4346	18.8563		19.0	-.5392	115.7824
2.8260	6.3371		7.0	.2205 ⁻¹	7.4495	10.9374	19.8569		20.0	-.5403	126.5343
3.3114	7.3232		8.0	.2374 ⁻¹	10.4510	11.4405	20.8576		22.0	-.5415	149.4905
3.8017	8.3142		9.0	.2494 ⁻¹	13.9235	12.9419	23.8579		24.0	-.5420	174.4106
						13.9425	25.8581		26.0	-.5422	201.3130
						14.9428	27.8582		28.0	-.5423	230.2069
						15.9429	29.8582		30.0	-.5424	261.0966

TABLE IV.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER PLANT
WITH ZERO - Continued

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$
$\zeta = 1.02; \omega_n T_1 = 8.0$						$\zeta = 1.5; \omega_n T_1 = 0$					
0.5244 ⁻¹	0.1523		0.2	-0.2345 ⁻⁴	0.1998 ⁻²	7.5	16.7713	18.8055	19.6684		53.1826
.1099	.3092		.4	-.1847 ⁻³	.1593 ⁻¹	8.0	17.7839	19.8306	20.0936		60.8881
.1728	.4703		.6	-.6127 ⁻³	.5349 ⁻¹	8.5	18.7953	20.8536	21.1166		69.0974
.2411	.6354		.8	-.1425 ⁻²	.1259	9.0	19.8032	21.8695	22.1325		77.8152
.3151	.8042		1.0	-.2724 ⁻²	.2436	9.5	20.8115	22.8861	23.1492		87.0380
.5259	1.2413		1.5	-.8648 ⁻²	.7964	10.0	21.8176	23.8982	24.1613		96.7672
.7733	1.6970		2.0	-.1907 ⁻¹	1.8074	$\zeta = 1.5; \omega_n T_1 = 0.5$					
1.3684	2.6519		3.0	-.5414 ⁻¹	5.4243	.5119 ⁻¹	.1513		.2	0.4098 ⁻³	.1252 ⁻³
2.0663	3.6460		4.0	-.1051	11.1110	.1045	.3052		.4	.2700 ⁻²	.9964 ⁻³
2.8248	4.6615		5.0	-.1658	18.4949	.1596	.4615		.6	.7527 ⁻²	.3342 ⁻²
3.6105	5.6870		6.0	-.2309	27.1654	.2161	.6198		.8	.1479 ⁻¹	.7865 ⁻²
4.4024	6.7161		7.0	-.2966	36.8144	.2739	.7799		1.0	.2403 ⁻¹	.1523 ⁻¹
5.1880	7.7457		8.0	-.3606	47.2332	.4227	1.1857		1.5	.5217 ⁻¹	.5014 ⁻¹
5.9597	8.7743		9.0	-.4214	58.2750	.5765	1.5970		2.0	.8180 ⁻¹	.1158
6.7128	9.8010		10.0	-.4780	69.8233	.8978	2.4306		3.0	.1316	.3718
7.4436	10.8254		11.0	-.5296	81.7747	1.2369	3.2782		4.0	.1651	.8452
8.1494	11.8471		12.0	-.5756	94.0330	1.5942	4.1408		5.0	.1863	1.5923
8.8282	12.8660		13.0	-.6156	106.5100	1.9691	5.0190		6.0	.1999	2.6639
9.4792	13.8819		14.0	-.6493	119.1329	2.3604	5.9126		7.0	.2087	4.1043
10.1027	14.8949		15.0	-.6769	131.8521	2.7667	6.8204		8.0	.2145	5.9508
10.7005	15.9053		16.0	-.6988	144.6486	3.1866	7.7412		9.0	.2186	8.2340
11.2755	16.9133		17.0	-.7156	157.5345	3.6183	8.6737		10.0	.2214	10.9782
11.8314	17.9192		18.0	-.7280	170.5485	4.0605	9.6164		11.0	.2235	14.2023
12.3718	18.9235		19.0	-.7371	183.7453	4.5118	10.5680		12.0	.2250	17.9206
12.9005	19.9266		20.0	-.7435	197.1851	4.9707	11.5273		13.0	.2261	22.1434
13.4343	20.9303		22.0	-.7510	225.0144	5.4364	12.4931		14.0	.2269	26.8780
14.9501	23.9320		24.0	-.7545	254.3928	5.9076	13.4645		15.0	.2276	32.1290
15.9572	25.9328		26.0	-.7561	285.5386	6.3836	14.4406		16.0	.2281	37.8990
16.9604	27.9331		28.0	-.7568	318.5694	6.8636	15.4207		17.0	.2285	44.1898
17.9619	29.9333		30.0	-.7571	353.5449	7.3469	16.4041		18.0	.2288	51.0015
$\zeta = 1.02; \omega_n T_1 = 16.0$						7.8331	17.3904		19.0	.2291	58.3337
.5253 ⁻¹	.1524		.2	-.1359 ⁻⁴	.3995 ⁻²	8.3216	18.3789		20.0	.2293	66.1855
.1103	.3095		.4	-.1077 ⁻³	.3186 ⁻¹	9.3042	20.3616		22.0	.2296	83.4430
.1737	.4708		.6	-.3594 ⁻³	.1070	10.2922	22.3497		24.0	.2298	102.7627
.2429	.6361		.8	-.8406 ⁻³	.2516	11.2840	24.3415		26.0	.2299	124.1329
.3181	.8052		1.0	-.1616 ⁻²	.4868	12.2784	26.3359		28.0	.2300	147.5430
.5339	1.2423		1.5	-.5200 ⁻²	1.5894	13.2746	28.3321		30.0	.2301	172.9842
.7900	1.6971		2.0	-.1161 ⁻¹	3.5997	$\zeta = 1.5; \omega_n T_1 = 1.0$					
1.4159	2.6465		3.0	-.3367 ⁻¹	10.7362	.5243 ⁻¹	.1525		.2	.2258 ⁻³	.2496 ⁻³
2.1635	3.6313		4.0	-.6653 ⁻¹	21.7946	.1094	.3102		.4	.1629 ⁻²	.1982 ⁻²
2.9887	4.6356		5.0	-.1066	35.8876	.1703	.4725		.6	.4943 ⁻²	.6613 ⁻²
3.8557	5.6498		6.0	-.1507	52.1135	.2346	.6398		.8	.1051 ⁻¹	.1544 ⁻¹
4.7415	6.6683		7.0	-.1967	69.8518	.3016	.8108		1.0	.1839 ⁻¹	.2960 ⁻¹
5.6337	7.6886		8.0	-.2431	88.7412	.4774	1.2509		1.5	.4696 ⁻¹	.9442 ⁻¹
6.5257	8.7094		9.0	-.2895	108.5830	.6606	1.7027		2.0	.8428 ⁻¹	.2100
7.4139	9.7302		10.0	-.3355	129.2639	1.0373	2.6208		3.0	.1670	.6245
8.2962	10.7506		11.0	-.3807	150.7110	1.4234	3.5452		4.0	.2437	1.3243
9.1711	11.7705		12.0	-.4251	172.8677	1.8193	4.4729		5.0	.3073	2.3534
10.0372	12.7899		13.0	-.4684	195.6816	2.2257	5.4047		6.0	.3580	3.7512
10.8933	13.8086		14.0	-.5106	219.0971	2.6429	6.3417		7.0	.3975	5.5519
11.7380	14.8266		15.0	-.5514	243.0519	3.0705	7.2846		8.0	.4281	7.7842
12.5695	15.8439		16.0	-.5907	267.4746	3.5079	8.2337		9.0	.4518	10.4718
13.3860	16.8603		17.0	-.6282	292.2822	3.9540	9.1891		10.0	.4701	13.6335
14.1856	17.8758		18.0	-.6637	317.7395	4.4080	10.1502		11.0	.4844	17.2838
14.9658	18.8902		19.0	-.6969	342.6547	4.8690	11.1168		12.0	.4956	21.4337
15.7242	19.9035		20.0	-.7276	367.9898	5.3361	12.0883		13.0	.5044	26.0910
17.1663	21.9262		22.0	-.7800	418.3243	5.8083	13.0640		14.0	.5114	31.2612
18.4972	23.9431		24.0	-.8193	467.4505	6.2851	14.0435		15.0	.5170	36.9476
19.7169	25.9545		26.0	-.8453	514.9228	6.7656	15.0263		16.0	.5214	43.1522
20.8452	27.9611		28.0	-.8605	561.0995	7.2493	16.0118		17.0	.5250	49.8758
21.9122	29.9646		30.0	-.8684	606.9106	7.7358	16.9997		18.0	.5279	57.1184
$\zeta = 1.5; \omega_n T_1 = 0$						8.2245	17.9896		19.0	.5303	64.8793
.5	1.4788	2.1565	2.3554		.6055 ⁻¹	8.7151	18.9812		20.0	.5322	73.1575
1.0	2.7742	3.7852	4.0222		.4570	9.7009	20.9684		22.0	.5350	91.2614
1.5	3.9937	5.2372	5.4870		1.3250	10.6911	22.9596		24.0	.5369	111.4194
2.0	5.1628	6.5809	6.8362		2.7136	11.6844	24.9535		26.0	.5382	133.6210
2.5	6.2944	7.8469	8.1050		4.6418	12.6798	26.9494		28.0	.5391	157.8570
3.0	7.3991	9.0578	9.3176		7.1112	13.6767	28.9465		30.0	.5396	184.1196
3.5	8.4824	10.2255	10.4863		10.1212	$\zeta = 1.5; \omega_n T_1 = 2.0$					
4.0	9.5492	11.3598	11.6212		13.6696	.5310 ⁻¹	.1531		.2	.2965 ⁻⁴	.4987 ⁻³
4.5	10.6033	12.4683	12.7301		17.7484	.1122	.3125		.4	.2241 ⁻³	.3997 ⁻²
5.0	11.6466	13.5552	13.8175		22.3612	.1770	.4777		.6	.7114 ⁻³	.1318 ⁻¹
5.5	12.6819	14.6263	14.8887		27.4890	.2468	.6484		.8	.1580 ⁻²	.3068 ⁻¹
6.0	13.7120	15.6865	15.9491		33.1393	.3212	.8240		1.0	.2880 ⁻²	.5861 ⁻¹
6.5	14.7354	16.7335	16.9963		39.3103	.5221	1.2802		1.5	.8093 ⁻²	.1846
7.0	15.7552	17.7730	18.0358		45.9928						

TABLE IV.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER PLANT
WITH ZERO - Continued

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_m$	b/B	$R_0 \omega_n^2 / BK$
$\zeta = 1.5; \omega_n T_1 = 2.0$						$\zeta = 1.5; \omega_n T_1 = 8.0$					
0.7375	1.7533		2.0	0.1582 -1	0.4037	7.3854	11.8636		12.0	-0.3804	75.0732
1.1884	2.7245		3.0	.3616 -1	1.1492	8.0230	12.8727		13.0	-.4126	86.1493
1.6489	3.7080		4.0	.5908 -1	2.3214	8.6483	13.8808		14.0	-.4419	97.6521
2.1134	4.6951		5.0	.8163 -1	3.9341	9.2613	14.8880		15.0	-.4683	109.5515
2.5812	5.6836		6.0	.1025	6.0019	9.8622	15.8943		16.0	-.4920	121.8253
3.0523	6.6734		7.0	.1210	8.5386	10.4516	16.8998		17.0	-.5130	134.4581
3.5270	7.6642		8.0	.1372	11.5562	11.0302	17.9046		18.0	-.5314	147.4404
4.0050	8.6562		9.0	.1512	15.0638	11.5987	18.9087		19.0	-.5475	160.7682
4.4862	9.6493		10.0	.1631	19.0684	12.1580	19.9123		20.0	-.5614	174.4423
4.9701	10.6433		11.0	.1731	23.5750	$\zeta = 1.5; \omega_n T_1 = 16.0$					
5.4565	11.6381		12.0	.1816	28.5870	.5371 -1	.1536		.2	-.1265 -4	.3990 -2
5.9450	12.6338		13.0	.1887	34.1066	.1150	.3142		.4	-.9957 -4	.3161 -1
6.4354	13.6301		14.0	.1947	40.1350	.1837	.4811		.6	-.3291 -3	.1051
6.9273	14.6270		15.0	.1996	46.6728	.2599	.6538		.8	-.7601 -3	.2442
7.4206	15.6244		16.0	.2038	53.7202	.3430	.8315		1.0	-.1440 -2	.4654
7.9150	16.6222		17.0	.2072	61.2769	.5788	1.2933		1.5	-.4433 -2	1.4538
8.4103	17.6204		18.0	.2101	69.3424	.8480	1.7725		2.0	-.9446 -2	3.1428
8.9065	18.6189		19.0	.2124	77.9161	1.4608	2.7579		3.0	-.2536 -1	8.6825
9.4032	19.6176		20.0	.2144	86.9975	2.1428	3.7591		4.0	-.4795 -1	16.8844
9.9084	21.6157		22.0	.2174	106.6803	2.8714	4.7664		5.0	-.7562 -1	27.3811
10.4139	23.6144		24.0	.2194	128.3854	3.6319	5.7763		6.0	-.1069	39.8194
10.9202	25.6135		26.0	.2208	152.1080	4.4137	6.7875		7.0	-.1408	53.8951
11.4273	27.6129		28.0	.2217	177.8442	5.2087	7.7991		8.0	-.1762	69.3584
11.9350	29.6125		30.0	.2224	205.5907	6.0110	8.8107		9.0	-.2125	86.0090
$\zeta = 1.5; \omega_n T_1 = 4.0$						6.8159	9.8222		10.0	-.2492	103.6886
.5345 -1	.1534		.2	-.1900 -4	.9972 -3	7.6199	10.8334		11.0	-.2858	122.2709
.1138	.3135		.4	-.1470 -3	.7907 -2	8.4205	11.8442		12.0	-.3220	141.6543
.1807	.4798		.6	-.4775 -3	.2631 -1	9.2156	12.8545		13.0	-.3576	161.7549
.2540	.6518		.8	-.1084 -2	.6118 -1	10.0035	13.8643		14.0	-.3924	182.5011
.3330	.8290		1.0	-.2020 -2	.1167	10.7829	14.8736		15.0	-.4262	203.8297
.5519	1.2900		1.5	-.5977 -2	.3659	11.5527	15.8824		16.0	-.4588	225.6834
.7938	1.7693		2.0	-.1226 -1	.7945	12.3118	16.8907		17.0	-.4901	248.0084
1.3179	2.7567		3.0	-.3058 -1	2.2191	13.0594	17.8984		18.0	-.5201	270.7539
1.8679	3.7600		4.0	-.5391 -1	4.3741	13.7948	18.9056		19.0	-.5487	293.8707
2.4272	4.7682		5.0	-.7949 -1	7.2064	14.5174	19.9123		20.0	-.5756	317.3113
2.9884	5.7776		6.0	-.1054	10.6667	$\zeta = 2.0; \omega_n T_1 = 0$					
3.5477	6.7868		7.0	-.1305	14.7142	.5	1.4819	2.1243	2.2851		.0443
4.1029	7.7953		8.0	-.1538	19.3159	1.0	2.8138	3.8071	3.9862		.3690
4.6531	8.8029		9.0	-.1751	24.4458	1.5	4.0798	5.3431	5.5264		1.1165
5.1980	9.8095		10.0	-.1942	30.0840	2.0	5.3099	6.8051	6.9899		2.3150
5.7374	10.8153		11.0	-.2110	36.2158	2.5	6.5017	8.1884	8.3737		4.0314
6.2717	11.8203		12.0	-.2257	42.8306	3.0	7.6579	9.5009	9.6864		6.2888
6.8013	12.8246		13.0	-.2384	49.9215	3.5	8.7910	10.7675	10.9530		9.0772
7.3267	13.8282		14.0	-.2492	57.4843	4.0	9.9023	11.9901	12.1757		12.4183
7.8482	14.8313		15.0	-.2585	65.5168	4.5	10.9965	13.1782	13.3638		16.3031
8.3665	15.8338		16.0	-.2663	74.0185	5.0	12.0706	14.3269	14.5126		20.7466
8.8819	16.8360		17.0	-.2729	82.9901	5.5	13.1471	15.4800	15.6657		25.6714
9.3948	17.8378		18.0	-.2785	92.4329	6.0	14.2075	16.6005	16.7863		31.1648
9.9056	18.8393		19.0	-.2832	102.3487	6.5	15.2583	17.7021	17.8879		37.1886
10.4147	19.8405		20.0	-.2871	112.7397	7.0	16.3028	18.7911	18.9769		43.7366
10.9202	20.8427		22.0	-.2930	134.9561	7.5	17.3408	19.8673	20.0529		50.8034
11.4273	21.8437		24.0	-.2971	159.0998	8.0	18.3685	20.9228	21.1085		58.4224
11.9350	22.8446		26.0	-.3000	185.1863	8.5	19.4042	21.9941	22.1797		66.5052
12.4421	23.8453		28.0	-.3019	213.2292	9.0	20.4227	23.0311	23.2168		75.1658
12.9492	24.8457		30.0	-.3033	243.2389	9.5	21.4511	24.0878	24.2735		84.2782
$\zeta = 1.5; \omega_n T_1 = 8.0$						10.0	22.4654	25.1160	25.3016		93.9667
.5362 -1	.1535		.2	-.1971 -4	.1994 -2	$\zeta = 2.0; \omega_n T_1 = 0.5$					
.1146	.3139		.4	-.1544 -3	.1581 -1	.5231 -1	.1526		.2	-.1225 -2	.1260 -3
.1827	.4807		.6	-.5073 -3	.5258 -1	.1084	.3104		.4	.8028 -2	.9865 -3
.2579	.6532		.8	-.1165 -2	.1222	.1671	.4726		.6	.2214 -1	.3270 -2
.3396	.8308		1.0	-.2195 -2	.2330	.2274	.6381		.8	.4292 -1	.7580 -2
.5693	1.2927		1.5	-.6667 -2	.7285	.2886	.8057		1.0	.6868 -1	1.4444 -1
.8286	1.7725		2.0	-.1403 -1	1.5773	.4430	1.2288		1.5	.1430	.4559 -1
1.4077	2.7608		3.0	-.3674 -1	4.3735	.5979	1.6518		2.0	.2157	.1014
2.0370	3.7657		4.0	-.6784 -1	8.5432	.9098	2.4912		3.0	.3255	.3086
2.6949	4.7767		5.0	-.1045	13.9274	1.2290	3.3262		4.0	.3890	.6780
3.3683	5.7899		6.0	-.1446	20.3725	1.5591	4.1648		5.0	.4230	1.2518
4.0485	6.8037		7.0	-.1862	27.7439	1.9019	5.0121		6.0	.4408	2.0714
4.7293	7.8173		8.0	-.2280	35.9273	2.2579	5.8705		7.0	.4504	3.1758
5.4061	8.8303		9.0	-.2690	44.8263	2.6269	6.7408		8.0	.4557	4.6013
6.0758	9.8424		10.0	-.2084	54.3601						
6.7361	10.8535		11.0	-.3456	64.4612						

TABLE IV.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER PLANT
WITH ZERO - Concluded

$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_M$	b/B	$P_0 \omega_n^2 / BK$	$\omega_n T_1$	$\omega_n T_2$	$\omega_n T_3$	$\omega_n T_M$	b/B	$P_0 \omega_n^2 / BK$
$\zeta = 2.0; \omega_n T_1 = 0.5$						$\zeta = 2.0; \omega_n T_1 = 4.0$					
3.0082	7.6229		9.0	0.4587	6.3803	0.5466 -1	0.1547		0.2	-0.3791 -5	0.9943 -3
3.4012	8.5163		10.0	.4604	8.5422	.1184	.3183		.4	-.2909 -4	.7820 -2
3.8050	9.4204		11.0	.4615	11.1123	.1903	.4901		.6	-.9330 -4	.2569 -1
4.2188	10.3344		12.0	.4622	14.1120	.2692	.6688		.8	-.2085 -3	.5880 -1
4.6418	11.2574		13.0	.4627	17.5608	.3538	.8531		1.0	-.3816 -3	.1102
5.0731	12.1888		14.0	.4630	21.4749	.5827	1.3306		1.5	-.1077 -2	.3295
5.5120	13.1278		15.0	.4632	25.8665	.8256	1.8213		2.0	-.2118 -2	.6865
5.9577	14.0735		16.0	.4634	30.7474	1.3267	2.8167		3.0	-.4985 -2	1.8134
6.4096	15.0255		17.0	.4635	36.1269	1.8339	3.8169		4.0	-.8526 -2	3.4781
6.8670	15.9829		18.0	.4636	42.0089	2.3427	4.8178		5.0	-.1242 -1	5.6704
7.3293	16.9453		19.0	.4637	48.4018	2.8518	5.8190		6.0	-.1643 -1	8.3846
7.7960	17.9119		20.0	.4637	55.3059	3.3609	6.8201		7.0	-.2042 -1	11.6161
						3.8697	7.8212		8.0	-.2427 -1	15.3610
						4.3780	8.8222		9.0	-.2793 -1	19.6160
						4.8858	9.8231		10.0	-.3135 -1	24.3781
						5.3930	10.8240		11.0	-.3451 -1	29.6449
						5.8996	11.8248		12.0	-.3740 -1	35.4144
						6.4056	12.8255		13.0	-.4004 -1	41.6849
						6.9110	13.8262		14.0	-.4242 -1	48.4551
						7.4159	14.8268		15.0	-.4456 -1	55.7241
						7.9202	15.8273		16.0	-.4648 -1	63.4910
						8.4241	16.8278		17.0	-.4819 -1	71.7553
						8.9276	17.8282		18.0	-.4971 -1	80.5167
						9.4307	18.8286		19.0	-.5107 -1	89.7748
						9.9334	19.8289		20.0	-.5227 -1	99.5295
$\zeta = 2.0; \omega_n T_1 = 1.0$						$\zeta = 2.0; \omega_n T_1 = 8.0$					
.5360 -1	.1538		.2	.4500 -3	.2498 -3	.5485 -1	.1548		.2	-.1582 -4	.1988 -2
.1137	.3153		.4	.3226 -2	.1961 -2	.1192	.3187		.4	-.1229 -3	.1563 -1
.1788	.4837		.6	.9676 -2	.6464 -2	.1925	.4909		.6	-.3988 -3	.5134 -1
.2477	.6578		.8	.2028 -1	.1486 -1	.2735	.6699		.8	-.9013 -3	.1174
.3189	.8363		1.0	.3488 -1	.2800 -1	.3612	.8545		1.0	-.1668 -2	.2198
.5017	1.2943		1.5	.8521 -1	.8544 -1	.6021	1.3327		1.5	-.4834 -2	.6561
.6854	1.7589		2.0	.1470	.1829	.8634	1.8243		2.0	-.9758 -2	1.3634
1.0496	2.6878		3.0	.2764	.5172	1.4190	2.8223		3.0	-.2416 -1	3.5783
1.4129	3.6089		4.0	.3920	1.0680	1.9997	3.8263		4.0	-.4335 -1	6.8033
1.7795	4.5224		5.0	.4858	1.8706	2.5960	4.8320		5.0	-.6598 -1	10.9719
2.1516	5.4303		6.0	.5574	2.9567	3.2023	5.8384		6.0	-.9097 -1	16.0212
2.5305	6.3353		7.0	.6095	4.3557	3.8146	6.8450		7.0	-.1174	21.8912
2.9171	7.2402		8.0	.6461	6.0954	4.4297	7.8516		8.0	-.1445	28.5253
3.3121	8.1477		9.0	.6712	8.2028	5.0451	8.8581		9.0	-.1718	35.8715
3.7157	9.0598		10.0	.6882	10.7035	5.6587	9.8643		10.0	-.1986	43.8822
4.1280	9.9779		11.0	.6998	13.6210	6.2690	10.8703		11.0	-.2247	52.5152
4.5488	10.9026		12.0	.7077	16.9761	6.8749	11.8759		12.0	-.2498	61.7330
4.9776	11.8341		13.0	.7132	20.7865	7.4754	12.8811		13.0	-.2736	71.5032
5.4138	12.7723		14.0	.7171	25.0674	8.0701	13.8859		14.0	-.2961	81.7978
5.8570	13.7169		15.0	.7199	29.8313	8.6586	14.8903		15.0	-.3171	92.5928
6.3064	14.6674		16.0	.7219	35.0880	9.2408	15.8943		16.0	-.3366	103.8684
6.7615	15.6232		17.0	.7235	40.8455	9.8167	16.8980		17.0	-.3547	115.6083
7.2218	16.5841		18.0	.7246	47.1102	10.3863	17.9014		18.0	-.3712	127.7995
7.6867	17.5494		19.0	.7255	53.8872	10.9499	18.9044		19.0	-.3864	140.4320
8.1557	18.5188		20.0	.7262	61.1792	11.5079	19.9071		20.0	-.4001	153.4983
$\zeta = 2.0; \omega_n T_1 = 2.0$						$\zeta = 2.0; \omega_n T_1 = 16.0$					
.5430 -1	.1544		.2	.8857 -4	.4980 -3	.5494 -1	.1549		.2	-.1164 -4	.3976 -2
.1167	.3174		.4	.6652 -3	.3914 -2	.1197	.3189		.4	-.9095 -4	.3126 -1
.1861	.4882		.6	.2086 -2	.1287 -1	.1936	.4912		.6	-.2968 -3	.1026
.2612	.6658		.8	.4561 -2	.2950 -1	.2758	.6704		.8	-.6745 -3	.2347
.3406	.8488		1.0	.8172 -2	.5535 -1	.3651	.8551		1.0	-.1255 -2	.4392
.5499	1.3219		1.5	.2191 -1	.1665	.6126	1.3331		1.5	-.3687 -2	1.3094
.7656	1.8066		2.0	.4106 -1	.3494	.8847	1.8243		2.0	-.7544 -2	2.7182
1.1975	2.7861		3.0	.8860 -1	.9422	1.4747	2.8212		3.0	-.1919 -1	7.1173
1.6261	3.7668		4.0	.1408	1.8557	2.1068	3.8242		4.0	-.3533 -1	13.4920
2.0542	4.7469		5.0	.1926	3.1159	2.7703	4.8290		5.0	-.5515 -1	21.6771
2.4846	5.7265		6.0	.2419	4.7496	3.4580	5.8347		6.0	-.7791 -1	31.5114
2.9189	6.7062		7.0	.2874	6.7814	4.1645	6.8410		7.0	-.1029	42.8396
3.3577	7.6864		8.0	.3287	9.2329	4.8849	7.8475		8.0	-.1297	55.5164
3.8014	8.6672		9.0	.3658	12.1224	5.6150	8.8541		9.0	-.1576	69.4099
4.2500	9.6489		10.0	.3989	15.4658	6.3513	9.8608		10.0	-.1863	84.4019
4.7034	10.6315		11.0	.4281	19.2762						
5.1613	11.6153		12.0	.4540	23.5645						
5.6234	12.6001		13.0	.4766	28.3398						
6.0895	13.5860		14.0	.4965	33.6092						
6.5592	14.5731		15.0	.5139	39.3787						
7.0321	15.5613		16.0	.5292	45.6528						
7.5080	16.5505		17.0	.5424	52.4351						
7.9867	17.5407		18.0	.5540	59.7281						
8.4677	18.5319		19.0	.5641	67.5339						
8.9510	19.5239		20.0	.5730	75.8536						

TABLE V.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER UNSTABLE PLANT WITH ZERO

T_1/τ_2	T_2/τ_2	T_3/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2^2$	T_1/τ_2	T_2/τ_2	T_3/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2^2$
$\tau_2/\tau_3 = 0.10; \tau_1/\tau_2 = 0$						$\tau_2/\tau_3 = 0.10; \tau_1/\tau_2 = 2.0$					
0.2936	-1	0.1002	0.1708	0.2	0.3446	1.7798	5.5502		7.0	0.2703	1.7297
.6008	-1	.2030	.3430	.4	.5733	1.8886	5.9333		7.5	.2947	2.0868
.9120	-1	.3068	.5156	.6	.8102	1.9946	6.3120		8.0	.3174	2.4852
.1230		.4120	.6890	.8	.1052	2.0979	6.6864		8.5	.3385	2.9266
.1555		.5186	.8632	1.0	.2600	2.1987	7.0565		9.0	.3576	3.4125
.2395		.7917	1.3022	1.5	.1305	2.2970	7.4224		9.5	.3755	3.9444
.3272		1.0734	1.7462	2.0	.4089	2.3929	7.7843		10.0	.3914	4.5239
.4182		1.3633	2.1951	2.5	.9880	2.5783	8.4967		11.0	.4184	5.8321
.5119		1.6608	2.6488	3.0	.2025	2.7556	9.1952		12.0	.4396	7.3504
.6081		1.9653	3.1072	3.5	.3699	2.9255	9.8813		13.0	.4558	9.0926
.7062		2.2760	3.5698	4.0	.6212	3.0887	10.5567		14.0	.4680	11.0733
.8058		2.5924	4.0366	4.5	.9781	3.2456	11.2227		15.0	.4771	13.3069
.9065		2.9136	4.5070	5.0	.1464	3.3966	11.8804		16.0	.4838	15.8086
1.0080		3.2390	4.9810	5.5	.2101	3.5422	12.5307		17.0	.4885	18.5924
1.1099		3.5679	5.4580	6.0	.2916	3.6824	13.1744		18.0	.4920	21.6745
1.2120		3.8998	5.9378	6.5	.3931	3.8177	13.8121		19.0	.4944	25.0663
1.3139		4.2341	6.4201	7.0	.5173	3.9460	14.4441		20.0	.4961	28.7834
1.4156		4.5703	6.9047	7.5	.6665						
1.5167		4.9080	7.3913	8.0	.8432						
1.6172		5.2467	7.8796	8.5	1.0498						
1.7168		5.5862	8.3694	9.0	1.2888						
1.8155		5.9262	8.8606	9.5	1.5625						
1.9133		6.2663	9.3530	10.0	1.8732						
2.0104		6.6461	10.3407	11.0	2.6149						
2.2927		7.6243	11.3316	12.0	3.5318						
2.4749		8.2997	12.3248	13.0	4.6414						
2.6518		8.9716	13.3198	14.0	5.9605						
2.8232		9.6394	14.3162	15.0	7.5054						
2.9893		10.3028	15.3135	16.0	9.2918						
3.1499		10.9616	16.3116	17.0	11.3350						
3.3052		11.6154	17.3102	18.0	13.6498						
3.4552		12.2644	18.3092	19.0	16.2503						
3.5999		12.9084	19.3085	20.0	19.1503						
$\tau_2/\tau_3 = 0.10; \tau_1/\tau_2 = 0.5$						$\tau_2/\tau_3 = 0.10; \tau_1/\tau_2 = 3.0$					
.4875	-1	.1485		.2	-0.8623	.5069	.1507	.2	.2328	.7493	.7493
.9544	-1	.2940		.4	-.5701	.1027	.3030	.4	.1800	.5992	.5992
.1406		.4366		.6	-.1601	.1559	.4567	.6	.5870	.2019	.2019
.1847		.5767		.8	-.3182	.2101	.6121	.8	.1344	.4775	.4775
.2280		.7149		1.0	-.5245	.2653	.7691	1.0	.2533	.9299	.9299
.3342		1.0544		1.5	-.1194	.4064	1.1632	1.5	.7811	.3107	.3107
.4393		1.3898		2.0	-.1982	.5507	1.5760	2.0	1.6877	.7662	.7662
.5443		1.7242		2.5	-.2801	.6962	1.9911	2.5	.2995	.1394	.1394
.6495		2.0595		3.0	-.3600	.8414	2.4119	3.0	.4696	.2362	.2362
.7549		2.3962		3.5	-.4351	.9853	2.8366	3.5	.6756	.3668	.3668
.8606		2.7346		4.0	-.5041	1.1268	3.2637	4.0	.9129	.5348	.5348
.9663		3.0747		4.5	-.5664	1.2654	3.6917	4.5	.1176	.7428	.7428
1.0718		3.4162		5.0	-.6222	1.4006	4.1194	5.0	.1459	.9934	.9934
1.1769		3.7590		5.5	-.6717	1.5323	4.5458	5.5	.1757	1.2886	1.2886
1.2815		4.1027		6.0	-.7154	1.6602	4.9699	6.0	.2064	1.6304	1.6304
1.3854		4.4470		6.5	-.7538	1.7845	5.3911	6.5	.2377	2.0203	2.0203
1.4885		4.7917		7.0	-.7874	1.9052	5.8088	7.0	.2691	2.4600	2.4600
1.5907		5.1365		7.5	-.8167	2.0222	6.2226	7.5	.3002	2.9508	2.9508
1.6918		5.4812		8.0	-.8422	2.1358	6.6319	8.0	.3308	3.4940	3.4940
1.7917		5.8257		8.5	-.8643	2.2460	7.0367	8.5	.3605	4.0906	4.0906
1.8906		6.1697		9.0	-.8835	2.3529	7.4365	9.0	.3891	4.7415	4.7415
1.9881		6.5131		9.5	-.9001	2.4566	7.8313	9.5	.4165	5.4478	5.4478
2.0844		6.8558		10.0	-.9145	2.5573	8.2208	10.0	.4424	6.2101	6.2101
2.2731		7.5387		11.0	-.9375	2.7498	8.9840	11.0	.4694	7.9055	7.9055
2.4563		8.2176		12.0	-.9546	2.9315	9.7299	12.0	.5296	9.8329	9.8329
2.6340		8.8922		13.0	-.9672	3.1032	10.4475	13.0	.4629	11.9980	11.9980
2.8062		9.5621		14.0	-.9764	3.2658	11.1501	14.0	.5896	14.4076	14.4076
2.9729		10.2271		15.0	-.9831	3.4202	11.8358	15.0	.6103	17.0714	17.0714
3.1341		10.8871		16.0	-.9880	3.5674	12.5066	16.0	.6261	20.0008	20.0008
3.2898		11.5420		17.0	-.9915	3.7080	13.1648	17.0	.6379	23.2090	23.2090
3.4403		12.1918		18.0	-.9940	3.8427	13.8123	18.0	.6464	26.7113	26.7113
3.5855		12.8366		19.0	-.9958	3.9719	14.4508	19.0	.6526	30.5212	30.5212
3.7256		13.4763		20.0	-.9970						
3.8603		14.1108		21.0	-.9980						
3.9906		14.7499		22.0	-.9986						
$\tau_2/\tau_3 = 0.10; \tau_1/\tau_2 = 2.0$						$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 0$					
.5048	-1	.1505		.2	.3567	.5086	.1509	.2	.1176	.1250	.1250
.1018		.3021		.4	.2714	.1034	.3036	.4	.9212	.9985	.9985
.1540		.4549		.6	.8709	.1574	.4582	.6	.3043	.3364	.3364
.2068		.6087		.8	.1963	.2129	.6147	.8	.7055	.7954	.7954
.2601		.7637		1.0	.3643	.2697	.7731	1.0	.1347	.1549	.1549
.3951		1.1559		1.5	.1082	.4166	1.1773	1.5	.4283	.5169	.5169
.5311		1.5536		2.0	.2255	.5688	1.5926	2.0	.9531	.1207	.1207
.6669		1.9556		2.5	.3869	.7243	2.0178	2.5	.1742	.2313	.2313
.8014		2.3601		3.0	.5872	.8811	2.4513	3.0	.2809	.3909	.3909
.9340		2.7657		3.5	.8188	1.0377	2.8915	3.5	.4152	.6056	.6056
1.0638		3.1711		4.0	.1074	1.1929	3.3368	4.0	.5760	.1880	.1880
1.1908		3.5752		4.5	.1344	1.3455	3.7857	4.5	.7610	1.2184	1.2184
1.3148		3.9770		5.0	.1622	1.4949	4.2368	5.0	.9678	1.6232	1.6232
1.4357		4.3759		5.5	.1902	1.6406	4.6890	5.5	.1194	2.0969	2.0969
1.5534		4.7713		6.0	.2178	1.7823	5.1414	6.0	.1436	2.6417	2.6417
1.6681		5.1628		6.5	.2446	1.9200	5.5930	6.5	.1692	3.2592	3.2592
						2.0535	6.0434	7.0	.1959	3.9511	3.9511
						2.1830	6.4919	7.5	.2236	4.7188	4.7188
						2.3084	6.9381	8.0	.2519	5.5638	5.5638
						2.4299	7.3816	8.5	.2807	6.4873	6.4873
						2.5476	7.8220	9.0	.3098	7.4904	7.4904
						2.6616	8.2592	9.5	.3390	8.5743	8.5743
						2.7721	8.6927	10.0	.3682	9.7398	9.7398
						2.8828	9.1248	11.0	.4260	12.3175	12.3175
						2.9828	9.5478	12.0	.4819	15.2257	15.2257
						3.0771	9.9687	13.0	.5349	18.4614	18.4614
						3.1667	10.3852	14.0	.5838	22.0170	22.0170
						3.2514	10.7971	15.0	.6278	25.8807	25.8807
						3.3314	11.2048	16.0	.6662	30.0388	30.0388
$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 0$						$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 0$					
.2962	-1	.1007		.2	.3567	.2962	.1007	.2	.1176	.1250	.1250
.5980	-1	.2025		.4	.2714	.5980	.2025	.4	.9212	.9985	.9985
.9056	-1	.3056		.6	.8709	.9056	.3056	.6	.3043	.3364	.3364
.1219		.4100		.8	.1963	.1219	.4100	.8	.7055	.7954	.7954

TABLE V.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER
UNSTABLE PLANT WITH ZERO - Continued

T_1/τ_2	T_2/τ_2	T_3/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2^2$	T_1/τ_2	T_2/τ_2	T_3/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2^2$
$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 0$						$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 3.0$					
0.1537	0.5155	0.8619	1.0		0.6484 -3	0.5050 -1	0.1505		0.2	0.3134 -4	0.1875 -3
.2352	.7846	1.2995	1.5		.3261 -2	.1019	.3022		.4	.2423 -3	.1498 -2
.3191	1.0608	1.7418	2.0		.1022 -1	.1540	.4552		.6	.7901 -3	.5048 -2
.4047	1.3436	2.1889	2.5		.2469 -1	.2067	.6094		.8	.1808 -2	.1194 -1
.4917	1.6324	2.6408	3.0		.5057 -1	.2599	.7650		1.0	.3409 -2	.2324 -1
.5792	1.9266	3.0973	3.5		.9234 -1	.3936	1.1593		1.5	.1050 -1	.7761 -1
.6669	2.2253	3.5584	4.0		.1550	.5268	1.5608		2.0	.2265 -1	.1813
.7542	2.5280	4.0238	4.5		.2438	.6576	1.9678		2.5	.4013 -1	.3475
.8406	2.8337	4.4931	5.0		.3645	.7843	2.3785		3.0	.6275 -1	.5876
.9259	3.1420	4.9661	5.5		.5226	.9061	2.7910		3.5	.8997 -1	.9106
1.0095	3.4520	5.4426	6.0		.7240	1.0220	3.2036		4.0	.1211	1.3238
1.0912	3.7633	5.9221	6.5		.9744	1.1318	3.6145		4.5	.1552	1.8326
1.1708	4.0751	6.4043	7.0		1.2793	1.2352	4.0224		5.0	.1915	2.4412
1.2481	4.3872	6.8890	7.5		1.6443	1.3323	4.4260		5.5	.2291	3.1526
1.3230	4.6989	7.3799	8.0		2.0746	1.4233	4.8243		6.0	.2673	3.9689
1.3953	5.0100	7.8647	8.5		2.5752	1.5084	5.2164		6.5	.3053	4.8912
1.4650	5.3202	8.3552	9.0		3.1509	1.5879	5.6018		7.0	.3426	5.9199
1.5320	5.6291	8.8471	9.5		3.8062	1.6619	5.9798		7.5	.3786	7.0547
1.5963	5.9366	9.3403	10.0		4.5455	1.7310	6.3502		8.0	.4128	8.2947
1.7170	6.5467	10.3297	11.0		6.2917	1.7952	6.7127		8.5	.4450	9.6388
1.8271	7.1495	11.3223	12.0		8.4184	1.8551	7.0671		9.0	.4747	11.0854
1.9273	7.7445	12.3172	13.0		10.9509	1.9109	7.4135		9.5	.5018	12.6330
2.0179	8.3317	13.3137	14.0		13.9106	1.9628	7.7521		10.0	.5262	14.2801
2.0988	8.9111	14.3114	15.0		17.3160	2.0563	8.4070		11.0	.5671	17.8693
2.1734	9.4832	15.3098	16.0		21.1825	2.1380	9.0349		12.0	.5980	21.8503
2.2395	10.0483	16.3088	17.0		25.5232	2.2095	9.6401		13.0	.6204	26.2292
2.2988	10.6069	17.3091	18.0		30.3487	2.2725	10.2266		14.0	.6360	31.0194
$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 0.5$						$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 5.0$					
.4857 -1	.1483	.2	-0.9237 -3	.3128 -4	.5066 -1	.1507		.2	.1764 -4	.3124 -3	
.9475 -1	.2932	.4	-.6106 -2	.2510 -3	.1026	.3029		.4	.1382 -3	.2496 -2	
.1391	.4348	.6	-.1715 -1	.8519 -3	.1555	.4567		.6	.4564 -3	.8410 -2	
.1821	.5736	.8	-.3405 -1	.2035 -2	.2095	.6121		.8	.1058 -2	.1989 -1	
.2240	.7100	1.0	-.5611 -1	.4016 -2	.2641	.7692		1.0	.2020 -2	.3871 -1	
.3257	1.0438	1.5	-.1276	.1400 -1	.4031	1.1692		1.5	.6420 -2	.1292	
.4245	1.3716	2.0	-.2116	.3447 -1	.5434	1.5791		2.0	.1427 -1	.3013	
.5213	1.6966	2.5	-.2968	.7008 -1	.6826	1.9977		2.5	.2605 -1	.5769	
.6165	2.0206	3.0	-.3836	.8186	.8186	2.4234		3.0	.4191 -1	.9736	
.7100	2.3442	3.5	-.4630	.2076	.9500	2.8545		3.5	.6177 -1	1.5050	
.8017	2.6678	4.0	-.5356	.3206	1.0796	3.2890		4.0	.8538 -1	2.1813	
.8913	2.9911	4.5	-.6008	.4708	1.1946	3.7255		4.5	.1123	3.0095	
.9787	3.3140	5.0	-.6586	.6640	1.3066	4.1622		5.0	.1422	3.9941	
1.0636	3.6362	5.5	-.7094	.9058	1.4116	4.5978		5.5	.1745	5.1378	
1.1459	3.9575	6.0	-.7537	1.2018	1.5096	5.0313		6.0	.2087	6.4419	
1.2256	4.2776	6.5	-.7920	1.5573	1.6008	5.4616		6.5	.2443	7.9067	
1.3024	4.5962	7.0	-.8251	1.9776	1.6855	5.8879		7.0	.2810	9.5315	
1.3765	4.9131	7.5	-.8533	2.4675	1.7640	6.3095		7.5	.3182	11.3147	
1.4476	5.2283	8.0	-.8774	3.0318	1.8367	6.7258		8.0	.3556	13.2540	
1.5159	5.5415	8.5	-.8979	3.6748	1.9039	7.1362		8.5	.3929	15.3462	
1.5814	5.8526	9.0	-.9151	4.4009	1.9660	7.5402		9.0	.4297	17.5871	
1.6440	6.1616	9.5	-.9297	5.2141	2.0234	7.9373		9.5	.4656	19.9715	
1.7039	6.4684	10.0	-.9419	6.1178	2.0762	8.3269		10.0	.5003	22.4935	
1.8156	7.0754	11.0	-.9607	8.2111	2.1698	9.0825		11.0	.5651	27.9217	
1.9170	7.6735	12.0	-.9736	10.7061							
2.0087	8.2631	13.0	-.9825	13.6247							
2.0914	8.8443	14.0	-.9885	16.9857							
2.1699	9.4178	15.0	-.9925	20.8049							
2.2328	9.9840	16.0	-.9951	25.0956							
2.2929	10.5437	17.0	-.9969	29.8660							
$\tau_2/\tau_3 = 0.25; \tau_1/\tau_2 = 2.0$						$\tau_2/\tau_3 = 0.5; \tau_1/\tau_2 = 0$					
.5029 -1	.1503	.2	.4459 -2	.1250 -3	.3055		.1004	0.1709	.2	.2192 -3	
.1011	.4533	.4	.3392 -3	.9989 -3	.3825		1.3114	2.1789	2.5	.4919 -1	
.1522	.4533	.6	.1088 -2	.3367 -2	.4582		1.5862	2.6281	3.0	.1005	
.2035	.6860	.8	.2452 -2	.7964 -2	.5318		1.8639	3.0821	3.5	.1831	
.2549	.7995	1.0	.4551 -2	.1551 -1	.6029		2.1440	3.5411	4.0	.3063	
.3089	1.1464	1.5	.1350 -1	.5188 -1	.6708		2.4256	4.0048	4.5	.4799	
.3570	1.5370	2.0	.2809 -1	.1214	.7352		2.7083	4.4731	5.0	.7140	
.4034	1.9295	2.5	.4807 -1	.2334	.7959		2.9914	4.9455	5.5	1.0181	
.4469	2.3222	3.0	.7270 -1	.3961	.8525		3.2744	5.4218	6.0	1.4017	
.4866	2.7136	3.5	.1010	.6162	.9051		3.5568	5.9017	6.5	1.8733	
.5203	3.1021	4.0	.1318	.8999	.9536		3.8383	6.3846	7.0	2.4407	
.5474	3.4854	4.5	.1640	1.2519	.9982		4.1185	6.8703	7.5	3.1108	
.5690	3.8657	5.0	.1967	1.6766	1.0388		4.3973	7.3584	8.0	3.8898	
.5850	4.2391	5.5	.2290	2.1773	1.0758		4.6744	7.8486	8.5	4.7828	
.5961	4.6062	6.0	.2602	2.7571	1.1093		4.9498	8.3405	9.0	5.7941	
.6020	4.9667	6.5	.2897	3.4184	1.1395		5.2234	8.8339	9.5	6.9271	
.6031	5.3203	7.0	.3172	4.1634	1.1667		5.4952	9.3285	10.0	8.1847	
.6048	5.6672	7.5	.3424	4.9941	1.2130		6.0336	10.3206	11.0	11.0820	
.6072	6.0074	8.0	.3652	5.9123	1.2500		6.5655	11.3155	12.0	14.4978	
.6097	6.3412	8.5	.3856	6.9198	1.2793		7.0916	12.3122	13.0	18.4381	
.6125	6.6689	9.0	.4035	8.0185	1.3025		7.6127	13.3102	14.0	22.9050	
.6158	6.9909	9.5	.4191	9.2102	1.3207		8.1297	14.3089	15.0	27.8983	
.6190	7.3075	10.0	.4326	10.4971							
.6224	7.6263	10.5	.4459	13.3647							
.6260	7.9473	11.0	.4600	16.6389							
.6298	8.2704	11.5	.4754	20.3381							
.6337	8.5953	12.0	.4865	24.4790							
.6378	8.9219	12.5	.4913	29.0785							
$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 0.5$						$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 0.5$					
.4827 -1	.1480	.2	-.1026 -2	.6256 -4	.4827 -1	.1480	.2	-.1026 -2	.6256 -4		
.9360 -1	.2919	.4	-.6780 -2	.5018 -3	.9360 -1	.2919	.4	-.6780 -2	.5018 -3		
.1366	.4319	.6	-.1902 -1	.1703 -2	.1366	.4319	.6	-.1902 -1	.1703 -2		
.1778	.5683	.8	-.3774 -1	.4066 -2	.1778	.5683	.8	-.3774 -1	.4066 -2		

TABLE V.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER
UNSTABLE PLANT WITH ZERO - Continued

T_1/τ_2	T_2/τ_2	T_3/τ_2	T_M/τ_2	b/B	$R_0/BK\tau_2^2$	T_1/τ_2	T_2/τ_2	T_3/τ_2	T_M/τ_2	b/B	$R_0/BK\tau_2^2$	
$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 0.5$						$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 5.0$						
0.2174	0.7019	1.0	-0.6213	-1	0.8019	0.8960	3.2148	4.0	0.1275	4.2061		
.3115	1.0263	1.5	-.1409		.2793	.9685	3.6328	4.5	.1657	5.7370		
.3999	1.3416	2.0	-.2330		.6868	1.0313	4.0486	5.0	.2069	7.5184		
.4835	1.6514	2.5	-.3281		.1393	1.0852	4.4607	5.5	.2502	9.5402		
.5625	1.9576	3.0	-.4198		.2498	1.1311	4.8677	6.0	.2946	11.7899		
.6370	2.2608	3.5	-.5048		.4101	1.1701	5.2688	6.5	.3395	14.2534		
.7068	2.5614	4.0	-.5814		.6304	1.2031	5.6629	7.0	.3839	16.9152		
.7719	2.8596	4.5	-.6492		.9207	1.2309	6.0495	7.5	.4275	19.7592		
.8322	3.1551	5.0	-.7081		1.2902	1.2543	6.4279	8.0	.4694	22.7689		
.8878	3.4481	5.5	-.7588		1.7474	1.2740	6.7976	8.5	.5094	25.9273		
.9389	3.7384	6.0	-.8018		2.2999	1.2905	7.1581	9.0	.5470	29.2180		
.9855	4.0260	6.5	-.8381		2.9546	1.3045	7.5091	9.5	.5819	32.6249		
1.0279	4.3108	7.0	-.8683		3.7174							
1.0663	4.5930	7.5	-.8934		4.5932							
1.1010	4.8725	8.0	-.9140		5.5862							
1.1323	5.1496	8.5	-.9309		6.6999							
1.1604	5.4242	9.0	-.9447		7.9372							
1.1856	5.6966	9.5	-.9558		9.3001							
1.2081	5.9669	10.0	-.9648		10.7906							
1.2461	6.5017	11.0	-.9778		14.1585							
1.2764	7.0298	12.0	-.9862		18.0473							
1.3002	7.5523	13.0	-.9914		22.4608							
1.3190	8.0703	14.0	-.9947		27.3986							
$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 2.0$						$\tau_2/\tau_3 = 1.0; \tau_1/\tau_2 = 0$						
.4997	.1500	.2	.5944	-4	.2499	.2927	1.0000	0.1707	.2	.0408	-4	
.9975	.3002	.4	.4521	-3	.1997	.5846	2.0000	.3415	.4	.6660	-4	
.1492	.4506	.6	.4450	-2	.6729	.8747	3.0000	.5125	.6	.3364	-4	
.1981	.6013	.8	.3265	-2	.1591	1.162	4.0000	.6838	.8	.1061	-2	
.2463	.7523	1.0	.6054	-2	.3098	1.446	5.0000	.8554	1.0	.2582	-2	
.3629	1.1308	1.5	.1790	-1	.1033	2.136	7.5000	1.2864	1.5	.1294	-1	
.4725	1.5096	2.0	.3706	-1	.2409	2.788	1.0000	1.7212	2.0	.4028	-1	
.5739	1.8869	2.5	.6299	-1	.4609	3.392	1.2500	2.1608	2.5	.9646	-1	
.6665	2.2610	3.0	.9447	-1	.7768	3.943	1.5000	2.6057	3.0	.1953		
.7502	2.6301	3.5	.1299		1.1991	4.434	1.7500	3.0566	3.5	.3517		
.8251	2.9926	4.0	.1675		1.7350	4.866	2.0000	3.5134	4.0	.5806		
.8917	3.3475	4.5	.2057		2.3890	5.239	2.2500	3.9761	4.5	.8962		
.9507	3.6939	5.0	.2432		3.1637	5.556	2.5000	4.4444	5.0	1.3111		
1.0027	4.0315	5.5	.2788		4.0595	5.822	2.7500	4.9178	5.5	1.8360		
1.0484	4.3603	6.0	.3118		5.0763	6.042	3.0000	5.3958	6.0	2.4793		
1.0886	4.6803	6.5	.3416		6.2131	6.223	3.2500	5.8777	6.5	3.2474		
1.1239	4.9920	7.0	.3681		7.4688	6.369	3.5000	6.3631	7.0	4.1447		
1.1549	5.2960	7.5	.3911		8.8424	6.487	3.7500	6.8513	7.5	5.1739		
1.1821	5.5929	8.0	.4108		10.3334	6.581	4.0000	7.3419	8.0	6.3366		
1.2060	5.8835	8.5	.4275		11.9415	6.656	4.2500	7.8344	8.5	7.6335		
1.2271	6.1685	9.0	.4414		13.6669	6.715	4.5000	8.3285	9.0	9.0644		
1.2456	6.4485	9.5	.4529		15.5104	6.762	4.7500	8.8238	9.5	10.6291		
1.2619	6.7243	10.0	.4623		17.4727	6.799	5.0000	9.3201	10.0	12.3266		
1.2891	7.2653	11.0	.4762		21.7577	6.851	5.5000	10.3149	11.0	16.1173		
1.3103	7.7954	12.0	.4851		26.5308	6.882	6.0000	11.3118	12.0	20.4300		
1.3270	8.3177	13.0	.4907		31.7921	6.902	6.5000	12.3098	13.0	25.2587		
						6.913	7.0000	13.3087	14.0	30.5987		
							7.5000	14.3080	15.0	36.4466		
							8.0000	15.3075	16.0	42.7998		
							8.5000	16.3073	17.0	49.6566		
							9.0000	17.3071	18.0	57.0158		
							9.5000	18.3070	19.0	64.8766		
							10.0000	19.3069	20.0	73.2384		
$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 3.0$						$\tau_2/\tau_3 = 1.0; \tau_1/\tau_2 = 0.5$						
.5017	.1502	.2	.4477	-4	.3749	.4768	.1474	.2	-.1231	-2	.1250	-3
.1006	.3011	.4	.3461	-3	.2996	.9131	.2893	.4	-.8118	-2	.1002	-2
.1509	.4526	.6	.1128	-2	.1009	.1316	.4259	.6	-.2272	-1	.3396	-2
.2011	.6050	.8	.2580	-2	.2385	.1692	.5779	.8	-.4496	-1	.8006	-1
.2509	.7582	1.0	.4860	-2	.4642	.2044	.6899	1.0	-.7379	-1	.1594	-1
.3724	1.1448	1.5	.1493	-1	.1547	.2836	.9922	1.5	-.1659		.5517	-1
.4878	1.5359	2.0	.3207	-1	.3599	.3524	1.2844	2.0	-.2719		.1346	
.5952	1.9299	2.5	.5647	-1	.6865	.4119	1.5671	2.5	-.3789		.2704	
.6933	2.3247	3.0	.8799	-1	1.1531	.4629	1.8430	3.0	-.4796		.4784	
.7817	2.7183	3.5	.1244		1.7724	.5061	2.1136	3.5	-.5700		.7738	
.8604	3.1085	4.0	.1655		2.5517	.5423	2.3801	4.0	-.6489		.9423	
.9297	3.4937	4.5	.2054		3.4940	.5723	2.6433	4.5	-.7159		1.1759	
.9903	3.8723	5.0	.2547		4.5982	.5968	2.9038	5.0	-.7719		1.6758	
1.0430	4.2430	5.5	.3000		5.8606	.6167	3.1622	5.5	-.8180		2.3013	
1.0886	4.6051	6.0	.3444		7.2755	.6327	3.4188	6.0	-.8555		3.0519	
1.1280	4.9579	6.5	.3866		8.8375	.6455	3.6741	6.5	-.8857		4.9428	
1.1619	5.3010	7.0	.4263		10.5324	.6557	3.9282	7.0	-.9099		6.0869	
1.1910	5.6395	7.5	.4622		12.3585	.6638	4.1815	7.5	-.9292		8.7747	
1.2162	5.9584	8.0	.5039		14.3061	.6701	4.4340	8.0	-.9444		10.7747	
1.2378	6.2732	8.5	.5236		16.3688	.6751	4.6860	8.5	-.9565		13.3178	
1.2564	6.5794	9.0	.5487		18.5415	.6791	4.9376	9.0	-.9659		15.9931	
1.2726	6.8778	9.5	.5701		20.8205	.6822	5.1888	9.5	-.9734		18.7995	
1.2865	7.1689	10.0	.5883		23.2038	.6846	5.4398	10.0	-.9792		21.7366	
1.2992	7.4531	11.0	.6029		25.6815	.6880	5.6911	11.0	-.9873		24.7754	
							.6900	12.0	-.9923		29.9991	
							.6913	13.0	-.9953		36.0610	
$\tau_2/\tau_3 = 0.50; \tau_1/\tau_2 = 5.0$						$\tau_2/\tau_3 = 1.0; \tau_1/\tau_2 = 2.0$						
.5034	.1504	.2	.2744	-4	.6248	.4933	.1494	.2	.8913	-4	.4997	-3
.1012	.3018	.4	.2149	-3	.4992	.9714	.2978	.4	.6773	-3	.3990	-2
.1524	.4542	.6	.7096	-3	.1681	.1432	.4454	.6	.2169	-2	.1343	-1
.2037	.6078	.8	.1644	-2	.3974	.1873	.5922	.8	.4871	-2	.3168	-1
.2549	.7627	1.0	.3137	-2	.7733	.2293	.7383	1.0	.9003	-2	.6150	-1
.3810	1.1558	1.5	.9948	-2	.2575	.3242	1.1006	1.5	.2632	-1	.2032	
.5019	1.5570	2.0	.2203	-1	.5988	.4042	1.4578	2.0	.5360	-1	.4669	
.6153	1.9653	2.5	.3998	-1	1.1406	.4696	1.8089	2.5	.8924	-1	.8767	
.7194	2.3790	3.0	.6386	-1	1.9121	.9217	2.1523	3.0	.1306		1.4456	
.8130	2.7962	3.5	.9327	-1	2.9311	.5625	2.4872	3.5	.1747		2.2771	
						.6938	2.8128	4.0	.2189		3.0684	
							3.1289	4.5	.2612		4.1122	

TABLE V.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER
UNSTABLE PLANT WITH ZERO - Continued

T_1/τ_2	T_2/τ_2	T_3/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2^2$	T_1/τ_2	T_2/τ_2	T_3/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2^2$		
$\tau_2/\tau_3 = 1.0; \tau_1/\tau_2 = 2.0$						$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 0.5$							
0.6358	3.4359		5.0	0.3001	5.3000	0.1793	0.6555		1.0	-0.9524	-1	0.3106	-1
.6495	3.7342		5.5	.3347	6.6233	.2326	.9305		1.5	-.2084		.1049	
.6598	4.0247		6.0	.3648	8.0747	.2703	1.1873		2.0	-.3320		.2483	
.6677	4.3081		6.5	.3905	9.6487	.2965	1.4340		2.5	-.4500		.4816	
.6736	4.5855		7.0	.4119	11.3413	.3142	1.6755		3.0	-.5547		.8207	
.6781	4.8577		7.5	.4296	13.1500	.3259	1.9150		3.5	-.6436		1.2768	
.6816	5.1256		8.0	.4440	15.0736	.3335	2.1543		4.0	-.7170		1.8571	
.6842	5.3899		8.5	.4557	17.1118	.3384	2.3943		4.5	-.7765		2.5656	
.6862	5.6513		9.0	.4651	19.2652	.3415	2.6354		5.0	-.8242		3.4044	
.6878	5.9103		9.5	.4725	21.5341	.3434	2.8779		5.5	-.8621		4.3742	
.6890	6.1674		10.0	.4784	23.9198	.3446	3.1216		6.0	-.8921		5.4750	
.6906	6.4774		11.0	.4868	29.0453	.3454	3.3665		6.5	-.9156		6.7064	
						.3458	3.6123		7.0	-.9341		8.0679	
						.3461	3.8590		7.5	-.9486		9.5588	
						.3463	4.1063		8.0	-.9599		11.1784	
						.3464	4.3543		8.5	-.9687		12.9263	
						.3465	4.6026		9.0	-.9756		14.8021	
$\tau_2/\tau_3 = 1.0; \tau_1/\tau_2 = 3.0$						$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 2.0$							
.4955	1.496		.2	.7160	1.7494	.4805	1.1482		.2	.1484	-3	.9983	-3
.9790	1.790		.4	.5531	1.5985	.4918	1.2931		.4	.1123	-2	.7943	-2
.1448	1.4475		.6	.1800	1.2013	.1315	.4351		.6	.3573	-2	.2657	-1
.1900	.5962		.8	.4109	1.4750	.1666	.5746		.8	.7954	-1	.6217	-1
.2333	.7449		1.0	.7716	1.9220	.1973	.7118		1.0	1.194	-1	.1194	
.3317	1.1169		1.5	.2345	1.3042	.2562	1.0471		1.5	.4093	-1	.3807	
.4149	1.4893		2.0	.4958	1.6978	.2939	1.3734		2.0	.7930	-1	.8362	
.4827	1.8610		2.5	.8551	1.3079	.3166	1.6923		2.5	1.257		1.4927	
.5361	2.2303		3.0	1.294	2.1432	.3298	2.0046		3.0	1.748		2.3383	
.5770	2.5952		3.5	1.788	3.2088	.3372	2.3101		3.5	2.229		3.3544	
.6078	2.9540		4.0	.2308	4.4906	.3413	2.6091		4.0	2.678		4.5224	
.6305	3.3055		4.5	.2833	5.9705	.3436	2.9015		4.5	3.079		5.8271	
.6472	3.6485		5.0	.3342	7.6281	.3446	3.1877		5.0	3.429		7.2572	
.6594	3.9825		5.5	.3821	9.4432	.3456	3.4682		5.5	3.726		8.8052	
.6682	4.3073		6.0	.4261	11.3977	.3462	4.0143		6.0	3.975		10.4662	
.6746	4.6230		6.5	.4655	13.4763	.3464	4.2813		6.5	4.181		12.2376	
.6794	4.9297		7.0	.5003	15.6669	.3465	4.5449		7.0	4.349		14.1186	
.6828	5.2282		7.5	.5303	17.9603	.3465	4.8058		7.5	4.484		16.1092	
.6854	5.5190		8.0	.5558	20.3509				8.0				
.6872	5.8030		8.5	.5772	22.8353								
.6887	6.0810		9.0	.5949	25.4125								
.6897	6.3539		9.5	.6095	28.0833								
.6905	6.6224		10.0	.6213	30.8496								
$\tau_2/\tau_3 = 1.0; \tau_1/\tau_2 = 5.0$						$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 3.0$							
.4969	1.198		.2	.4702	1.1249	.4824	1.1484		.2	.1252	-3	.1457	-2
.9853	1.492		.4	.3680	.9975	.9265	1.2941		.4	.9633	-3	.1192	-4
.1462	.4492		.6	.1214	.3355	.1329	.4376		.6	.3116	-2	.3985	-1
.1923	.5994		.8	.2807	.7916	.1687	.5793		.8	.7052	-2	.9326	-2
.2367	.7501		1.0	.5341	1.536	.2002	.7198		1.0	1.110	-1	1.1791	
.3384	1.1303		1.5	.1677	.5069	.2604	1.0679		1.5	.3836	-1	.5702	
.4247	1.5162		2.0	.3658	1.1618	.2984	1.4144		2.0	.7730	-1	1.2480	
.4949	1.9075		2.5	.6505	2.1705	.3207	1.7603		2.5	1.264		2.2151	
.5496	2.3029		3.0	.1013	3.5516	.3307	2.1049		3.0	1.813		3.4438	
.5908	2.7004		3.5	.1438	5.2954	.3330	2.4466		3.5	2.381		4.8968	
.6209	3.0978		4.0	.1908	7.3734	.3395	2.7836		4.0	2.938		6.5391	
.6425	3.4932		4.5	.2403	9.7484	.3429	3.1145		4.5	3.466		8.3415	
.6578	3.8849		5.0	.2908	12.3821	.3446	3.4383		5.0	3.952		10.2814	
.6685	4.2714		5.5	.3412	15.2387	.3456	3.7544		5.5	4.389		12.3416	
.6759	4.6517		6.0	.3903	18.2865	.3460	4.0626		6.0	4.776		14.5094	
.6810	5.0249		6.5	.4375	21.4981	.3463	4.3631		6.5	5.111		16.7756	
.6846	5.3903		7.0	.4823	24.8495	.3464	4.6562		7.0	5.398		19.1344	
.6871	5.7472		7.5	.5241	28.3197								
.6888	6.0953		8.0	.5626	31.8906								
$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 0$						$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 0$							
.2884	.9917	0.1703	.2		.0823	.4840	1.1486		.2	.8611	-4	.2496	-2
.5668	1.1967	.3400	.4		.1329	.9321	1.2949		.4	.6716	-3	.1986	-1
.8335	1.2926	.5093	.6		.6698	.1340	1.4395		.6	.2202	-2	.6644	-1
.1087	.3870	.6783	.8		.2104	.1705	.5831		.8	.5051	-2	.1555	
.1327	.4799	.8473	1.0		.5098	.2026	.7264		1.0	.9509	-2	.2987	
.1860	.7069	1.2709	1.5		.2513	.2641	1.0860		1.5	.2878	-1	.9506	
.2294	.9280	1.6986	2.0		.7656	.3024	1.4520		2.0	.5981	-1	2.0766	
.2635	1.1459	2.1324	2.5		.1785	.3242	1.8258		2.5	1.006		3.6708	
.2891	1.3628	2.5737	3.0		.3504	.3356	2.2062		3.0	1.482		5.6741	
.3077	1.5809	3.0232	3.5		.6102	.3413	2.5903		3.5	1.996		8.0141	
.3208	1.8014	3.4806	4.0		.9725	.3441	2.9753		4.0	.2525		10.6271	
.3298	2.0251	3.9454	4.5		1.4485	.3454	3.3585		4.5	.3053		13.4641	
.3358	2.2524	4.4166	5.0		2.0462	.3460	3.7381		5.0	.3568		16.4884	
.3397	2.4832	4.8935	5.5		2.7705	.3463	4.1125		5.5	.4065		19.6720	
.3422	2.7173	5.3750	6.0		3.6245	.3464	4.4804		6.0	.4536		22.9924	
.3439	2.9542	5.8604	6.5		4.6095	.3465	4.8408		6.5	.4977		26.4307	
.3449	3.1937	6.3488	7.0		5.7263								
.3455	3.4352	6.8396	7.5		6.9746								
.3459	3.6784	7.3325	8.0		8.3541								
.3462	3.9230	7.8269	8.5		9.8643								
.3463	4.1688	8.3225	9.0		11.5044								
.3464	4.4155	8.8190	9.5		13.2740								
.3465	4.6628	9.3163	10.0		15.1723								
$\tau_2/\tau_3 = 2.0; \tau_1/\tau_2 = 0.5$						$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 0$							
.4648	1.1461		.2	-.1638	-.2497	.2754	1.1735		.2			.2063	-4
.6675	1.2841		.4	-.1075	-.1994	.5135	1.3777		.4			.3272	-3
.1218	.4143		.6	-.2988	-.6720	.7131	1.5902		.6			.1619	-2
.1525	.5378		.8	-.5861	-.1591	.8757	1.8100		.8			.4965	-2
						.1005	2.0360		1.0			.1169	-1
						.1212	2.2394		1.5			.5295	-1
						.1312	2.4604		2.0			.1474	
						.1357	2.6926		2.5			.3153	
						.1375	2.9360		3.0			.5730	
						.1382	3.1777		3.5			.9330	
						.1385	3.4517		4.0			1.4049	
						.1385	3.9214		4.5			1.9955	

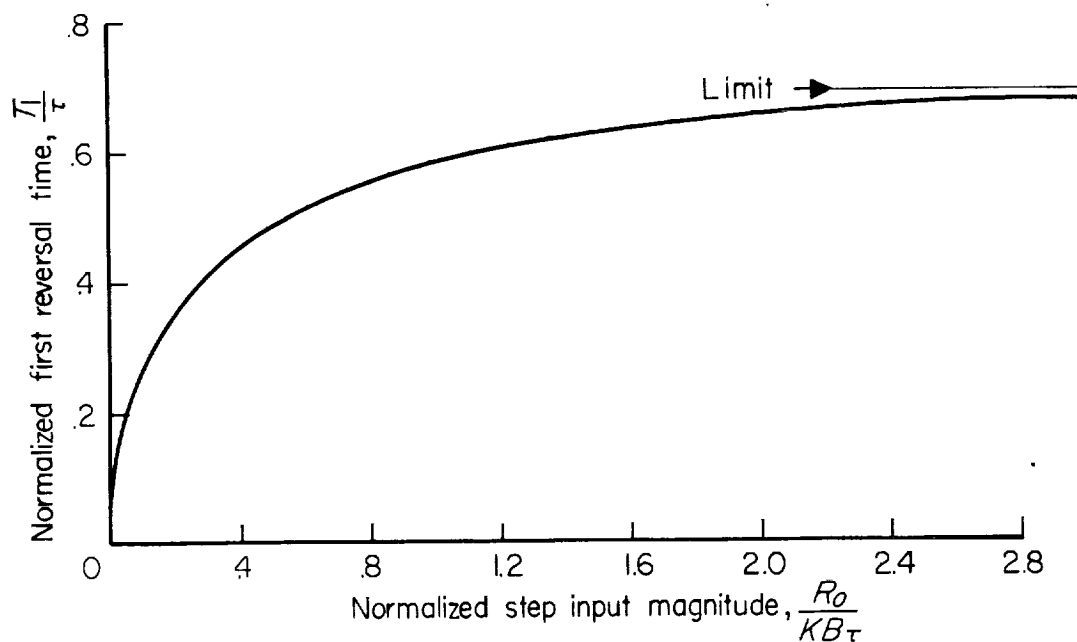
TABLE V.- NORMALIZED OPTIMUM RESPONSES FOR A TYPE 2 FOURTH-ORDER
UNSTABLE PLANT WITH ZERO - Concluded

T_1/τ_2	T_2/τ_2	T_3/τ_2	T_m/τ_2	b/B	$R_0/BK\tau_2^2$
$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 0.5$					
0.4295 ⁻¹	0.1422		0.2	-0.2838 ⁻²	0.6188 ⁻³
.7370 ⁻¹	.2692		.4	-.1813 ⁻¹	.4824 ⁻²
.9509 ⁻¹	.3830		.6	-.4851 ⁻¹	.1570 ⁻¹
.1097	.4869		.8	-.9117 ⁻¹	.3572 ⁻¹
.1195	.5840		1.0	-.1419	.6679 ⁻¹
.1320	.8116		1.5	-.2813	.2024
.1364	1.0327		2.0	-.4147	.4338
.1379	1.2553		2.5	-.5303	.7728
.1384	1.4818		3.0	-.6264	1.2273
.1386	1.7124		3.5	-.7044	1.8029
$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 2.0$					
.4428 ⁻¹	.1446		.2	.3236 ⁻³	.2475 ⁻²
.7729 ⁻¹	.2797		.4	.2389 ⁻²	.1922 ⁻¹
.1003	.4076		.6	.7313 ⁻²	.6192 ⁻¹
.1155	.5310		.8	.1552 ⁻¹	.1382
.1251	.6519		1.0	.2689 ⁻¹	.2519
.1353	.9514		1.5	.6612 ⁻¹	.7009
.1379	1.2522		2.0	.1143	1.3654
.1385	1.5536		2.5	.1651	2.2166
.1386	1.8532		3.0	.2146	3.2311
$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 3.0$					
.4444 ⁻¹	.1449		.2	.2837 ⁻³	.3713 ⁻²
.7775 ⁻¹	.2809		.4	.2130 ⁻²	.2884 ⁻¹
.1011	.4110		.6	.6634 ⁻²	.9293 ⁻¹
.1164	.5378		.8	.1431 ⁻¹	.2074
.1259	.6637		1.0	.2520 ⁻¹	.3778
.1358	.9822		1.5	.6426 ⁻¹	1.0462
.1381	1.3102		2.0	.1147	2.0235
.1385	1.6447		2.5	.1708	3.2571
.1386	1.9812		3.0	.2284	4.7049
$\tau_2/\tau_3 = 5.0; \tau_1/\tau_2 = 5.0$					
.4457 ⁻¹	.1451		.2	.2018 ⁻³	.6188 ⁻²
.7813 ⁻¹	.2820		.4	.1537 ⁻²	.4809 ⁻¹
.1017	.4138		.6	.4853 ⁻²	.1550
.1171	.5436		.8	.1062 ⁻¹	.3461
.1266	.6739		1.0	.1894 ⁻¹	.6300
.1362	1.0106		1.5	.4977 ⁻¹	1.7398
.1382	1.3664		2.0	.9128 ⁻¹	3.3468
.1386	1.7368		2.5	.1393	5.3515

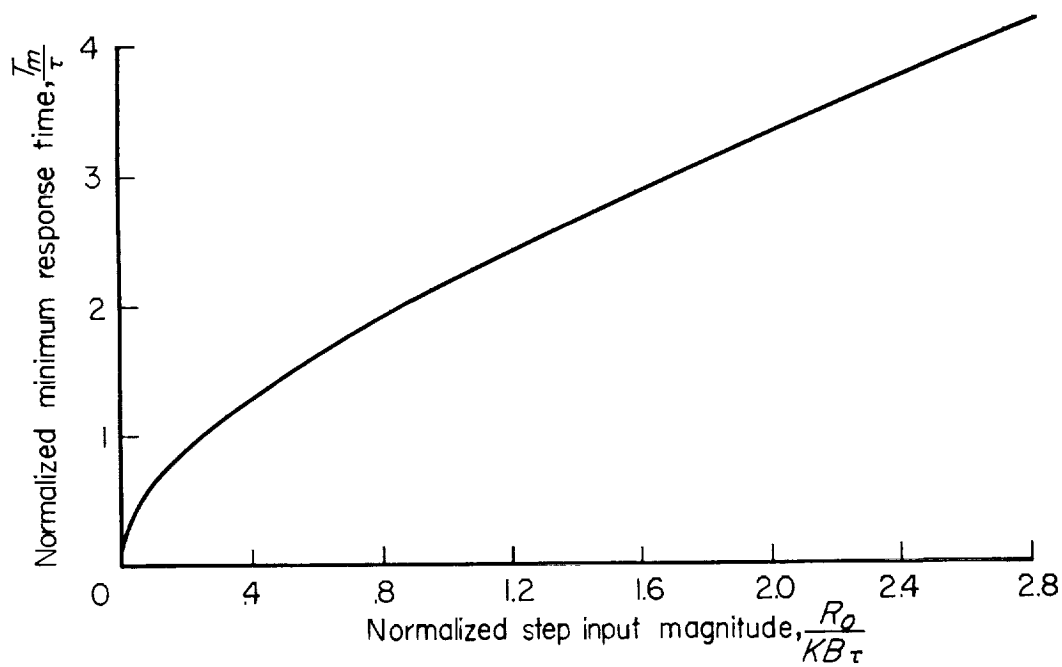
TABLE VI.- REFERENCES FOR OPTIMUM SYSTEMS

Case	Plant transfer function	References	Results or comments
1	$\frac{K}{s}$		$x(t) = B \operatorname{sgn} e(t)$
2	$\frac{K(\tau_1 s + 1)}{s(\tau_2 s + 1)}$	11 for $\tau_1 = 0$; for $\tau_1/\tau_2 \geq 0.5$ optimum derived here	for $\tau_1 = 0$ $x(t) = B \operatorname{sgn} \left[e(t) + \frac{\tau_2 \dot{e}(t)}{KB} \left[\frac{1 - \ln \left(1 + \frac{ \dot{e} }{KB} \right)}{ \dot{e} KB} \right] \right]$
3	$\frac{K}{s[(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1]}$	12	$x(t) = B \operatorname{sgn} f(e, \dot{e}, \ddot{e})$ complex function
4	$\frac{K}{s^2}$	11	$x(t) = B \operatorname{sgn} \left(e + \frac{\dot{e}}{2KB} \right)$
5	$\frac{K(\tau_1 s + 1)}{s^2(\tau_2 s + 1)}$	13 for $\tau_1 = 0$	$x(t) = B \operatorname{sgn} f(e, \dot{e}, \ddot{e})$ for $\tau_1 = 0$
6	$\frac{K}{s^2[(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1]}$		$x(t) = B \operatorname{sgn} f(e, \dot{e}, \ddot{e}, \ddot{\ddot{e}})$
7	$\frac{K}{s^3}$	14	$x(t) = B \operatorname{sgn} f(e, \dot{e}, \ddot{e})$
8	$\frac{K}{s(\tau s - 1)}$	Approximate optimum derived here	
9	$\frac{K(\tau_1 s + 1)}{s[(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1]}$	None	
10	$\frac{K(\tau_1 s + 1)}{s(\tau_2 s + 1)(\tau_3 s - 1)}$		
11	$\frac{K(\tau_1 s + 1)}{s^2[(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1]}$	Approximate optimum derived here	
12	$\frac{K(\tau_1 s + 1)}{s^2(\tau_2 s + 1)(\tau_3 s - 1)}$		

Note: A number of the references use output derivative feedbacks rather than error derivatives. As has been pointed out, error derivatives are necessary if the optimum is to be obtained for arbitrary inputs.



(a) Optimum first reversal time.



(b) Minimum response time.

Figure 1.- Optimum response times for a type 1 second-order unstable plant.

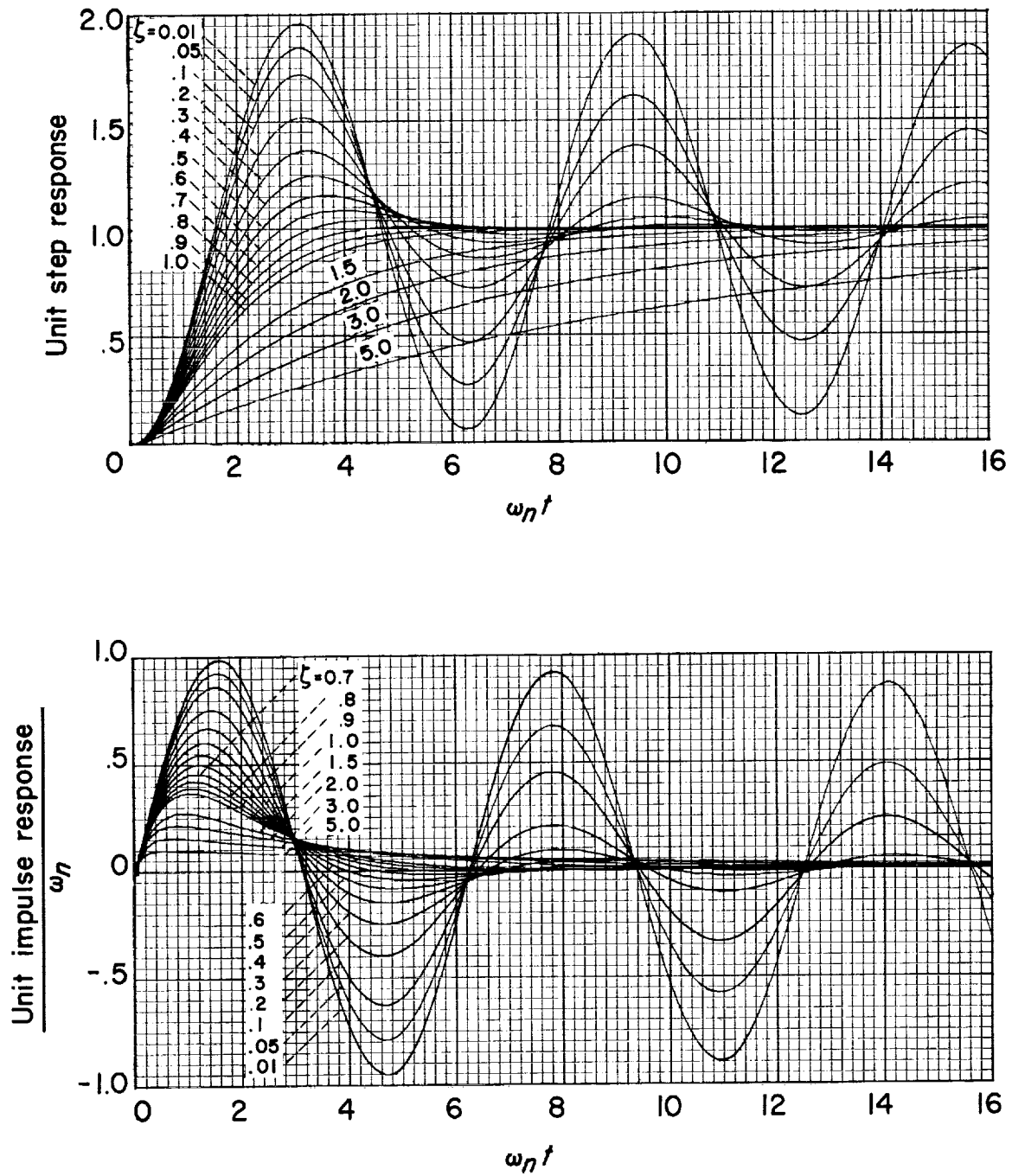
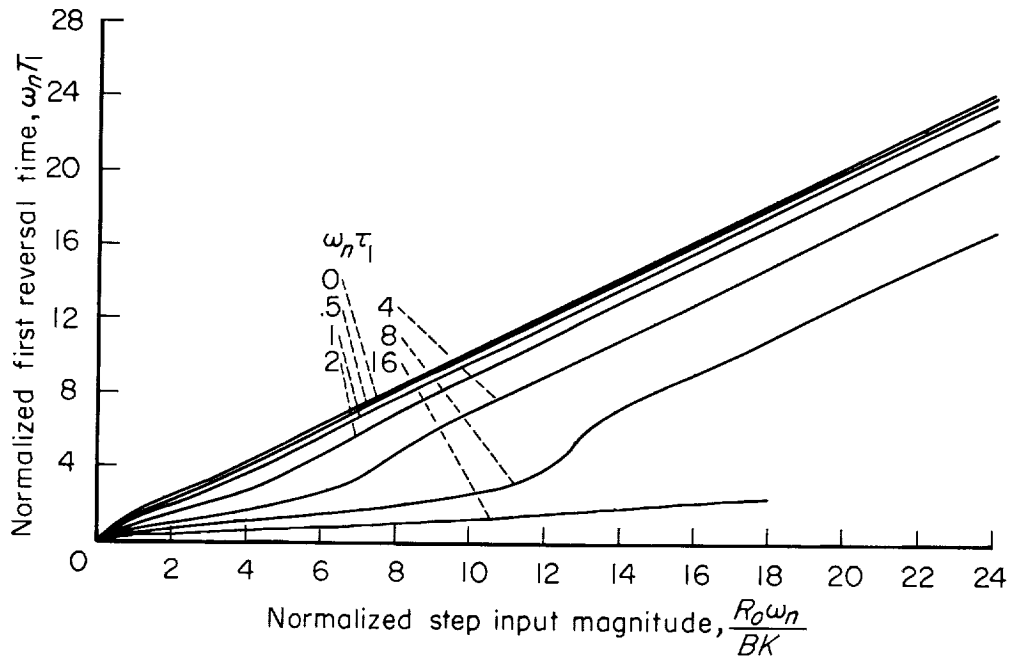
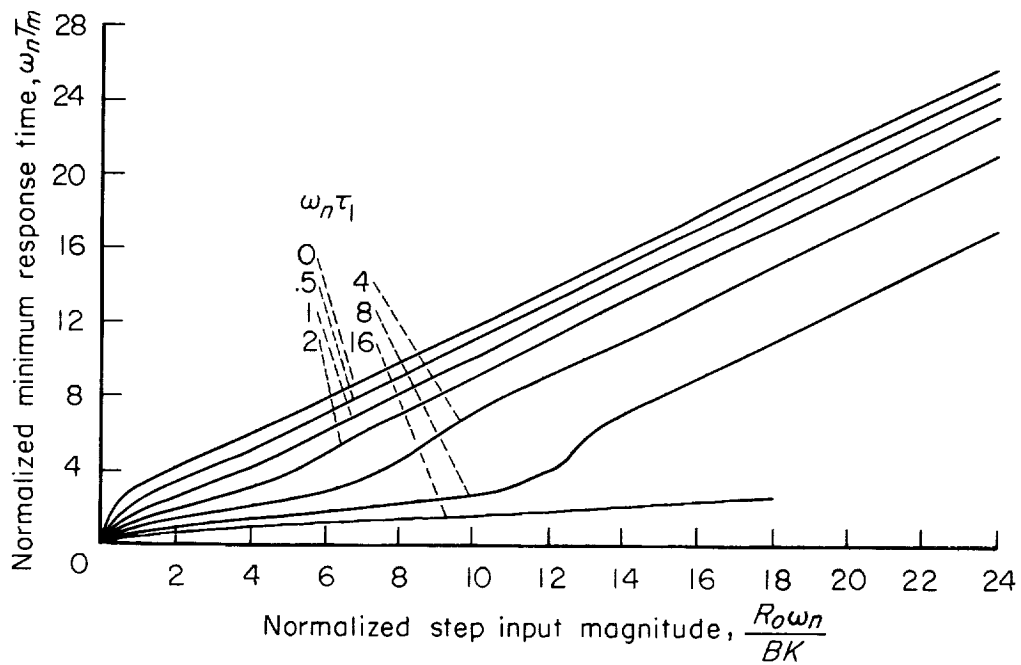


Figure 2.- Responses of a second-order system.

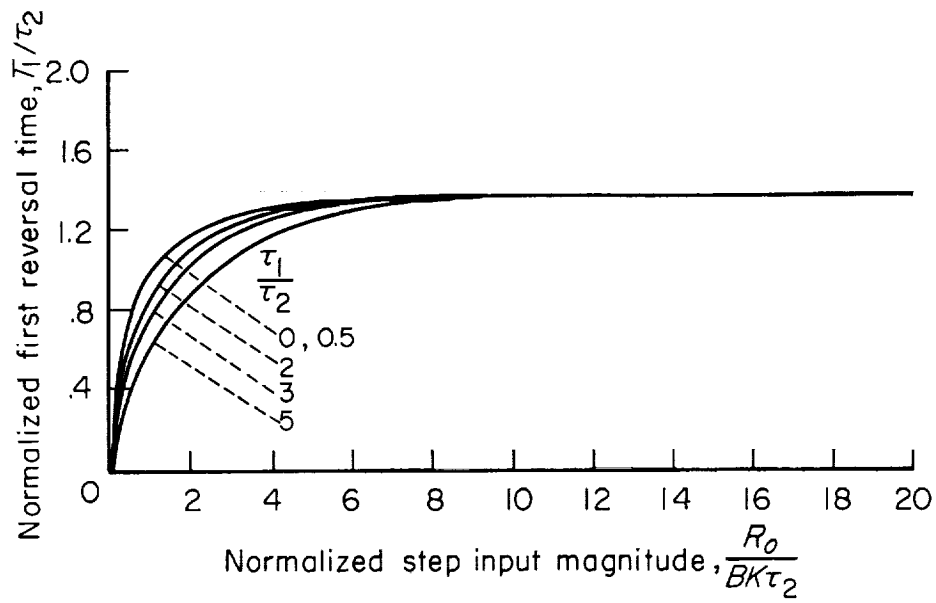


(a) Optimum first reversal time.

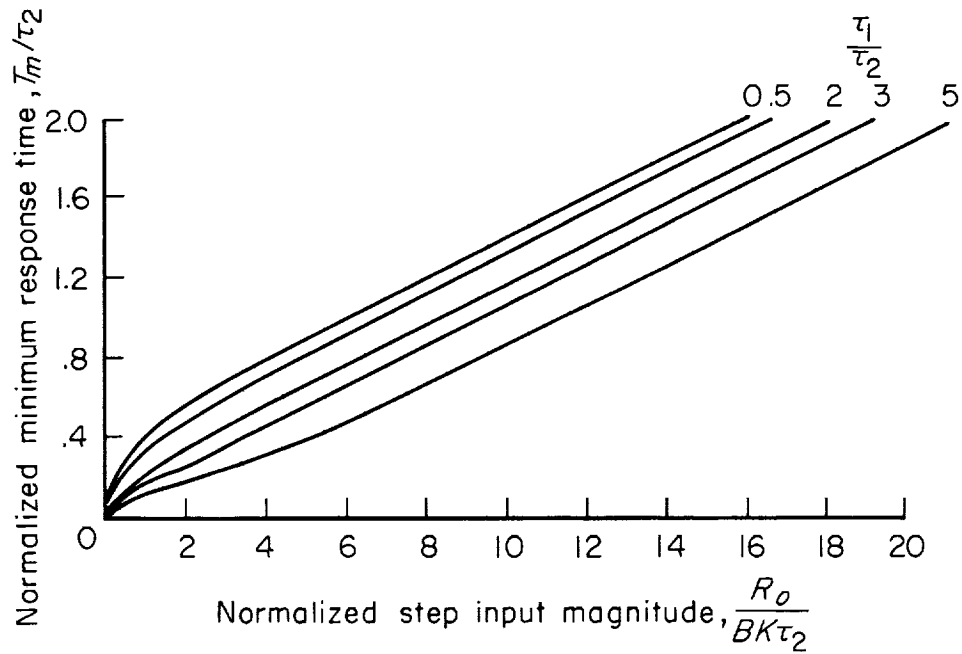


(b) Minimum response time.

Figure 3.- Optimum response times for a type 1 third-order plant with zero ($\zeta = 0.5$).

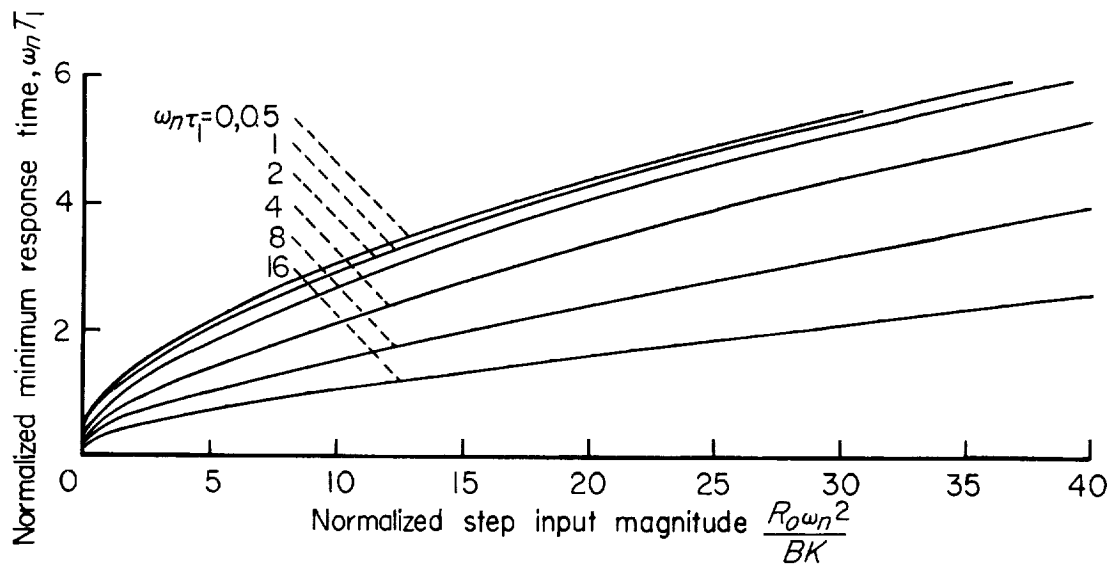


(a) Optimum first reversal time.

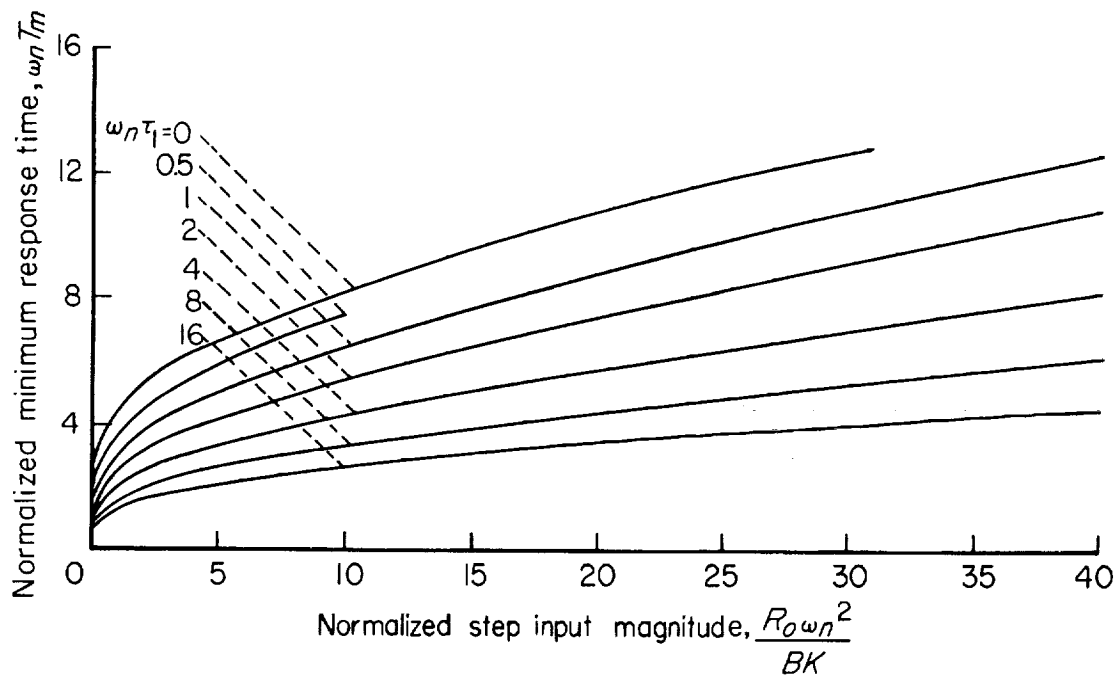


(b) Minimum response time.

Figure 4.- Optimum response times for a type 1 third-order unstable plant with zero ($\tau_2/\tau_3 = 0.5$).

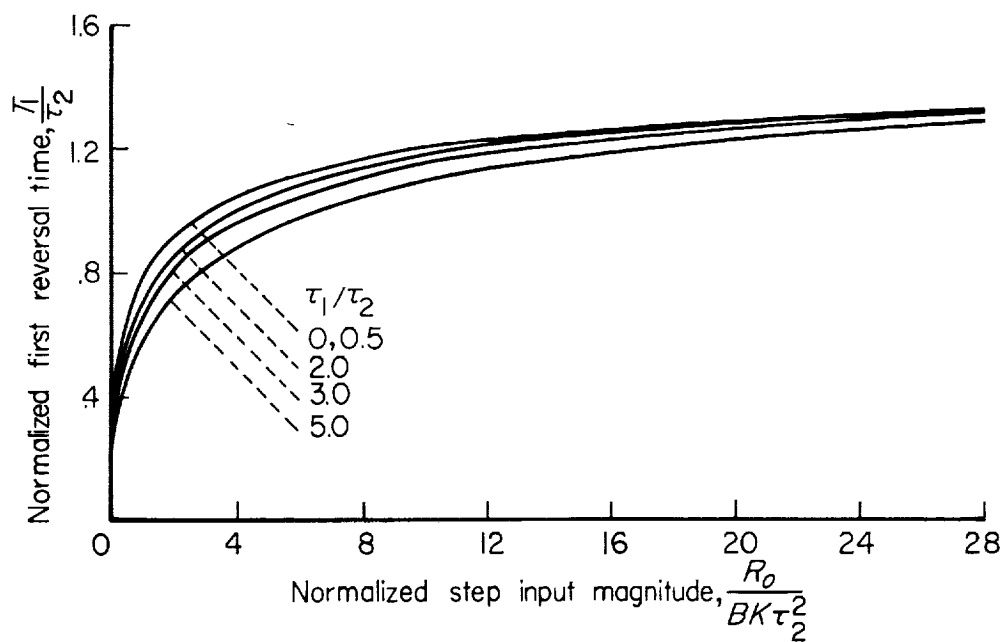


(a) Optimum first reversal time.

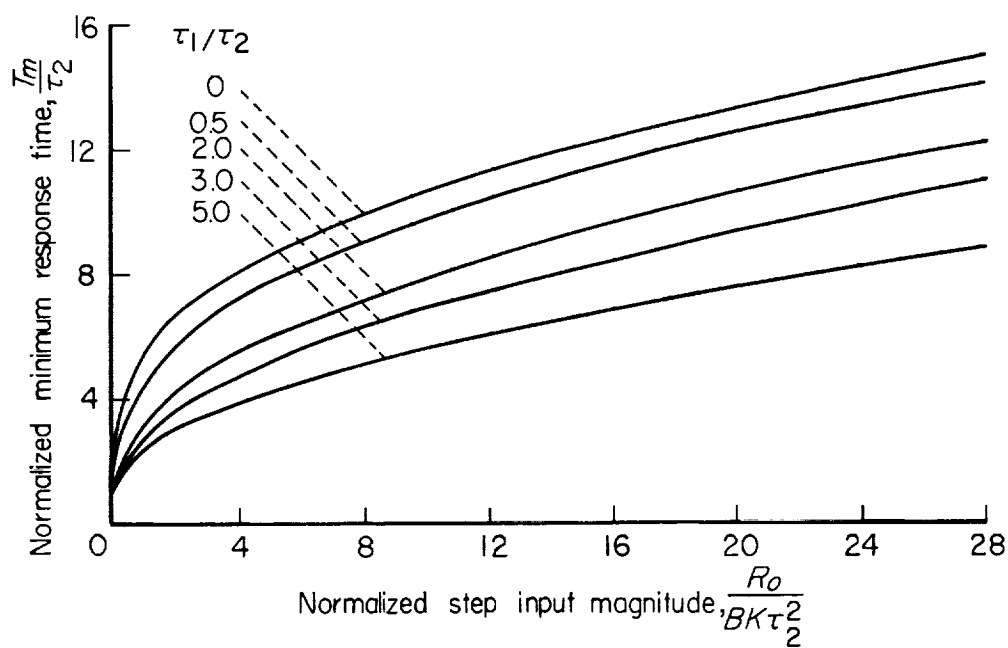


(b) Minimum response time.

Figure 5.- Optimum response times for a type 2 fourth-order plant with zero ($\zeta = 0.5$).



(a) Optimum first reversal time.



(b) Minimum response time.

Figure 6.- Optimum response times for a type 2 fourth-order unstable plant with zero ($\tau_2/\tau_3 = 0.5$).

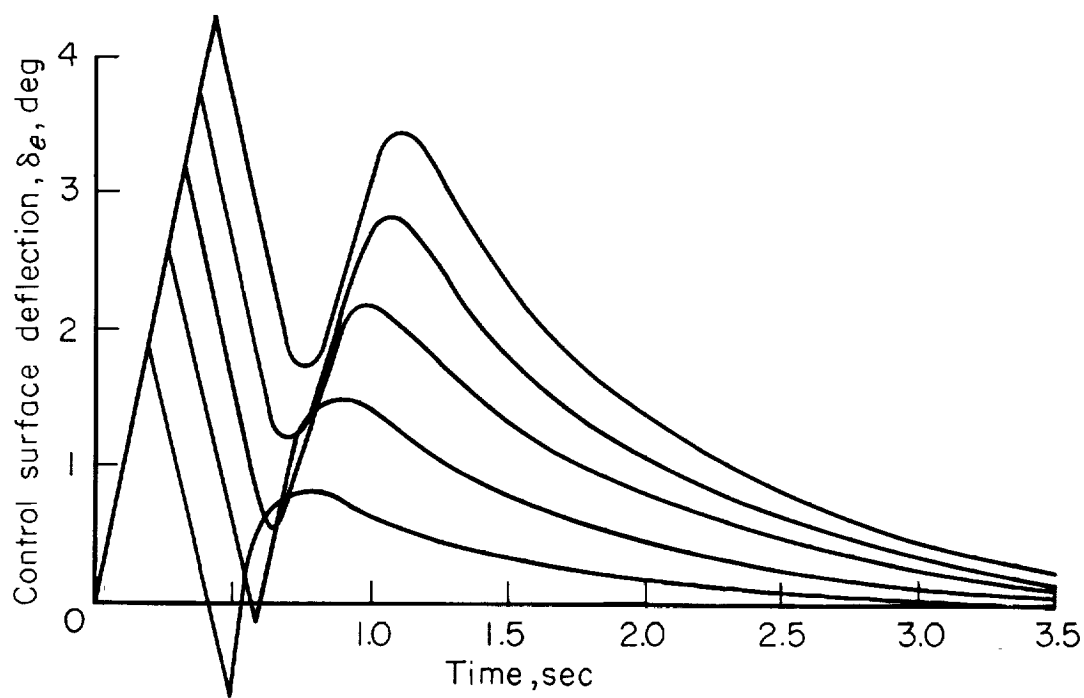
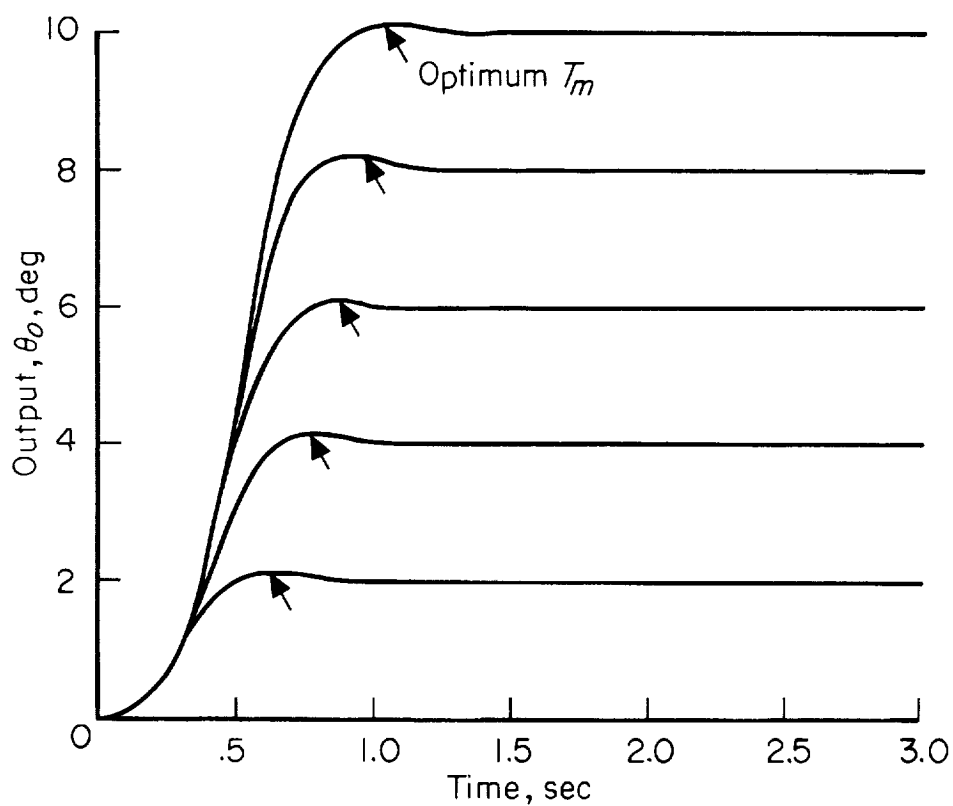


Figure 7.- Responses of attitude control system with stable aerodynamics.

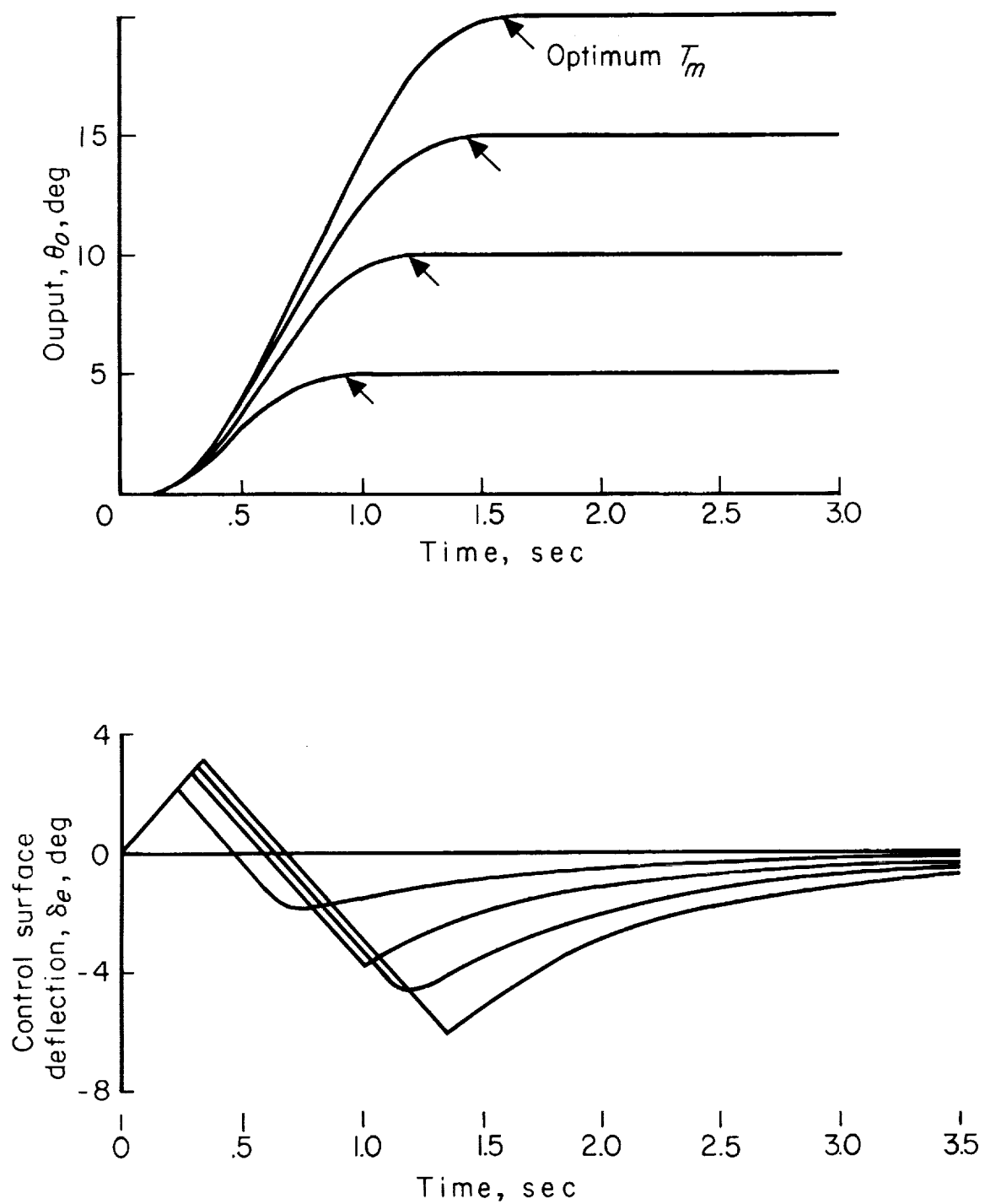
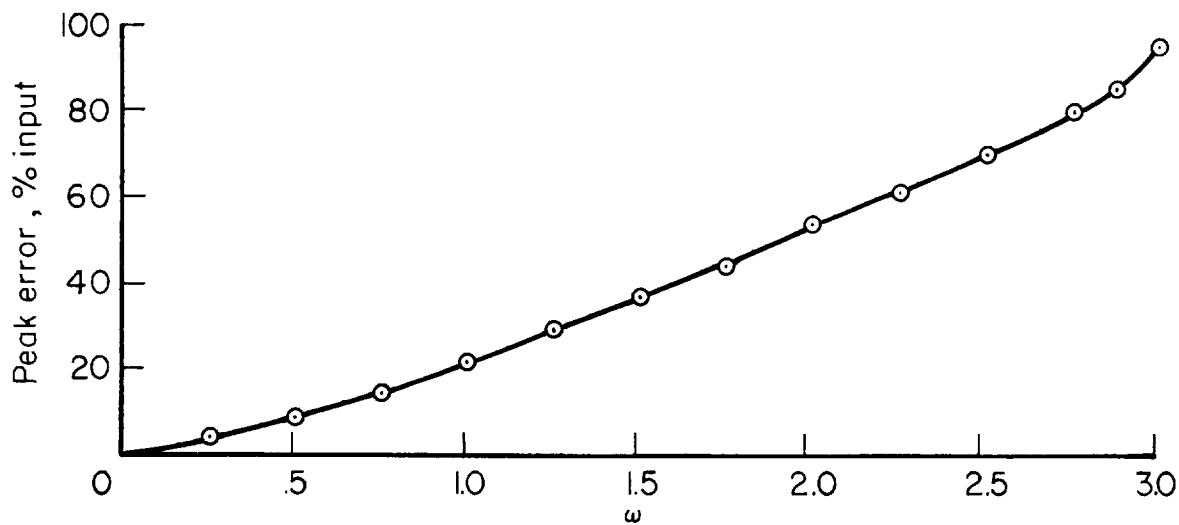
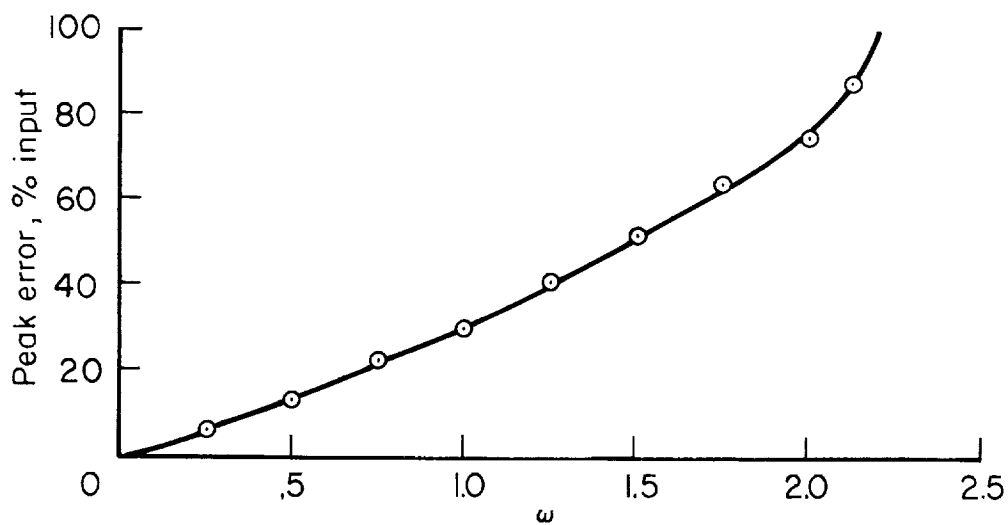


Figure 8.- Responses of attitude control system for unstable aerodynamics.

Input = $10 \sin \omega t$ 

(a) Stable.



(b) Unstable.

Figure 9.- Error magnitude versus frequency for the two example attitude control systems.

