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NASA TN D-883

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NASA TN D-883

TECHNICAL NOTE

D-883

ANALYTICAL EVALUATION OF A METHOD OF MIDCOURSE GUIDANCE FOR RENDEZVOUS WITH EARTH SATELLITES

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON June 1961

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

A digital-computer simulation was made of the midcourse or ascent phase of a rendezvous between a ferry vehicle and a space station. The simulation involved a closed-loop guidance system in which both the relative position and relative velocity between ferry and station are measured (by simulated radar) and the relative-velocity corrections required to null the miss distance are computed and applied. The results are used to study the effectiveness of a particular set of guidance equations and to study the effects of errors in the launch conditions and errors in the navigation data. A number of trajectories were investigated over a variety of initial conditions for cases in which the space station was in a circular orbit and also in an elliptic orbit. Trajectories are described in terms of a rotating coordinate system fixed in the station.

As a result of this study the following conclusions are drawn. Successful rendezvous can be achieved even with launch conditions which are substantially less accurate than those obtained with present-day techniques. The average total-velocity correction required during the midcourse phase is directly proportional to the radar accuracy but the miss distance is not. Errors in the time of booster burnout or in the position of the ferry at booster burnout are less important than errors in the ferry velocity at booster burnout. The use of dead bands to account for errors in the navigational (radar) equipment appears to depend upon a compromise between the magnitude of the velocity corrections to be made and the allowable miss distance at the termination of the midcourse phase of the rendezvous. When approximate guidance equations are used, there are limits on their accuracy which are dependent on the angular distance about the earth to the expected point of rendezvous.

INTRODUCTION

This paper is concerned with midcourse guidance of a vehicle launched from the earth in an attempt to rendezvous with a target in a near-earth orbit. The midcourse phase is considered to start at the time of booster burnout (where the payload and target are several hundred miles apart) and to continue until the launched vehicle or payload is within 5 to 10 miles of the target. During this time the payload is to be guided onto a collision course with the target. At the end of the midcourse phase the payload is injected into orbit in close proximity of the target. However, this latter maneuver, generally referred to as the terminal phase, is treated elsewhere in the literature (for example, ref. 1) and is not covered herein.

During the midcourse or ascent phase, the guidance could be basically inertial (ref. 2) or a closed-loop system in which both the relative position and the relative velocity of the payload with respect to the target are measured and the relative equations of motion are solved to compute the corrections required to null the miss distance. Reference 3 proposes this latter method for the terminal phase of rendezvous. Since equipment capable of measuring relative position and velocity will probably be required for the terminal phase, such equipment is investigated herein for use during the midcourse phase.

In the study of this paper, a guidance logic system based on relative position and velocity data is developed. By simulating this guidance logic system and computing the trajectories of the target and payload, the effects of errors in launch conditions, errors in the radar system, dead bands in the guidance system, and so forth, are investigated to determine their effect on target miss distance and fuel for velocity corrections. In so doing it is considered that the guidance equations are simply a tool, and the results are generally applicable to a "typical" guidance system for the midcourse phase of rendezvous.

Although "target" and "payload" are general terms in frequent use, it is felt that some ambiguity may result in subsequent sections. Therefore, in the remainder of this paper the target will be designated as a "space station" in orbit about the earth and the payload will be referred to as a "ferry vehicle."

SYMBOLS

Any consistent set of units may be used. In this report, it is assumed that

$$g_e = 32.17 \text{ feet per second per second}$$

$$r_e = 3{,}960 \text{ statute miles}$$

1 statute mile = 5,280 feet

1 foot = 0.3048 meter

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C_s	angular momentum per unit mass (Kepler's constant)
g_e	earth gravitational constant
h	height above surface of earth
m	mass of ferry vehicle
R	radial distance from space station to ferry vehicle
r	radial distance from center of earth
T	thrust
t	time, measured from booster burnout
V	velocity
X,Y,Z	axes system with center fixed in space station; the Z-axis is perpendicular to plane of station's orbit and X- and Y-axes lie in plane of orbit with Y-axis always pointing away from center of earth
x,y,z	orthogonal components of rectangular coordinate system
Θ	angle measured about earth from some reference position (When station is in a circular orbit, $\Theta_s = 0$ when $t = 0$; when station is in an elliptic orbit, $\Theta_s = 0$ at perigee.)
θ	angle formed by X-axis and projection of R on orbital plane of space station
λ	a determinant defined by equation (18)
σ	root-mean-square error (standard deviation)
τ	elapsed time until rendezvous
τ_1	time until $x = y = 0$ or until $x = y = z = 0$
τ_2	time until $z = 0$
ψ	angle formed by R and orbital plane of space station

ω computed angular velocity of space station (When station is in a circular orbit, $\omega = \dot{\Theta}_s$ = Constant.)

Subscripts:

a apogee

c computed value

e earth

f ferry vehicle

L launch

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o particular value of a variable at instant at which indicated computation is made and, thus, designates initial condition at each instant of time

p perigee

$\dot{R}, R\dot{\psi}, R\dot{\theta} \cos \psi$ orthogonal components of relative velocity measured by spherical coordinates centered in space station

s space station

v resultant velocity

Primed symbols denote quantities which are in error due to radar inaccuracies.

Dots over quantities denote derivatives with respect to time.

COORDINATES AND EQUATIONS OF MOTION

Assumptions

In the equations of motion used in this study, the following assumptions were made. The earth has an inverse-square gravitational field (spherically symmetric). All motions take place at an altitude where the effects of the air resistance can be completely neglected. Instantaneous changes in the velocity can be made through the use of small pulses in thrust and the total fuel expenditure is sufficiently small to make the changes in vehicle weight negligible.

Coordinates

The motion of the orbiting space station is defined in terms of polar coordinates Θ_s and r_s with the origin fixed at the center of the earth. (See fig. 1.) Since the earth's gravitational field is assumed to be spherically symmetric, the motion of the station is planar and is not dependent upon the orientation with respect to the earth of the orbital plane.

The position of the ferry is defined with respect to a rectangular coordinate system fixed in the space station. This coordinate system, shown in figure 1, is constrained to move with the station at all times. The X- and Y-axes lie in the plane of the station's orbit while the Z-axis is perpendicular to this plane. The X- and Y-axes rotate about the Z-axis so that the Y-axis always points away from the center of the earth. To accomplish this rotation the angular velocity of the X- and Y-axes about the Z-axis is always identical to the instantaneous angular velocity of the space station in its orbit about the earth. This angular velocity Θ_s varies with time when the station is in an elliptic orbit and is constant (and denoted by the symbol ω) when the station is in a circular orbit.

For reasons to be explained later the position of the ferry with respect to the rotating coordinate system of the station is also defined in terms of spherical coordinates. These coordinates are denoted by R, θ, and ψ and their relationship to the x,y,z coordinate system is also shown in figure 1.

Equations of Motion

In this section, the equations used to define the exact motion of the station and ferry are given. The first equations give the position and velocity of the station with respect to the center of the earth and the second set of equations define the relative position and velocity of the ferry with respect to the instantaneous position of the station.

<u>Motions of station</u>.- The Keplerian equations of motion were used to compute the position and velocity of the space station. These equations are, in their usual form,

$$\left.\begin{aligned} &\ddot{r}_s - r_s\dot{\Theta}_s^{\,2} + \frac{g_e r_e^{\,2}}{r_s^{\,2}} = 0 \\ &r_s^{\,2}\dot{\Theta}_s = C_s \end{aligned}\right\} \tag{1}$$

where C_s is the constant angular momentum per unit mass of the station. It was found convenient to specify the conic section in terms of its characteristic constants and the initial (t = 0) position of the station. For an ellipse, the angle θ_s is defined to be zero at perigee. In addition, the apogee radius $r_{s,a}$, the perigee radius $r_{s,p}$, and the angular position of the station at time zero $\theta_{s,L}$ are defined. From these four conditions, the following relationships can be obtained:

The semimajor axis

$$A = \frac{1}{2}\left(r_{s,p} + r_{s,a}\right) \tag{2}$$

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the eccentricity

$$\epsilon = \frac{r_{s,a} - r_{s,p}}{2A} \tag{3}$$

and the angular momentum per unit mass

$$C_s = \left[g_e r_e^2 A\left(1 - \epsilon^2\right)\right]^{1/2} \tag{4}$$

With equations (1) to (4) all necessary information on the station trajectory was obtained.

Motions of ferry.- The equations used to define the relative motions of the ferry are derived in reference 4 for the general case of a station in an elliptic orbit. For the particular case of a station in a circular orbit, the equations are also given in reference 3. The general equations in terms of x,y,z coordinate system are

$$\left.\begin{aligned}
&\ddot{x} - (y + r_s)\ddot{\theta}_s - 2(\dot{y} + \dot{r}_s)\dot{\theta}_s - x\dot{\theta}_s^{\,2} + g_e r_e^2 \frac{x}{r_f^3} = 0 \\
&\ddot{y} + x\ddot{\theta}_s + 2\dot{x}\dot{\theta}_s + \ddot{r}_s - (y + r_s)\dot{\theta}_s^{\,2} + g_e r_e^2 \frac{y + r_s}{r_f^3} = 0 \\
&\ddot{z} + g_e r_e^2 \frac{z}{r_f^3} = 0
\end{aligned}\right\} \tag{5}$$

where

$$r_f = \left[x^2 + (y + r_s)^2 + z^2\right]^{1/2} \tag{6}$$

The thrust term T/m does not appear in these equations since only impulsive thrust was assumed and its effect is inserted as an instantaneous change in velocity.

For the purpose of simulating the radar or other positional-rate measuring devices, it was found advantageous to transform equations (5) to spherical coordinates. To effect this change the following transformations were used:

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$$\left.\begin{aligned}
x &= R\cos\psi\cos\theta \\
y &= -R\cos\psi\sin\theta \\
z &= R\sin\psi
\end{aligned}\right\} \tag{7}$$

These transformations applied to equations (5) yield

$$\left.\begin{aligned}
&\ddot{R} - R\dot{\psi}^2 - R\left(\dot{\theta} - \dot{\theta}_s\right)^2\cos^2\psi + \left[r_s\left(\dot{\theta}_s^{\,2} - \frac{g_e r_e^{\,2}}{r_f^{\,3}}\right) - \ddot{r}_s\right]\cos\psi\sin\theta \\
&\qquad - \left(r_s\ddot{\theta}_s + 2\dot{r}_s\dot{\theta}_s\right)\cos\psi\cos\theta + \frac{g_e r_e^{\,2} R}{r_f^{\,3}} = 0 \\
&\left[R\ddot{\theta} + 2\dot{R}(\dot{\theta} - \dot{\theta}_s) - R\ddot{\theta}_s\right]\cos\psi - 2R\dot{\psi}(\dot{\theta} - \dot{\theta}_s)\sin\psi \\
&\qquad + \left(r_s\ddot{\theta}_s + 2\dot{r}_s\dot{\theta}_s\right)\sin\theta + \left[r_s\left(\dot{\theta}_s^{\,2} - \frac{g_e r_e^{\,2}}{r_f^{\,3}}\right) - \ddot{r}_s\right]\cos\theta = 0 \\
&R\ddot{\psi} + 2\dot{R}\dot{\psi} + R\left(\dot{\theta} - \dot{\theta}_s\right)^2\sin\psi\cos\psi + \left(r_s\ddot{\theta}_s + 2\dot{r}_s\dot{\theta}_s\right)\sin\psi\cos\theta \\
&\qquad - \left[r_s\left(\dot{\theta}_s^{\,2} - \frac{g_e r_e^{\,2}}{r_f^{\,3}}\right) - \ddot{r}_s\right]\sin\psi\cos\theta = 0
\end{aligned}\right\} \tag{8}$$

Likewise equation (6) becomes

$$r_f = \left(R^2 + r_s^{\,2} - 2Rr_s\cos\psi\sin\theta\right)^{1/2} \tag{9}$$

In the special case where the station is in a circular orbit, r_s and $\dot{\theta}_s = \omega$ are constants and equations (8) reduce to the form:

$$\left.\begin{aligned}
&\ddot{R} - R\dot{\psi}^2 - R(\dot{\theta} - \omega)^2\cos^2\psi + \left(\omega^2 - \frac{g_e r_e^2}{r_f^3}\right)r_s \cos\psi \sin\theta + \frac{g_e r_e^2 R}{r_f^3} = 0 \\
&\left[R\ddot{\theta} + 2\dot{R}(\dot{\theta} - \omega)\right]\cos\psi - 2R\dot{\psi}(\dot{\theta} - \omega)\sin\psi + \left(\omega^2 - \frac{g_e r_e^2}{r_f^3}\right)r_s \cos\theta = 0 \\
&R\ddot{\psi} + 2\dot{R}\dot{\psi} + R(\dot{\theta} - \omega)^2\sin\psi\cos\psi - \left(\omega^2 - \frac{g_e r_e^2}{r_f^3}\right)r_s \sin\psi\sin\theta = 0
\end{aligned}\right\} \tag{10}$$

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Approximate Equations Used for Guidance

For the purpose of guiding the ferry to the station, it is desirable to calculate on board one of the vehicles the proper relative velocity required for rendezvous, based on the measured instantaneous relative positions. Since the nonlinear differential equations used to define the exact motions are not particularly suitable for simple on-board computations, a simplified approximate form of these equations is used for guidance computations. The development of these approximate equations is given in both references 3 and 4 and hence is only briefly outlined herein.

For the case of the station in a circular orbit, the equations of motion in terms of the rectangular coordinate system (eqs. (5)) may be linearized, and, therefore, solved in closed form, by approximating the gravity difference between the two vehicles. If the gravity potential term

$$\frac{g_e r_e^2}{r_f^3} = g_e r_e^2\left[x^2 + (y + r_s)^2 + z^2\right]^{-3/2} \tag{11}$$

is expanded into a Taylor series about the origin of the rotating coordinate system and all terms higher than the first order are dropped, then the gravity difference between the two vehicles is a function of only one variable y and is given by the expression:

$$\frac{g_e r_e^2}{r_f^3} \approx \frac{g_e r_e^2}{r_s^3}\left(1 - 3\frac{y}{r_s}\right) \tag{12}$$

The physical significance of this approximation is discussed in reference 4 and it is sufficient to note here that the approximation represents a rotating parallel gravitational field rather than the spherical gravitational field of equations (5).

By expanding each of the gravitational difference terms in the equations of motion (eqs. (5)) and by making the approximation that r_s and $\dot{\Theta}_s$ are constant (exact if the station is in a circular orbit), a set of linear approximate equations for nonthrusting flight is obtained:

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$$\left.\begin{aligned} \ddot{x} - 2\omega\dot{y} &= 0 \\ \ddot{y} + 2\omega\dot{x} - 3\omega^2 y &= 0 \\ \ddot{z} + \omega^2 z &= 0 \end{aligned}\right\} \tag{13}$$

These equations have the solutions:

$$\left.\begin{aligned} x &= 2\left(2\frac{\dot{x}_o}{\omega} - 3y_o\right)\sin \omega t - 2\frac{\dot{y}_o}{\omega}\cos \omega t + \left(6y_o - 3\frac{\dot{x}_o}{\omega}\right)\omega t + 2\frac{\dot{y}_o}{\omega} + x_o \\ y &= \left(2\frac{\dot{x}_o}{\omega} - 3y_o\right)\cos \omega t + \frac{\dot{y}_o}{\omega}\sin \omega t + 4y_o - 2\frac{\dot{x}_o}{\omega} \\ z &= z_o \cos \omega t + \frac{\dot{z}_o}{\omega}\sin \omega t \end{aligned}\right\} \tag{14}$$

For any instantaneous relative position of the ferry x_o, y_o, z_o, it may be shown that the instantaneous relative velocity components required to achieve interception after a time τ_1 are given by the following expressions:

$$\frac{\dot{x}_c}{\omega} = \frac{x_o\left(\sin \omega\tau_1\right) + y_o\left[6\omega\tau_1 \sin \omega\tau_1 - 14(1 - \cos \omega\tau_1)\right]}{\lambda} \tag{15}$$

$$\frac{\dot{y}_c}{\omega} = \frac{2x_o\left(1 - \cos \omega\tau_1\right) + y_o\left(4 \sin \omega\tau_1 - 3\omega\tau_1 \cos \omega\tau_1\right)}{\lambda} \tag{16}$$

$$\frac{\dot{z}_c}{\omega} = \frac{-z_o}{\tan \omega\tau_1} \tag{17}$$

where

$$\lambda = 3\omega\tau_1 \sin \omega\tau_1 - 8(1 - \cos \omega\tau_1) \tag{18}$$

In terms of the spherical coordinates R, θ, and ψ, equations (15), (16), and (17) become, respectively,

$$\frac{\dot{R}_c}{\omega R_o} = \left(K_2 + K_3 \sin^2\theta_o - K_4 \sin \theta_o \cos \theta_o\right)\cos^2\psi_o + K_1 \sin^2\psi_o \tag{19}$$

$$\frac{\dot{\psi}_c}{\omega} = \left(K_1 - K_2 - K_3 \sin^2\theta_o + K_4 \sin \theta_o \cos \theta_o\right)\sin \psi_o \cos \psi_o \tag{20}$$

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$$\frac{\dot{\theta}_c}{\omega} = K_3 \sin \theta_o \cos \theta_o + K_4 \sin^2\theta_o - K_5 \tag{21}$$

where

$$K_1 = -\cot \omega\tau_1$$

$$K_2 = \frac{1}{\lambda} \sin \omega\tau_1$$

$$K_3 = \frac{3}{\lambda}\left(\sin \omega\tau_1 - \omega\tau_1 \cos \omega\tau_1\right)$$

$$K_4 = \frac{6}{\lambda}\left(\omega\tau_1 - 2 + 2 \cos \omega\tau_1\right)$$

$$K_5 = \frac{2}{\lambda}(1 - \cos \omega\tau_1)$$

SIMULATION

General

The problem situation for the midcourse phase of the rendezvous is shown schematically in figure 2. The station is in a fixed orbit and the ferry is launched and guided onto an intersecting trajectory. Navigational data in the form of relative position and velocity between the two vehicles are obtained by radar or by some optical equipment. In order to simulate this physical situation, the equations of motion of the station (eqs. (1)) and the ferry (eqs. (8) or (10)) were solved by using a high-speed digital computer. The computer was also used to generate random errors for the simulation of the ferry-borne radar and to simulate the ferry-borne guidance system. Each of the components and the information flow lines are shown in figure 3.

To the correct relative position and velocity of the ferry are added random errors based on assumed characteristics which are discussed in the next section. The position and velocity data, modified by radar errors, are fed into the ferry guidance system where any corrections in velocity are computed. The guidance system also computes when these corrections should be made and triggers a switch at the desired time which initiates the indicated action. In the simulation the proper orientation and thrust control are assumed so that the velocity change that would have taken place is fed back into the equations of motion of the ferry vehicle. In the integration of these equations, the velocity change was made instantaneously, which implies an impulsive thrust.

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Although not included in the simulation, the attitude or orientation and the thrust-control channels are indicated in figure 3 with dashed lines. The orientation channel is placed inside the switch that activates the corrective thrust. This arrangement allows the vehicle to maintain the proper attitude for thrusting at all times. Filters can be used to allow only low-frequency motion and the vehicle will always be properly oriented at the time the corrective thrust is applied.

At the cost of additional complication, the orientation and thrust-control dynamics could be included for some specific vehicle and rocket engine. However, it was felt that the inclusion of such items would mask the effects of the guidance system.

Assumed Radar Characteristics

In this study radar contact was assumed to exist between the station and the ferry. In order to arrive at a reasonable estimate for navigational errors, the following characteristics were ascribed to the system. A pulse Doppler radar utilizing a dish antenna was to be located on the ferry vehicle.[1] This radar was to transmit with a peak power output of 250 kilowatts. The return signal was given a power boost by means of a transponder located on the station. Measurements of the

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[1]Whether the radar and guidance-computer system is located on the ferry or on the station is only of academic interest here. If it is located on the station, a radio data link would be required to transmit the guidance information to the ferry.

line-of-sight range, range rate, directional angle, and angular rate were made at a rate of 200 samples per second and then averaged over 1/2-second intervals. It was estimated that such a system would result in root-mean-square errors of

$$\sigma_R = 30 \text{ ft in range}$$

$$\sigma_{\dot{R}} = 3 \text{ ft/sec in range rate}$$

$$\sigma_\theta = \sigma_\psi = 5 \times 10^{-4} \text{ radians in angle}$$

$$\sigma_{\dot{\theta}} = \sigma_{\dot{\psi}} = 10^{-5} \text{ radians/sec in angular rate}$$

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For the simulation the distributions of these errors were assumed to be Gaussian. At the time of each correction, a Monte Carlo technique (such as that used in refs. 5 and 6) was used to pick errors for each of the position and velocity components, based on the foregoing σ values. Depending upon the sign, these errors were then added to or subtracted from the true position and velocity components given by the equations of motion. These quantities, primed symbols in figure 3, are then fed into the ferry-borne guidance system.

GUIDANCE LOGIC SYSTEM

General

Equations (15) to (21) define, to a degree of approximation, the relative velocity required to rendezvous as a function of the instantaneous relative position of the ferry and a desired time of rendezvous. How these equations were utilized to effect the rendezvous and the determination of a desired time to rendezvous is the subject of this section. To aid in its description a block diagram of the guidance logic system is shown in figure 4. Each of the component parts are described in following subsections.

Calculation of Time to Intercept

It is not unreasonable to assume that at the time of launch some nominal trajectory for the ferry will be specified so that near-rendezvous conditions will be achieved. A number of such trajectories are given in reference 4. It is therefore assumed in this study that at the time of booster burnout some approximate time to intercept is denoted as $(\tau_1)_L$. At any subsequent time t the approximate time to intercept is then computed with the following equation:

$$\tau_1(t) = (\tau_1)_L - t \tag{22}$$

As the ferry approaches the station, however, there seem to be two inherent disadvantages in making this approximate computation. One objection is that this calculation does not provide a sufficiently accurate value of τ_1 upon which to base the velocity computations. The other objection is that it is possible for τ_1 to become negative before rendezvous is achieved, which would result in a breakdown of the guidance equations. In this study it was, therefore, decided to compute τ_1 over the final part of the trajectory from the instantaneous relative position and velocity of the ferry and station. The equation which was used for this computation was

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$$\tau_1(t)\Big]_{\text{optimum}} = \frac{R(t)}{-\dot{R}(t)} \tag{23}$$

The derivation of the expression for the desired time to intercept is given in appendix A. However, as a result of the approximations which were made in deriving the equation, equation (23) is inaccurate for large values of θ and cannot be applied over the entire range of interest. In this study τ_1 was computed by equation (22) until the value of $\omega\tau_1$ became equal to 30° at which time equation (23) was used until rendezvous occurred. Physically the condition $\omega\tau_1 = 30^\circ$ means that the angle, measured about the center of the earth, between the instantaneous position of the station and the expected point of rendezvous was 30°.

Calculation of ω

When the space station is in an elliptic orbit, the angular velocity $\dot{\theta}_s$ of the station is not a constant and, therefore, it is necessary to calculate a nonconstant value for ω to be used in the approximate guidance equations. The assumption was made that ω varies approximately as a sinusoidal function of t. If, when $\theta_s = 0$, $\omega = \omega_p$ and, when $\theta_s = \pi$, $\omega = \omega_a$, then

$$\omega = \frac{\omega_p + \omega_a}{2} + \frac{\omega_p - \omega_a}{2} \cos\left(\theta_{s,L} + \frac{\omega_p + \omega_a}{2} t\right) \tag{24}$$

When the station is in a circular orbit, ω reduces to the correct constant.

Computation of Velocity Corrections

Once the desired time to intercept has been specified, equations (15) to (17) or (19) to (21) may be used to calculate the relative velocity components required to achieve the interception at that time. However, several factors must be considered when the equations are implemented into a guidance system (that is, guidance logic). One factor is the L
method of measuring position and velocity (navigation system). If the 1
navigation system were perfect, a comparison of the computed and meas- 4
ured velocity components would then yield the velocity corrections. 7
However, in practice, the radar used for navigation will have some 6
inherent errors: some random and some due to alinement or bias errors. It is presumed that the root mean square of these errors will be known and the computed velocity corrections should take into account the fact that the measured positions and velocities may be in error to within some specified limits.

A second factor involves the approximate equations used to compute the velocity corrections. These equations may be sufficiently accurate only after the two vehicles achieve a certain relative position. Thus, if, the computed velocity corrections have a sphere of validity, they should be used only in that sphere.

A third factor involves the independence of the in-plane and out-of-plane velocity corrections. Since it may not be desirable or necessary to make both in-plane and out-of-plane corrections simultaneously, a provision for separate computations should be provided in the system.

Taking into account these three factors, a guidance system for computing the velocity corrections was devised and this system is shown schematically in figure 4. As indicated in the figure, the radar-measured position of the ferry is used to compute the desired velocity of the ferry for rendezvous at time τ_1. When the ferry is on a trajectory that will intersect the plane of the station's orbit, the time of this intersection τ_2 is determined and is used to compute changes in the out-of-plane velocity. These computed velocities are compared with the radar-measured velocities and the differences are then determined. The differences are multiplied by a reduction factor, to account for the σ error of the radar, and these corrected values are then sent as command changes to the guidance system. The equations used for each of these steps and the times at which these velocity corrections are made are detailed in the following sections.

Coordinate conversion. - The computation of the desired velocity corrections in terms of the rectangular coordinates x,y,z or in terms of the spherical coordinates R,θ,ψ is one of choice. The desired velocities in terms of the former are given by equations (15), (16), and (17). For this study it was considered to be more desirable to describe the motions and velocity changes in terms of the rectangular coordinates. Therefore, the radar-measured position of the ferry was first converted from spherical to rectangular coordinates by using the relations of equations (7). After computing the desired velocity components, these components were converted back to spherical form by using the inverse of equations (7). These inverse relations are

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$$\left.\begin{aligned} R &= \sqrt{x^2 + y^2 + z^2} \\ \theta &= \tan^{-1} \frac{-y}{x} \\ \psi &= \tan^{-1} \frac{z}{\sqrt{x^2 + y^2}} \end{aligned}\right\} \tag{25}$$

These conversions are indicated in figure 4 although it should be emphasized that they could be eliminated by computing the desired velocities directly in spherical coordinates.

In-plane velocity components. - It is desired to bring the two in-plane positional coordinates to zero simultaneously after some time interval τ_1. The velocities required to do this are computed by using equations (15) and (16). These desired in-plane velocities were computed at a finite number of points along the trajectory. The choice of times or positions at which corrections should be made is somewhat controversial, since the total amount of velocity change required will be dependent upon how many velocity changes are made, upon the accuracy of the navigational equipment, and upon the line-of-sight range at the time at which the corrections are made. (See refs. 6, 7, and 8.) It was decided to make a correction as soon as radar acquisition could be obtained since the velocity required to correct a given miss distance increases as an inverse function of the range. It was also considered desirable to correct more frequently as the range decreased since the accuracy of the guidance equations and (possibly) that of the radar also increase as inverse functions of the range. Therefore, in this study, the initial in-plane velocity correction was made at a distance assumed to be that of radar acquisition and subsequent corrections were made each time the line-of-sight range was reduced by a factor of 2 $\left(R_o,\ R_o/2,\ R_o/4,\ R_o/8,\right.$ and so forth$\left.\right)$ until the range became less than 5 miles. This description is not meant to imply, however, that the last correction occurred at

5 miles exactly. It merely states that the last correction occurred at a range greater than 5 miles and less than 10 miles.

Out-of-plane velocity components. - When the ferry is not launched into the plane of the station's orbit, the motion of the ferry normal to the plane (herein referred to as out-of-plane motion) will be essentially sinusoidal. Computation of the desired out-of-plane velocity is based upon the premise that the out-of-plane position should be brought to zero prior to or simultaneous with the in-plane position. The alternative, that of injecting the ferry into a non-coplanar orbit and later making the orbits coplanar, appears to have two major disadvantages: L
(1) The guidance to match closely the in-plane velocity and position 1
will be more difficult if the two vehicles are not in close proximity 4
and (2) since the out-of-plane velocity correction will increase as the 7
inertial velocity of the vehicle is increased, injection into orbit prior 6
to making the orbits coplanar will usually be more expensive in fuel.

The out-of-plane motion of the ferry is approximated by (see eqs. (14))

$$z = z_o \cos \omega t + \frac{\dot{z}_o}{\omega} \sin \omega t$$

With the instantaneous relative position and velocity of the ferry and a value of ω specified, the time at which z will become zero τ_2 may be computed:

$$\tau_2 = \frac{1}{\omega} \tan^{-1} \frac{z_o \omega}{-\dot{z}_o} \tag{26}$$

There are then three possibilities: that τ_2 will be greater than, equal to, or less than τ_1 (the time when the in-plane position will be zero). If at the initial or any subsequent correction point $\tau_2 \leqq \tau_1$, then no change in the out-of-plane velocity is made. If, however, $\tau_2 > \tau_1$, then a new out-of-plane velocity is computed so that x, y, and z will become zero simultaneously (that is, so that $\tau_1 = \tau_2$). Thus, the guidance equations for the out-of-plane velocity are as follows:

$$\text{If} \quad \begin{Bmatrix} \tau_1 \geqq \tau_2 \\ z_o \neq 0 \end{Bmatrix} \qquad \text{keep} \qquad \dot{z}_c = \dot{z}_o$$

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$$\text{If} \quad \left\{ \begin{matrix} \tau_2 > \tau_1 \\ z_o \neq 0 \end{matrix} \right\} \qquad \text{make} \qquad \dot{z}_c = \frac{-z_o \omega}{\tan \omega \tau_1}$$

$$\text{If} \quad z_o = 0 \qquad \text{make} \qquad \dot{z}_c = 0$$

These three instructions are used throughout the entire rendezvous where the subscript o with the variable denotes the instantaneous value of the variable at the time the correction is made. If, for instance, at the time of radar acquisition $\tau_2 < \tau_1$, no correction is computed until the ferry passes through the plane of the station's orbit $(z = 0)$. At the time the radar indicates that $z = 0$, the guidance system attempts to bring $\dot{z}$ to zero. However, due to radar inaccuracies in position and velocity, some small error may be incurred. This error will show up the next time the in-plane velocity is corrected. If the newly computed time τ_2 is then greater than the computed time τ_1, a second value of $\dot{z}_c$ is computed from equation (17) which will make $\tau_2 = \tau_1$. If, however, $\tau_2 < \tau_1$, no $\dot{z}$ correction is computed until z again passes through zero, at which time the process is repeated. Thus, out-of-plane corrections are called for any time $z = 0$ and at any time the in-plane velocity is corrected, if $\tau_2 > \tau_1$.

<u>Reduction in computed velocity changes to allow for radar inaccuracy</u>.- As shown in figure 4, the desired values of in-plane velocity and out-of-plane velocity are converted to spherical coordinates and the difference between the measured and computed values is determined. If the radar (excluding inaccuracies in the guidance equations) were perfectly accurate, these values would represent the correct velocity-component changes. In practice, however, the radar will be subject to errors which will not be removed by data smoothing and these errors may lead to excessive corrections in velocity.

To compensate for such errors, so-called "dead bands" have been considered previously by others (for example, ref. 8). If the expected standard deviation of the navigational error in determining velocity σ_V is known and the computed error in velocity is less than σ_V, then a $1\sigma_V$ dead band would imply that no correction should be made. If, however, the measured velocity error were greater than the dead band, then (a) the total computed velocity correction could be made or (b) a velocity correction derived from the difference between the magnitude of the computed velocity change and absolute magnitude of σ_V could be made. The merits and disadvantages of such dead bands have not been completely established, but the principal is that of undercorrecting the computed velocity change by an amount based on the mean uncertainty of navigational-measurement devices. In the remainder of this paper, dead bands are referred to as having option (a) or (b).

In order to investigate such dead bands, it was assumed that the predominant errors in the radar data, after smoothing, will be due to "play," hysteresis, or changes in bias and that these errors will be essentially statistical. It was further assumed that the root mean square (or standard deviation σ) of these errors was known. If σ for each component of velocity were known and subtracted[2] from the corresponding component of the corrective velocity, then the change in velocity (with the allowance for radar errors) would be

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$$\Delta V = \left[\left(\left|\Delta\dot{R}\right| - \sigma_{\dot{R}}\right)^2 + \left(\left|\Delta R\dot{\psi}\right| - \sigma_{R\dot{\psi}}\right)^2 + \left(\left|\Delta R\dot{\theta}\cos\psi\right| - \sigma_{R\dot{\theta}\cos\psi}\right)^2\right]^{1/2}$$

However, for simplicity, it is assumed herein that the standard deviation of each velocity component is in direct proportion to the magnitude of the component change over the total velocity change ΔV_c and that the constant of proportionality is some total expected deviation σ_V. Thus,

$$\sigma_{\dot{R}} = \left|\frac{\Delta(\dot{R})}{\Delta V_c}\right|\sigma_V$$

$$\sigma_{R\dot{\psi}} = \left|\frac{\Delta(R\dot{\psi})}{\Delta V_c}\right|\sigma_V$$

$$\sigma_{R\dot{\theta}\cos\psi} = \left|\frac{\Delta(R\dot{\theta}\cos\psi)}{\Delta V_c}\right|\sigma_V$$

where

$$\Delta V_c = \left[(\Delta\dot{R})^2 + (\Delta R\dot{\psi})^2 + (\Delta R\dot{\theta}\cos\psi)^2\right]^{1/2}$$

With this assumption each computed component of corrective velocity is reduced by the factor

$$\left|1 - \frac{\sigma_V}{|\Delta V_c|}\right| \qquad \left(\left|\Delta V_c\right| > \sigma_V\right) \tag{27}$$

[2]The stipulation is that, if the computed change in velocity is less than or equal to the specified σ, a zero correction is made.

and the total computed change in velocity is reduced by the same factor

$$\Delta V = \left|1 - \frac{\sigma_V}{|\Delta V_c|}\right| \Delta V_c \qquad \left(\left|\Delta V_c\right| > \sigma_V\right) \qquad (28)$$

As indicated in the diagram of figure 4, the difference between the total measured velocity and computed velocity is obtained, the factor of equation (27) is determined, and each velocity component is multiplied by this factor $\left(\text{if } \sigma_V \geqq \left|\Delta V_c\right|\right.$, a zero correction is specified$\left.\right)$. In practice these corrections would then serve as inputs to the guidance system. In the simulation the velocity corrections are fed back to the equations of motion as instantaneous velocity changes whenever such changes are indicated.

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When $\left|\Delta V_c\right| > \sigma_V$, the foregoing procedure is essentially option (b). To simulate option (a), the factor of equation (27) was made unity so that the following relations were used in the guidance logic system:

$$\Delta V = \Delta V_c \qquad \left(\left|\Delta V_c\right| > \sigma_V\right)$$

$$\Delta V = 0 \qquad \left(\left|\Delta V_c\right| \leqq \sigma_V\right)$$

Thus, whenever the measured error was greater than or equal to the predicted mean navigation error, the total computed velocity correction was made. If, however, the mean error was greater than or equal to the computed velocity change, no correction was made.

RESULTS

General

In order to investigate the characteristics and effectiveness of the ferry guidance system employed in this study, rendezvous with a space station in a circular orbit and with a station in an elliptic orbit were attempted. At the start of each attempted rendezvous, the ferry was assumed to be in coasting flight (nonthrusting) after separation from the last stage of the boost vehicle. Nominal initial conditions were obtained from the studies of reference 4. Errors in the initial position or velocity or both were then added to simulate ferry launch errors. The ferry was then guided toward the station by the system described in the preceding section. The results and some modifications brought about by these results are given in the following sections.

Rendezvous With a Station in a Near-Earth Circular Orbit

Typical trajectories.- With the space station in a circular orbit 300 statute miles above the earth, rendezvous was attempted over a wide variety of nominal trajectories. Four of these trajectories are shown in figure 5.

Presented in figures 5(a) to 5(d) are the variations of the range rate $\dot{R}$ and the three components of the velocity in rectangular coordinates $\dot{x}$, $\dot{y}$, and $\dot{z}$ as a function of the range. Ticks have been used on each of these velocity time histories to emphasize the small discontinuities due to the impulsive velocity corrections. The magnitude and direction of these corrections are given in a detailed breakdown in table I for each time history.

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Also given in these figures are the variations of the spherical coordinates θ and ψ with R, the variation of the height for the ferry h_f and station h_s with distance around the earth Θ, and the variation of the rectangular coordinates y and z with x.

Figure 5(a) shows a trajectory obtained when the ferry was launched at approximately the correct position and velocity. The initial values for x, y, $\dot{x}$, and $\dot{y}$ were taken from reference 4 for a case in which the ferry trajectory was coplanar with the station orbit. In addition, initial values of 50 miles and -230 ft/sec were used for z and $\dot{z}$, respectively. Any errors in the initial values of x, y, $\dot{x}$, and $\dot{y}$ which result from extending the initial conditions for an in-plane trajectory to an out-of-plane trajectory are shown in reference 4 to be relatively small and therefore were not taken into account. The initial range from ferry to station was 457 miles. In this particular case, the rendezvous occurred 146° downrange from the position of the station at the time of ferry launch (booster burnout) and the midcourse phase took 38 minutes.

The trajectory of figure 5(b) was obtained by adding relatively large initial errors in position and velocity to the initial conditions used for figure 5(a). The ferry was launched 8 miles too low and 8 miles behind the desired trajectory. In addition, the ferry was launched with both the x and y velocity components 200 ft/sec too low. This magnitude of error does not necessarily represent the kind of accuracy that has actually been achieved in launches to date. It is intended to represent an extreme case. Even with such initial errors, rendezvous was still accomplished.

Figures 5(c) and 5(d) show two other typical trajectories. In figure 5(c) the ferry approached the station from above and in figure 5(d) the approach is made from below the station. In both of these trajectories it was necessary to make a z velocity correction immediately since τ_2 was greater than τ_1. However, in figure 5(c), the ferry did not pass through the station's orbital plane until after the last correction was made (R = 5.4 miles) and the correction required to make the two vehicles coplanar does not appear in the time history. In figure 5(d), this planar intersection occurred when the ferry was 10.6 miles from the station, and the appropriate correction was made at that point.

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The velocity corrections made by the simulator in performing the rendezvous trajectories of figure 5 are listed in table I. The table lists the times in seconds at which each velocity correction was made after the initiation of the midcourse phase, the angle about the center of the earth through which the station has traveled at the time of the correction, and the range at which the velocity corrections were made. Each velocity correction is shown in terms of components in the station-centered rectangular coordinate system and in terms of the (station-centered) spherical coordinate system. The vector sum of these components, representing the total velocity change required at each correction point, is shown in the last column. The sum of the terms in this last column represents the total velocity correction required for the mission. It should be noted, however, that a 1σ dead band was used in the guidance system for all the trajectories of figure 5.

<u>Accuracy of approximate guidance equations</u>.- In the time histories of figure 5, it may be noted that no in-plane velocity correction was made until the range between the station and ferry was 75 to 150 miles. Although this line-of-sight range exceeds the upper limit of any currently known airborne radar, the limitation was not imposed solely for this reason. The principal reason for the limitation is found in the approximate guidance equations. As pointed out in reference 4, linearization of the exact equations of motion has the practical effect of changing the representative spherical gravitational field of the earth to an approximate gravitational field. Thus, the motions computed by two integrations of these approximate equations becomes less accurate with increasing values of relative range and Θ_s (or ωt). Furthermore, the guidance equations, which are the inverse of the motion equations, become less accurate with relative range and $-\Theta_s$ (or $+\omega\tau$, the distance to be traveled around the earth until rendezvous occurs).

This inaccuracy can be seen in figure 6(a) which shows $\dot{x}$, $\dot{y}$, $R\dot{\theta}$, and $\dot{R}$ as computed from the guidance equations and the correct velocity components $\dot{x}$, $\dot{y}$, $R\dot{\theta}$, and $\dot{R}$ required to rendezvous at each point on a typical trajectory (obtained from numerical solutions of the exact equations as described in ref. 4). The ferry trajectory lay entirely within the station's orbital plane. The figure shows that, even if the ferry is on a collision trajectory with the correct velocity, the guidance equations will compute that an error exists and this error increases with both R and $\omega\tau$.

In figure 6(b), the difference between the correct and computed values of $\dot{x}$, $\dot{y}$, $\dot{R}$, and $R\dot{\theta}$ is plotted as a function of $\omega\tau$ (or $-\dot{\theta}_s$). It may be seen that the errors are small when $\omega\tau < 90^{\circ}$ but become quite large as $\omega\tau$ increases beyond 90°. This effect was investigated with a number of trajectories where the relative ranges at the condition $\omega\tau = 90^{\circ}$ were different. It was found that this effect was dependent primarily on $\omega\tau$ rather than on R but an increase in R produced an increase in the magnitude of the error.

When the approximate equations were used for ferry guidance at values of $\omega\tau > 90^{\circ}$, it was found that large errors in the velocity correction were obtained. A typical result is shown in figure 6(c). Plotted in the figure are the velocity corrections made during an in-plane rendezvous where the ferry was initially on exactly the right trajectory and the correct time to rendezvous was initially specified. At the start of the trajectory, the range was 457 miles and the value of $\omega\tau$ was 146°. If no corrections had been made, the trajectory indicated with a solid line in figure 6(a) would have been followed. It may be seen that initially a very large velocity change was made followed at the second correction point ($R_0 \div 2$) by an even larger correction of the opposite sign. Subsequent corrections were smaller in magnitude. The actual trajectory (not shown) resulted in a miss of 350 feet with the last correction made 17 miles from the station. Even though the guidance system still produced a rendezvous course, the corrections were completely unnecessary.

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In view of these results, a limit was imposed on the guidance system which prevented a velocity correction from being made until the estimated value of $\omega\tau_1$ was 90° or less.

Error studies.- In order to evaluate the sensitivity of the guidance system to radar accuracy, a study was made of the effect of varying the magnitude of the assumed radar errors. This study was made by changing the set of values assumed for the standard deviations of the radar errors given in the section entitled "Assumed Radar Characteristics." For each set of σ values, simulated rendezvous trajectories were computed from the same initial conditions a number of times. The process was performed with no radar error and at multiples of 1, 2, and 3 times the standard deviation. The sum total of the velocity corrections and the miss distance for each trajectory were then plotted against multiples of the standard deviation of the radar error. The results are shown in figure 7 as a rough envelope of the velocities required with a faired line through the average values. The envelope of points shows a continually greater spread of required velocity corrections for less accurate radar with a linear increase in the average value. However, the miss distance showed virtually no correlation or trend with radar error. In all cases corrections were called for at the same values of range, with the last velocity correction being made at 8.5 miles from the station. In this study the two vehicles were coplanar, no dead band was used, and the velocity changes were required only to correct inaccuracies in the initial (launch) conditions and any errors arising from the use of the approximate guidance equations.

A study was also made of varying the value of $\tau_{1,L}$ initially specified for the guidance equations. This variation in $\tau_{1,L}$ is equivalent to varying the position of the station at the time of booster burnout. For instance, in a period of 2 seconds a space station in a 300-mile orbit would travel approximately 10 miles. The results of the study shown in figure 8 show virtually no trend at all for an error in the time of launch of up to 6 seconds. This result, at first, might seem surprising, but it must be realized that for the last part of the trajectory the time to intercept is computed by the guidance logic system. Along the first part of the flight path, a small error in the time to intercept does not produce a significant change in the velocity corrections. Along later parts of the flight path ($\omega\tau < 30^{\circ}$) the time to intercept, as computed by the guidance logic system, is dependent only on the instantaneous conditions and only indirectly dependent on the initially specified time. Any differences in the total velocity required to intercept are apparently so small that they are masked by the radar errors.

A study was made of the effect of varying the velocity of the ferry vehicle at booster burnout. Trajectories were simulated where the ferry was launched with an initial velocity error of ±10, ±20, and ±30 ft/sec. In each case, this initial error was added to the velocity components in rough proportion to the magnitude of the components and thus represents a general velocity error. The results shown in figure 9(a) show a definite dependence of the velocity change required upon the velocity of the ferry at booster burnout.

It may be noted that the minimum of the curve of figure 9(a) occurs for a velocity slightly higher than the nominal value (obtained from ref. 4). Thus, it might appear that a slightly higher launch velocity is desirable. That this is not so, however, can be seen by inspection of figure 9(b). Figure 9(b) gives the relative closing velocity of the ferry and station at the time of rendezvous as a function of the error in launch velocity for the same trajectories of figure 9(a). It can be seen from this figure that a higher launch velocity also results in a higher closing velocity. Since this higher closing velocity must be removed during the terminal phase when the ferry is injected into the station's orbit, there seems to be no particular advantage in a higher velocity at launch. In fact, when the launch velocity, the midcourse velocity corrections, and the velocity required to inject the ferry into the station's orbit (the relative closing velocity) are added, it is seen that the minimum total velocity is obtained with the nominal launch conditions of reference 4.

Dead bands.- With a value of σ_V of 3 ft/sec, a dead band with option (b) was employed in all the trajectories of the preceding sections. Thus, no velocity correction was made if the computed total velocity change ΔV_c was less than or equal to 3 ft/sec. Whenever $\Delta V_c > 3$ ft/sec, each component of the computed velocity was multiplied by the factor given by equation (27). Since the assumed sources of the errors in the radar (hysteresis, and so forth) were not functions of range or time, the value of σ_V was kept constant throughout the entire trajectory.

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The effectiveness of a dead band with option (b) was evaluated by repeatedly launching the ferry with the same initial conditions but with different values of σ_V in the guidance logic circuit. Values of σ_V of 0, 3, 6, 9, and 12 ft/sec were used. With each value of σ_V, 10 rendezvous trajectories were made. The initial conditions were the same as those used for the trajectory of figure 5(a). The average target miss distance and the average of the total velocity changes were computed for each set of 10 trajectories. The results of using a dead band with option (b) are shown in figure 10. The process was repeated using a dead band with option (a) and these results are also shown in figure 10. When option (a) was used, the factor given by equation (27) was replaced by unity.

The results show that the miss distance increases almost linearly with the magnitude of the dead band, whereas the total corrective velocity requirements approach a limiting value. It should be noted, however, that a dead band of 3 ft/sec increased the miss distance by a factor of 10 for option (b) and a factor of 5 (using the faired value) for option (a) over that obtained with a zero dead band. With a zero dead band the average miss distance was only 0.035 statute mile (185 ft) even though the last correction in every case was made when the ferry was 8.4 miles from the station. (See table I(a) which gives the relative position for each velocity correction.)

Rendezvous With a Station in a Near-Earth Elliptic Orbit

Typical trajectories.- Rendezvous of the ferry with a station in an elliptic orbit (perigee 100 and apogee 500 statute miles above the earth) was investigated for a variety of nominal trajectories. Based on the launch conditions given in reference 4, rendezvous was attempted with the station at three positions in its orbit: $\Theta_s = 90^\circ$, 180° (apogee), and 270°. Three typical trajectories, one for each of the three station positions at the time of rendezvous, are shown in figure 11. In addition to the variables used in figure 5 to describe the relative motion of the vehicles when the station was in a circular orbit, one additional variable is shown: $\dot{\Theta}_s - \omega$. Since ω is an approximate computation for the angular velocity (eq. (24)) of the station and is used in the guidance equations, a measure of the difference between $\dot{\Theta}_s$ and ω is plotted for each trajectory of figure 11 as a function of the relative range during the rendezvous. Specific data on the magnitude and direction of the velocity corrections applied during each trajectory are given in table II.

<u>Effect of computed value of ω</u>.- Before arriving at the expression of equation (24) for computing ω as a function of Θ_s, a number of constants for ω were investigated. Although $\dot{\Theta}_s$ varies during the rendezvous for a station in an elliptic orbit, rendezvous was attempted by using ω equal to (a) the initial value of $\dot{\Theta}_s$, (b) the expected final value of $\dot{\Theta}_s$, and (c) the expected average value of $\dot{\Theta}_s$ during the rendezvous. Each of these approximations worked for some initial launch conditions and not for others. In cases where the ferry failed to rendezvous, several velocity corrections were made until the vehicles were some 20 to 30 miles apart. Before the range could be halved again, the range rate became positive and the nearest point of approach was some 15 to 20 miles. If corrections had been made more often during this portion of the trajectory, smaller miss distances would have been achieved. However, the results point up the fact that this type of an approximation for ω is not very accurate for ellipses with an eccentricity of 0.047. For even smaller values of eccentricity and more frequent velocity corrections, these approximations might be sufficient.

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In all cases investigated with the station in a 100-to-500-mile elliptic orbit, equation (24) gave an approximate value for the angular velocity of the station, which was sufficiently accurate to guide the ferry to a rendezvous with the same accuracy as was obtained when the station was in a 300-mile circular orbit. It is anticipated that more eccentric orbits would probably require better approximations but the limits have not been investigated.

<u>Guidance to within 200 feet of station</u>.- Several trajectories were simulated with guidance corrections applied until the vehicle was between 100 and 200 feet of the station. One typical trajectory is shown in figure 12 and the details of the corrections are given in table III. A $1\sigma_V$ dead band was used so that at several correction points the computed corrections were less than σ_V and, therefore, no correction was made. During the last 5 miles, however, several corrections totaling 99 ft/sec were made. In this trajectory, rendezvous occurred at about 280° where the error in ω and $\dot{\Theta}_s$ was relatively large. The station is accelerating as it approaches perigee and the ferry must constantly increase its velocity to achieve rendezvous. It may be noted that most

of the corrections are in the $-\dot{x}$ direction due to the acceleration of the station. Although R is less then one-tenth of a mile when the last two corrections are made, ψ is close to 80° so that the directions of $R\dot{\psi}$ and $-\dot{x}$ are nearly parallel. Thus, some of these latter corrections have the effect of injecting the ferry into the same orbit as the station. The final miss distance was 16.9 feet and the injection velocity that would have been required at this position was 183 ft/sec.

IMPLICATIONS OF RESULTS

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The foregoing results have illustrated the effectiveness of a guidance technique in which the inherent errors converge to some small value or zero as the target is approached. The study is not complete in that all possible errors and facets of the guidance system were not investigated, but it does illustrate some of the more important gross effects. In addition to the finite results already given, there are some observations and intuitive results which merit attention here.

The results are not especially favorable toward dead bands. However, it should be pointed out that, if the velocity had been corrected every time the measured error exceeded the dead band, the results might have been more favorable. In the method studied, corrections were made only at finite intervals along the trajectory and, thus, errors were allowed to build up from one correction to the next.

It is interesting to note that, when dead bands were used, most of the velocity corrections were applied in the same general direction; therefore, a ferry with a single rocket engine could have made most of these corrections without resorting to large angular changes in orientation. The velocity deficiency, usually incurred with dead bands, required the largest corrections to be made in the $-\dot{x}$ directions except when major orbital plane changes were made. Furthermore, these corrections in the $-\dot{x}$ direction tend to inject the ferry into the station's orbit and, hence, reduce the injection (or closing) velocity at the time of rendezvous.

When the standard deviation of the radar errors was varied, the average miss distance was not noticeably changed but the average velocity requirement increased linearly with increased radar error. It would appear that fairly large radar errors could be tolerated during the midcourse phase provided velocity corrections are applied at frequent intervals.

When the ferry is launched from out of the orbital plane of the station, the two planes may intersect before radar acquisition is obtained. In this case, stored or ground-transmitted data will have to be used to determine when and how much out-of-plane velocity should be applied. This correction should not represent any great difficulty. If, for instance, the ferry is launched out of plane but on to an initially parallel heading, intersection will occur 90° downrange from the launch position and a clock could be used to measure a quarter of the orbital period of the station. The results given herein are possibly more applicable to errors arising from such methods since the error in the time interval between orbital intersection ($z = 0$) and the time at which the $\dot{x}$ component of velocity was corrected was intentionally made relatively large. The digital computer could have been forced to pick small time intervals of integration whenever x approached zero, and a condition of $z = 0$, $\dot{z} = 0$ could have been obtained. However, artifically forcing this condition would not have proved the validity of the guidance equations or the ability of the guidance system to bring z to zero prior to or simultaneous with the x and y components.

Since the purpose of the simulation was to study gross effects, the angular motions of the ferry about its center of mass were not simulated and the thrust was applied impulsively. It is felt that the vehicle should be continuously oriented so that when the thrust is called for, it may be applied instantly without delays for reorienting the ferry. If a low-pass filter were used between the guidance and orientation networks, small amplitude, high-frequency excitation could be avoided. The assumption of impulsive thrust does not appear unreasonable since most of the in-plane velocity changes were on the order of 20 to 30 feet per second. Only the out-of-plane corrections were relatively large, being on the order of 100 ft/sec for every 17 miles of out-of-plane distance (plus any initial out-of-plane velocity) at the time of launch. However, these plane changes were not made at precisely $z = 0$ and, thus, the time required to make large velocity changes was probably adequately simulated.

CONCLUSIONS

As a result of the foregoing studies of the midcourse phase of rendezvous, the following general conclusions may be drawn:

1. The approximate guidance equations lead to large velocity errors in the in-plane velocity components when the earth angle until rendezvous $\omega\tau_1$ is more than 90°. The errors are small, however, when $\omega\tau_1 \leqq 90^{\circ}$ and approach zero as the angle $\omega\tau_1$ approaches zero. This restriction does not apply to the out-of-plane velocity components.

2. Successful rendezvous are obtained even when the velocity errors existing at the time of booster burnout are substantially worse than can be expected from present-day launch accuracies. Although these errors are expensive in terms of velocity changes for guidance, the expense is not excessive.

3. Errors in the time of booster burnout of up to 5 seconds do not have an appreciable effect upon the total velocity correction required. This result is attributed to the method of guidance which is primarily dependent upon the instantaneous conditions and only indirectly dependent upon the specified time to rendezvous. Also, for similar reasons, moderate errors in the launch position are relatively unimportant in the accuracy or fuel requirements for rendezvous.

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4. The average total velocity correction for rendezvous is directly proportional to the accuracy of the radar used. However, no correlation is found between radar error and miss distance.

5. When a dead band is used and corrections are made only at definite positions on the trajectory, the miss distance increases linearly with the magnitude of the dead band while the total amount of corrective velocity approaches a limiting value. The use of dead bands appears to be based on a trade off between miss distance and total velocity correction.

6. When the space station is in a near-earth elliptic orbit, rendezvous is always achieved provided a sufficiently accurate instantaneous value of the station angular velocity is used in the guidance equations. If velocity corrections are continued to within 200 feet of the station, most of the final correction has the effect of injecting the ferry into orbit.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., March 22, 1961.

APPENDIX

COMPUTATION OF τ_1 FROM RELATIVE POSITION AND VELOCITY OF FERRY AND STATION

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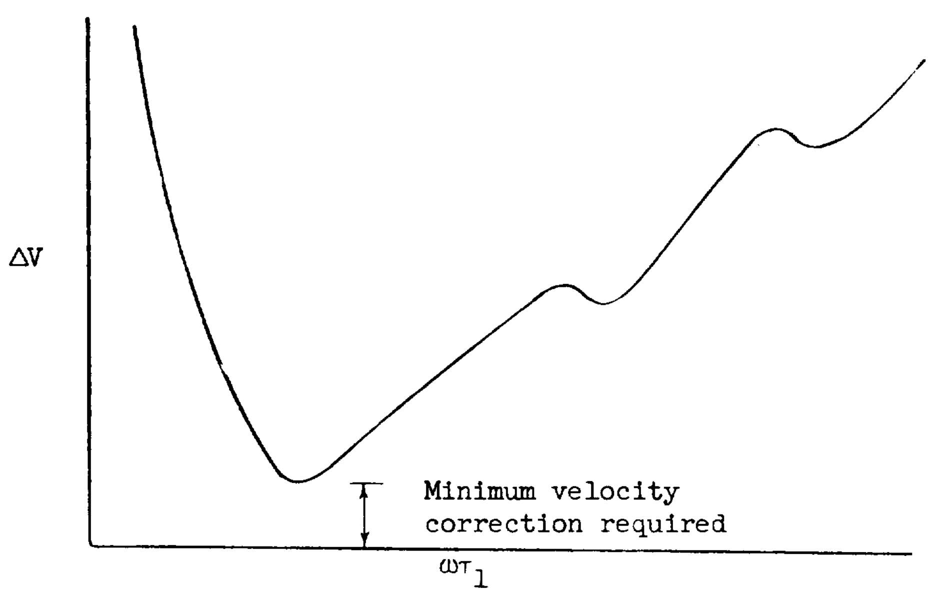
Over the final part of the trajectory the optimum time to intercept τ_1 was computed in the following manner. At any particular instant during the approach of the ferry to the space station, x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, and $\dot{z}_o$ are specified (from radar or some other measurement). If, at that instant equations (15), (16), and (17) are computed for a particular value of τ_1 (along with the specified values of x_o, y_o, and z_o), then the total change of velocity that must be made in order to intercept at time τ_1 may be computed from the following equation:

$$\Delta V = \sqrt{\left[\dot{x}_o - \dot{x}_c(\tau_1)\right]^2 + \left[\dot{y}_o - \dot{y}_c(\tau_1)\right]^2 + \left[\dot{z}_o - \dot{z}_c(\tau_1)\right]^2} \qquad \text{(A1)}$$

Now, if the process is repeated for a whole series of values of τ_1, then a variation of ΔV with τ_1 may be plotted for that particular set of initial conditions (x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, and $\dot{z}_o$). A typical plot is shown in sketch (a).



Sketch (a)

It may be noted that the curve will have a number of maximums and minimums (one for each successive orbit) but the minimum of interest is the lowest minimum which will also usually be the first minimum. Thus, one method of computing the optimum time to intercept would be to generate such a process in the ferry vehicle and have the computer stop when the first minimum occurred. Another method would be to give the pilot an oscilloscope presentation of the plot and have him pick the minimum. In either case this particular value of τ_1 or $\omega\tau_1$ would be used to compute the velocity corrections that must be made to effect the rendezvous.

Because these are an infinite number of extremums, the condition for directly computing the extremums - namely, L1476

$$\Delta V \frac{\partial \Delta V}{\partial \tau_1} = \left[\dot{x}_o - \dot{x}_c(\tau_1)\right]\frac{\partial \dot{x}_c}{\partial \tau_1} + \left[\dot{y}_o - \dot{y}_c(\tau_1)\right]\frac{\partial \dot{y}_c}{\partial \tau_1}$$

$$+ \left[\dot{z}_o - \dot{z}_c(\tau_1)\right]\frac{\partial \dot{z}_c}{\partial \tau_1} = 0 \tag{A2}$$

leads to a very lengthy, complicated transcendental equation. However, if the optimum time to intercept is "small enough" (to be defined later) so that

$$\sin \omega\tau_1 = \omega\tau_1 - \frac{(\omega\tau_1)^3}{3!} + \frac{(\omega\tau_1)^5}{5!} - \dots$$

$$\cos \omega\tau_1 = 1 - \frac{(\omega\tau_1)^2}{2!} + \frac{(\omega\tau_1)^4}{4!} - \dots$$

then this condition limits the infinite number of solutions to a finite number of solutions, only one of which will have any physical significance.

Such a calculation has been carried out and the condition for the extremums found to be

$$0 = a_o + a_1(\omega\tau_1) + a_2(\omega\tau_1)^2 + a_3(\omega\tau_1)^3 + \dots \tag{A3}$$

where

$$a_o = x_o^2 + y_o^2 + z_o^2$$

$$a_1 = x_o \frac{\dot{x}_o}{\omega} + y_o \frac{\dot{y}_o}{\omega} + z_o \frac{\dot{z}_o}{\omega}$$

Although the smallest positive roots of the cubic and quadratic expressions were computed, it was found that the simplest and best results were obtained from using the one unique solution obtained from the linear expression:

$$0 = a_o + a_1\omega\tau_1$$

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or

$$\tau_1\Big]_{opt} = \frac{x_o^2 + y_o^2 + z_o^2}{-\left(x_o\dot{x}_o + y_o\dot{y}_o + z_o\dot{z}_o\right)} = \frac{R_o}{-\dot{R}_o} \tag{A4}$$

where R_o is the line-of-sight range defined by

$$R_o = \left(x_o^2 + y_o^2 + z_o^2\right)^{1/2}$$

Equation (A4) then is an expression for the optimum time to intercept as determined from $\left(x_o,\ y_o,\ z_o,\ \dot{x}_o,\ \dot{y}_o,\text{ and }\dot{z}_o\right)$ or $\left(R_o\text{ and }\dot{R}_o\right)$.

In order to determine the accuracy of expression (A4), the exact equations (that is, from eq. (A1)) were solved for the case in which the ferry vehicle was ejected from a space station (in a 300-mile circular orbit) at relative velocities of 400, 600, and 800 ft/sec at $t = 0$. The equations were solved in negative time so that the trajectories of the ferry vehicle prior to intercept were computed. Thus, for each value of $\omega\tau_1$ the solutions gave the values of (x, y, z, $\dot{x}$, $\dot{y}$, and $\dot{z}$) and (R and $\dot{R}$). The variation of $\omega\tau_1$ with R is shown in figure 13. It should be noted that $\omega\tau_1$ represents the angular travel of the station around the earth and as such is more significant than just elapsed time. Also shown in figure 13 by dashed lines are the values of $\omega\tau_1$, obtained by use of equation (A4), plotted as a function of R. It can be seen in figure 13 that equation (A4) gave very good results out to a value of $\omega\tau_1$ of 40^o and within the last 30^o continually improved as the time to intercept grew smaller. It can also be seen that the range of the ferry from the station was not important.

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TABLE I. - DETAILED BREAKDOWN OF VELOCITY CORRECTIONS FROM FIGURE 5

(a) Figure 5(a)

t, sec	θ, deg	R, miles	$\Delta\dot{x}$, ft/sec	$\Delta\dot{y}$, ft/sec	$\Delta\dot{z}$, ft/sec	$\Delta\dot{R}$, ft/sec	$\Delta R\dot{\theta}\cos\psi$, ft/sec	$\Delta R\dot{\psi}$, ft/sec	$\sqrt{\sum_i \Delta V_i^2}$, ft/sec
831	52.0	155.0	0.0	0.0	360.9	0.9	0.0	360.8	360.9
595	57.11	140.0	-7.2	-64.4	22.7	57.2	28.8	24.5	68.6
1,375	87.7	71.3	.6	3.3	1.7	-3.4	.5	1.5	3.7
1,807	115.2	34.3	7.0	-3.9	5.3	7.5	.9	6.0	9.7
2,055	131.0	17.1	-2.7	-9.8	18.8	5.2	-8.5	2.3	10.3
2,175	138.6	8.4	-.1	2.6	.4	.5	-2.4	.4	2.6
									Total: 456

TABLE I.- DETAILED BREAKDOWN OF VELOCITY CORRECTIONS FROM FIGURE 5 - Continued

(b) Figure 5(b)

t, sec	θ, deg	R, miles	$\Delta\dot{x}$, ft/sec	$\Delta\dot{y}$, ft/sec	$\Delta\dot{z}$, ft/sec	$\Delta\dot{R}$, ft/sec	$\Delta R\dot{\theta}\cos\psi$, ft/sec	$\Delta R\dot{\psi}$, ft/sec	$\sqrt{\sum_i \Delta V_i^2}$, ft/sec
768	48.9	144.8	0.0	0.0	380.0	14.9	0.0	379.8	380.1
895	57.1	109.2	-139.9	-501.4	34.6	515.5	-36.8	77.4	522.6
1,408	89.7	54.2	3.4	15.0	.0	10.5	-11.2	.1	15.4
1,871	119.3	26.5	12.8	21.1	.3	18.4	-16.4	1.5	24.7
2,087	133.0	13.4	3.1	9.6	.1	3.4	-9.5	.1	10.1
2,195	140.0	6.7	4.0	12.0	36.0	.9	-12.4	-36.1	38.2
									Total: 991

TABLE I.- DETAILED BREAKDOWN OF VELOCITY CORRECTIONS FROM FIGURE 5 - Continued

(c) Figure 5(c)

t, sec	θ, deg	R, miles	$\Delta\dot{x}$, ft/sec	$\Delta\dot{y}$, ft/sec	$\Delta\dot{z}$, ft/sec	$\Delta\dot{R}$, ft/sec	$\Delta R\dot{\theta}\cos\psi$, ft/sec	$\Delta R\dot{\psi}$, ft/sec	$\sqrt{\sum_i \Delta V_i^2}$, ft/sec
0	0.0	179.4	19.3	5.3	-28.4	-9.8	-19.9	-30.0	37.3
799	50.9	89.1	5.0	-4.3	.0	4.2	4.9	-1.5	6.7
1,112	70.8	43.7	7.6	-15.2	.0	9.8	13.5	-3.4	17.0
1,247	79.5	22.4	-8.0	.6	.0	-5.2	9.0	1.7	10.5
1,320	84.1	10.9	-17.2	-2.4	.0	-16.2	4.3	4.4	17.4
1,353	86.3	5.4	-21.0	-9.0	.0	-22.5	.6	3.9	22.8
									Total: 122

TABLE I.- DETAILED BREAKDOWN OF VELOCITY CORRECTIONS FROM FIGURE 5 - Concluded

(d) Figure 5(d)

t, sec	θ, deg	R, miles	$\Delta\dot{x}$, ft/sec	$\Delta\dot{y}$, ft/sec	$\Delta\dot{z}$, ft/sec	$\Delta\dot{R}$, ft/sec	$\Delta R\dot{\theta}\cos\psi$, ft/sec	$\Delta R\dot{\psi}$, ft/sec	$\sqrt{\sum_i \Delta V_i^2}$, ft/sec
0	0.0	240.5	21.7	41.2	-19.8	26.8	-14.1	-87.2	92.4
511	32.6	120.0	8.4	-66.6	.0	-57.5	29.3	18.4	67.1
9,115	58.1	57.9	7.4	-7.6	.0	3.2	10.1	-9.7	10.6
1,095	69.8	28.8	-6.0	-35.7	.0	-14.5	33.0	33.6	36.2
1,183	75.4	14.5	-14.9	-6.1	.0	-15.6	3.6	1.5	16.1
1,207	76.9	10.6	-.1	.0	295.1	-1.7	.0	295.1	295.1
1,235	78.2	7.3	-----	-----	-----	-----	-----	-----	-----
									Total: 517

TABLE II.- DETAILED BREAKDOWN OF VELOCITY CORRECTIONS FROM FIGURE 11

(a) Figure 11(a)

t, sec	θ, deg	R, miles	$\Delta\dot{x}$, ft/sec	$\Delta\dot{y}$, ft/sec	$\Delta\dot{z}$, ft/sec	$\Delta\dot{R}$, ft/sec	$\Delta R\dot{\theta}\cos\psi$ ft/sec	$\Delta R\dot{\psi}$, ft/sec	$\sqrt{\sum_i \Delta V_i^2}$, ft/sec
0	3.9	130.4	1.6	8.3	2.9	-5.3	7.2	0.4	9.0
784	58.0	64.1	-31.4	-53.2	0	4.7	-61.5	3.0	61.8
1,032	74.4	31.8	-.5	-2.7	0	-1.2	-2.4	-.8	2.8
1,152	82.2	15.8	-2.7	-12.3	0	-6.1	-10.4	-3.9	12.7
1,212	86.1	7.8	.1	-17.4	0	-11.7	-10.7	-7.1	17.3
									Total: 103.5

TABLE II.- DETAILED BREAKDOWN OF VELOCITY CORRECTIONS FROM FIGURE 11 - Continued

(b) Figure 11(b)

t, sec	θ , deg	R, miles	$\Delta\dot{x}$, ft/sec	$\Delta\dot{y}$, ft/sec	$\Delta\dot{z}$, ft/sec	$\Delta\dot{R}$, ft/sec	$\Delta R\dot{\theta}\cos\psi$ ft/sec	$\Delta R\dot{\psi}$, ft/sec	$\sqrt{\sum_i \Delta V_i^2}$, ft/sec
1,152	94.1	118.6	-25.8	0.2	0	-21.1	14.8	1.7	25.8
1,408	110.1	86.9	0	0	306.8	-20.0	0	306.1	306.8
1,680	126.7	57.9	26.0	61.5	12.8	63.2	16.4	18.9	68.0
2,080	149.6	29.1	16.9	-21.5	.1	25.4	-9.6	3.2	27.4
2,384	168.3	14.5	1.4	15.1	-.1	-15.0	-1.4	-1.8	15.1
2,560	178.5	7.1	.7	2.3	0	-2.4	-.2	-.2	2.5
									Total: 445.6

TABLE II. - DETAILED BREAKDOWN OF VELOCITY CORRECTIONS FROM FIGURE 11 - Concluded

(c) Figure 11(c)

t, sec	θ , deg	R, miles	$\Delta\dot{x}$, ft/sec	$\Delta\dot{y}$, ft/sec	$\Delta\dot{z}$, ft/sec	$\Delta\dot{R}$, ft/sec	$\Delta R\dot{\theta}\cos\psi$, ft/sec	$\Delta R\dot{\psi}$, ft/sec	$\sqrt{\sum_i \Delta V_i^2}$, ft/sec
1,408	144.9	162.3	0.06	0.04	251.7	-7.2	0	251.5	251.6
2,336	119.1	46.3	-29.7	-52.2	9.1	48.2	30.8	20.5	60.8
3,008	238.9	22.6	25.4	30.5	15.0	-41.2	10.3	1.2	42.5
3,344	259.8	11.1	-3.1	-2.3	0	3.2	-1.7	1.1	3.8
3,512	269.4	5.6	------	------	-----	-----	----	-----	-----
									Total: 358.7

TABLE III.- DETAILED BREAKDOWN OF VELOCITY CORRECTIONS FROM FIGURE 12

t, sec	θ , deg	R, miles	$\Delta\dot{x}$, ft/sec	$\Delta\dot{y}$, ft/sec	$\Delta\dot{z}$, ft/sec	$\Delta\dot{R}$, ft/sec	$\Delta R\dot{\theta}\cos\psi$, ft/sec	$\Delta R\dot{\psi}$, ft/sec	$\sqrt{\sum_i \Delta V_i^2}$, ft/sec
1,408	144.9	162.3	0	0	253.2	-7.1	0	253.1	253.2
2,304	199.1	48.4	-28.5	-52.4	6.7	49.1	29.5	18.0	60.1
3,008	238.9	22.7	27.2	33.4	15.2	-44.3	11.3	.29	45.7
3,344	259.8	11.1	-3.7	.3	0	3.5	.44	1.29	3.7
3,496	269.3	5.6	-----	-----	-----	-----	------	------	-----
3,568	274.0	2.8	-----	-----	-----	-----	------	------	-----
3,606	276.4	1.4	-----	-----	-----	-----	------	------	-----
3,627	277.7	.70	-----	-----	-----	-----	------	------	-----
3,634	278.2	.35	-----	-----	-----	-----	------	------	-----
3,639	278.6	.17	-22.5	-19.7	22.8	0	-26.1	27.2	37.7
3,642	278.7	.09	-9.2	-6.7	7.2	-.1	-9.9	9.1	13.5
3,643	278.8	.04	-28.9	-15.8	16.4	-.1	-29.1	22.5	36.8
3,644	278.9	.02	-7.3	-7.2	4.3	-2.5	-10.3	3.7	11.2
									Total: 461.7

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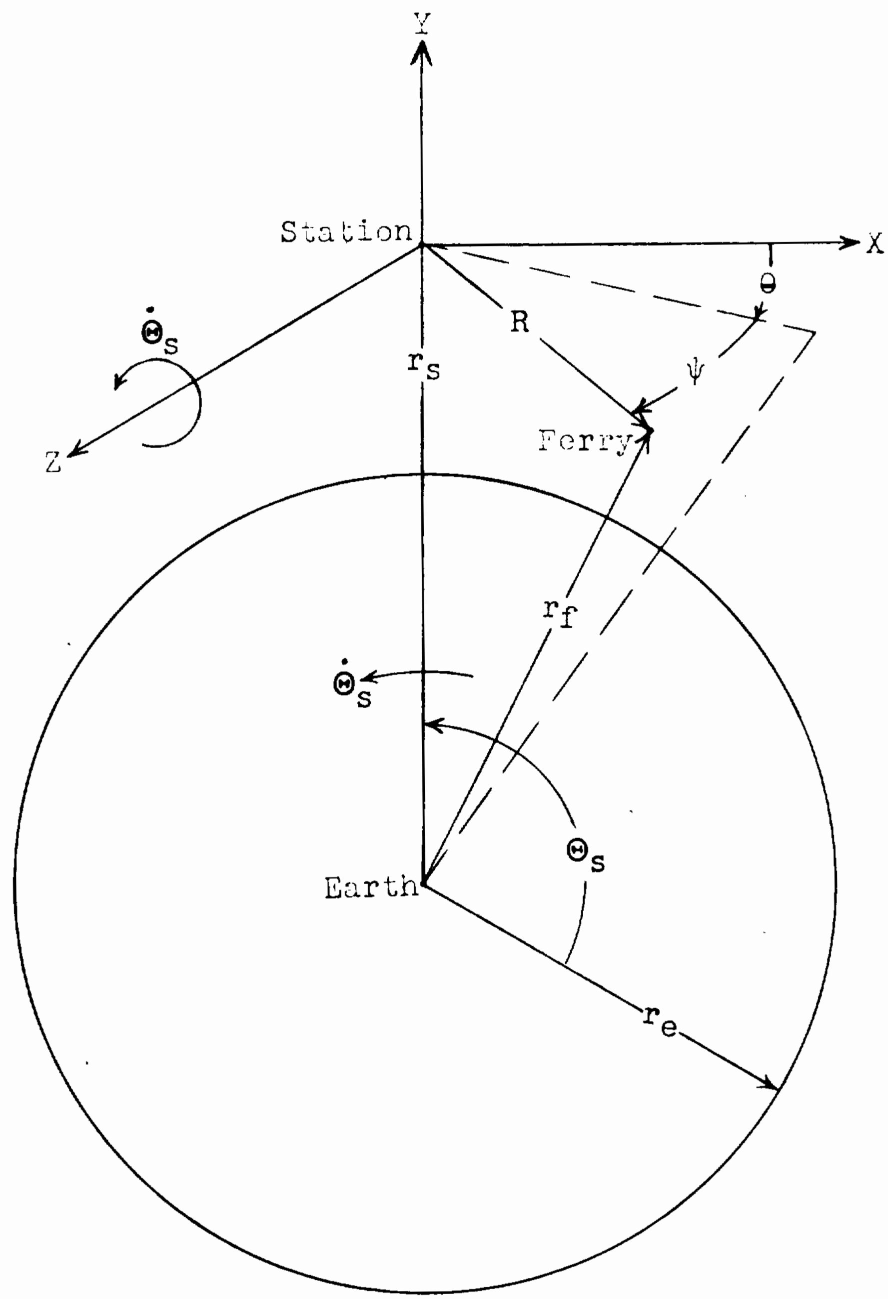

Figure 1.- Coordinates employed in describing the motions of the space station and ferry.

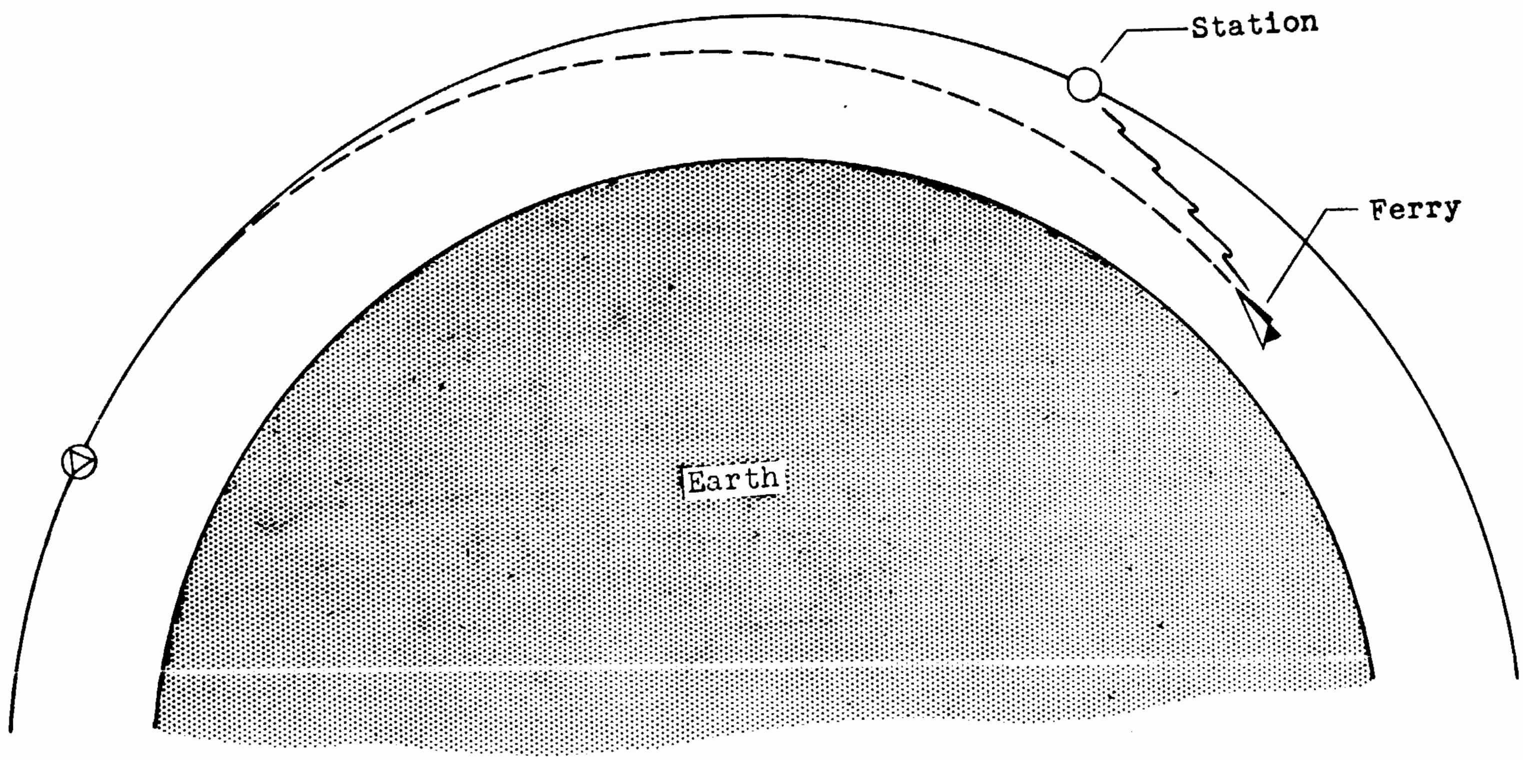

Figure 2.- Schematic drawing of the problem situation for the midcourse phase of rendezvous.

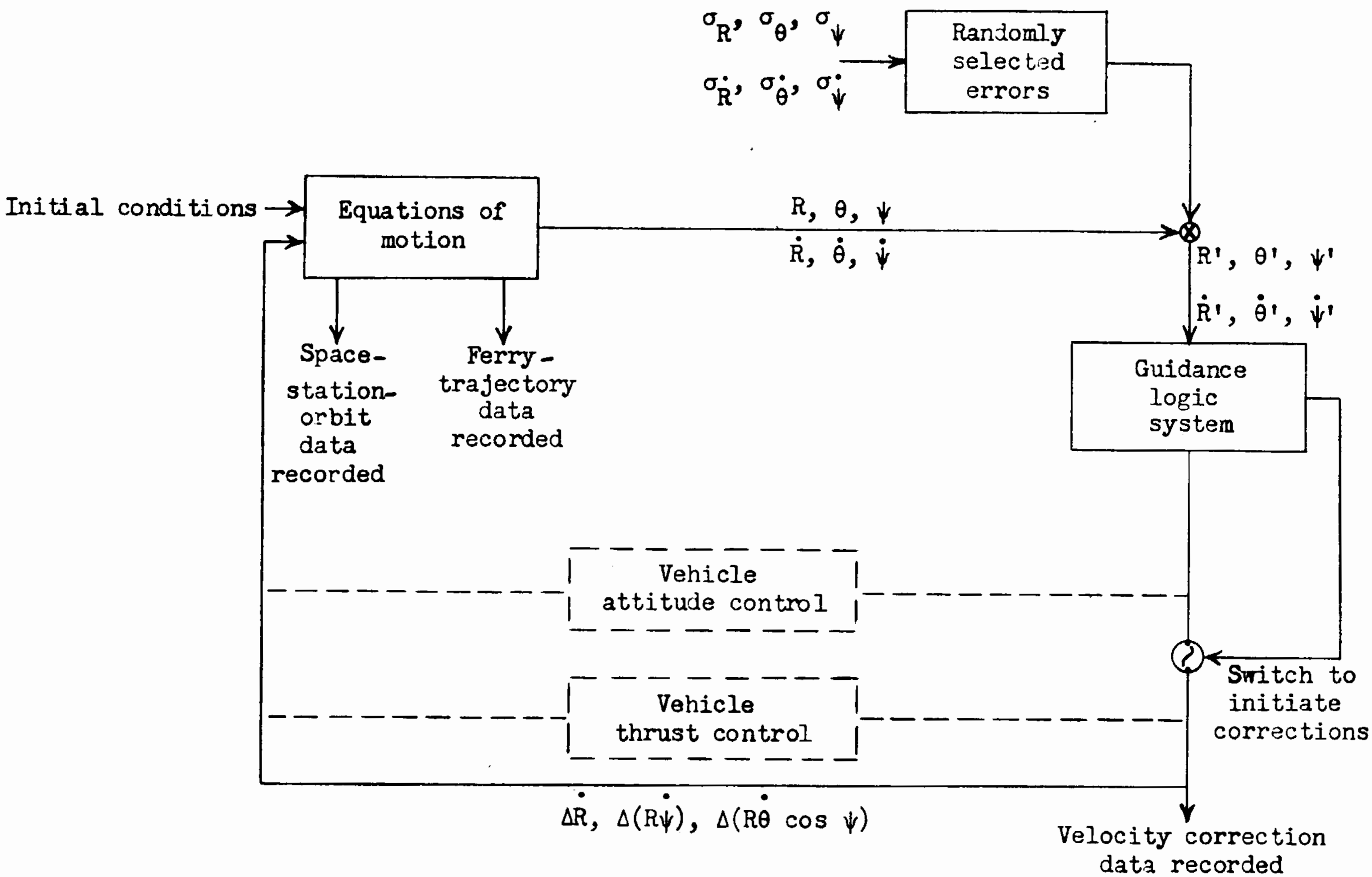

Figure 3.- Block diagram of simulation of rendezvous guidance. Components in dashed lines were not simulated but were assumed to exist.

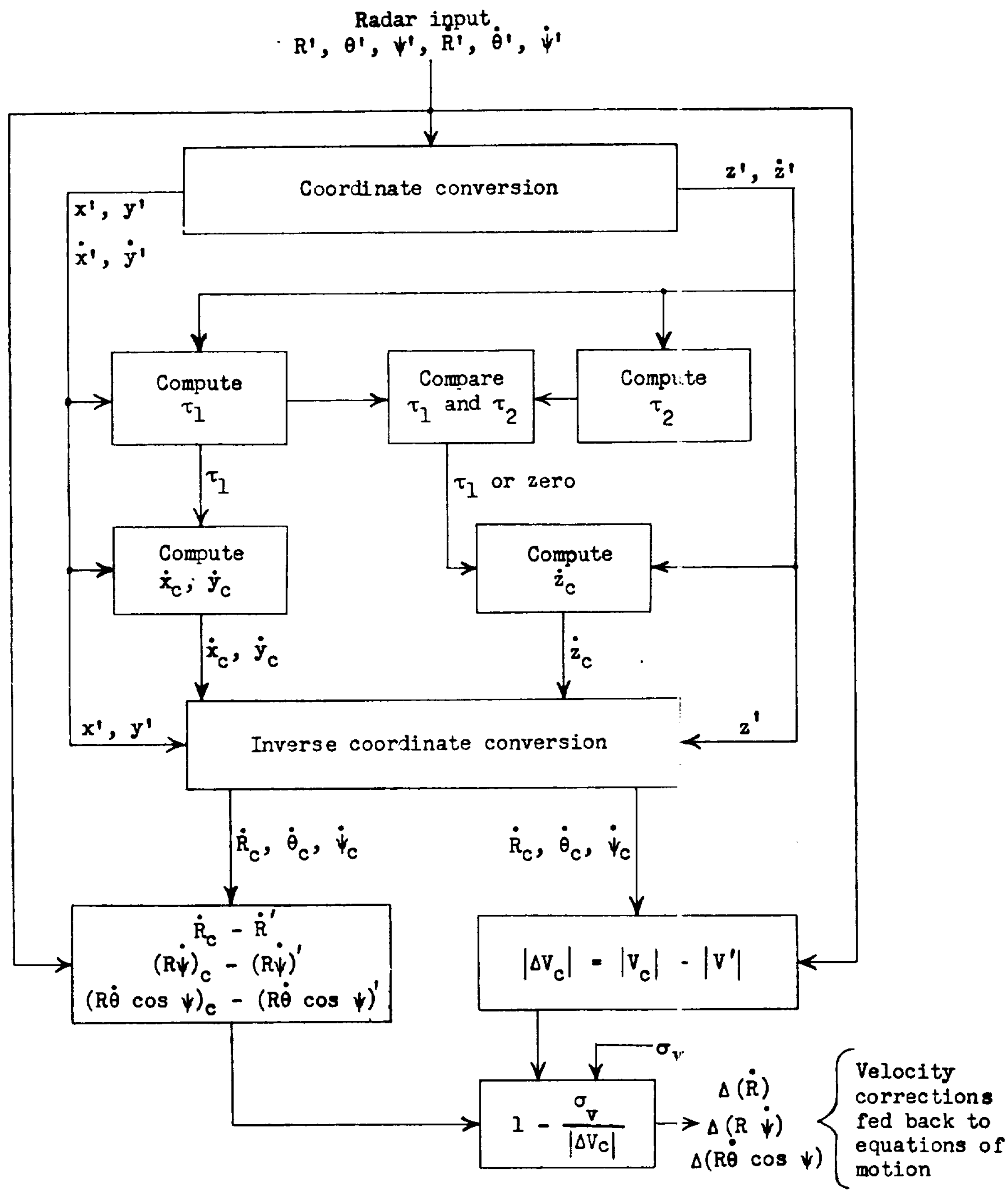

Figure 4.- Block diagram of guidance system for the midcourse phase of rendezvous.

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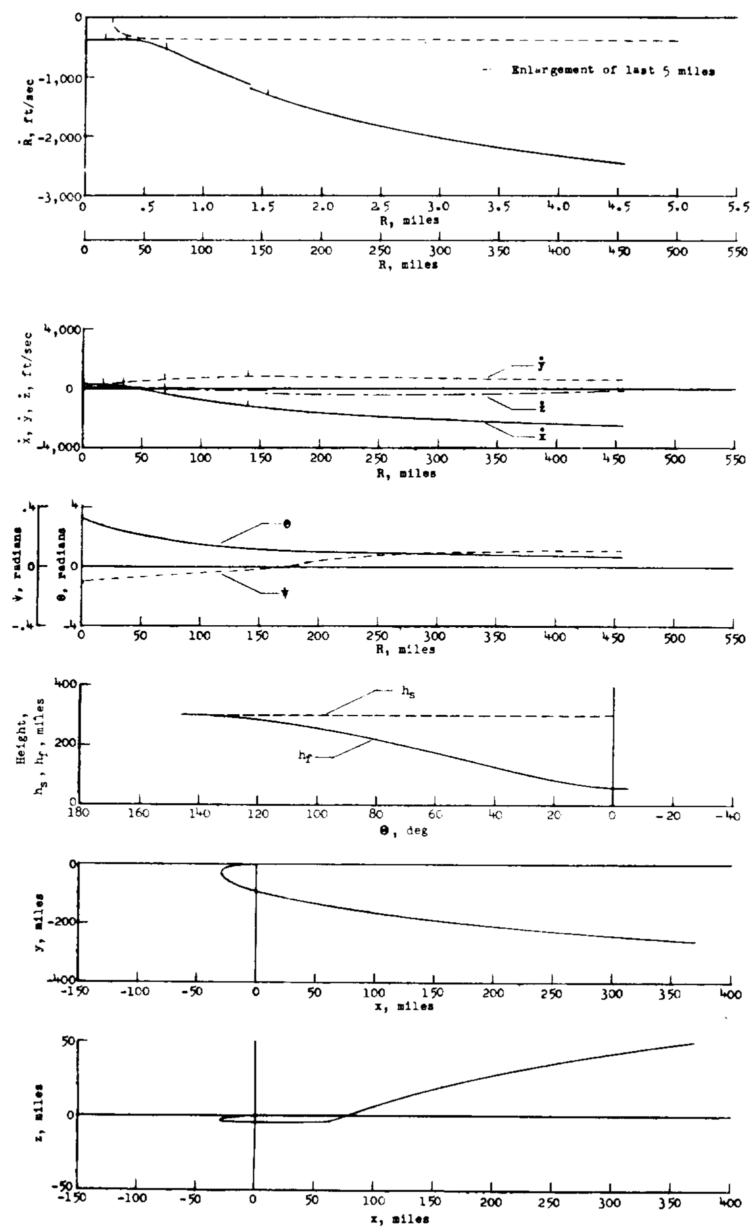
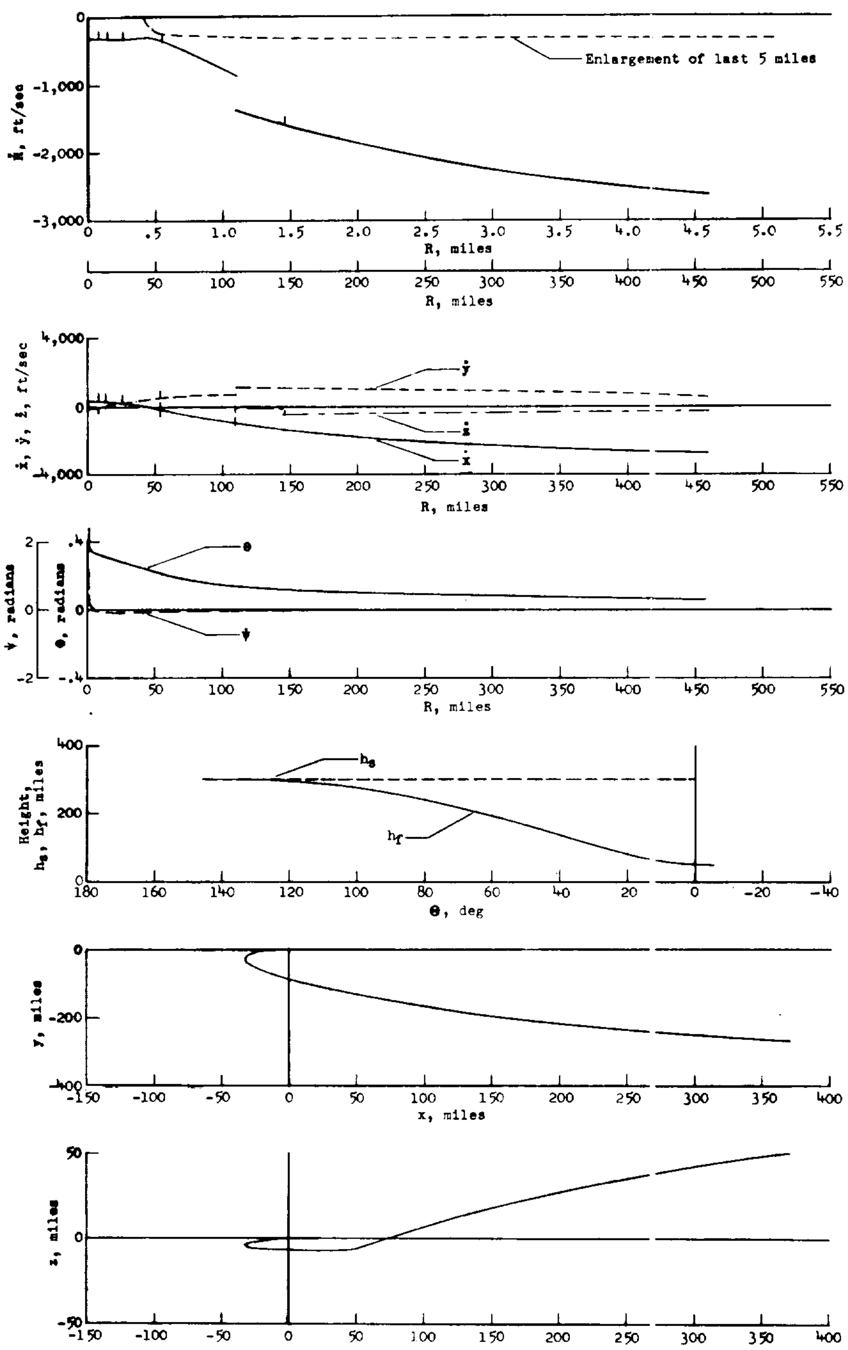


(a) Ferry launched with approximately the correct position and velocity. Interception occurs from below and in front of station.

Figure 5.- Typical trajectories for rendezvous with a station in a 300-mile circular orbit. Ferry launch occurs 50 miles out of plane.



(b) Ferry launched with large errors in position and velocity. Interception occurs from below and in front of station.

Figure 5.- Continued.

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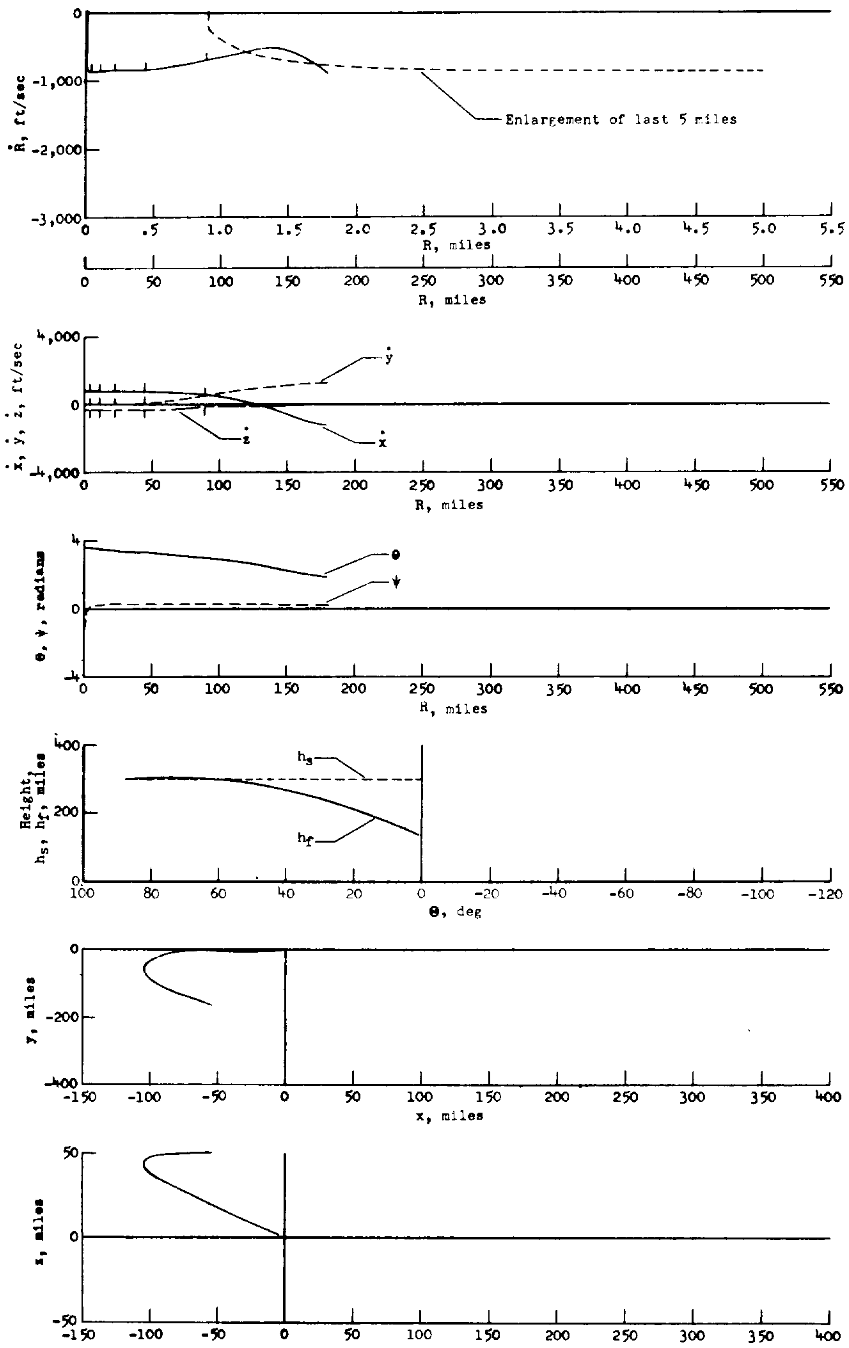
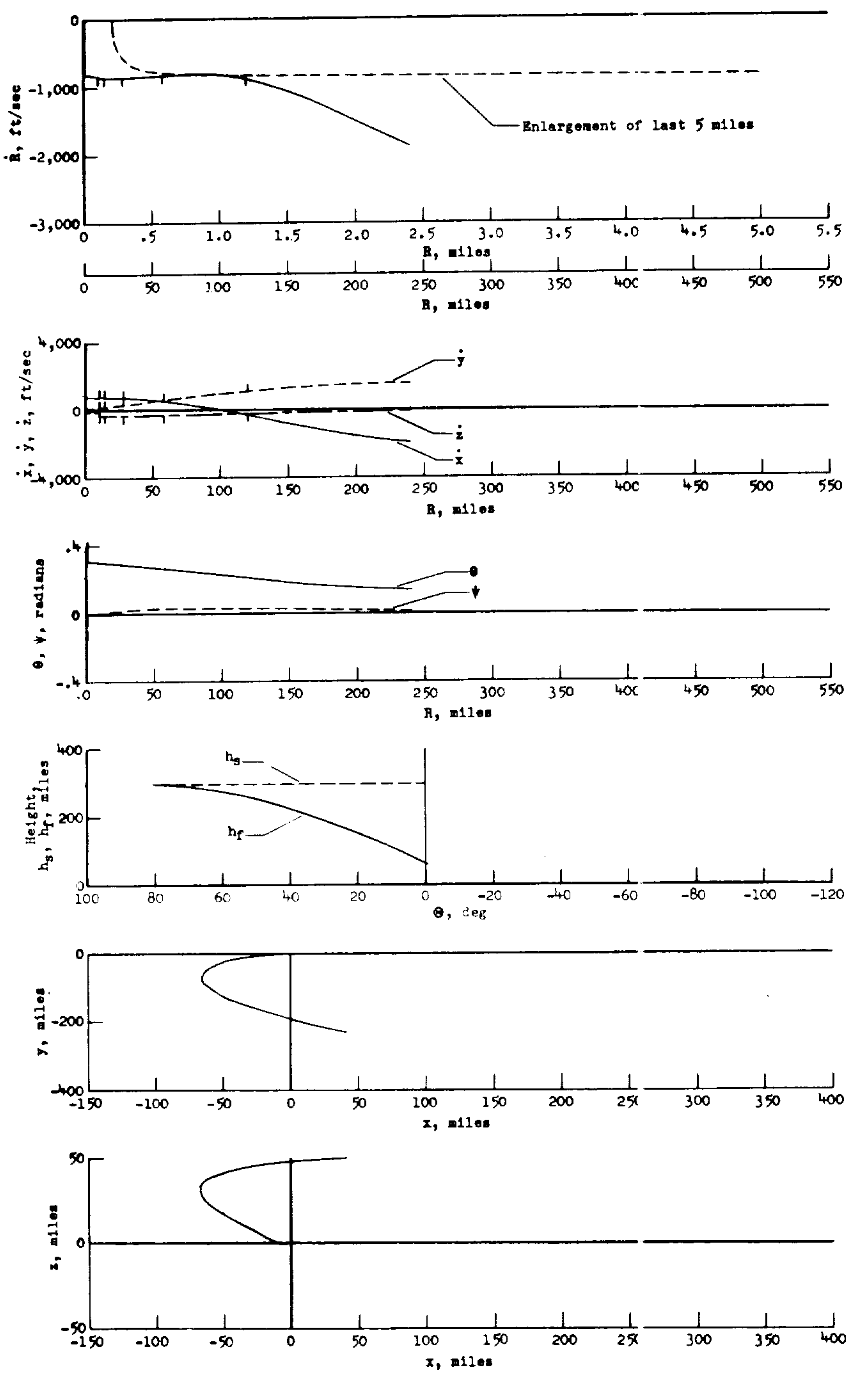


(c) Ferry launched with small error in out-of-plane velocity. Interception occurs from above and in front of station.

Figure 5.- Continued.



(d) Ferry launched with small error in out-of-plane velocity. Interception occurs from below and in front of station.

Figure 5.- Concluded.

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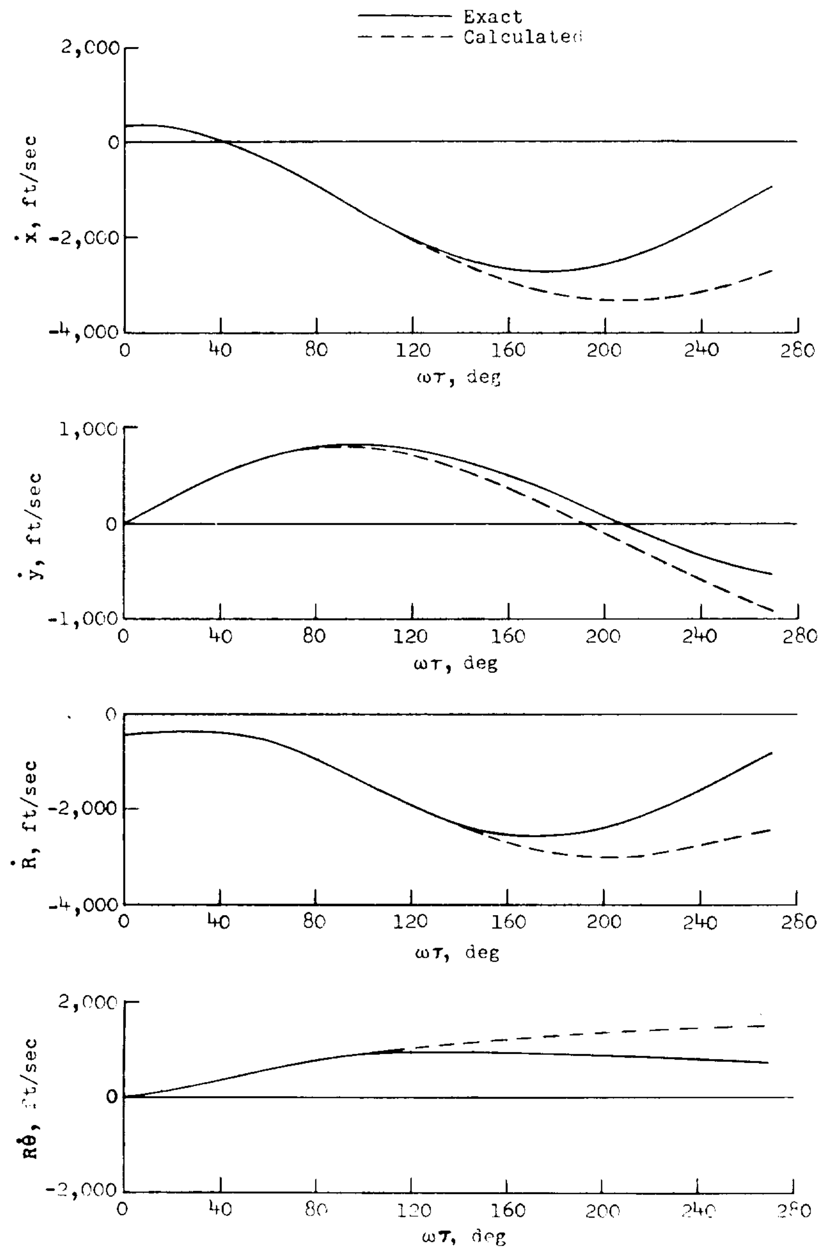
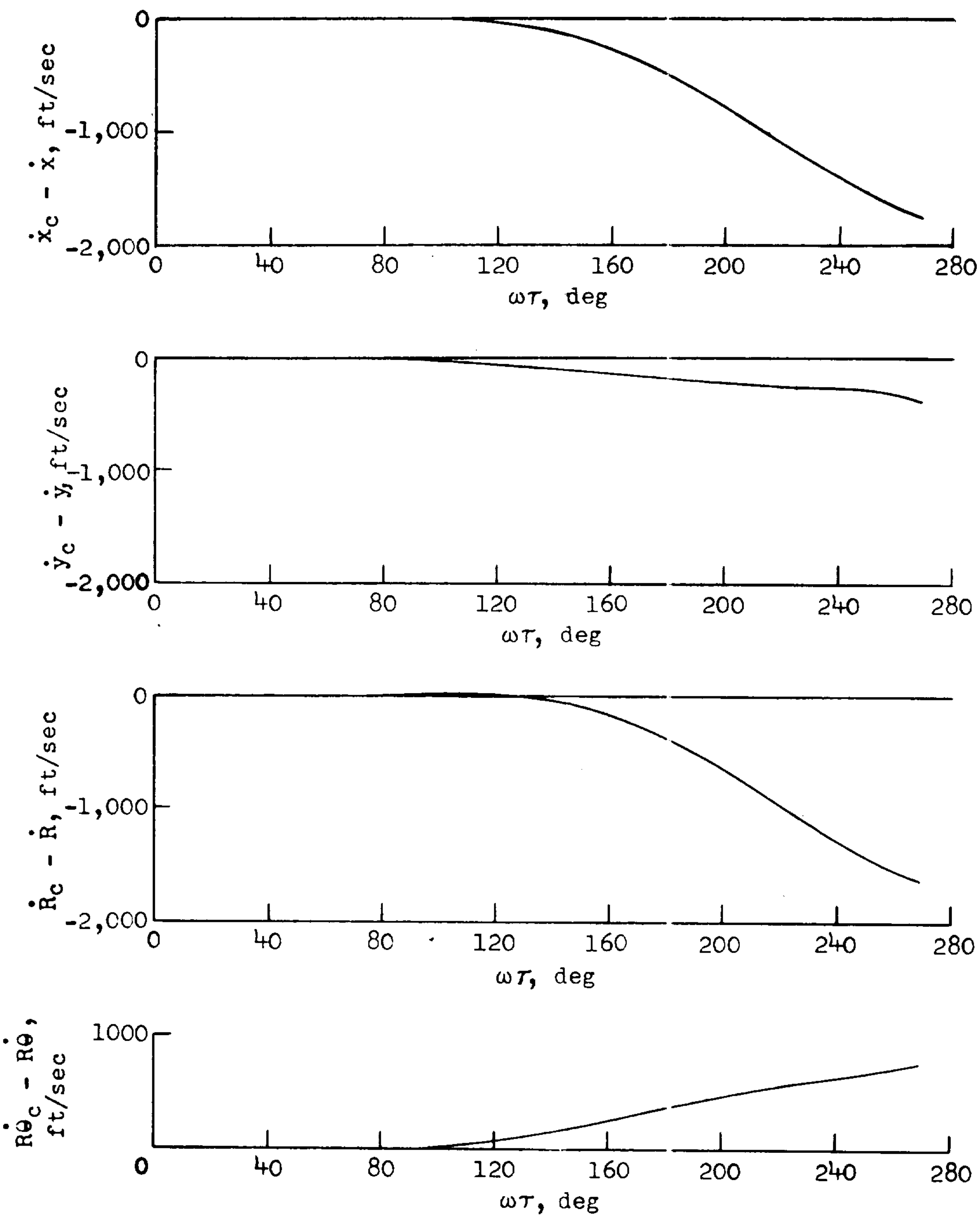


(a) Magnitude of the velocity components.

Figure 6.- The variation with position of the velocity components computed by the approximate guidance equation and the exact velocity components required to intercept. Variations are given in both rectangular and polar coordinates. Ferry and station are in coplanar orbits.



(b) Difference in magnitude between exact and computed velocity components.

Figure 6.- Continued.

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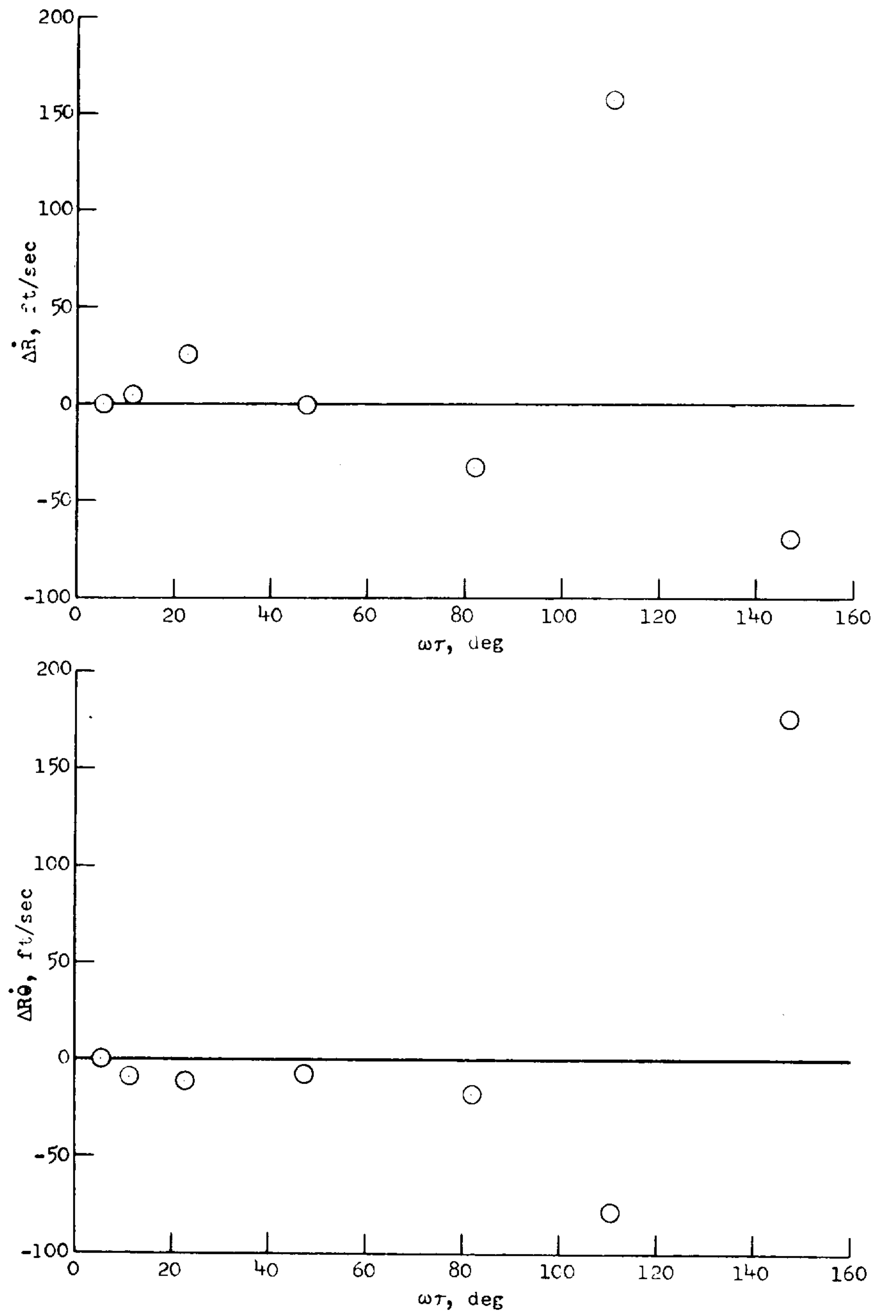

(c) Corrections actually made on a trajectory which required no corrections to intercept.

Figure 6.- Concluded.

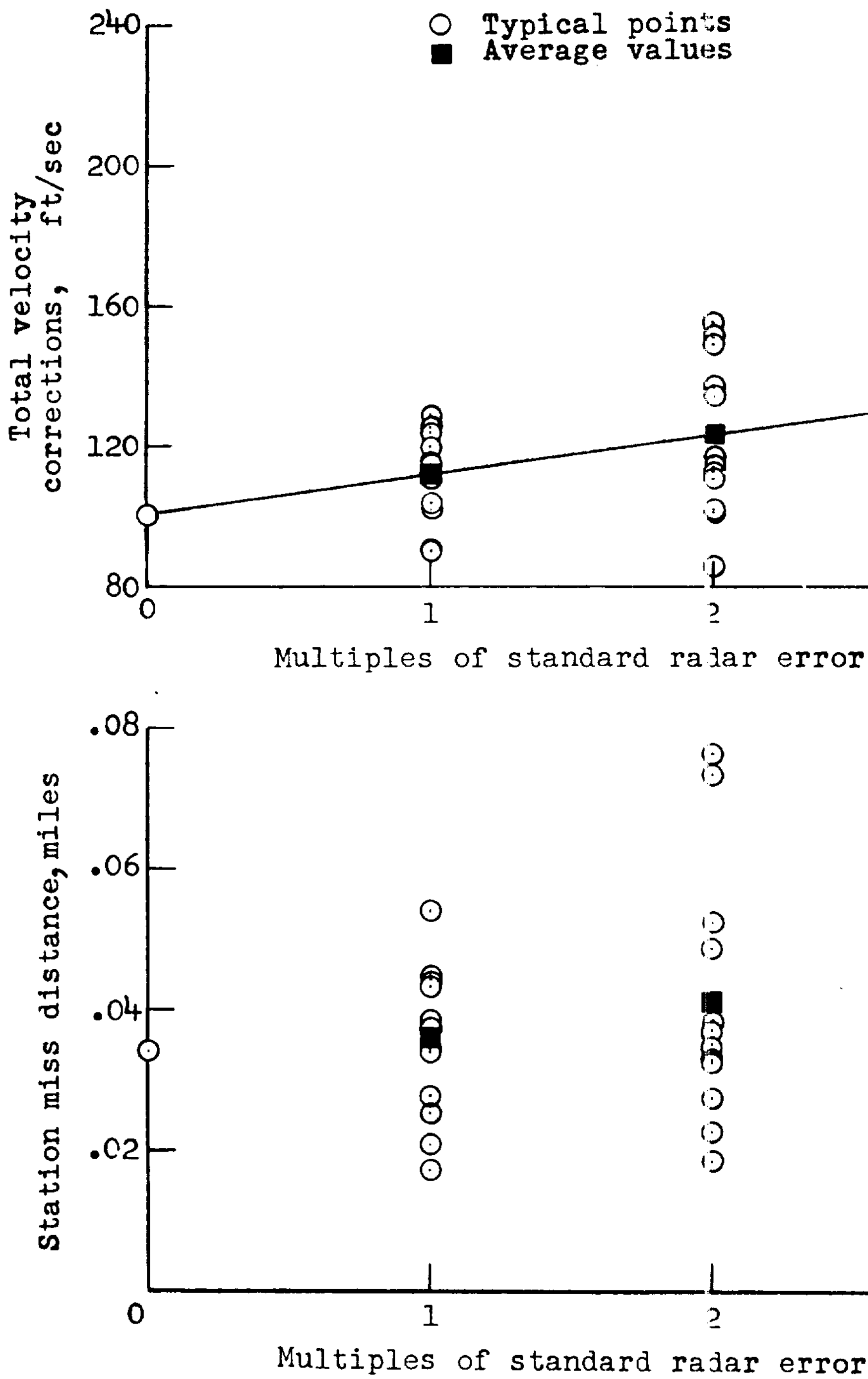

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Figure 7.- Distribution and averaged values of the total velocity corrections and miss distances obtained during rendezvous as a function of the standard deviation of the radar errors.

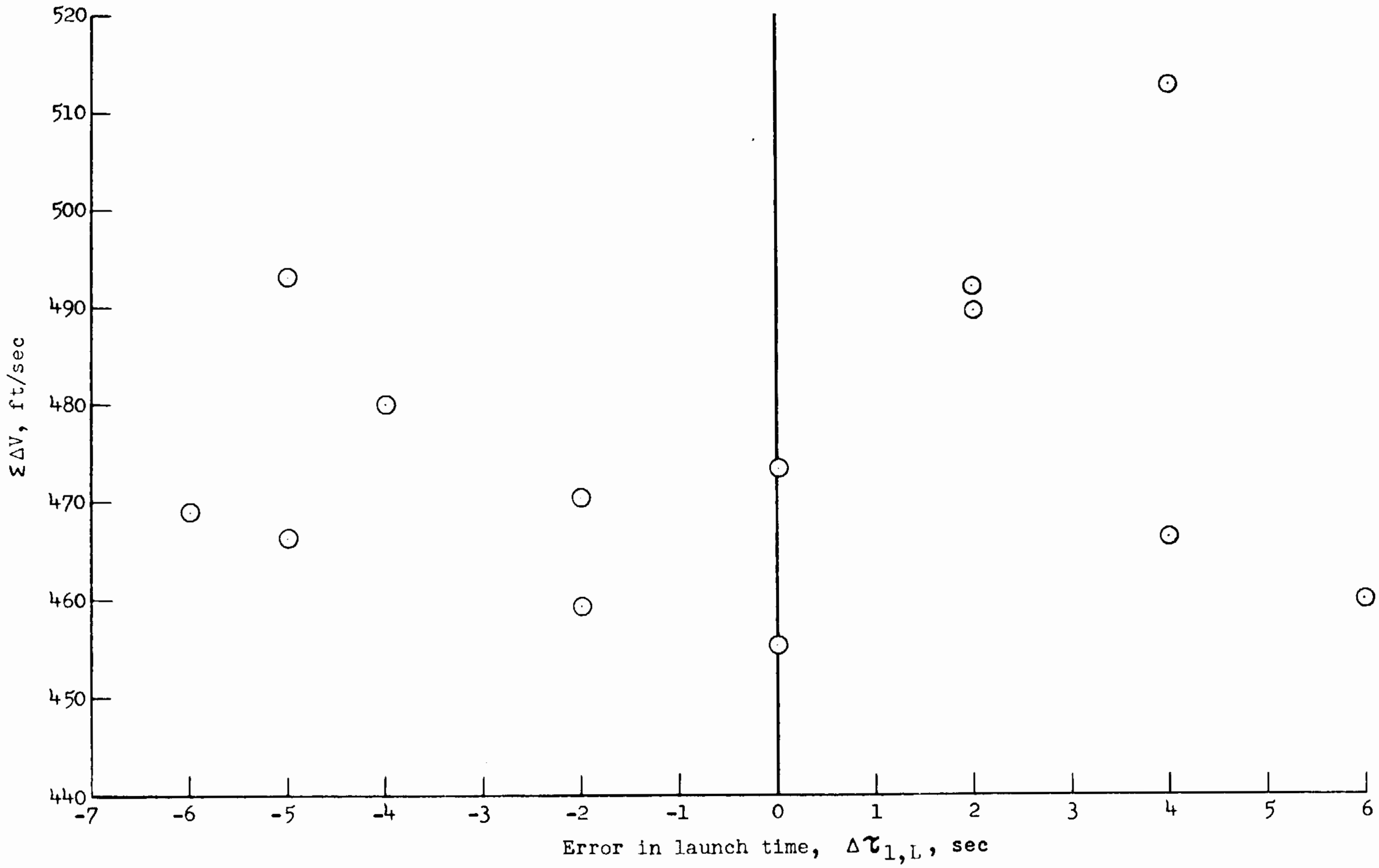
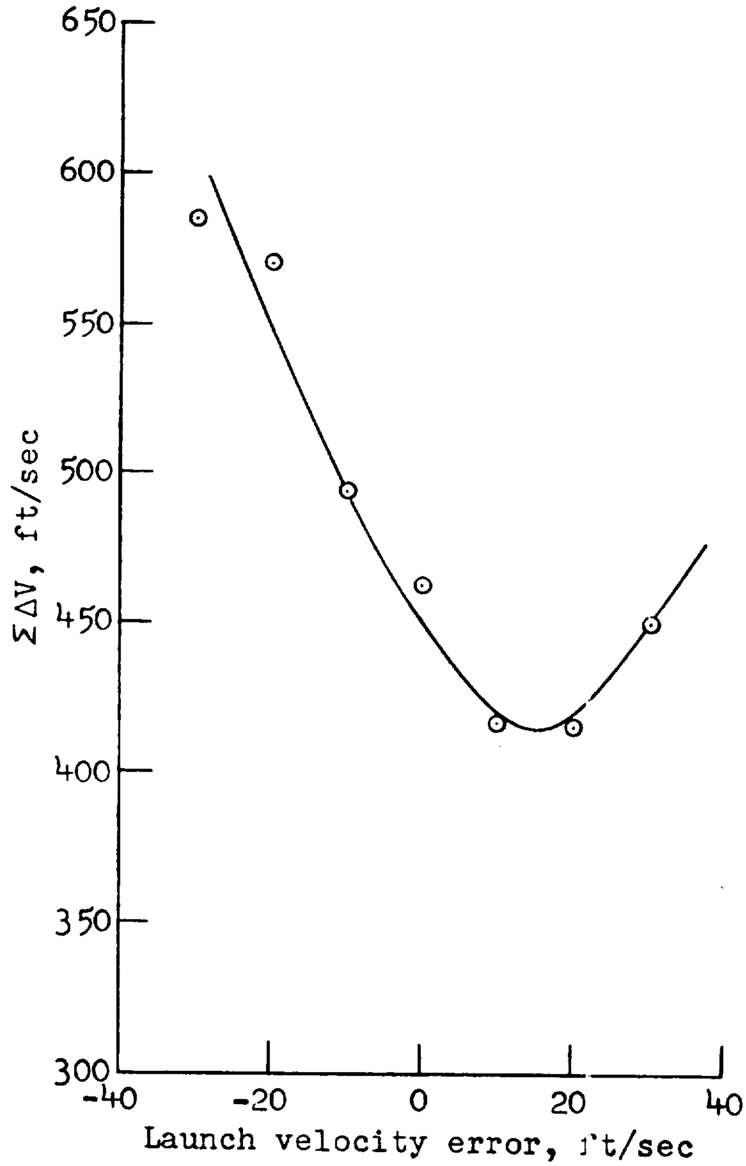


Figure 8.- Variation of the total velocity corrections due to errors in launch times or errors in the predicted time to intercept.



(a) Total of midcourse velocity changes as a function of launch velocity error.

Figure 9.- Variation of the total velocity corrections and relative closing velocity with errors in the launch velocity.

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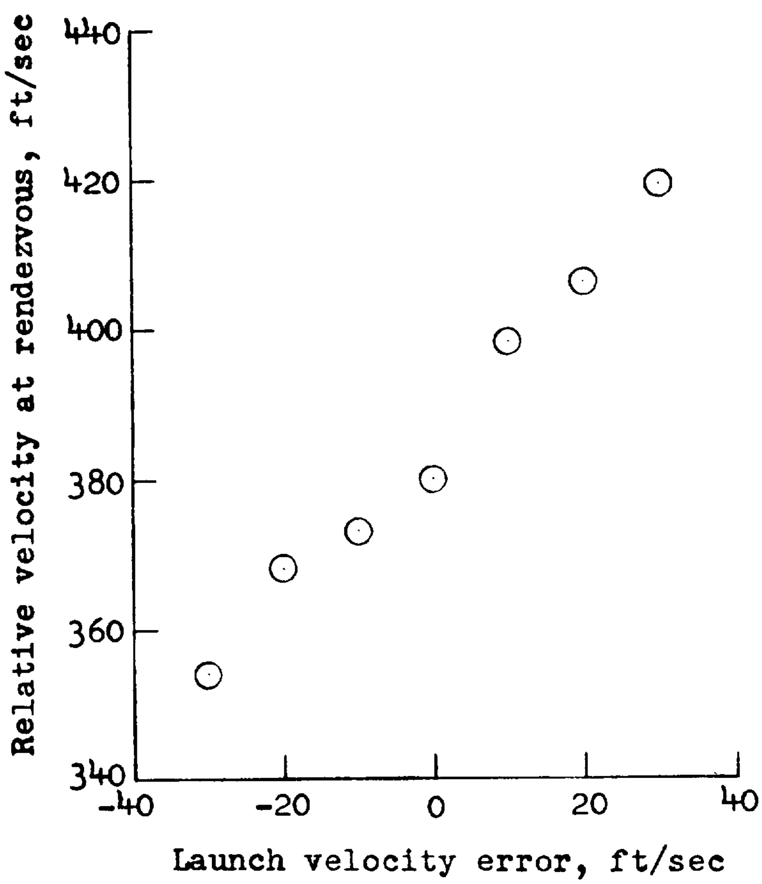
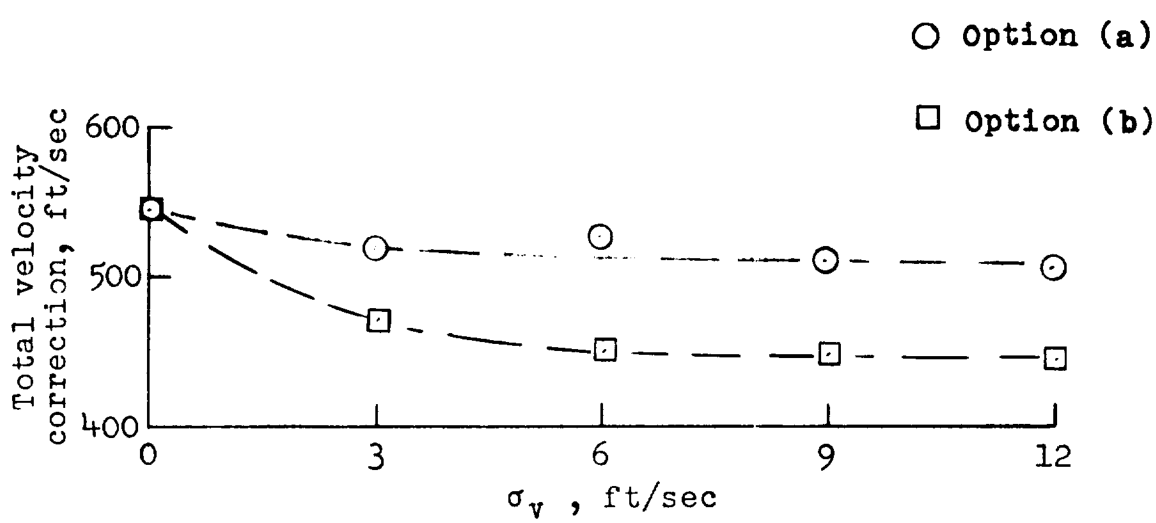


(b) Terminal velocity as a function of launch velocity error.

Figure 9.- Concluded.



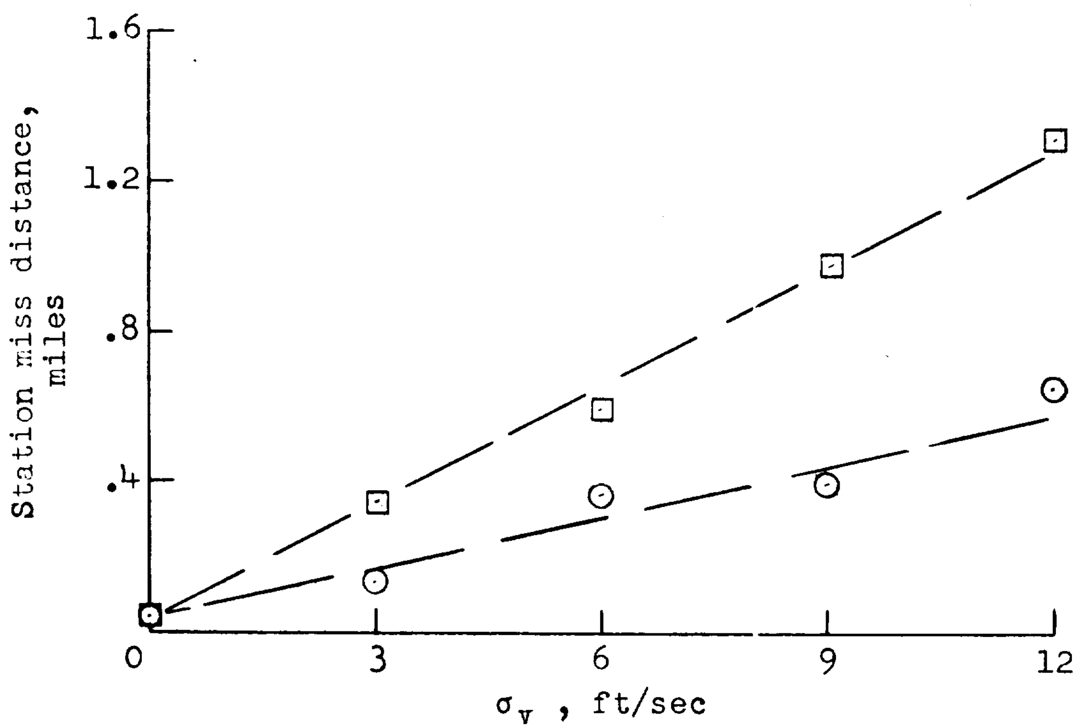


Figure 10.- Averaged total velocity corrections and miss distances as a function of dead-band width. For the radar simulated, $1\sigma_v$ = 3 feet per second.

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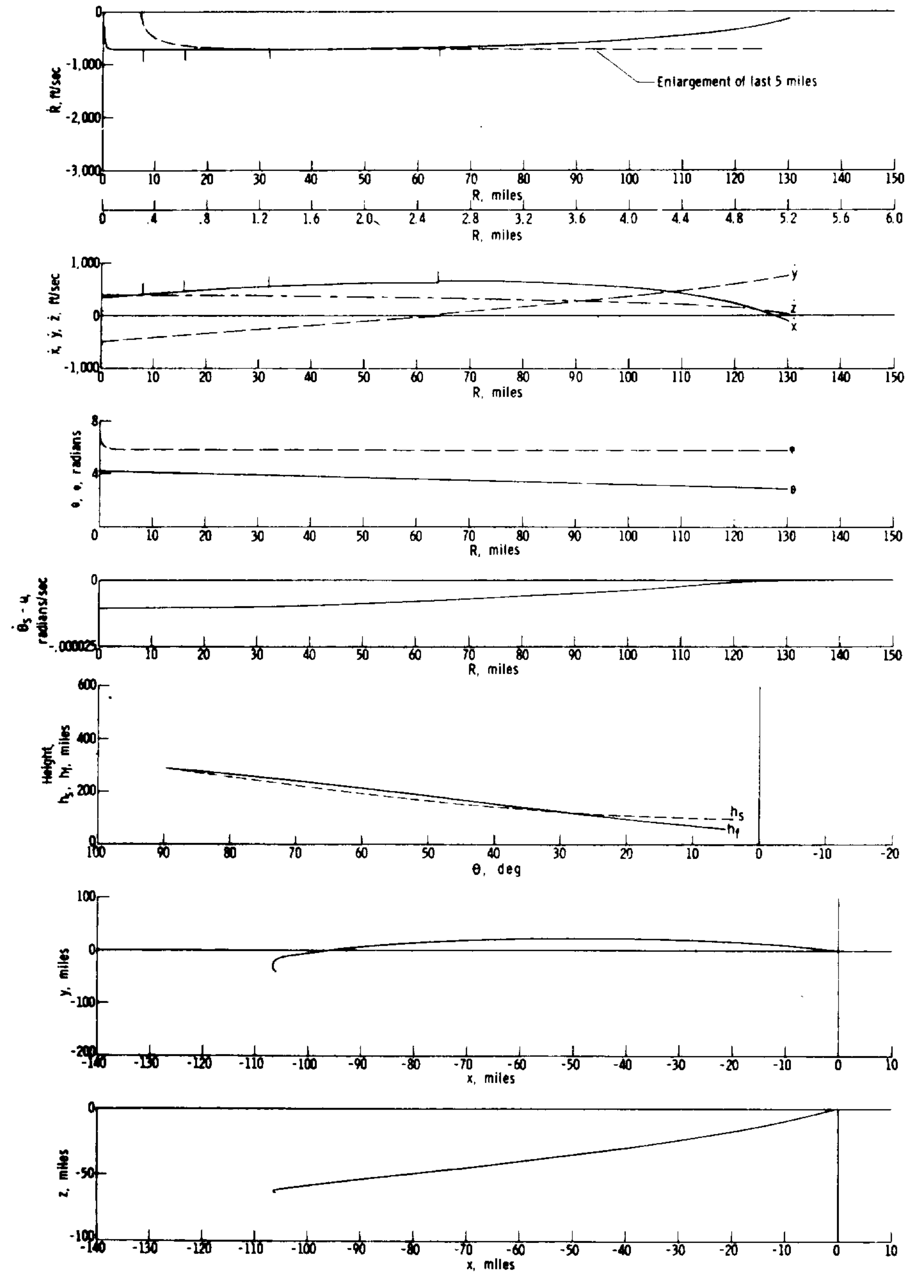
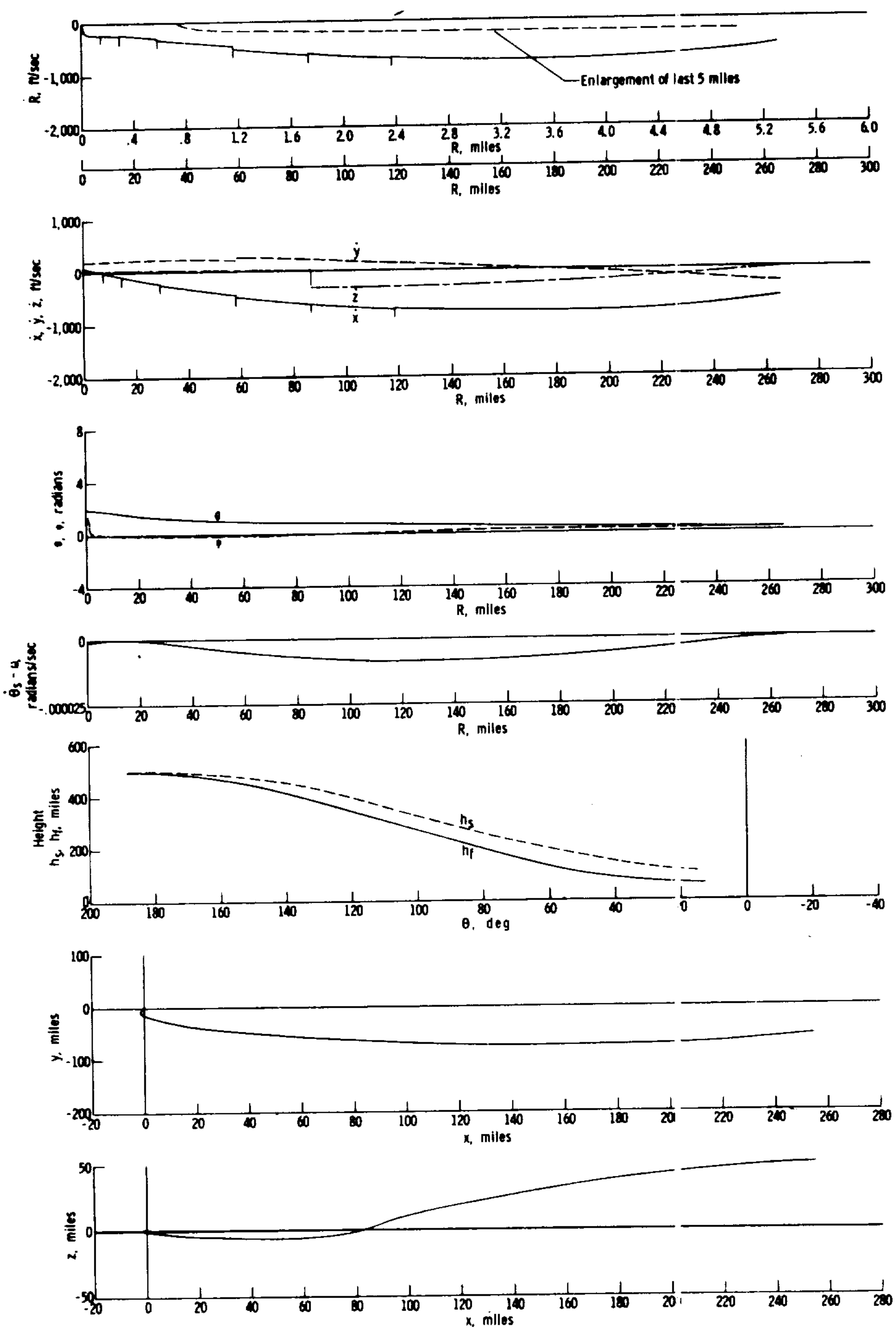


(a) Rendezvous occurs at $\Theta_s \approx 90^\circ$.

Figure 11.- Several trajectories showing the ferry guided to a rendezvous with an orbiting space station. Station is in 100-to-500-mile elliptic orbit with perigee at $\Theta_s = 0^\circ$. Velocity corrections were terminated when ferry was 5 miles from the station.



(b) Rendezvous occurs at $\Theta_s \approx 180^\circ$.

Figure 11.- Continued.

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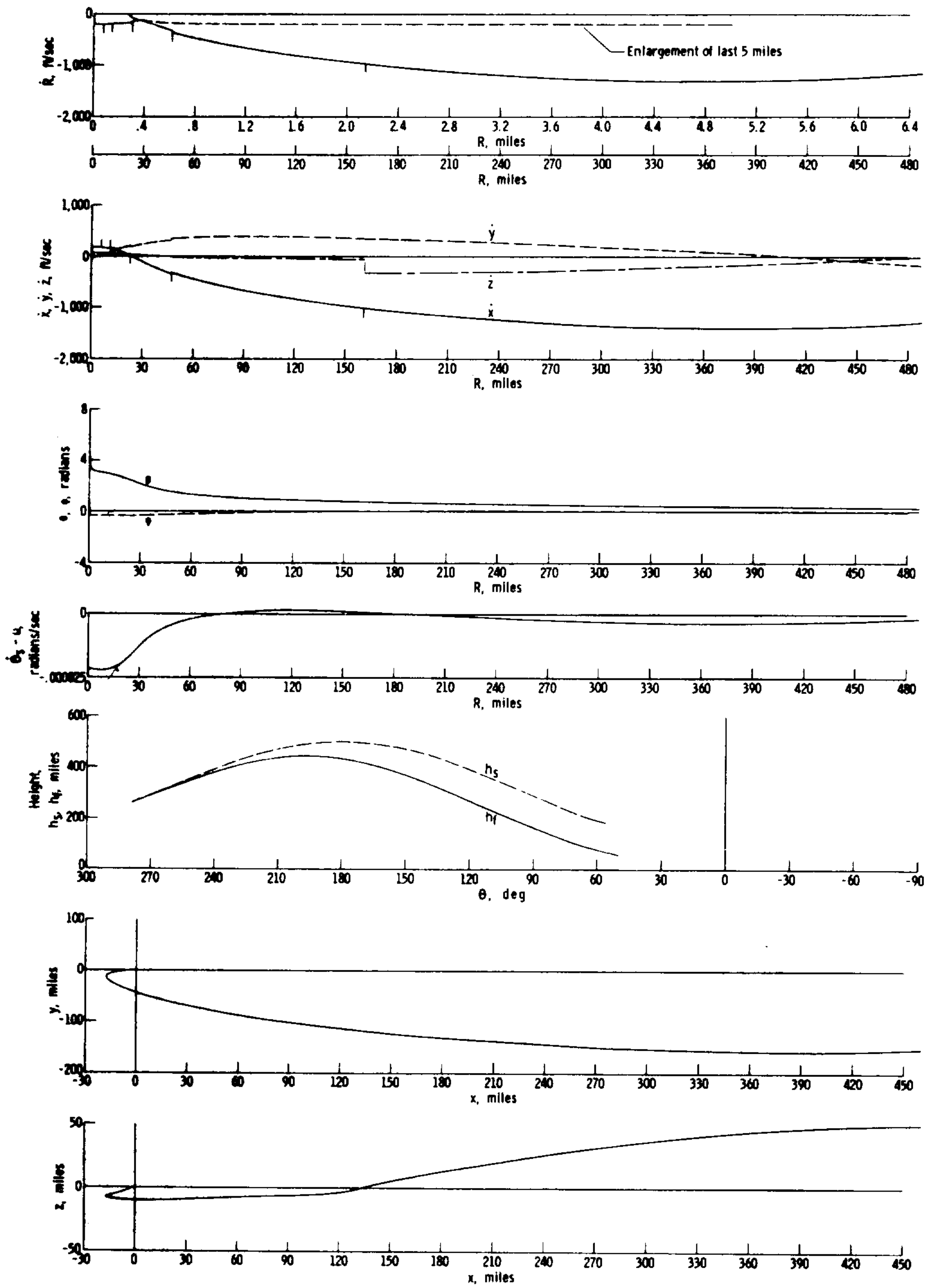

(c) Rendezvous occurs at $\theta_s \approx 270^\circ$.

Figure 11.- Concluded.

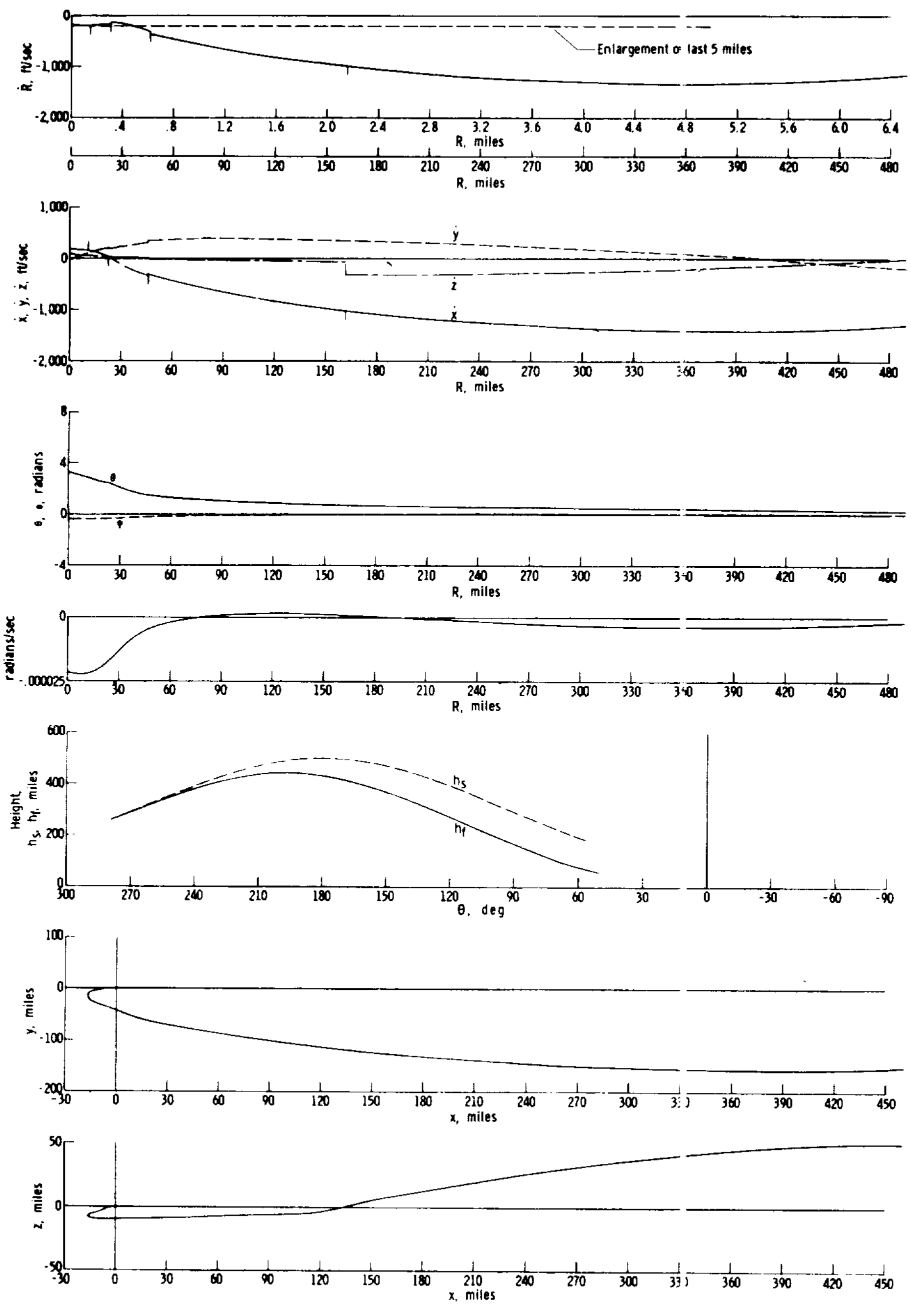

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Figure 12.- A trajectory of a ferry guided to a rendezvous with an orbiting space station. Station is in a 100-to-500-mile elliptic orbit with perigee at $\theta_s = 0^\circ$. Rendezvous occurs at $\theta_s \approx 280^\circ$. Velocity corrections were terminated when ferry was 200 feet from the station.

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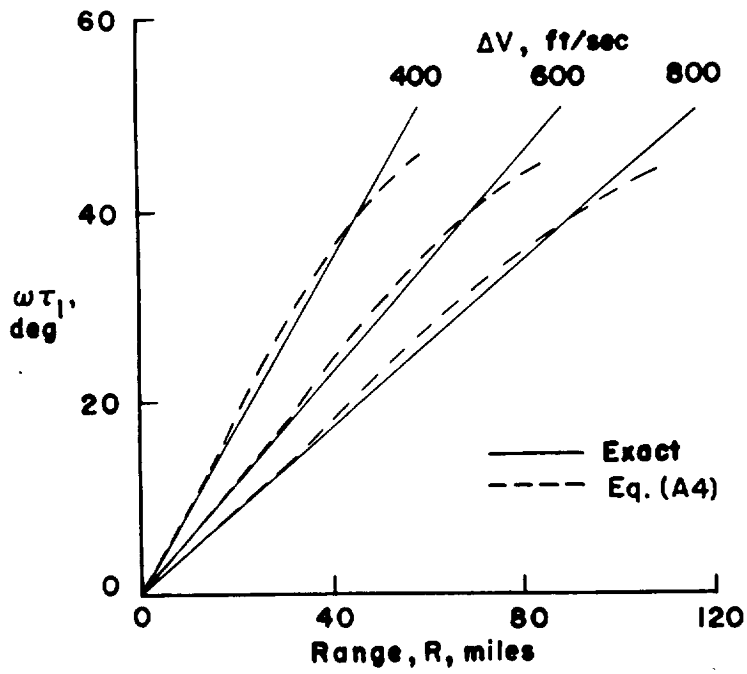

Figure 13.- Comparison of the exact angular distance $\omega\tau_1$ to intercept (no corrections required) and the value computed from $(\omega R/-\dot{R})$. Without terminal thrust control, collision would occur at relative velocity ΔV.