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RADIATION SHIELDING OF THE STAGNATION REGION BY

TRANSPIRATION OF AN OPAQUE GAS

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SUMMARY

The laminar compressible boundary layer in the two-dimensional and axisymmetric stagnation regions has been analyzed to show the effects of the injection of a radiation absorbing foreign gas on an incident radiation field, and on the enthalpy profiles across the boundary layer. Total heat transfer to the stagnation region is evaluated for numerous cases and the results are compared with the no shielding case. Required absorption properties of the foreign gas are determined and compared with properties of known gases.

INTRODUCTION

In some regimes of high-speed flight, the shock layer on a blunt body emits thermal radiation which is incident on the vehicle. This phenomenon becomes more prominent with increasing flight speed, diminishing altitude, and increasing nose radius. Kivel (ref. 1) analyzes the radiation problem in the inviscid shock layer. His results show, for example, that the radiation heat transferred to the stagnation region exceeds the convective heat transferred if the nose radius of curvature is greater than about 1.7 feet and the vehicle is traveling at escape speed at 200,000 feet altitude. The surface is assumed to be black and to accept all of the incident radiation.

The problem of shielding the vehicle surface from this thermal radiation becomes important. It is well known (ref. 2) that ordinary convective heating in the stagnation region can be greatly diminished by the injection of gas into the boundary layer. The possibility exists that the radiative heat transfer to the vehicle surface can also be diminished if the gas injected into the boundary layer is opaque to radiation. Of course, the absorption of radiation by the opaque gas raises its temperature and thus the temperature gradient of the boundary layer at the wall, and therefore increases the convective heat transferred to the vehicle. Thus the question arises, is a net saving of heat transfer achieved by injection of the opaque gas, and if so, what absorption properties need this gas have in order to be effective? The purpose of this paper is to study the effects of an opaque gas injected into the boundary layer on the combination of radiative and convective heat transfer in the stagnation region of bluff bodies traveling at hypersonic speeds. However, an exact analysis of the interaction of a radiation field with a mixture of air and a foreign gas in the compressible laminar boundary layer is a very difficult problem. First, because it requires the solution of a set of nonlinear, coupled, partial differential integral equations. Secondly, in the boundary layer where large temperature gradients exist, it is doubtful that Kirchoff's law can be used to relate emission and absorption properties of the mixture locally. For these reasons, an exploratory analysis has been made in which simplifying assumptions make the problem more tractable. It is expected that the results of the approximate analysis will retain the significant qualitative aspects of the actual physical behavior. If these results show that gains are achieved by use of an opaque gas, one would be encouraged to investigate the problem further, experimentally or theoretically.

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Programming the problem for solution on the electronic computer was done by Mrs. Yvonne Sheaffer.

SYMBOLS

a	slope of lines in figure 4 and equation (35)
c_p	specific heat at constant pressure
C	Chapman-Rubesin function, $\frac{\rho\mu}{\rho_e\mu_e}$
D	coefficient of diffusion of foreign species into air
f, F	dimensionless stream functions
$\vec{F}$	radiation flux
g	ratio of total enthalpy to enthalpy exterior to the boundary layer
h	static enthalpy of the mixture
I	radiation intensity (integrated over wave length)
j	total enthalpy, $h + \frac{u^2}{2}$
k	thermal conductivity
K	absorption coefficient, defined by equation (7)
l	dimensionless radiation intensity defined by equation (21)

Le	Lewis number, $\frac{Pr}{Sc}$
n	exponent in equation (1), zero for two-dimensional case and unity for axisymmetric case
p	pressure
Pr	Prandtl number, $\frac{c_p\mu}{k}$
q	total heat-transfer rate (convective and radiative)
r_o	radius of cross section of body of revolution
s	dimensionless transformed independent variable parallel to body surface (eq. (9))
Sc	Schmidt number, $\frac{\mu}{\rho D}$
u	velocity component parallel to surface
v	velocity component normal to surface
W	mass fraction of foreign species, $\frac{\rho_1}{\rho}$
x	distance along body surface measured from stagnation point
y	distance normal to body surface
α	dimensionless absorption coefficient defined by equation (26)
β	constant in velocity relationship (eq. (17))
η	dimensionless transformed independent variable normal to body surface (eq. (10))
μ	coefficient of viscosity
ρ	gas density
ψ	stream function
ξ	$\frac{\eta}{\sqrt{2}}$

Superscripts

', '', '''	derivative with respect to the independent variable concerned

Subscripts

e	flow exterior to the boundary layer
w	quantities at the wall
1	foreign absorbing gas in the boundary layer
2	air

ANALYSIS

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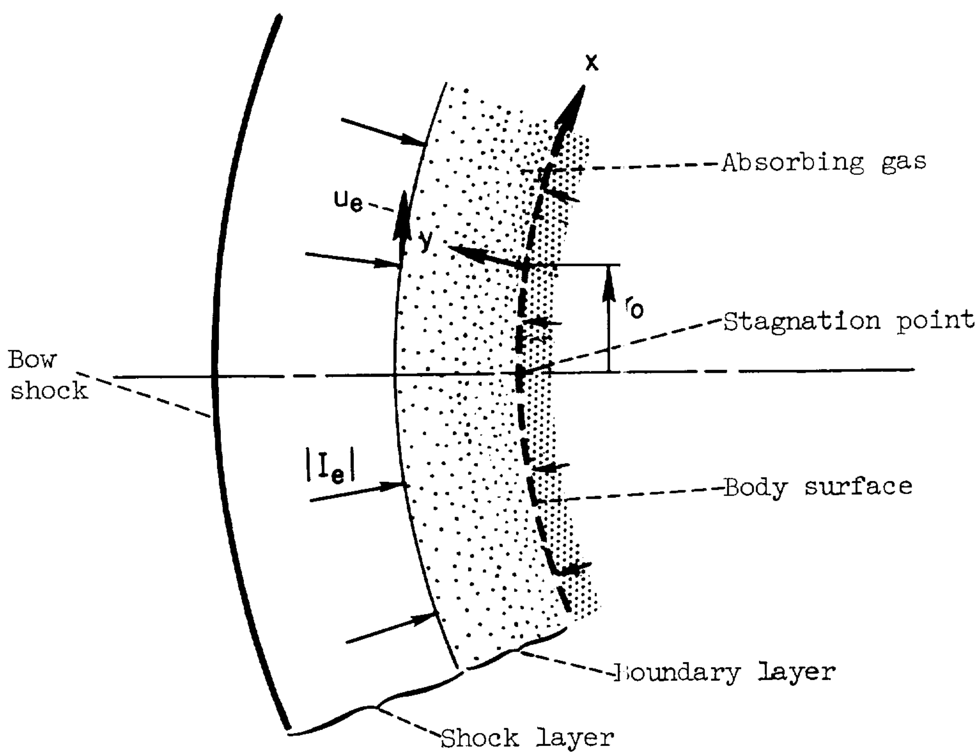
The physical model chosen for analysis is represented by sketch (a). The region between the shock wave and the body is divided into a shock layer of hot radiating air and a boundary layer consisting of a mixture of air and a foreign absorbing gas being injected through and normal to the porous body surface.



Sketch (a)

The following assumptions are made regarding the properties of these regions. Additional assumptions will appear and be discussed where they are needed in the analysis.

1. The radiant energy being emitted from the hot shock layer is incident on the outer edge of the boundary layer in beams of radiation of integrated (over wavelength) intensity I_e (ref. 3, eq. (5) and footnote of ref. 4, p. 53), all assumed to be traveling normal to the wall.

2. The air within the boundary layer is assumed to be transparent to the incident radiation and its emitted radiation is neglected in comparison with the incident radiation flux.

3. The injected foreign gas, which will absorb a portion of the incident radiation before it reaches the body surface, has absorption properties which are assumed to be independent of the wavelength of the radiation (i.e., it is a gray gas).

4. The mixture of air and the injected foreign gas in the boundary layer is assumed to be chemically inert.

5. The surface of the vehicle is assumed to be cold and black; thus it absorbs all incident radiation which reaches the surface and it emits no radiation.

Before proceeding to the mathematics of the problem, it should be noted that the assumption of a black vehicle surface is a limiting case. Ideally, if the surface of the vehicle were purely a reflector, it would not accept the incident radiation, and a nonabsorbing gas could be injected for the usual convective heating protection. Indeed, in that case an absorbing gas would be undesirable in that it would trap radiation energy in the boundary layer and probably increase the convective heat transferred to the reflective surface by raising the temperature gradient at the wall. However, the characteristics of the actual vehicle surface will be between those of a perfect reflector and of a black surface and at some degree of surface absorptivity it will become desirable to inject an absorbing foreign gas. To simplify the mathematical problem, the analysis which follows considers the limiting case of the black surface.

The partial differential equations describing the laminar compressible boundary layer of a binary mixture of gases in the presence of a radiation field are statements of continuity, the momentum theorem, conservation of energy, and diffusion of foreign species and are expressed, respectively, as

$$\frac{\partial}{\partial x}(\rho u r_o^n) + \frac{\partial}{\partial y}(\rho v r_o^n) = 0 \qquad (1)$$

$$\rho u \left(\frac{\partial u}{\partial x}\right) + \rho v \left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{\partial p}{\partial x} \qquad (2)$$

$$\rho u \frac{\partial j}{\partial x} + \rho v \frac{\partial j}{\partial y} = \frac{\partial}{\partial y}\left(\frac{k}{c_p}\frac{\partial j}{\partial y}\right) + \frac{\partial}{\partial y}\left[\mu\left(1 - \frac{1}{Pr}\right)\frac{\partial u^2/2}{\partial y}\right]$$

$$+ \frac{\partial}{\partial y}\left[\rho D \frac{\partial W}{\partial y}(h_1 - h_2)\right] - \mathrm{div}\,\vec{F} \qquad (3)$$

$$\rho u \frac{\partial W}{\partial x} + \rho v \frac{\partial W}{\partial y} - \frac{\partial}{\partial y}\left(\rho D \frac{\partial W}{\partial y}\right) = 0 \qquad (4)$$

The boundary conditions are

at $y = 0$

$$v = v_w,\ u = 0,\ j = j_w,\ W = W_w \qquad (5)$$

at $y \to \infty$

$$u \to u_e,\ j \to j_e,\ W \to C \qquad (6)$$

The exponent n in equation (1) is zero for two-dimensional flow and is unity for axisymmetric flow over a body of revolution. The third term on the right-hand side of equation (3) can be derived from reference 5, page 457, and arises from molecular diffusion of the two gases. It will be neglected by saying $C_{p_2} \approx C_{p_1}$ (and therefore $h_1 \approx h_2$) in order to simplify the treatment of the problem.

The last term in the energy equation (3) is the rate of gain of energy per unit volume due to the radiation flux. For the case where the element of gas volume is exchanging radiant energy (by absorption and emmission) with all other element gas volumes inside and outside the boundary layer, the radiation flux is expressed by integration over all space. Then the energy equation (3) is a partial differential integral equation which is exceedingly difficult to solve. The problem will be simplified by neglecting the radiant energy emitted by the absorbing gas in comparison with the radiant energy abosrbed. An energy balance is still maintained of course; enthalpy and other forms of energy are simply not diminished by emmission of radiation. This simplification is justifiable if (1) either the absorbing gas is a good absorber and poor emitter, or (2) if the rate of energy emmission from the foreign gas is small compared with the rate of energy absorption. The second instance above can be visualized under some circumstances. It can be expected a priori that a large mass fraction of the dense boundary-layer region (near the wall) will be absorbing gas, and a small mass fraction of the much less dense region (near the shock layer) will be absorbing gas. Therefore, the bulk of the absorbing gas is in the dense region which is cold compared with the shock layer. If the shock layer and boundary layer are behaving like black bodies and because they both emit energy at a rate proportional to the fourth power of their respective temperatures, then it is clear that the energy emitted by the shock layer (and thus absorbed by the foreign gas) is large compared with energy emitted by the boundary layer, and the latter can be neglected. For shock-layer emission deviating from black body conditions, the argument is weakened but not necessarily invalidated. For purposes of this exploratory analysis, the radiant energy emitted by the absorbing gas will be neglected in favor of the energy absorbed, and the results apply whenever this situation exists.

In order to express the last term in equation (3) we consider that the radiation traveling in the negative y direction (toward the wall) is absorbed by the foreign gas such that the fraction of the local integrated intensity absorbed in traveling the small distance $-\Delta y$ is proportional to the local density of the absorbing gas and to the path length; or in the form of reference 6 (pp. 5 and 24), for any fixed x,

$$\frac{\partial I}{\partial y} = K\rho_1 I \tag{7}$$

where K can be defined as an absorption coefficient. Because we have assumed that there is no scattering and have neglected emission in the boundary layer, there is only a y component of the radiation flux and thus the last term in the energy equation (3) is simply

$$\operatorname{div} \vec{F} = \frac{\partial I}{\partial y} \tag{8}$$

Equations (1), (2), (3), (4), and (7) are to be transformed from x and y as independent variables to s and η making use of the Levy transformation (ref. 7), a stream function, several definitions and assumptions and some exterior flow relationships as follows.

The Levy transformation is

$$s = \int_0^x \rho_e u_e \mu_e r_o^{2n} dx \tag{9}$$

$$\eta = \frac{u_e r_o^{\,n}}{\sqrt{2sC}} \int_0^y \rho \, dy \tag{10}$$

A stream function is defined so that

$$\frac{\partial\psi}{\partial y} = \rho u r_o^{\,n} \,, \qquad -\frac{\partial\psi}{\partial x} = \rho v r_o^{\,n} \tag{11}$$

and the continuity equation (1) is satisfied. The following quantities are defined

$$g(\eta) = j/j_e \tag{12}$$

$$\frac{\rho\mu}{\rho_e\mu_e} = C \qquad \text{(ref. 8)} \tag{13}$$

where C is assumed constant and

$$f'(\eta) = u/u_e \tag{14}$$

from which

$$f(\eta) = \psi/\sqrt{2sC} \tag{15}$$

In the axisymmetric blunt-body stagnation region it is assumed that

$$r_o = x \tag{16}$$

At the outer edge of the boundary layer, the external velocity is described by

$$u_e = \beta x \tag{17}$$

and it is assumed that

$$\rho_e\mu_e = (\rho_e\mu_e)_{\text{stagnation}} \tag{18}$$

Use of equations (16), (17), and (18) in equations (9) and (10) yields

$$s = \frac{\beta \rho_e \mu_e}{2(n + 1)} x^{2(n + 1)} \tag{19}$$

$$\eta = \sqrt{\frac{n + 1}{\beta C \rho_e \mu_e}} \int_0^y \rho \, dy \tag{20}$$

Thus for two-dimensional flow and axisymmetric flows, s is proportional to x^2 and x^4, respectively, while η is proportional to y modified by compressibility. A dimensionless-radiation flux is defined by

$$l(\eta) = \frac{I}{j_e \rho_e \mu_e u_e r_o^n} \sqrt{\frac{2s}{C}} \tag{21}$$

Formally transforming equations (2), (3), (4), and (7) to the new independent variables s and η by means of the newly defined quantities and the assumptions results in the following set of ordinary differential equations (with constant Pr and Sc)

$$f''' + ff'' = \frac{2s}{u_e}\left(\frac{du_e}{ds}\right)\left(f'^2 - \frac{\rho_e}{\rho}\right) \tag{22}$$

$$fg' + \frac{1}{Pr} g'' - l' = - \frac{u_e^2}{j_e} \frac{\partial}{\partial \eta}\left[\left(1 - \frac{1}{Pr}\right) f'f''\right] \tag{23}$$

$$fW' + \frac{1}{Sc} W'' = 0 \tag{24}$$

$$l' - \alpha W l = 0 \tag{25}$$

where

$$\alpha = K \sqrt{\frac{\rho_e \mu_e C}{(n + 1)\beta}} \tag{26}$$

The corresponding boundary conditions (5) and (6) transform to at $\eta = 0$

$$f = f_w,\ f' = 0,\ g = 0,\ W = W_w \tag{27}$$

at $\eta \to \infty$

$$f' \to 1,\ g \to 1,\ W \to 0,\ l \to l_e \tag{28}$$

where f_w is proportional to the injection rate (as will be shown subsequently) and the third of conditions (27) comes from the fact that the wall is much colder than the flow at the outer edge of the boundary layer. The last of conditions (28) is the boundary condition on equation (25), indicating that the local radiation intensity approaches the incident radiation intensity near the outer edge of the boundary layer. A4

The set of differential equations (22) through (25) would be more tractable if the right-hand sides of equations (22) and (23) were zero. 11 Then the equations would contain functions of only one independent variable η and therefore similarity solutions could be obtained. This approach will be used and the right-hand side of equation (22) will be neglected by virtue of the qualitative physical argument of reference 9 (based on the fact that the surface temperature is much lower than T_e). Now, of course, equation (22) with the right-hand side equal to zero is the familiar Blasius equation (ref. 10) where η is related to the Blasius ξ, and $f(\eta)$ and its derivatives are related to the Blasius $F(\xi)$ and its derivatives by

$$\left.\begin{aligned}
\eta &= \sqrt{2}\ \xi \\
f(\eta) &= F(\xi)/\sqrt{2} \\
f'(\eta) &= F'(\xi)/2 \\
f''(\eta) &= F''(\xi)/2\sqrt{2}
\end{aligned}\right\} \tag{29}$$

Equations (29) cause the boundary conditions on $f(\eta)$ to be compatible with the boundary conditions on $F(\xi)$ (ref. 11). Thus a solution of equation (22) with the right-hand side equal to zero can be obtained at once from reference 11.

Next, the right-hand side of equation (23) will be neglected because $u_e^2 \ll j_e$ for the stagnation region in hypersonic flow.

The boundary conditions (27) and (28) on the diffusion equation are mathematically sufficient, but are not very useful for initiating the numerical integration at $\eta = 0$. For this reason, it is necessary to choose initial values of W_w and W'_w so that (1) a mass balance on the air at the surface is satisfied for a given injection rate and (2) $W(\infty) \to 0$. The assumption that the air does not penetrate the wall through which the absorbing gas is injected, leads to the following mass balance on the air at the wall

$$\rho_w D_w \left(\frac{\partial W}{\partial y}\right)_w + \rho_w v_w (1 - W_w) = 0 \tag{30}$$

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which when transformed to the independent variable η becomes

$$W'_w = Sc\ f_w (1 - W_w) \tag{31}$$

Formally integrating equation (24) twice, making use of equation (22), the boundary conditions (27) and (28), and equation (31), leads to

$$W_w = \frac{1}{1 - \left[(f'_w)^{Sc} / f_w Sc \int_0^\infty (f'')^{Sc} d\eta\right]} \tag{32}$$

Equation (32) gives the correct value of W_w so that $W(\infty) \to 0$. Equation (31) gives the corresponding W'_w used to begin the numerical integration of equation (24).

In the radiation absorption law (eq. (25)), the absorption coefficient α (and therefore K) is assumed to be constant although in principle K could be any given function of temperature without destroying similarity.

Method of Solution

Equations (22) through (25) (with the right-hand sides equal to zero) subject to boundary conditions (27) and (28) were integrated numerically by the Adams-Moulton (ref. 12, p. 200, eqs. 6.6.2) predictor corrector method using the IBM 704 electronic data processing machine. Briefly, the sequence of machine computation was as follows: the quantities f_w, f'_w, f''_w, g_w, l_w, Pr, Sc, and α are specified (f_w and f''_w are obtained from reference 11 by use of equations (29)). Equation (22) (with the right-hand side equal to zero) was integrated numerically as a convenience for the machine computation and simultaneously the integral $\int_0^\infty (f'')^{Sc} d\eta$

was evaluated for use in equation (32). Initial values W_w and W'_w were obtained from equations (32) and (31), respectively. Two values of g'_w were arbitrarily chosen. Each value of g'_w was used separately to integrate equations (24), (25), and (23) simultaneously (working in that order). The values of $g(\infty)$ resulting from the two integrations were used to interpolate linearly to give an improved value on g'_w so that $g(\infty) \to 1$. The integration was repeated until the boundary conditions were satisfied.

In all of the numerical examples, Schmidt number and Prandtl number were taken to be 0.72 (and implicitly Lewis number is unity).

DISCUSSION OF RESULTS

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Let our first objective be to determine the radiation intensity at the wall in terms of that incident on the outer edge of the boundary layer. This information is obtained from the solution of equations (22), (24), and (25). The ratio of the intensity of radiation at the wall to that incident on the outer edge of the boundary layer is plotted as a function of α for two values of f_w in figure 1. The symbol f_w is the value of the stream function at the wall and can be shown to be directly proportional to the mass injection rate by virtue of equations (11), (15), (9), and (10), which are combined to yield

$$\rho_w v_w = - f_w \left(\rho_e u_e r_o^n \mu_e \sqrt{\frac{C}{2\xi}}\right) \tag{33}$$

An injection rate corresponding to $f_w = -1.238/\sqrt{2}$ leads at once to laminar separation (ref. 11 and eqs. (29)). This corresponds to an injection rate of about 0.023 pound per square foot second for a vehicle with a nose radius of 1.7 feet, flying at escape speed at 200,000 feet altitude. Therefore the injection rate shown in the figure corresponding to $f_w = -1/\sqrt{2}$ is substantial. The equation for the two curves shown in figure 1 is obtained by writing the formal solution of equation (25)

$$\frac{l_w}{l_e} = e^{-\alpha \int_0^\infty W \, d\eta} \tag{34}$$

The integral $\int_0^\infty W \, d\eta$ has the values of 2.73 and 0.968 for $f_w = -1/\sqrt{2}$ and $-1/2\sqrt{2}$, respectively.

It is seen that at a given value of the absorption coefficient α for the higher injection rate ($f_w = -1/\sqrt{2}$), much more radiation is absorbed in the boundary layer before reaching the wall. It can be noted that if α is about 2 or greater, almost all of the incident radiation is absorbed before reaching the wall.

Next, it is interesting to examine in detail how the radiation intensity diminishes from the outer edge of the boundary layer to the wall. Profiles of the radiation intensity across the boundary layer are shown in figure 2 for various values of the absorption coefficient and for the stronger blowing case $f_w = -1/\sqrt{2}$. For increasing values of α, the region in which most of the absorption takes place tends to move away from the wall ($\eta = 0$).

The absorption of radiant energy within the boundary layer raises the local enthalpy level. The influence of absorption on enthalpy can be shown from solutions of the energy equation (23) which yields profiles of total enthalpy across the boundary layer. Approximately 40 examples were computed; some typical results are shown in figure 3. The lower curve corresponds to either no incident radiation or no absorption. For either increasing incident radiation intensity and a given absorption coefficient or increasing absorption coefficient and a given incident radiation intensity, the total enthalpy level at a given location (away from the wall) increases.

The dashed curve corresponds to a very large specified l_e. It "overshoots" (i.e., $g > 1$) indicating that the local total enthalpy (and temperature if it is taken to be roughly proportional to total enthalpy) in the boundary layer exceeds that exterior to the boundary layer. This situation is not compatible with the physical model being analyzed. In the first place, energy cannot be transferred from a lower to higher temperature without an input of work. Secondly, for higher values of $-l_e$, the overshoot would be larger and would penetrate farther into the boundary layer, violating the condition that the bulk of the absorbing gas is at a temperature considerably less than that of the exterior flow. Finally, the neglected reradiation would tend to smooth out and diminish the overshoot. There is probably a region around the onset of overshoot where the solutions are of diminishing value. However, for lack of other criteria, solutions are presented up to, but not into, the overshoot region. A different analysis would be necessary to study the behavior of the high $-l_e$ conditions.

It can be seen in figure 3 that the enthalpy gradient at the wall increases with increasing α and $-l_e$. This gradient is of major significance because it is proportional to the convective rate of heat transfer to the wall. It is obvious that the reduction of radiant heat transfer resulting from absorption tends to be offset by an increase of convective heat transfer. Before evaluating this situation, it is advantageous to look at the influence of absorption coefficient and incident radiation intensity on the enthalpy gradient at the wall in figure 4. Two features of this figure are of particular interest. First, for a given α and f_w, g'_w appears to be linear with l_e; that is,

$$g'_w = (g'_w)_{l_e=0} - a l_e \qquad (35)$$

where $a = a(\alpha)$ is the slope of the straight line corresponding to a given α. Secondly, the curves for $\alpha = 1.0$ and 1.5 appear to lie out of order, between the $\alpha = 0.1$ and 0.5 curves rather than above them. That is, for a given incident radiation flux $(-l_e)$, increasing the absorption coefficient (α) first increases the enthalpy gradient at the wall (g'_w), but for higher values of α, the wall enthalpy gradient tends to diminish toward the no absorption value. Physically, this decrease in wall enthalpy gradient for higher values of the absorption coefficient indicates that the incident radiation energy is being absorbed farther out in the boundary layer and influences the wall enthalpy gradient less.

The single point for $\alpha = 0.3$ shown in the figure was computed to see if it lies in order, which it does (between the $\alpha = 0.1$ and 0.5 curves).

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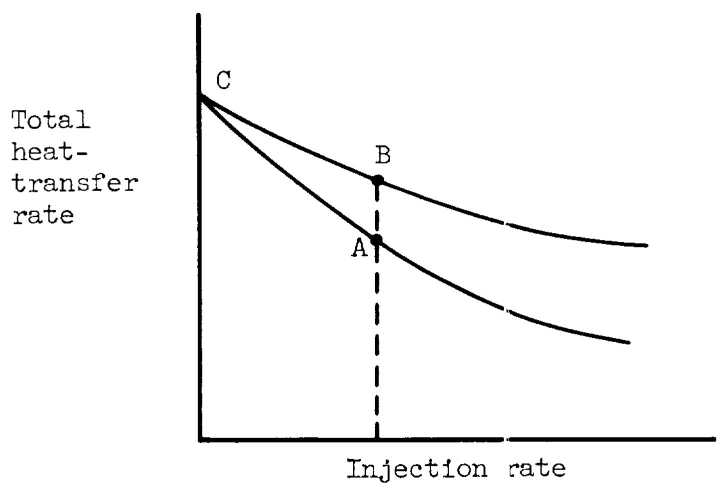
So far, it has been shown that in shielding the wall from radiant energy the convective heat transfer to the wall is increased. It now must be shown whether or not a net saving in heat transfer results from injection of an absorbing gas. Injecting a nonabsorbing gas into the boundary layer will diminish the total heat-transfer rate to the wall by reducing the convective part of the heat-transfer rate, as for example, is diagrammed by the upper curve in sketch (b). The injection of an absorbing



Sketch (b)

gas might be expected to diminish the total heat-transfer rate even further (such as shown by lower curve in sketch (b)). The total heat-transfer rate with injection of an absorbing gas will be compared with that with injection of a nonabsorbing gas at the same rate (A will be compared with B). The dimensionless total heat-transfer rate to a wall through which an absorbing gas is injected is obtained by summing the convective and radiative heat-transfer rates and dividing by $j_e\sqrt{(n + 1)C\beta(\rho_e\mu_e)_{stagnation}}$.

Thus

$$\frac{q \text{ with absorption}}{j_e\sqrt{(n+1)C\beta(\rho_e\mu_e)_{\text{stagnation}}}} = l_w + \left(\frac{-g'_w}{Pr}\right) \tag{36}$$

Similarly, the dimensionless total heat-transfer rate to a wall through which a nonabsorbing gas is injected at the same rate is

$$\frac{q \text{ no absorption}}{j_e\sqrt{(n+1)C\beta(\rho_e\mu_e)_{\text{stagnation}}}} = l_e + \frac{(-g'_w)_{\alpha=0}}{Pr} \tag{37}$$

Comparing the heat-transfer rate with absorption to that without absorption at a given injection rate, by use of equations (36) and (37), yields

$$\frac{q \text{ with absorption}}{q \text{ no absorption}} = \frac{l_w Pr - g'_w}{l_e Pr - (g'_w)_{\alpha=0}} \tag{38}$$

The comparison is shown in figure 5(a) for an injection rate corresponding to $f_w = -1/\sqrt{2}$. The figure shows that a definite advantage is obtained by injecting an absorbing gas. The effectiveness of this method of shielding against excessive heating increases with increasing absorption coefficient and increasing incident radiation intensity. For $\alpha = 1.5$ and $l_e = -0.5$, the total heat-transfer rate at the wall is diminished by approximately 2/3. The region to the right of these curves corresponds to the enthalpy overshoot condition and, because the analysis is not valid in that region, the curves are not extended beyond the overshoot boundary.

It is worth noting that a combination of equations (38), (35), and (34) yields an expression for the curves of figure 5(a) (although the curves were not obtained by this expression). It is

$$\frac{q \text{ with absorption}}{q \text{ no absorption}} = \frac{l_e\left[a(\alpha) + Pr\ e^{-\alpha\int_0^\infty W\,d\eta}\right] - (g'_w)_{l_e=0}}{l_e Pr - (g'_w)_{l_e=0}} \tag{39}$$

Using for example the value 2.73 for the integral (see below eq. (34)), 0.72 for Pr, 0.0714 for $(g'_w)_{l_e=0}$ and, from figure 4, the value $a = 0.160$ corresponding to $\alpha = 1$ yields the simple relationship

$$\frac{q \text{ with absorption}}{q \text{ no absorption}} = \frac{0.287\, l_\epsilon - 0.0992}{l_e - 0.0992} \qquad (40)$$

which can be used to describe the appropriate curve in figure 5.

It is also of interest to compare the total heat-transfer rate during injection of an absorbing gas to the total heat-transfer rate without any injection (A vs C in sketch (b)). The dimensionless total heat-transfer rate at the wall without injection is

$$\frac{q \text{ no injection}}{j_e \sqrt{(n+1)C\beta(\rho_e\mu_e)_{\text{stagnation}}}} = l_e - \frac{(g'_w)_{f_w=0}}{Pr} \qquad (41)$$

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The ratio of equation (36) to (41) is

$$\frac{q \text{ with absorption}}{q \text{ no injection}} = \frac{l_w Pr - g'_w}{l_e Pr - (g'_w)_{f_w=0}} \qquad (42)^1$$

The comparison is shown in figure 5(b). The injection rate for the absorption condition corresponds to $f_w = -1/\sqrt{2}$. Here, of course, the effect is very pronounced. Examination of the figure shows first of all that it is advantageous to inject a gas whether it absorbs radiation or not. As seen before, the heat-transfer rate will be diminished more by injection of an absorbing gas. However, for low levels of incident radiation intensity, a large absorption coefficient is not much more effective than a small absorption coefficient in reducing the total heat-transfer rate. Again, for large values of incident radiation intensity, large absorption coefficients are very advantageous. In particular, for $\alpha = 1.5$, the heat-transfer rate is only 1/5 that for no injection at all for all values of incident radiation intensity, and less than 1/2 that for injecting a nonabsorbing gas if $l_e = -0.5$.

Finally, something should be said about the actual absorption coefficient K. The required K is dependent on the flight condition by virtue of its relationship with α (eq. (26)). For a flight speed of 34,000 feet per second at 175,000 feet altitude, and body-nose radii of 1 to 10 feet, the required K is of the order of 10^5 to 10^7 ft^2/slug for values of α from 0.1 to 1.5. Little is known of the absorption properties of even ordinary gases. Measurements by Eckert and others (ref. 5, pp. 384-386) can be interpreted to give an approximate "grey" K for purposes of this discussion. It appears that carbon dioxide has an absorption coefficient of the order of 10 sq ft per slug at ordinary pressures, for temperatures up to 1600° C. The coefficient for water

[1]Equations (42), (35), and (34) could be combined to give the heat-transfer comparison of injection with absorption to the no injection case corresponding to equation (39).

vapor is roughly an order of magnitude higher than that of carbon dioxide, and so the gas we want to blow into the boundary layer should be 3 to 5 orders of magnitude better than water vapor in this regard.

It is encouraging to find that at least one gas has a high absorption coefficient. That gas is cesium vapor, which has an absorption coefficient of the order of 10^8 square feet per slug at 0° C and 1-mm Hg in the wavelength region 2000 to 3000 Å (which can be shown from ref. 13). Demonstrably then, such opaque gases exist, and the problem is to find a suitable one.

CONCLUDING REMARKS

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There are several notable results of this exploratory analysis of the effects of the injection of an opaque gas in the stagnation region of a blunt body traveling at hypersonic speed. At a black vehicle surface the reduction of the radiation heat-transfer rate by radiation absorption in an injected opaque gas is accompanied by an increase in the convective heat-transfer rate. However, the net effect is that a saving in total heat-transfer rate (radiative plus convective) of as much as 2/3 can be achieved by injecting the absorbing gas into the boundary layer. The opaque gas must have a high absorption coefficient (3 to 5 orders of magnitude higher than water vapor) in order to effectively reduce the total heat-transfer rate to the body. It is pointed out that under some conditions, cesium vapor has an absorption coefficient 6 orders of magnitude higher than water vapor. It would be worthwhile to search for gases that have, for actual flight conditions of interest, suitably high absorption coefficients as well as desirable injection properties.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., May 17, 1960

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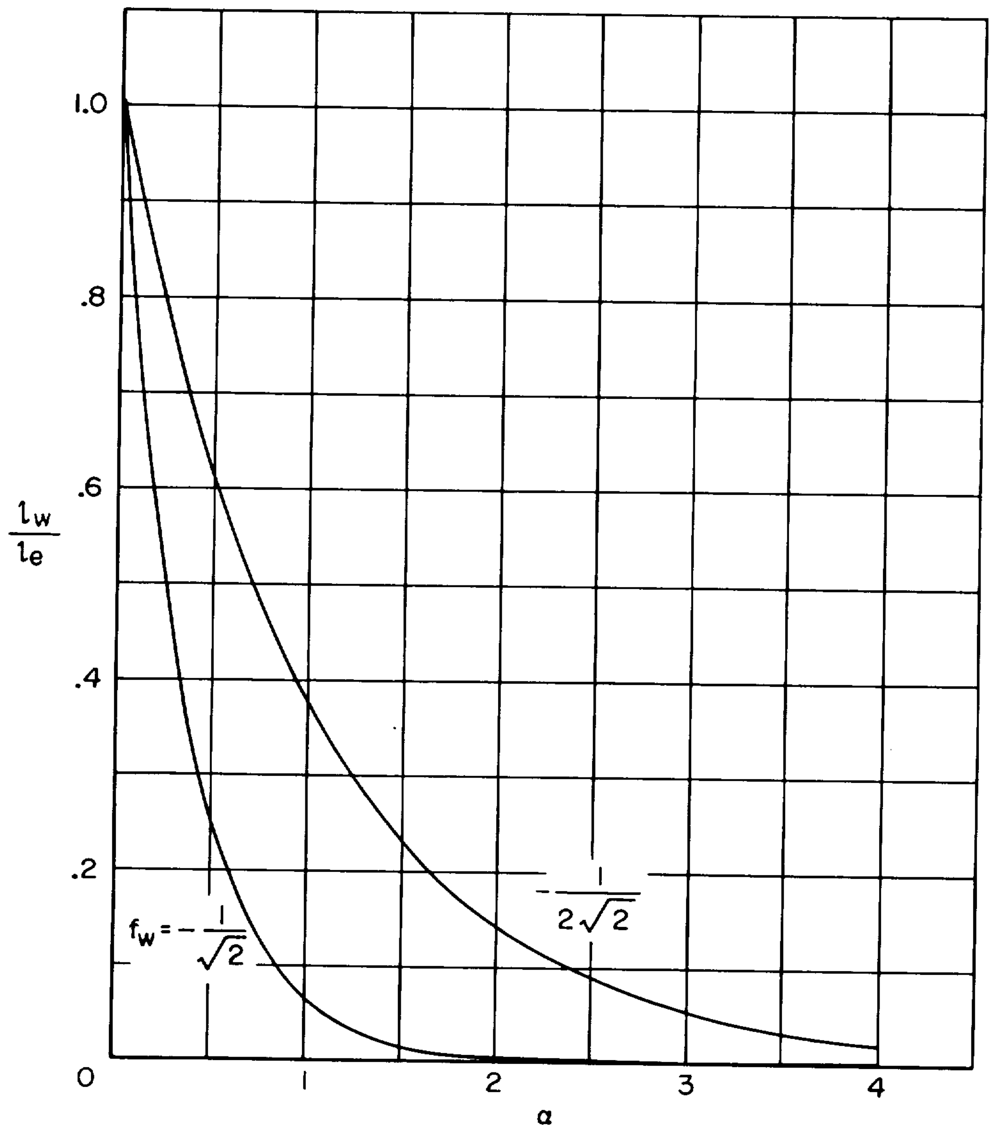

Figure 1.- Influence of absorption coefficient on radiation intensity at the wall.

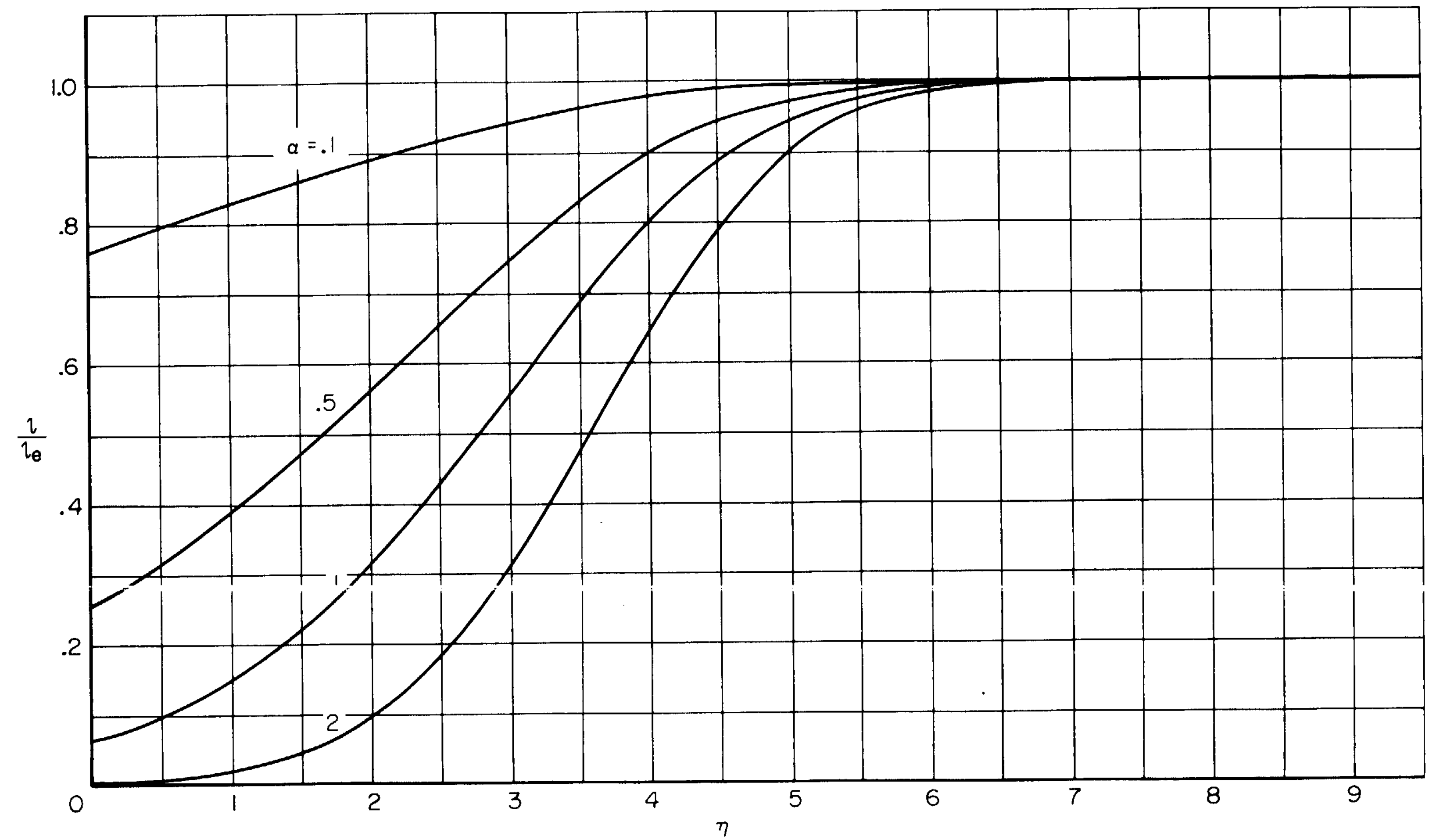

Figure 2.- Influence of absorption coefficient on radiation intensity profiles for $f_w = -1/\sqrt{2}$.

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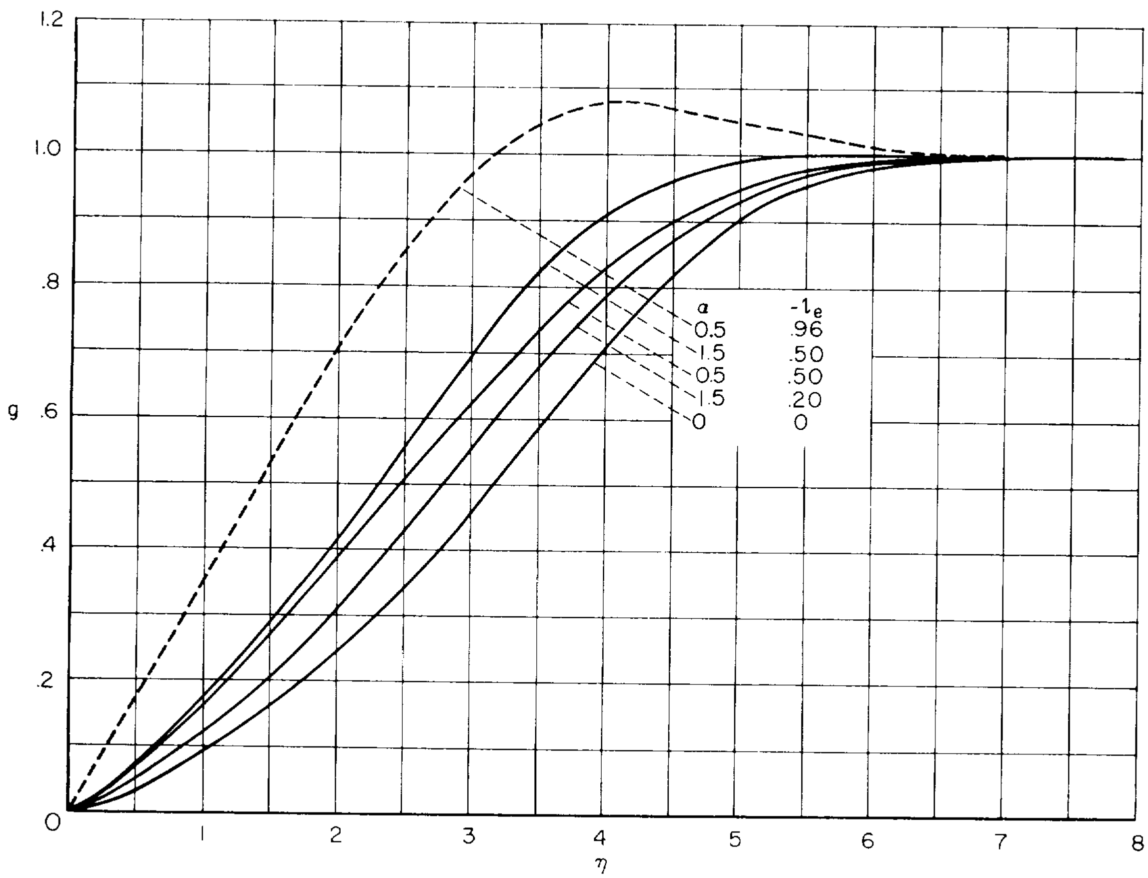

Figure 3.- Influence of radiation absorption on enthalpy profiles for $f_w = -1/\sqrt{2}$.

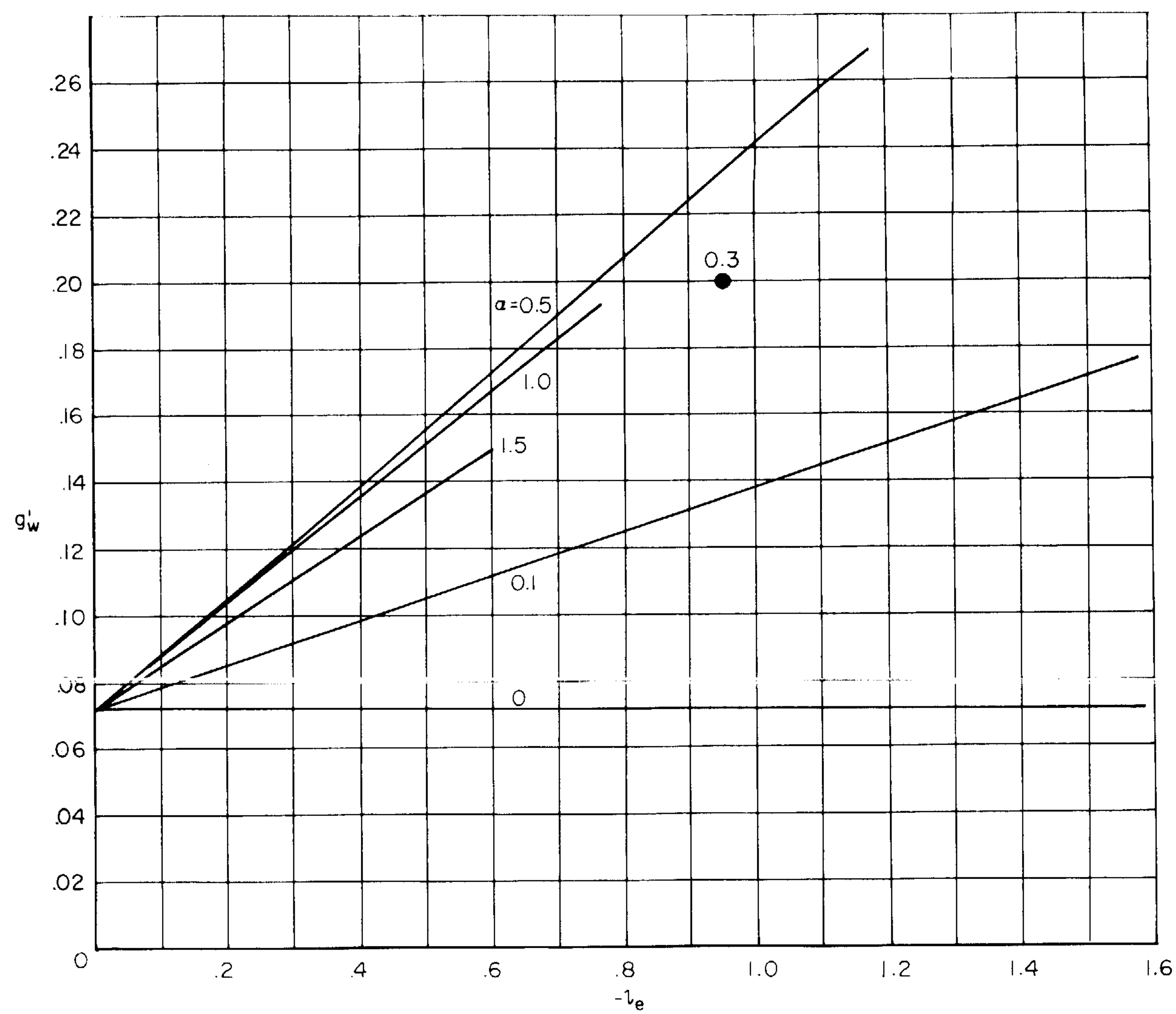

Figure 4.- Influence of radiation absorption on enthalpy gradient at the wall for $f_w = -1/\sqrt{2}$.

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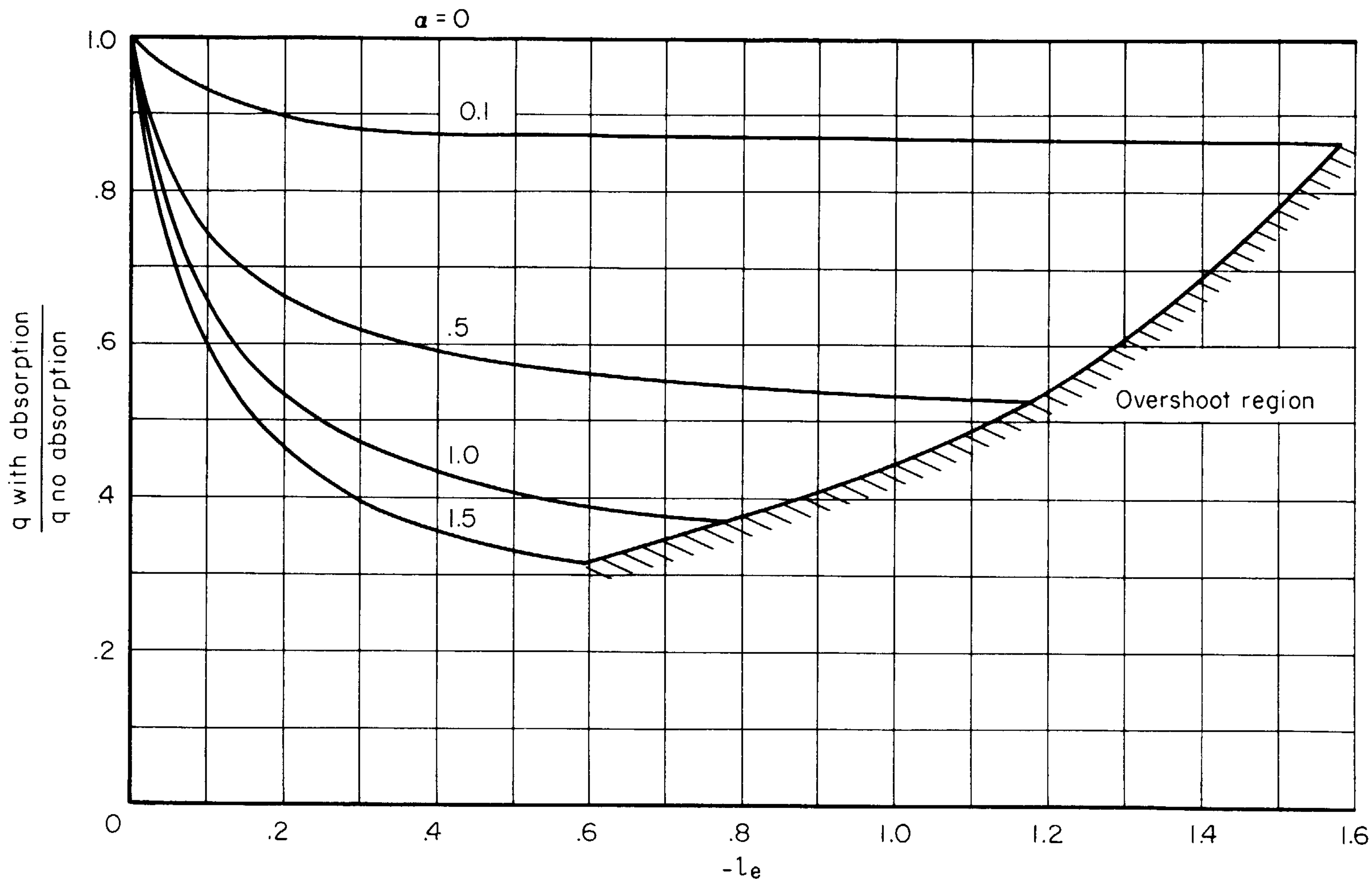
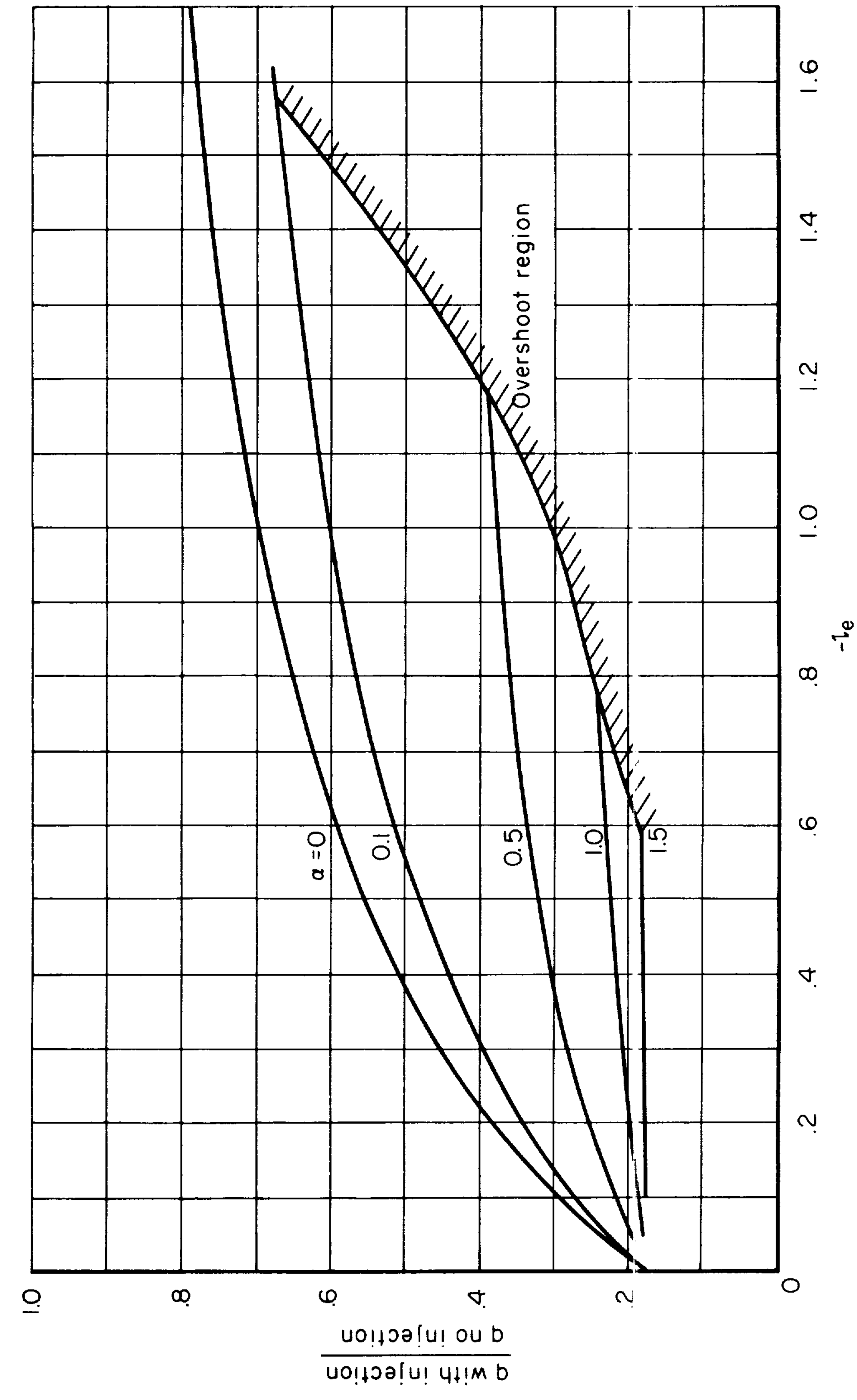


(a) Comparison of injecting absorbing gases with injecting a nonabsorbing gas.

Figure 5.- The effect of gas injection at a rate corresponding to $f_w = -1/\sqrt{2}$ on the total heat-transfer rate.



(b) Comparison of injecting an absorbing gases with the noninjection case.

Figure 5.- Concluded.

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National Aeronautics and Space Administration.
RADIATION SHIELDING OF THE STAGNATION REGION BY TRANSPIRATION OF AN OPAQUE GAS. John Thomas Howe. September 1960. 24p. OTS price, $0.75. (NASA TECHNICAL NOTE D-329)

The laminar compressible boundary layer in the two-dimensional and axisymmetric stagnation regions has been analyzed to show the effects of the injection of a radiation-absorbing foreign gas on an incident radiation field, and on the enthalpy profiles across the boundary layer. Total heat transfer to the stagnation region is evaluated for numerous cases and the results are compared with the no shielding case. Required absorption properties of the foreign gas are determined and compared with properties of known gases.

(Initial NASA distribution: 2, Aerodynamics, missiles and space vehicles; 15, Chemistry, physical; 20, Fluid mechanics.)

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