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A SELF-ADAPTIVE MISSILE GUIDANCE SYSTEM

FOR STATISTICAL INPUTS

By H. Rodney Peery

Ames Research Center
Moffett Field, Calif.

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## SUMMARY

A method of designing a self-adaptive missile guidance system is presented. The system inputs are assumed to be known in a statistical sense only. Newton's modified Wiener theory is utilized in the design of the system and to establish the performance criterion. The missile is assumed to be a beam rider, to have a g limiter, and to operate over a flight envelope where the open-loop gain varies by a factor of 20. It is shown that the percent of time that missile acceleration limiting occurs can be used effectively to adjust the coefficients of the Wiener filter. The result is a guidance system which adapts itself to a changing environment and gives essentially optimum filtering and minimum miss distance.

## INTRODUCTION

The analytical design of missile guidance systems is in general complicated by many factors. Three of the most troublesome factors are the random character of the input signals, the inherent g limitation of the missile, and the wide variation in open-loop gain of the system.

The first of these three factors, the random character of the input signals, clearly suggests that the design should be carried out on a statistical basis. Stewart (ref. 1) was the first to report on the statistical design of a beam-rider missile guidance system. This was an application of the classic paper by Wiener (ref. 2). Stewart's initial effort (ref. 1) showed one method of approximating the ideal Wiener filter. It also illustrated the fact that a considerable reduction in mean-square miss distance could be obtained by using statistical design methods rather than transient response methods.

The second design problem is created by the inherent g limitation of the missile and is obviously nonlinear in character. A direct analytical approach to this problem is extremely difficult because of the lack of an optimum general nonlinear design method. However, Stewart (ref. 3) applied the modified Wiener theory, which was suggested by Newton (ref. 4),

to this problem. This was an indirect approach because it depended upon placing a constraint on the mean-square acceleration rather than the actual acceleration of the missile. Under certain conditions, this indirect approach has proved to be a very useful design tool. Using Newton's method, Stewart, et al.(ref. 5), showed that the three most important factors affecting miss distance are target acceleration, glint noise, and the ratio of missile acceleration to target acceleration. He also showed that there is only a small increase in miss distance due to missile dynamics. Thus, for purposes of analysis, the missile can, in general, be treated as a simple gain constant, the value of which is dependent upon Mach number and altitude.

The third design problem considered is created by the large variation in the open-loop gain of the system. The predominant cause of this gain variation is the large change in dynamic pressure, q, which the missile encounters over its flight envelope. The fact that the controlled system (i.e., missile) changes complicates the problem to a large degree and also focuses attention on the need for a self-adaptive guidance system. Considerable effort has recently been expended on the problem of open-loop gain variation in the design of self-adaptive autopilots for aircraft (ref. 6). Some of the concepts which have been useful in the design of the self-adaptive autopilots will be utilized here. For example, interest will be centered upon the use of internal measurements of system dynamic performance to effect continuous adjustment of the controller parameters. Optimum operation of the system, as defined by a predetermined performance criterion, will be the primary design objective.

Thus, it is the scope of this paper to consider the design of a simplified beam-rider missile guidance system where the inputs are known in a statistical sense, where saturation (g limiting) exists, and where the open-loop gain changes as a function of dynamic pressure. A method is presented for designing systems that will automatically adjust the Wiener filter part of the guidance system as a function of the percent of time of acceleration limiting. The result is a system that is self-adaptive; that is, the system automatically adjusts itself in such a manner that essentially minimum mean-square miss distance results even though the gain of the missile varies over a wide range.

NOMENCLATURE

$a_M$    missile acceleration, ft/sec$^2$

$a_{MC}$    missile acceleration command signal, ft/sec$^2$

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| $\overline{a_{MD}}$ | missile acceleration, ft/sec$^2$ rms (bar indicates rms quantity) |
| $a_{ML}$ | missile acceleration limit, ft/sec$^2$ ($a_{ML}$ = maximum value of $a_M$ = 20 g) |
| D(s) | optimum Wiener filter transfer function (s is complex variable, $\sigma + j\omega$) |
| g | acceleration of gravity, 32.2 ft/sec$^2$ |
| $K_F$ | open-loop gain of Wiener filter |
| $K_M$ | open-loop gain of missile |
| KG(s) | open-loop transfer function of optimum Wiener filter and missile combination |
| N | magnitude of white noise spectral density, ft$^2$-sec |
| $p_i$ | numerator coefficients of optimum Wiener filter |
| $q_i$ | denominator coefficients of optimum Wiener filter |
| q | dynamic pressure, lb/ft$^2$ |
| w | input signal to servo which adjusts Wiener filter |
| $\bar{x}$ | (percent time limiting)/100 |
| y | reference signal in percent time limiting detector |
| $y_M$ | displacement of missile from reference, ft |
| $y_N$ | displacement of radar beam due to noise, ft |
| $y_T$ | displacement of target from reference, ft |
| $\ddot{y}_T$ | acceleration of target, ft/sec$^2$ |
| $\delta$ | control surface deflection, deg |
| $\epsilon$ | system error signal, $y_T - y_M$, ft |
| $\bar{\epsilon}$ | rms error (or rms miss distance), ft rms |
| $\epsilon_A$ | apparent error, $y_N + y_T - y_M$, ft |
| $\Phi_{NN}(s)$ | power spectral density of "white" radar noise, ft$^2$-sec |
| $\Phi_{y_T y_T}(s)$ | power spectral density of target displacement, ft$^2$-sec |

## DESCRIPTION OF THE PROBLEM

### Guidance Problem

Stewart (ref. 1) has shown that a beam-rider guidance system can be represented by the block diagram of figure 1 if certain simplifying assumptions are made. The inputs to the system are the target position, $y_T$, and the radar glint noise, $y_N$. The output of the system is missile position, $y_M$. The error of the system, $\epsilon$, is a quantity which cannot be measured because $y_T$ and $y_N$ actually come in at the same point and are physically inseparable. However, the error $\epsilon$, which can be expressed as miss distance in feet, is the quantity that we desire to minimize through proper design. The apparent error, $\epsilon_a$, is the only error quantity that can be physically measured. It is seen that $\epsilon_a = y_T + y_N - y_M$.

It is assumed that the quantities $y_N$ and $y_T$ are known only in a statistical sense and that they are uncorrelated. To be more specific, units and input magnitudes have been chosen for design purposes so that the power spectral densities of the noise and target position are given, respectively, by

$$\Phi_{NN}(s) = N, \text{ a constant}$$
$$= 100 \text{ ft}^2 \text{ sec} \tag{1}$$

$$\Phi_{y_T y_T}(s) = \frac{829.47}{s^4(0.16 - s^2)} \text{ ft}^2 \text{ sec} \tag{2}$$

It is apparent that the noise signal is assumed to be white noise with a constant spectral density. The value of the noise given in equation (1) corresponds approximately to glint noise from a medium sized bomber. The spectral density of the target signal, $y_T$, has been derived from the assumption that the target maneuvers laterally with a maximum acceleration of 1g first in one direction, then in the other. Thus the target acceleration signal is a rectangular wave of ±1g with zero crossings which have a Poisson distribution and an average period of 5 seconds.

The missile is also assumed to have a velocity advantage over the target. Thus a hit will be assured if the error in following the ±1g random maneuver of the target is made small enough. This error can never be zero because of the noise which contaminates the input signal. However, the analytical design technique (modified Wiener theory) used in this report does yield systems which have a smoothing effect on the noise; that is, the effect of the noise is minimized by proper design.

It should be noted that the above assumptions are essentially the same as those used by Stewart in previous beam-rider studies (refs. 1, 3, and 5). A minor variation, however, occurs in the form of the power

spectral densities of the target maneuver and the noise as a result of using slightly different definitions of the Fourier and inverse Fourier integrals.

The main justification for using the previously described inputs is that they can be described statistically in convenient mathematical forms. However, they are also believed to be good approximations for this design problem.

Figure 1 shows that the apparent error, $\epsilon_a$, is the input to the Wiener filter, $D(s)$. The function of $D(s)$ is to provide the desired filtering and also stability to the over-all system. Initially $D(s)$ is considered as a linear network. This restriction will be removed later. The output of $D(s)$ is the missile acceleration command signal, $a_{MC}$,[1] which is the input to the missile-autopilot combination; $\delta$ is the control surface excursion; and the $\pm L$ limits on the control surface servo indicate that control surface excursions are limited. For design purposes, it is assumed that the acceleration of the missile is limited to $\pm 20g$ by the structural load limit of the missile. The open-loop gain of the missile, $K_M$, will be allowed to vary as a function of dynamic pressure, q. The range of variation of $K_M$ will be from 20 to 1 ft/sec$^2$/degree and the servo limit, L, will be assumed to be $32.2^\circ$.

## Missile Autopilot Problem

The missile autopilot block shown in figure 1 is a much simplified approximation to a real system. Consider the slightly more complicated version shown in figure 2(a). The control requirements of the autopilot can now be defined as follows:

(a) If $a_{MC} > 20g$ and $K_M L > 20g$, then $a_{ML} = 20g$

(b) If $a_{MC} < 20g$ and $K_M L > a_{MC}$, then $a_M = a_{MC}$

(c) If $a_{MC} < 20g$ and $K_M L < a_{MC}$, then $a_M = K_M L$

To meet these requirements, the autopilot must provide a transfer function, $a_M/a_{MC}$, which is independent of the gain, $K_M$, in the linear region; furthermore, it must limit the maximum acceleration of the missile to 20g, the assumed structural load limit. It can be seen from figure 2(a) that the product of missile gain, $K_M$, and the limit level, L, is the maximum maneuvering capability of the missile. If $K_M L > 20$, the missile can exceed the structural load limit. Requirement (a) prevents this occurrence. The fact that $K_M$ will be allowed to vary by a factor of 20 adds considerable difficulty to the problem of meeting the above autopilot control requirements. However, McLean and Schmidt (ref. 7) have recently shown

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[1]It should be noted that the units of $a_M$, $a_{MC}$, and $a_{ML}$ are given in ft/sec$^2$ in figure 1 and subsequent figures. However, for convenience, all acceleration quantities in the text and also in the figures will be given in gravitational units or g's.

that the effect of gain changes in the controlled element (missile) can
be reduced by means of a very high gain in the forward loop. Their
solution is shown in figure 2(b). A study shows that the configuration
in figure 2(b) meets the requirements previously set forth. Furthermore,
it is apparent that the entire missile autopilot can now be treated as a
unity gain limiter with variable limits rather than considered as a fixed
limit level and a changing gain, $K_M$. Viewed in this manner, figure 2(b)
is equivalent to that shown in figure 2(c). The limit level, $a_{ML}$, is
variable from 1g up to 20g, the assumed structural load limit.

This treatment of the missile autopilot is admittedly quite simple.
The dynamics of the missile, instruments, servo, etc., certainly must be
considered in any practical system; however, it is believed that the high
gain system described in reference 7, if properly designed, could result
in a missile autopilot which has the property of giving a transfer func-
tion, $a_M/a_{MC}$, which is independent of flight conditions in the linear
region. The model of such an autopilot might then be reasonably
approximated by the limiter of figure 2(c), followed by a linear, time-
invariant, transfer function.

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## DESIGN OF THE GUIDANCE SYSTEM

In view of the above discussion concerning inputs, limits, and
autopilot design, the guidance system design problem can be restated as
follows. Design the controller of figure 3 so that the mean-square
error, $\overline{\epsilon^2}$, will be minimized when the statistical inputs are given by
equations (1) and (2) and the limit level, $a_{ML}$, varies by a factor of 20.

The method of design used in this paper relies mainly upon Newton's
modified Wiener theory. Because of this, the limit level $a_{ML}$, can not
be used explicitly in the design. Instead, design of the controller is
effected by placing a constraint on the mean-square acceleration of the
missile $\overline{a_{MD}^2}$ in the classic manner. It will be shown that if the design
value of $\overline{a_{MD}^2}$ is too large, the system becomes unstable for small values
of limit level $a_{ML}$. Conversely, if the design value of $\overline{a_{MD}^2}$ is small,
one has unnecessarily large $\overline{\epsilon^2}$ for the large values of limit level, $a_{ML}$.

It will be shown in the sequel that the value of design acceleration
$\overline{a_{MD}}$ should vary as a function of the limit level, $a_{ML}$, in order to obtain
the optimum response (minimum $\overline{\epsilon^2}$). Since we are interested in a rather
large range in limit level, it is obvious that the values of $\overline{a_{MD}}$ chosen
for design purposes will also span a large range. Furthermore, each value
of $\overline{a_{MD}}$ defines a unique controller or Wiener filter, D(s). Hence, for
the guidance system to operate, it will be necessary to change the con-
troller with limit level. There are at least two methods of accomplishing
this: (1) Measure the outside environment (e.g., q) and program the con-
troller to change as $a_{ML}$ changes with the environment; and (2) measure
the dynamic performance internally (e.g., percent of time that the servo

of figure 1 is at is maximum) and use the internally measured signals to adjust D(s) continuously in an optimum fashion.[2] Both methods yield essentially the same response when the inputs are restricted to those described by equations (1) and (2). However, the second method will be concentrated upon in this paper because it yeilds a system that is superior for certain deviations in the magnitude of these statistical inputs. In this sense, the system adapts itself to both changes in missile open-loop gain and inputs.

## Classical Modified Wiener Design

The initial step in the design procedure is to design several optimum systems, corresponding to various points in the flight regime, by conventional Newton modified Wiener design theory and then calculate the rms error for each of the optimum systems. Therefore, four optimum linear systems having the same statistical inputs, but different mean-square acceleration constraints, were designed.[3] In all four cases it was assumed that the rms acceleration of the missile would be one-half the acceleration capability for that particular point in the flight regime. Since for structural reasons the peak acceleration of the missile has been fixed at 20g, the rms design acceleration $\overline{a_{MD}}$ corresponding to this case is 10g rms. The other three designs were based on $a_{ML} = 10$, 4, and 3g with corresponding rms design accelerations of $\overline{a_{MD}} = 5$, 2, and 1-1/2g rms. The choice of the design acceleration, $\overline{a_{MD}}$, to be one-half the acceleration available, $a_{ML}$, at each point in the flight regime is rather arbitrary; that is, no exact rule exists for selecting the design rms acceleration as a function of the peak acceleration capability. The hypothesis has been made that $\overline{a_{MD}}/a_{ML} = 1/2$ is a good choice because it can be shown that the acceleration of the missile, $a_M$, will be limited approximately 4.56 percent of the time (see appendix). In other words the response will be essentially linear when $\overline{a_{MD}}/a_{ML} = 1/2$.

All four of the optimum linear systems, discussed above, have open-loop transfer functions of the same form. This transfer function is given in general terms by

$$KG(s) = \frac{K_F(p_2 s^2 + p_1 s + 1)}{s^2(q_3 s^3 + q_2 s^2 + q_1 s + 1)} \tag{3}$$

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[2]The fact that internal measurements of dynamic performance are used to adjust the controller to compensate for changes in missile gain leads to the viewpoint that the guidance system is self-adaptive in nature.

[3]The calculations involved in the analytic design of optimum systems are considered redundant to this paper. For examples of these calculations the reader is referred to the early works of Stewart (refs. 1 and 3) and to the particularly lucid description of the method given by Newton (ref. 8)

Table I shows how the parameters of the Wiener filter, $D(s)$, in equation (3) vary as a function of the design acceleration, $\overline{a_{MD}}$.[4] The block diagram for the optimum systems is then given by figure 4.

The rms error corresponding to each of the four optimum systems can now be obtained. It can either be calculated by using equation (3) as the optimum linear system and standard integral tables (ref. 8) or it can be measured on an analog computer by simulating the system shown in figure 4. The calculated rms errors for these systems are given in figure 5. Furthermore, simulation studies on an analog computer checked these points to well within the expected engineering tolerances when the limit level ($\pm a_{ML}$ in fig. 4) was set equal to twice the rms design acceleration, $\overline{a_{MD}}$, in each case.

Analog computer studies were also made to determine the effect of the limit level on rms error for each of the four optimum systems. The results of these studies are shown in figure 6. For comparison purposes the curve of figure 5 is also plotted in figure 6. Thus the performance of linear systems, given in figure 5, is compared to that of nonlinear systems where the nonlinearity is saturation.

Inspection of figure 6 reveals that a design based on $\overline{a_{MD}}/a_{ML} = 1/2$ does not necessarily yield the minimum rms error when the system has a physical limiter. For example, consider points A and B in figure 6. These points correspond to two systems which have the same acceleration limit, $a_{ML} = 6g$. Point A corresponds to an optimum system design based on $\overline{a_{MD}} = 3g$ rms ($\overline{a_{MD}}/a_{ML} = 1/2$) and point B corresponds to an optimum design based on $\overline{a_{MD}} = 5g$ rms ($\overline{a_{MD}}/a_{ML} = 5/6$). However, the rms error at point A is 28 feet rms and at point B it is 25.5 feet rms. At point A the system is operating essentially as a linear system because saturation occurs only a small percentage of the time (approximately 4.56 percent). At point B the system operation is nonlinear and saturation occurs approximately 23 percent of the time (see appendix). Thus it is obvious that system performance can be improved (in one case, at least) by simply forcing limiting to occur a greater percentage of the time than is required for linear response.

## Stability Studies by Root Locus Method

Booton, et al. (ref. 9), suggested a method of calculating the rms error curves of figure 6. The method depends upon the fact that a limiter can be treated as an equivalent gain which is a function of $\overline{a_{MD}}/a_{ML}$. This concept of treating a limiter as an equivalent gain suggests the use of root locus plots of the four optimum systems as an aid in analyzing the problem. Root loci of the four optimum systems are shown in figure 7. Table I shows that there is a particular value of gain associated with each

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[4]The classic Wiener design (i.e., infinite acceleration limits) is also given for comparison purposes.

optimum linear system. For $\overline{a_{MD}}$ = 10g rms, $K_F$ = 4.45; for $\overline{a_{MD}}$ = 5g rms, $K_F$ = 3.25; for $\overline{a_{MD}}$ = 2g rms, $K_F$ = 1.15; and for $\overline{a_{MD}}$ = 1-1/2g rms, $K_F$ = 0.50. These particular values of gain fix the closed-loop poles for the respective systems and at these values the systems will yield the minimum rms error for the assumed inputs. Note, however, (see fig. 7(a)) that as the gain of the system designed for $\overline{a_{MD}}$ = 10g rms is reduced from the design value of 4.45 to 3.25, 1.15, and 0.50 (corresponding to the gains of the other optimum systems), the system actually becomes unstable. Thus it is apparent that for stability reasons alone it is necessary to change the pole-zero locations of the filter when the forward gain of the system is reduced by limiting. A composite plot of the closed-loop pole-zero locations of the four optimum systems shows how the poles and zeros should vary as the equivalent gain is reduced in order to maintain optimum linear response (minimum rms error). This is shown in figure 8. Note that one pair of complex poles and the real pole are fixed in the s plane and do not depend upon $\overline{a_{MD}}$. The complex zeros and the other pair of complex poles do, however, vary as a function of $\overline{a_{MD}}$ in an orderly manner. Inspection of the root loci (fig. 7) in the neighborhood of the dominant poles (i.e., those nearest the origin) shows that the optimum systems do not all have the same relative stability. For example, figure 9 shows a composite root locus plot of the dominant poles of the four optimum systems. The gains have been normalized for each system so that the values of gain shown correspond to a per unit of "design value of gain" in each case. Note that the systems designed for the lower values of $\overline{a_{MD}}$ are less sensitive to a reduction in gain than those designed for the higher values of $\overline{a_{MD}}$. Thus it is obvious that the increase in miss distance (rms error) as the limit level is lowered (see fig. 6) is in part due to a reduction in stability; and furthermore, the degradation in performance is a function of the design acceleration, $\overline{a_{MD}}$. It may be recalled that figure 6 showed that system performance could be improved by simply forcing limiting to occur a greater percentage of the time than is required for linear response (i.e., 23 percent rather than 4.56 percent of the time for the case where $a_{ML}$ = 6g).

## Design of the Self-Adaptive Loop

Previous considerations have shown that it is desirable to have a control system which measures the percent of time limiting. Furthermore, the control system should adjust the modified Wiener filter parameters in such a manner that the percent of time limiting is the optimum for the particular value of $a_{ML}$. A method of doing this is shown in figure 10.

Switch S is closed when $a_{MC}$ > $a_{ML}$; thus, $\bar{x}$ is equal to percent time limiting per 100 (where $\bar{x}$ is equal to time average value of x). Note that x is compared to the reference y and the difference drives a servo which adjusts the parameters of the Wiener filter according to figure 11. Figure 11 was derived from the design data given in table I. The signal w was arbitrarily chosen to be a linear function of the rms

design acceleration, $\overline{a_{MD}}$. Thus the points in figure 11 where $w = 0$ and $w = 1$ correspond, respectively, to $\overline{a_{MD}} = 1.5g$ rms and $\overline{a_{MD}} = 10g$ rms; and the intermediate values of $w$ between 0 and 1 correspond to optimum modified Wiener design where $\overline{a_{MD}}$ lies between $1.5g$ and $10g$ rms.

With the signs shown in figure 10, if $x > y$ the servo adjusts the parameters of the filter toward a lower $\overline{a_{MD}}$ design. Conversely, if $x < y$ the servo adjusts the parameters toward a higher $\overline{a_{MD}}$ design. Thus the servo seeks a null where $x = y$, the point where the system has a constant percent time limiting. Since $x$ can have only the values of one or zero, the servo hunts continuously about the quantity $y$.

Tests were made of the complete system on an analog computer with the statistical inputs given in equations (1) and (2). With the servo gain set at 20, as shown in figure 10, it was found that the self-adaptive loop had very good characteristics. The gain was held constant during the remainder of the study.

Analog computer studies were made to evaluate the effect of $y$ (i.e., fixed values of percent time limiting) on the mean-square error of the system $\bar{\epsilon}^2$, for various values of limiting $a_{ML}$. Figure 12 shows a block diagram of the system, including the self-adaptive loop, that was used during these studies.

The results of the computer studies are given in figure 13(a). The curves show multivalued characteristics very similar to those discovered by Booton, et al. (ref. 9). However, it is apparent from a cursory inspection of figure 13(a) that on each curve there is a minimum point (minimum rms error) which is determined by a particular percent time limiting for each value of acceleration limiting, $a_{ML}$. These minimum points were used to determine the curve plotted in figure 13(b) which shows how the missile acceleration limit, $a_{ML}$, should vary as a function of percent time limiting. The results shown in figure 13(b) seem almost intuitive. Certainly it is obvious that as the limit level, $a_{ML}$, is reduced to a level approaching the magnitude of the input signals, it is necessary for the system to be operating in the saturated region nearly 100 percent of the time in order to follow the inputs. This implies that the percent time limiting should increase as $a_{ML}$ decreases. It should be noted however, that limits were placed on the variations of the Wiener filter parameters to keep the variation within the design values given in figure 11. Since the system operates at the extreme values part of the time, it is obvious that slightly different results would be obtained if the design values were extended by an order of magnitude.

In order to use the results of figure 13(b), it is necessary to measure the acceleration capability at each point in the flight regime. The only practical way the author knows to do this is to measure external environment. This suggests that in order to obtain the optimum performance it would be necessary to relax the design requirement of adaptive control.

However, if an estimate of the average $a_{ML}$ can be made for a typical missile attack, then figure 13(b) can be used to determine the optimum percent time limiting, and measurement of dynamic pressure will not be mandatory.

## Simulation Tests With Varying $K_M$

So far only fixed missile gain, $K_M$, has been considered in both the analytic design phase and in the analog computer studies which have been described. The problem of a changing missile gain, $K_M(t)$ has been analyzed only on the analog computer. The computer tests involving $K_M(t)$ (see fig. 14) were made with the statistical inputs given by equations (1) and (2) with $K_M(t)$ decreasing linearly with time over a 20 to 1 range in a 2-minute interval. Typical time history curves of the system error $\epsilon$, the Wiener filter adjusting signal $w$, and the missile acceleration $a_M$, are shown in figure 15. The fact that the error $\epsilon$ does not increase greatly near the end of the run indicates that the gain variation is slow enough for the problem to be considered essentially time invariant. Thus the time average error $(\bar{\epsilon})$ for one run is essentially the same as the ensemble average error $(\tilde{\epsilon})$ for a number of runs. (The reader will recognize this as an assumption that is often made in missile studies.) No claim is made as to the conformity to actual missile gain variation for this particular gain changing program. However, it was necessary to present the guidance system with the same inputs and the same variation of $K_M(t)$ in order to compare results in the following studies. Analog tests were made to determine how the rms error, $\bar{\epsilon}$, varied with the rms design acceleration, $\overline{a_{MD}}$, (i.e., D(s) was held fixed at various design values) when $K_M(t)$ changed by a factor of 20 to 1. The results of the tests are shown in figure 16(a). For this particular variation of $K_M(t)$ there is an optimum modified Wiener design at $\overline{a_{MD}} \approx 5g$. This corresponds to a g limit of roughly 10g or the mid-range of the $K_M(t)$ variation.

Tests were also made to determine the effect of percent time limiting when $K_M(t)$ varied over the 20 to 1 range; D(s) was allowed to vary to maintain a fixed percent time limiting. The results are shown in figure 16(b). Note the minimum rms error point of the curves occurs at approximately 20-percent time limiting. Since 20-percent time limiting corresponds to an optimum g limit of 10g in figure 13(b) and the average g limit (based on the chosen $K_M(t)$) is also 10g, it is apparent that optimum response can be obtained if the average g limit is known. This means that there is the practical problem of anticipating the average g limit during a missile flight. Selecting the average g limit should be no more difficult than estimating the statistical character of the inputs to the missile. The average g limit can probably be chosen to correspond to the mid-point in the flight regime in most cases.

The curves shown in figure 6 are quite heuristic in nature because they show that under certain conditions performance better than that obtained with Newton's modified Wiener system is possible. In other words,

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there should be a curve that is tangent to the curves of figure 6 and corresponds to all possible $\overline{a_{MD}}$ designs from $\overline{a_{MD}} = 10g$ down to $1\text{-}1/2g$. The curve would like to the left of the dashed curve[5] in figure 6. Analog computer studies show that with $K_M$ fixed and with the percent time limiting chosen to correspond to the $g$ limit according to figure 13(b), the performance is better than that with an optimum modified Wiener system. The curve in figure 17 labeled "optimum percent time limiting" shows this performance, and for comparison purposes the 4.56-percent time limiting (or optimum Newton modified Wiener) curve is presented. Tests were also made with the $g$ limit fixed ($K_M$ = constant) and the percent time limiting fixed at 20 percent. This performance curve in figure 17 shows that 20-percent time limiting is better than 4.56 percent over the range.

It is also interesting to consider the operation of the missile system when it is assumed that dynamic pressure is measured. Such operation means that design requirements have been relaxed so that adaptive performance is not required. However, it also means that the system can now operate with the percent time limiting changing as a function of $g$ limit level (i.e., dynamic pressure). In other words, the miss distance (rms error) that can now be expected is that described by the optimum percent time limiting curve of figure 17 rather than the 20-percent time limiting curve. Analog computer tests show that the rms error, $\bar{\epsilon}$, is 22.41 feet rms under these conditions when $K_M(t)$ varies in the manner previously considered. The average $g$ limit, it should be recalled is 10g in this case. The value of $\bar{\epsilon} = 22.41$ feet rms compares quite closely to the value of $\bar{\epsilon} = 23.04$ feet rms which is obtained when $a_{ML}$ is fixed at 10g and the optimum percent time limiting of 20 percent is used (see fig. 13(b)).

Thus it can be seen that two closely related designs have evolved from this investigation. One is an adaptive type design that depends upon the designer's being able to estimate the average $g$ limit level of the beam-rider missile during a typical attack. The other is an "optimum" design whose response depends upon maintaining the optimum percent time limiting at all times. In both cases, performance, as measured by rms error (or rms miss distance), is superior to that realized by Newton's modified Wiener design. Figure 18 shows the block diagrams of the two systems. It should be noted that a percent time limiting computer (detector plus servo) is the heart of both designs. Furthermore the Wiener filter, $D(s)$, has variable coefficients that are adjusted by the percent time limiting computer in both cases.

## INPUT VARIATIONS

In all of the preceding discussions, the inputs have remained fixed at the original design values. Roberts (ref. 10) has shown that control

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[5]The dashed curve corresponds to optimum Newton modified Wiener system performance.

systems with a certain class of randomly varying inputs can be made self-optimizing. In general, his method depends upon synthesizing the Wiener filter part of the design in such a manner that the spectral density of one of the state variables of the system has the same form as the noise when the system is operating in an optimum manner. The method works very well for simple systems where the signal and/or noise change slowly with time. Roberts also shows that his method leads to a complicated function that is extremely difficult to realize in the case of the problem we have been studying.

Any change in the inputs, either in magnitude or shape of power spectral density, will change the miss distance of an optimum system; that is, the system will be optimum only for the exact inputs that were assumed at the start of the design. Analog computer studies were made to determine the effects of changing target and noise magnitudes. The results are shown in figure 19. The "solid" curves show rms errors for a modified Wiener design of $\overline{a_{MD}} = 5g$ when it is subjected to a 10g limit, and inputs other than those for which it was designed. For the tests the Wiener filter coefficients were held constant at the 5g design values. The dashed curves show the results of computer tests which were made with $a_{ML} = 10g$, a fixed 20-percent time limiting, and the Wiener filter, $D(s)$, changing to maintain the 20-percent time limiting. The design values of input magnitudes are $\Phi_N(s) = 100$ and $\Phi_{y_T y_T}(s) = 1.00$ per unit.

A study of figure 19 reveals that the performance of the self-adaptive system is superior to that of a system which has a fixed modified Wiener filter except in the region where both the target and noise inputs are much greater than the design values. A study of time histories of the adaptive system variables (which is not presented here) shows that the performance of the system could be improved in this region by forcing the system to operate with more than 20-percent time limiting. Hence it is obvious that the self-adaptive design presented can adapt itself to some input magnitude changes as well as changes in missile gain.

CONCLUDING REMARKS

A method of designing a self-adaptive missile guidance system has been presented. To illustrate the technique, the design has been effected for a simplified beam-rider missile where the inputs are assumed to be known in a statistical sense only, the missile is g limited, and the gain of the missile varies by a factor of 20 over its flight envelope. It has been shown that the dynamic performance (i.e., percent time limiting) can be measured internally and the measurements, in turn, can be used to adjust the controller (modified Wiener filter) so that the missile performs in an optimum manner over its flight envelope.

The initial design phase has been accomplished by the analytical method suggested by Newton's modified Wiener theory. Analog computer studies have been used in the subsequent design phases to show that the

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performance criterion of the missile (minimum mean-square error) changes throughout the flight envelope as a function of the missile's acceleration capability. Computer studies have also shown that the percent of time that missile acceleration limiting occurs can be measured and that optimum values of percent time limiting are associated with each value of limit level.

It has been shown that the guidance system can be made self-adaptive in the case where one particular value of percent time limiting can be chosen as a reference. This reference signal can then be compared to the measured value of percent time limiting and the difference between these two signals can be used to drive a servo which, in turn, adjusts the Wiener filter. In this study, the reference value of percent time limiting was chosen to correspond to the mid-range limit level. Analog studies were also made to determine the performance of a beam-rider missile where it was assumed that dynamic pressure, q, could be measured; that is, the self-adaptive feature was dropped and it was assumed that the reference signal to the percent time limiting detector varied as a function of external environment measurements. In this case, it was shown that optimum performance (minimum $\bar{\epsilon}^2$) could be maintained throughout the entire flight envelope.

Studies were also made to determine the effect of input magnitudes on system performance. The results of these studies show that the adaptive loop adjusts the system to give good operation when the input magnitudes are varied from 25 percent to as much as 225 percent of the design values.

It is believed that the concept of using percent time limiting to adjust the controller in a guidance system is a significant contribution to, and may extend the usefulness of, classical design methods.

Further research areas may include the application of this concept to the design of homing missile guidance systems in which some of the parameters are time varying. The concept may also be useful in designing certain optimum nonlinear systems which are subject to deterministic inputs.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., July 26, 1960

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APPENDIX


BASIS FOR INTUITIVE FEELING THAT CONTROL OF RMS VALUE OF A

SIGNAL COMES CLOSE TO CONTROLLING PEAK VALUE


The intuitive feeling that control of the rms value of a signal comes close to controlling the peak value is based on the following reasoning. First, the assumption is made that the input signal to a limiter is Gaussian and has a zero mean value. This can be represented by the Gaussian distribution function shown in figure 20.

The command acceleration signal to the missile is $a_M$, and $\sigma$ is the standard deviation or the rms value of the missile acceleration $(\sigma = \overline{a_{MD}})$. For example, if we select the value of $\overline{a_{MD}}$ for design purposes so that it is equal to $a_{ML}/2$, the area under the curve from $-2\sigma$ to $+2\sigma$ will be a measure of the percent of the time that the missile guidance will be operating in the linear or unsaturated range. For calculation purposes it is somewhat more convenient to determine the value of the integral from $-\infty$ to $-2\sigma$ and from $+2\sigma$ to $+\infty$; that is, it is easier to calculate the percent of the time that the acceleration required of the missile exceeds the available acceleration. This expression is defined by

$$P(|a_M| > a_{ML}) = 2 \left[ 1 - \int_{-\infty}^{\frac{a_{ML}}{a_{MD}}} \frac{\exp - (a_M{}^2/2)}{\sqrt{2\pi}} \, d(a_M) \right] \qquad (A1)$$

Integrals of this type are tabulated extensively in mathematical and engineering literature (ref. 11). For this example, $a_{MD} = a_{ML}/2$, evaluation of the above equation yields a value of 0.0456. This means that if the design value of rms missile acceleration, $a_{MD}$, is chosen to be one-half the maximum acceleration available, $a_{ML}$, then the probability that the required acceleration $(a_M)$ will exceed the limit level $(a_{ML})$ is 0.0456; or, in other words, the system will be operating in the saturated range only 4.56 percent of the time. This small percentage of time during which saturation is expected has lead to the aforementioned intuitive feeling that control of the rms value of a signal comes close to controlling the peak value.

Equation (A1) is valid only in the case of a cascade system. By rationalization only (through use of the central limit theorem) can equation (A1) be extended to the feedback system which is of interest in this report. Hence, percent time limiting calculations for feedback systems that are based on equation (A1) are rough approximations only.

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TABLE I.- SUMMARY OF OPTIMUM NEWTON MODIFIED

WIENER SYSTEM PARAMETERS

| Design parameter | Missile rms design acceleration, $\overline{a_{MD}}$, g | | | | |
|---|---|---|---|---|---|
| | 1.5 | 2.0 | 5.0 | 10.0 | Wiener |
| $K_F$ | 0.4969 | 1.1526 | 3.2492 | 4.4506 | 7.1366 |
| $p_2$ | 4.1561 | 2.4768 | 1.3198 | 1.1051 | .8601 |
| $p_1$ | 3.5073 | 2.5826 | 1.8019 | 1.6287 | 1.4143 |
| $q_3$ | .3787 | .2738 | .0849 | .0356 | 0 |
| $q_2$ | 1.4542 | 1.2580 | .6829 | .4346 | 0 |
| $q_1$ | 2.7606 | 2.8680 | 2.7389 | 2.6495 | 2.4756 |

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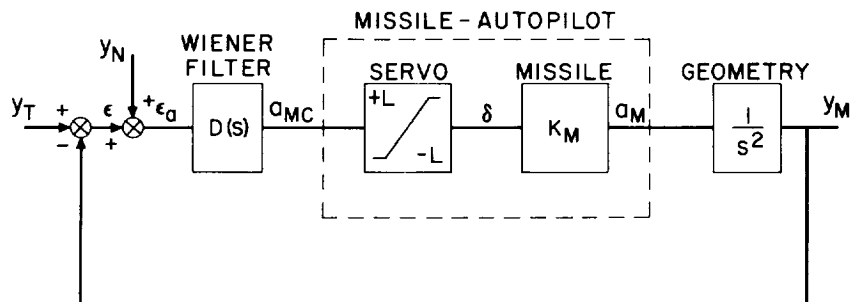
Figure 1.- Typical beam-rider guidance system.
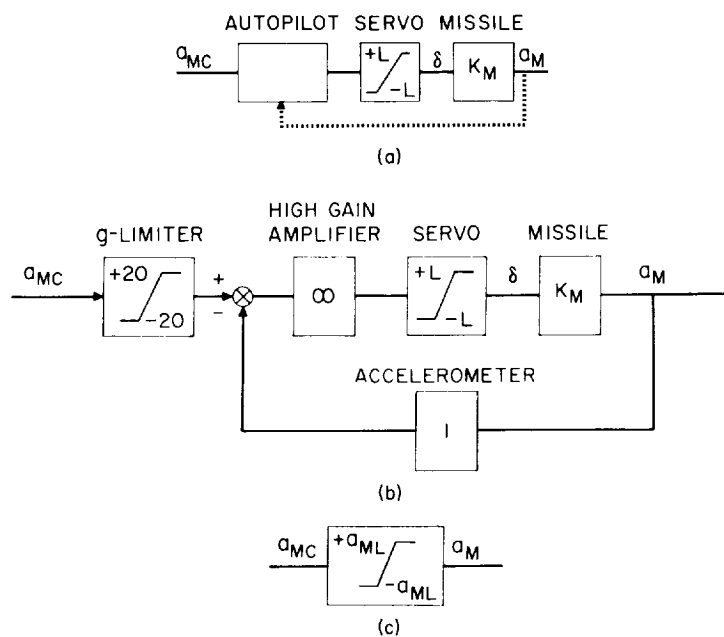


(a)  Classical representation.
(b)  High-gain autopilot design.
(c)  Variable limit level configuration.

Figure 2.- Missile autopilot block diagrams.
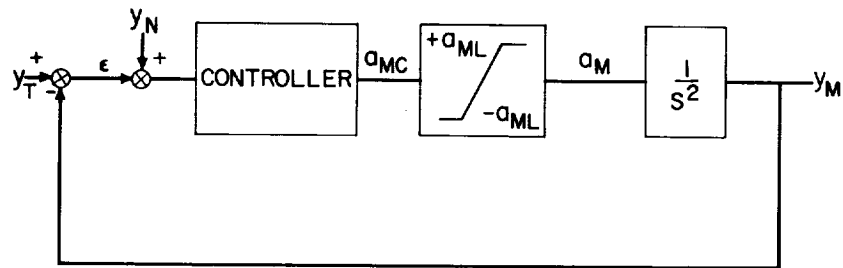
Figure 3.- Beam-rider control system with variable limit level autopilot.
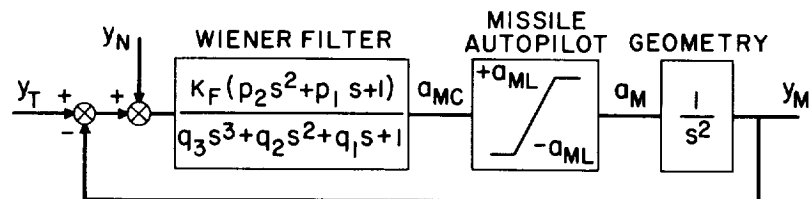
Figure 4.- Block diagram of beam-rider missile control system with modified Wiener filter and variable limit level autopilot.
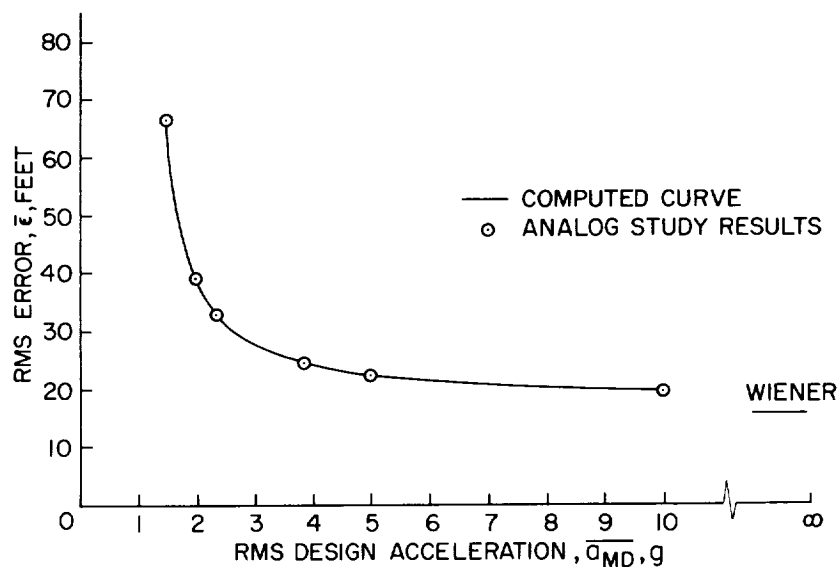
Figure 5.- RMS error for various Newton modified Wiener beam-rider
missile designs.
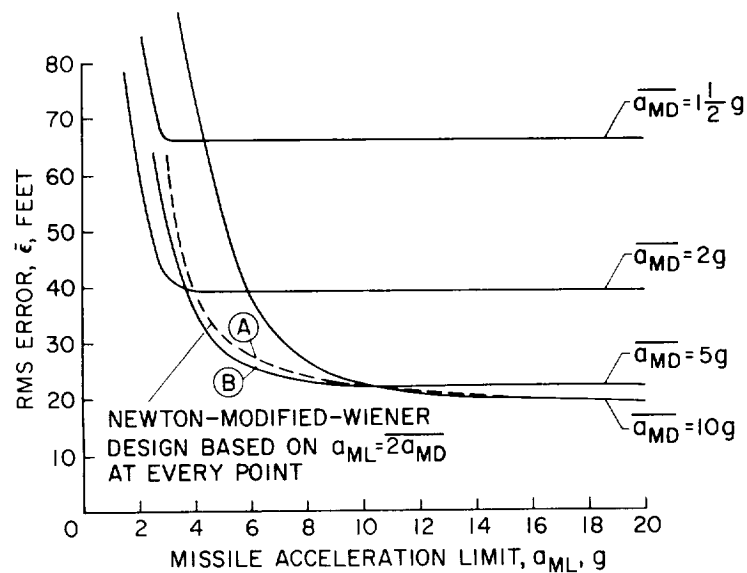


Figure 6.- RMS error for various Newton modified Wiener missile design
accelerations ($\overline{a_{MD}}$) as a function of missile acceleration limiting
($a_{ML}$).
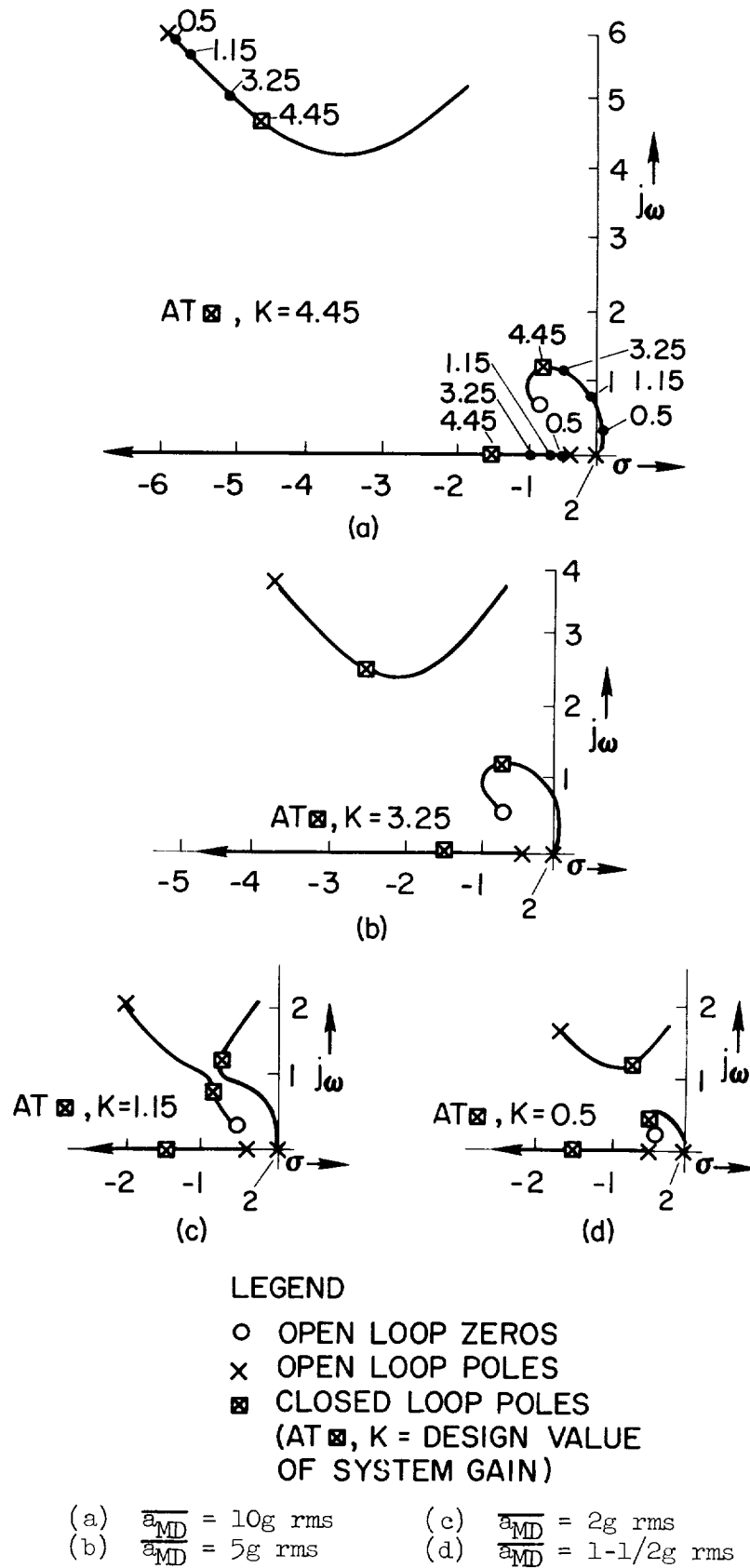
Figure 7.- Root loci of four optimum modified Wiener systems.
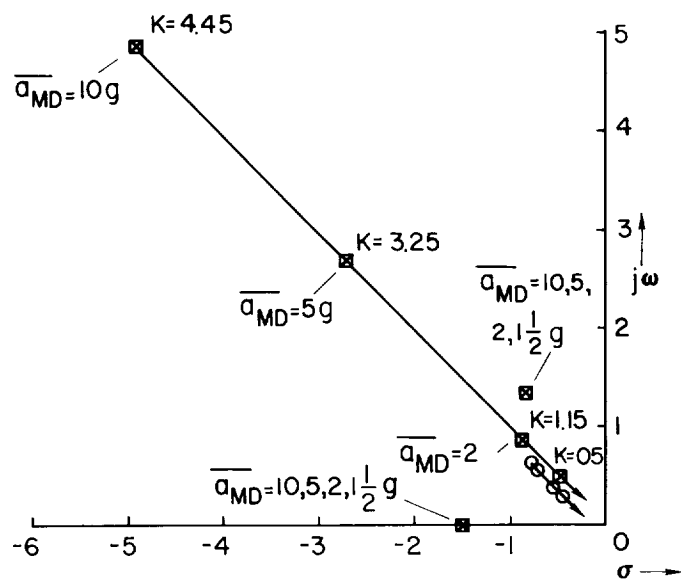
Figure 8.- Composite root loci plot of closed-loop poles and zeros of the four optimum systems.
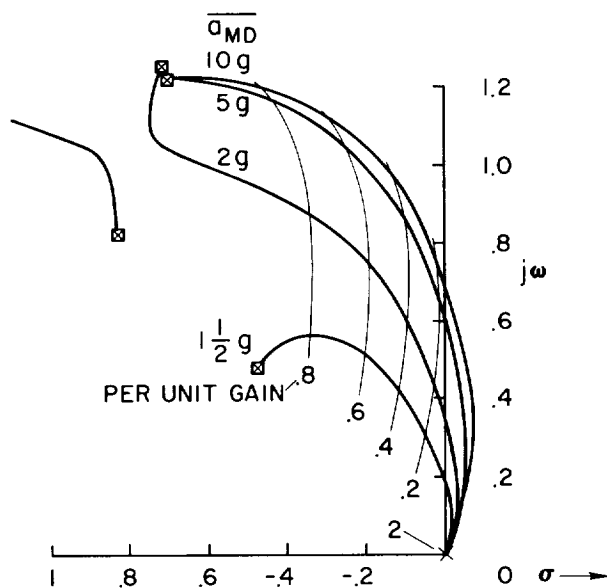


Figure 9.- Composite root loci plot of the dominant poles of the four optimum systems.
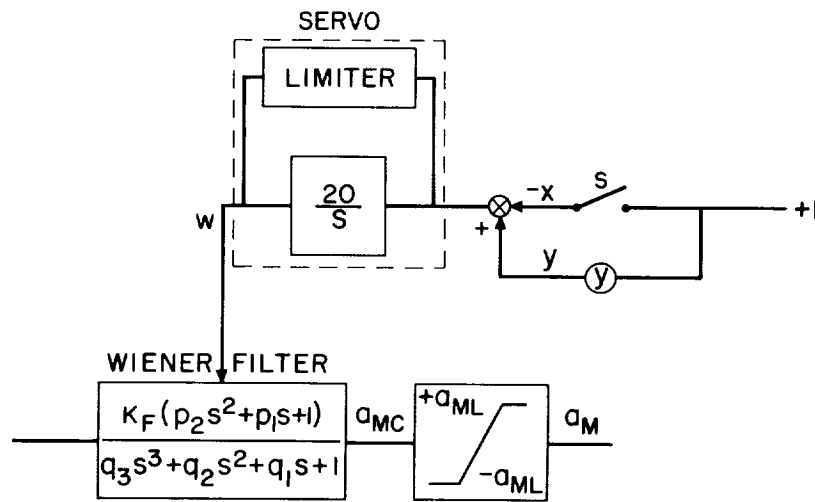
Figure 10.- Percent time limiting self-adaptive loop.
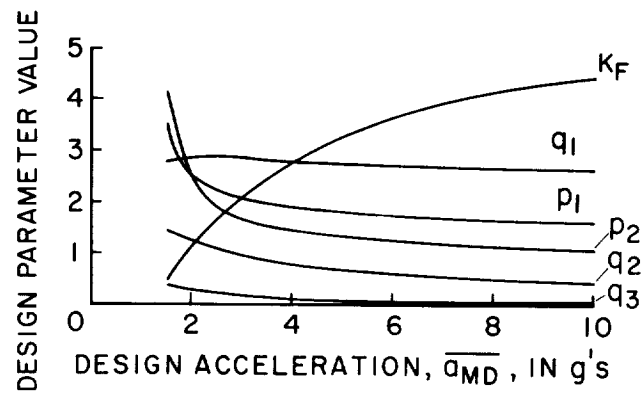


Figure 11.- Variation of modified Wiener filter parameters with rms design acceleration.
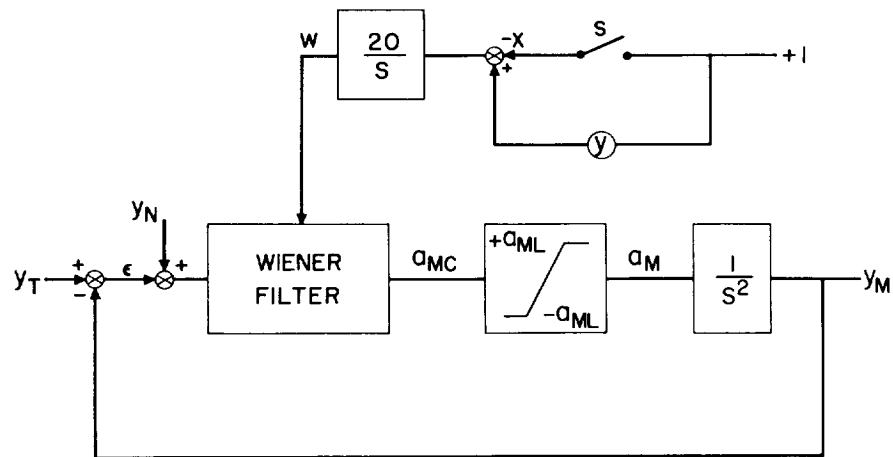
Figure 12.- Block diagram of self-adaptive beam-rider missile control system.
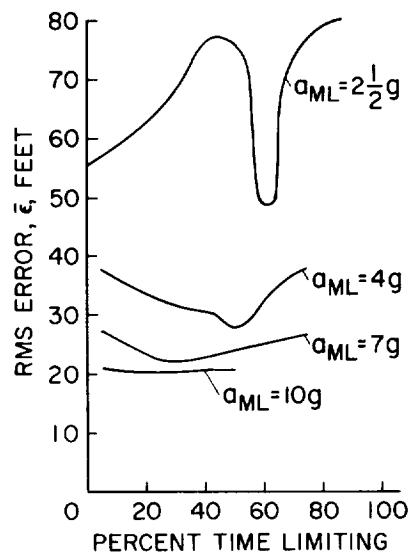


(a)   RMS error for various  g  limits as a function of percent time limiting.

Figure 13.- Effect of  g  limit on rms error.
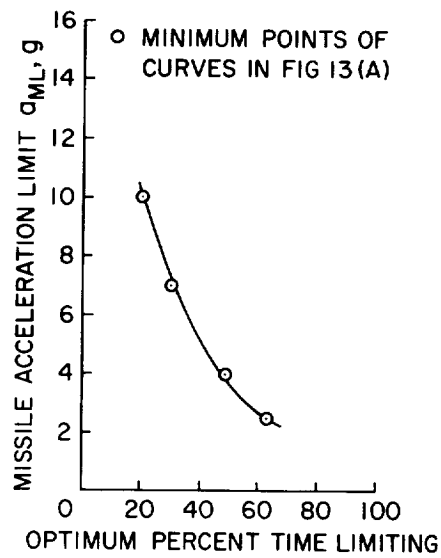
(b) Acceleration limit versus optimum percent time limiting.

Figure 13.- Concluded.
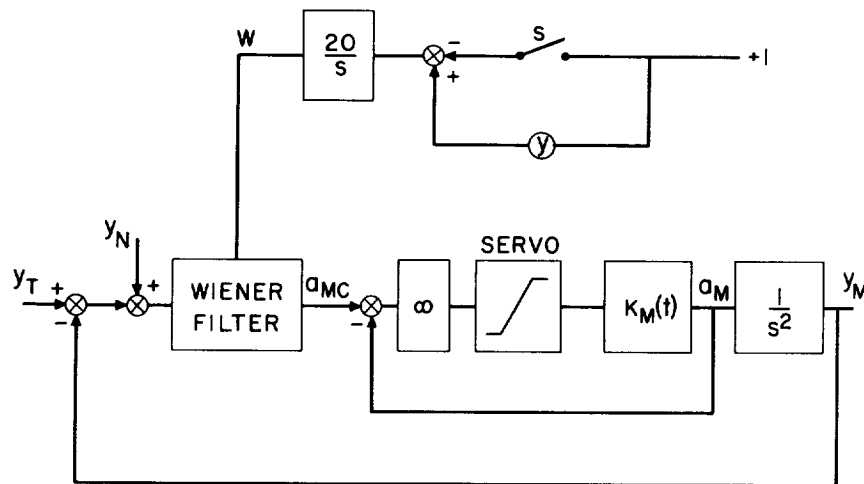


Figure 14.- Block diagram of beam-rider system with time varying missile gain, $K_M(t)$.
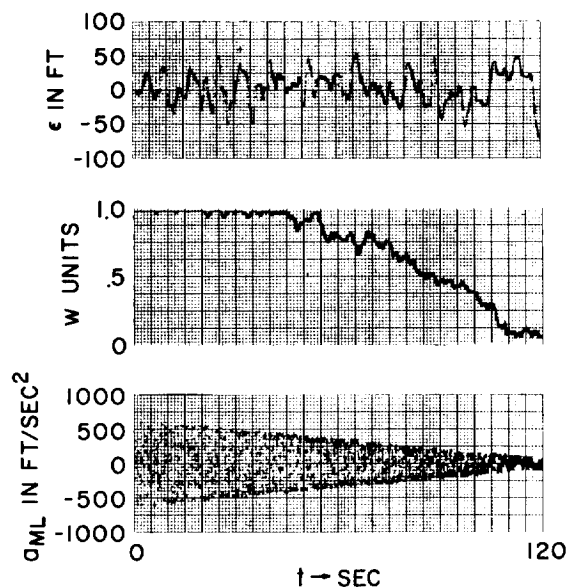
Figure 15.- Analog computer time history plots of $\bar{\epsilon}$, w, and $a_M$ for adaptive guidance system where $K_M(t)$ varies.
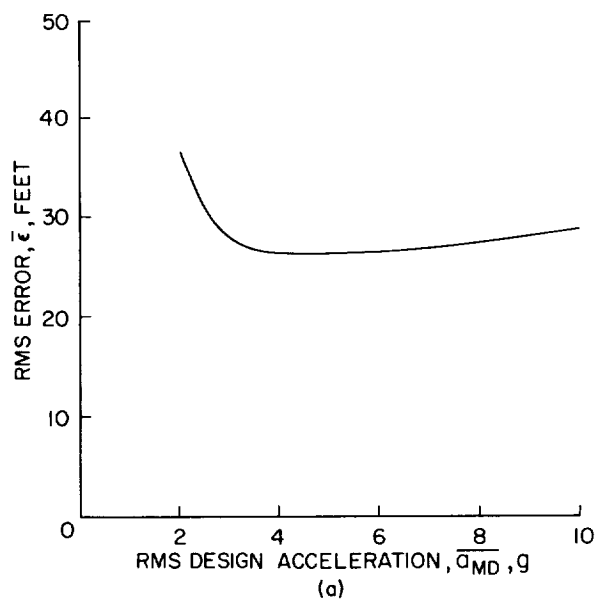


(a) RMS error as a function of rms design acceleration.

Figure 16.- RMS error variations when missile gain decreases linearly with time.
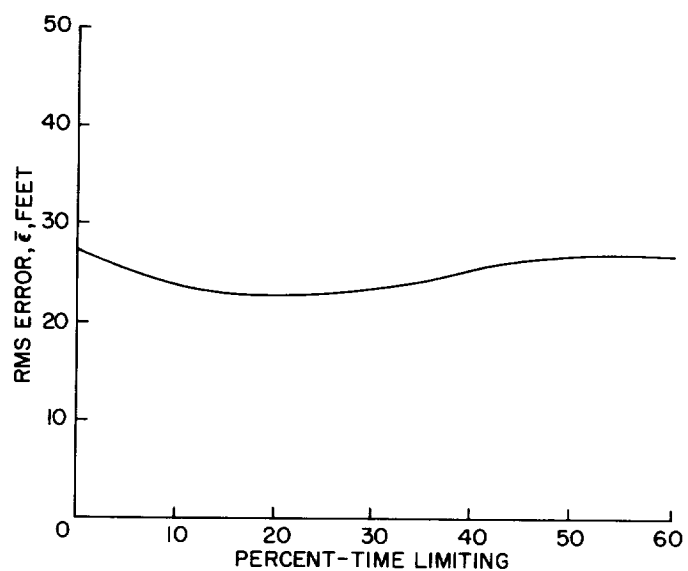
(b)   RMS error as a function of percent time limiting.
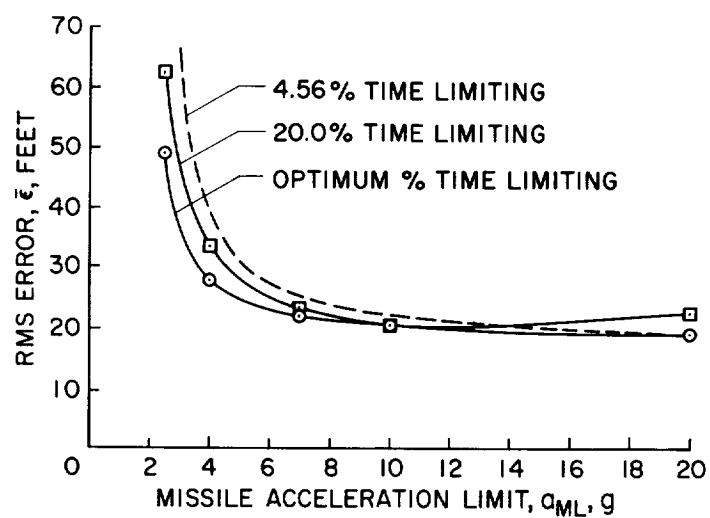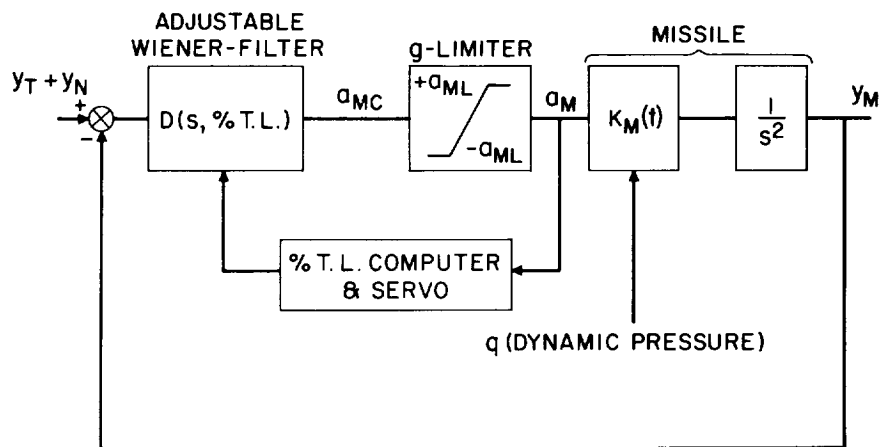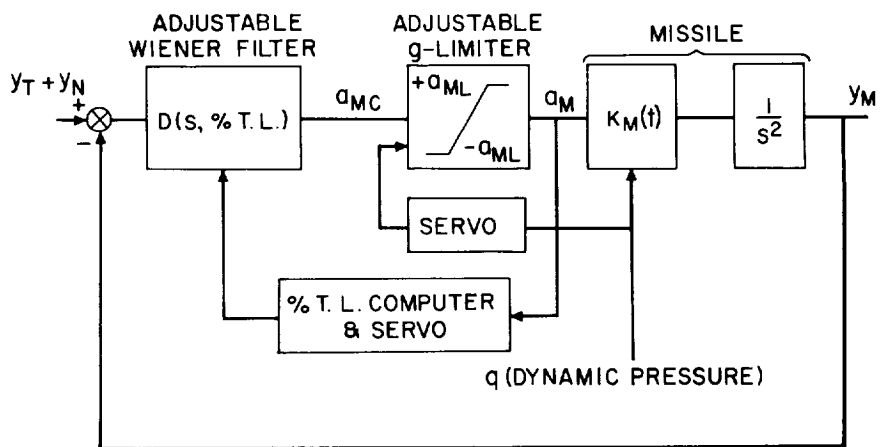
Figure 16.- Concluded.



Figure 17.- RMS error variation with missile acceleration limit level.

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(a) Self-adaptive system.



(b) Self-optimizing system.

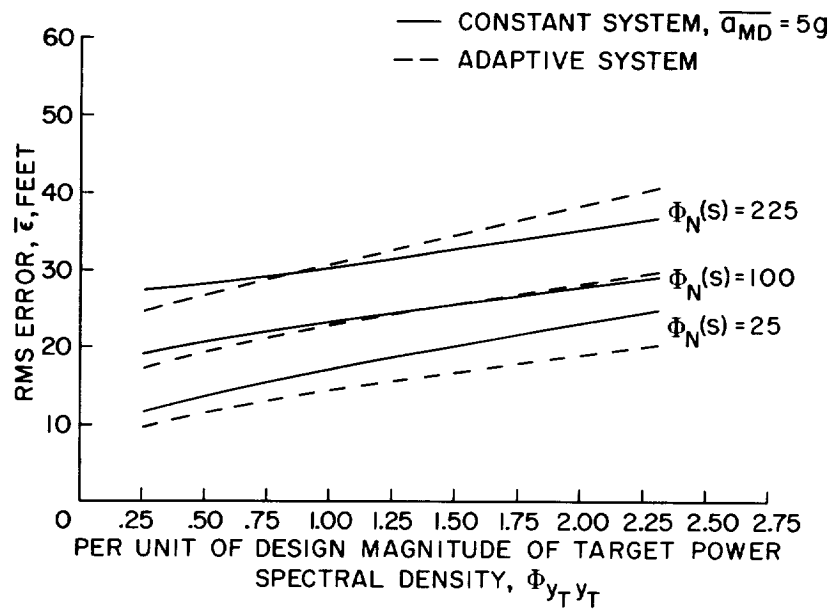Figure 18.- Block diagrams of beam-rider missile guidance systems.

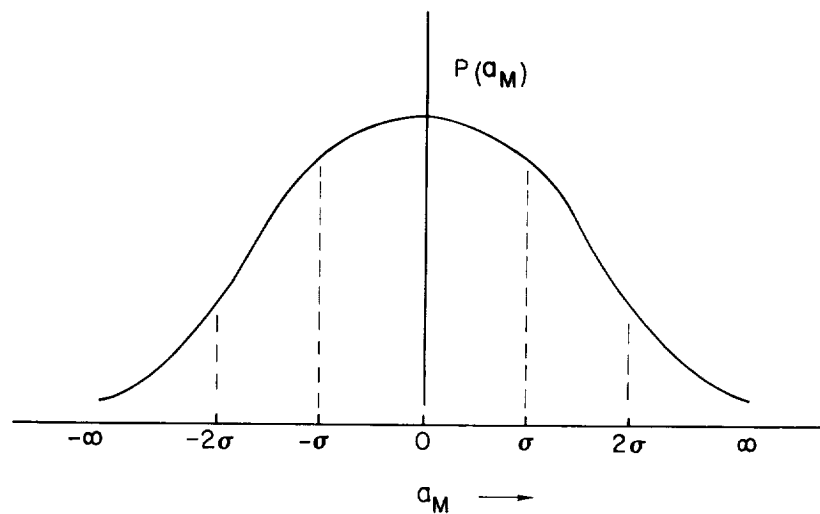Figure 19.- Variation of rms error with input magnitudes.



Figure 20.- Gaussian distribution function.

NASA TN D-343
National Aeronautics and Space Administration.
A SELF-ADAPTIVE MISSILE GUIDANCE SYSTEM
FOR STATISTICAL INPUTS. H. Rodney Peery.
November 1960. 30p. OTS price, $0.75.
(NASA TECHNICAL NOTE D-343)

A method of designing a self-adaptive missile guidance
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modified Wiener theory is utilized in the design of the
system and to establish the performance criterion.
The missile is assumed to be a beam rider, to have a
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where the open-loop gain varies by a factor of 20.
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