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LAMINAR BOUNDARY LAYER BEHIND A STRONG SHOCK MOVING INTO AIR

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SUMMARY

The laminar wall boundary layer behind a strong shock advancing into stationary air has been determined. Numerical results have been obtained for shock Mach numbers up to 14 using real gas values for density and viscosity and assuming Prandtl and Lewis numbers of 0.72 and 1, respectively. The numerical results for shear and heat transfer agree, within 4 percent, with a previously presented approximate analytical expression for these quantities. A slight modification of this expression results in agreement with the numerical data to within 2.5 percent. Analytical expressions for boundary-layer thickness and displacement thickness, correct to within 4 percent for the present data, have also been obtained.

INTRODUCTION

Until 1957, all studies of the compressible laminar wall boundary layer behind a moving shock had assumed that the product of viscosity times density $\rho\mu$ was constant throughout the boundary layer (e.g., refs. 1 to 5). This assumption becomes less valid as shock strength increases. Consequently a study was undertaken, at that time, to evaluate the effect of variable $\rho\mu$ on the laminar boundary behind a strong shock moving into air. Real gas properties (ref. 6) were used. The boundary layer was assumed to be in thermodynamic equilibrium, and the Lewis number was assumed to be 1. Shock Mach numbers from 4 to 14 were considered.

The initial phase of the investigation was for a Prandtl number of 1. Numerical results for shear and heat transfer were reported in reference 7. Analytical expressions were used therein which correlated the shear and heat-transfer data to within 3 percent. The analytical expressions were generalized (by comparison with constant $\rho\mu$ solutions) to account for Prandtl number not equal to 1, but the validity of this generalization was not established.

The second phase of the investigation was for a Prandtl number of 0.72. Numerical results were obtained prior to the spring of 1958. These results were not published at that time in the hope that more extensive data would be obtained. However, the press of other research projects prevented the continuation of the study. Recent informal discussions with Dr. R. Hartunian, of the Cornell Aeronautical Laboratory, and Dr. N. Kemp, of the Avco Everett Research Laboratory, have indicated that the unpublished data are of current interest. In particular, the numerical data are of interest for correlating experimental wall heat-transfer measurements in shock tubes and for estimating the flow nonuniformity in low-density shock tubes. Hence the present publication of the data was undertaken.

The present report includes the Prandtl number 1 data previously reported in reference 7, in order to make that data more generally available. It is also shown herein that the approximate analytical expression for shear and heat transfer, developed in reference 7, is correct to within 4 percent for Prandtl number 0.72. Finally, approximate expressions are developed herein for boundary-layer thickness and displacement thickness which agree with the numerical results for Prandtl number 0.72 to within 4 percent. The latter expressions are useful for estimating flow nonuniformity in shock tubes (e.g., refs. 8 and 9).

Numerical solutions for the wall laminar boundary layer behind strong shocks in oxygen recently have been presented in reference 10. Real gas properties, including the effects of Lewis number other than 1, are considered. The correlation of theoretically derived heat transfer with experimentally observed heat transfer in a shock tube (for both laminar and turbulent cases) is also discussed therein and in reference 11. In addition, reference 10 presents approximate analytical expressions for shear and heat transfer which are similar to those developed in reference 7 but which include the effect of Lewis number.

The author is indebted to Richard J. Wisniewski for aid in obtaining the boundary-layer thickness correlations of the present report.

ANALYSIS

Consider the laminar boundary layer behind a shock moving into a stationary fluid. The problem is a steady-state one in a coordinate system fixed with respect to the shock (fig. 1(a)). In this coordinate system the wall moves with the shock velocity $u_w = M_s a_1$.

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Equations of Motion

If the boundary layer is assumed to be in thermodynamic equilibrium with a Lewis number of 1, the equations of motion for $x > 0$ are (e.g., ref. 12)

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{1a}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{1b}$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu}{\sigma} \frac{\partial H}{\partial y}\right) + \frac{\partial}{\partial y}\left[\frac{\mu}{2}\left(1 - \frac{1}{\sigma}\right)\left(\frac{\partial u^2}{\partial y}\right)\right] \tag{1c}$$

$$p = z\,\rho RT \tag{1d}$$

where H is the local stagnation enthalpy

$$H = h + \left(\frac{u^2}{2}\right) \tag{2}$$

The other symbols are defined in the appendix. From equation (1a), a stream function exists such that

$$\left.\begin{aligned} \frac{\partial\psi}{\partial y} &= \frac{\rho}{\rho_w} u \\ \frac{\partial\psi}{\partial x} &= -\frac{\rho}{\rho_w} v \end{aligned}\right\} \tag{3}$$

Introducing new variables

$$\left.\begin{aligned} \xi &= x \\ \eta &= \sqrt{\frac{u_e}{2x\nu_w}}\int_0^y \frac{\rho}{\rho_w}\,dy \end{aligned}\right\} \tag{4}$$

such that

$$\left.\begin{aligned} \frac{\partial}{\partial x} &= \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta} \\ \frac{\partial}{\partial y} &= \frac{\rho}{\rho_w}\sqrt{\frac{u_e}{2x\nu_w}}\frac{\partial}{\partial \eta} \end{aligned}\right\} \qquad (5)$$

and assuming ψ to have the form

$$\psi = \sqrt{2u_e \xi \nu_w}\, f(\eta) \qquad (6)$$

reduces equations (1b) and (1c) to the form

$$[Cf'']' + ff'' = 0 \qquad (7a)$$

$$\left[\frac{C}{\sigma}\, g'\right]' + fg' + \frac{u_e^2}{2H_e}\left[2C\left(1 - \frac{1}{\sigma}\right)f'f''\right]' = 0 \qquad (7b)$$

with boundary conditions

$$\left.\begin{aligned} &f(0) = 0 \qquad && g(0) = \frac{h_w}{H_e} + \frac{u_e^2}{2H_e}U^2 \\ &f'(0) = U && \\ &f'(\infty) = 1 && g(\infty) = 1 \end{aligned}\right\} \qquad (8)$$

where

$$\left.\begin{aligned} C &\equiv \frac{\rho\mu}{(\rho\mu)_w} \\ U &\equiv \frac{u_w}{u_e} \\ g &\equiv \frac{H}{H_e} \equiv \frac{h}{H_e} + \frac{u_e^2}{2H_e}(f')^2 \end{aligned}\right\} \qquad (9)$$

In order to integrate equations (7), it is necessary to express C and σ as functions of f and g. This is generally done by determining C and σ as functions of h/h_w (for each free-stream pressure) and noting that

$$\frac{h}{h_w} = \frac{g - \frac{u_e^2}{2H_e}(f')^2}{g(0) - \frac{u_e^2}{2H_e}U^2} \tag{10}$$

The boundary-layer quantities of interest are

$$\tau_w \equiv \left(\mu \frac{\partial u}{\partial y}\right)_w = \mu_w u_e \sqrt{\frac{u_e}{2\xi \nu_w}}\, f''(0) \tag{11a}$$

$$q_w \equiv -\left(k \frac{\partial T}{\partial y}\right)_w = -\left(\frac{\mu}{\sigma}\right)_w \sqrt{\frac{u_e}{2\xi \nu_w}}\, h'(0) \tag{11b}$$

$$y = \frac{\rho_w}{\rho_e} \sqrt{\frac{2\xi \nu_w}{u_e}}\, [\eta - I(\eta)] \tag{11c}$$

$$\frac{v_e}{u_e} = \frac{\delta^*}{2\xi} = \frac{\rho_w}{\rho_e} \sqrt{\frac{\nu_w}{2\xi u_e}} \left[\lim_{\eta \to \infty} (\eta - f) - I(\infty)\right] \tag{11d}$$

where

$$I(\eta) = \int_0^\eta \left(1 - \frac{\rho_e}{\rho}\right) d\eta \tag{12}$$

The integral $I(\eta)$ can be evaluated, after integrating equations (7), if ρ/ρ_w is known as a function of h/h_w.

Integration of the Energy Equation for $\sigma = 1$

For $\sigma = 1$, equations (7) and (8) become

$$[Cf'']' + ff'' = 0 \tag{13a}$$

$$[Cg']' + fg' = 0 \tag{13b}$$

$$\left.\begin{array}{ll} f(0) = 0 & g(0) = H_w/H_e \\ f'(0) = U & g(\infty) = 1 \\ f'(\infty) = 1 & \end{array}\right\} \tag{13c}$$

Equation (13b) can be integrated to yield

$$g - g(0) = \left[\frac{f' - U}{1 - U}\right](1 - g(0)) \tag{14}$$

which is a form of the Crocco relation between velocity and stagnation enthalpy. If $g(0) = 1$, then

$$g = 1 \tag{15}$$

so that the stagnation enthalpy is constant across the boundary layer. Recall that in a shock fixed coordinate system, stagnation enthalpy is conserved across the shock wave. It can then be shown that $g(0) = 1$ provided $h_w = h_1$. The latter relation is approximately valid in the shock tube because the relatively high heat capacity and conductivity of the wall tends to maintain the wall at its original temperature (e.g., ref. 7), and the pressure effect on enthalpy is small. When equation (15) applies, $h'(0) \neq 0$ and there is heat transfer to the wall (in contra distinction to the semi-infinite flat-plate problem where there is no heat transfer for the $g = 1$ case).

Numerical Solutions

Numerical solutions of equations (7) and (8) have been obtained which correspond to shocks of strength up to $M_s = 14$ propagating into air at $T_1 = 522^\circ$ R and $p_1 = 0.001$ and 0.01 atmosphere. The Runge-Kutta method of integration was used with a step size $\Delta\eta = 0.02$.[1] The wall temperature was assumed to remain at 522° R for these calculations. Conditions behind the shock were determined by using thermodynamic charts for equilibrium air (ref. 6).

In the integration of equations (7) it was assumed that σ was constant across the boundary layer. This is reasonable in view of the

[1]This step size was established for the relatively low shock Mach number cases by decreasing the step size until the shear and heat transfer results were insensitive to a further change. Since η_δ decreases with increase in shock Mach number, a further check of the validity of this step size should have been made at $M_s = 14$ - but this was not done. However, Dr. Richard Hartunian, in a private communication, has stated that he checked the effect of step size at $M_s = 14$ and found that $\Delta\eta = 0.02$ leads to accurate results for shear and heat transfer. He also stated that the computations of reference 10 were also made with $\Delta\eta = 0.02$ (by error, ref. 10 notes the step size as being $\Delta\eta = 0.01$).

rough computations of reference 13. The following analytic expression for C was used:

$$C = \frac{1.5481}{\sqrt{h/h_w}} - \frac{0.5481}{h/h_w} + 0.0028\left(\frac{h}{h_w} - 1\right) - 5.74\times10^{-5}\left(\frac{h}{h_w} - 1\right)^2 \qquad (16)$$

Equation (16) agrees to within 3 percent (for $T_w = T_1 = 522^\circ$ R and the specified ranges of p_1 and M_s) with computations based on the thermodynamic charts for equilibrium air and the Sutherland viscosity-temperature relation. The thermodynamic charts provided ρ/ρ_w and T/T_w as a function of h/h_w (for each specified T_w and free-stream pressure) and C could then be found by using the Sutherland relation

$$\frac{\mu}{\mu_w} = \frac{1 + \frac{198.6}{T_w}}{\frac{T}{T_w} + \frac{198.6}{T_w}}\left(\frac{T}{T_w}\right)^{3/2} \qquad (17)$$

The constants in equation (16) were chosen so that C has specified values at $h/h_w = 1$, 10, 40 and has the correct slope at $h/h_w = 1$. As a result, equation (16) provides an accurate representation of C for $1 \le h/h_w \le 42$, which is the range required for the present investigation. Equation (16) tends to underestimate C for h/h_w greater than 42.

Similarly, ρ/ρ_w can be approximated by

$$\frac{\rho}{\rho_w} = \frac{0.02586\,\frac{h}{h_w} + 0.94828}{\frac{h}{h_w} - 0.02586} \qquad (18)$$

for the range of p_1 and M_s under consideration. The constants in equation (18) were chosen so that ρ/ρ_w has specified values at $h/h_w = 1$, 10 and has the correct slope at $h/h_w = 1$. Equation (18) is correct to within a few percent for $1 \le h/h_w \le 30$ but is correct only to within 10 percent for h/h_w near 42. It should be remembered that equation (18) is used only to evaluate $I(\eta)$.

In order to determine the effects of Prandtl number, equations (7) were integrated for $\sigma = 1.0$ and $\sigma = 0.72$. To determine the effect of variable C, equations (7) were integrated for $C = 1$ and for C as defined by equation (16). The results are summarized in table I. The results for $C = 1$ represent an extension of the results of references 2 and 3 to values of U beyond 6. The shear and heat-transfer results for $\sigma = 1$ and C = equation (16) have been reported in reference 7. The results for $\sigma = 0.72$ and C = equation (16) are new and represent the primary numerical contribution of the present paper.

CORRELATION OF DATA

Reference 7 proposed that the laminar-boundary-layer shear and heat transfer for variable C and constant σ could be estimated from

$$\frac{f''(0)}{U - 1} = -0.489\sqrt{1 + 1.665\ U}\ C_e^{0.29} \tag{19a}$$

$$\frac{h'(0)}{h_r - h_w} = 0.489\sqrt{1 + 1.665\ U}\ C_e^{0.29}\ \sigma^{(0.48+0.022\ U)} \tag{19b}$$

where h_r is found from

$$\frac{h_r}{h_e} = 1 + (U - 1)^2\ \frac{u_e^2}{2h_e}\ \sigma^{(0.39-0.023\ U)} \tag{19c}$$

A comparison of equations (19a) and (19b) with the numerical results obtained by integrating equations (7) is also given in table I. For $\sigma = 1$, equations (19a) and (19b) are correct to within 1 percent for C a constant and to within 3 percent for C varying according to equation (16). (The accuracy of eqs. (19a) and (19b), for $\sigma = 1$, was previously pointed out in ref. 7.) The present numerical solutions for $\sigma = 0.72$ indicate that equations (19a) and (19b) are correct to within about 4 percent for the data considered herein.

Equations (19a) and (19b) would agree with the $\sigma = 0.72$ data to within 2.5 percent if the factor $C_e^{0.29}$ were replaced by $C_e^{0.265}$ in these equations. The exponent 0.29 was proposed in reference 7 on the basis of the numerical solutions for the $\sigma = 1$ cases. The use of the exponent 0.265 improves the correlation with the $\sigma = 0.72$ data, but at the expense of a poorer correlation of the $\sigma = 1$ data. Since the $\sigma = 0.72$ solution is the more realistic one, for air, the exponent 0.265 should be used in equation (19b) when making estimates of the wall heat transfer due to strong shocks moving through air.

Reference 10 obtained numerical solutions for the shear and heat transfer associated with strong shocks moving through oxygen, assuming

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$\sigma = 0.72$ and different Lewis numbers. The data for Lewis number equal to 1 were correlated to within 1 percent by equations (19a) and (19b) with C_e raised to the 0.24 power. The difference between the exponents 0.265 and 0.24 is relatively small, considering that the viscosity law, the shock solutions, and the real gas properties used in reference 10 were different from those used herein.

The displacement thickness δ^* in the steady coordinate system (fig. 1(a)) is of interest for computing attenuation and nonuniformities in shock tubes (e.g., refs. 8 and 9). The displacement thickness is found from $I(\infty)$ and $\lim\limits_{\eta \to \infty} (f - \eta)$ by substituting into equation (11d). Values for these quantities, obtained from the numerical integration of equations (7), are given in table I. The following approximate formula can be used to estimate δ^*:

$$\delta^* \sqrt{\frac{u_e}{2x\nu_w}} \frac{\rho_e}{\rho_w} = C_e^{0.37} \left[\lim_{\eta \to \infty} (\eta - f) - I(\infty)\right]_{\text{Ref. 3}} \tag{20}$$

where

$$\left[\lim_{\eta \to \infty} (\eta - f)\right]_{\text{Ref. 3}} \equiv \frac{1.134(1 - U)}{\sqrt{1 + 1.022\ U}} \tag{21}$$

$$[I(\infty)]_{\text{Ref. 3}} \equiv \frac{1.134}{\sqrt{1 + 1.022\ U}} \frac{\frac{h_r}{h_e} - \frac{h_w}{h_e}}{\sigma^{0.47+0.029\ U}} - \frac{1.569}{\sqrt{1 + 0.993\ U}} \frac{(U - 1)^2 \frac{u_e^2}{2h_e}}{\sigma^{0.045(U-1)}} \tag{22}$$

Equations (21) and (22) were obtained in reference 3 and correlate the constant property solutions to within 1 percent. The factor $C_e^{0.37}$, in equation (20), corrects these expressions for the effect of variable C. The ratio of the value of δ^*, as obtained from equation (20), to the value of δ^* obtained by numerical integration is given in table I. It is seen that, for the cases considered herein, equation (20) is correct to within 3 percent.

The boundary-layer thickness δ in the unsteady (wall stationary) coordinate system (fig. 1(b)) may be defined as the value of y at which the velocity relative to the wall reaches 99 percent of its free-stream value. That is, it corresponds to

$$
\left.\begin{aligned}
\frac{\bar{u}}{\bar{u}_e} = \frac{U - f'}{U - 1} = 0.99 \\
\\
f' = 0.99 + 0.01\,U
\end{aligned}\right\} \tag{23}
$$

or

This boundary-layer parameter is of interest, in experimental shock-tube studies, as a measure of the extent to which the boundary layer extends into the free stream.

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The value of η corresponding to $y = \delta$ is denoted herein by η_δ. The boundary-layer thickness δ can then be computed from equation (11c) by using the values of η_δ and $I(\infty)$ given in table I. (More properly, $I(\eta_\delta)$ should be used when computing δ from equation (11c), but the use of $I(\infty)$ should introduce a negligible error.) The resulting values for δ are included in table I.

The following approximate formula can be used to estimate δ:

$$\delta \sqrt{\frac{u_e}{2x\nu_w}}\,\frac{\rho_e}{\rho_w} = C_e^{0.48}\left[\eta_\delta - I(\infty)\right]_{\mathrm{Ref.}\ 3} \tag{24}$$

where

$$\left[\eta_\delta\right]_{\mathrm{Ref.}\ 3} = \frac{3.20}{\sqrt{1 + 0.543\,U}} \tag{25}$$

and $I(\infty)$ is found from equation (22). (Again, $I(\eta)$ has been evaluated, for convenience, at $\eta = \infty$ rather than η_δ. The error should be unimportant.) Equation (25) was found by assuming η_δ to be of the form $A/\sqrt{1 + BU}$ and determining A and B from the $U = 1$ and $U = 6$ constant property results of reference 3. (Eq. (25) differs slightly from a similar formula given in appendix D of ref. 8 and is the more accurate expression if the edge of the boundary layer is taken to be $\bar{u}/\bar{u}_e \equiv 0.990$.) The coefficient $C_e^{0.48}$, in equation (24), is the approximate correction to account for variable C. The ratio of δ, as obtained from equation (24), to the value obtained from the numerical integration of equations (7) is given in table I. Equation (24) is correct to 4 percent for the cases noted herein.

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APPENDIX - SYMBOLS

a	local speed of sound
C	$\rho\mu/(\rho\mu)_w$
f	function of η defined by eq. (6)
g	H/H_e
H	stagnation enthalpy in shock stationary coordinate system
h	static enthalpy
$I(\eta)$	integral defined by eq. (12)
M_s	Mach number of the shock
p	pressure
q_w	heat-transfer rate
R	gas constant
T	absolute static temperature
U	u_w/u_e
u,v	velocities parallel to x,y axis
$\overline{u},\overline{v}$	velocities parallel to $\overline{x},\overline{y}$ axis
$\overline{u}_s$	velocity of shock in $\overline{x},\overline{y}$ coordinate system
u_w	velocity of wall in x,y coordinate system
x,y	steady coordinate system (fig. 1(a))
$\overline{x},\overline{y}$	unsteady coordinate system (fig. 1(b))
z	molecular weight ratio
δ	fluid velocity - boundary-layer thickness (value of y corresponding to $\overline{u}/\overline{u}_e \equiv (u - u_w)/(u_e - u_w) = 0.99$)
δ^*	fluid-boundary-layer displacement thickness in steady coordinate system, $\int_0^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right)dy$

η similarity parameter, eq. (4)

η_δ value of η corresponding to $y = \delta$

μ coefficient of viscosity

ν kinematic viscosity

ξ function defined by eq. (4)

ρ mass density

σ Prandtl number

τ_w local shear stress exerted by fluid on wall

ψ stream function, eq. (3)

Subscripts:

e flow external to fluid boundary layer

r quantity evaluated for zero heat transfer

w quantity evaluated at the wall

1 undisturbed flow ahead of shock

Superscript:

' denotes differentiation with respect to η

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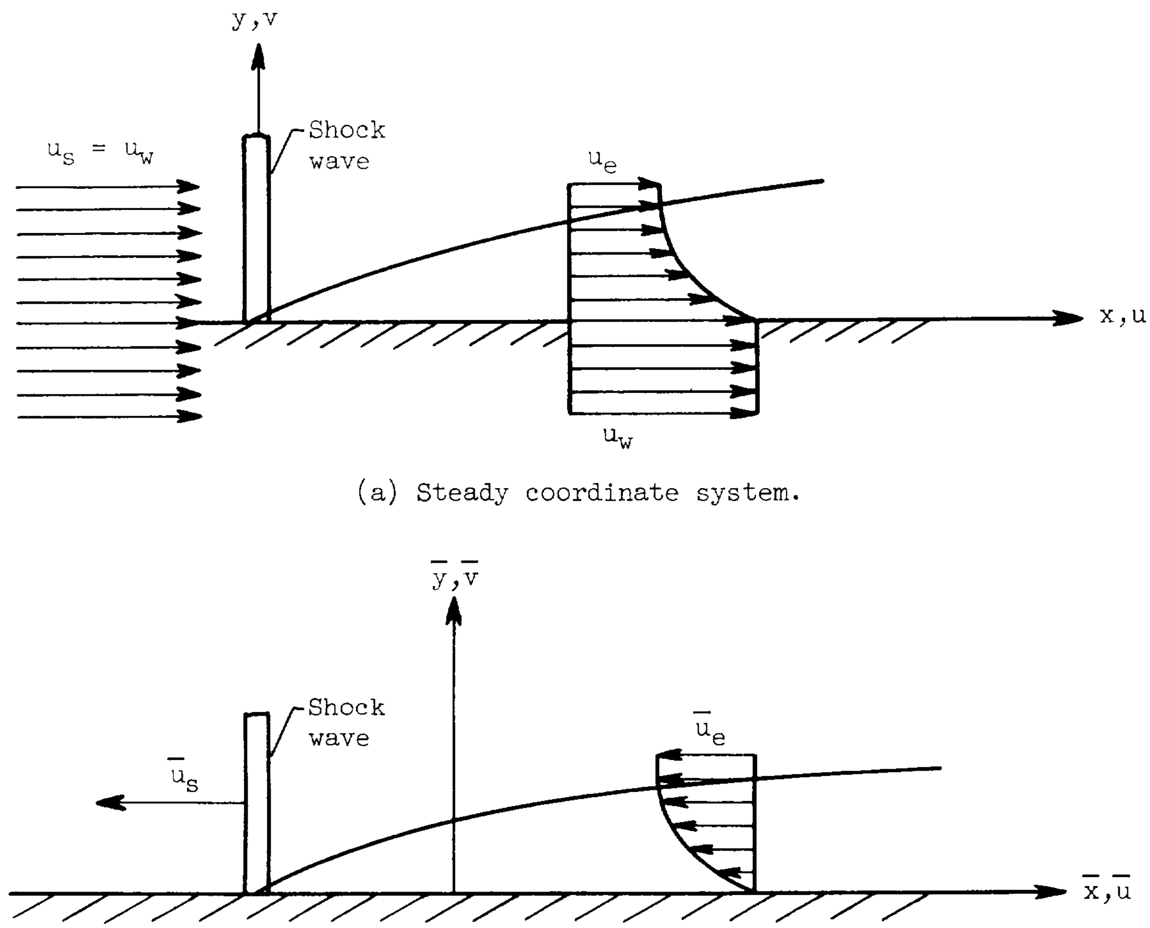
TABLE I. - SUMMARY OF LAMINAR-BOUNDARY-LAYER RESULTS

[$T_1 = T_w = 522^{\circ}$ R; $g(0) = 1$; δ and η_δ correspond to $\bar{u}/\bar{u}_e = 0.99$.]

M_s	p_1, atmos.	σ	U	$\frac{u_e^2}{2H_e}$	$\frac{h_e}{h_w}$	C	C_e	Results of numerical integration of equation (7)								Accuracy of approximate formulas			
								$-f''(0)$	$-g'(0)$	$\frac{h'(0)}{h_w}$	η_δ	$\lim_{\eta\to\infty}(f-\eta)$	$I(\infty)$	$\delta^*\sqrt{\frac{u_e}{2x\nu_w}}\frac{\rho_e}{\rho_w}$	$\delta\sqrt{\frac{u_e}{2x\nu_w}}\frac{\rho_e}{\rho_w}$	$\frac{[f''(0)]_{Eq. (19a)}}{f''(0)}$	$\frac{[h'(0)]_{Eq. (19b)}}{h'(0)}$	$\frac{(\delta^*)_{Eq. (20)}}{\delta^*}$	$\frac{(\delta)_{Eq. (24)}}{\delta}$
4	0.001	1	4.875	0.032072	4.066	1	1	5.721	0	7.514	1.665	1.794	------	------	------	1.000	1.000	-----	-----
6	↓	↓	6.190	.022920	8.022	↓	↓	8.528	↓	19.87	1.528	2.176	------	------	------	1.001	1.001	-----	-----
8	↓	↓	8.060	.014279	13.62	↓	↓	13.09	↓	41.63	1.382	2.643	------	------	------	1.001	1.001	-----	-----
10	↓	↓	10.11	.009318	20.82	↓	↓	18.77	↓	74.33	1.263	3.085	------	------	------	1.002	1.002	-----	-----
12	↓	↓	10.93	.008090	29.58	↓	↓	21.23	↓	112.0	1.224	3.247	------	------	------	1.002	1.002	-----	-----
14	↓	↓	11.33	.007597	40.05	↓	↓	22.46	↓	156.0	1.206	3.324	------	------	------	1.002	1.002	-----	-----
4	0.001	1	4.875	0.032072	4.066	Eq. (16)	0.6410	5.155	0	6.770	1.338	1.560	0.2118	-1.772	1.126	0.976	0.976	0.983	1.018
6	↓	↓	6.190	.022920	8.022	↓	.4951	7.101	↓	16.54	1.073	1.710	.1877	-1.898	.8853	.980	.980	.989	1.025
8	↓	↓	8.060	.014279	13.62	↓	.4055	10.15	↓	32.26	.8723	1.901	.1504	-2.051	.7219	.995	.995	1.002	1.026
10	↓	↓	10.11	.009318	20.82	↓	.3459	13.67	↓	54.13	.7319	2.061	.1204	-2.181	.6115	1.011	1.011	1.017	1.023
12	↓	↓	10.93	.008090	29.58	↓	.2992	14.65	↓	77.27	.6582	2.039	.1042	-2.143	.5540	1.024	1.024	1.026	1.018
14	↓	↓	11.33	.007597	40.05	↓	.2528	14.75	↓	102.5	.5963	1.965	.0921	-2.057	.5042	1.024	1.024	1.026	1.015
4	0.001	0.72	4.875	0.032072	4.066	Eq. (16)	0.6410	5.228	0.3454	5.416	1.351	1.584	0.2820	-1.866	1.069	0.962	0.971	0.972	1.008
6	.001	↓	6.190	.022920	8.022	↓	.4951	7.258	.4466	13.24	1.086	1.749	.2537	-2.002	.8322	.959	.967	.974	1.009
6	.01	↓	6.150	.023220	8.022	↓	.4951	7.181	.4447	13.19	1.089	1.741	.2546	-1.995	.8346	.959	.968	.974	1.008
8	.001	↓	8.060	.014279	13.62	↓	.4055	10.43	.5305	25.82	.8825	1.951	.2056	-2.157	.6769	.968	.973	.985	1.002
10	.001	↓	10.11	.009318	20.82	↓	.3459	14.10	.5947	43.33	.7404	2.120	.1658	-2.286	.5746	.981	.980	.998	.988
10	.01	↓	9.40	.010760	20.83	↓	.3458	12.57	.5689	41.66	.7641	2.023	-----	------	-----	.980	.982	----	----
12	.001	↓	10.93	.008090	29.58	↓	.2992	15.15	.6035	61.93	.6673	2.102	.1434	-2.246	.5239	.989	.988	1.007	.973
14	.001	↓	11.33	.007597	40.05	↓	.2528	15.31	.5964	82.29	.6081	2.035	.1264	-2.161	.4817	.987	.985	1.003	.957
14	.01	↓	10.65	.008598	40.00	↓	.2530	13.89	.5749	79.46	.6259	1.960	-----	------	-----	.987	.987	-----	----

E-975



(a) Steady coordinate system.

(b) Unsteady coordinate system.

Figure 1. - Coordinate systems used to study boundary layer behind a shock wave advancing into a stationary fluid.

NASA TN D-291
National Aeronautics and Space Administration.
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The laminar wall boundary layer behind a strong shock advancing into stationary air has been determined. Numerical results have been obtained for shock Mach numbers up to 14, using real gas values for density and viscosity and assuming Prandtl and Lewis numbers of 0.72 and 1, respectively. The numerical results for shear and heat transfer agree, within 4 percent, with a previously presented approximate analytical expression for these quantities. A slight modification of this expression results in agreement with the numerical data to within 2.5 percent. Analytical expressions for boundary-layer thickness and displacement thickness, correct to within 4 percent for the present data, have also been obtained.

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