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THE LIMITING SIZES OF THE HABITABLE PLANETS

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by
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SUMMARY

The astrobiological problem of the occurrence of life in the universe is discussed from the standpoint of the size and nature of planets upon which living organisms might arise. The conclusion is tentatively drawn that the most likely radius of a habitable planet lies between 10^8 cm and 2×10^9 cm. Conditions of temperature and density also bear upon the occurrence of life; thus the moon and Mercury, although both fall within the range of favorable radii, are nevertheless believed uninhabited by indigeneous life.

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INTRODUCTION

In a number of papers (e.g. References 1 and 2) the astrobiological problem of the occurrence of life in the universe has been studied from the standpoint of the nature of stars. By some simple and direct arguments it has been concluded that the emergence of life, especially of an advanced form, has its highest probability in the neighborhood of single main-sequence stars of spectral types F, G and K. In this paper the problem will be discussed from the standpoint of the nature of planets on which living organisms might arise.

FUNDAMENTAL CONDITIONS

In order to derive some limits to the sizes of the habitable planets, it is necessary first to examine the fundamental conditions under which life may emerge. Although there is little definite information regarding the origin of life even on the earth, authorities in this field generally agree that coacervates in an aqueous medium represent the earliest stage in the evolution of organic substances into living organisms (Reference 3). Therefore an atmosphere over a solid crust of the planet is necessary in order to prevent water from evaporating away rapidly. Indeed, even if life did not originate in an aqueous medium, living organisms without the aid of some substances in the liquid form for their maintenance are virtually inconceivable.

Since stars and planets are formed out of the interstellar medium, which is composed predominantly of hydrogen, it is reasonable to theorize that in the early stages of its evolution a planet (or more properly a proto-planet) must have a high percentage of hydrogen, especially in its outer envelope (Reference 4). Life may first appear under reducing conditions (Reference 3), but an atmosphere composed mainly of hydrogen does not favor its development into a high form. Thus, the hydrogen atmosphere must first be at least partially dissipated. Indeed, the present atmosphere of the earth is actually a secondary one formed as the result of chemical processes that took place subsequent to the formation of the planet (Reference 5). If the condition is imposed that the hydrogen atmosphere be dissipated completely or partially in a time less than a certain value (say 3×10^9 years), an upper limit for the total mass of the planet can be derived because a very massive planet (e.g., Jupiter) will retain the hydrogen atmosphere for a time much longer than this value. On the other hand, if life of an advanced form is to be found on the surface of

a planet, the secondary atmosphere must not be dissipated in less than 3×10^9 years, which is believed to be the time-scale of biological evolution on the earth. If living organisms elsewhere exist by the same chemical processes as do terrestrial ones, the oxygen must not be lost in the time-scale of biological evolution. This necessity for maintaining a secondary atmosphere composed of oxygen and other gases for a certain period of time places a lower limit on the mass of the habitable planet. Urey has qualitatively discussed the habitability of a planet from the foregoing point of view (Reference 6), and a quantitative estimate for the sizes of habitable planets can now be developed from these considerations.

DISSIPATION OF PLANETARY ATMOSPHERES

It should be emphasized that by the dissipation of a hydrogen atmosphere is not meant so large a loss of mass by a planet (or proto-planet) as to change drastically its relative chemical composition. Shklovsky has conclusively illustrated (Reference 7) that such a change is not feasible. On the other hand, hydrogen must be a dominant ingredient in the upper layers of a planet in the early stages because of its high cosmic abundance and its lightness.

Jeans has derived the rate of mass-loss of planetary atmospheres by computing the number of molecules which cross unit area of a sphere in an outward direction in unit time with a speed greater than the escape velocity there (Reference 8). He gives also the time-scale, t_1, for complete dissipation of the atmosphere. Since he assumes a constant rate of dissipation, the time-scale for the loss of one half of the atmosphere is simply $t_1/2$. Several other refined calculations for the escape rate have since been performed but all without any significant difference from Jeans' original formula (Reference 9).

If we denote the radius and the mean density of the planet by a and ρ, the root-mean-square velocity of molecules by C, and the gravitational constant by G, we can express t_1 of Jeans as follows:

$$t_1 = \frac{4.35}{4\pi G\rho} \frac{C^3}{a(C^2 + 4\pi G\rho a^2)} \exp\left(\frac{4\pi G\rho a^2}{C^2}\right) \text{seconds.} \qquad (1)$$

By introducing two parameters of the dimension of time

$$\tau_o = (4\pi G\rho)^{-\frac{1}{2}}, \qquad \tau_1 = \frac{a}{C}, \qquad (2)$$

and by writing

$$\frac{\tau_1}{\tau_o} = x, \qquad (3)$$

Equation 1 can be reduced to the dimensionless form

$$\frac{t_1}{4.35\,\tau_o} = \frac{e^{x^2}}{x(1+x^2)}\ , \tag{4}$$

which illustrates its physical implications more clearly.

For $t_1 = 3 \times 10^9$ years and 6×10^9 years respectively, x has been determined for three values of τ_o corresponding to three values of ρ. The results are given in Table 1. It is significant that the value of x derived from Equation 4 does not change greatly with the time t_1 or with ρ; this is due to the exponential factor in the equation. A star of spectral type F, G or K stays on the main sequence for a time varying from 10^9 to 10^{11} years, and if life is to be found on any planet associated with such a star, t_1 must be within this time range. The corresponding values of x lie between 5.8 and 6.4; consequently, for all practical purposes, x may be regarded as a constant. Physically, this means that whether a planet retains an atmosphere depends not so much on the time-scale of its existence as on its radius and mean density. In other words, there exist critical pairs of values of mass and radius which completely determine the ability of a planet to retain an atmosphere of some given gas. This fact, indeed, is what makes the present estimate of the limiting sizes of habitable planets significant.

PLANETARY RADIUS AND DENSITY

The upper limit of a for the habitable planet is, by Equation 2,

$$a = C\tau_o x\ , \tag{5}$$

where C represents the mean velocity of molecular or atomic hydrogen as the case may be. In Equation 5, the value of x which may be regarded as constant is about 6. Also, τ_o varies within a reasonable range because it is inversely proportional to the square root of the mean density of the planet, and the mean density does not vary greatly from one celestial body to another. In the case of planets in the solar system, for example, the

Table 1
Values of x Determined for Three Values of τ_o Corresponding to Three Values of ρ

Mean Density ρ (gm/cm^3)	$\tau_o = (4\pi G\rho)^{-\frac{1}{2}}$	Values of $x = \tau_1/\tau_o$	
		$t_1 = 3 \times 10^9$ years	$t_1 = 6 \times 10^9$ years
3.5	5.838×10^2	6.056	6.116
4.5	5.149×10^2	6.067	6.126
5.5	4.658×10^2	6.076	6.135

mean density lies between 0.7 (Saturn) and 5.5 (Earth). Hence the corresponding highest and lowest values of τ_o have a ratio of less than 3. Actually there is reason to believe that hydrogen may be dissipated during the early stages of the planet's evolution when the mean density is lower than its final value. Even so, no change in the order of magnitude of τ_o is to be expected. Consequently the upper limit of a is predominantly determined by C, which is the thermal velocity in the outermost atmosphere (or "exosphere" as Spitzer calls it; see Reference 9) and is difficult to estimate, especially in the early stages of the planet's evolution. Until the detailed atomic processes occurring in the exosphere during the dissipation of hydrogen are understood, a range of reasonable temperatures must be assumed in order to compute the upper limit of a. The results are given in Table 2, where the molecular velocity of hydrogen has been used. If the atomic state of hydrogen is dominant, the limits of a in the table should be increased by a factor of $\sqrt{2}$. The corresponding upper limit of the total mass M of the planet can be calculated directly from a and ρ.

The lower limits of both the radii and total masses for the habitable planets of different mean densities can be calculated in a similar way. If the condition is imposed that oxygen should not be completely lost in 3×10^9 years, the lower limit of a is given also by Equation 5 with the same values of x as given in Table 1; however, C should now be understood as the velocity of oxygen molecules or atoms, depending on the physical conditions in the exosphere. Since the molecular weight of oxygen is about 16 times that of hydrogen, the thermal velocity of oxygen molecules is only one-fourth that of hydrogen molecules. Thus, the lower limit of a is one-fourth the value derived for the upper limit and the lower limit of M is 1/64 the value for the upper limit.

Equation 5 may also be written in the following form:

$$a = \left(\frac{3kT}{4\pi G\rho m}\right)^{\frac{1}{2}} x \,, \tag{6}$$

Table 2
Limiting Radii of Habitable Planets

Absolute Temperature, T (°K)	Mean Molecular Velocity, C (cm/sec)	Planetary Radius, a (cm)		
		$\rho = 3.5$ gm/cm^3	$\rho = 4.5$ gm/cm^3	$\rho = 5.5$ gm/cm^3
173	1.47×10^5	5.2×10^8	4.6×10^8	4.2×10^8
273	1.84×10^5	6.5×10^8	5.8×10^8	5.2×10^8
373	2.15×10^5	7.6×10^8	6.7×10^8	6.1×10^8
573	2.66×10^5	9.4×10^8	8.3×10^8	7.5×10^8
1000	3.51×10^5	12.4×10^8	11.0×10^8	9.9×10^8
2000	4.96×10^5	17.5×10^8	15.5×10^8	14.0×10^8

where m is the mass of molecules concerned. Since x may be taken as a constant, bot the upper and lower limits of a are directly proportional to $\sqrt{T/\rho}$. In other words, th probability of life on a planet depends only on its mean density and the temperature of i atmosphere.

CONCLUDING REMARKS

Tentatively we may conclude that a habitable planet has most likely a radius lying between 10^8 cm and 2×10^9 cm. This is a generous estimate since that range includes the moon and Mercury, both of which are known to be uninhabitable because of their unfavorable position in the solar system and their unfavorable densities.

Poveda has recently suggested that the rate of biological evolution on a planet may increase with its surface area if other conditions are equal. If this is so, the earth has not only a favorable position in the solar system but also a favorable size for developing living organisms.

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