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STUDY OF AN ACTIVE CONTROL SYSTEM FOR A SPINNING BODY

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SUMMARY

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The mission requirements for some satellites require that they spin continuously and at the same time maintain a precise direction of the spin axis. An analog-computer study has been made of an attitude control system which is suitable for such a satellite. The control system provides the necessary attitude control through the use of a spinning wheel, which will provide precession torques, commanded by an automatic closed-loop servomechanism system. The sensors used in the control loop are rate gyroscopes for damping of any wobble motion and a sun seeker for attitude control. The results of the study show that the controller can eliminate the wobble motion of the satellite resulting from a rectangular pulse moment disturbance and then return the spin axis to the reference space axis. The motion is damped to half amplitude in less than one cycle of the wobble motion. The controller can also reduce the motion resulting from a step change in product of inertia both by causing the new principal axis to be steadily alined with the spin vector and by reducing the cone angle generated by the reference body axis. These methods will reduce the motion whether the satellite is a disk, sphere, or rod configuration.

INTRODUCTION

The mission requirements of some satellites can best be satisfied if they spin continuously. An example is a manned space station which spins in order to provide the occupants with a simulated gravity field. It has been shown that if such a spinning body is disturbed it will wobble with an undamped motion. (See refs. 1 to 4.) Therefore, such spinning satellites require an attitude control system. For example, if the manned space station is equipped with an auxiliary power unit which uses a parabolic solar collector, it will be necessary for this collector to point continuously at the sun with fairly good accuracy. In order to provide control for such continuous pointing, the control system should damp the wobble which will result from any outside disturbance and from internal movements that result in changes in product of inertia of the station with respect to the body axes. It will also be necessary to provide a means of erecting the station to the sunline initially and each time the station comes out of the earth's shadow.

Presented herein are results of an analog-computer study of a control system which can satisfy these control requirements. The control torques are derived from the precession torques generated by a spinning wheel. It is assumed that the control of this wheel is automatic through the use of closed-loop control, with the command information supplied by rate gyroscopes for rate control and a sun seeker for control of the position of the satellite axis.

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SYMBOLS

X,Y,Z body axes

X_i,Y_i,Z_i axes fixed in inertial space

X',Y',Z' principal axes; identical to body axes when there is no product of inertia

I_X,I_Y,I_Z moments of inertia about body X-, Y-, and Z-axis, respectively, slug-ft^2

I_{XY},I_{XZ},I_{YZ} products of inertia, slug-ft^2

T_X,T_Y,T_Z body-axis control torques about X-, Y-, and Z-axis, respectively, ft-lb

M disturbance moment, ft-lb

p,q,r rates of rotation about X-, Y-, and Z-axis, respectively, radians/sec

$\omega_X,\omega_Y,\omega_Z$ body-axis components of total rotational rate of control-wheel angular-momentum vector, radians/sec

H angular momentum due to spin of control wheel, ft-lb-sec

H' constant defined by equations (38), slug-ft^2

ψ,θ,ϕ Euler angles, radians

$\bar{\eta}$ constant vector on X_i-axis (denotes only magnitude when bar is removed), used with no dimensions specified

l,m,n components of η along X-, Y-, and Z-axis, respectively, used with no dimensions specified

δ_Z outer gimbal deflection, radians unless otherwise specified

δ_Y inner gimbal deflection, radians unless otherwise specified

K_1 control gain, radians/radian/sec

K_1' constant defined by equations (38)

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K_2 control gain, radians/unspecified dimension

s Laplacian variable, per sec

t time, sec

Ω frequency, radians/sec

$T_{1/2}$ time to damp to half amplitude, sec

P period, sec

c angle between body axes and principal axes, radians

Subscripts:

o initial condition

X,Y,Z component in X-, Y-, or Z-axis directions

X'Y' principal axes

A dot over a quantity indicates differentiation with respect to time.

DESCRIPTION OF CONTROL SYSTEM

General Description

In order to facilitate a proper visualization of the control system and its operation, a general description will be given followed by a more exact definition with equations. The torque used by this control system is the precession torque produced by a spinning wheel. Precession torques arise when a spinning wheel is forced to rotate about an axis other than its spin axis.

A spinning satellite provides a natural situation for the application of a spinning wheel to provide precession torques. In this case the spinning of the satellite provides a constantly available angular-rate vector which can be combined with the angular-momentum vector of the control wheel to provide a continuous torque. The operation of the assumed mechanism is as follows. When no torque is required, the control-wheel angular-momentum vector is alined with the spin vector of the satellite. When torque is required about a particular body axis of the satellite, the control wheel is rotated on an axis parallel to that body axis, thereby a component of the control-wheel angular-momentum vector is
produced along an axis that is perpendicular to both the satellite spin L
vector and the satellite body axis for which the torque is required. 1
This situation produces the desired torque, which can be expressed as 5
a cross product of the satellite spin rate and the control-wheel angular 1
momentum. This torque is a nearly proportional function of the tilt of 9
the control wheel, with a constant spin rate of the station and a constant angular momentum of the control wheel assumed.

A sketch of the mechanism studied in this investigation is shown in figure 1. In this case a single control wheel mounted in a double gimbal is assumed. This type of gimbal mounting will produce a conflict between the operation of control about one axis and that about the other - that is, if the inner gimbal is deflected 90°, rotation of the outer gimbal will not change the direction of the control-wheel momentum vector. For small deflections of the inner gimbal, the effect is less pronounced. It would also be possible to use two control wheels mounted in single, mutually perpendicular gimbals and eliminate this conflict. However, the single-wheel configuration offers weight-saving possibilities.

In this study it is assumed that the control-wheel gimbal angles are commanded by signals from rate gyroscopes mounted on the body axes of the satellite and by signals from a sun seeker mounted on the spin axis of the satellite and rigidly attached to the satellite. The rate-gyroscope signals provide damping, and the sun seeker provides the necessary signal for alining the satellite with the sunline.

Equations of Motion

The general equations of motion are simplified by making the following assumptions:

no coupling exists between the force equations and the moment equations

$$I_{XZ} = I_{YZ} = 0$$

$$M_X = M_Z = 0$$

the product of inertia terms $I_{XY}\dot{p}$ and $I_{XY}\dot{q}$ can be neglected Then the rotational equations of motion for a rigid body are:

$$I_X\dot{p} + \left(I_Z - I_Y\right)qr + I_{XY}pr = T_X \tag{1}$$

$$I_Y\dot{q} + \left(I_X - I_Z\right)rp - I_{XY}qr + M_Y = T_Y \tag{2}$$

$$I_Z\dot{r} + \left(I_Y - I_X\right)pq - I_{XY}p^2 + I_{XY}q^2 = T_Z \tag{3}$$

The problems studied herein include the effect of introducing a rectangular pulse disturbance about the Y-axis and the effects of products of inertia on the motion of the satellite. Products of inertia exist when the X, Y, and Z body axes are not the principal axes of the body. For convenience and clarity in the preceding equations only an I_{XY} product of inertia is included, although I_{XZ} and I_{ZY} products could also exist.

Presumably, a manned space station such as is being considered herein will be built so that either the maximum or minimum principal axis of inertia will be parallel to the center line of the solar collector. The movement of an occupant will therefore cause the product of inertia with respect to these body axes to change from zero to some finite value. In order to investigate this situation, tests were made in which a product of inertia I_{XY} was introduced. The movement of an occupant would also cause small changes in the body-axis moments of inertia, but these effects are neglected herein because they would have only a small effect on the results.

In all the cases considered, it is assumed that initially the satellite is spinning about the X-axis, so that the initial spin rate is equal to p_o, and that it is the X-axis that should point in the reference direction.

Control Equations

The precession torque generated by a spinning wheel is equal to the product of the moment of inertia of the wheel, its spin velocity, and the rate-of-rotation components which are perpendicular to the spin axis of the wheel. For this investigation the moment of inertia of the control wheel and its spin velocity are assumed to be constant, and only their product, known as the angular momentum of the control wheel H, will be specified. It is assumed that the angular momentum of the controller is due entirely to the spin of the control wheel, that is, angular momentum due to rotation of the controller on axes perpendicular to the wheel spin axis is neglected. As described before, this momentum vector of the control wheel will be alined with whichever body axis is prescribed, and therefore it is the body-axis components of the control-wheel momentum, H_X, H_Y, and H_Z, that are of concern. The components of the precession torque can be assumed equal to the product of these angular-momentum components and the orthogonal rates to which they are subjected. The equations for the body-axis components of the precession torque are

$$T_X = H_Y\omega_Z - H_Z\omega_Y \tag{4}$$

$$T_Y = H_Z\omega_X - H_X\omega_Z \tag{5}$$

$$T_Z = H_X\omega_Y - H_Y\omega_X \tag{6}$$

The rate terms ω_X, ω_Y, and ω_Z, which are the X, Y, and Z components of the total rotation of the control-wheel angular-momentum vector, are given by the equations

$$\omega_X = p - \dot{\delta}_Y \sin \delta_Z \tag{7}$$

$$\omega_Y = q + \dot{\delta}_Y \cos \delta_Z \tag{8}$$

$$\omega_Z = r + \dot{\delta}_Z \tag{9}$$

In order to derive the gimbal rates, $\dot{\delta}_Y$ and $\dot{\delta}_Z$, first it will be necessary to define the gimbal-angle equations.

The purpose of the control system is to supply damping torques and torques which will aline the satellite X-axis with the reference sun-line. The torques which are desired are torques about the Y- and Z-axis. Since ω_X will be a large and steady value as compared with ω_Y and ω_Z, the Y and Z torques can be obtained by calling for momentum components H_Z and H_Y, respectively, which, it can be seen, are multiplied by ω_X. These momentum components are obtained by commanding a proper orientation of the gimbals carrying the control wheel. The proper orientation of the gimbals is a function of rate-gyroscope and sun-seeker signals.

A practical consideration of the output signals from rate gyroscopes leads to the conclusion that, for analytical purposes, these signals can be considered equal to the body-axis rates q and r. The analytical representation of the sun-seeker signals is as follows: A constant vector $\bar{\eta}$, coincident with light rays from the sun, is assumed to exist on the reference X_i-axis. Body-axis components of this constant vector can be defined by the equations:

$$l = (\cos\theta \cos\psi)\eta \tag{10}$$

$$m = (\cos\psi \sin\theta \sin\phi - \sin\psi \cos\phi)\eta \tag{11}$$

$$n = (\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi)\eta \tag{12}$$

where ψ, θ, and ϕ are Euler angles, taken in that order. A sketch showing these components is given in figure 2. The components m and n are used to represent the sun-seeker signals. The fact that the quantities m and n have the same characteristics that sun-seeker signals will have is illustrated by the following two special examples. If the satellite is spinning about its X-axis but with the X-axis displaced from the reference X_i-axis, then m and n will oscillate about zero with a frequency equal to the spin frequency and with a peak amplitude equal to the displacement. If the satellite is spinning with the total resultant rotation vector on the reference X_i-axis but with the X-axis displaced from the reference line, then m and n will be constant with $\sqrt{m^2 + n^2}$ equal to the half angle of the cone generated by the X-axis.

It is not necessary to determine the Euler angles in order to determine l, m, and n. These components can be determined from the following relationships:

$$\dot{l} = mr - nq \tag{13}$$

$$\dot{m} = np - lr \tag{14}$$

$$\dot{n} = lq - mp \tag{15}$$

$$l = \eta + \int \dot{l} \, dt \tag{16}$$

$$m = \int \dot{m} \, dt \tag{17}$$

$$n = \int \dot{n} \, dt \tag{18}$$

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It is now assumed that the desired damping and attitude control torques will be obtained if the gimbal deflections are defined by the control equations

$$\delta_Z = K_1 r + K_2 m \tag{19}$$

$$\delta_Y = K_1 q - K_2 n \tag{20}$$

where K_1 and K_2 are the constant control gains. K_1 has the dimensions of radians per radian per sec, whereas K_2 is a nondimensional number that, for small deflections, can be thought of as expressing radians (of gimbal angle) per radian (of angular displacement of body axes from reference space axes). The mathematical signs given in these control equations are for the situation where the control-wheel angular-momentum vector is nominally in the same direction as the spin vector of the satellite, as shown in figure 1. The control should be such that the gimbal always runs ahead of the satellite, that is, the gimbal should tilt further than the satellite.

The body-axis components of the control-wheel angular-momentum vector can now be defined as

$$H_X = H \cos \delta_Y \cos \delta_Z \tag{21}$$

$$H_Y = H \cos \delta_Y \sin \delta_Z \tag{22}$$

$$H_Z = -H \sin \delta_Y \tag{23}$$

Also, the rate factors, ω_X, ω_Y, and ω_Z that are a part of the precession-torque terms can now be defined. As was stated previously, these rate factors are made up of the body-axis rates and the gimbal rates, which are added vectorially. Since the gimbal angles are functions of body-axis rates and m and n, the gimbal rates are functions of the derivatives of these factors:

$$\dot{\delta}_Z = K_1\dot{r} + K_2\dot{m} \tag{24}$$

$$\dot{\delta}_Y = K_1\dot{q} - K_2\dot{n} \tag{25}$$

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Substituting equations (24) and (25) into equations (7), (8), and (9) gives

$$\omega_X = p - \left(K_1\dot{q} - K_2\dot{n}\right)\sin\left(K_1 r + K_2 m\right) \tag{26}$$

$$\omega_Y = q + \left(K_1\dot{q} - K_2\dot{n}\right)\cos\left(K_1 r + K_2 m\right) \tag{27}$$

$$\omega_Z = r + K_1\dot{r} + K_2\dot{m} \tag{28}$$

Substituting the expressions for the body-axis components of the precession torque (eqs. (4) to (6)) into equations (1) to (3), and then making the substitutions for gimbal deflections and angular-momentum vector from equations (19) to (23) and the substitutions for the rate factors from equations (26) to (28) yields

$$\begin{aligned} I_X\dot{p} + \left(I_Z - I_Y\right)qr + I_{XY}pr = {} & H\left[\cos\left(K_1 q - K_2 n\right)\sin\left(K_1 r + K_2 m\right)\right]\left(r + K_1\dot{r} + K_2\dot{m}\right) \\ & + H\left[\sin\left(K_1 q - K_2 n\right)\right]\left[q + \left(K_1\dot{q} - K_2\dot{n}\right)\cos\left(K_1 r + K_2 m\right)\right] \end{aligned} \tag{29}$$

$$\begin{aligned} I_Y\dot{q} + \left(I_X - I_Z\right)rp - I_{XY}qr + M_Y = {} & -H\left[\sin\left(K_1 q - K_2 n\right)\right]\left[p - \left(K_1\dot{q} - K_2\dot{n}\right)\sin\left(K_1 r + K_2 m\right)\right] \\ & - H\left[\cos\left(K_1 q - K_2 n\right)\cos\left(K_1 r + K_2 m\right)\right]\left(r + K_1\dot{r} + K_2\dot{m}\right) \end{aligned} \tag{30}$$

$$\begin{aligned} I_Z\dot{r} + \left(I_Y - I_X\right)pq - I_{XY}p^2 + I_{XY}q^2 = {} & H\left[\cos\left(K_1 q - K_2 n\right)\cos\left(K_1 r + K_2 m\right)\right]\left[q + \left(K_1\dot{q} - K_2\dot{n}\right)\cos\left(K_1 r\right.\right. \\ & \left.\left. + K_2 m\right)\right] - H\left[\cos\left(K_1 q - K_2 n\right)\sin\left(K_1 r + K_2 m\right)\right]\left[p - \left(K_1\dot{q} - K_2\dot{n}\right)\sin\left(K_1 r + K_2 m\right)\right] \end{aligned} \tag{31}$$

The original intention was that, for example, a body rate q would call for a Y torque. Generally, this intention is carried out, but the preceding equations show that other torques are also commanded as a result of cross-coupling effects. These cross-coupling effects make it difficult to predict the effect of a given control command and make it desirable to conduct an analytical study of the system.

The use of simple constants K_1 and K_2 for the control gains means that perfect servomechanism operation is being assumed. Since the highest frequencies that will appear in the solutions are equal to the spin frequency, and since servomechanisms can be made with natural frequencies much higher than this spin frequency, it is felt that this assumption is adequate for predicting the operation of an actual system.

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Linear Equations

It is possible to write linear equations for a system restricted to the principal body axes and the controller used as a damper by making some simplifying assumptions. A general solution of the simplified equations gives the natural frequency and damping of the system. The necessary assumptions are:

$$p = \text{constant}$$

$$\tau_{XY} = 0$$

$$K_2 = 0$$

$$\cos\left(K_1 q\right) = 1$$

$$\sin\left(K_1 q\right) = K_1 q$$

$$\cos\left(K_1 r\right) = 1$$

$$\sin\left(K_1 r\right) = K_1 r$$

$$p \gg \left(K_1 \dot{q}\right)\left(K_1 r\right)$$

The equations (30) and (31) then become

$$I_Y\dot{q} + \left(I_X - I_Z\right)rp = -HK_1qp - H\left(r + K_1\dot{r}\right) \tag{32}$$

$$I_Z\dot{r} + \left(I_Y - I_X\right)pq = H\left(q + K_1\dot{q}\right) - HK_1rp \tag{33}$$

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The solutions of these equations for an uncontrolled body have been treated in other studies, for example, references 1 and 2, but will be repeated herein to facilitate comparison with solutions for a controlled body. The characteristic equation for the uncontrolled body is

$$s^2 + \frac{\left(I_Z - I_X\right)\left(I_Y - I_X\right)}{I_YI_Z}p^2 = 0 \tag{34}$$

which, for $I_X > I_Y,I_Z$ or $I_X < I_Y,I_Z$ defines an undamped oscillation with a period given by the expression

$$\Omega = \sqrt{\frac{\left(I_Z - I_X\right)\left(I_Y - I_X\right)p^2}{I_YI_Z}} \tag{35}$$

These solutions can be nondimensionalized by differentiating with respect to nondimensional time pt instead of dimensional time t. The results will then be expressed in terms of spin revolutions instead of seconds.

The characteristic equation for the controlled body is

$$s^2 + \left[\frac{2I_XHK_1p + 2H^2K_1}{I_YI_Z + H^2K_1^{\,2}}\right]s + \frac{\left(I_Z - I_X\right)\left(I_Y - I_X\right)p^2 + H^2K_1^{\,2}p^2 + \left(2I_X - I_Z - I_Y\right)Hp + H^2}{I_YI_Z + H^2K_1^{\,2}} = 0 \tag{36}$$

The time to damp to half amplitude is given by the expression

$$T_{1/2} = \frac{0.692\left(I_YI_Z + H^2K_1^{\,2}\right)}{2I_XHK_1p + 2H^2K_1} \tag{37}$$

The solutions for the controlled body can be nondimensionalized by making these additional substitutions:

$$\left.\begin{aligned} H &= pH' \\ K_1 &= \frac{K_1'}{p} \end{aligned}\right\} \tag{38}$$

The period and time to half amplitude would then be expressed as revolutions instead of seconds, and the values obtained for a given configuration would apply for any initial spin velocity.

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RESULTS

Linear Equations

The period and time to half amplitude obtained by using the linear equations are shown in figure 3. The conditions made in these calculations are as follows. The rolling velocity p is assumed to be constant at a value of 1 radian per second. The curves given in figure 3 represent the results for a variety of axially symmetric configurations. The variation in shape is defined in terms of the ratios I_Y/I_X or I_Z/I_X. The moment of inertia I_X is assumed to be constant and the moments of inertia I_Y and I_Z are assumed to be equal and to vary so that the ratios I_Z/I_X and I_Y/I_X vary from 0.5 (a flat disk) to 2 (a rod). The control wheel is assumed to have an angular momentum equal to 1/20 of the spin momentum of the satellite body $\left(\frac{1}{20}I_X p\right)$. This value represents a wheel size and wheel spin rate that could easily be carried in a satellite. The gain K_1 between the body-axis rates q and r and the control-wheel gimbal angles δ_Z and δ_Y is assumed to be 1 radian per radian per second.

For the uncontrolled body the time to half amplitude is always infinite. For the disk configuration the period of the wobble is equal to the spin period, 6.28 seconds. As the ratio I_Y/I_X is increased to 1 (this value represents a sphere) the period increases to infinity. Further increase in I_Y and I_Z causes a decrease in period, and this result represents the wobble of a rod spinning about its minimum axis of inertia.

The addition of the damper control brings about very short times to half amplitude, which vary from 3 seconds to 53 seconds. The longer time to half amplitude for the large values of I_Y and I_Z as compared with that for the small values of I_Y and I_Z is a reflection of the fact that under the assumptions made for these calculations the larger values of I_Y and I_Z represent larger bodies, but the momentum of the control wheel is held constant. Increasing either the angular momentum of the control wheel or the gain of the control will shift the $T_{1/2}$ curve to smaller values. The period of the characteristic motion with the damper is very nearly the same as for the body alone, with the exception that the peak asymptote is shifted to a higher value of I_Y/I_X. This shift reflects the fact that the addition of the control-wheel momentum to the spin momentum of the body effectively increases I_X a small amount.

Nonlinear Equations

Particular solutions for the nonlinear equations of motion were obtained by using an analog computer. Three different configurations were studied. The results for a nearly spherical satellite $\left(I_X = 9{,}500 \text{ slug-ft}^2;\ I_Y = I_Z = 9{,}000 \text{ slug-ft}^2\right)$, with an initial spin rate of 0.6 radian per second, are given in figures 4 to 9. The results obtained for a disk configuration $\left(I_X = 9{,}500 \text{ slug-ft}^2;\ I_Y = I_Z = 4{,}750 \text{ slug-ft}^2\right)$ spinning on the axis of greatest inertia are shown in figure 10. The results for a rod shape $\left(I_X = 9{,}500 \text{ slug-ft}^2;\ I_Y = I_Z = 14{,}250 \text{ slug-ft}^2\right)$ spinning on the axis of least inertia are shown in figure 11. These figures are tracings of analog records.

In all cases in which a control wheel was included, the angular momentum of the control wheel is assumed to be 200 ft-lb-sec. This angular momentum could be obtained by using a 32-pound flywheel with a 7-inch radius turning at 5,000 rpm. The control-system gains were varied from 5 to 20 radians per radian per second for the rate signals, and 0.33 to 3.33 for the attitude signal in various runs. It should also be noted that the ordinate scales used in the figures vary from figure to figure.

Nearly Spherical Configuration

The response of the uncontrolled nearly spherical body subjected to a 10-second rectangular pitching-moment pulse M_Y of 15 foot-pounds is shown in figure 4. In these tests in which step moments are used to disturb the system, the moments are probably too large to represent any particular event that might happen to a satellite, but the large moments are used as a severe test of the stability and performance of the system. With the use of the linear equations, a wobble period of 188 seconds is predicted for this example, and this prediction is in good agreement with the low frequency mode in the particular solution shown.

The displacement of the X-axis from the reference X_i-axis is equal to $\sin^{-1}\left(\sqrt{n^2 + m^2/\eta}\right)$. For convienence in using the analog computer, η was given a value of 100. The magnitude of the maximum values of m and n of ±2.6, therefore, indicates angular displacements of ±1.5°. The small oscillations superimposed on the 188-second wobble oscillation, which have a period equal to the spin period of 10.4 seconds, indicate that the resultant rotation (spin) vector is displaced from the X_i-axis by a small amount, approximately 0.15°. Such a result would be expected as a result of the pitching-moment disturbance. The displacement of the X-axis from the instantaneous spin axis is equal to $\tan^{-1}\left(\sqrt{q^2 + r^2/p}\right)$. The pitch and yaw rates are shown to have maximum values of ±0.014 radian per second, and therefore the X-axis is displaced from the spin vector by an angle of 1.35°. The phase relation of q and r indicates that the X-axis displacement from the spin vector alternates from the XY-plane to the XZ-plane in quarter periods. A sketch giving a pictorial representation of these results is shown in figure 5.

Next the control system was included (fig. 6(a)), and the system was subjected to a pitching-moment pulse of 15 foot-pounds, which this time was held on for 36 seconds. The gain on the rate signal in this case is 10 radians per radian per second, and the gain on the attitude signal is 0.33 radian per radian. The action of the damper control is to hold the X-axis close to the spin vector, thus the body rates r and q are kept low. When the disturbance is removed, the X-axis quickly alines with the spin vector, and q and r become exactly zero. At this time the spin vector is displaced from the X_i-axis by 0.23° as is indicated by the ±0.4 oscillations in n and m with the frequency of the spin frequency. It should also be pointed out that the variations of n and m appear in quadrature, so that $\sqrt{m^2 + n^2} = \pm 0.4$ also. In this case the attitude control is too weak to bring about any reduction in this amplitude in the short time of the test.

An example in which a pitching moment of 120 foot-pounds was applied for about 90 seconds is shown in figure 6(b). In this example, the gain on the rate signal is 5 radians per radian per second, and on the attitude signal is 3.33 radians per radian. When the disturbance is removed the body-axis rates and m and n go to zero, and this result indicates that both the spin vector and the X-axis are realined with the reference X_i-axis. It should be noted that in this example gimbal deflections of nearly 60° are encountered. Figure 6(c) illustrates more clearly the manner in which m and n are brought to zero. Close examination of the analog-computer record shows that the short period mode is being slowly damped out.

These examples demonstrate that the controller is capable of removing the wobble that results from a step disturbance and can realine a given body axis with the reference sunline after it has been displaced.

A disturbance which is likely to occur in a manned space station is a movement of the occupant, which will cause a change in product of inertia. Presumably, the station would be built so that either the maximum or minimum principal axis of inertia will coincide with the center line of the solar collector. The movement of an occupant would therefore cause the product of inertia with respect to the body axes to change from zero to some value. Therefore, tests were made in which a product of inertia I_{XY} of 50 slug-ft^2 was introduced. The results for the uncontrolled spherical body are shown in figure 7.

With the body-axis moments of inertia of the satellite as given $\left(I_X = 9{,}500 \text{ slug-ft}^2;\ I_Y = I_Z = 9{,}000 \text{ slug-ft}^2\right)$, the product of inertia of 50 slug-ft^2 indicates that the principal axis is displaced 5.73° from the X-axis in the XY-plane. The relationship defining this shift is

$$I_{X'Y'} = \frac{I_X - I_Y}{2} \sin 2c + I_{XY} \cos 2c$$

With X' and Y' taken as the principal axes of inertia, $I_{X'Y'}$ is zero. Setting $I_{X'Y'}$ equal to zero allows the determination of the angle c, which is then the angle between the body X- or Y-axis and the principal X'- or Y'-axis, respectively.

A spinning body has a preference for spinning on a principal axis of inertia. Thus, in the present example, with small factors neglected, the yawing torque $-I_{XY}p^2$ produced by the product of inertia will be balanced by the precession torque $\left(I_Y - I_X\right)pq$ when the body yaws 5.73° so that the principal axis is under the spin vector. However, when there is no damping in the system, an overshoot equal to the initial movement will occur. Thus variations in m, which indicate angular displacements from 0° to -11.5°, and variations in q from 0 to -0.12 radian per second appear in figure 7. The precession coupling terms bring about ±5.73° oscillations about the Y-axis, and cause corresponding variations in n and r.

The locus of points on the body which come under the spin vector trace a circle around the principal X'-axis. Therefore it can be seen that the solution presented herein is in agreement with the development known as Poinsot's Construction. (See, for example, ref. 1.)

The addition of a damping control tends to bring the principal axis into alignment with the spin vector, with the result that the X-axis has a steady displacement of 5.73° from the spin vector - that is to say, the X-axis describes a 5.73° cone about the spin vector. An additional effect takes place, however. By a process which will be described, the control-wheel angular-momentum vector reaches a steady-state deflection which causes a steady-state torque to be applied which opposes the torque produced by a product of inertia, and the angle of the generated cone is reduced in size.

A description of this process, which is defined mathematically by equations (29) to (31), follows. At this point it is convenient to write down the significant parts of equations (30) and (31):

$$I_Y\dot{q} + \left(I_X - I_Z\right)rp = -HK_1 qp$$

$$I_Z\dot{r} + \left(I_Y - I_X\right)pq - I_{XY}p^2 = -HK_1 rp$$

The main torque produced by the product of inertia is a torque on the Z-axis described by the term $I_{XY}p^2$. This torque can be opposed by the torque created when an angle δ_Z (that is, $K_1 r$) is commanded. The relation is $I_{XY}p^2 - H(K_1 r)p$ = the unbalanced torque about the Z-axis. In this example, with $H = 200$ ft-lb-sec and $p = 0.6$ radian per second, a value of δ_Z of 8° is required to completely balance the torque produced by the product of inertia.

When the rate gyroscopes alone are used to command the gimbal angles, the following process takes place. The product of inertia produces a torque about the Z-axis which, in the steady state, causes a pitching rate q to be measured. This pitching rate will call for an angle δ_Y which will produce a torque about the Y-axis. This torque will cause a yawing velocity to be measured in the steady state. This yawing velocity will call for an angle δ_Z which will produce a torque about the Z-axis which will oppose the product-of-inertia torque and thus reduce the cone angle. It is not possible, with the assumed control equations, to reduce the cone angle to zero, of course, because the situation is similar to that of a weak-position servomechanism acting against a considerable load. The possibility exists, however, of including the integral of the attitude error in the control equation and thereby reducing the cone angle to zero.

With a gain of 5 radians per radian per second for the rate signals, the steady-state deflection of the X-axis in the XY-plane is 1.6°, and in the XZ-plane is 1.3°, for a resultant cone angle of 2.04° (fig. 8(a)). With a gain of 20 radians per radian per second, the cone angle is 0.7° (fig. 8(b)).

The transient motion brought about by a step increase in I_{XY}, and the action of the controller, cause the spin vector to be displaced from the reference X_1-axis, and a small oscillation with a period equal to the spin period is superimposed on the steady-state values of n and m, as is shown in figures 8(a) and 8(b). The addition of the attitude signal causes this superimposed oscillation to be slowly eliminated, with this elimination indicating that the spin vector is being alined with the reference axis. The larger the rate gain, the longer the time required to attenuate this oscillation. The attitude signal also causes changes in the steady-state gimbal angles, which result in a slightly altered cone angle. These results are shown in figure 8(c).

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If the rate-gyroscope input axes were rotated 90° with respect to the body axes, so that

$$\delta_Z = -K_1 q$$

$$\delta_Y = K_1 r$$

then the measured effect of a product of inertia would call for gimbal deflections such that the control torque would directly oppose the disturbance torque. However, there would be no damping in the system. A case was tried in which it was assumed that the gyroscopes were advanced through a lead angle of 60°, with a gain of 20 radians per radian per second, so that

$$\delta_Z = 20(0.5r - 0.866q)$$

$$\delta_Y = 20(0.5q + 0.866r)$$

The results are shown in figure 9. A comparison with the case in which the gyroscopes were not rotated shows a reduction in cone angle from 0.7° to 0.515° with the gyroscope rotated. Further comparison shows a slight reduction in damping with the gyroscopes rotated. A linear analysis predicts such a reduction in damping. Again, the addition of the attitude signals reduces the superimposed oscillation to zero.

Disk and Rod Configurations

The motion of a disk, with a product of inertia I_{XY} of 50 slug-ft^2, is shown in figure 10. In this case the product of inertia of 50 slug-ft^2 indicates that the principal axes are shifted 0.5° from the body axes, and therefore the variation in m indicates an oscillation from 0° to -1°, and a pitching-rate oscillation from 0 to -0.012 radian per second occurs. The wobble frequency is equal to the spin frequency in this case, and the combination of wobble motion and spin motion results in a space-oriented trace of the X-axis which is a straight line rather than a circle as was obtained with the nearly spherical body. This result is indicated by the fact that n remains at zero.

The addition of the controller brings the principal axis into steady alinement with the spin vector, and the X-axis moves in a 0.5° cone. The controller is not as effective in reducing the cone angle in this case as it was in the case of the sphere. The small cone results in small body rates q and r, and a very high rate gain would be required to bring about the δ_Z needed to reduce the size of the cone. As can be seen in figure 10, the combination of rate signal and attitude signal results in a δ_Z of 0, and the cone angle is not reduced by any significant amount.

The results obtained with the rod configuration (fig. 11) are very similar to those obtained with the disk. In this case, again, the principal axis is shifted 0.5° from the body axis by a product of inertia of 50 slug-ft^2. With the rod, the wobble period is 32 seconds. The combination of the wobble period and the spin period of 10.4 seconds results in the peculiar wave shape for m and n. For a given rate gain, the time to half amplitude is larger for the rod than for the disk for the reasons given in the discussion of the linear equations.

CONCLUSIONS

An analytical study of a wide variety of cases of the use of a controller, which utilizes precession torques produced by a spinning wheel and which is commanded by rate-gyroscope and sun-seeker signals, to control a spinning body shows that the controller will perform the following functions:

1. The controller reduces the motion of the body resulting from a rectangular pulse moment disturbance by causing the reference body axes to be steadily alined with the spin vector, and by realining the reference body axes with the space reference line.

2. The controller reduces the motion resulting from a step increase in product of inertia by bringing the new principal axes into steady alinement with the spin vector, and by reducing the cone angle generated by the reference body axis.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., April 11, 1961.

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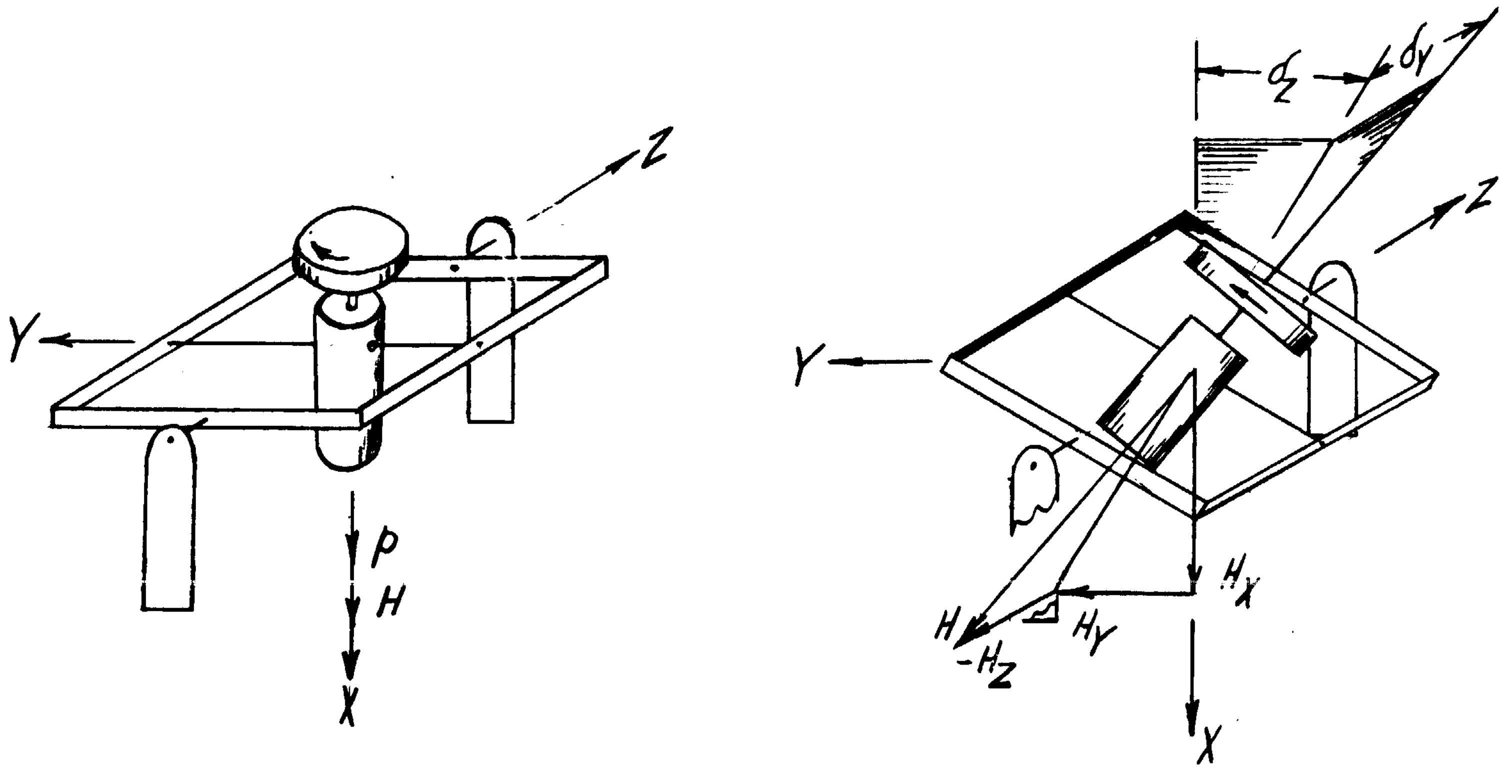

(a) Zero torque position.

(b) Deflected position.

Figure 1.- Sketch of controller for a spinning body.

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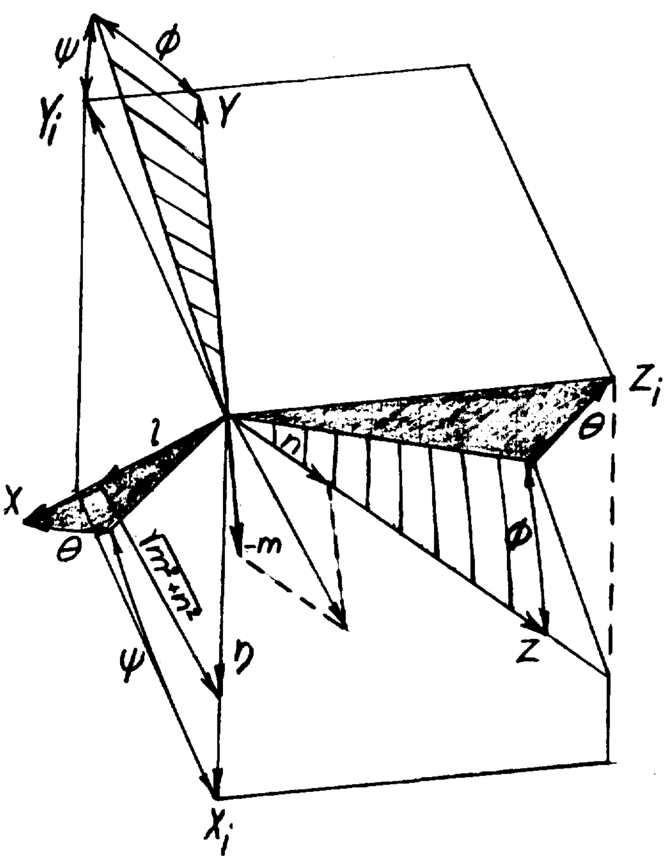


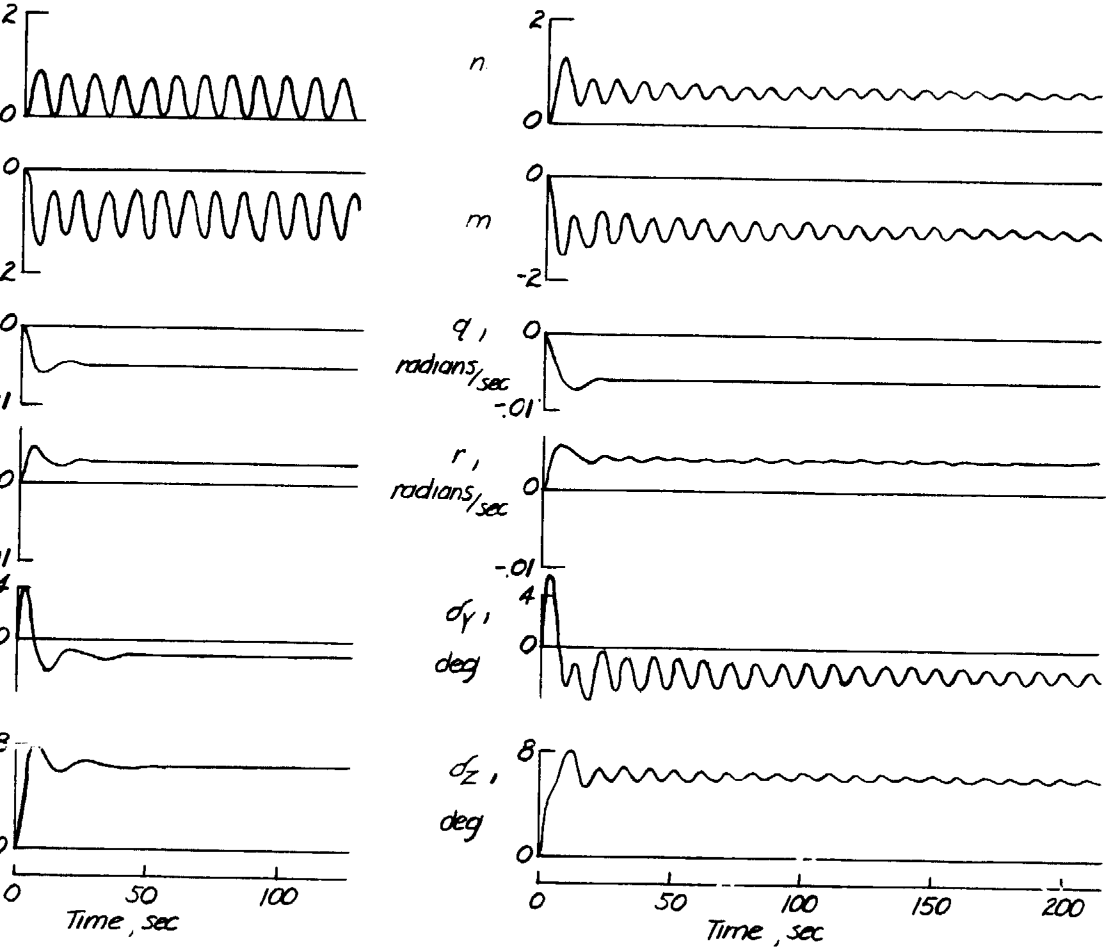
Figure 2.- Sketch showing relation of inertial axes and body axes.

These points come under the spin vector in sucession, 188 seconds required for complete traversal.

's trace, second required each circle.

sturbance.

ectangular M_Y pulse.



= 20(0.5q + 0.866r);
20(0.5r - 0.866q).

(b) $\delta_Y = 20(0.5q - 0.866r) - 3.33n$;
$\delta_Z = 20(0.5r - 0.866q) + 3.33m$.

tion of nearly spherical body with $I_{XY} = 50$ slug-ft^2; with control system; H = 200 ft-lb-sec.

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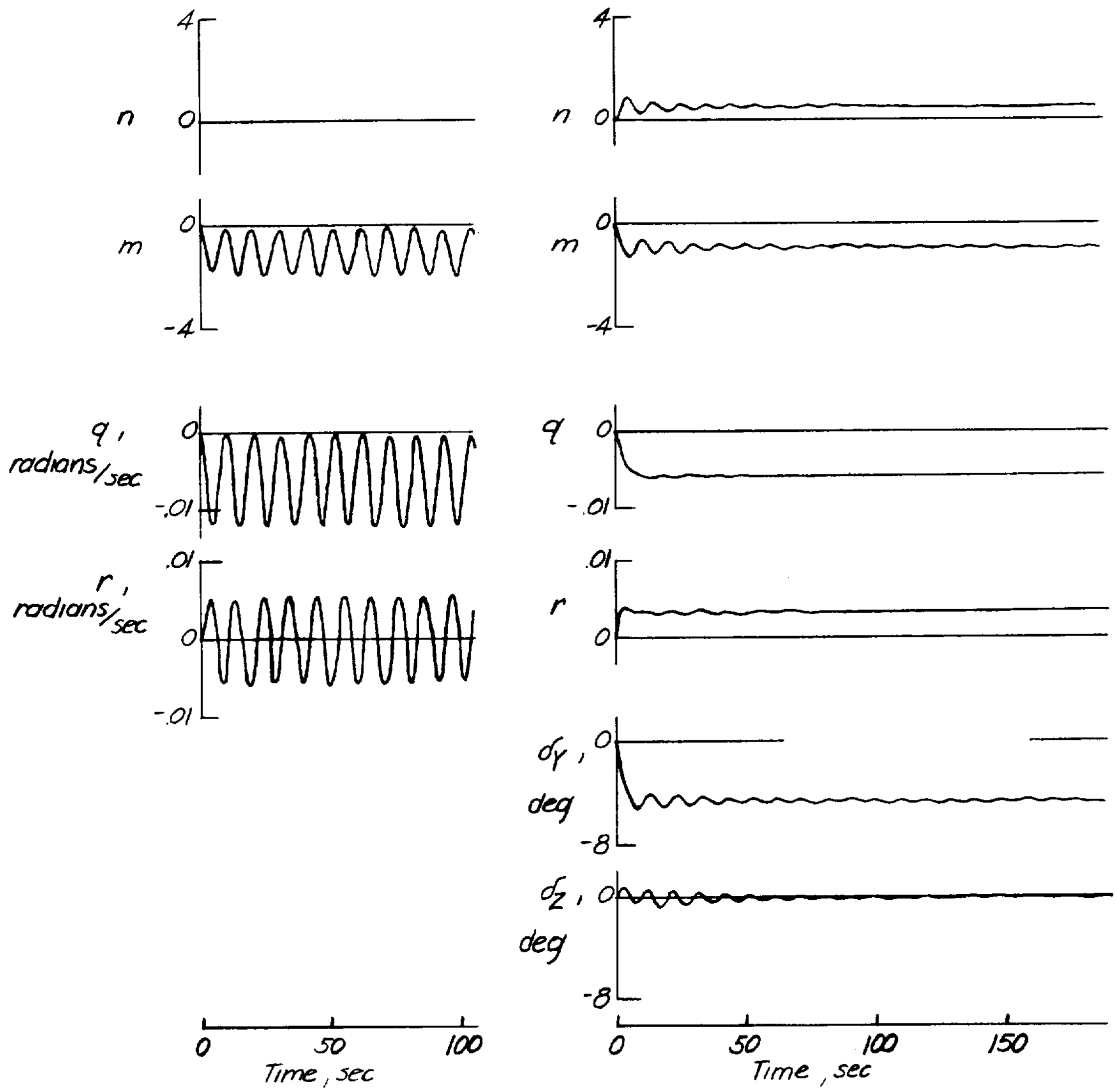
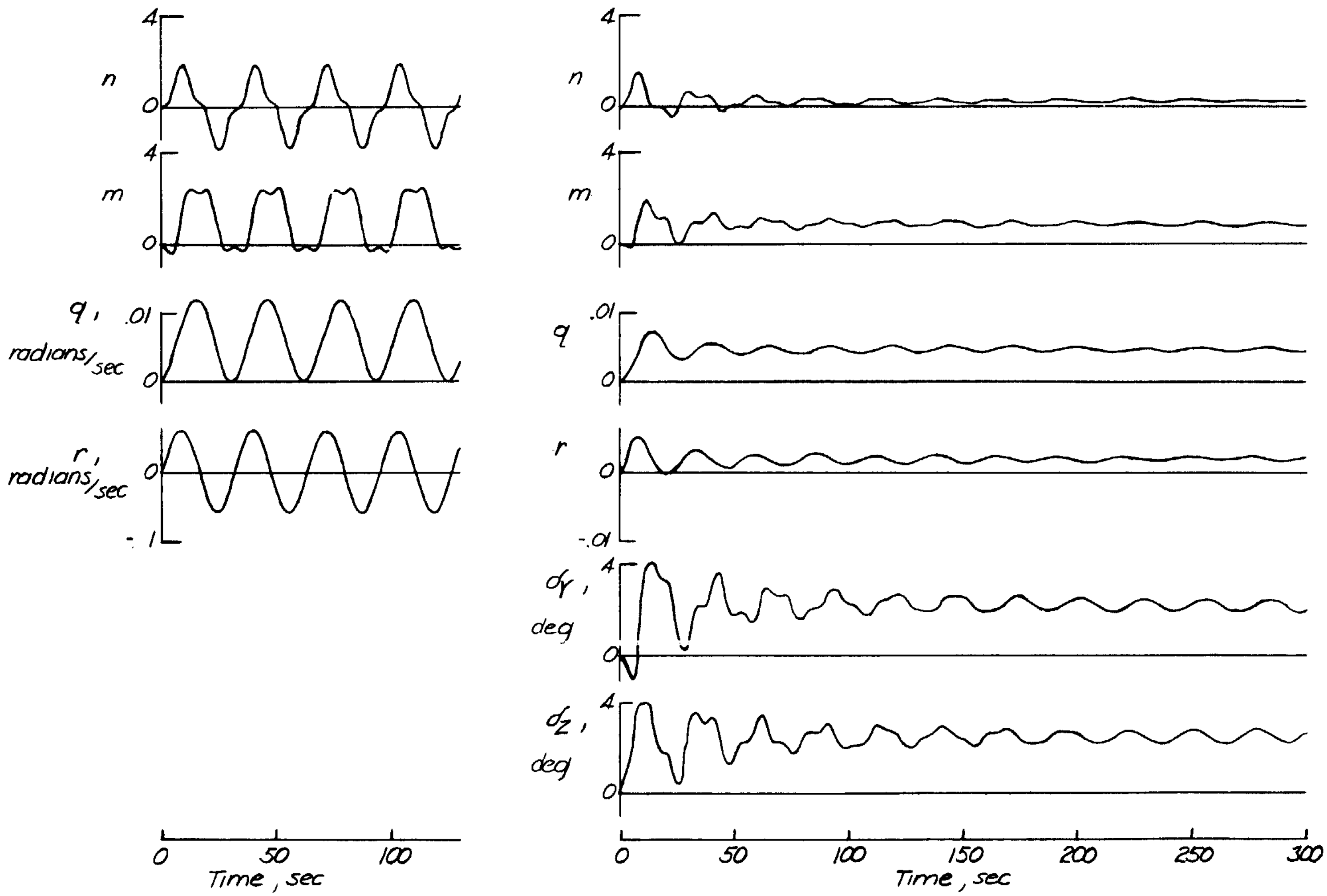


(a) $\delta_Y = 0$; $\delta_Z = 0$. (b) $\delta_Y = 10q - 3.33n$; $\delta_Z = 10r + 3.33m$.

Figure 10.- Motion of a disk with $I_{XY} = 50$ slug-ft^2.



(a) $\delta_Y = 0$; $\delta_Z = 0$.

(b) $\delta_Y = 10q - 3.33n$; $\delta_Z = 10r + 3.33m$.

Figure 11.- Motion of a rod with $I_{XY} = 50$ slug-ft^2.

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