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TECHNICAL NOTE

D-886

THEORY OF AN ELECTROMAGNETIC MASS ACCELERATOR FOR ACHIEVING HYPERVELOCITIES

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON June 1961

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SUMMARY

It is shown that for any electromagnetic accelerator which employs an electromagnetic force for driving the projectile and uses the projectile as the heat sink for the energy dissipated in it by ohmic heating, the maximum velocity attainable without melting is a function of the mass of the projectile. Therefore, for hypervelocities a large projectile mass is required and thus a power supply of very large capacity is necessary. It is shown that the only means for reducing the power requirement is maximizing the gradient of the mutual inductance. In the scheme of the sliding-coil accelerator investigated herein, the gradient of the mutual inductance is continuously maintained at a high value. It is also shown that for minimum length of the accelerator, the current must be kept constant despite the rise in induced voltage during acceleration. The use of a capacitor bank as an energy source with the condition that the current be kept constant is investigated.

Experiments at low velocities are described.

INTRODUCTION

The experimental investigation of impacts at hypervelocities (from about 15 km/sec to 72 km/sec) is currently of much interest. Various schemes for achieving such velocities in the laboratory have been investigated. Systems where the electromagnetic force is employed for an acceleration are reviewed, for instance, in reference 1. In such electromagnetic systems the projectile is basically a conductor in a magnetic field, and it is propelled by the interaction of the current which flows through it and a magnetic field which is generated by another part of the electrical circuit. The ohmic heating of the projectile thus appears as the main problem of electromagnetic acceleration, and it is because of this heating effect that hypervelocities have not yet been achieved through electromagnetic acceleration. The theoretical treatments of the previous electromagnetic accelerators, however, did not proceed to a general relation between the effect of ohmic heating and the maximum velocity which can be achieved. Deriving such a general relation is the main purpose of this paper. From this relation the limitations of electromagnetic acceleration should be derivable, and the main features of a feasible scheme should be deducible.

SYMBOLS

a	coil diameter, m
A	area, m^2
b	distance of separation of coil ends, m
B	magnetic flux density, webers/m^2
c	distance of separation of coil centers, m
C	capacitance, farads
d	diameter of wire, m
E	electric field strength, volts/m
F	force, newtons
H	heat, joules
I	current, amp
l	length, m
L	self-inductance, henries
m	mass, kg
M	mutual inductance, henries
N	number of turns
n	number of turns per unit length, 1/m
P	power, watts
R	ohmic resistance, ohms
t	time, sec

T temperature, $^{\circ}K$

v velocity (also $\dot{x}$), m/sec

V voltage, volts

W energy, joules

x,y,z coordinate of system, with x as distance in direction of acceleration, m

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α,β constants (defined in eqs. (10))

γ specific heat, joules/kg $^{\circ}K$

Δ difference

μ permeability, henries/m

σ electrical conductivity, mhos/m

σ_t tensile strength, newtons/m^2

τ integral multiple of separation time of coils, sec

ϕ magnetic flux, webers

Subscripts:

a accelerator

e end

i induced

k mechanical

m magnetic

0 applied, initial

max maximum

1 refers to coil 1, the driving coil

2 refers to coil 2, the driven coil

1,2,3 . . . number of steps in divided capacitor bank

A bar over a symbol indicates mean or average value. A dot over a symbol indicates differentiation with respect to time t. A prime on a symbol indicates differentiation with respect to distance x.

GENERAL RELATIONS

The general relation between the ohmic heating of the projectile and the maximum velocity which can be achieved by electromagnetic acceleration is derived in the present section. Basically, in all schemes of electromagnetic acceleration the projectile is a conductor in a magnetic field, and it is propelled by the electromagnetic force. The current flowing through the projectile heats it. Thus, acceleration is possible only as long as the projectile does not melt.

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Assume that the process of acceleration is short in time, so that no cooling mechanism can become effective. The projectile then is the heat sink. The case where the projectile is extremely small is excluded. The rate of ohmic heating of the projectile is then given by the equation

$$RI^2 dt = \gamma m \, dT \tag{1}$$

where R stands for the ohmic resistance, dt is the increment of time, γ is the specific heat, m is the mass of the projectile, and dT is the rise in temperature.

The driving force derives from the gradient of the magnetic field energy. If the driving circuit and the circuit represented by the projectile are connected in series and if the force acts in the x-direction, the force is given by the equation

$$F = \frac{1}{2} I^2 \frac{dL}{dx} \tag{2}$$

where the total inductance of the system is $L = L_1 + L_2 \pm 2M$ and consists of the self-inductance L_1 of the driving circuit, the self-inductance L_2 of the projectile, and twice their mutual inductance M.

The velocity of the projectile is obtained by integrating the force equation (eq. (2)):

$$\begin{aligned} \dot{x} &= \int_0^t \frac{F}{m} \, dt \\ &= \frac{1}{2} \int_0^t I^2 \frac{dL}{dx} \frac{dt}{m} \end{aligned} \tag{3}$$

From equation (1) the time variable can be replaced by the change of temperature. Inserting this parameter into equation (3) yields

$$\dot{x} = \frac{1}{2} \int_{T_0}^{T} \frac{dL}{dx} \frac{\gamma}{R} \, dT \tag{4}$$

The velocity now appears as a function of the temperature, and the maximum velocity attainable is readily obtained by extending the integral to the melting temperature of the projectile.

With $R = \frac{l}{\sigma A}$, where l is the length of the conductor, A is the cross-section area, and σ is the conductivity, the velocity is

$$\dot{x} = \frac{1}{2} \int_{T_0}^{T_{melt}} \frac{dL}{dx} \frac{A}{l} \sigma\gamma \, dT$$

The greatest velocity obviously is obtained, when the expression $\frac{dL}{dx} \frac{A}{l}$ continuously remains at its highest possible value. Then, for maximum velocity

$$\dot{x}_{max} = \frac{1}{2} \left(\frac{dL}{dx} \frac{A}{l}\right)_{max} \int_{T_0}^{T_{melt}} \sigma\gamma \, dT \tag{5}$$

The integral on the right-hand side depends only on the properties of the material of which the projectile consists. The integral, therefore, has to be considered as a material constant. Since the expression in front of the integral determines the geometrical configuration of the accelerator, including its size, it is seen from equation (5) that there is no similarity in electromagnetic acceleration.

If the magnitude of the parenthesized expression is to be found as a function of the velocity desired, equation (5) can be divided by the material constant $\Delta(\sigma\gamma T)$ to obtain

$$\frac{2\dot{x}}{\Delta(\sigma\gamma T)} = \left(\frac{dL}{dx} \frac{A}{l}\right) \tag{6}$$

In the electromagnetic cgs system of units, inductance has the dimension of length, expressed in centimeters. The right-hand side of equation (6) thus represents a characteristic length, and this length appears as a linear function of the velocity desired.

For a certain velocity the expression $\left(\frac{dL}{dx}\frac{A}{l}\right)$ must be constant. It may be concluded, therefore, that the characteristic length expressed by equation (6), can be considered as the cube root of a volume that the projectile must have in order to achieve a certain desired velocity. For copper and for $\dot{x}_{max} = 2 \times 10^{6}$ cm/sec, $\left(\frac{dL}{dx}\frac{A}{l}\right) \approx 0.2$ cm. (For this evaluation, the dependency of σ and γ on the temperature has not been considered. A rigorous computation will result in a somewhat larger value for the characteristic length.) The minimum weight of the projectile is then of the order of grams. In the hypervelocity range the kinetic energy that must be delivered to the projectile is in the megajoule range. Such a large quantity of energy should be transferred in a very short time, if the accelerator is to have a tolerable length. An electromagnetic hypervelocity mass accelerator, therefore, fundamentally requires an enormous power input.

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The derivation of a characteristic length for the electromagnetic acceleration (eq. (6)) represents, of course, an order-of-magnitude evaluation. A closer study of the expression $\left(\frac{dL}{dx}\frac{A}{l}\right)$ reveals that a fundamental improvement for electromagnetic acceleration can be achieved only through maximizing the $\frac{dL}{dx}$ factor with regard to its dependency on x. Increases in the inductance gradient by increasing the length, for instance, would basically be cancelled by the increase of the length l in the denominator of the right-hand side in equation (6). An efficient electromagnetic accelerator, therefore, requires a scheme which provides a continuous high value of the $\frac{dL}{dx}$ parameter; that is, one in which the driving and the driven conductors continuously stay close together.

From integration of the force equation (2), it is seen immediately that the shortest distance over which the coil is accelerated can be obtained when the current continuously remains at its highest possible value. There is an upper limit for the current, because the magnetic stresses have to be balanced by the strength of the material.

From the requirements for maximum velocity for the smallest projectile possible and for the shortest length of the accelerator, it may be concluded that an electromagnetic accelerator should keep the conductors of the driving and driven circuits continuously close together and furnish a continuous maximum current. Or, from a more practical point of view, a good electromagnetic accelerator should feature a constant maximum value of the gradient of the mutual inductance, and a voltage matching should be provided in order to maintain the current at its highest permissible magnitude.

THEORY OF THE SLIDING COIL ACCELERATOR

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In the attempt to design an apparatus which comes close to these requirements, a scheme is suggested which can be called a "sliding-coil accelerator." (See fig. 1.) Basically, the sliding-coil accelerator employs two coils, which attract or repel each other. One coil, coil 1, is the stationary one, the long driving coil. The driven coil is identified as coil 2; it is short and slightly larger in cross section than the driving coil so that it is free to move across the stationary coil.

The driven coil 2 picks up the current from a parallel rail by means of a sliding contact. Via another sliding contact, coil 2 feeds the current at its end into the driving coil. The driving coil then completes the circuit. By this arrangement, the driving coil is energized continuously up to the position of the driven coil. Both coils magnetically stay close together; thus a high and nearly constant gradient of the inductance is provided.

If the two coils are carefully examined to see how the sliding contact moves from one turn of the driving coil to the next one, it is noted that the gradient of the inductance is not constant. It varies, instead, with the small distance by which the two coils are separated, until a new turn of the driving coil is energized and restores the original condition.

From computations of the inductance of the system it is found that, under the condition of small variations of the distance, the average value of the gradient of the inductance can be defined with sufficient accuracy by the arithmetic mean:

$$\left(\overline{\frac{dL}{dx}}\right) = \frac{1}{2}\left[\left(\frac{dL}{dx}\right)_{b=b_{max}} + \left(\frac{dL}{dx}\right)_{b=0}\right]$$

where b is the distance separating the ends of the two coils.

The maximum distance of separation is here assumed to be the thickness of the wire of which the driving coil is wound. It should be realized, however, that the arcing at the sliding contact may create an effective distance greater than that. The average gradient of the inductance then decreases, and the performance of the system, consequently, will decrease also. The problem of arcing may not be too serious, however, because the interaction of the longitudinal current in the arc with the radial component of the magnetic field will produce a magnetic blowout effect.

The maximum current for the sliding-coil accelerator has to be derived from the requirement that the magnetic forces will not rupture the coils. A formula, which relates the tensile strength of the wire to the magnetic stresses has been developed by Miller, Dow, and Haddad (ref. 2) and is shown in a subsequent summary of the equations for the sliding-coil accelerator.

The current can be kept constant by controlling the applied voltage. As the projectile becomes accelerated, it generates an electromotive force. The induced voltage and the condition for constant current can be derived from the power equation

$$V_0 I = I^2 R + \frac{\partial W_m}{\partial t} + F\dot{x} \tag{7}$$

where V_0 is the applied voltage, W_m is the magnetic-energy storage, and $\dot{x}$ is the velocity. On the average (see appendix A) the magnetic energy storage of the system is assumed to be constant. Then $\partial W_m/\partial t$ is zero. With the force equation (2), the power equation (7) yields

$$V_0 = I\left(R + \frac{1}{2}\frac{dL}{dx}\dot{x}\right) \tag{8}$$

which is the condition to be used for keeping the current constant. Multiplying the parenthetical expression by the current I gives as the second term the induced voltage, averaged over several cycles of coil separation and subsequent switching.

POWER SOURCES

If equation (6) is used for achieving a velocity of some 20 km/sec, a minimum mass of the projectile of the order of several grams is seen to be required. To accelerate this mass up to meteorite velocities over a length suitable for laboratory work requires a power input of the order of millions of kilowatts. Obviously, an energy storage device is more economical than a generator for such high power. A capacitor bank for energy storage seems to be advantageous and its use as power source is investigated.

For keeping the current constant, as required by equation (8), the voltage must increase along the path of acceleration. On the other hand, a capacitor bank will decrease its voltage while expending energy.

In order to match the requirements of equation (8) with the characteristic of a capacitor bank, two schemes of arrangement are suggested. The first scheme may be called the "divided capacitor bank." It consists of a number of capacitors hooked up in series. At each subsequent junction of two capacitors, an increasing potential against ground is established. The second scheme makes use of one big capacitor. The increasing voltage according to equation (8) is matched by an arrangement of resistors.

The Divided Capacitor Bank

The scheme of the divided capacitor bank is represented in figure 2. The path of acceleration is divided into a large number of small steps of length dx. The length dx shall be larger than the distance between adjacent turns on the driving coil; it is small as compared with the length of the accelerator. The amount of energy transferred to the system while the projectile travels the distance dx will correspondingly be a small fraction of the total energy expenditure. The treatment of the energy relations by using calculus, therefore, can be expected to be a sufficiently good approximation.

The increment of energy added to the system while the projectile travels the distance dx is obtained from the power equation as

$$dW = I^2\left(\frac{R}{\sqrt{\alpha x}} + \beta\right)dx \tag{9}$$

where

$$\left.\begin{aligned} \alpha &= \frac{2F}{m} \\ \beta &= \frac{1}{2}\,\frac{dL}{dx} \end{aligned}\right\} \tag{10}$$

This energy must be furnished by discharging the capacitors; thus,

$$dW = CV\,dV \tag{11}$$

The combination of equations (9) and (11) yields the voltage drop across the capacitor bank due to the energy expenditure for moving the propelled coil the distance dx at a total distance x from the origin:

$$dV = \frac{I^2}{CV}\left(\frac{R}{\sqrt{\alpha x}} + \beta\right)dx \tag{12}$$

The voltage which the capacitor bank furnishes at a distance x from the origin then is the difference between the initial voltage to which it was charged at the position x and the voltage drop due to the acceleration of the projectile up to the position x,

$$V(x) = V_0(x) - \int_0^x dV \tag{13}$$

On the other hand, equation (8) must be fulfilled

$$V(x) = I(R + \beta\sqrt{\alpha x})$$

Since equation (8) requires an increase of the voltage with increasing x, $V_0(x)$ must be a function of x such that the difference $V_0(x) - \int_0^x dV$ results in the required $V(x)$ according to equation (8).

Differentiating equation (13) and substituting from equation (8) gives

$$V' = V_0' - \frac{1}{C\sqrt{\alpha x}} \tag{14}$$

Differentiating equation (8) and substituting in the left side of equation (14) yields the capacitance as a function of the length:

$$C(x) = \frac{1}{\frac{\alpha\beta}{2} + V_0' \frac{\sqrt{\alpha x}}{I}} \tag{15}$$

The function V_0' in the denominator of equation (15) is still arbitrary. Evidently the charging voltage V_0 depends on the size of the capacitors. The smaller the capacitance, the higher is V_0 in order to satisfy equation (14). A steep function V_0' consequently, according to equation (15), results in a steeper decrease of the added capacitance with the length. From consideration of the increase of the induced voltage with increasing velocity, the limitation of the increase of V_0 becomes apparent.

If V_0' is assumed to increase with $\sqrt{x}$, the capacitance C decreases approximately with $1/x$. For larger values of x the curve C has a very small slope. In this case, the decrease of the total capacitance will be very slight as the distance increases:

$$\frac{1}{C_{l+1}} = \frac{1}{C_l} + \frac{1}{C} \tag{16}$$

and because of the small slope

$$C_{l+1} \approx C_l \tag{16a}$$

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It follows, then, that $C \to \infty$.

When C_{l+1} is the total capacitance at step $l+1$, then C_l is the total capacitance at step l and C is the individual capacitance which is added when the propelled coil passes the $(l+1)$th step.

From equations (16) and (16a), it can be seen that the capacitances C_l have to become increasingly large. The scheme of the divided capacitor bank appears feasible. Whether it is economical will depend on particular requirements.

The Capacitor Bank With a Resistor in Line

The capacitor bank with a resistor in line (fig. 3) avoids the disadvantage of the divided capacitor bank, the requirement of greatly increasing capacitances.

In this arrangement, the voltage V_0 is the voltage across the single unit capacitor bank and is also a function of the position of coil 2. In contrast to the scheme described for the divided capacitor bank, the capacitance is constant. Instead, the resistance is a function of the length. If the employed resistance is considered much larger than the resistance of the coils, apart from the conditions at the very end of the accelerator, the resistance of the coils may be neglected. If the condition for constant current (eq. (8)) is utilized, it is seen that the function V_0 coincides with V as used before:

$$V = I(R + \beta\sqrt{\alpha x}) \tag{17}$$

The power input into the system by discharging the capacitor bank, by accelerating the projectile, and by generating heat in the resistor R is

$$-CV \frac{dV}{dt} = RI + F\dot{x}$$

$$-CV \frac{dV}{dx} = I^2\left(\frac{R}{\sqrt{\alpha x}} + \beta\right) \tag{18}$$

Differentiating equation (17) gives

$$\frac{dV}{dx} = I\left(\frac{\alpha\beta}{2\sqrt{\alpha x}} + R'\right) \tag{19}$$

The combination of equations (18) and (19) delivers a differential equation for the resistance R as a function of the length

$$R' = -\frac{1}{\sqrt{\alpha}}\left(\frac{\alpha\beta}{2} + \frac{1}{C}\right)\frac{1}{\sqrt{x}} \tag{20}$$

and by integration

$$R = R_0 - \frac{2}{\sqrt{\alpha}}\left(\frac{\alpha\beta}{2} + \frac{1}{C}\right)\sqrt{x} \tag{21}$$

The integration constant R_0 is the resistance of R at $x = 0$

$$R_0 = \frac{V_0}{I}$$

where V_0 is the initial charging voltage. At the end of the accelerator R is zero. Then, when l denotes the length of the accelerator

$$0 = \frac{V_0}{I} - \frac{2}{\sqrt{\alpha}}\left(\frac{\alpha\beta}{2} + \frac{1}{C}\right)\sqrt{l} \tag{22}$$

or

$$V_0 = \frac{2I}{\sqrt{\alpha}}\sqrt{l}\left(\frac{\alpha\beta}{2} + \frac{1}{C}\right) \tag{23}$$

Equation (23) delivers the charging voltage of the capacitor bank. Since the magnitudes α, β, l, and I are fixed by the demand for obtaining a desired velocity, the charging voltage V_0 depends on the capacitance. At the limit as $C \to \infty$, equation (23) yields

$$V_0 = \frac{2I}{\sqrt{\alpha}}\sqrt{l}\,\frac{\alpha\beta}{2} \tag{24}$$

and the initial resistance R_0 for the case of a very large capacitor bank is

$$R_0 = \beta\sqrt{\alpha l} \tag{25}$$

Apparently the initial resistance of the system must equal the final impedance. Thus, 50 percent or more of the energy stored in the capacitor bank is lost when the scheme described here is used.

According to equation (25), there is a freedom of choice between the size of the capacitor bank and the amount of the initial voltage to which the bank must be charged. The choice will depend on the requirements of practical cases and will basically represent a compromise between high voltage problems and problems of efficiency.

Equation (8) requires that the voltage at the end of the acceleration be

$$V_e = I\beta\sqrt{\alpha l} \tag{26}$$

Dividing equation (23) by equation (26) and solving for C yields an equation for the capacitance depending on the ratio of the initial voltage V_0 and the voltage after the acceleration is completed:

$$C = \frac{2}{\alpha\beta} \frac{1}{\left(\frac{V_0}{V_e} - 1\right)} \tag{27}$$

If, for instance, V_0 is chosen to be twice V_e, the capacitance has to be $2/\alpha\beta$. A greater ratio V_0/V_e apparently does not improve the system; with $\frac{V_0}{V_e} = 2$, the bank will be discharged by 75 percent.

SUMMARY OF FORMULAS AND SAMPLE CALCULATION

Provided friction forces can be neglected, and provided the arcing at the sliding contacts has no serious effects, the sliding-coil accelerator now can be designed.

For the sliding-coil accelerator, the previous relation of maximum velocity and permissible heating (eq. (5)) becomes:

$$\dot{x}_{max} = \left(\frac{dL}{dx}\,\frac{1}{8aN_2}\right)(\sigma\gamma\ \Delta T)\,i^2 \tag{28}$$

The term within the first parentheses on the right-hand side contains the gradient of the mutual inductance, the diameter of the coils, and the number of turns on the projected coil, N_2. This expression must be maximized simultaneously. The term within the second parentheses is maximized by selecting the best material. Equation (28) thus requires a certain wire diameter d for achieving a particular velocity. (Note that this procedure requires the assumption that $a \gg d$, because then the dependency of L on d is essentially eliminated. In the case where the diameter of the coils a and the wire diameter d are comparable, $\frac{dL}{dx}\frac{1}{8aN_2}d^2$ has to be maximized. This case is, however, not considered favorable for the sliding-coil accelerator because the dL/dx parameter would decrease appreciably.)

The maximum current under the condition that the magnetic stresses will not rupture the coils may be obtained from a formula derived by Dow, Miller, and Haddad in reference 2:

$$\sigma_t \geqq \frac{\mu_0 a I^2}{\pi d^2}\left[\frac{1}{\pi a}\left(\log_e \frac{8a}{d} + 1\right) + n\right] \tag{29}$$

where n is the number of turns per unit length.

With the magnitude of the current known, the length l of the accelerator can be determined from the following equation:

$$l = \frac{m}{I^2\,\frac{dL}{dx}}\,\dot{x}_{max}^2 \tag{30}$$

Using a capacitor bank as power supply and a variable resistor for the voltage matching, the capacitance must be determined from equation (27)

$$C = \frac{2}{\alpha\beta}\left(\frac{1}{\frac{V_0}{V_e} - 1}\right)$$

where the final voltage V_e is determined by equation (26). The resistance as a function of the length is given by equation (21). The charging voltage V_0 can be determined from compromising between high-voltage problems and the desire to keep the capacitor bank small.

With copper selected as the wire material, a sample calculation for achieving a velocity of 20 km/sec, has been carried out. The characteristic values are

Current, amp	1.76×10^4
Diameter of coils, m	2.5×10^{-2}
Thickness of wire, m	2×10^{-3}
Weight of projectile, kg	3×10^{-3}
Length of accelerator, m	40
Capacitor bank, f	4×10^{-3}
Charging voltage, v	3.5×10^4

From these values it can be seen that the construction of a full-size accelerator would require considerable technical effort.

EXPERIMENTAL WORK

As the energy relations indicate, a full-size accelerator would require an enormous energy storage device. In addition, the accelerator should consist of a large track built in a highly precise manner. For extremely high velocities the track must be constructed inside a vacuum tank. Because of the technical problems involved, construction of a full-size accelerator was not undertaken. Instead a small-scale model of the sliding-coil accelerator has been built for verifying the principles derived.

The Apparatus

For energy storage, a capacitor bank was used. Its capacitance was 2,500 microfarads and it was capable of storing 5,000 joules at a voltage of 2,000 volts.

The driving coil of the accelerator was built by winding an AWG 23 enameled copper wire on a bakelite rod of 3/4-inch (1.9 cm) diameter. The length of this coil was 10 inches (25 cm).

The driven coils were wound of the same wire. Samples incorporating 5, 10, and 15 turns have been fired. The driven coils had a diameter slightly greater than that of the driving coil, so as to allow them to slide freely along the driving coil.

From the equations of reference 3, it was found that the mutual inductance of the two coils is not greatly sensitive to a slight difference of the mutual diameters. Thus, the driven coil could be made with generous tolerances in regard to the diameter.

Three types of circuits have been tried. In the first type, the current was fed through one rail and a sliding contact into the driven coil. From the driven coil, the current was led into the driving coil. A pickup pulled by the driven coil carried the current from the driving coil into a second rail. (See fig. 4.) In this case the projectile consisted of the driven coil and a body carrying four sliding contacts.

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In a second arrangement only one rail was used (fig. 5). Here the circuit was closed by the driving coil. There were two possibilities of feeding the current from the rear end of the driving coil up to the driven coil and from the driven coil through a rail back to the capacitor bank. The driven coil was pushed along the driving coil so that the number of turns to be energized and also the ohmic resistance were increased. However, when the current was fed into the front end of the driving coil, the driven coil was pulled and a decreasing number of turns of the driving coil were energized; thus, the ohmic resistance was decreased. The best results were obtained when 5- and 10-turn coils were used as projectiles with only a 10 centimeter length of the driving coil.

Results and Discussion

Figures 6 to 12, which were obtained from the equations of reference 3, show the inductance gradient as $\partial M/\partial c$. As regards the acceleration, attention is given to the distance between the ends of the coils and no telescoping is allowed to occur. When $x = 0$, the ends of the coils coincide. The variable c, however, represents the distance between coil centers. In figure 8 where $\partial M/\partial c$ is plotted against c, a maximum is shown to occur when the ends of the coils coincide. The accelerator design is concerned only with that portion of the curve to the right of the maximum. The relation between x and c is simply taken into account and $\partial L/\partial x$ is deduced from figures 6 to 12.

The gradient of the inductance for the arrangement with a 5-turn coil was computed to be

$$\frac{dL}{dx} = 5 \times 10^{-4} \frac{h}{m}$$

The initial resistance was 1 ohm. Thus, the circuit was an overdamped system giving a current of 2,000 amperes shortly after switching.

The linear dependency of the resistance in the experimental setup does not deviate too much from the second-order dependency as required by the theory outlined before, so long as the distances involved are not too large. The actual variation of the current is estimated in the order of not greater than 1:2.

The approximation of a constant force in the experimental setup may furthermore be justified by the fact that the gradient of the mutual inductance eventually decreases as the coil approaches the end of the path of acceleration. This effect will, to a certain degree, cancel the effect of the increase of current. With a constant current of 2,000 amperes and a weight of the projectile of 1 gram

$$F = \frac{1}{2}\frac{dL}{dx}I^2 = \frac{5}{2} \times 10^{-4} \times 4 \times 10^6 = 10^3 \text{ newtons}$$

$$\frac{2F}{m}x = 2 \times 10^6 \times 10^{-1} = 20 \times 10^4$$

and

$$\dot{x} = \sqrt{\frac{2F}{m}x} = 450 \frac{m}{sec}$$

Thus, velocities up to 450 meters per second were to be expected.

For a large number of shots made with the test accelerator, the velocities have been measured simultaneously by a photocell arrangement and by a ballistic pendulum. Both measurements have been in good coincidence, the pendulum always indicating velocities about 10 percent smaller than those measured by the photocells. There was, however, a great spread in the values obtained. Most of the shots gave velocities in the range of 200 to 300 meters per second. For a number of shots, velocities were between 300 and 400 meters per second. The velocity for one shot was measured at 420 meters per second.

This variation in velocity must be attributed to the rather provisional experimental setup. Experience has shown that careful consideration must be given design and construction details, particularly with regard to the sliding contacts. As a matter of fact, it was observed that in the cases of failures the projectile was hindered somewhere along its path of acceleration. As a result, it came out "off-schedule," and in consequence, both the driven coil and the driving coil were overloaded and were destroyed.

Another irregularity results from the arcing at the sliding contacts. It is believed that occasionally so much of the wires vaporized that short circuits occurred as a result of a breakdown through the arc. In such cases almost no energy was transferred into mechanical work of acceleration. It should be mentioned that the provisional setup did not afford any features to prevent the vaporization of the copper wire at the sliding contacts.

According to the theory, an induced voltage must develop across the accelerated coil as

$$V_i = \frac{1}{2}\frac{dL}{dx}\dot{x} = 10^3 \times 5 \times 10^{-4} \times \dot{x} = \frac{\dot{x}}{2} \text{ volts}$$

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This voltage had to be expected as a residual voltage across the capacitor bank after the shot.

After most of the shots no charge was left in the capacitor bank, but after a number of firings a voltage was left. The voltage left in these cases was found to be in a good agreement with the values derived by the theory, in all cases slightly lower (10 percent) than the computed values. The failure of the system to produce this voltage reading must again be attributed to a breakdown due to plasma generation at the sliding contacts.

After having made numerous shots, an observer could, by watching the brightness of the arcs at the sliding contacts, predict the success of a shot in advance of the reading of the velocity measurements.

More information of the above-mentioned breakdown phenomena and on the forces involved in the acceleration could certainly have been obtained by a direct-current measurement. For safety reasons, such a direct-current measurement could not have been made without applying considerably more effort than had been planned.

CONCLUDING REMARKS

It has been shown that for any electromagnetic accelerator which employs electromagnetic forces for driving the projectile and where the projectile is the heat sink for the energy dissipated in it by ohmic heating, the maximum velocity obtainable without melting is a function of the mass of the projectile. Therefore, for hypervelocities a larger projectile mass is required. It has been shown that the only means for reducing the power requirements is maximizing the gradient of the mutual inductance. In the scheme of the sliding-coil accelerator the gradient of the mutual inductance is continuosly maintained at a high value. Theoretical treatment has shown that the principles of the sliding-coil accelerator can be used to achieve hypervelocities through comparatively moderate technical efforts. A problem which has not been solved is the effect of arcing at the sliding contacts at hypervelocities. More information on these effects is needed before construction of a full-scale sliding-coil accelerator can be attempted. A comprehensive design for the construction of a hypervelocity electromagnetic mass accelerator has not been anticipated in the scope of this paper; instead, the basic relations of scaling are revealed and a suggestion has been developed which projects the problem of electromagnetic acceleration to hypervelocities into the realm of realistic technical possibilities.

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1 National Aeronautics and Space Administration,
5 Langley Field, Va., March 24, 1961.
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APPENDIX A

DERIVATION OF THE INDUCED VOLTAGE

The induced voltage for a conducting body moving with the velocity v with respect to a magnetic field is derived from

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \text{curl}(\vec{v} \times \vec{B}) \tag{A1}$$

when the curl of the vector product is expanded and v is assumed constant for a short but finite period of time τ, then, with $\text{div } B = 0$, equation (A1) becomes

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{v} \text{ grad} \cdot \vec{B} \tag{A2}$$

If the velocity has an x-component only, the x-component of the vector gradient has to be taken; thus,

$$v_x(\text{grad} \cdot B)_x = v_x \text{ grad } B_x$$

The induced voltage across either one of the two coils of the sliding-coil accelerator is obtained by applying Stokes' theorem in equation (A2):

$$V_i = -\iint \left(\frac{\partial B_x}{\partial t}\right) dy\ dz - \iint \left(\frac{\partial B_x}{\partial x}\right) v_x\ dy\ dz \tag{A3}$$

The integration has to be taken over the cross-sectional area of the coil in a plane perpendicular to x, the direction of propagation. Thus,

$$V_i = -\frac{\partial \phi}{\partial t} - v_x \frac{\partial \phi}{\partial x} \tag{A4}$$

Two cases must now be distinguished. The first case describes the type of accelerator, where four sliding contacts are employed. In this accelerator, the length of the driving coil does not change. (See previous section "Experimental work," first type of accelerator.) The second case is applicable to the sliding-coil accelerator with two sliding contacts, where the length of the driving coil changes with the position of the driven coil.

In the first case, the length of the driving coil does not change with the position of the driven coil. With constant current, no net change takes place in the amount of magnetic flux of the system. The amount of magnetic energy storage, on the average, is constant.

The induced voltage of the system is the sum of the voltage induced in coil 2 (the driven coil) and that in coil 1 (the driving coil). In a system moving with the velocity v of the propelled coil the switching mechanism serves to reestablish the magnetic flux after it has changed because of the separation of the coils. The change of flux through the driven coil thus is periodic. On the average, over complete cycles of coil separation and subsequent switchings, there is no net change of flux through coil 2. The driven coil, therefore, does not show a net induced voltage.

The induced voltage across the driving coil is

$$\left(V_i\right)_1 = -\frac{\partial \phi_1}{\partial t} - v\frac{\partial \phi_1}{\partial x} \tag{A5}$$

where the subscript 1 refers to coil 1. Since in this system the amount of flux does not change, on the average, due to the switching mechanism, $\frac{\partial \phi_1}{\partial t} = 0$. There is, however, a displacement of flux with respect to the driving coil; the expression $v\frac{\partial \phi_1}{\partial x}$ is different from zero.

The flux through coil 1 is

$$\phi_1 = I\left(L_1 \pm M_{12}\right) \tag{A6}$$

where L_1 is the self-inductance of coil 1 and M_{12} is the mutual inductance. In a system where L_1 is constant,

$$\frac{\partial \phi_1}{\partial x} = \pm \frac{\partial M_{12}}{\partial x} I$$

where the plus or minus sign applies for alike or opposite sense of winding, respectively. If τ is an integral multiple of the time of 1 cycle of coil separation and subsequent switching, the average induced voltage in coil 1 is

$$\overline{V_i} = \frac{1}{\tau}\int_0^\tau V_i\,dt = \pm \frac{v}{\tau}\int_0^\tau I(t)\left(\pm \frac{\partial M_{12}}{\partial x}\right)dt$$

With

$$\frac{\partial M_{12}}{\partial x} = \text{Constant}$$

and

$$\bar{I} = \frac{1}{\tau} \int_0^{\tau} I \, dt$$

$$\bar{V}_i = vI \frac{\partial M_{12}}{\partial x} = \frac{1}{2} vI \frac{dL}{dx} \tag{A7}$$

which is also the induced voltage over the whole system, in accordance with equation (8).

In the case of the two-sliding-contact systems, the length of the driving coil (coil 1) depends upon the position of the propelled coil, coil 2. Apart from the conditions at the end or the beginning of the accelerator (depending on the cases where the principles of repelling or attracting coils are used), coil 1 can be considered as a long solenoid. Then the mutual inductance does not change with the length of coil 1.

From the definition of the mutual inductance

$$M_{21} = \frac{V_2}{\dot{I}_1}$$

it follows that

$$V_2 = -n_2 \iint \dot{B} \, dy \, dz$$

and for the coils close together

$$V_2 \approx \frac{1}{2} n_2 r^2 \pi \dot{B}_1 \approx n_2 \left(\frac{N_1}{l_1}\right) r^2 \pi \mu_0 \frac{1}{2} \frac{dI_1}{dt}$$

where $2r = a$ is the diameter of the coils and l_1 is the length of coil 1. The mutual inductance then is

$$M_{21} \approx r^2 \pi \mu_0 \left(\frac{N_1}{l_1}\right) N_2 \frac{1}{2} \tag{A8}$$

An elongation of coil 1 does not change the ratio N_1/l_1. Consequently, a variation of the length of this coil does not change the mutual inductance. (This result is unaffected by the approximation in equation (A8).) Therefore, by the same reasoning as given for the previous system, there is no net change of flux through the driven coil 2, and on the average there is no induced voltage across coil 2 in the case of a two-sliding-contact system.

The induced voltage across the driving coil is composed of

$$V_i = -\frac{\partial \phi_1}{\partial t} - v\,\frac{\partial \phi_1}{\partial x} \tag{A9}$$

the parameter $\frac{\partial \phi_1}{\partial t} = \frac{\partial L_1}{\partial t} I$ because the mutual inductance does not vary with time, and $\frac{\partial \phi_1}{\partial x} = I \frac{\partial M}{\partial x}$ because the change of the self-inductance is not caused by the separation of the coils but by a subsequent switching in time.

The second term on the right-hand side of equation (A9) is therefore of the same magnitude as in equation (A7) for the four-sliding-contact system. In the two-sliding-contact system an additional term $I\,\partial L_1/\partial t$ appears for computing the induced voltage. Equation (8) of the main text has to be corrected correspondingly.

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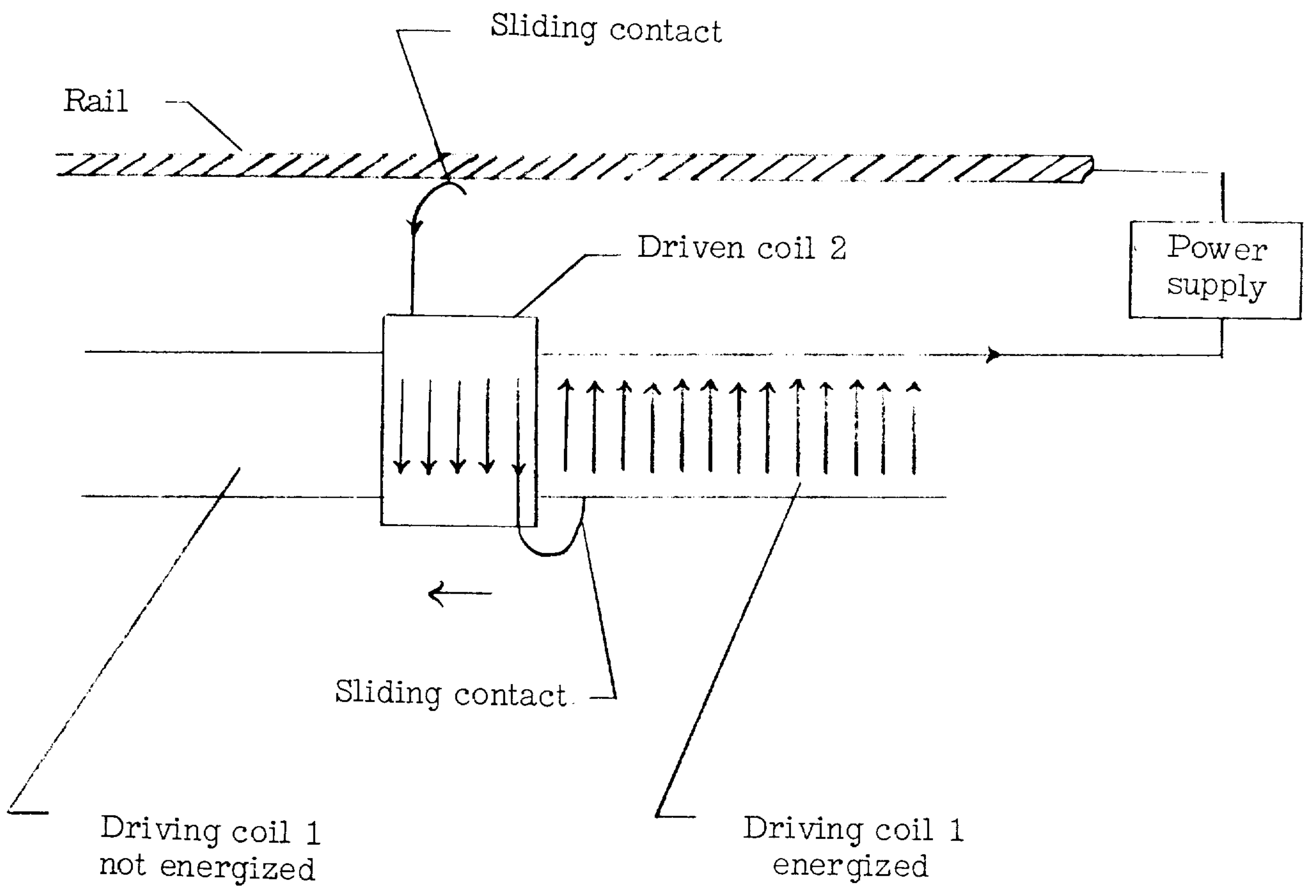

Figure 1.- Schematic diagram of sliding coil accelerator.

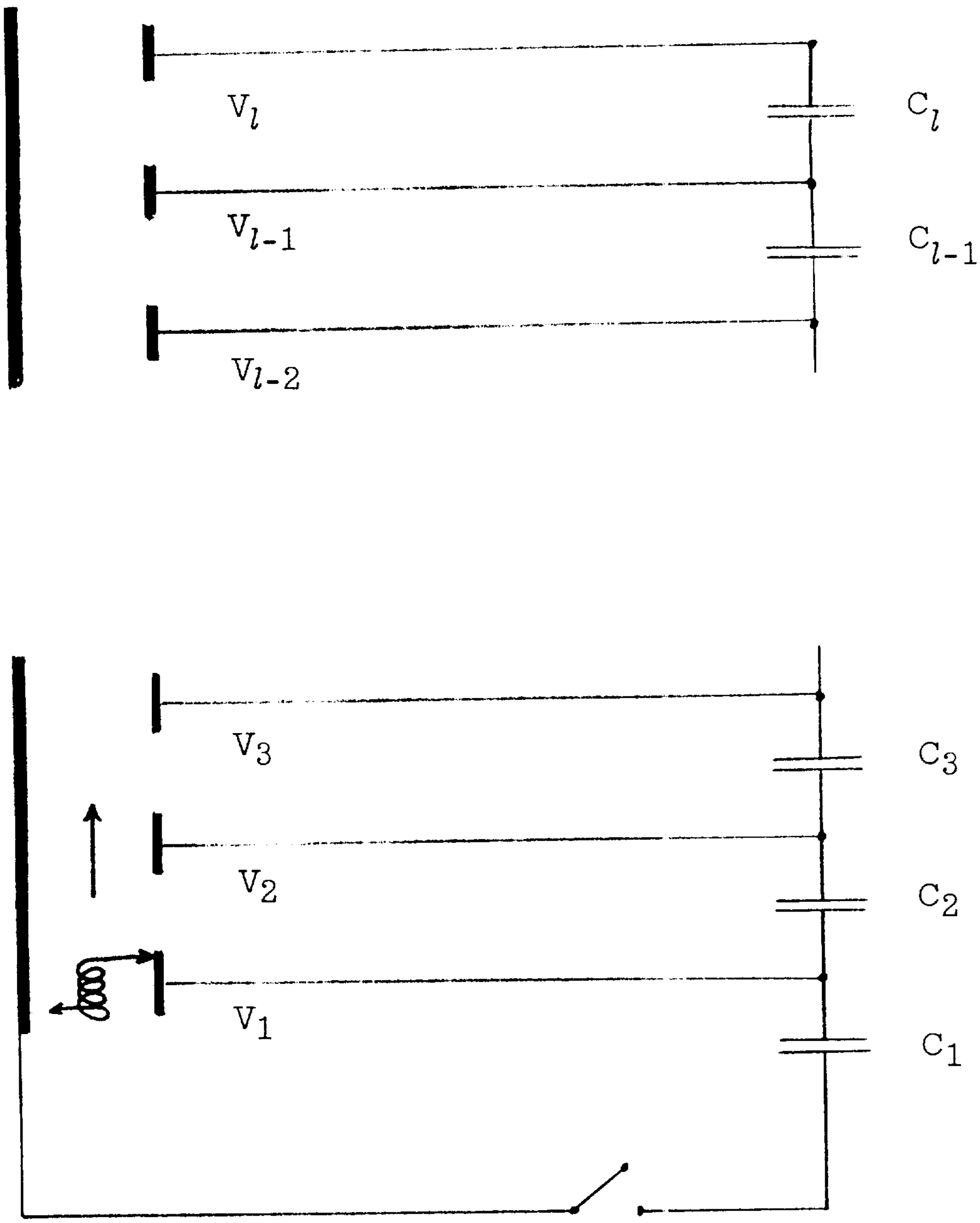

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Figure 2.- The divided capacitor bank.

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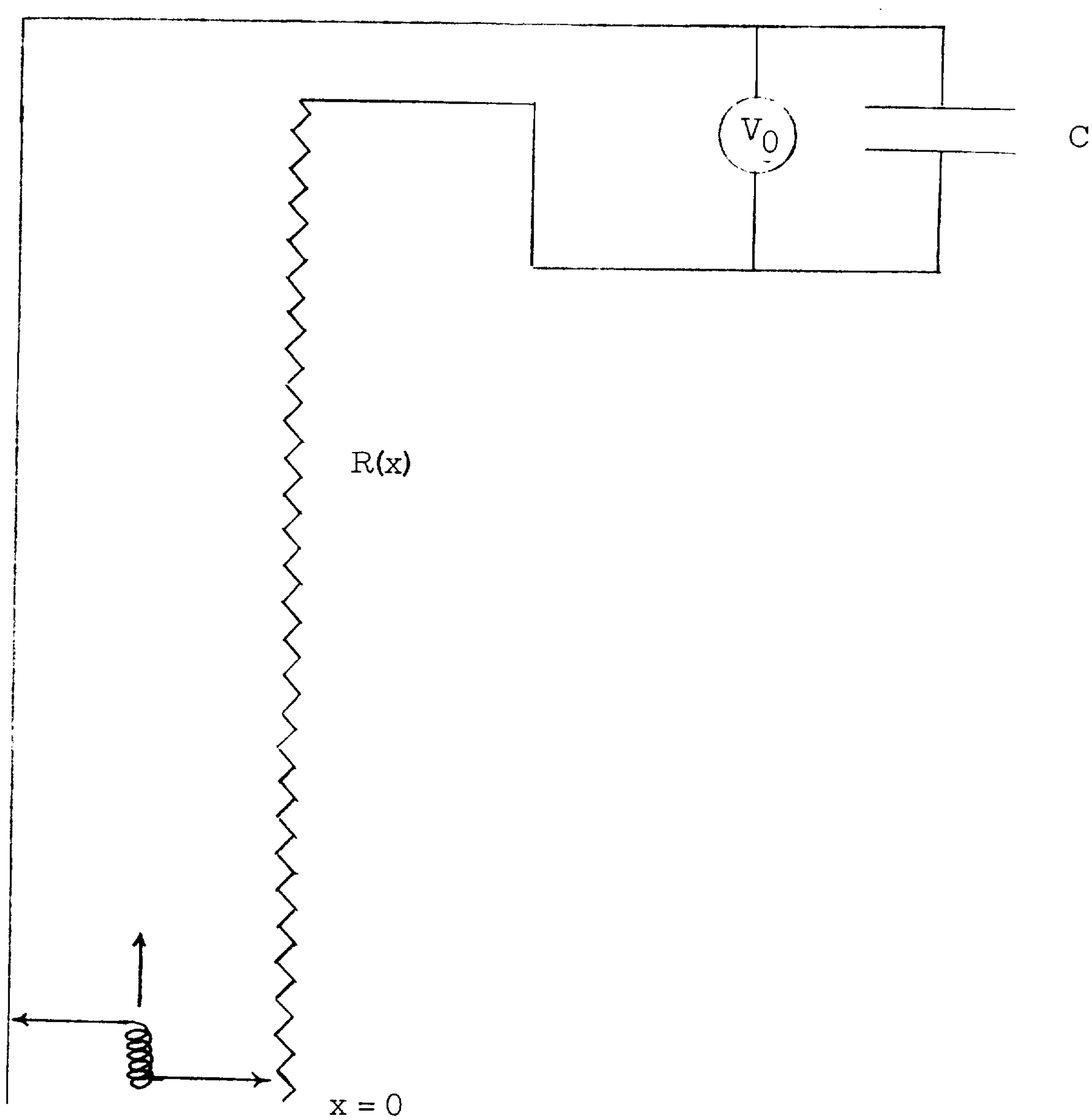

Figure 3.- Capacitor bank with a resistor in line.

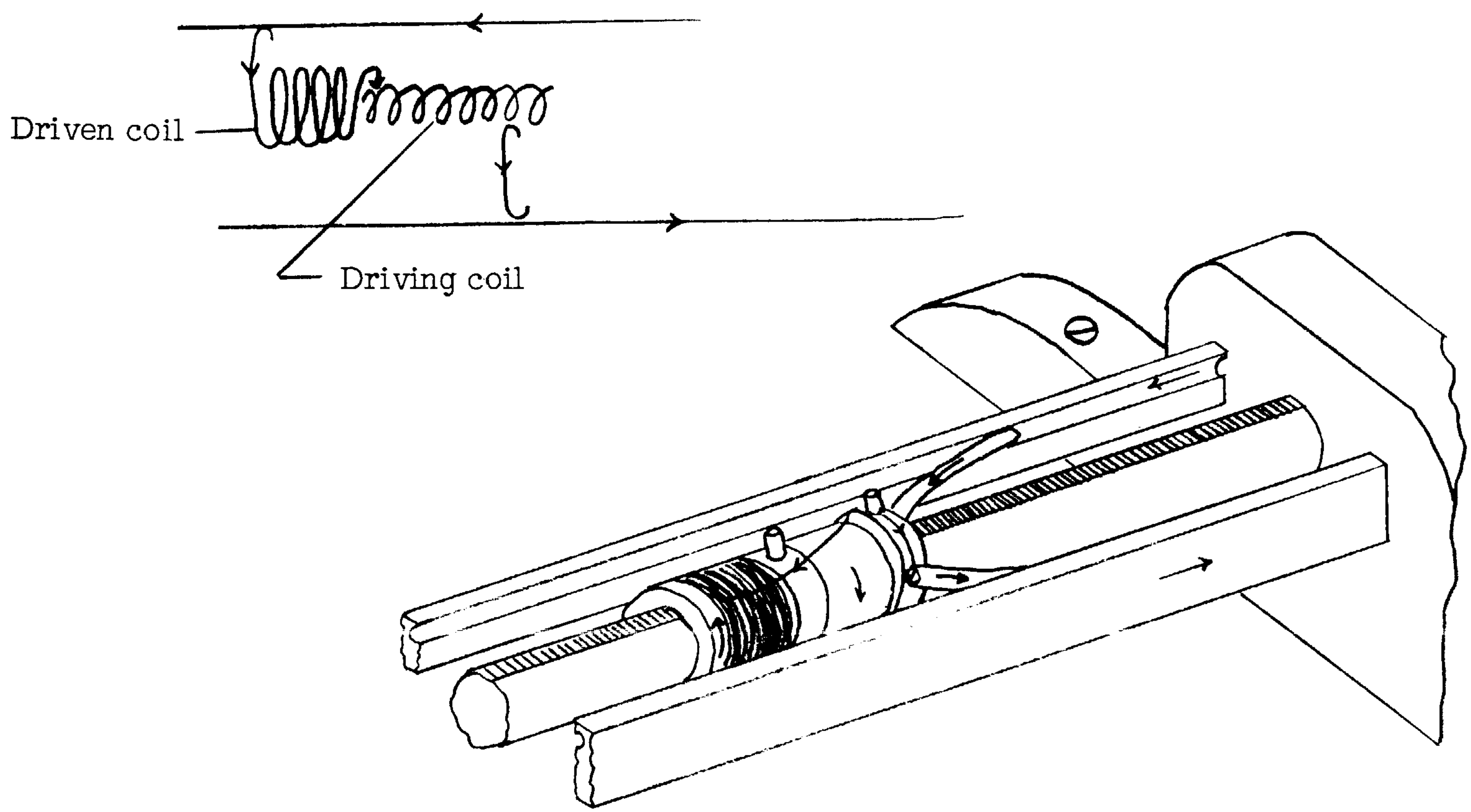

Figure 4.- Double-rail accelerator.

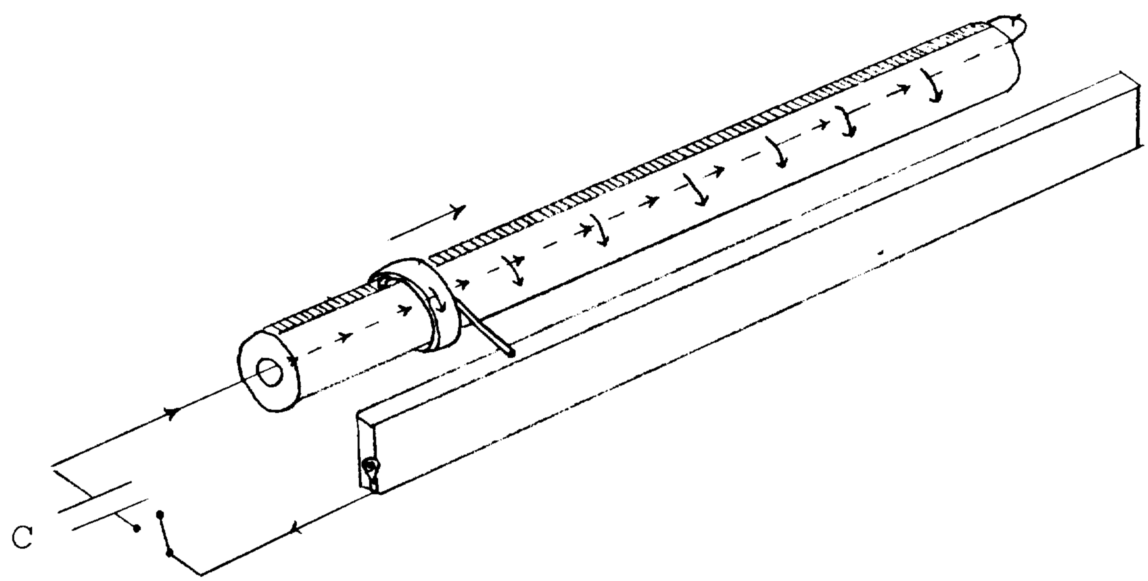

Figure 5.- Single-rail accelerator.

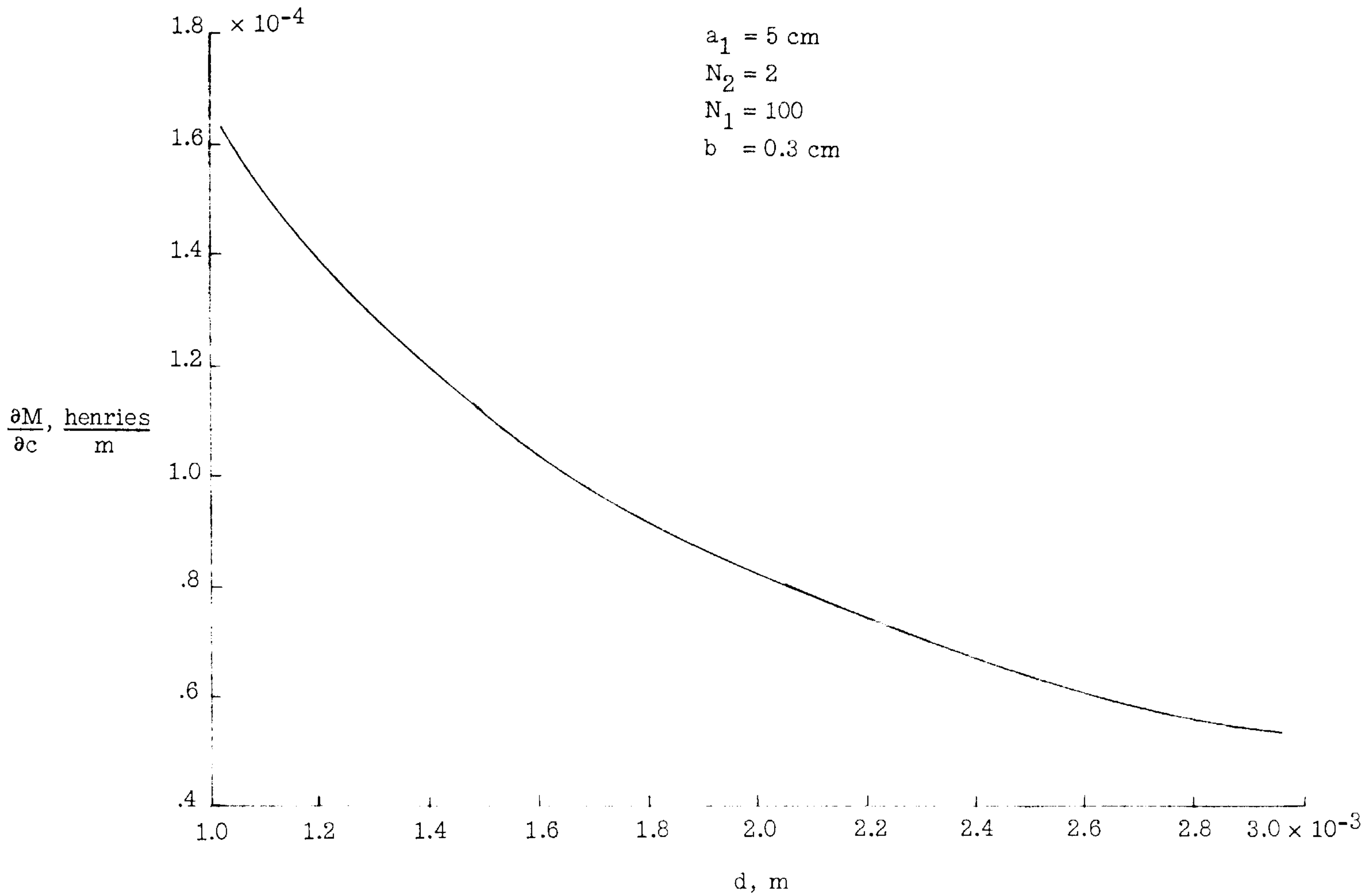

Figure 10.- The inductive gradient as a function of wire diameter from 0.1 centimeter to 0.3 centimeter.

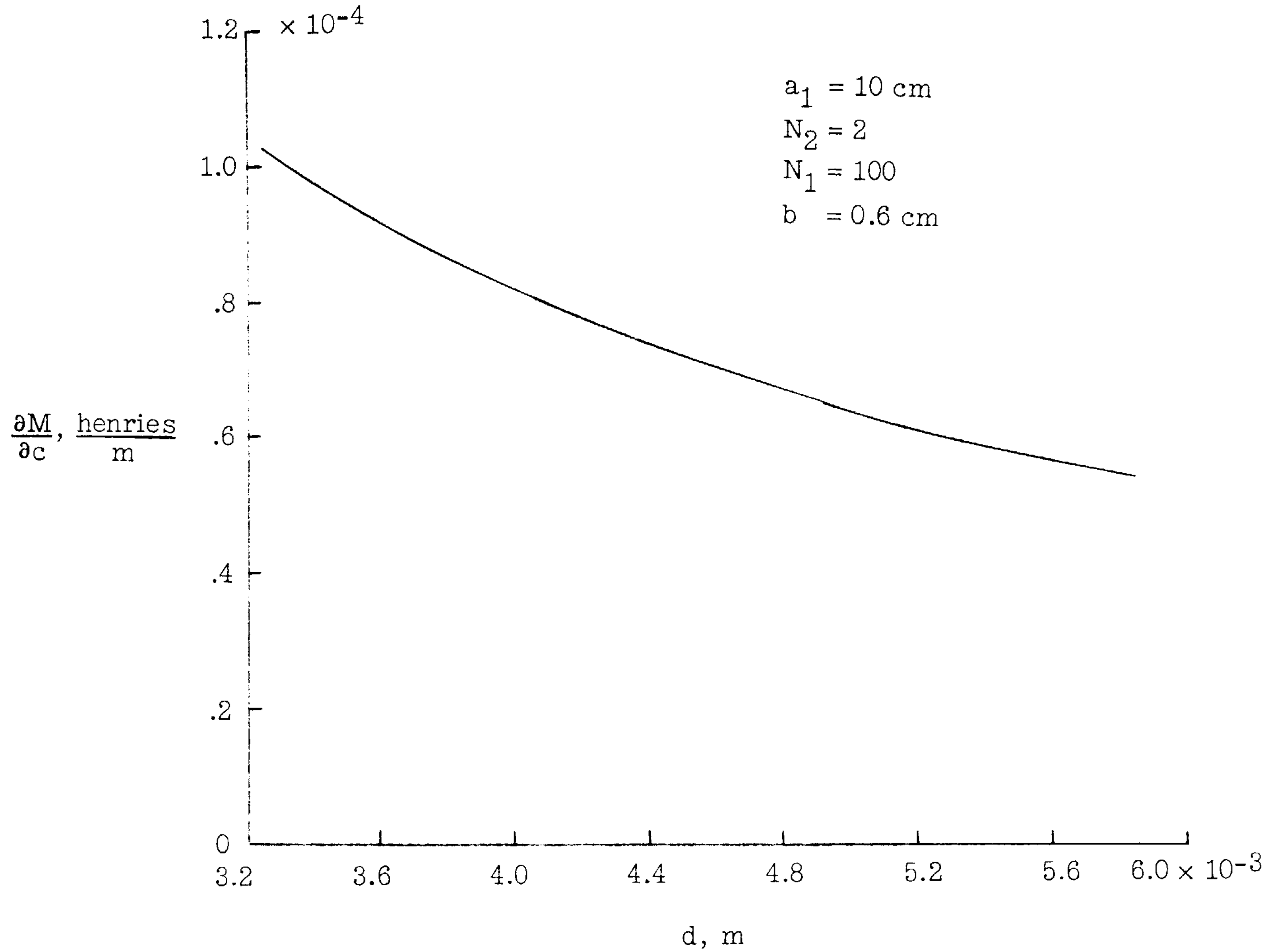

Figure 11.- The inductive gradient as a function of wire diameter from 0.32 centimeter to 0.6 centimeter.

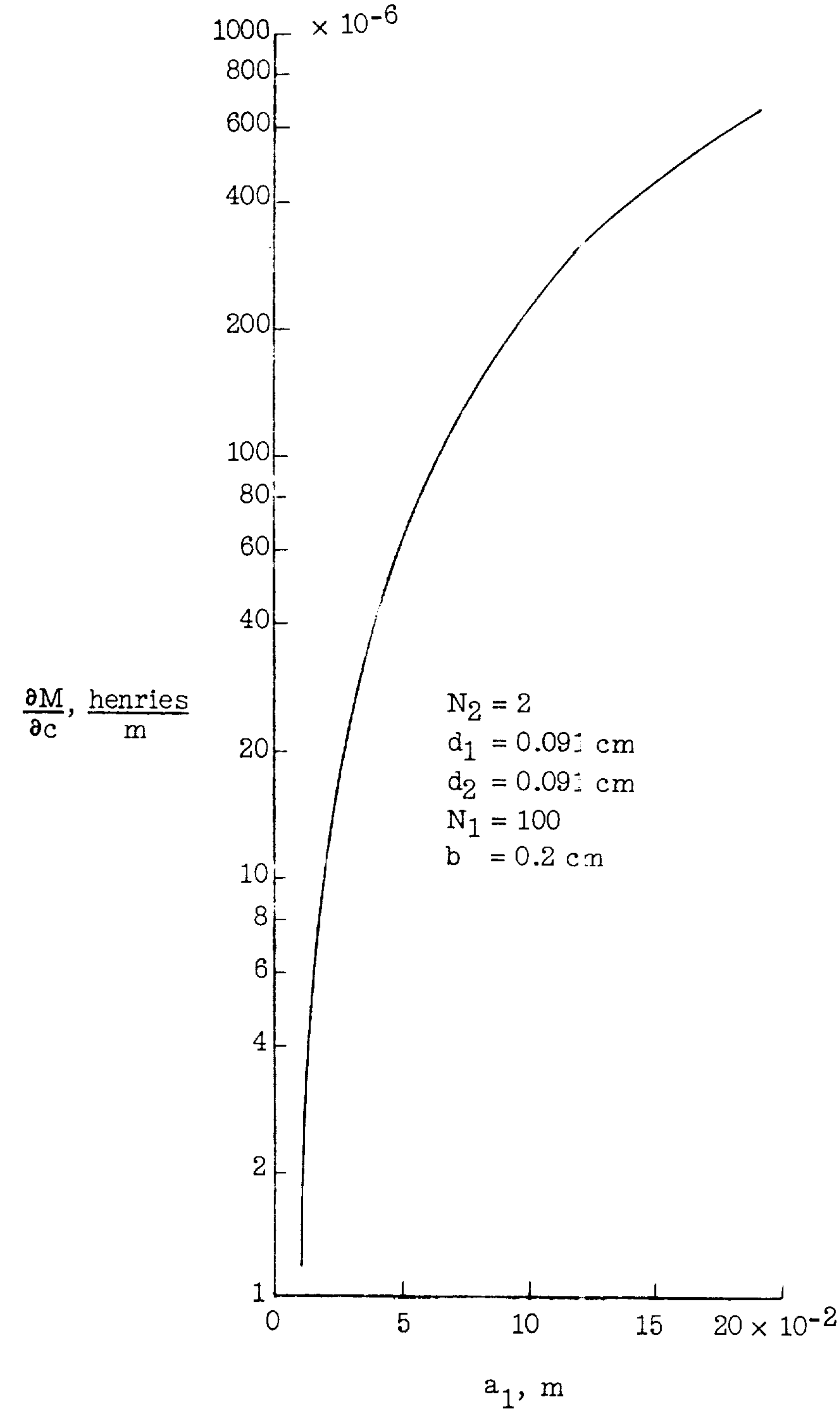

Figure 12.- The inductive gradient as a function of diameter of driving coil.

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