NASA TN D-1099

NASA TN D-1099

IN-18
388 516

TECHNICAL NOTE

D-1099

EARTH REFLECTED SOLAR RADIATION INPUT TO SPHERICAL SATELLITES

F. G. Cunningham

Goddard Space Flight Center
Greenbelt, Maryland

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON October 1961

G

D-1099

EARTH REFLECTED SOLAR RADIATION INPUT TO SPHERICAL SATELLITES

by
F. G. Cunningham
Goddard Space Flight Center

SUMMARY

A general calculation is given for the earth's albedo input to a spherical satellite, with the assumption that the earth can be considered a diffusely reflecting sphere. The results are presented in general form so that appropriate values for the solar constant and albedo of the earth can be used as more accurate values become available. The results are also presented graphically; the incident power is determined on the assumption that the mean solar constant is 1.353×10^6 erg cm^{-2} sec^{-1} and the albedo of the earth is 0.34.

D-1099

CONTENTS

D-1099

EARTH REFLECTED SOLAR RADIATION INPUT TO SPHERICAL SATELLITES

by
F. G. Cunningham
Goddard Space Flight Center

INTRODUCTION

Several recent papers have considered the thermal behavior of satellites and space vehicles in near-earth orbits. Goldman and Singer, considering the temperature environment of a spherical satellite, made the approximation that a satellite receives reflected solar radiation from only that portion of the earth directly beneath it (Reference 1). This does not take into account the diffusiveness of reflection from the earth. Wood and Carter calculated the reflected solar radiation input to a sphere located on the earth-sun line, but not for any other orientations (Reference 2). Hibbs' paper (Reference 3), which includes the basic integral expression for reflected solar radiation, gives a result which is erroneous. This error will be reviewed presently.

CALCULATIONS

The geometry of the problem is shown in Figure 1. The associated definitions are:

- r = The distance of the satellite from the center of the earth, in mean earth radii.
- ρ = The distance from the satellite to the element of area ds on the earth's surface, in mean earth radii.
- θ = The co-latitudinal angle defining the position of ds with respect to r.
- ϕ = The azimuthal angle of integration defining ds with respect to the plane formed by r and the solar vector.
- ξ = The angle between ρ and the normal to the surface element ds.

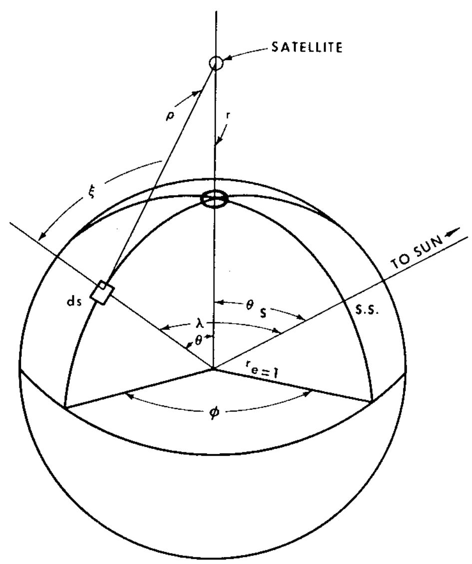

Figure 1 - Geometry of the problem

λ = The angle between the solar vector and the normal to the surface element ds.
θ_S = The angle between the solar vector and r.
$\mathcal{S}$ = The solar constant.
α = The mean albedo of the earth.

The general expression for the reflected solar radiation incident upon a satellite, P_{SE}, can be written as

$$P_{SE} = \frac{\mathcal{S}\alpha}{\pi} \int \frac{\cos \lambda \cos \xi \, ds}{\rho^2} \quad , \tag{1}$$

where the cross sectional area of the satellite is assumed to be unity. From Figure 1, the quantities in Equation 1 are:

$$\rho^2 = r^2 + 1 - 2r \cos \theta \quad ;$$

$$ds = \sin \theta \, d\theta \, d\phi \quad ; \tag{2}$$

$$\cos \xi = \frac{r \cos \theta - 1}{(r^2 + 1 - 2r \cos \theta)^{\frac{1}{2}}}$$

and

$$\cos \lambda = \cos \theta \cos \theta_S + \sin \theta \sin \theta_S \cos \phi \quad . \tag{3}$$

Equation 3 follows from the relationships of the angles and sides of a spherical triangle. Substituting Equations 2 and 3 into Equation 1 gives the complete expression for the reflected solar radiation,

$$P_{SE} = \frac{2\mathcal{S}\alpha}{\pi} \int_0^{\theta_m} \int_0^{\phi_m} \frac{(r \cos \theta - 1)(\cos \theta \cos \theta_S + \sin \theta \sin \theta_S \cos \phi) \sin \theta \, d\theta \, d\phi}{(r^2 + 1 - 2r \cos \theta)^{\frac{3}{2}}} \quad . \tag{4}$$

Equation 4 is to be integrated over the area of all the terrestrial surface elements which both "see" the satellite and are illuminated by the sun. The limits of integration for θ and ϕ in Equation 4 depend on the relations between the two relevant regions: Region 1, the hemisphere of the earth that is illuminated by the sun; and Region 2, the spherical cap, consisting of all the elements of the earth which see the satellite. All elements of Region 1 satisfy the condition $\lambda \leq \pi/2$. The great circle which constitutes the boundary between the sunlit and unlit hemispheres, i.e., the terminator, is the locus of points for which $\lambda = \pi/2$. All elements of Region 2 satisfy the condition $0 \leq \theta \leq \theta_m$, where $\theta_m = \cos^{-1} 1/r$.

There are three possible relations between Regions 1 and 2 which define the areas of integration:

a. Region 2 lies completely within Region 1;
b. Region 2 lies partly within Region 1; or
c. Region 2 lies completely outside of Region 1.

D-1099

For condition a, $0 \leq \theta_S \leq \pi/2 - \theta_m$, and ϕ takes on all values from 0 to 2π; but because of symmetry with respect to the plane determined by the earth-satellite and earth-sun lines, the integration can be performed between the limits 0 and π and the result multiplied by 2. In this case $0 \leq \theta \leq \theta_m$. In condition c, where $\theta_S > \pi/2 + \theta_m$, no reflected sunlight is incident on the satellite; and this condition is of no further interest.

Condition b presents two possibilities: b_1, in which $\pi/2 - \theta_m < \theta_S \leq \pi/2$; and b_2, in which $\pi/2 < \theta_S \leq \pi/2 + \theta_m$. For b_1 the area of integration is divided into two parts: b_{11} for which $0 \leq \theta \leq \pi/2 - \theta_S$ and $0 \leq \phi \leq \pi$; and b_{12} for which $\pi/2 - \theta_S \leq \theta \leq \theta_m$. In part b_{12} of subregion b_1, and in subregion b_2 where $\theta_S - \pi/2 \leq \theta \leq \theta_m$, the upper limit of the ϕ integration, ϕ_m, may be determined by solving Equation 3 for $\cos \phi_m$ after substituting the value $\lambda = \pi/2$:

$$\cos \phi_m = -\frac{\cos \theta \cos \theta_S}{\sin \theta \sin \theta_S} \quad . \tag{5}$$

The fact that the range of ϕ is not 0 to 2π (that is, 0 to π in the present paper) for all values of θ in the range $0 \leq \theta \leq \theta_m$ was overlooked in the paper by Hibbs (Reference 3). The result, upon integration, is that the term in Equation 4 containing $\cos \phi$ is eliminated for all values of θ_S. Consider Equation 4 when $\theta_S = \pi/2$. If the $\cos \phi$ term is zero upon integration and the $\cos \theta_S$ term equals zero by definition of the problem, then the incident power is identically zero. But, clearly, when $\theta_S = \pi/2$, half of Region 2 lies within Region 1 and the satellite does indeed see illuminated portions of the earth.

In summary, the regions and subregions of integration are:

$$\text{a.} \quad 0 \leq \theta_S \leq \frac{\pi}{2} - \theta_m, \text{ for which } \begin{array}{l} 0 \leq \theta \leq \theta_m, \\ 0 \leq \phi \leq \pi \ , \end{array} \tag{6}$$

and the incident power $\equiv P_{SE_1}$;

$$b_1. \quad \frac{\pi}{2} - \theta_m < \theta_S \leq \frac{\pi}{2}, \text{ where } b_1 = b_{11} + b_{12} \ , \tag{7}$$

$$b_{11}. \quad 0 \leq \theta \leq \frac{\pi}{2} - \theta_S \text{ and } 0 \leq \phi \leq \pi \ ,$$

$$b_{12}. \quad \frac{\pi}{2} - \theta_S \leq \theta \leq \theta_m \text{ and } 0 \leq \phi \leq \cos^{-1}(-\cot \theta \cot \theta_S) \ ,$$

and the incident power $\equiv P_{SE_2}$;

$$b_2. \quad \frac{\pi}{2} < \theta_S \leq \frac{\pi}{2} + \theta_m, \text{ for which } \begin{array}{l} \theta_S - \frac{\pi}{2} \leq \theta \leq \theta_m \\ 0 \leq \phi \leq \cos^{-1}(-\cot \theta \cot \theta_S) \end{array} \tag{8}$$

and the incident power $\equiv P_{SE_3}$.

In consideration of the foregoing, and with the inclusion of Equation 3, Equation 4 yields

$$P_{SE_1} = \frac{2\delta\alpha}{\pi}\int_0^{\theta_m}\int_0^{\pi} \frac{(r\cos\theta - 1)(\cos\theta\cos\theta_S + \sin\theta\sin\theta_S\cos\phi)\sin\theta\, d\theta\, d\phi}{(r^2 + 1 - 2r\cos\theta)^{\frac{3}{2}}}, \quad (9)$$

where

$$0 \le \theta_S \le \frac{\pi}{2} - \theta_m \quad ;$$

$$P_{SE_2} = \frac{2\delta\alpha}{\pi}\int_0^{\left(\frac{\pi}{2} - \theta_S\right)}\int_0^{\pi} \frac{(r\cos\theta - 1)(\cos\theta\cos\theta_S + \sin\theta\sin\theta_S\cos\phi)\sin\theta\, d\theta\, d\phi}{(r^2 + 1 - 2r\cos\theta)^{\frac{3}{2}}}$$

$$+ \frac{2\delta\alpha}{\pi}\int_{\left(\frac{\pi}{2} - \theta_S\right)}^{\theta_m}\int_0^{\phi_m} \frac{(r\cos\theta - 1)(\cos\theta\cos\theta_S + \sin\theta\sin\theta_S\cos\phi)\sin\theta\, d\theta\, d\phi}{(r^2 + 1 - 2r\cos\theta)^{\frac{3}{2}}}, \quad (10)$$

where

$$\frac{\pi}{2} - \theta_m < \theta_S \le \frac{\pi}{2} \quad ;$$

and

$$P_{SE_3} = \frac{2\delta\alpha}{\pi}\int_{\left(\theta_S - \frac{\pi}{2}\right)}^{\theta_m}\int_0^{\phi_m} \frac{(r\cos\theta - 1)(\cos\theta\cos\theta_S + \sin\theta\sin\theta_S\cos\phi)\sin\theta\, d\theta\, d\phi}{(r^2 + 1 - 2r\cos\theta)^{\frac{3}{2}}} \quad (11)$$

where

$$\frac{\pi}{2} < \theta_S \le \frac{\pi}{2} + \theta_m \quad .$$

Upon integration, Equations 9, 10, and 11 become:

$$P_{SE_1} = \frac{2\delta\alpha}{3}\left[\left(2r + \frac{1}{r^2}\right) - \left(2 + \frac{1}{r^2}\right)\left(r^2 - 1\right)^{\frac{1}{2}}\right]\cos\theta_S \quad ; \quad (12)$$

$$P_{SE_2} = \frac{2\delta\alpha\cos\theta_S}{3}\left[\frac{\left(1 + \frac{1}{r^2}\right)\left(-2r^2 + 1\right) + \left(2r - \frac{1}{r}\right)\sin\theta_S + \sin^2\theta_S}{(r^2 + 1 - 2r\sin\theta_S)^{\frac{1}{2}}} + \frac{1}{r^2} + 2r\right]$$

$$+ \frac{2\delta\alpha}{\pi}\int_{\left(\frac{\pi}{2} - \theta_S\right)}^{\theta_m} \frac{(r\cos\theta - 1)\cos\theta_S\sin\theta\cos\theta}{(r^2 + 1 - 2r\cos\theta)^{\frac{3}{2}}}\cos^{-1}\left(-\frac{\cos\theta\cos\theta_S}{\sin\theta\sin\theta_S}\right) d\theta$$

$$+ \frac{2\delta\alpha}{\pi}\int_{\left(\frac{\pi}{2} - \theta_S\right)}^{\theta_m} \frac{(r\cos\theta - 1)\sin\theta\left(\sin^2\theta - \cos^2\theta_S\right)^{\frac{1}{2}}}{(r^2 + 1 - 2r\cos\theta)^{\frac{3}{2}}}\, d\theta \quad ; \quad (13)$$

D-1099

and

$$P_{SE_3} = \frac{2\delta\alpha}{\pi} \int_{\left(\theta_S - \frac{\pi}{2}\right)}^{\theta_m} \frac{(r \cos \theta - 1) \cos \theta_S \sin \theta \cos \theta}{\left(r^2 + 1 - 2r \cos \theta\right)^{\frac{3}{2}}} \cos^{-1} \left(- \frac{\cos \theta \cos \theta_S}{\sin \theta \sin \theta_S}\right) d\theta$$

$$+ \frac{2\delta\alpha}{\pi} \int_{\left(\theta_S - \frac{\pi}{2}\right)}^{\theta_m} \frac{(r \cos \theta - 1) \sin \theta \left(\sin^2 \theta - \cos^2 \theta_S\right)^{\frac{1}{2}}}{\left(r^2 + 1 - 2r \cos \theta\right)^{\frac{3}{2}}} d\theta \quad . \tag{14}$$

SIMPLIFICATION OF THE EQUATIONS

Equations 12, 13, and 14 may be simplified by defining the following quantities:

$$A = \frac{2}{3} \cos \theta_S \left[\frac{\left(1 + \frac{1}{r^2}\right)\left(-2r^2 + 1\right) + \left(2r - \frac{1}{r}\right) \sin \theta_S + \sin^2 \theta_S}{\left(r^2 + 1 - 2r \sin \theta_S\right)^{\frac{1}{2}}} + \frac{1}{r^2} + 2r \right] , \tag{15}$$

$$B = \frac{2}{\pi} \int_{\left(\frac{\pi}{2} - \theta_S\right)}^{\theta_m} \frac{(r \cos \theta - 1) \cos \theta_S \sin \theta \cos \theta}{\left(r^2 + 1 - 2r \cos \theta\right)^{\frac{3}{2}}} \cos^{-1} \left(- \frac{\cos \theta \cos \theta_S}{\sin \theta \sin \theta_S}\right) d\theta , \tag{16}$$

$$C = \frac{2}{\pi} \int_{\left(\frac{\pi}{2} - \theta_S\right)}^{\theta_m} \frac{(r \cos \theta - 1) \sin \theta \left(\sin^2 \theta - \cos^2 \theta_S\right)^{\frac{1}{2}}}{\left(r^2 + 1 - 2r \cos \theta\right)^{\frac{3}{2}}} d\theta , \tag{17}$$

where θ_S has the range indicated in Equation 7;

$$D = \frac{2}{\pi} \int_{\left(\theta_S - \frac{\pi}{2}\right)}^{\theta_m} \frac{(r \cos \theta - 1) \cos \theta_S \sin \theta \cos \theta}{\left(r^2 + 1 - 2r \cos \theta\right)^{\frac{3}{2}}} \cos^{-1} \left(- \frac{\cos \theta \cos \theta_S}{\sin \theta \sin \theta_S}\right) d\theta , \tag{18}$$

$$E = \frac{2}{\pi} \int_{\left(\theta_S - \frac{\pi}{2}\right)}^{\theta_m} \frac{(r \cos \theta - 1) \sin \theta \left(\sin^2 \theta - \cos^2 \theta_S\right)^{\frac{1}{2}} d\theta}{\left(r^2 + 1 - 2r \cos \theta\right)^{\frac{3}{2}}} , \tag{19}$$

where θ_S now has the range indicated in Equation 8; and

$$F = \frac{2}{3}\left[\left(2r + \frac{1}{r^2}\right) - \left(2 + \frac{1}{r^2}\right)\left(r^2 - 1\right)^{\frac{1}{2}}\right]\cos\theta_S \quad , \tag{20}$$

where the range of θ_S is given in Equation 6.

With these definitions, the inputs can be written simply:

$$P_{SE_1} = \mathcal{S}\alpha F \quad ; \tag{21}$$

$$P_{SE_2} = \mathcal{S}\alpha(A + B + C) \quad ; \tag{22}$$

and

$$P_{SE_3} = \mathcal{S}\alpha(D + E) \quad . \tag{23}$$

The error introduced by the assumption of a perfectly spherical earth should be small and is neglected. However, the value used for the average albedo (α) can introduce a sizeable variation into the result. The hemispherical yearly average of the earth's albedo seems to vary between 0.34 and 0.40, depending upon the investigator (Reference 4).

RESULTS

Figures 2 and 3 show the results of the preceding calculations, with the value 0.34 for the average albedo of the earth and the value 0.1353 watt/cm^2 for the solar constant. The power input (for unit absorptivity) per unit cross-sectional area of the satellite is plotted as a function of altitude above the surface of the earth. The angle θ_S is the parameter which generates the family of curves. Figure 2 presents the results in the range from 200 to 3200 kilometers and Figure 3 covers the range from 200 to 32,000 kilometers.

ACKNOWLEDGMENTS

The Author wishes to thank Mr. E. Monasterski of the Goddard Space Flight Center for the IBM 7090 evaluations of the A, B, C, D, E, and F functions.

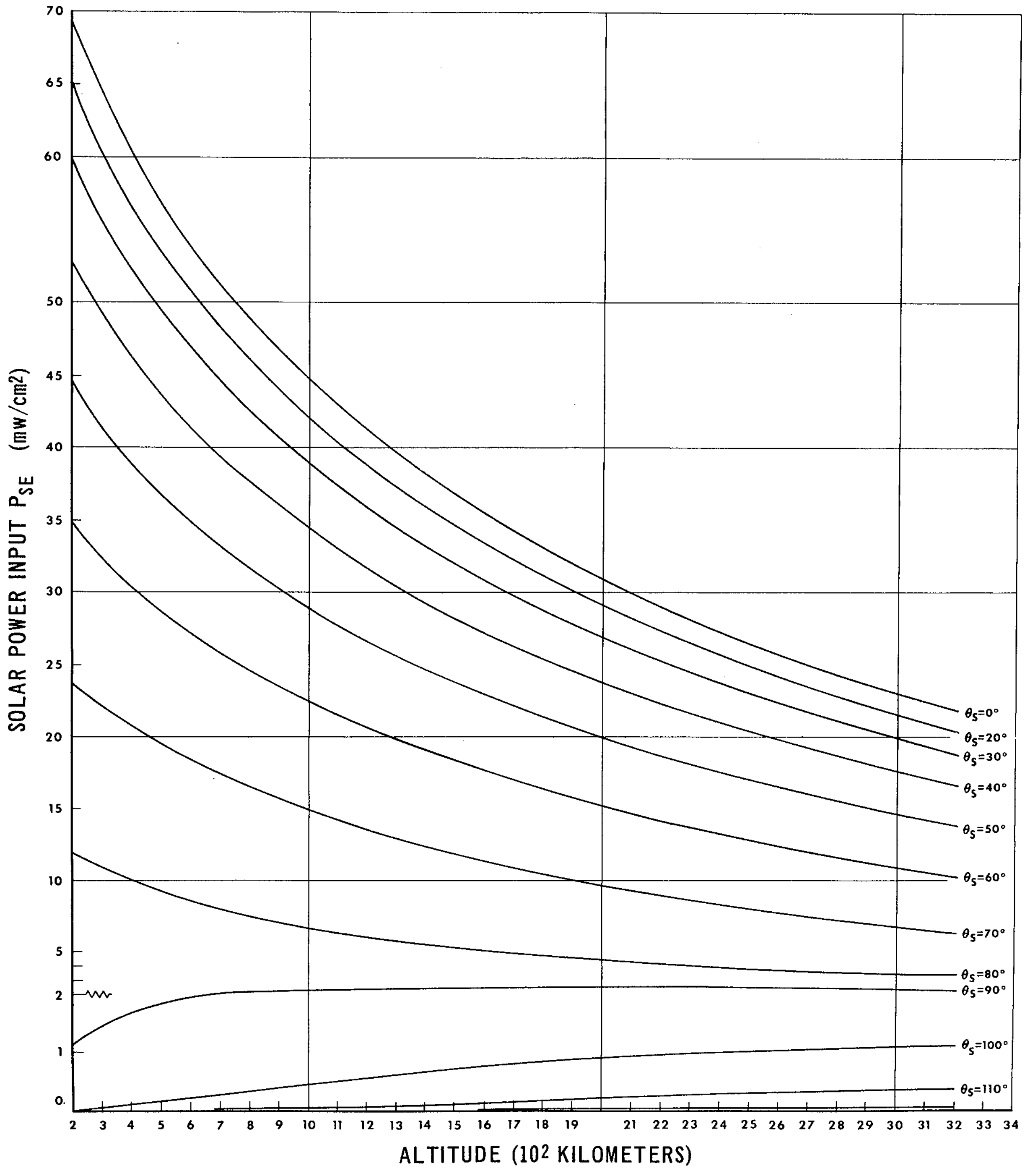

Figure 2 - Power input per unit cross-sectional area of a satellite plotted as a function of altitude for the range 200–3200 miles

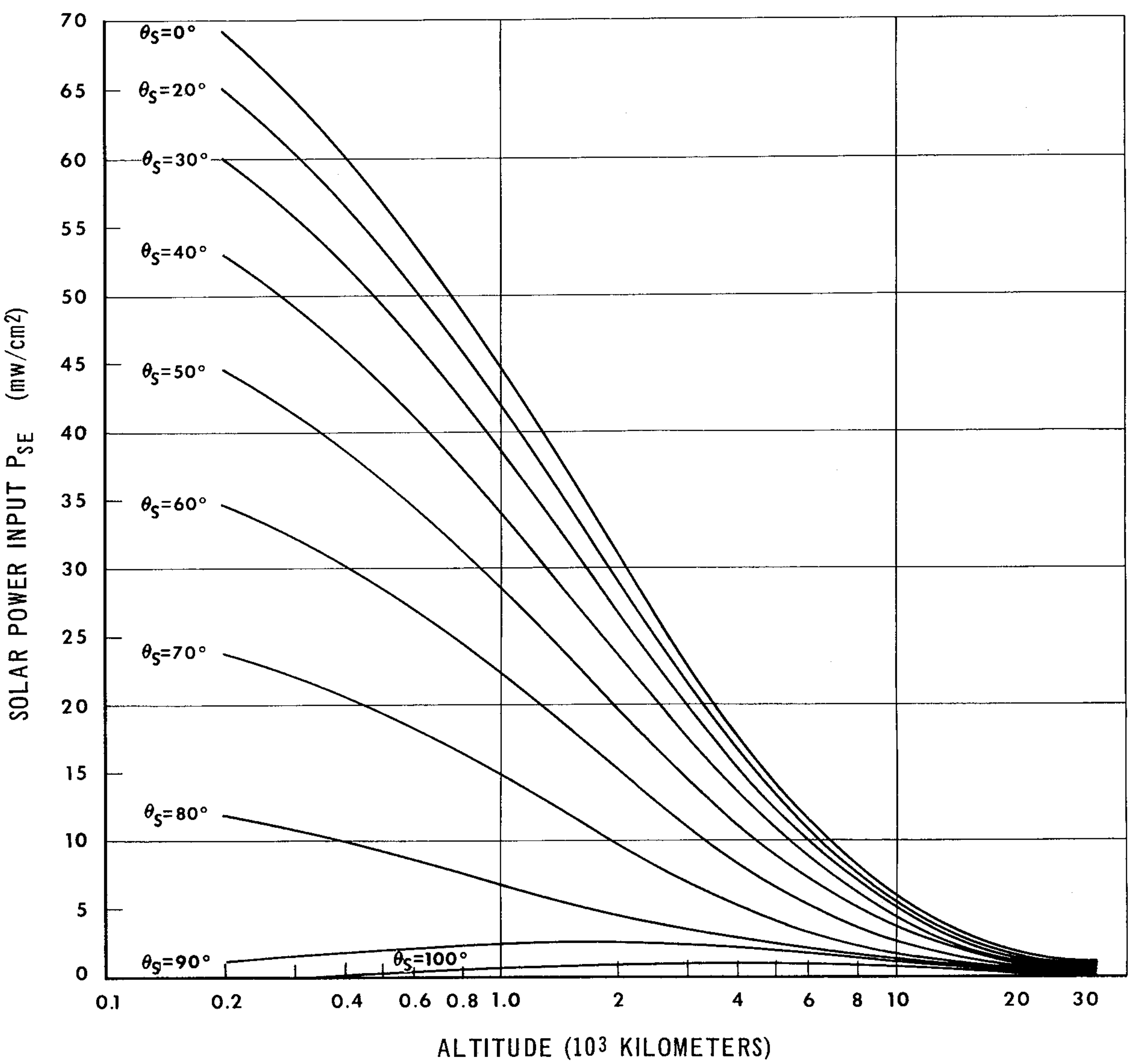

Figure 3 - Power input per unit cross-sectional area of a satellite plotted as a function of altitude for the range 200–32,000 miles

REFERENCES

1. Goldman, D. T., and Singer, S. F., "Studies of a Minimum Orbital Unmanned Satellite of the Earth; Part IV, Radiation Equilibrium and Temperature", Physics Dept. Technical Report No. 46, University of Maryland, College Park, Maryland, June 1956

2. Wood, G. P., and Carter, A. F., "Predicted Characteristics of An Inflatable Aluminized-Plastic Spherical Earth Satellite With Regard to Temperature, Visibility, Reflection of Radar Waves, and Protection from Ultraviolet Radiation", NASA Technical Note D-115, October 1959, pp. 12 ff

3. Hibbs, A. R., "The Temperature of an Orbiting Missile", Jet Propulsion Laboratory, Pasadena, California, Progress Report No. 20-294, March 28, 1956, p. 3

4. Johnson, J. C., "Physical Meteorology", New York: Wiley, 1954, p. 171

D-1099

NASA-Langley, 1961

NASA TN D-1099
National Aeronautics and Space Administration.
EARTH REFLECTED SOLAR RADIATION INPUT TO SPHERICAL SATELLITES. F. G. Cunningham. October 1961. 9p. OTS price, $0.50. (NASA TECHNICAL NOTE D-1099)

A general calculation is given for the earth's albedo input to a spherical satellite, with the assumption that the earth can be considered a diffusely reflecting sphere. The results are presented in general form so that appropriate values for the solar constant and albedo of the earth can be used as more accurate values become available. The results are also presented graphically; the incident power is determined on the assumption that the mean solar constant is 1.353×10^6 erg cm^{-2} sec^{-1} and the albedo of the earth is 0.34.

Copies obtainable from NASA, Washington

I. Cunningham, F. G.
II. NASA TN D-1099

(Initial NASA distribution: 7, Astrophysics; 21, Geophysics and geodesy; 26, Materials, other; 35, Power sources, supplementary; 47, Satellites; 52, Structures.)

NASA

NASA TN D-1099
National Aeronautics and Space Administration.
EARTH REFLECTED SOLAR RADIATION INPUT TO SPHERICAL SATELLITES. F. G. Cunningham. October 1961. 9p. OTS price, $0.50. (NASA TECHNICAL NOTE D-1099)

A general calculation is given for the earth's albedo input to a spherical satellite, with the assumption that the earth can be considered a diffusely reflecting sphere. The results are presented in general form so that appropriate values for the solar constant and albedo of the earth can be used as more accurate values become available. The results are also presented graphically; the incident power is determined on the assumption that the mean solar constant is 1.353×10^6 erg cm^{-2} sec^{-1} and the albedo of the earth is 0.34.

Copies obtainable from NASA, Washington

I. Cunningham, F. G.
II. NASA TN D-1099

(Initial NASA distribution: 7, Astrophysics; 21, Geophysics and geodesy; 26, Materials, other; 35, Power sources, supplementary; 47, Satellites; 52, Structures.)

NASA

NASA TN D-1099
National Aeronautics and Space Administration.
EARTH REFLECTED SOLAR RADIATION INPUT TO SPHERICAL SATELLITES. F. G. Cunningham. October 1961. 9p. OTS price, $0.50. (NASA TECHNICAL NOTE D-1099)

A general calculation is given for the earth's albedo input to a spherical satellite, with the assumption that the earth can be considered a diffusely reflecting sphere. The results are presented in general form so that appropriate values for the solar constant and albedo of the earth can be used as more accurate values become available. The results are also presented graphically; the incident power is determined on the assumption that the mean solar constant is 1.353×10^6 erg cm^{-2} sec^{-1} and the albedo of the earth is 0.34.

Copies obtainable from NASA, Washington

I. Cunningham, F. G.
II. NASA TN D-1099

(Initial NASA distribution: 7, Astrophysics; 21, Geophysics and geodesy; 26, Materials, other; 35, Power sources, supplementary; 47, Satellites; 52, Structures.)

NASA

NASA TN D-1099
National Aeronautics and Space Administration.
EARTH REFLECTED SOLAR RADIATION INPUT TO SPHERICAL SATELLITES. F. G. Cunningham. October 1961. 9p. OTS price, $0.50. (NASA TECHNICAL NOTE D-1099)

A general calculation is given for the earth's albedo input to a spherical satellite, with the assumption that the earth can be considered a diffusely reflecting sphere. The results are presented in general form so that appropriate values for the solar constant and albedo of the earth can be used as more accurate values become available. The results are also presented graphically; the incident power is determined on the assumption that the mean solar constant is 1.353×10^6 erg cm^{-2} sec^{-1} and the albedo of the earth is 0.34.

Copies obtainable from NASA, Washington

I. Cunningham, F. G.
II. NASA TN D-1099

(Initial NASA distribution: 7, Astrophysics; 21, Geophysics and geodesy; 26, Materials, other; 35, Power sources, supplementary; 47, Satellites; 52, Structures.)

NASA