NASA TN D-1143

NASA TN D-1143

IN-12
398 324

TECHNICAL NOTE

D-1143

ORBITAL PAYLOAD REDUCTIONS RESULTING FROM BOOSTER AND TRAJECTORY MODIFICATIONS FOR RECOVERY OF A LARGE ROCKET BOOSTER

By Alan D. Levin and Edward J. Hopkins

Ames Research Center
Moffett Field, Calif.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON December 1961

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-1143

ORBITAL PAYLOAD REDUCTIONS RESULTING FROM BOOSTER AND TRAJECTORY MODIFICATIONS FOR RECOVERY OF A LARGE ROCKET BOOSTER

By Alan D. Levin and Edward J. Hopkins

SUMMARY

An analysis was made to determine the reduction in payload for a 300 nautical mile orbit resulting from the addition of inert weight, representing recovery gear, to the first-stage booster of a three-stage rocket vehicle. The values of added inert weight investigated ranged from 0 to 18 percent of gross weight at lift off. The study also included the effects on the payload in orbit and the distance from the launch site at burnout and at impact caused by variation in the vertical rise time before the programmed tilt. The vertical rise times investigated ranged from 16.7 to 100 percent of booster burning time.

For a vertical rise of 16.7 percent of booster burning time it was found that a 50-percent increase in the weight of the empty booster resulted in only a 10-percent reduction of the payload in orbit. For no added booster weight, increasing vertical rise time from 16.7 to 100 percent of booster burning time (so that the spent booster would impact in the launch area) reduced the payload by 37 percent. Increasing the vertical rise time from 16.7 to 50 percent of booster burning time resulted in about a 15-percent reduction in the impact distance, and for vertical rise times greater than 50-percent the impact distance decreased rapidly.

INTRODUCTION

Recovery of the first-stage booster[1] may be desirable when there are frequent launchings of rocket vehicles and has been investigated by many authors (e.g., see refs. 1 to 3). Recovery by any technique results in a weight penalty which must be absorbed by the rocket vehicle. The weight of the recovery gear that must be added to recover a booster successfully means a reduction in the orbital payload and may result in unacceptable payload reductions. For booster recovery, it is advantageous to reduce the distance from the launch site to the point of atmosphere entry. One

[1]Hereinafter the term "booster" shall mean the first stage of the multiple-stage rocket vehicle.

method of reducing this distance, though at some loss in payload, involves an increase in the burning time of the booster during vertical ascent before the programmed tilt occurs.

The present investigation was undertaken to determine the reduction of payload that would result from the addition of inert weight to the booster of a large three-stage rocket. A portion of the study was devoted to the determination of the effect of varying the ratio of vertical rise time to booster burning time on the payload in orbit and on the booster distance from the launch site at burnout and at impact. Since the aerodynamic forces are small above about 100,000 feet, the impact distance represents the maximum distance for which a glide capability would have to be provided.

A
5
2
2

NOTATION

A reference area, $\frac{\pi d^2}{4}$, ft^2

A_j nozzle exit area, ft^2

C_D drag coefficient, $\frac{D}{(1/2)\rho V_e^{2} A}$

D drag, lb

d booster diameter, ft

F total external force, $(F_r^2 + F_\lambda^2 + F_\psi^2)^{1/2}$, lb

g acceleration of gravity, ft/sec^2

h altitude above the earth's surface, naut. mi. or ft

h_n altitude at end of n seconds vertical rise, ft

I_{sp} specific impulse, sec

L lift, lb

$\frac{L}{D}$ lift-drag ratio

m instantaneous mass, slugs

m_n vehicle mass after n seconds burning time, $m_o - \dot{m}t_n$, slugs

m_o initial vehicle mass, slugs

$\dot{m}$ — mass flow, slugs/sec

p — ambient pressure, lb/ft^2

p_j — nozzle exhaust pressure, lb/ft^2

Q_k — generalized force in Lagrange equation

q_k — generalized coordinate in Lagrange equation

R — radius of the earth, ft

A522

r — distance from earth's center to mass point, R + h, ft

s — range traveled over earth's surface, ft

T — thrust, lb

T_{KE} — kinetic energy, $slug\text{-}ft^2/sec^2$

t — time, sec

t_n — burning time of n seconds vertical rise, sec

V — absolute velocity, ft/sec

V_e — relative velocity, ft/sec

V_n — velocity at end of n seconds vertical rise, ft/sec

W_a — added inert weight, lb

W_f — fuel weight required to inject payload, lb

W_g — basic gross weight plus added inert weight, lb

W_p — useful payload weight, lb

W_s — structural weight, lb

W_t — vehicle weight at time t, lb

x,y,z — Cartesian coordinates

α_T — thrust angle, measured from the relative velocity vector, positive up, deg

β — heading angle, measured from North, positive East, deg

γ flight-path angle, measured from the local horizontal, positive up, deg

λ longitude, measured from the Greenwich meridian, positive East, deg

μ earth's mass times the universal gravitational constant, ft^3/sec^2

ρ atmospheric density, $slug/ft^3$

ψ latitude, measured from the equator, positive North, deg

ω_e angular velocity of the earth, radians/sec

Subscripts

H local horizontal component

i initial conditions

o sea level

r radial component

λ transverse component (tangent to latitude)

ψ meridian component (tangent to longitude)

METHOD OF ANALYSIS

Trajectory

An altitude 300 nautical miles above the earth's surface was selected as the payload orbit. An IBM 704 digital computer was used for the analysis.

The basic assumptions made in regard to the present study were:

1. A three-dimensional, rotating, spherical earth
2. No atmosphere
3. 100-percent fuel consumption in stages 1 and 2
4. An impulsive injection of the payload into orbit

In addition to the basic assumptions listed above, the assumptions made in regard to the equations of motion are presented in appendix A.

Launch and staging procedure.- The procedure used for the launch and staging sequence throughout the analysis, except for the case of a vertical rise time to booster burnout, was programmed as follows:

1. A launch from latitude 28.48° North, longitude 80.50° West, with a vertical rise for a specified number of seconds

2. A 1.0° impulsive tilt from the vertical in a due east direction

3. A constant thrust-vectoring angle to booster burnout

4. Separation of the booster and second-stage ignition at the time of booster burnout

5. A gravity turn to second-stage burnout

6. Separation of the second-stage and third-stage ignition at the time of second-stage burnout

7. A third-stage gravity turn for 150 seconds

8. A coast to orbital altitude

9. Injection of the payload into orbit (restart of the third-stage engine)

For the case of a vertical rise time to booster burnout, the constant thrust-vectoring program was applied to the second stage. This then omitted steps 3 and 5 as given above.

Vertical ascent program.- The ascent program used throughout the analysis started with a vertical climb for the first n seconds of thrusting. The velocity and altitude of the vehicle at the end of the vertical rise portion of the trajectory were obtained from the equations of motion presented in appendix A. This was done by holding the vehicle in a vertical position throughout the required vertical rise time. This in essence solved the relations given below, taking into account the change of I_{sp} with altitude.

$$V_n = g_o I_{sp} \ \ln \frac{m_o}{m_n} - \int_o^{t_n} g \ dt \tag{1}$$

$$h_n = \frac{g_o I_{sp}}{\dot{m}} \left(\dot{m} t_n - m_n \ \ln \frac{m_o}{m_n} \right) - \int_o^{t_n} gt \ dt \tag{2}$$

At the end of the n seconds vertical rise time, the flight-path angle[2] was decreased impulsively before the constant thrust-vectoring program was initiated. A flight-path angle of 89° was the largest angle that could be used, except for the vertical ascent portion of the trajectory. At the end of the vertical rise, the velocity taken along the flight path was the component of the vertical velocity ($V_n \sin \gamma$). The trajectory was then computed by the equations of motion presented in appendix A.

Parameter Variations

Inert weight, ranging in value from 0 to 18 percent of gross weight at lift off, was added to the weight of the empty booster. The only rocket vehicle characteristics changed were empty booster weight and the gross weight which were increased by the added inert booster weight. To analyze the effect of added booster weight, the ascent trajectory was modified only by the changes required in the thrust-vector angle.[3] This angle change was necessary to insure that the payload would be placed into the 300 nautical mile orbit.

The study also included modifications to the trajectory by varying the vertical rise time of the booster. The vertical rise times investigated were 16.7, 25, 50, 75, and 100 percent of total booster burning time. This phase of the study also included the effects on payload in orbit of added inert booster weight ranging in value from 0 to 18 percent of gross weight at lift off.

A complete booster trajectory for the case of no added inert weight was determined. The range covered from booster burnout to impact was added to the range traveled from lift off to burnout to obtain the total distance from the launch site at impact. The range was obtained from equation (A34).

Drag and Impulsive Injection Analysis

To ascertain the effect of drag on the payload in orbit and on the distance from the launch site, a constant value of $C_D = 0.5$, based on maximum cross-sectional area of the booster, was assumed throughout the entire flight trajectory. The value of C_D was approximately 2-1/2 times the integrated value for typical missiles throughout the range of velocities encountered by the booster, and represents the approximate peak

---

[2]Measured from the local horizontal to the relative velocity vector.

[3]The angle between the thrust direction and the relative velocity vector, positive up.

value of C_D for a missile at transonic speed. Using this value for C_D, it was found that the error in impact distance was about 10.5 percent for a vertical rise of 16.7 percent of total booster burning time, decreasing to approximately 1.5 percent for a vertical rise of 75 percent of booster burning time. The effect of drag resulted in an error of less than 1.5 percent on the distance from the launch site at booster burnout. The effect of drag on the payload in orbit resulted in a maximum error of less than 4 percent, and on this basis it was assumed that the no-atmosphere assumption was justified. However, range data were obtained for both the no-drag and drag conditions.

A In actual practice it is not possible to impulsively inject a payload
5 into orbit. However, it was found that the loss in altitude due to a
2 finite injection time resulted in a maximum error of less than 2 percent
2 in the prescribed orbital altitude. Although to maintain a prescribed altitude during the injection phase there would be a payload loss, the loss was considered to be small since the maximum injection time was less than 3.5 minutes. Therefore, the assumption of an impulsive injection appears justified.

Rocket Vehicle Characteristics

The characteristics of the basic three-stage rocket vehicle used throughout the analysis are presented in table I. The weight values given are for the individual stages and do not include any upper stage weight. Calculated results are presented in dimensionless form since they apply to any vehicle with all weights in the same proportion.

RESULTS AND DISCUSSION

Payload

The effects on the payload in orbit resulting from variations in the vertical rise time of the booster and from the addition of inert weight are presented in figure 1. With no added booster weight, variation of the vertical rise time from 16.7 to 100 percent of booster burning time resulted in a maximum reduction of 37 percent of the payload in orbit. From the figure it can be seen that the rate of change of payload with added inert booster weight decreased slightly with increasing vertical rise time.

A first-order analysis of landing speed indicated that there would be little advantage to adding more weight for wings than that equal to about 50 percent of the empty booster weight ($W_a/(W_g - W_a) \approx 0.04$), since any further addition of weight would produce relatively small changes in the landing speed. The addition of 50 percent of the empty booster weight results in landing speeds ranging between 75 to 140 miles per hour, corresponding to wing weights based on exposed area of 5 to 30 pounds per square foot, respectively. A lift coefficient of 1.0 based on wing area[4] was assumed in the analysis.

Range

The effect of the time of vertical rise on the distance from the launch site at both booster burnout and impact is presented in figure 2. The distances presented are for the booster with no added inert weight. Two curves of distance from the launch site at booster impact are presented, one for no drag and the other for a $C_D = 0.5$ throughout the entire booster trajectory. Since the effect of drag on the burnout distance was negligible, only the zero drag results are presented. It is seen that because the burnout distance is small in comparison with the impact distance, the effect of vertical rise time on the burnout distance is also small. However, increasing the vertical rise from 16.7 to 50 percent of booster burning time resulted in about a 15-percent reduction in impact distance, and for vertical rise times greater than 50 percent of booster burning time, the impact distance decreased rapidly. It may be further noted that for $C_D = 0.5$ the impact distance is about 10 percent lower than in the no-drag case for a vertical rise of 16.7 percent of booster burning time, decreasing to about 1.5 percent lower for a vertical rise of 75 percent of booster burning time.

Shown in figure 3 is the variation of the thrust-vectoring angle required for placing the payload into a 300 nautical mile orbit. The curves shown are for added inert booster weights from 0 to 18 percent of the basic gross weight. These curves are shown only for vertical rise times from 16.7 to 75 percent of booster burning time. The points presented for the vertical rise of 100 percent of booster burning time represent results for which vectoring was applied to the second stage instead of the booster stage. From the figure it can be seen that the required thrust-vector angle becomes excessively large with increasing vertical rise time. Based upon present day rocket configurations which use a maximum nozzle gimbling of about 10°, the vertical rise times would be limited to about 38 percent of booster burning time for the case of no added inert weight and to about 61 percent of booster burning time for an addition of 18 percent of the basic gross weight.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Sept. 6, 1961

---

[4] Wing area included area blanketed by the booster. The plan form assumed for the wing was triangular and had an aspect ratio of 2.0.

APPENDIX A

EQUATIONS OF MOTION

The equations of motion used for the present investigation were programmed on an IBM 704 digital computer. The basic assumptions made in obtaining the equations of motion were:

1. The earth and atmosphere rotate as one body.

A
5
2
2

2. The earth is a homogeneous sphere.

3. The angular velocity of the earth is constant.

4. The gravitational field has an inverse square variation.

5. The atmosphere is that described in reference 4.

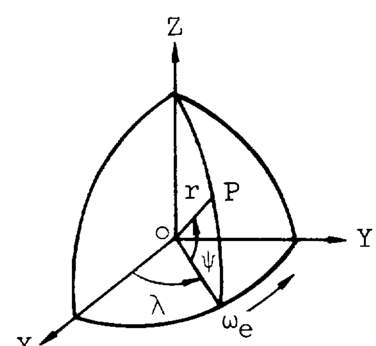
6. The acceleration of the center of the earth is negligible.

DERIVATION OF EQUATIONS

The equations of motion for a point mass, located at a point P, are presented using a spherical coordinate system. The geometry of the coordinate system is presented in sketch (a). In this system ox, oy, and oz are axes fixed in the earth, where oxy is taken to be the equatorial plane; oz the polar axis; ozx the meridian of Greenwich; λ the longitude, positive east of Greenwich; and ψ is the latitude, positive north of the equatorial plane.

The transformation equations from the Cartesian coordinate system to the spherical coordinate system are given by

$$r = (x^2 + y^2 + z^2)^{1/2} \tag{A1}$$



Sketch (a)

$$\psi = \text{arc sin}\,\frac{z}{r} = \text{arc tan}\,\frac{z}{(x^2 + y^2)^{1/2}} \tag{A2}$$

$$\frac{F_\lambda}{m} = \frac{T}{m} \cos(\gamma + \alpha_T) \sin \beta - \frac{1}{2} \rho \frac{C_D A}{m} V_e^2 \cos \gamma \sin \beta \qquad (A26)$$

$$V_e^2 = \dot{r}^2 + r^2\dot{\psi}^2 + r^2\cos^2\psi\dot{\lambda}^2 \qquad (A27)$$

$$D = \frac{1}{2} \rho C_D A V_e^2 \qquad (A28)$$

$$g = \frac{\mu}{r^2} \qquad (A29)$$

The constants used in the above equations are

$$\mu = 1.40775 \times 10^{16} \text{ ft}^3/\text{sec}^2$$

$$\omega_e = 7.2921158 \times 10^{-5} \text{ radians/sec (at the equator)}$$

$$R = 20.926428 \times 10^6 \text{ ft}$$

The variation of thrust with altitude was taken into account according to the relation

$$T = T_o + A_j(p_o - p) \qquad (A30)$$

The amount of fuel required to accelerate the payload to orbital speed was computed on the basis of an impulsive injection, which results in

$$W_f = W_t[1 - \exp(-\Delta V/g_o I_{sp})] \qquad (A31)$$

where

$$\Delta V = \sqrt{\frac{\mu}{r}} - V \qquad (A32)$$

The useful payload in orbit was then determined from the relation

$$W_p = (W_t - W_s) - W_f \qquad (A33)$$

The distance from the launch site was obtained from

$$s = R \text{ arc} \cos[\cos(\psi_i - \psi)\cos(\lambda_i - \lambda)] \qquad (A34)$$

The range given by equation (A34) gives the actual distance from the launch point, that is, it accounts for the movement of the launch point due to the earth's rotation.

REFERENCES

1. Koelle, H. H.: On the Optimum Size of Orbital Carrier Vehicles Based on Overall Economy. Paper presented to 11th International Astronautical Congress of the International Astronautical Federation, Stockholm, Sweden, Aug. 1960.

2. Buschmann, R. P.: Recoverable Boosters - The Next Big Step in Satellite and Space Systems. American Rocket Society Paper No. 1012-59, Nov. 1959.

3. Shipps, Paul R.: On the Methods and Economics of Recovering Boosters for Orbital Vehicles. IAS Paper No. 60-29, Jan. 1960.

4. Minzner, R. A., and Ripley, W. S.: The ARDC Model Atmosphere 1956. Air Force Surveys in Geophysics no. 86, Dec. 1956. (AFCRC TN 56-204)

5. Housner, G. W., and Hudson, D. E.: Applied Mechanics - Dynamics. Vol. 2. Second ed., N. Y., D. Van Nostrand Co., Aug. 1959.

TABLE I.- ROCKET VEHICLE CHARACTERISTICS

	Stage 1	Stage 2	Stage 3
Gross weight, lb	789,500	232,000	60,000
Fuel weight, lb	699,000	219,600	W_f
Payload weight,[1] lb	---	---	53,000 - W_f
Empty weight, lb	90,500	12,400	7,000
Weight flow, lb/sec	5,825	1,200	108
Burning time, sec	120	183	---
Sea-level thrust, lb	1,500,000	---	---
Vacuum thrust, lb	1,700,000	363,000	45,000
Sea level I_{sp}, sec	257.5	---	---
Vacuum I_{sp}, sec	291.8	302.5	416.7
Diameter, d, ft	21.42	13.33	10.00

[1]After applying the impulsive velocity increment required to reach orbital speed, the weight of the remaining fuel was considered to represent useful payload in orbit.

A
5
2
2

A
5
2
2

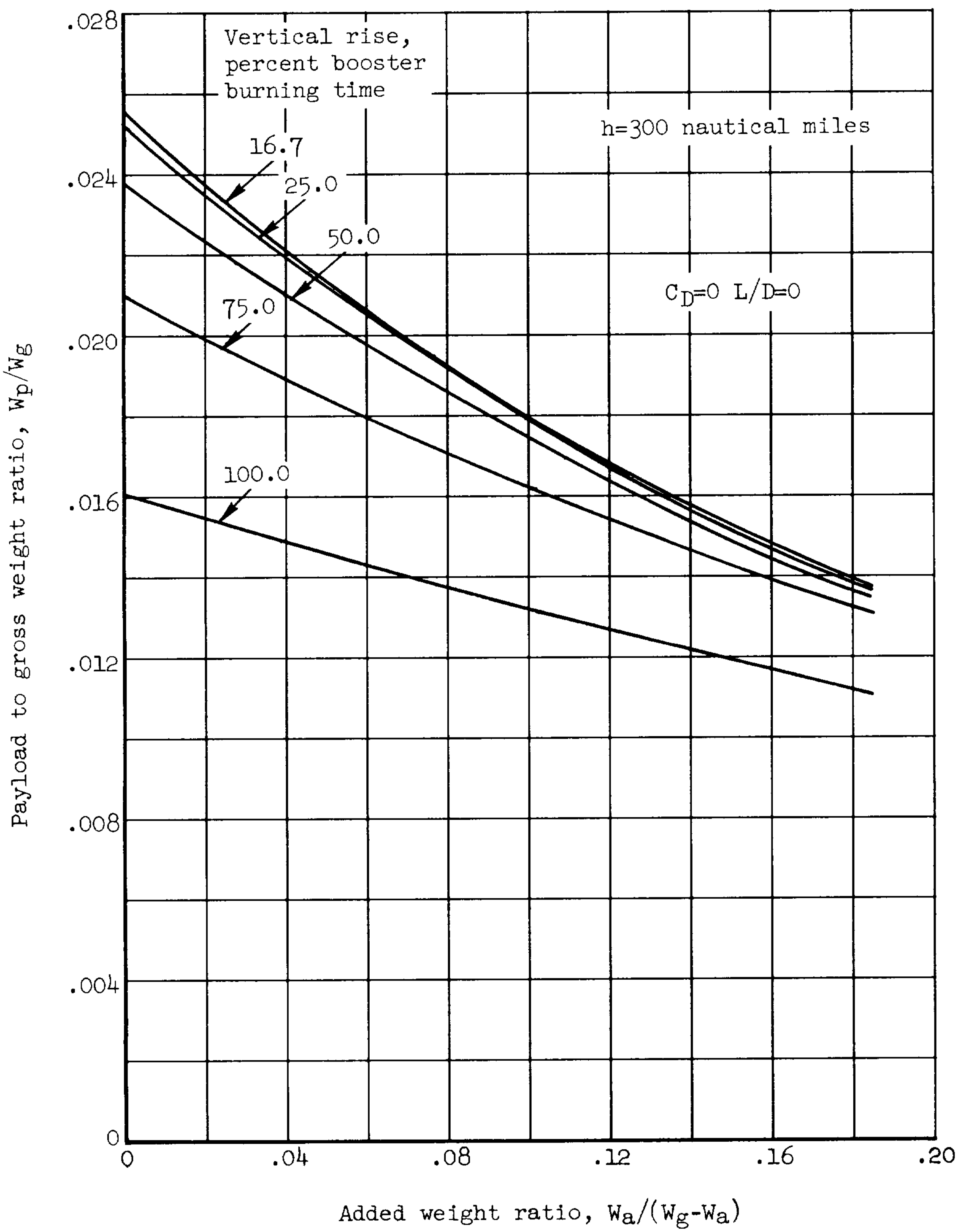


Figure 1.- Effect of vertical rise time and added weight ratio on the payload to gross weight ratio.

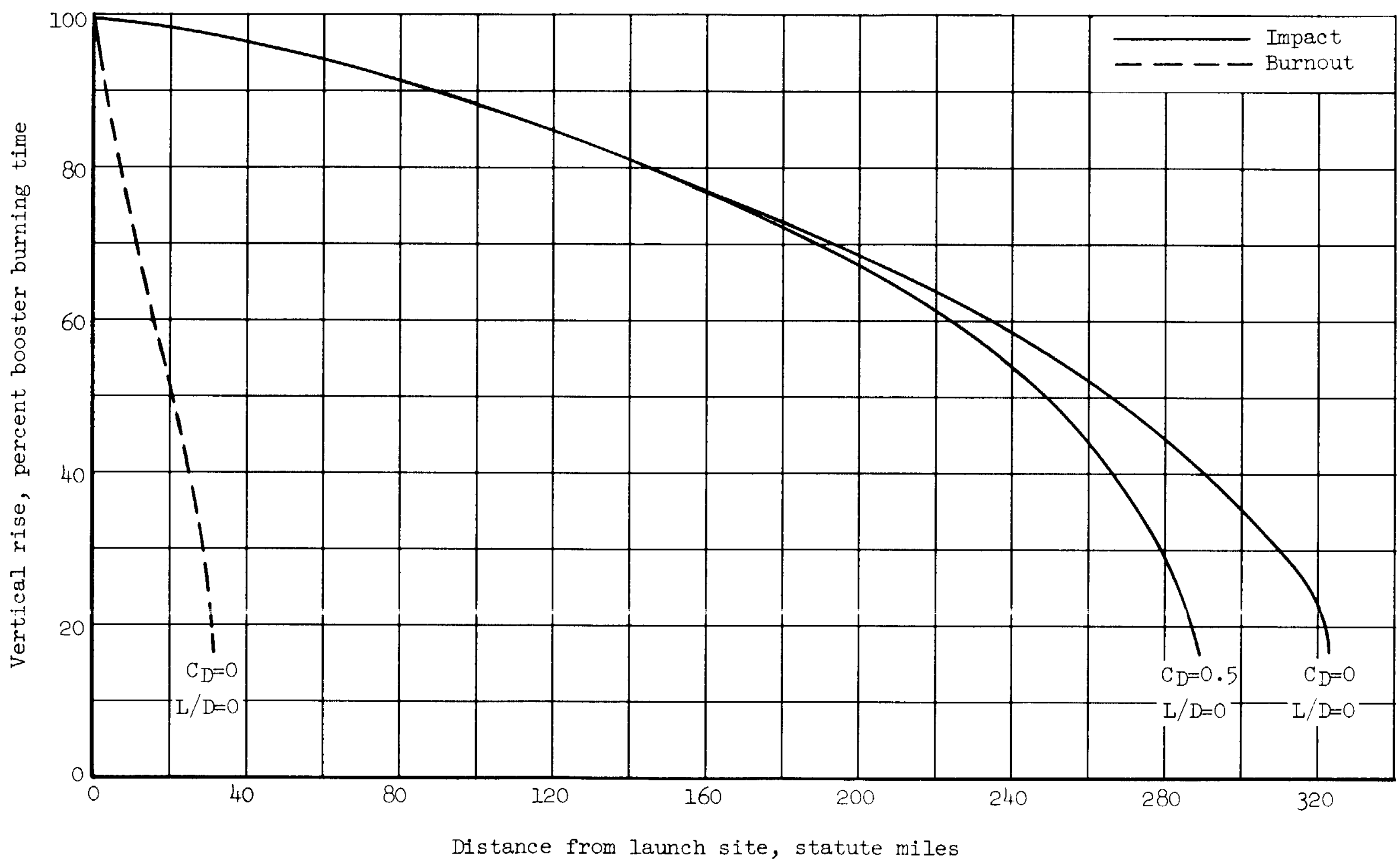


Figure 2.- Effect of vertical rise time on booster distance from the launch site at burnout and at impact of empty booster.

A
5
2
2

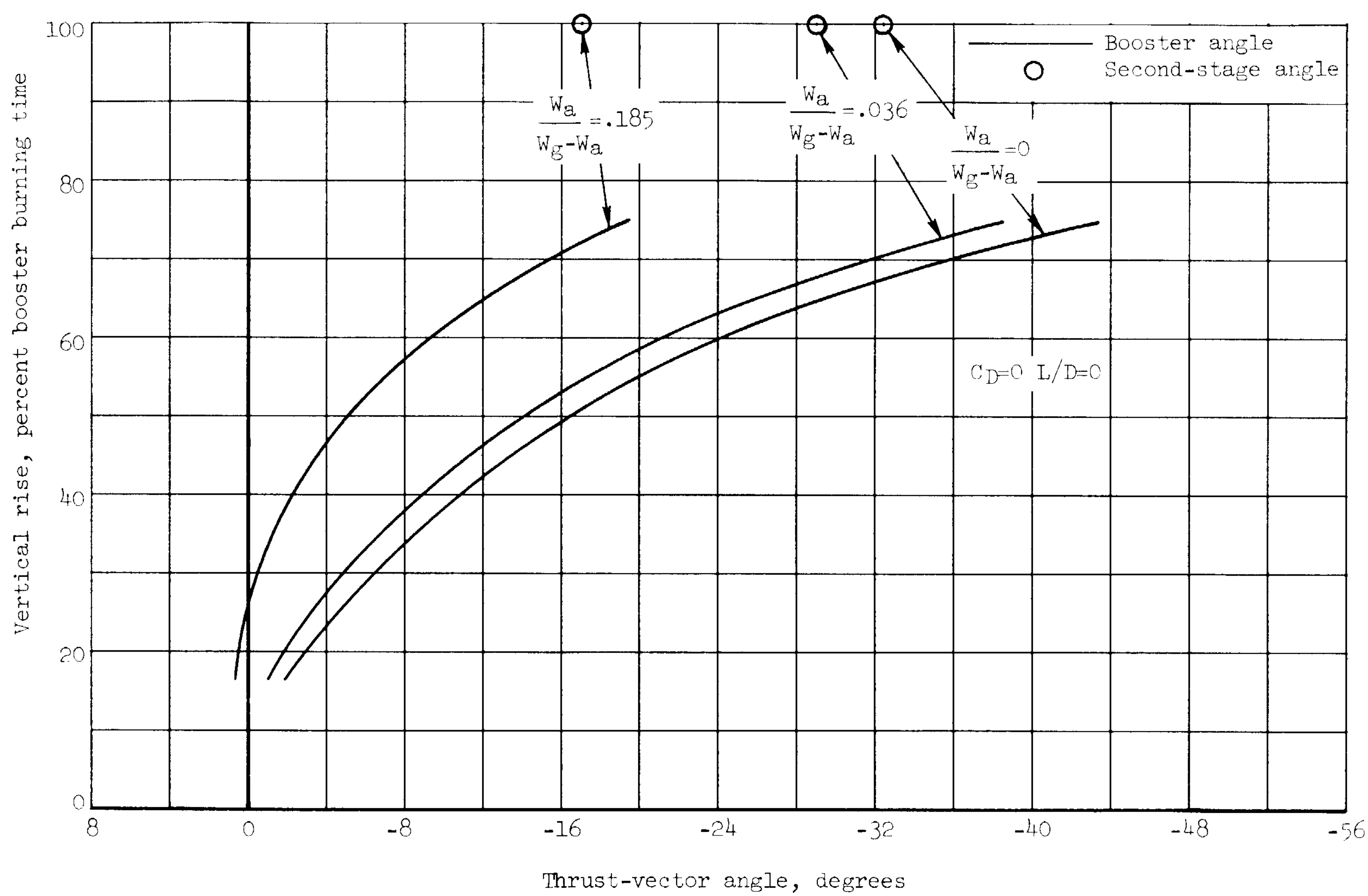


Figure 3.- Variation of required thrust-vector angle for a 300 nautical mile orbit.