



W-18
393 027
1

TECHNICAL NOTE

D-1063

THE EFFECT OF SOLAR RADIATION PRESSURE ON THE MOTION OF AN ARTIFICIAL SATELLITE

by

Robert W. Bryant

Goddard Space Flight Center
Greenbelt, Maryland

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON

September 1961

1. Introduction
2. Methodology
3. Results
4. Discussion
5. Conclusion

**THE EFFECT OF SOLAR RADIATION
PRESSURE ON THE MOTION OF AN
ARTIFICIAL SATELLITE**

by

Robert W. Bryant
Goddard Space Flight Center

SUMMARY

The effects of solar radiation pressure on the motion of an artificial satellite are obtained, including the effects of the intermittent acceleration which results from the eclipsing of the satellite by the earth. Vectorial methods have been utilized to obtain the nonlinear equations describing the motion, and the method of Kryloff-Bogoliuboff has been applied in their solution.



CONTENTS

Summary	i
INTRODUCTION	1
THE KRYLOFF-BOGOLIUBOFF METHOD	1
THE CHANGE IN ENERGY PRODUCED BY RADIATION PRESSURE	2
THE COMPLETE SYSTEM OF EQUATIONS FOR THE OSCULATING ELEMENTS	5
ACKNOWLEDGMENT	8
References	8

D-1063



THE EFFECT OF SOLAR RADIATION PRESSURE ON THE MOTION OF AN ARTIFICIAL SATELLITE

by

Robert W. Bryant
Goddard Space Flight Center

D-1063

INTRODUCTION

In the motion of artificial satellites with large area-to-mass ratios, such as the Echo I balloon satellite, the magnitude of the solar radiation pressure effect is substantial and must be taken into account in the analysis of the tracking data. It is of fundamental importance in any study attempting to determine atmospheric density from satellite drag data to distinguish the energy loss (or gain) produced by radiation pressure from the loss due to drag. Furthermore, the effect is important since the orbital lifetime of a satellite will be substantially affected by the changes in energy resulting from radiation pressure. Several authors have considered the effects of solar radiation pressure on the motion of an artificial satellite (References 1, 2, and 3); it is the purpose of this paper to present these effects, including that of the earth's shadow, in the form of nonlinear equations derived vectorially and suitable for use with electronic computers.

THE KRYLOFF-BOGOLIUBOFF METHOD

H. Lass and J. Lorell (Reference 4) have applied the method of Kryloff-Bogoliuboff (Reference 5) in the following low-thrust cases: constant radial, constant transverse, and intermittent.

The method may be described simply: A nonlinear equation

$$\frac{dx}{dt} = \mu f(x, \sin t),$$

where μ is a small number, becomes, upon integration,

$$\frac{x(t+2\pi) - x(t)}{2\pi} = \frac{\mu}{2\pi} \int_t^{t+2\pi} f(x, \sin \tau) d\tau. \quad (1)$$

Since μ is small, x may be considered constant during the integration from $\tau = t$ to $\tau = t+2\pi$. Since $\sin t$ is periodic, Equation 1 becomes

$$\frac{x(t+2\pi) - x(t)}{2\pi} \approx \frac{\mu}{2\pi} \int_0^{2\pi} f(x, \sin \tau) d\tau = \mu F(x). \quad (2)$$

Replacing the left side of Equation 2 with dx/dt , since the slope of the secant line is approximately that of the tangent, gives

$$\frac{dx}{dt} \approx \frac{\mu}{2\pi} \int_0^{2\pi} f(x, \sin \tau) d\tau = \mu F(x).$$

This result may be obtained simply by averaging the original equation over one cycle of the motion with respect to time.

THE CHANGE IN ENERGY PRODUCED BY RADIATION PRESSURE

The effect of radiation pressure, if the earth's shadow is neglected, will provide only short period terms in the development for the semi-major axis of the orbit (Reference 6). However, the inclusion of the shadow effect results in quite a different phenomenon.

From the energy integral and Brown's δ/dt operator (Reference 6), we obtain

$$\frac{da}{dt} = \frac{2a^2 \mathbf{v} \cdot \mathbf{F}}{\mu}. \quad (3)$$

The velocity vector \mathbf{v} may be represented in terms of a unit vector \mathbf{P} directed from the center of the earth to perigee and a vector \mathbf{Q} at right angles to \mathbf{P} such that $\mathbf{P}, \mathbf{Q}, \mathbf{R}$, with \mathbf{R} the unit vector in the direction of the angular momentum vector, form a right handed system. The vector \mathbf{F} is the disturbing acceleration, a is the semi-major axis, and $\mu = GM$ is the gravitational constant multiplied by the mass of the earth.

We have

$$\mathbf{v} = \frac{\mathbf{Q}\sqrt{\mu a(1-e^2)} \cos E - \mathbf{P}\sqrt{\mu a} \sin E}{r}$$

and

$$\mathbf{F} = -F\mathbf{u}^0,$$

where

e = the eccentricity,

E = the eccentric anomaly,

F = the magnitude of the disturbing acceleration,

\mathbf{u}^0 = the unit vector in the direction of the Sun, and

$r = a(1 - e \cos E)$;

and we have

$$\mathbf{P} = \begin{bmatrix} -\cos i \sin \omega \sin \Omega + \cos \omega \cos \Omega \\ +\cos i \sin \omega \cos \Omega + \cos \omega \sin \Omega \\ +\sin i \sin \omega \end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix} -\cos i \cos \omega \sin \Omega - \sin \omega \cos \Omega \\ +\cos i \cos \omega \cos \Omega - \sin \omega \sin \Omega \\ +\sin i \cos \omega \end{bmatrix},$$

$$\mathbf{R} = \begin{bmatrix} +\sin i \sin \Omega \\ -\sin i \cos \Omega \\ +\cos i \end{bmatrix}.$$

Hence we obtain for Equation 3:

$$\frac{da}{dt} = \frac{-2a^2 F}{r\sqrt{\mu a}} \left(\mathbf{Q} \cdot \mathbf{u}^0 a \sqrt{1-e^2} \cos E - \mathbf{P} \cdot \mathbf{u}^0 a \sin E \right). \quad (4)$$

Considering the slowly varying quantities to be constant during one revolution, and using

$$\frac{d}{dE} = \frac{r}{an} \frac{d}{dt},$$

we obtain

$$\begin{aligned} \Delta a &= - \frac{2a^2 F}{n\sqrt{\mu a}} \left(\mathbf{Q} \cdot \mathbf{u}^0 \sqrt{1-e^2} \int_{E_1}^{E_2} \cos E \, dE - \mathbf{P} \cdot \mathbf{u}^0 \int_{E_1}^{E_2} \sin E \, dE \right) \\ &= - \frac{2a^3 F}{\mu} \left(\mathbf{Q} \cdot \mathbf{u}^0 \sqrt{1-e^2} \sin E + \mathbf{P} \cdot \mathbf{u}^0 \cos E \right)_{E_1}^{E_2}. \end{aligned} \quad (5)$$

The quantities E_1 and E_2 are the eccentric anomaly at the exit and entrance of the shadow, respectively, and are obtained from the equation

$$\mathbf{r} \cdot \mathbf{u}^0 = -\sqrt{r^2 - \rho^2} \quad (6)$$

or, in terms of E ,

$$\mathbf{P} \cdot \mathbf{u}^0 (\cos E - e) + \mathbf{Q} \cdot \mathbf{u}^0 \sqrt{1-e^2} \sin E = -\sqrt{(1-e \cos E)^2 - \left(\frac{\rho}{a}\right)^2}, \quad (7)$$

where ρ is the radius of the earth. The solution of Equation 7 is readily obtained by the iterative technique and the entrance and exit roots are easily identifiable. We may write the expression for Δa as follows:

$$\Delta a = - \frac{4a^3 F}{\mu} \sin \frac{E_2 - E_1}{2} \sqrt{(\mathbf{Q} \cdot \mathbf{u}^0)^2 (1-e^2) + (\mathbf{P} \cdot \mathbf{u}^0)^2} \sin \left(\frac{E_1 + E_2}{2} + \phi \right), \quad (8)$$

where

$$\phi = \arctan \frac{\mathbf{Q} \cdot \mathbf{u}^0 \sqrt{1-e^2}}{-\mathbf{P} \cdot \mathbf{u}^0}.$$

An examination of this expression shows that: (1) if there is no shadowing, $\sin (E_2 - E_1)/2 = 0$, and therefore $\Delta a = 0$; and (2) if $e = 0$, then ϕ becomes $-(E_1 + E_2)/2$ from a consideration of Equation 7, and again $\Delta a = 0$.

THE COMPLETE SYSTEM OF EQUATIONS FOR THE OSCULATING ELEMENTS

The equations for the elements may be obtained (Reference 7) as follows:

$$\frac{da}{dt} = \frac{2a^2 \mathbf{v} \cdot \mathbf{F}}{\mu} ;$$

$$\frac{de}{dt} = \sqrt{\frac{a}{\mu} (1 - e^2)} (T \cos E + N) ;$$

$$\frac{di}{dt} = \frac{rW}{\sqrt{\mu a (1 - e^2)}} \cos (\omega + f) ;$$

$$\sin i \frac{d\Omega}{dt} = \frac{rW}{\sqrt{\mu a (1 - e^2)}} \sin (\omega + f) ;$$

$$\frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt} = \frac{1}{\sqrt{\mu} e} \left(T \sqrt{a} \sin E - \sqrt{a(1 - e^2)} L \right) ;$$

where

$$T = \mathbf{F} \cdot \mathbf{R} \times \mathbf{r}^0 ,$$

$$N = \mathbf{F} \cdot \mathbf{Q} ,$$

$$W = \mathbf{F} \cdot \mathbf{R} ,$$

f = the true anomaly,

$$L = \mathbf{F} \cdot \mathbf{P}, \text{ and}$$

\mathbf{r}^0 = the unit vector in the direction of \mathbf{r} .

The complete procedure will be carried out for only one equation; the others may be obtained in exactly the same way:

$$\frac{de}{dt} = \sqrt{\frac{a}{\mu} (1 - e^2)} (T \cos E + N), \quad (9)$$

where

$$\begin{aligned} T &= -F \mathbf{u}^0 \cdot \mathbf{R} \times \mathbf{r}^0 = \frac{-F \mathbf{u}^0 \cdot \mathbf{R} \times \mathbf{r}}{r} \\ &= \frac{-Fa}{r} \left[\mathbf{Q} \cdot \mathbf{u}^0 (\cos E - e) - \mathbf{P} \cdot \mathbf{u}^0 \sqrt{1 - e^2} \sin E \right], \end{aligned}$$

$$N = -F \mathbf{u}^0 \cdot \mathbf{Q} = -F \mathbf{Q} \cdot \mathbf{u}^0 \frac{a}{r} (1 - e \cos E),$$

$$\mathbf{r} = \mathbf{P}a (\cos E - e) + \mathbf{Q}a \sqrt{1 - e^2} \sin E.$$

Substituting the above values into Equation 9, we have

$$\begin{aligned} \frac{de}{dt} &= -\sqrt{\frac{a}{\mu} (1 - e^2)} F \left[\mathbf{Q} \cdot \mathbf{u}^0 \frac{a}{r} \left(\frac{3}{2} - 2e \cos E + \frac{1}{2} \cos 2E \right) \right. \\ &\quad \left. - \mathbf{P} \cdot \mathbf{u}^0 \frac{a}{r} \sqrt{1 - e^2} \sin E \cos E \right]; \quad (10) \end{aligned}$$

from which

$$\begin{aligned} \frac{e(t+\tau) - e(t)}{\tau} &= -\frac{F}{\tau} \int_t^{t+\tau} \sqrt{\frac{a}{\mu} (1 - e^2)} \left[\mathbf{Q} \cdot \mathbf{u}^0 \frac{a}{r} \left(\frac{3}{2} - 2e \cos E + \frac{1}{2} \cos 2E \right) \right. \\ &\quad \left. - \mathbf{P} \cdot \mathbf{u}^0 \frac{a}{r} \sqrt{1 - e^2} \sin E \cos E \right] dt, \end{aligned}$$

where τ is the period of the satellite.

Since F is small we may consider slowly varying quantities constant during the integration from t to $t + \tau$. By changing the independent variable for the integration and replacing the secant by the tangent on the left side of the above equation we obtain

$$\frac{de}{dt} = -\frac{1}{2\pi} \sqrt{\frac{a}{\mu}} (1-e^2) F \left[\mathbf{Q} \cdot \mathbf{u}^0 \int_{E_1}^{E_2} \left(\frac{3}{2} - 2e \cos E + \frac{1}{2} \cos 2E \right) dE \right. \\ \left. - \mathbf{P} \cdot \mathbf{u}^0 \sqrt{1-e^2} \int_{E_1}^{E_2} \sin E \cos E dE \right]. \quad (11)$$

The limits are E_1 to E_2 since F is zero outside this range. Therefore Equation 11 becomes

$$\frac{de}{dt} = -\frac{1}{2\pi} \sqrt{\frac{a}{\mu}} (1-e^2) F \left[\mathbf{Q} \cdot \mathbf{u}^0 \left(\frac{3}{2} E - 2e \sin E + \frac{1}{4} \sin 2E \right) \right. \\ \left. + \frac{\mathbf{P} \cdot \mathbf{u}^0 \sqrt{1-e^2}}{4} \cos 2E \right]_{E_1}^{E_2}.$$

The complete set of equations for the osculating elements is:

$$\frac{da}{dt} = -\frac{a^2 F}{\pi \sqrt{\mu a}} \left[\mathbf{Q} \cdot \mathbf{u}^0 \sqrt{1-e^2} \sin E + \mathbf{P} \cdot \mathbf{u}^0 \cos E \right]_{E_1}^{E_2};$$

$$\frac{de}{dt} = -\frac{\sqrt{a(1-e^2)}}{2\pi \sqrt{\mu}} F \left[\mathbf{Q} \cdot \mathbf{u}^0 \left(\frac{3E}{2} - 2e \sin E + \frac{1}{4} \sin 2E \right) \right. \\ \left. + \frac{\mathbf{P} \cdot \mathbf{u}^0 \sqrt{1-e^2}}{4} \cos 2E \right]_{E_1}^{E_2};$$

$$\frac{di}{dt} = -\frac{a F \mathbf{R} \cdot \mathbf{u}^0}{2\pi \sqrt{\mu a(1-e^2)}} \left\{ \left[-\frac{3e}{2} E + (1+e^2) \sin E - \frac{e}{4} \sin 2E \right] \cos \omega \right. \\ \left. + \sqrt{1-e^2} \left[\cos E - \frac{e}{4} \cos 2E \right] \sin \omega \right\}_{E_1}^{E_2};$$

$$\sin i \frac{d\Omega}{dt} = -\frac{a F \mathbf{R} \cdot \mathbf{u}^0}{2\pi \sqrt{\mu a(1-e^2)}} \left\{ \left[-\frac{3e}{2} E + (1+e^2) \sin E - \frac{e}{4} \sin 2E \right] \sin \omega \right. \\ \left. - \sqrt{1-e^2} \left[\cos E - \frac{e}{4} \cos 2E \right] \cos \omega \right\}_{E_1}^{E_2};$$

$$\frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt} = -\frac{\sqrt{a} F}{2\pi \sqrt{\mu} e} \left[\mathbf{Q} \cdot \mathbf{u}^0 \left(e \cos E - \frac{1}{4} \cos 2E \right) - \mathbf{P} \cdot \mathbf{u}^0 \sqrt{1-e^2} \left(\frac{3E}{2} - e \sin E - \frac{1}{4} \sin 2E \right) \right]_{E_1}^{E_2}$$

The integration of this system of equations can now be carried out with a large scale computer.

ACKNOWLEDGMENT

The author wishes to express his gratitude to Mrs. Ann Bailie for her work in verifying the developments.

REFERENCES

1. Parkinson, R. W., Jones, H. M., and Shapiro, I. I., "Effects of Solar Radiation Pressure on Earth Satellite Orbits," *Science* 131(3404):920-921, March 25, 1960
2. Musen, P., Bryant, R., and Bailie, A., "Perturbations in Perigee Height of Vanguard I," *Science* 131(3404):935-936, March 25, 1960
3. Musen, P., "The Influence of Solar Radiation Pressure on the Motion of an Artificial Satellite," *J. Geophys. Res.* 65(5):1391-1396, May 1960
4. Lass, H., and Lorell, J., "Low Acceleration Takeoff from a Satellite Orbit," *Amer. Rocket Soc. J.* 31:24-28, January 1961
5. Kryloff, N., and Bogoliuboff, N., "Introduction to Non-Linear Mechanics," (translation) Princeton: Princeton Univ. Press, 1947, Chapters 10-12
6. Musen, P., "Special Perturbations of the Vectorial Elements," *Astron. J.* 59(7):262-267, August 1954
7. Stromgren, B., *Pub. Med. Kobnnavns Obs.* No. 65, 5, 1929