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DETERMINATION OF SATELLITE ORBITS FROM RADAR DATA

W. F. Cahill and I. Harris

Goddard Space Flight Center

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SUMMARY

An optimum method for determining satellite orbits from radar data is presented in this report. Offering a good combination of speed and accuracy, the method makes use of orbit inclination and orbit elements in the plane, and proceeds with a differential correction of the orbit elements.

Rapid, accurate methods of computing orbit elements are required to predict satellite positions for acquisition by other radars at points later along the orbit. In some cases the data are limited to a single pass over the observing station. The dynamical method is described in detail, and its accuracy is compared with those of two other methods: the purely geometrical, and the least-squares geometrical.

By this optimum method the computing time, including the differential correction time, is 1 minute. Without differential correction, the rough determination takes from 10 to 20 seconds with approximately 5 miles positional uncertainty.

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DETERMINATION OF SATELLITE ORBITS FROM RADAR DATA*

INTRODUCTION

An investigation of methods for determining satellite orbits from radar is described in this paper. Particular emphasis is placed on problems that require rapid calculation of the orbit elements from limited data. In some cases the available information may be restricted to the data obtained from a single pass, over a period of 2 to 4 minutes, as the satellite goes by the observing station. The solution to this computing problem requires rather different methods than those previously developed for calculating orbits based on several satellite revolutions.

A procedure which offers a good combination of speed and accuracy is described in some detail in the following pages. This method requires between 10 and 20 seconds for a rough determination, and yields orbit elements in that time with an accuracy of 0.001 in eccentricity and 0.05 minute in period—a degree of accuracy sufficient to determine whether the satellite is in orbit. The program then proceeds automatically with a correction routine that operates on the preliminary orbit elements to yield a more accurate prediction of the satellite position at later points along the orbit. The accuracy of these corrected predictions is 500 yards or better, which is sufficient for acquisition by other radars at later points along the orbit. This correction routine requires an additional 40 seconds of computer time.

PROCEDURE

The procedure chosen as optimum involves the following steps:

(1) Orbit inclination

A plane is passed through the center of the earth and through all the points along the radar track. The inclination of the plane and the position of the line of nodes are adjusted to the full set of data by a least-squares calculation. The plane of the orbit is then determined.

(2) Orbit elements in the plane

After the plane of the orbit has been fixed, the three orbit elements in the plane must be determined. These may be taken as the period, perigee altitude, and argument of perigee. They can be computed from any three items of information along the track. The most accurate way found was to choose two points on the track and the time between

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An IBM-704 time of 1 minute is required for the determination of the orbit elements from one radar pass including twenty data points. By elimination of the differential correction routine, the computing time can be reduced to between 10 and 20 seconds at the sacrifice of a factor of 10 in the accuracy of the orbit determination. In this case, the them as the three fundamental items. The procedure for calculating the orbit elements depends in an essential way on the dynamics of the satellite motion and may be called the "dynamical" method.

Full use is made of the data by dividing the track into halves and pairing the points

DETAILS OF THE ANALYSIS

The x, y, z Cartesian coordinates of the satellite are determined from the slant range, bearing, and elevation. The orbit plane is then determined by a least-squares adjustment of the angle of inclination and the position of the line of nodes at a given time. Appendix A gives the equations used for the reduction of the radar data.

The second step in the procedure consists in the determination of the planar orbit elements, that is, the period, eccentricity, and argument of perigee. The dynamical, or Gauss-Olbers, method has been found to give the most accurate results in this determination.

GAUSS-OLBERS METHOD FOR DETERMINATION OF THE PLANAR ELEMENTS

The equation of an ellipse expressed in terms of the eccentric anomaly is

$$r = a(1 - e \cos E) , \tag{1}$$

and the Keplerian equation is

$$k(t - T)\, a^{-\frac{3}{2}} = E - e \sin E , \tag{2}$$

where a is the mean radius, e the eccentricity, and T the time at which the eccentric anomaly vanishes. The relation between eccentric anomaly and true anomaly is given by

$$\cos v = - \frac{e - \cos E}{1 - e \cos E} ,$$

where v is the true anomaly.

Gauss devised a method for solving these transcendental equations for the orbit parameters in terms of two points on the ellipse and the time difference between them. The method developed by Gauss was extended to other orbit classes by Olbers. Let r_1, β_1 and r_2, β_2 be two positions of the satellite at times t_1 and t_2, respectively; r is the distance from the center of the earth, and β is the angular position of the satellite in the plane of the orbit measured with respect to the line of nodes. In terms of these quantities the Gauss solution gives the following expression for the mean radius:

$$a = \frac{r_1 + r_2 - 2\sqrt{r_1 r_2} \cos g \cos f}{2 \sin^2 g}$$

The eccentricity is given by

In this equation p is the semilatus rectum

$$p = \left(\frac{\eta\, r_1 r_2 \sin 2f}{k(t_2 - t_1)}\right)^2$$

where $f = (\beta_2 - \beta_1)/2$, $k = \sqrt{MG}$, G is the gravitational constant, and M is the mass of the earth. The quantities g and η are calculated as follows:

$$\sin\frac{g}{2} = \left(\frac{m^2}{\eta^2} - \ell\right)^{\frac{1}{2}}$$

where

$$\ell = \frac{r_1 + r_2}{4\sqrt{r_1 r_2}\cos f} - \frac{1}{2},$$

$$m = \frac{k(t_2 - t_1)}{\left(2\sqrt{r_1 r_2}\cos f\right)^{\frac{3}{2}}},$$

and η is the root of the following equation:

$$\eta^3 - \eta^2 - h\eta - \frac{h}{g} = 0 .$$

The quantity h in this equation is given by

$$h = \frac{m^2}{\frac{5}{6} + \ell + \xi},$$

where

$$\xi = \frac{2}{35}x^2 + \frac{52}{1575}x^3$$

with

$$x = \frac{m^2}{\eta^2} - \ell .$$

An iterative method was used to solve these equations.

APPROXIMATION FOR SMALL ECCENTRICITY

If the eccentricity is less than 0.01, it is sometimes convenient to replace the preceding equations by new ones based on the assumption that a circular orbit is a good approximation for the motion. The development can be carried out either by expanding the Gauss-Olbers solution in powers of terms of the order of the eccentricity, or by expanding the original Equations 1 and 2 in the same manner. The results of this development are as follows:

The mean motion is calculated from

$$N = N_0 - \frac{2}{t_2 - t_1} \tan \frac{N_0}{2} \left(1 - \frac{r_2 + r_1}{2a_0}\right),$$

where

$$N_0 = \frac{2f}{t_2 - t_1}$$

and

$$a_0 = \left[\frac{k(t_2 - t_1)}{2f}\right]^{\frac{2}{3}} .$$

The eccentricity is then obtained from

$$e = \frac{r_2 - r_1}{2a} \csc \frac{g_2 - g_1}{2} \left\{1 + \tan^2 \frac{g_2 - g_1}{2} \left[\frac{2(a - r_1)}{r_2 - r_1} - 1\right]^2\right\}^{\frac{1}{2}} ,$$

in which

$$g_2 - g_1 = N(t_2 - t_1).$$

GEOMETRICAL METHOD

In the geometrical method the working equations are much simpler, although they yield less accurate results. The purely geometrical statement that three points and the focus completely determine an ellipse is used here. If x and y are the Cartesian coordinates of the points of the ellipse in the plane of the orbit, then these must satisfy the following equation:

$$\sqrt{(x + 2ae \cos \theta)^2 + (y + 2ae \sin \theta)^2} = 2a - \sqrt{x^2 + y^2} ,$$

which constitutes a statement of the basic property of the ellipse. Here

$$\sqrt{x^2 + y^2} + bx + dy - \bar{R} = 0 , \tag{3}$$

with

$$b = e \cos \theta, \quad d = e \sin \theta, \quad \bar{R} = a(1 - e^2) ,$$

where e is the eccentricity, θ the argument of perigee, and a the mean radius.

Equation 3 is linear in the three unknowns b, d, and $\bar{R}$, and may be solved directly if three points (x, y) are given by the radar observations. If there is a redundancy of data points, the unknowns may be fitted by least squares or by direct averages.

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Appendix A

REDUCTION OF RADAR DATA

Let S_i, E_i, B_i denote slant range, elevation, and bearing at time t_i. Let R_0 be the distance from the center of the earth to the radar station of latitude φ_0 and longitude λ_0. Then, the distance of the satellite from the center of the earth is

$$r_i^2 = S_i^2 + R_0^2 + 2R_0S_i \sin E_i .$$

The latitude φ_i and longitude λ_i of the satellite are computed by means of

$$\sin \varphi_i = \sin \varphi_0 \cos Q_i + \cos \varphi_0 \sin Q_i \cos B_i ,$$

$$\frac{\sin (\lambda_i - \lambda_0)}{\sin Q_i} = \frac{\sin B_i}{\cos \varphi_i} .$$

Next, transfer to a set of ordinary spherical coordinates, correcting for the earth's rotation:

$$\lambda_i' = \lambda_i + \omega_e(t_i - t_k) ,$$

$$\varphi_i' = \frac{\pi}{2} - \varphi_i .$$

Here ω_e is the angular velocity of the earth, and t_k is the time of the particular observation at which the line of nodes is computed. Then the Cartesian coordinates in the fixed frame are given by

$$X_i = r_i \sin \varphi_i' \cos \lambda_i' ,$$

$$Y_i = r_i \sin \varphi_i' \sin \lambda_i' ,$$

$$Z_i = r_i \cos \varphi_i' ,$$

where the X-axis passes through the Greenwich meridian at time t_k and the Z-axis is the polar axis. The plane of the orbit is found by a least-squares fitting.

Next, coordinates in the plane of the orbit are calculated by means of

$$\overline{X}_i = r_i \cos \beta_i ,$$

$$\overline{Y} = r_i \sin \beta_i ,$$

where $\sin \beta_i = \frac{\cos \varphi_i'}{\sin \alpha}$ and α is the inclination angle.

The planar elements of the ellipse are now computed by using the Cartesian coordinates $\overline{X}_i$, $\overline{Y}_i$ for the geometrical method and the polar coordinates r_i, β_i for the dynamical method.

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