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TECHNICAL NOTE

D-959

MEASUREMENT OF FLOW ANGULARITY AT

SUPERSONIC AND HYPERSONIC SPEEDS WITH THE

USE OF A CONICAL PROBE

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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TECHNICAL NOTE D-959

MEASUREMENT OF FLOW ANGULARITY AT

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SUMMARY

The characteristics of a 40° half-angle cone for measuring flow angularity were determined experimentally. Tests were conducted at a Mach number of 21 in helium with check points at Mach number 3.55 in air for angles of pitch up to 5°. The air tests confirmed theoretical indications of small or negligible Mach number and test-medium effects for the case of air and helium. The instrument is capable of measuring flow angularity at high Reynolds numbers and speeds greater than that necessary for shock attachment to within $\pm 1/3^\circ$.

INTRODUCTION

An instrument capable of measuring flow angularity is of considerable value for both flight and wind-tunnel applications. The uses in flight testing are of course obvious. In the wind tunnel, the exact flow conditions existing in the test section must be known in order to evaluate and give validity to the data. This knowledge is becoming more important in hypersonic tunnels since many of them use conical nozzles which produce flows of higher angularity than contoured nozzles. Many devices have been employed to determine flow angularity. The simplest were various types of vanes which would weathercock into the wind and thus show direction. However, the difficulty of supporting the vane, measuring the angles, and minimizing bearing friction left much to be desired. The method chosen for this investigation is to determine flow angularity by measuring the difference between the pressure coefficients of diametrically opposed orifices. The orifices may be located on the surface of one of several geometric configurations - hemisphere, cone, or power body. In this investigation a cone with four equally spaced static-pressure orifices around the surface was used. The early investigations of conical flow angularity probes were restricted to subsonic or low supersonic Mach numbers and, in most cases, to small flow angles.

The range was increased in reference 1 to large flow angles at Mach numbers of 1.72, 1.95, and 2.46. In this report the speed range was extended to include hypersonic Mach numbers. In addition, the effects of both Mach number and the ratio of specific heats on the calibration curves were investigated.

SYMBOLS

C_p surface pressure coefficient, $\dfrac{p_s - p_\infty}{q_\infty}$

$\dfrac{\Delta p}{q_\infty}$ difference in pressure coefficient between diametrically opposed orifices

$\left(\dfrac{\Delta p}{q_\infty}\right)_\epsilon$ difference in pressure coefficient between orifices 2 and 4 (fig. 2), $\dfrac{p_{s,2} - p_{s,4}}{q_\infty}$

$\left(\dfrac{\Delta p}{q_\infty}\right)_\sigma$ difference in pressure coefficient between orifices 1 and 3 (fig. 2), $\dfrac{p_{s,1} - p_{s,3}}{q_\infty}$

$$K = \left(\frac{\gamma + 1}{\gamma - 1}\right)\left(\frac{1}{\gamma M_\infty^2 \sin^2 \tau}\right)$$

$m = \tan \tau$

M_∞ Mach number

$\bar{n}$ unit normal to local surface

$\bar{i}, \bar{j}, \bar{k}$ unit vectors along X, Y, and Z coordinate axes, respectively

p_s static pressure on cone surface

p_∞ free-stream static pressure

q_∞ free-stream dynamic pressure

R   Reynolds number

u,v,w   velocities in X, Y, and Z directions (fig. 2)

V   resultant velocity

$\overline{V}$ unit vector in direction of resultant velocity

X,Y,Z   Cartesian coordinates of body axes

α angle of attack (fig. 2), deg

β angle of sideslip (fig. 2), deg

γ ratio of specific heats

δ angle between free stream and local tangent to body surface,
             deg

ϵ angle of downwash (fig. 2), deg

θ angle of pitch of cone axis (fig. 2), deg

$\lambda = \dfrac{\gamma - 1}{\gamma + 1}$

$\xi = \dfrac{\sin \theta}{\sin \tau}$

σ angle of sidewash (fig. 2), deg

τ cone half-angle, deg

ϕ angle of roll (fig. 2), deg

ϕ_{ϵ} angle of roll for $\left(\dfrac{\Delta p}{q_{\infty}}\right)_{\epsilon}$, deg

ϕ_{σ} angle of roll for $\left(\dfrac{\Delta p}{q_{\infty}}\right)_{\sigma}$, deg

Subscripts:

1,2,3,4   position of orifices on cone surface (fig. 2)

∞ free-stream conditions

MODEL AND APPARATUS

Wind Tunnel

The tests were conducted in a 3-inch-diameter helium blowdown tunnel at Langley Research Center. The calibration curve for the tunnel and a diagram of it are presented in reference 2.

Model and Starting Device

The model was a cone-cylinder combination with an 80° included angle cone. Four orifices, 0.020 inch in diameter, were spaced equally around the model in a plane 0.071 inch from the nose. The model was sting mounted in the tunnel and held at each roll position by means of two set screws located at the attachment point of the model and sting. A simple screw mechanism acted as the model-support strut and permitted pitch and translation of the model in a vertical plane from outside the test section. Details of the model and support system are shown in figure 1.

A spike was necessary to start the flow and was made from two spring-steel rods 0.040 inch in diameter. The rods met 2 inches in front of the model and formed a 20° angle. They were attached to a cylindrical sleeve which was free to slide on the model and sting. Two wires leading out of the tunnel were fastened to the sleeve and thus permitted extension or retraction of the starting device. As the device was retracted, the rods separated and rested against the sting.

An included angle of 80° was chosen as a compromise between the following considerations:

1. A cone with a large included angle has a greater pressure difference across two diametrically opposed orifices at given flow angles than a slender cone and thus is more sensitive (ref. 1).

2. A large-angle cone would avoid the increased viscous effects present at hypersonic speeds on slender cones.

3. Unpublished data on a series of cone models showed that the pressure coefficient on a 40° half-angle cone was constant and invariant within the Reynolds numbers range of the tests. The tests referred to were conducted over a Mach number range of 20.60 to 21.44, and the corresponding Reynolds numbers were 194,300 and 330,000 based on body diameter. The results of these tests were used as a basis for selection of orifice locations on the probe model.

Pressure Measurement

The difference in pressures between diametrically opposed orifices was measured by 1 lb/sq in. inductance-type differential gages. The differential gages were used instead of individual gages for each orifice because of the increased accuracy they offered. In order to prevent overloading, the circuit was designed so that the gages would only be open to the model during the period between the starting and stopping shock loads. This was accomplished by four solenoid valves which were operated simultaneously by a single switch. These valves were placed in each of the orifice leads between the model and gage.

TESTS

The tests were made at a Mach number of 21 and a Reynolds number of 251,000 based on body diameter. In each case the stagnation pressure was 2,015 lb/sq in. abs and the stagnation temperature was approximately 90° F. The model was pitched in 1° increments until 5° had been attained. At each pitch angle the model was rolled through 90° in 10° increments. The orifices on the probe were located as close as possible to the nose and the nose was always kept on the tunnel center line. The flow angle notation is shown in figure 2.

The estimated experimental accuracy is given in the following table:

θ, deg . . . . ±0.10
ϕ, deg . . . . ±0.5
$\left(\dfrac{\Delta p}{q_\infty}\right)$, percent . . . . ±2

The precision with which flow angles can be determined by use of the methods of this report is as follows:

	At $M_\infty = 21$	At other Mach numbers
α, deg	±0.25	±0.33
σ, deg	±0.25	±0.33

RESULTS AND DISCUSSION

Probe Calibration at $M_\infty = 21$ in Helium

Figure 3 shows typical plots of the difference in pressure coefficient between opposed orifices plotted against the angle of pitch for

constant roll angles. Note that all of the curves do not pass through the origin. By first rolling the model 180° with respect to the tunnel and then rolling the tunnel 180° with respect to the model it was determined that the main cause was support misalinement. All the data are shown in figure 4 after shifting each curve through the origin to correct for this misalinement. Because of model symmetry, the data for each set of opposed orifices should agree for complementary roll angles at the same angle of attack. For example, $\left(\dfrac{\Delta p}{q_\infty}\right)_\epsilon$ at $\theta = 1°$ and $\phi = 30°$ should be the same as $\left(\dfrac{\Delta p}{q_\infty}\right)_\sigma$ at $\theta = 1°$ and $\phi = 60°$. By plotting $\dfrac{\Delta p}{q_\infty}$ on the ordinate and having two abscissa scales for ϕ such that they are complementary, another check on the data may be made. Such a plot is shown in figure 5. Inspection shows that the data at the higher pressure do not agree and the disagreement increases with angle of pitch. This discrepancy was caused by orifice 3 which was located $1\frac{1}{2}°$ away from its proper position. This mislocation caused the pressure difference between orifices 1 and 3 to be less than the true value. It is interesting to note that the mislocation only affects a small part of the data. Such a result is to be expected since the pressure on a cone at angle of attack is most sensitive to roll angle changes in the region giving maximum pressure.

The surface pressure coefficient as given in reference 3 to a second-order approximation is

$$
C_p = 2\sin^2\tau\left\{1 + \frac{\lambda}{4} - 2\xi\cos\tau\cos\phi + \frac{5}{4}\lambda K + \lambda^2\left(\frac{3}{32} + \frac{\tan^2\tau}{4}\right)\right.
$$
$$
\left. + \xi^2\left[\cos 2\tau - \sin^2\phi\left(\cos^2\tau + \frac{1}{4}\right)\right] + \lambda\xi\frac{\cos\phi}{\cos\tau}\left(\frac{\sin^2\tau}{2} - \frac{4}{15}\right)\right\} - \frac{p_\infty}{q_\infty} \qquad (1)
$$

The difference in pressure coefficient for diametrically opposed orifices is

$$
\Delta C_p = 2\sin^2\tau\left[4\xi\cos\tau\cos\phi - \frac{2\lambda\xi\cos\phi}{\cos\tau}\left(\frac{\sin^2\tau}{2} - \frac{4}{15}\right)\right] \qquad (2)
$$

In this equation, ϕ denotes the meridian angle of either one of the two orifices. Note that in deriving the above expression the limit $K \to 0$ (hypersonic speeds) was not needed to obtain the result shown. The terms containing K as a factor cancel each other and give an expression which

is independent of Mach number and should apply at supersonic as well as hypersonic speeds.

In order to apply Newtonian theory, the angle between the free stream and local surface tangent at combined angle of attack and roll must be determined. For the coordinate system shown in figure 2, the equation for a cone can be written as

$$X^2 = \frac{1}{m^2}\left(Y^2 + Z^2\right) \tag{3}$$

The local unit normal is

$$\bar{n} = \frac{m^2 X\bar{i} - Y\bar{j} - Z\bar{k}}{\sqrt{m^4 X^2 + Y^2 + Z^2}} \tag{4}$$

or

$$\bar{n} = \sin \tau \bar{i} - \cos \tau \sin \phi \bar{j} - \cos \tau \cos \phi \bar{k} \tag{5}$$

The free-stream unit vector is

$$\bar{V}_\infty = \cos \theta \bar{i} + \sin \theta \bar{k} \tag{6}$$

By taking the dot product of the unit normal and free-stream unit vector, the angle is found to be

$$\sin \delta = \sin \tau \cos \theta - \cos \phi \cos \tau \sin \theta \tag{7}$$

The pressure coefficient according to Newtonian theory is

$$C_p = 2 \sin^2 \delta \tag{8}$$

and the pressure-difference coefficient for two diametrically opposed orifices is

$$\Delta C_p = \frac{p_{s,2} - p_{s,4}}{q_\infty} = 8 \sin \tau \cos \tau \sin \theta \cos \theta \cos \phi \tag{9}$$

A comparison of the corrected data with values calculated with the use of Newtonian theory and the theory of reference 3 showed excellent agreement at small yaw (or pitch) angles and an error increasing with pitch angle for each theory. Cheng's theory always gave the best agreement and in most cases underpredicted by less than 2 percent (which is within the experimental accuracy) with a few instances of 13-percent error. Newtonian theory is not quite as good, usually giving an error of 3 percent at $\theta = 5^\circ$ and occasionally an error of 15 percent. Typical plots illustrating the best and worst cases are shown in figure 6.

Effects of Mach Number and Test Medium

In reference 1 it was concluded that Mach number had only a small or negligible effect on the difference in pressure coefficient between diametrically opposed orifices and, consequently, on flow angularity. For the tests conducted, Newtonian theory and the theory of Cheng can both be shown to substantiate that conclusion and further predict a near independence of the test medium. Newtonian theory is, of course, independent of γ and Cheng's theory shows only a slight variation between γ of 1.4 and 1.67 at small angles. (See fig. 7.) Excellent agreement of these theories with hypersonic helium data has already been shown in figure 6 and a comparison with the supersonic air data of reference 1 shows good agreement. (See fig. 8.) Note that Cheng's theory coincides with Newtonian in some cases. This might be expected since the limit $\gamma \to 1$ applied to Cheng's expression for ΔC_p gives

$$\Delta C_p = 8 \sin \tau \cos \tau \sin \theta \cos \phi \tag{10}$$

which is essentially the same expression given by Newtonian theory (eq. 9) since at small angles of yaw, $\cos \theta \approx 1$. Therefore, the indications are that at higher Reynolds numbers and speeds greater than that necessary for shock attachment the difference in pressure coefficient is nearly independent of Mach number and test medium for the case of air and helium. As a check, tests were conducted at $M_\infty = 3.55$ and a Reynolds number of 356,000 in air in the Langley Mach 3.5 blowdown jet. The results showed good agreement with hypersonic helium data. (See fig. 9.)

Flow Angle Determination

In order to aid in the determination of θ and ϕ from measurements of $\left(\dfrac{\Delta p}{q_\infty}\right)_\sigma$ and $\left(\dfrac{\Delta p}{q_\infty}\right)_\epsilon$ the results of figures 4(a) and 5(b) have been combined in figure 10 to give a plot of $\left(\dfrac{\Delta p}{q_\infty}\right)_\sigma$ against $\left(\dfrac{\Delta p}{q_\infty}\right)_\epsilon$ for positive values. The flow inclination in terms of σ and α can be calculated by use of the following relations:

$$\left.\begin{aligned} \tan \alpha &= \tan \theta \, \cos \phi \\[2mm] \tan \sigma &= \tan \theta \, \sin \phi \end{aligned}\right\} \tag{11}$$

For convenience in obtaining the quantities directly, curves are presented in figure 11 for positive values from which these angles can be determined without recourse to the equations. The sign conventions for quadrants other than that shown are indicated in the figure.

CONCLUDING REMARKS

The experimental data for the 40° half-angle cone which showed good agreement at a Mach number of 21 in helium and a Mach number of 3.55 in air confirmed the theoretical indications. Thus, only one calibration curve is required to determine flow angles to within $\pm 1/3^\circ$ over a wide Mach number range in either air or helium.

Langley Research Center,
    National Aeronautics and Space Administration,
        Langley Field, Va., July 11, 1961.

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2. Johnston, Patrick J., and Witcofski, Robert D.: Effect of a Variable-Geometry Diffuser on the Operating Characteristics of a Helium Tunnel Designed for a Mach Number in Excess of 20. NASA TN D-237, 1960.

3. Cheng, Hsien K.: Hypersonic Shock-Layer Theory of a Yawed Cone and Other Three-Dimensional Pointed Bodies. WADC TN 59-335, U.S. Air Force, Oct. 1959; Errata, June 1960.

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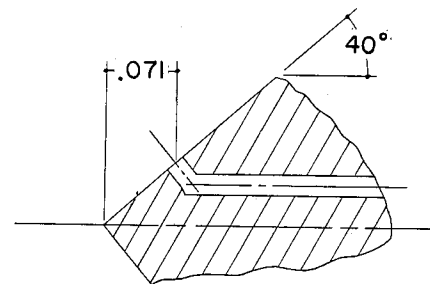
DETAIL A
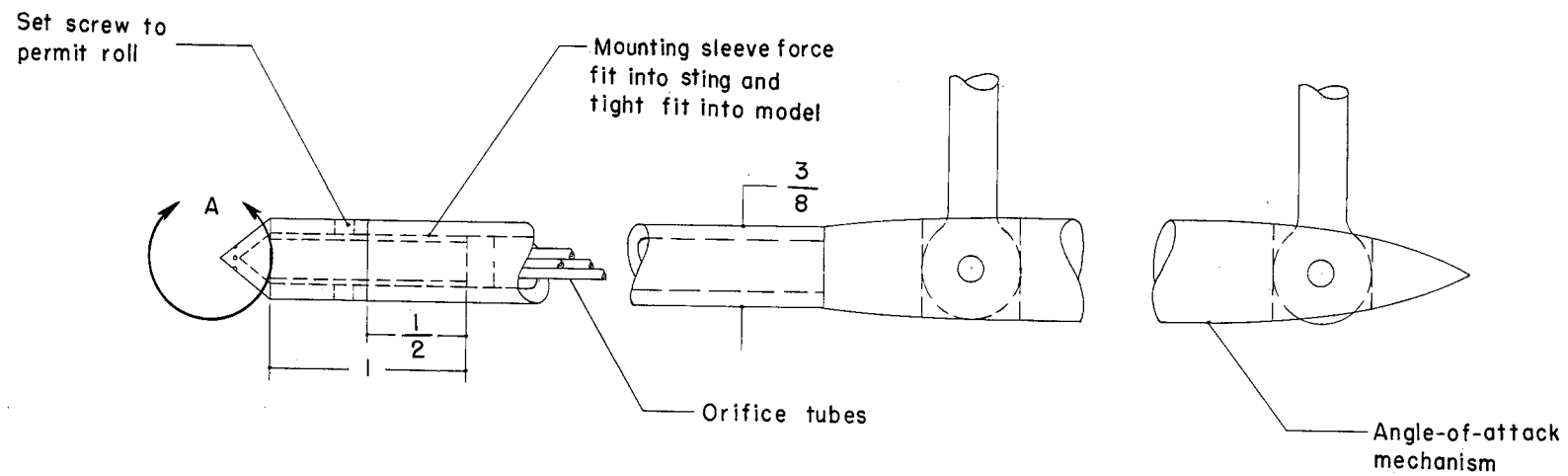
Figure 1.- Model and support. All dimensions are in inches.
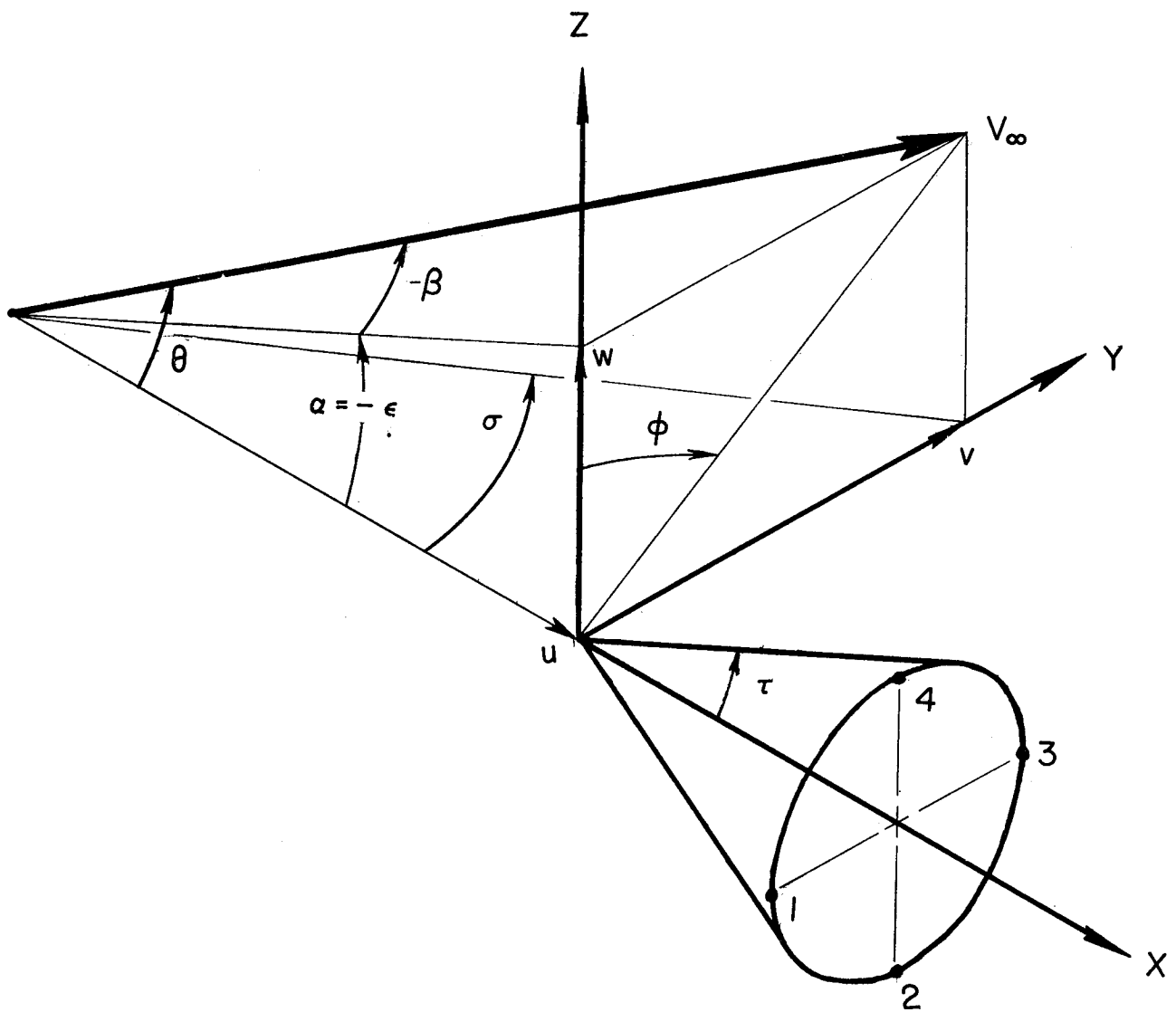
Figure 2.- Flow angle notation.
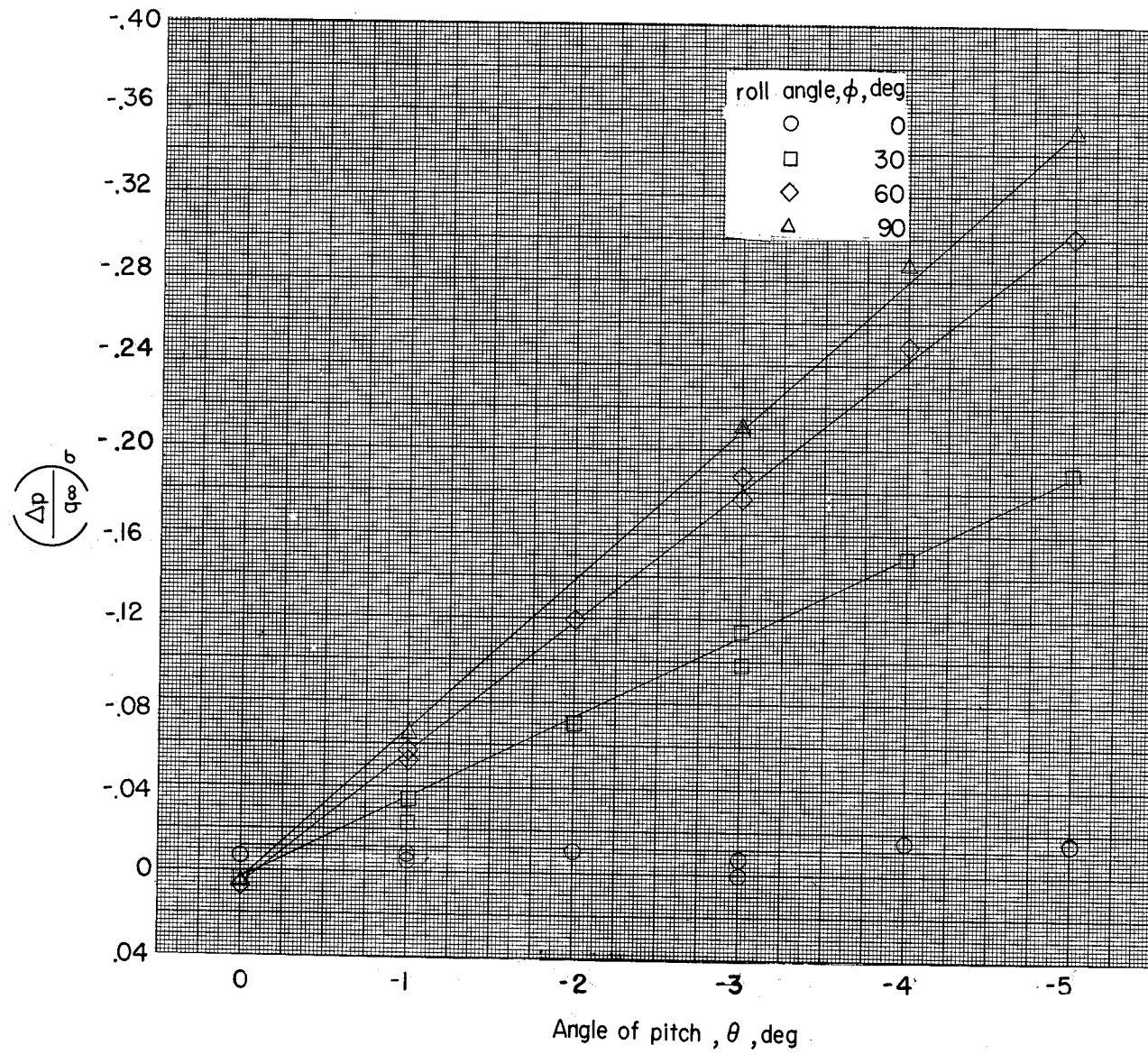
Figure 3.- Variation of difference in pressure coefficient with angle of pitch.
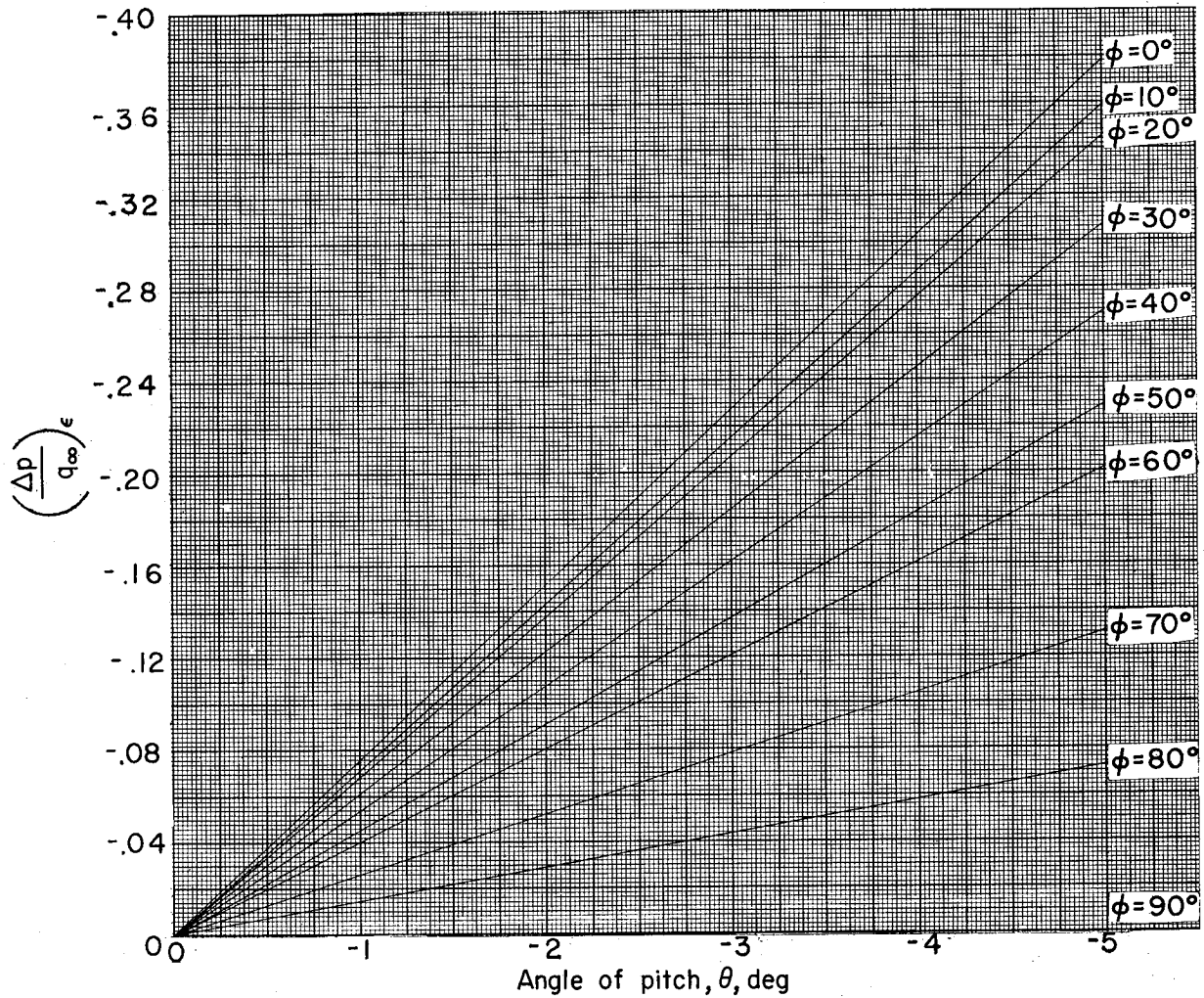
(a) $\left(\dfrac{\Delta p}{q_\infty}\right)_\epsilon$.

Figure 4.- Variation of difference in pressure coefficient with angle
of pitch (corrected data).
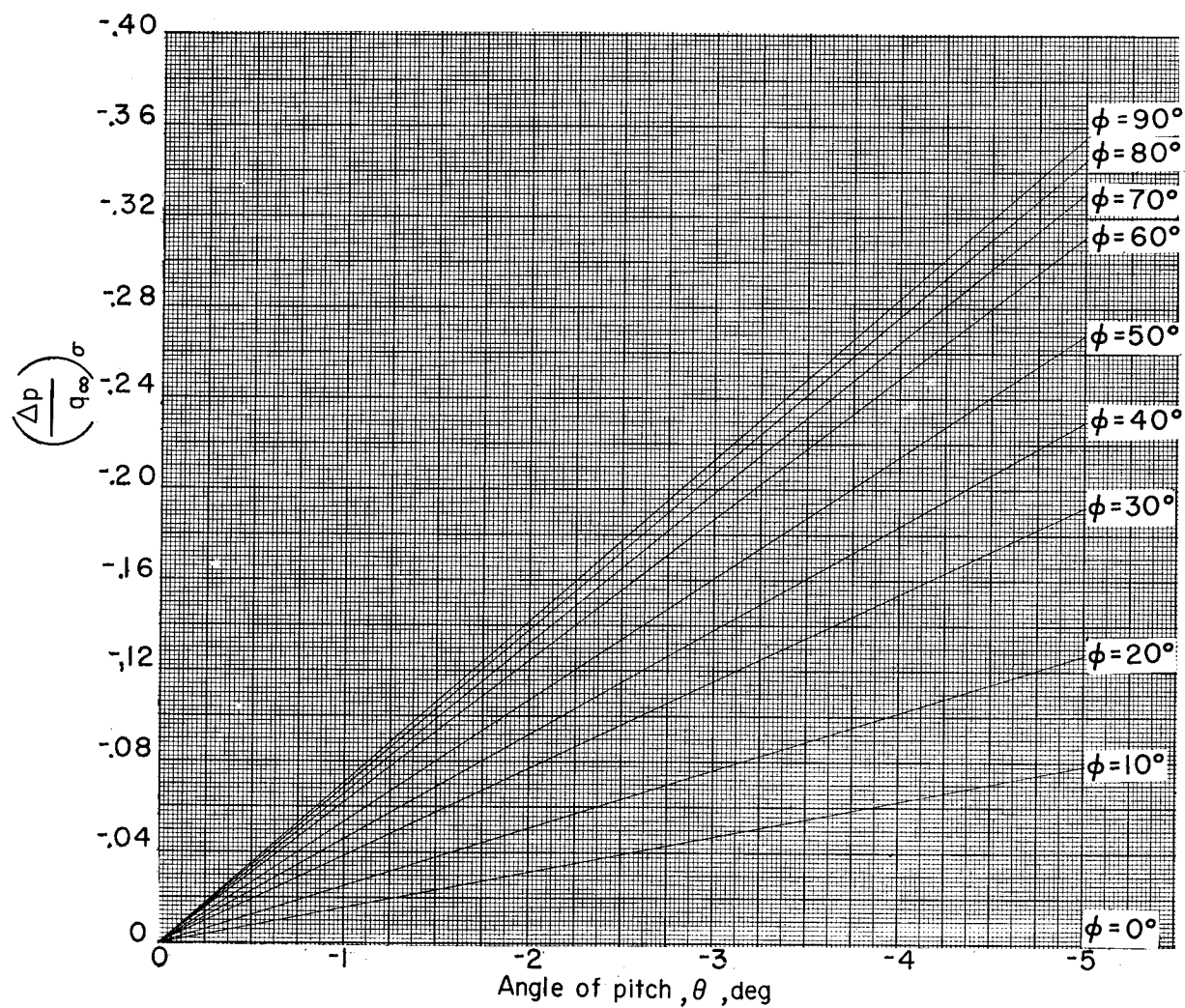
(b) $\left(\dfrac{\Delta p}{q_\infty}\right)_\sigma$.
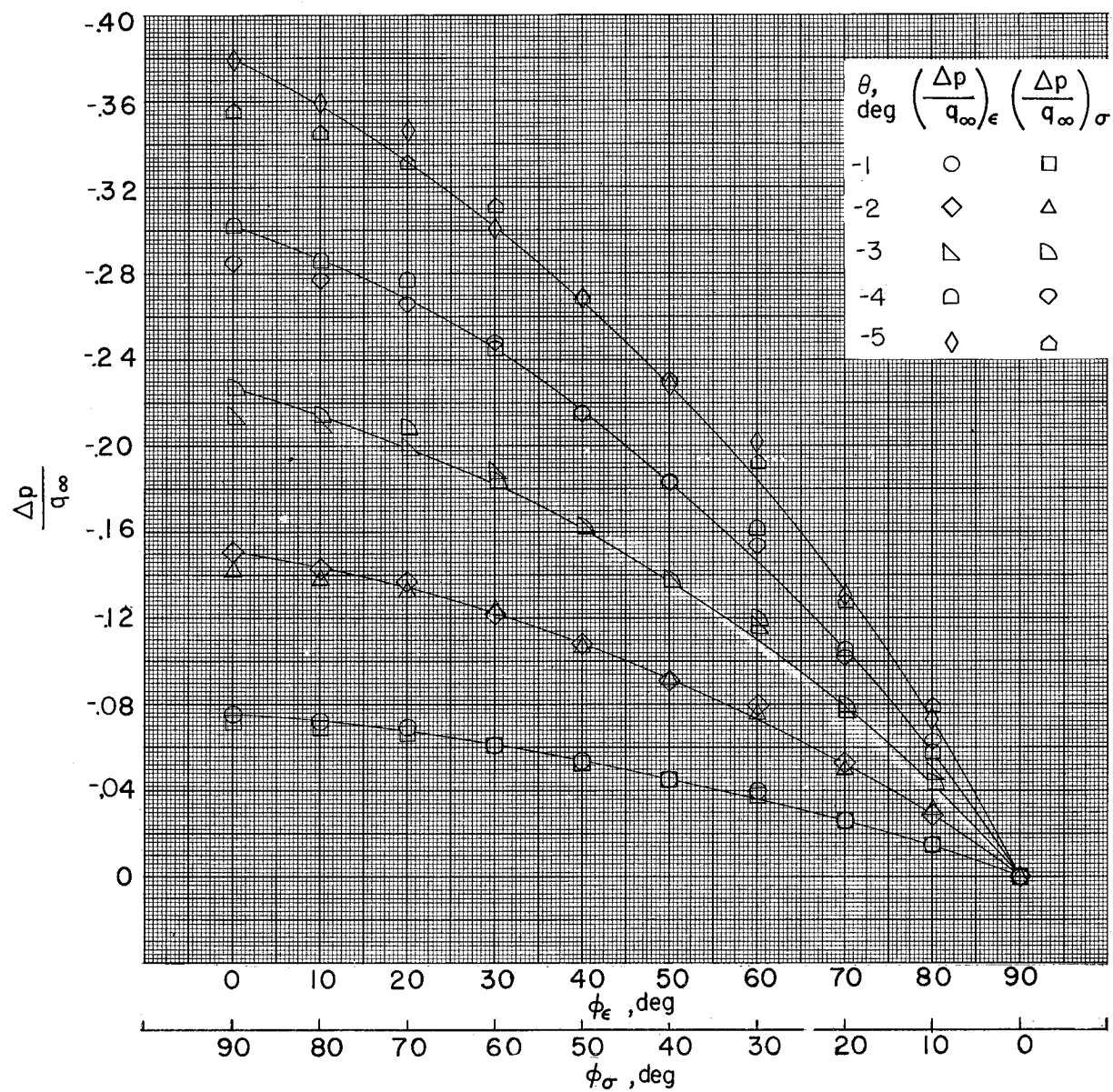
Figure 4.- Concluded.

Figure 5.- Variation of static-pressure difference coefficient
with angle of roll (corrected data).
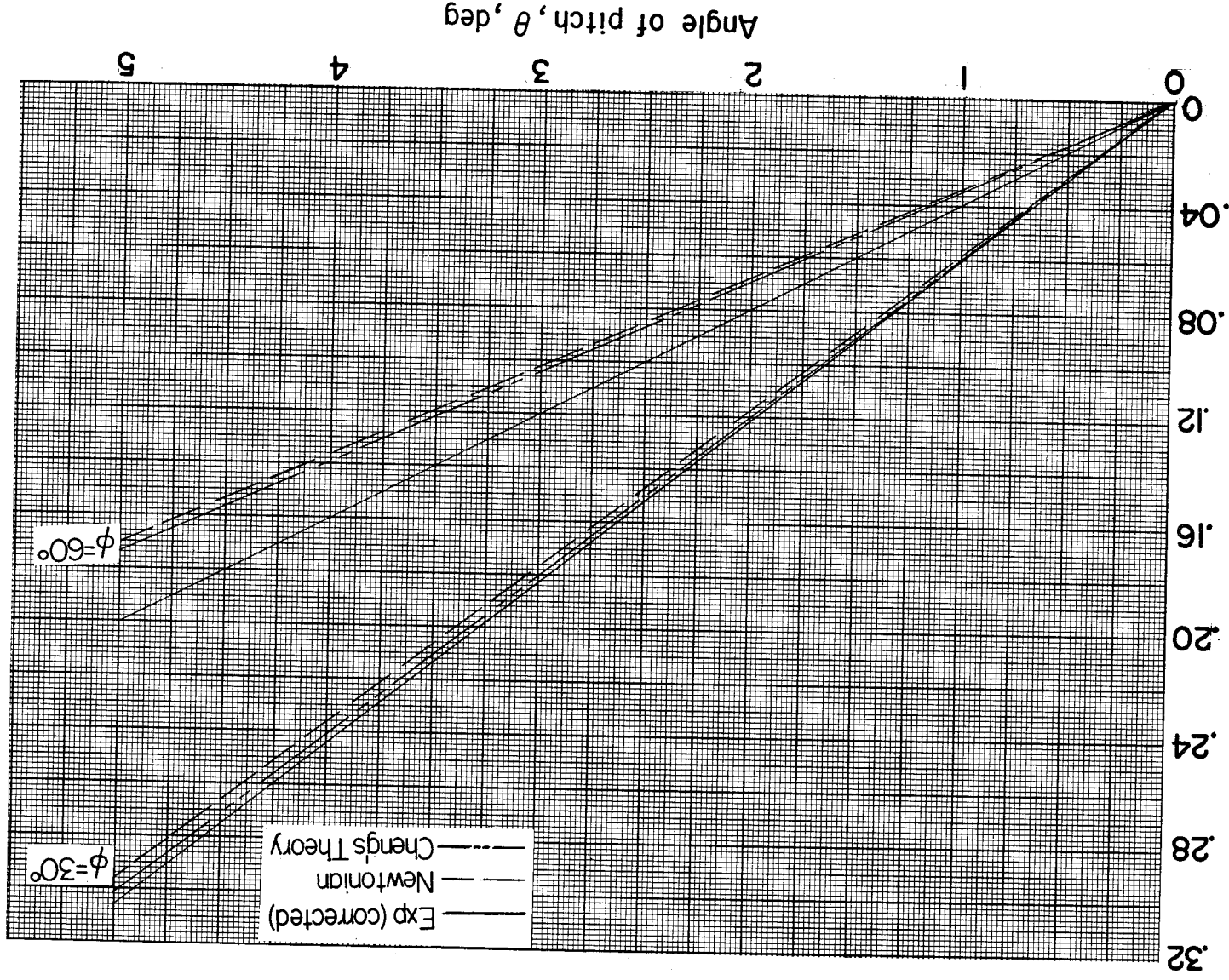
Figure 6.- Comparison of hypersonic experimental data with theory.
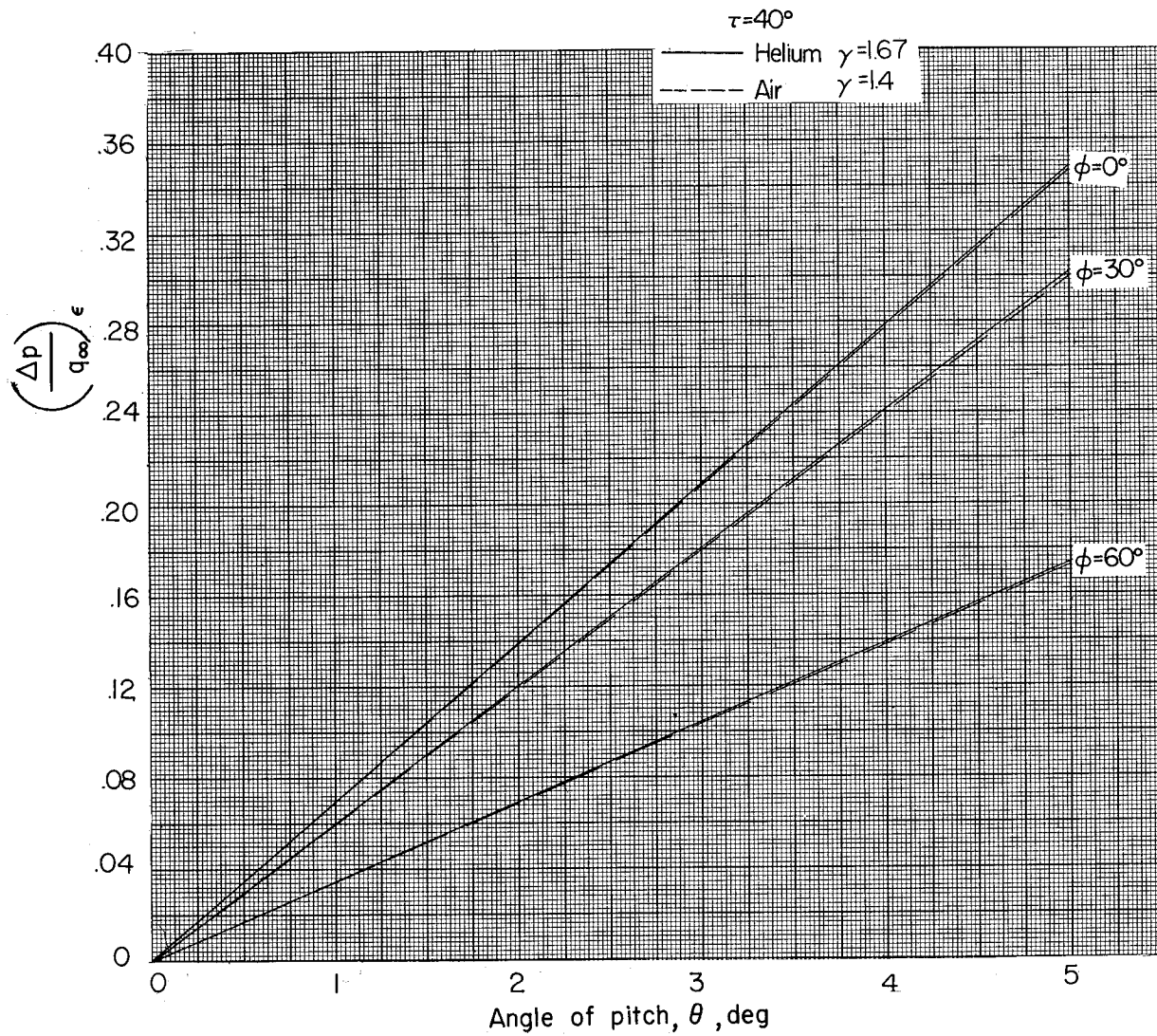
Figure 7.- Effect of γ on difference in pressure coefficient for air and helium (from Cheng's theory, ref. 3).
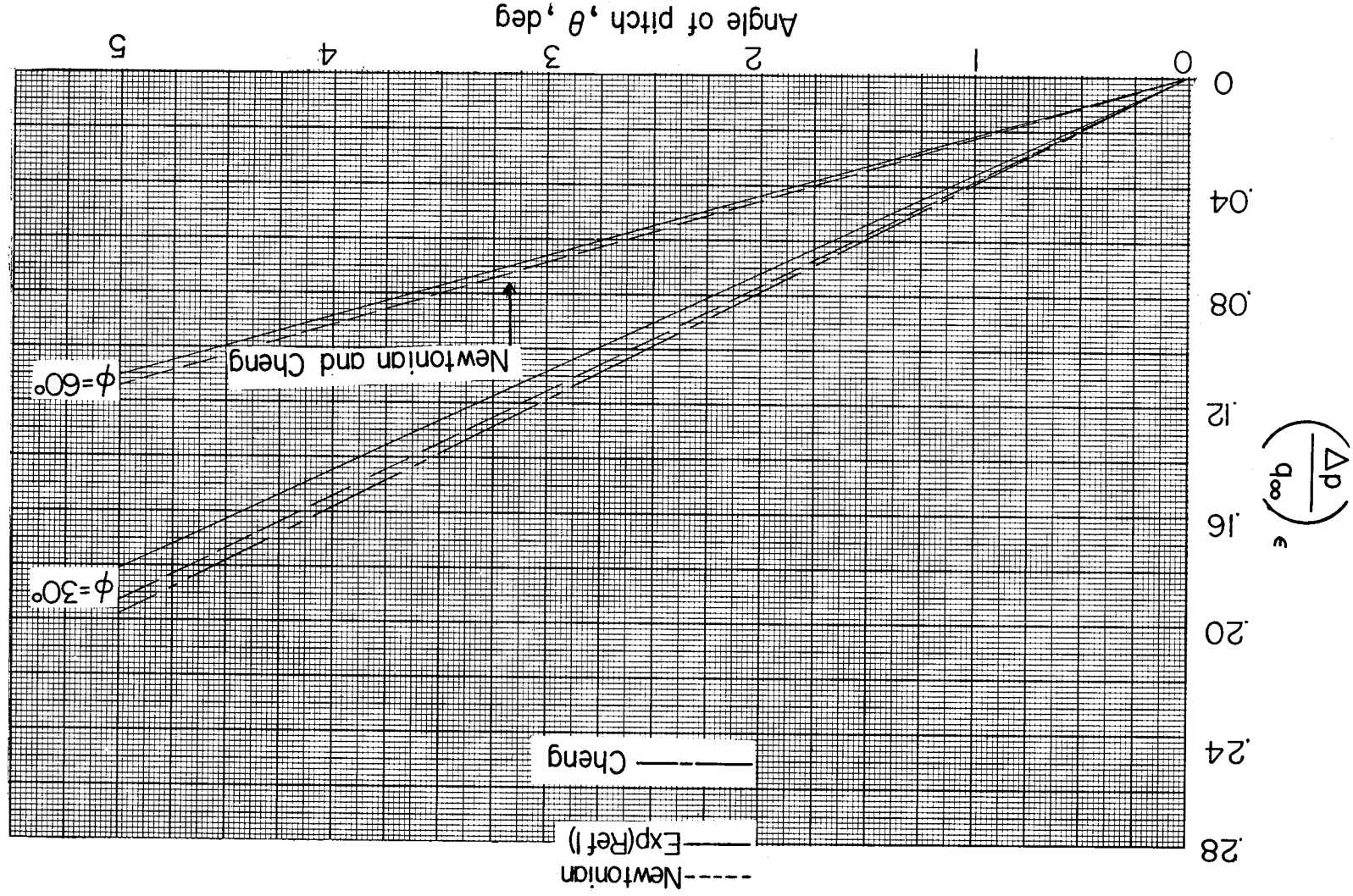
Figure 8.- Comparison of supersonic air data with theory ($\tau = 20°$).

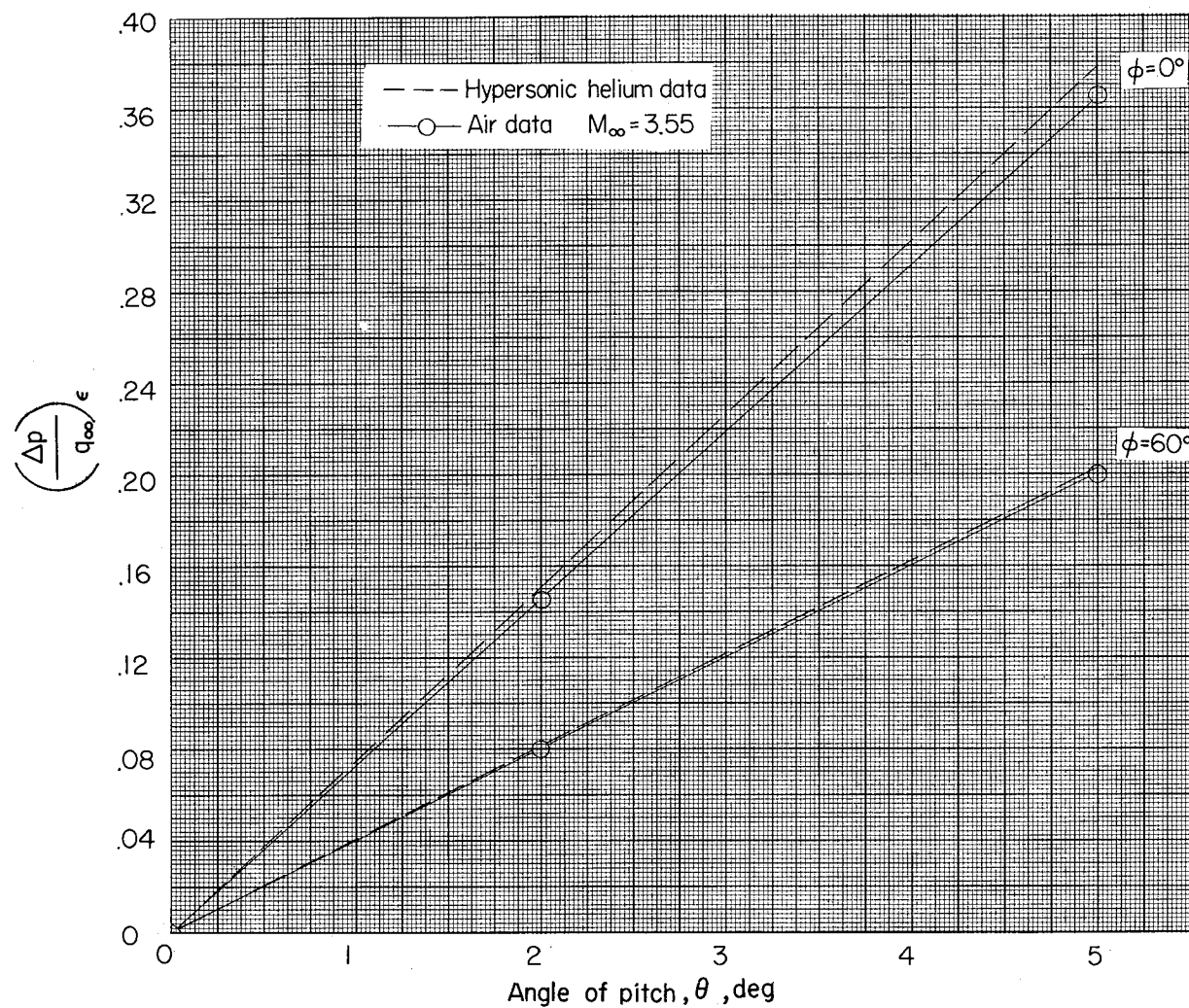Figure 9.- Comparison of hypersonic helium data with supersonic air data ($\tau = 40°$).
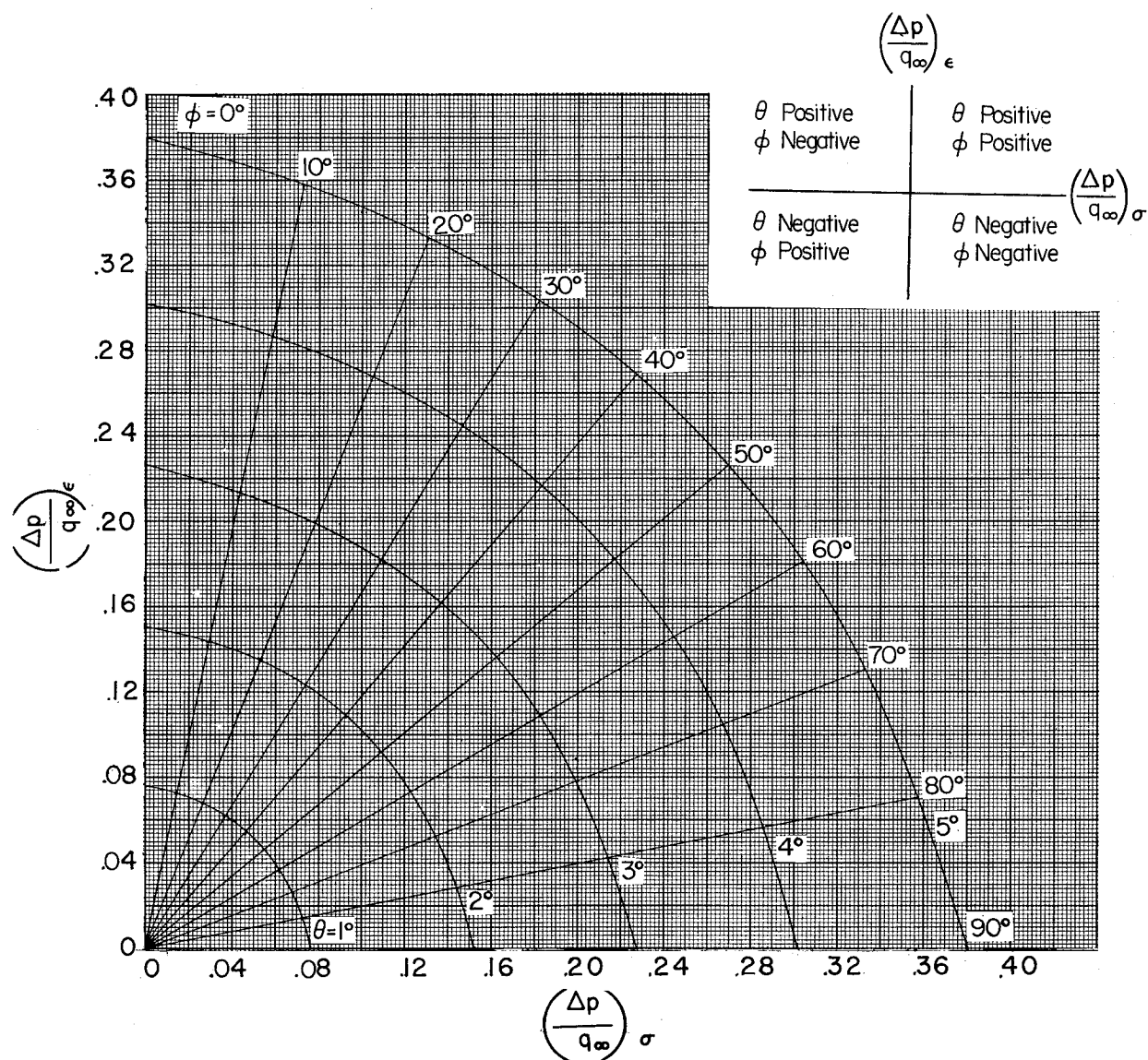
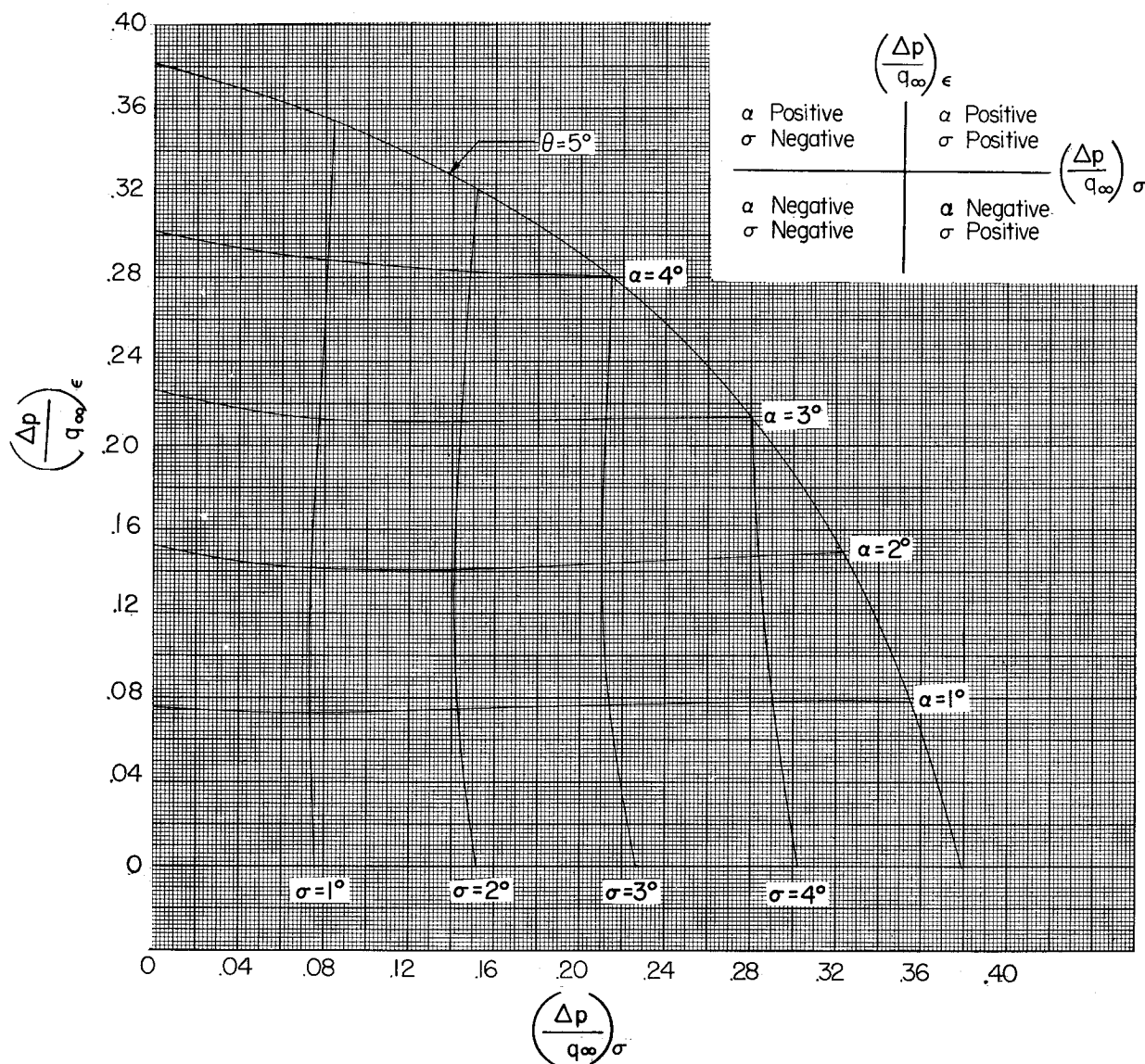Figure 10.- Chart for determination of pitch and roll angles.

Figure 11.- Chart for determination of sidewash and
angle-of-attack angles.

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