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MODIFIED MATRIX METHOD FOR CALCULATING STEADY-STATE SPAN LOADING ON FLEXIBLE WINGS IN SUBSONIC FLIGHT

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MODIFIED MATRIX METHOD FOR CALCULATING STEADY-STATE SPAN LOADING ON FLEXIBLE WINGS IN SUBSONIC FLIGHT

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SUMMARY

A method is presented for shortening the computations required to determine the steady-state span loading on flexible wings in subsonic flight. The method makes use of tables of downwash factors to find the necessary aerodynamic-influence coefficients for the application of lifting-line theory. Explicit matrix equations of equilibrium are converted into a matrix power series with a finite number of terms by utilizing certain characteristic properties of matrices. The number of terms in the series is determined by a trial-and-error process dependent upon the required accuracy of the solution. Spanwise distributions of angle of attack, airload, shear, bending moment, and pitching moment are readily obtained as functions of qm_R where q denotes the dynamic pressure and m_R denotes the lift-curve slope of a rigid wing. This method is intended primarily to make it practical to solve steady-state aeroelastic problems on the ordinary manually operated desk calculators, but the method is also readily adaptable to automatic computing equipment.

INTRODUCTION

As a result of experience in calculating the span loading on flexible wings, in which most of the work was done on a manually operated desk calculator, it has been found that results are most rapidly obtained by combining certain features from several methods. Reference 1 was found to be invaluable for its explicitness in defining the relationships between aerodynamic loadings and structural restraints in the form of matrix equations. Reference 2 furnished tables of downwash factors which can be used to eliminate most of the time spent in evaluating the elements of the aerodynamic-influence-coefficient matrices. References 3, 4, and 5 furnished an approach to the problem in which the overall problem was divided into two parts: (1) the determination of the airload produced by the various parameters in a particular equation of motion and the solution of the equation of motion for the relative amounts of each parameter; and (2) conversion of the matrix equation of equilibrium into a finite power series which is readily evaluated for any flight conditions of dynamic pressure and Mach number to obtain angle of attack, airload, shear, bending moment, and pitching moment resulting from any specified initial angle of attack.

The purpose of the present report is to combine the various features of these methods into a procedure which can readily be followed by the engineer. A sample problem including the effect of external stores is presented which illustrates in detail the procedures involved in solving for the additional root angle of attack, wing load, and other related quantities per unit of airplane load factor.

In order to maintain continuity in the development, the method for using the downwash tables from reference 2 is given in an appendix; and the pertinent portions of the downwash tables have been included in this report with slight modification.

SYMBOLS

b	wing span, ft
C_θ	twist-increment coefficient
c	chord, ft
c_r	wing-root chord (at airplane center line), ft
c_{n_α}	section normal lift-curve slope
$\bar{c}$	mean aerodynamic chord
EI	bending stiffness, lb-in.^{2}
F	downwash factor, $\frac{4\pi h}{\Gamma} w$
GJ	torsional stiffness, lb-in.^{2}
g	acceleration due to gravity
h	vortex semispan, ft
K	interpolating factor in downwash tables (tables I and II)
L	aerodynamic wing lift outboard of station in question, positive for up load, lb

L_{es} aerodynamic load acting on external store, lb

L_t aerodynamic load acting on horizontal tail, lb

l running aerodynamic wing load, lb/ft

M Mach number

M_b aerodynamic bending moment about axis parallel to airplane center line through a given wing station, positive for up load outboard of station, in-lb

M_Y aerodynamic pitching moment about axis perpendicular to airplane center line passing through quarter-chord point at a given wing station, negative for up load rearward of axis, in-lb

m_{es} lift-curve slope of external store in presence of wing per degree

m_o two-dimensional section lift-curve slope per degree

m_R lift-curve slope of rigid wing per degree

N_v number of vortices on both wings

n normal load factor at airplane center of gravity, positive when inertia loads are downward

q dynamic pressure, lb/sq ft

S wing area, sq ft

W gross airplane weight, lb

w downwash velocity, ft/sec

X,Y longitudinal and lateral body axes, respectively

x longitudinal coordinate, positive forward of reference line passing through wing quarter-chord at airplane center line

x_{cg} distance from pitching-moment reference axis to airplane center of gravity

x_t distance from pitching-moment reference axis to aerodynamic center of horizontal tail

y	lateral coordinate, positive outboard of airplane center line
α	angle of attack, deg
α_f	final angle of attack including deformation under airload, deg
α_i	initial angle of attack, deg
α_s	change in section angle of attack due to wing deformation under airload, deg
Γ	circulation, sq ft/sec
Δ	difference between two values
η_v	dimensionless vortex spanwise station
$\Lambda_{c/4}$	sweep angle of wing quarter-chord line, deg
$\Lambda_{3c/4}$	sweep angle of wing three-quarter-chord line, deg
λ	dominant latent root or eigenvalue

Matrix notation:

$[\ ]$	square matrix
$\lfloor\ \rfloor$	row matrix
$\{\ \}$	column matrix
$\lfloor\ \rceil$	diagonal matrix
$\Vert\ \Vert$	rectangular matrix, number of rows not necessarily equal to number of columns
$[\ ]^{-1}$	inverse matrix
$\{E\}$	error matrix
$\lfloor I \rceil$	unit diagonal matrix

$\|I_{int}\|$ integrating matrix

$[S_1]$ matrix of aerodynamic-influence coefficients per foot; obtained from lifting-line theory as derived in reference 1

$[S_2]$ structural elasticity matrix relating change in streamwise angle-of-attack distribution to airload distribution, as derived in reference 1, ft-deg/lb

$$[S_3] = \left[[S_2][S_1]^{-1}\left\lfloor 4\ \frac{m_o}{m_R}\right\rfloor\right]$$

Subscripts and superscripts:

ac aerodynamic center

add additional

adj adjusted

c chord

cp center of pressure

d downwash

es on external store

f final, at equilibrium

i initial

j exponent in power series

N number of terms

n quantity per unit of airplane load factor

O opposite wings

p parabolic

r at wing root (airplane center line)

S same wing

sta station

T trapezoidal

v vortex

y quantity at semispan station

α quantity per unit of change in root angle of attack

Primes indicate the next smaller value.

METHOD

The equation relating the normal forces which act on a rigid airplane to produce a change in normal load factor, with wing loads due to pitching velocity neglected, is as follows:

$$\Delta nW = qm_R S \, \Delta\alpha_r + \Delta L_t \tag{1}$$

When the wing is flexible, that part of the inertia load which acts on the wing will cause a change in wing angle-of-attack distribution and a resulting increment in aerodynamic lift which is proportional to Δn. Also, the initial aerodynamic load produced by $\Delta\alpha_r$ will cause the wing to deflect and twist with a resulting increment in aerodynamic load which is proportional to $\Delta\alpha_r$. An equation which is equivalent to equation (1) but which applies to a flexible wing with negligible pitching acceleration can, therefore, be written as

$$\Delta nW = qm_R S \, \Delta\alpha_r + \Delta L_\alpha \, \Delta\alpha_r + \Delta L_n \, \Delta n + \Delta L_t \tag{2}$$

where ΔL_α is the airload increment due to that part of the wing twist caused by a unit change in α_r and ΔL_n is the airload increment caused by the wing deflection under an inertia load of 1 g.

The quantities ΔL_α and ΔL_n can be calculated separately. By applying the equations of equilibrium between external load and internal restraint, the final angle of attack and aerodynamic load produced by any initial-angle-of-attack distribution $\left\{\alpha_i\right\}$ can be determined.

Equilibrium-Angle-of-Attack Distributions

From reference 1, if the final (at equilibrium) angle of attack $\left\{\alpha_f\right\}$ resulting from the application of $\left\{\alpha_i\right\}$ were known, the aerodynamic lift distribution could be calculated by the equation

$$\left\{l_f\right\} = \left[S_1\right]^{-1}\left\lfloor 4qm_o \right\rceil \left\{\alpha_f\right\} \tag{3}$$

For this report, $\left\{\alpha_f\right\}$ will be defined by

$$\left\{\alpha_f\right\} = \left\{\alpha_i\right\} + \left\{\alpha_s\right\} \tag{4}$$

where $\left\{\alpha_s\right\}$ is the change in section-angle-of-attack distribution caused by the load distribution $\left\{l_f\right\}$ acting on the flexible wing. As in reference 1, $\left\{\alpha_s\right\}$ is defined by

$$\left\{\alpha_s\right\} = \left[S_2\right]\left\{l_f\right\} \tag{5}$$

Substituting $\left\{\alpha_i\right\} + \left\{\alpha_s\right\}$ for $\left\{\alpha_f\right\}$ in equation (3) and substituting the resulting expression for $\left\{l_f\right\}$ in equation (5) gives

$$\left\{\alpha_s\right\} = \left[S_2\right]\left[S_1\right]^{-1}\left\lfloor 4qm_o \right\rceil \left\{\alpha_i + \alpha_s\right\} \tag{6}$$

Before continuing with the solution for $\left\{\alpha_s\right\}$, it will be desirable to factor both the dynamic pressure and compressibility variations out of the matrix $\left\lfloor 4qm_o \right\rceil$ and convert it to a matrix of constants. This operation is performed by multiplying and dividing the matrix by qm_R, where m_R is the overall lift-curve slope of the rigid wing. As a result,

$$\left\lfloor 4qm_o \right\rceil = qm_R\left\lfloor 4\,\frac{m_o}{m_R} \right\rceil \tag{7}$$

Each element of the matrix $\left\lfloor 4\,\frac{m_o}{m_R} \right\rceil$ can be considered constant at subsonic speeds. Equation (6) can, therefore, be written as

$$\left\{\alpha_s\right\} = qm_R\left[S_2\right]\left[S_1\right]^{-1}\left\lfloor 4\,\frac{m_o}{m_R} \right\rceil \left\{\alpha_i + \alpha_s\right\} \tag{8}$$

Now, solving equation (8) for $\{\alpha_s\}$ gives

$$\{\alpha_s\} = \left[\lfloor I \rceil - qm_R [S_2][S_1]^{-1} \left\lfloor 4\,\frac{m_o}{m_R} \right\rceil \right]^{-1} qm_R [S_2][S_1]^{-1} \left\lfloor 4\,\frac{m_o}{m_R} \right\rceil \{\alpha_1\} \tag{9}$$

The normal method of solution of equation (9) would be by matrix inversion. Since the matrix to be inverted $\left[\lfloor I \rceil - qm_R [S_2][S_1]^{-1} \left\lfloor 4\,\frac{m_o}{m_R} \right\rceil \right]$ is a function of qm_R, one inversion must be done for each value of qm_R to be investigated. Some method of iteration would be preferable, and a very useful method can be developed as follows: By the process usually known as "long division," equation (9) can be expanded into a power series of any desired number of terms and a remainder term. For convenience, let the matrix $\left[[S_2][S_1]^{-1} \left\lfloor 4\,\frac{m_o}{m_R} \right\rceil \right]$ be abbreviated as $[S_3]$. Then, equation (9) may be expanded to

$$\{\alpha_s\} = \Big[\lfloor I \rceil + qm_R [S_3] + (qm_R)^2 [S_3]^2 + \ldots + (qm_R)^j \left[\lfloor I \rceil - qm_R [S_3]\right]^{-1} [S_3]^j \Big] [S_3] \{\alpha_1\} qm_R \tag{10}$$

where the jth-power term is the exact remainder after j steps in the division. In this respect, equation (10) is analogous to the expansion

$$\frac{1}{1 - x} = (1 - x)^{-1} = 1 + x + x^2 + \ldots + (1 - x)^{-1} x^j \tag{11}$$

Equation (11) is true for all values of x and j, and equation (10) is likewise true for all values of j. Therefore, the answer obtained after j steps in the division must be equal to the answer after $j + 1$ steps. In equation form, this may be stated as

$$\left[\lfloor I \rceil + qm_R\left[S_3\right] + \cdot \cdot \cdot + \left(qm_R\right)^j \left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1} \left[S_3\right]^j\right] qm_R\left[S_3\right]\left\{\alpha_i\right\}$$

$$= \Big[\lfloor I \rceil + qm_R\left[S_3\right] + \cdot \cdot \cdot + \left(qm_R\right)^j \left[S_3\right]^j +$$

$$\left(qm_R\right)^{j+1}\left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1}\left[S_3\right]^{j+1}\Big] qm_R\left[S_3\right]\left\{\alpha_i\right\} \qquad (12)$$

Equation (12) reduces to

$$\left(qm_R\right)^j\left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1}\left[S_3\right]^j$$

$$= \left(qm_R\right)^j\left[S_3\right]^j + \left(qm_R\right)^{j+1}\left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1}\left[S_3\right]^{j+1} \qquad (13)$$

By solving equation (13) for $\left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1}$ in terms of $\left[S_3\right]^j$ and $\left[S_3\right]^{j+1}$, it will be shown that, under certain conditions, it will be possible to substitute an easily determined scalar quantity for the matrix $\left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1}$ for the purpose of calculating the remainder term (jth-power term) in equation (10). Thus, equation (13) can be written as

$$\left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1} = \left[S_3\right]^j\left[\left[S_3\right]^j - qm_R\left[S_3\right]^{j+1}\right]^{-1} \qquad (14)$$

When j is sufficiently large, $\left[S_3\right]^{j+1}$ is given to a high degree of accuracy by $\lambda\left[S_3\right]^j$, where λ is the dominant latent root or eigenvalue of $\left[S_3\right]$. (See ref. 6 for further discussion.) Substituting $\lambda\left[S_3\right]^j$ for $\left[S_3\right]^{j+1}$ in equation (14) gives

$$\left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1} = \left[S_3\right]^j\left[\left[S_3\right]^j\left(1 - \lambda qm_R\right)\right]^{-1} \tag{15}$$

However, since

$$\left[\left[S_3\right]^j\left(1 - \lambda qm_R\right)\right]^{-1} = \left[\left[S_3\right]^j\right]^{-1} \frac{1}{1 - \lambda qm_R}$$

and

$$\left[S_3\right]^j\left[\left[S_3\right]^j\right]^{-1} = \lfloor I \rceil$$

equation (15) reduces to

$$\left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1} = \lfloor I \rceil \frac{1}{1 - \lambda qm_R} \tag{16}$$

The jth-power term (the remainder) in equation (10) can now be written as

$$\begin{aligned}\left(qm_R\right)^j\left[\lfloor I \rceil - qm_R\left[S_3\right]\right]^{-1}\left[S_3\right]^j &= \frac{\left(qm_R\right)^j}{1 - \lambda qm_R}\lfloor I \rceil\left[S_3\right]^j \\ &= \frac{\left(qm_R\right)^j}{1 - \lambda qm_R}\left[S_3\right]^j\end{aligned}$$

Equation (10) in its entirety becomes

$$\left\{\alpha_s\right\} = \left[\lfloor I \rceil + qm_R\left[S_3\right] + \left(qm_R\right)^2\left[S_3\right]^2 + \ldots + \frac{\left(qm_R\right)^j}{1 - \lambda qm_R}\left[S_3\right]^j\right] qm_R\left[S_3\right]\left\{\alpha_i\right\} \tag{17}$$

It must be made clear that equation (16) applies only to operations on jth-power terms or higher powers of $\left[S_3\right]$. This does not appear to be a practical limitation in aeroelasticity calculations, since increasing powers of the typical $\left[\left[S_2\right]\left[S_1\right]^{-1}\left[4\ \frac{m_o}{m_R}\right]\right]$ matrix rapidly approach the required simple form. Equation (17) simplifies to

$$\left[\alpha_s\right] = qm_R\left[S_3\right]\left\{\alpha_i\right\} + (qm_R)^2\left[S_3\right]^2\left\{\alpha_i\right\} + \ . \ . \ . + \frac{(qm_R)^{j+1}}{1 - \lambda qm_R}\left[S_3\right]^{j+1}\left\{\alpha_i\right\} \tag{18}$$

If

$$\left[S_3\right]\left\{\alpha_i\right\} = \left\{C_{\theta,1}\right\}$$

and

$$\left[S_3\right]^2\left\{\alpha_i\right\} = \left\{C_{\theta,2}\right\} = \left[S_3\right]\left\{C_{\theta,1}\right\}$$

then, in general,

$$\left[S_3\right]^N\left\{\alpha_i\right\} = \left\{C_{\theta,N}\right\} = \left[S_3\right]\left\{C_{\theta,N-1}\right\}$$

It is seen that it is not necessary to raise $\left[S_3\right]$ to the various powers if $\left\{\alpha_i\right\}$ is specified. For a specific $\left\{\alpha_i\right\}$ distribution, equation (18) becomes

$$\left\{\alpha_s\right\} = \left\{C_{\theta,1}\right\}qm_R + \left\{C_{\theta,2}\right\}(qm_R)^2 + \ . \ . \ . + \left\{C_{\theta,N}\right\}\frac{(qm_R)^N}{1 - \lambda qm_R} \tag{19}$$

where N in equation (19) corresponds to $j + 1$ in equation (18).

In order to determine the number of terms N in equation (19) that will be required to give any specified accuracy for $\left\{\alpha_s\right\}$, a trial-and-error procedure is used. Essentially, this procedure consists of increasing N until two successive values are found which give the same answer to the desired accuracy. In other words, the equation with N terms must yield the same result as the equation with N + 1 terms. This condition is expressed as

$$\left\{C_{\theta,1}\right\}qm_R + \left\{C_{\theta,2}\right\}(qm_R)^2 + \ldots + \left\{C_{\theta,N}\right\}\frac{(qm_R)^N}{1 - \lambda qm_R} = \left\{C_{\theta,1}\right\}qm_R +$$

$$\left\{C_{\theta,2}\right\}(qm_R)^2 + \ldots + \left\{C_{\theta,N}\right\}(qm_R)^N + \left\{C_{\theta,N+1}\right\}\frac{(qm_R)^{N+1}}{1 - \lambda qm_R} + \left\{E\right\} \tag{20}$$

where $\left\{E\right\}$ is the difference between the two successive approximations to $\left\{\alpha_s\right\}$. Equation (20) reduces to

$$\left\{C_{\theta,N}\right\}\frac{(qm_R)^N}{1 - \lambda qm_R} = \left\{C_{\theta,N}\right\}(qm_R)^N + \left\{C_{\theta,N+1}\right\}\frac{(qm_R)^{N+1}}{1 - \lambda qm_R} + \left\{E\right\} \tag{21}$$

and equation (21) can be written as

$$\left\{\left\{C_{\theta,N}\right\} - \left\{C_{\theta,N+1}\right\}qm_R\right\}\frac{1}{1 - \lambda qm_R} = \left\{C_{\theta,N}\right\} + \frac{1}{(qm_R)^N}\left\{E\right\} \tag{22}$$

Equation (22) represents a system of simultaneous equations with unknowns $\frac{1}{1 - \lambda qm_R}$ and $\left\{E\right\}$. The number of equations in the system is equal to the number of elements in $\left\{E\right\}$. Therefore, there is one more unknown than there are equations. The required extra equation can be obtained in two ways: (1) by assuming the value of an element of $\left\{E\right\}$ at a particular semispan station to be zero, and (2) by imposing the condition that the sum of the squares of the errors must be a minimum (least-squares method). If the error is made to be zero at about the two-thirds-semispan station, the first method will give about the same results as the least-squares method and will be much simpler. The value of $\frac{1}{1 - \lambda qm_R}$ which will give zero error in equation (22) for one semispan station is

$$\frac{1}{1 - \lambda qm_R} = \frac{C_{\theta,N}}{C_{\theta,N} - qm_R C_{\theta,N+1}} = \frac{1}{1 - \frac{C_{\theta,N+1}}{C_{\theta,N}} qm_R} \tag{23}$$

where $C_{\theta,N}$ and $C_{\theta,N+1}$ are corresponding elements from the column matrices $\left\{C_{\theta,N}\right\}$ and $\left\{C_{\theta,N+1}\right\}$, respectively. The errors at the other semispan stations are easily checked by solving equation (22) for $\left\{E\right\}$. Thus,

$$\left\{E\right\} = (qm_R)^N \left\{\left(\frac{1}{1 - \lambda qm_R} - 1\right)\left\{C_{\theta,N}\right\} - \frac{1}{1 - \lambda qm_R}\left\{C_{\theta,N+1}\right\} qm_R\right\} \tag{24}$$

The elements of $\left\{E\right\}$ are then evaluated by inserting the value of $\frac{1}{1 - \lambda qm_R}$ obtained from equation (23) into equation (24). It is apparent that $\frac{1}{1 - \lambda qm_R}$ and the error $\left\{E\right\}$ both vary with qm_R. Therefore, in the trial-and-error procedure the highest value of qm_R to be analyzed should be used to calculate $\frac{1}{1 - \lambda qm_R}$ and the error $\left\{E\right\}$.

The error $\left\{E\right\}$ in equation (24) is not the actual error in the calculated value of $\left\{\alpha_s\right\}$ except when $\left\{E\right\} \equiv \left\{0\right\}$. It can be demonstrated that the largest of the errors $\left\{E\right\}$ is greater than the largest true error in the assumed solution for $\left\{\alpha_s\right\}$ in all cases where the $\left\{C_\theta\right\}$ terms alternate in sign.

When the columns $\left\{C_{\theta,N}\right\}$ and $\left\{C_{\theta,N} + 1\right\}$ have been found which make $\left\{E\right\}$ sufficiently small, $\left\{\alpha_s\right\}$ can be evaluated from equation (19) which is here repeated:

$$\left\{\alpha_s\right\} = \left\{C_{\theta,1}\right\} qm_R + \left\{C_{\theta,2}\right\}(qm_R)^2 + \ . \ . \ . + \left\{C_{\theta,N}\right\}\frac{(qm_R)^N}{1 - \lambda qm_R}$$

Each initial angle of attack will require a different equation for defining the resulting $\left\{\alpha_s\right\}$. As might be expected, the number of terms required to give any specified accuracy will depend on the shape of the initial-angle-of-attack distribution. It can be assumed that the initial-angle-of-attack distribution which consists of a unit angle of attack at each semispan station will require at least as many terms in its equation for $\left\{\alpha_s\right\}$ as required by any other distribution normally encountered. No harm will be done by carrying more than the minimum number of terms. Therefore, if N and $\dfrac{1}{1 - \lambda q m_R}$ are determined first for the unit symmetrical initial-angle-of-attack distribution, it will be unnecessary to repeat the trial-and-error process for any other initial angle of attack. It will be necessary only to compute the $\left\{C_\theta\right\}$ terms through $\left\{C_{\theta,N}\right\}$. The same value of $\dfrac{1}{1 - \lambda q m_R}$ applies to all equations for $\left\{\alpha_s\right\}$ since λ is a property of the matrix $\left[\left[S_2\right]\left[S_1\right]^{-1}\left\lfloor 4\,\dfrac{m_o}{m_R}\right\rceil\right]$.

The final-angle-of-attack distribution resulting from a particular initial distribution is given by equation (4):

$$\left\{\alpha_f\right\} = \left\{\alpha_i\right\} + \left\{\alpha_s\right\}$$

Thus, by combining equations (4) and (19) the final-angle-of-attack distribution is

$$\left\{\alpha_f\right\} = \left\{\alpha_i\right\} + qm_R\left\{C_{\theta,1}\right\} + \ . \ . \ . \ + \left\{C_{\theta,N}\right\}\frac{\left(qm_R\right)^N}{1 - \lambda q m_R} \tag{25}$$

Airload Distributions

The airload distribution resulting from $\left\{\alpha_f\right\}$ is calculated by substituting $\left\{\alpha_f\right\}$ from equation (25) into equation (3) and then by making use of the previously described conversion of $\left\lfloor 4qm_o\right\rceil$ into $qm_R\left\lfloor 4\,\dfrac{m_o}{m_R}\right\rceil$ to obtain the following equation:

$$\left\{l_f\right\} = qm_R\left[S_1\right]^{-1}\left\lfloor 4\ \frac{m_o}{m_R}\right\rfloor\left\{\left\{\alpha_i\right\} + qm_R\left\{C_{\theta,1}\right\} + \cdot\ \cdot\ \cdot + \frac{(qm_R)^N}{1 - \lambda qm_R}\left\{C_{\theta,N}\right\}\right\} \tag{26}$$

By performing the term-by-term multiplication indicated in equation (26), an equation for load distribution in terms of qm_R is obtained in the form

$$\left\{l_f\right\} = qm_R\left\{\frac{l_i}{qm_R}\right\} + \left(qm_R\right)^2\left\{\frac{l_1}{\left(qm_R\right)^2}\right\} + \cdot\ \cdot\ \cdot + \left(qm_R\right)^{N+1}\left\{\frac{l_N}{\left(qm_R\right)^{N+1}}\right\}\frac{1}{1 - \lambda qm_R} \tag{27}$$

where

$$\left\{\frac{l_i}{qm_R}\right\} = \left[S_1\right]^{-1}\left\lfloor 4\ \frac{m_o}{m_R}\right\rfloor\left\{\alpha_i\right\}$$

$$\left\{\frac{l_1}{\left(qm_R\right)^2}\right\} = \left[S_1\right]^{-1}\left\lfloor 4\ \frac{m_o}{m_R}\right\rfloor\left\{C_{\theta,1}\right\}$$

and so on for all the terms.

By integrating equation (27) term by term, an equation for aerodynamic shear (or total lift carried outboard of each station) resulting from $\left\{\alpha_i\right\}$ is obtained. The integration is done by premultiplying by a suitable integrating matrix, which is denoted here by $\left\|I_{int}\right\|$. The general form of the shear equation is

$$\left\{L_f\right\} = qm_R\left\{\frac{L_i}{qm_R}\right\} + \left(qm_R\right)^2\left\{\frac{L_1}{\left(qm_R\right)^2}\right\} + \cdot\ \cdot\ \cdot + \left(qm_R\right)^{N+1}\left\{\frac{L_N}{\left(qm_R\right)^{N+1}}\right\}\frac{1}{1 - \lambda qm_R} \tag{28}$$

where

$$\left\{\frac{L_i}{qm_R}\right\} = \left\|I_{int}\right\|\left\{\frac{l_i}{qm_R}\right\}$$

and so on for all the terms.

Now, the shear equation is integrated term by term along the Y-axis to obtain a bending- (or rolling-) moment equation and along the X-axis to obtain a pitching-moment equation. The same integrating matrix which was used to obtain shears may be used for the bending moments. If the wing quarter-chord locus is straight, the pitching-moment equation can be obtained by multiplying the bending-moment equation by $-\tan \Lambda_{c/4}$. In other cases, a different integrating matrix will have to be devised.

Application to Calculation of Additional Loading

The results of the calculations up to this point are equations by means of which the final angle of attack, airload, shear, bending moment, and pitching moment resulting from a given initial angle of attack can be evaluated at any of the chosen wing stations for any value of qm_R.

A particular flight condition may involve several different initial-angle-of-attack distributions. Suppose, for example, that it is desired to calculate the additional wing loading required to produce a certain change in airplane load factor Δn. The lift forces acting on the airplane are related to the inertia forces by equation (2)

$$\Delta nW = qm_R S \, \Delta\alpha_r + \Delta L_\alpha \, \Delta\alpha_r + \Delta L_n \, \Delta n + \Delta L_t$$

or by

$$\Delta nW = \left(qm_R S + \Delta L_\alpha\right)\Delta\alpha_r + \Delta L_n \, \Delta n + \Delta L_t$$

The quantity $qm_R S + \Delta L_\alpha$ is the total wing lift resulting from a unit change in wing angle of attack alone; that is, it is twice the aerodynamic shear at the center line calculated from equation (28) when $\left\{\alpha_i\right\}$ is equal to 1° at each semispan station. The quantity ΔL_n is twice the aerodynamic shear at the center line calculated when $\left\{\alpha_i\right\}$ is the twist due to an inertia loading of 1 g on the wing. Equation (2) may be written as

$$\Delta nW = 2L_{\alpha,r} \, \Delta\alpha_r + 2L_{n,r} \, \Delta n + \Delta L_t \tag{29}$$

where $L_{\alpha,r}$ and $L_{n,r}$ are the values of L_r at the center line calculated by equation (28) for the unit symmetrical and inertial twist initial angles of attack, respectively.

If a certain value of Δn is given, the quantities $\Delta\alpha_r$ and ΔL_t remain to be determined in equation (29). The additional equation necessary for this determination is the relationship between the aerodynamic and inertial pitching moments acting on the airplane. A convenient reference axis for the pitching moments will be that passing through the intersection of the wing quarter-chord line with the airplane center line. The aerodynamic pitching-moment components corresponding to each initial angle of attack will already have been calculated with respect to this axis by integrating the shear distributions along the X-axis as previously described. The required pitching-moment equation is, thus,

$$\Delta nWx_{cg} = 2M_{Y,\alpha,r}\,\Delta\alpha_r + 2M_{Y,n,r}\,\Delta n + \Delta L_t x_t \tag{30}$$

where x_{cg} is the distance from the pitching-moment reference axis to the airplane center of gravity and x_t is the distance from the pitching-moment reference axis to the horizontal-tail aerodynamic center.

By simultaneous solution of equations (29) and (30), $\Delta\alpha_r$ and ΔL_t are obtained, respectively, as

$$\Delta\alpha_r = \frac{\Delta n\left[W\left(1 - \frac{x_{cg}}{x_t}\right) + \frac{2}{x_t}M_{Y,n,r} - 2L_{n,r}\right]}{2L_{\alpha,r} - \frac{2}{x_t}M_{Y,\alpha,r}} \tag{31}$$

and

$$\Delta L_t = \Delta nW - 2L_{\alpha,r}\,\Delta\alpha_r - 2L_{n,r}\,\Delta n \tag{32}$$

The additional aerodynamic-shear distribution $\left\{L_{add}\right\}$ corresponding to the change in load factor Δn can now be evaluated from its components by using the value of $\Delta\alpha_r$ from equation (31). Thus,

$$\left\{L_{add}\right\} = \left\{L_\alpha\right\}\Delta\alpha_r + \left\{L_n\right\}\Delta n \tag{33}$$

Similar equations are used to evaluate distributions of angle of attack, airload, bending moment, and pitching moment.

Antisymmetrical Loadings

The same principles which have been applied in the previous sections to symmetrical loadings may also be applied to antisymmetrical loadings. The $[S_1]$ matrix for antisymmetrical loadings is different, as will be described in the appendix. For this reason, the corresponding matrix $\left[[S_2][S_1]^{-1}\left[4\frac{m_o}{m_R}\right]\right]$ is different and a different value of λ will be obtained for the antisymmetrical case.

Effect of Changing Wing Stiffness

Investigating the effects of changing the wing stiffness by a certain percentage which is constant along the span is a relatively simple matter. Such changes have the effect of multiplying $[S_2]$ by a constant. Each term in the angle-of-attack equations needs only to be multiplied by the appropriate power of the stiffness ratio. The changes are easily followed through the airload, shear, and pitching- and bending-moment equations.

External-Store Effects

If the wing carries external stores which produce some lift and pitching moment, the matrix $\left[[S_2][S_1]^{-1}\left[4\frac{m_o}{m_R}\right]\right]$ calculated for the wing alone is easily adjusted to account for the stores with the following provisions:

(1) The stores should not produce appreciable changes in the additional lift distribution of the rigid wing itself.

(2) The struts supporting the stores should be essentially rigid.

(3) The variations of store lift and pitching moment with angle of attack must be known.

Each store can be handled separately by the following procedure: The lift-curve slope of the external store in the presence of the wing, which will be called m_{es}, can be assumed to be a constant fraction of the rigid-wing lift-curve slope m_R so that $\frac{m_{es}}{m_R}$ is constant. The airload on the store will then be expressed by

$$L_{es} = qm_R \frac{m_{es}}{m_R} \frac{S}{2} \alpha_{es} \tag{34}$$

from which

$$\frac{dL_{es}}{d\alpha_{es}} = qm_R \frac{m_{es}}{m_R} \frac{S}{2} \tag{35}$$

The airload L_{es} acting at the store center of pressure will produce a change in angle of attack at each wing station because of wing twist of the flexible wing. The change in wing angle of attack at each station for each unit of store load is calculated by the methods described in reference 1. The results are arranged in a column matrix, each element of which is of the form $\frac{d\alpha_s}{dL_{es}}$. If each element of the column is multiplied by $\frac{dL_{es}}{d\alpha_{es}}$, the resultant column will consist of elements of the form $\frac{d\alpha_s}{d\alpha_{es}}$. Thus, equation (35) can be written as

$$\frac{d\alpha_s}{d\alpha_{es}} = qm_R \frac{d\alpha_s}{dL_{es}} \frac{m_{es}}{m_R} \frac{S}{2} \tag{36}$$

or

$$\frac{1}{qm_R} \frac{d\alpha_s}{d\alpha_{es}} = \frac{d\alpha_s}{dL_{es}} \frac{m_{es}}{m_R} \frac{S}{2}$$

The typical element of the column matrix $\frac{1}{qm_R}\left\{\frac{d\alpha_s}{d\alpha_{es}}\right\}$ has the same properties as an element of the matrix $\left[\left[S_2\right]\left[S_1\right]^{-1}\left\lfloor 4 \frac{m_o}{m_R}\right\rfloor\right]$. Under the assumption that the external-store strut is rigid, α_{es} will be the same as the wing angle of attack at the store semispan station. If the store station coincides with one of the vortex stations, the column matrix $\frac{1}{qm_R}\left\{\frac{d\alpha_s}{d\alpha_{es}}\right\}$ can be added directly to the column of the matrix

$\left[[S_2][S_1]^{-1}\left[4\ \frac{m_o}{m_R}\right]\right]$ which corresponds to that vortex station. In the general case, where the store is between two stations, the column matrix $\frac{1}{qm_R}\left\{\frac{d\alpha_s}{d\alpha_{es}}\right\}$ is divided between the two corresponding columns so that the greater portion is given to the column of $\left[[S_2][S_1]^{-1}\left[4\ \frac{m_o}{m_R}\right]\right]$ corresponding to the closer vortex station. For instance, if the store is located at a point one-third of the distance between two stations, two-thirds of the value of $\frac{1}{qm_R}\left\{\frac{d\alpha_s}{d\alpha_{es}}\right\}$ is added to the nearest adjacent column and the remainder is added to the other adjacent column of $\left[[S_2][S_1]^{-1}\left[4\ \frac{m_o}{m_R}\right]\right]$. This procedure is equivalent to interpolating between two wing angles of attack to determine the store angle of attack. When more than one store is present, several columns of $\left[[S_2][S_1]^{-1}\left[4\ \frac{m_o}{m_R}\right]\right]$ will be affected.

The $\{C_\theta\}$ columns and all quantities derived from them must be recalculated by using the adjusted $\left[[S_2][S_1]^{-1}\left[4\ \frac{m_o}{m_R}\right]\right]$ matrix. After the equation for $\{\alpha_f\}$ has been determined, an interpolation is made between the column elements corresponding to the two wing stations adjacent to each store to allow evaluation of the final angle of attack at the store stations for any value of qm_R. The load acting on each external store at equilibrium is then

$$L_{es} = \alpha_{f,es} qm_R \frac{m_{es}}{m_R} \frac{S}{2} \tag{37}$$

This load must be added to the new shear equations at each station inboard of the store. Each station inboard of the store is also subjected to a bending moment and pitching moment caused by the load acting on each store. These moment increments are

$$\Delta M_{b,es} = L_{es}\,\Delta y_{es}$$

and

$$\Delta M_{Y,es} = L_{es} \Delta x_{es}$$

where Δy_{es} and Δx_{es} are measured from the external-store center of pressure to the wing quarter-chord point at each vortex station involved.

APPLICATION OF METHOD TO SAMPLE PROBLEM

Sample calculations of the additional wing loading have been worked out for the wing-nacelle combination shown in figure 1. The calculations are divided into three parts. The first part, entitled "Influence Coefficients," includes the determination of the $\left[S_1\right]$, $\left[S_2\right]$, and $\left[4 \frac{m_o}{m_R}\right]$ matrices and of the external-store effects. The second part, entitled "Load-Distribution Coefficients," includes the determination of equations for the required components of final angle of attack, airload, shear, bending moment, and pitching moment in terms of qm_R. The third part, entitled "Additional Airloads," consists of the determination of the wing-root airloads and tail airloads per unit of load factor for specific conditions of qm_R, gross weight, and center-of-gravity location.

For the sample problem, the lift of the wing was represented by 20 horseshoe vortices, each 10 percent of the semispan in width. (See fig. 1.) The center of each of the bound vortices is located on the wing quarter-chord locus. The EI and GJ distributions which were used to calculate the $\left[S_2\right]$ matrix by the method of reference 1 are plotted against nondimensional semispan station in figure 2. The dimensions of the sample wing are as follows:

Span, ft	116
Vortex semispan, h, ft	2.9
Area, sq ft	1,428
Root chord, ft	17.34
Taper ratio	0.42
Aspect ratio	9.42
Quarter-chord sweep angle, deg	35
Three-quarter-chord sweep angle, deg	31.5
External-store location, percent of semispan	38.2

Influence Coefficients

The inverse of the aerodynamic-influence-coefficient matrix $\left(\left[S_1\right]^{-1}\right)$, the matrix $\left\lfloor 4 \frac{m_o}{m_R} \right\rfloor$, and the structural-influence-coefficient matrix $\left[S_2\right]$ are required in the calculation of final airload distribution, as shown by equation (26):

$$\left\{l_f\right\} = qm_R\left[S_1\right]^{-1}\left\lfloor 4 \frac{m_o}{m_R} \right\rfloor \left\{\left\{\alpha_1\right\} + qm_R\left\{C_{\theta,1}\right\} + \ . \ . \ . \ + \frac{\left(qm_R\right)^N}{1 - \lambda qm_R}\left\{C_{\theta,N}\right\}\right\}$$

where

$$\left\{C_{\theta,1}\right\} = \left[S_3\right]\left\{\alpha_1\right\} = \left[\left[S_2\right]\left[S_1\right]^{-1}\left\lfloor 4 \frac{m_o}{m_R} \right\rfloor\right]\left\{\alpha_1\right\}$$

and

$$\left\{C_{\theta,N}\right\} = \left[S_3\right]\left\{C_{\theta,N-1}\right\}$$

The product $\left[\left[S_2\right]\left[S_1\right]^{-1}\left\lfloor 4 \frac{m_o}{m_R} \right\rfloor\right]$ is first calculated for the wing alone, and then the method of adding external-store effects is demonstrated.

Calculation of $\left[S_1\right]$.- The method of deriving the $\left[S_1\right]$ matrix from the downwash tables (tables I and II) is described in detail in the appendix. For the sample problem, the required $\left[\frac{\Delta y}{h}\right]$ matrices are those given in equations (A6) and (A7). The necessary $\left[\frac{\Delta x}{h}\right]$ matrix is calculated from equation (A11) as

$$\left[\frac{\Delta x}{h}\right] = \left[\left(\frac{y}{h}\right)_d\right] \tan 31.5^\circ + \left[\frac{c_r}{2h}\right] - \left[\left(\frac{y}{h}\right)_v\right] \tan 35^\circ$$

which becomes, numerically,

$$\left[\frac{\Delta x}{h}\right] = \begin{bmatrix}
1.3421 & 2.7425 & 4.1429 & 5.5434 & 6.9438 & 8.3442 & 9.7446 & 11.1450 & 12.5455 & 13.9459 \\
0.1150 & 1.5154 & 2.9158 & 4.3162 & 5.7167 & 7.1171 & 8.5175 & 9.9179 & 11.3183 & 12.7188 \\
-1.1121 & 0.2883 & 1.6887 & 3.0891 & 4.4895 & 5.8900 & 7.2904 & 8.6908 & 10.0912 & 11.4916 \\
-2.2763 & -0.9388 & 0.4616 & 1.8620 & 3.2624 & 4.6628 & 6.0633 & 7.4637 & 8.8641 & 10.2645 \\
-3.5664 & -2.1660 & -0.7655 & 0.6349 & 2.0353 & 3.4357 & 4.8361 & 6.2366 & 7.6370 & 9.0374 \\
-4.7935 & -3.3931 & -1.9927 & -0.5922 & 0.8082 & 2.2086 & 3.6090 & 5.0094 & 6.4099 & 7.8103 \\
-6.0206 & -4.6202 & -3.2198 & -1.8194 & -0.4189 & 0.9815 & 2.3819 & 3.7823 & 5.1827 & 6.5832 \\
-7.2477 & -5.8473 & -4.4469 & -3.0465 & -1.6461 & -0.2456 & 1.1548 & 2.5552 & 3.9556 & 5.3560 \\
-8.4749 & -7.0744 & -5.6740 & -4.2736 & -2.8732 & -1.4728 & -0.0723 & 1.3281 & 2.7285 & 4.1289 \\
-9.7020 & -8.3016 & -6.9011 & -5.5007 & -4.1003 & -2.6999 & -1.2995 & 0.1010 & 1.5014 & 2.9018
\end{bmatrix} \qquad (38)$$

By the use of $\left[\frac{\Delta y}{h}\right]_S$ from equation (A6) and $\left[\frac{\Delta x}{h}\right]$ from equation (38) as arguments in the downwash tables (tables I and II), the downwash factors for vortex and downwash stations on the same wing $\left[F_S\right]$ are determined to be as follows:

$$\left[F_S\right] = \begin{bmatrix}
4.4942 & -1.2369 & -0.2313 & -0.0963 & -0.0527 & -0.0332 & -0.0228 & -0.0163 & -0.0127 & -0.0100 \\
-0.7176 & 4.3964 & -1.2454 & -0.2331 & -0.0969 & -0.0530 & -0.0334 & -0.0229 & -0.0163 & -0.0128 \\
-0.0955 & -0.7919 & 4.3244 & -1.2530 & -0.2349 & -0.0976 & -0.0532 & -0.0335 & -0.0230 & -0.0164 \\
-0.0364 & -0.1010 & -0.8608 & 4.2835 & -1.2596 & -0.2364 & -0.0982 & -0.0535 & -0.0337 & -0.0231 \\
-0.0187 & -0.0373 & -0.1068 & -0.9224 & 4.2286 & -1.2656 & -0.2382 & -0.0987 & -0.0538 & -0.0338 \\
-0.0114 & -0.0192 & -0.0387 & -0.1125 & -0.9761 & 4.1955 & -1.2712 & -0.2393 & -0.0992 & -0.0540 \\
-0.0077 & -0.0117 & -0.0198 & -0.0401 & -0.1171 & -1.0222 & 4.1692 & -1.2756 & -0.2404 & -0.0997 \\
-0.0057 & -0.0078 & -0.0120 & -0.0204 & -0.0416 & -0.1246 & -1.0614 & 4.1479 & -1.2798 & -0.2415 \\
-0.0042 & -0.0057 & -0.0080 & -0.0122 & -0.0209 & -0.0431 & -0.1307 & -1.0949 & 4.1301 & -1.2833 \\
-0.0032 & -0.0042 & -0.0057 & -0.0081 & -0.0125 & -0.0215 & -0.0447 & -0.1368 & -1.1234 & 4.1154
\end{bmatrix} \qquad (39)$$

Similarly, by using $\left[\frac{\Delta y}{h}\right]_O$ in place of $\left[\frac{\Delta y}{h}\right]_S$, the downwash factors for vortex and downwash stations on opposite wings $\left[F_O\right]$ are found to be

$$
[F_O] = \begin{bmatrix}
-0.0014 & -0.0017 & -0.0019 & -0.0023 & -0.0027 & -0.0033 & -0.0040 & -0.0049 & -0.0064 & -0.0079 \\
-0.0016 & -0.0018 & -0.0021 & -0.0025 & -0.0031 & -0.0037 & -0.0047 & -0.0059 & -0.0075 & -0.0098 \\
-0.0017 & -0.0020 & -0.0024 & -0.0028 & -0.0035 & -0.0043 & -0.0054 & -0.0071 & -0.0092 & -0.0125 \\
-0.0018 & -0.0022 & -0.0026 & -0.0032 & -0.0040 & -0.0050 & -0.0065 & -0.0085 & -0.0117 & -0.0163 \\
-0.0020 & -0.0024 & -0.0029 & -0.0036 & -0.0045 & -0.0060 & -0.0079 & -0.0107 & -0.0152 & -0.0224 \\
-0.0021 & -0.0026 & -0.0032 & -0.0040 & -0.0052 & -0.0069 & -0.0095 & -0.0137 & -0.0206 & -0.0327 \\
-0.0023 & -0.0028 & -0.0036 & -0.0046 & -0.0061 & -0.0083 & -0.0121 & -0.0181 & -0.0295 & -0.0521 \\
-0.0025 & -0.0031 & -0.0039 & -0.0052 & -0.0071 & -0.0101 & -0.0154 & -0.0252 & -0.0461 & -0.0956 \\
-0.0026 & -0.0034 & -0.0043 & -0.0058 & -0.0082 & -0.0123 & -0.0200 & -0.0369 & -0.0814 & -0.2311 \\
-0.0028 & -0.0036 & -0.0047 & -0.0065 & -0.0094 & -0.0152 & -0.0266 & -0.0581 & -0.1826 & -1.2448
\end{bmatrix} \tag{40}
$$

The aerodynamic-influence-coefficient matrix $[S_1]$ for symmetrical airloads, which is found by dividing the vortex semispan h into the sum of the downwash-factor matrices (eqs. (39) and (40)), is written as

$$
[S_1] = \Big[[F_S] + [F_O]\Big] \frac{1}{h} \tag{41}
$$

or, numerically, as

$$
[S_1] = \begin{bmatrix}
1.5492 & -0.4271 & -0.0804 & -0.0340 & -0.0191 & -0.0126 & -0.0092 & -0.0073 & -0.0066 & -0.0062 \\
-0.2480 & 1.5154 & -0.4302 & -0.0812 & -0.0345 & -0.0196 & -0.0131 & -0.0099 & -0.0082 & -0.0078 \\
-0.0335 & -0.2738 & 1.4904 & -0.4330 & -0.0822 & -0.0351 & -0.0202 & -0.0140 & -0.0111 & -0.0100 \\
-0.0132 & -0.0356 & -0.2977 & 1.4760 & -0.4357 & -0.0832 & -0.0361 & -0.0214 & -0.0157 & -0.0136 \\
-0.0071 & -0.0137 & -0.0378 & -0.3193 & 1.4566 & -0.4385 & -0.0849 & -0.0377 & -0.0238 & -0.0194 \\
-0.0047 & -0.0075 & -0.0144 & -0.0402 & -0.3384 & 1.4444 & -0.4416 & -0.0872 & -0.0413 & -0.0299 \\
-0.0034 & -0.0050 & -0.0081 & -0.0154 & -0.0425 & -0.3553 & 1.4335 & -0.4461 & -0.0931 & -0.0523 \\
-0.0028 & -0.0038 & -0.0055 & -0.0088 & -0.0168 & -0.0464 & -0.3713 & 1.4216 & -0.4572 & -0.1162 \\
-0.0023 & -0.0031 & -0.0042 & -0.0062 & -0.0100 & -0.0191 & -0.0520 & -0.3903 & 1.3961 & -0.5222 \\
-0.0021 & -0.0027 & -0.0036 & -0.0050 & -0.0076 & -0.0127 & -0.0246 & -0.0672 & -0.4503 & 0.9899
\end{bmatrix} \tag{42}
$$

Calculation of $\left\lfloor 4 \frac{m_o}{m_R} \right\rceil$.- The $[S_1]$ matrix can now be used to calculate the matrix $\left\lfloor 4 \frac{m_o}{m_R} \right\rceil$ from wind-tunnel measurements of pressure distribution on rigid-wing models. This process is described in detail in reference 1, but, briefly, from the definition of $\left[S_1\right]$, the following equation is derived:

$$\left[S_1\right]\left\{l\right\} = \left\lfloor 4qm_o \right\rceil\left\{\alpha\right\}$$

When the model is rigid, any change in angle of attack can be assumed constant along the semispan. Taking the derivative with respect to α and factoring q out of $\left\lfloor 4qm_o \right\rceil$, thus, results in

$$\left[S_1\right]\left\{\frac{dl}{d\alpha}\right\} = q\left\lfloor 4m_o \right\rceil\left\{1\right\}$$

or

$$\left[S_1\right]\left\{\frac{dl}{d\alpha}\right\}\frac{1}{q} = \left\lfloor 4m_o \right\rceil\left\{1\right\} = \left\{4m_o\right\}$$

However,

$$\left\{\frac{dl}{d\alpha}\right\}\frac{1}{q} = \left\{c_{n_\alpha}c\right\}$$

the elements of which are easily calculated from the chordwise pressure distributions at the required semispan stations. (The chords c must be those of the full-scale airplane to agree with $\left[S_1\right]$.) Therefore,

$$\left[S_1\right]\left\{c_{n_\alpha}c\right\} = \left\lfloor 4m_o \right\rceil\left\{1\right\} = \left\{4m_o\right\} \tag{43}$$

Dividing equation (43) by the wing lift-curve slope m_R gives

$$\frac{1}{m_R}\left[S_1\right]\left\{c_{n_\alpha}c\right\} = \left\lfloor 4\frac{m_o}{m_R} \right\rceil\left\{1\right\} \tag{44}$$

The lift-curve slope m_R is calculated from $\left\{c_{n_\alpha}c\right\}$ as

$$m_R = \left\lfloor \text{Integrating row} \right\rfloor\left\{c_{n_\alpha}c\right\}\frac{2}{S} \tag{45}$$

where the row matrix is taken from the bottom row of the integrating matrix $\left\|I_{int,p}\right\|$ given in table III and is multiplied by the wing semi-span given in feet.

The measured values of $\left\{c_{n_\alpha}c\right\}$ for the sample wing are as follows:

$$\left\{c_{n_\alpha}c\right\} = \begin{Bmatrix} 0.4746 \\ 0.7034 \\ 0.8256 \\ 0.9153 \\ 0.9829 \\ 1.0402 \\ 1.0948 \\ 1.1364 \\ 1.1702 \\ 1.2001 \end{Bmatrix}$$

The value of m_R from equation (45) is, thus, 0.07681.

The diagonal matrix $\left\lfloor 4\,\frac{m_o}{m_R} \right\rceil$ is then calculated from equation (44) as

$$\left\lfloor 4\,\frac{m_o}{m_R}\right\rceil \left\{1\right\} = \begin{bmatrix} 3.5391 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.4665 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.7976 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.9967 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.8882 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.8159 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5.9028 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.8988 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.0459 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.8564 \end{bmatrix} \left\{1\right\} \qquad (46)$$

Calculation of $\left[\left[S_1\right]^{-1}\left\lfloor 4\,\frac{m_o}{m_R}\right\rceil\right]$.- In the calculation of $\left[\left[S_1\right]^{-1}\left\lfloor 4\,\frac{m_o}{m_R}\right\rceil\right]$, the aerodynamic-influence-coefficient matrix $\left[S_1\right]$ was inverted and postmultiplied by $\left\lfloor 4\,\frac{m_o}{m_R}\right\rceil$ with the result that

$$\left[\left[S_1\right]^{-1}\left\lfloor 4\,\frac{m_o}{m_R}\right\rceil\right] = \begin{bmatrix} 2.4208 & 1.1850 & 0.6784 & 0.4563 & 0.3274 & 0.2560 & 0.2208 & 0.2003 & 0.2002 & 0.2334 \\ 0.4507 & 4.0832 & 1.4780 & 0.8570 & 0.5662 & 0.4195 & 0.3478 & 0.3070 & 0.3007 & 0.3478 \\ 0.1671 & 0.8973 & 4.5407 & 1.6797 & 0.9592 & 0.6553 & 0.5148 & 0.4379 & 0.4190 & 0.4786 \\ 0.0860 & 0.3586 & 1.1221 & 4.8693 & 1.7919 & 1.0624 & 0.7717 & 0.6240 & 0.5791 & 0.6520 \\ 0.0543 & 0.1929 & 0.4782 & 1.3335 & 4.9754 & 1.9229 & 1.2178 & 0.9142 & 0.8120 & 0.8964 \\ 0.0392 & 0.1251 & 0.2660 & 0.5998 & 1.4858 & 5.0978 & 2.1242 & 1.3974 & 1.1614 & 1.2457 \\ 0.0312 & 0.0924 & 0.1772 & 0.3468 & 0.7073 & 1.6532 & 5.3869 & 2.3547 & 1.7317 & 1.7722 \\ 0.0266 & 0.0750 & 0.1343 & 0.2394 & 0.4312 & 0.8460 & 1.9104 & 5.6812 & 2.8230 & 2.6282 \\ 0.0235 & 0.0650 & 0.1115 & 0.1878 & 0.3134 & 0.5542 & 1.0766 & 2.2707 & 6.3971 & 4.2354 \\ 0.0216 & 0.0587 & 0.0989 & 0.1623 & 0.2611 & 0.4400 & 0.7970 & 1.5081 & 3.1719 & 9.1053 \end{bmatrix} \tag{47}$$

Calculation of $\left[\left[S_2\right]\left[S_1\right]^{-1}\left\lfloor 4\,\frac{m_o}{m_R}\right\rceil\right]$.- In the calculation of $\left[\left[S_2\right]\left[S_1\right]^{-1}\left\lfloor 4\,\frac{m_o}{m_R}\right\rceil\right]$, the structural elasticity matrix $\left[S_2\right]$ determined by using the method of reference 1 is

$$\left[S_2\right] \times 10^3 = \begin{bmatrix} -1.2282 & -1.1817 & -1.0408 & -0.8509 & -0.6538 & -0.4739 & -0.3234 & -0.1979 & -0.08814 & 0 \\ -0.7709 & -0.7772 & -0.7503 & -0.6561 & -0.5258 & -0.3923 & -0.2732 & -0.1696 & -0.07611 & 0 \\ -0.4739 & -0.4739 & -0.4786 & -0.4613 & -0.3978 & -0.3107 & -0.2232 & -0.1413 & -0.06404 & 0 \\ -0.2763 & -0.2763 & -0.2763 & -0.2801 & -0.2699 & -0.2292 & -0.1731 & -0.1130 & -0.05201 & 0 \\ -0.1488 & -0.1488 & -0.1488 & -0.1488 & -0.1521 & -0.1476 & -0.1230 & -0.08471 & -0.03998 & 0 \\ -0.07082 & -0.07082 & -0.07082 & -0.07082 & -0.07082 & -0.07395 & -0.07288 & -0.05640 & -0.02792 & 0 \\ -0.02596 & -0.02596 & -0.02596 & -0.02596 & -0.02596 & -0.02596 & -0.02858 & -0.02808 & -0.01589 & 0 \\ -0.00236 & -0.00236 & -0.00236 & -0.00236 & -0.00236 & -0.00236 & -0.00236 & -0.00439 & -0.00386 & 0 \\ 0.00302 & 0.00302 & 0.00302 & 0.00302 & 0.00302 & 0.00302 & 0.00302 & 0.00302 & 0.00150 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{48}$$

The product of equation (47) premultiplied by equation (48) gives

$$\left[\left[S_2\right]\left[S_1\right]^{-1}\left[4\ \frac{m_o}{m_R}\right]\right] \times 10^2 = \begin{bmatrix} -0.3435 & -0.5167 & -0.4529 & -0.3612 & -0.2652 & -0.1932 & -0.1469 & -0.1166 & -0.1072 & -0.1222 \\ -0.3326 & -0.5138 & -0.4507 & -0.3597 & -0.2640 & -0.1923 & -0.1461 & -0.1158 & -0.1064 & -0.1214 \\ -0.2973 & -0.4865 & -0.4392 & -0.3517 & -0.2583 & -0.1878 & -0.1423 & -0.1123 & -0.1030 & -0.1174 \\ -0.2468 & -0.4241 & -0.4050 & -0.3339 & -0.2458 & -0.1783 & -0.1343 & -0.1051 & -0.0958 & -0.1091 \\ -0.1921 & -0.3416 & -0.3426 & -0.2986 & -0.2258 & -0.1632 & -0.1217 & -0.0941 & -0.0850 & -0.0966 \\ -0.1407 & -0.2564 & -0.2664 & -0.2441 & -0.1949 & -0.1437 & -0.1056 & -0.0800 & -0.0714 & -0.0809 \\ -0.0969 & -0.1797 & -0.1914 & -0.1817 & -0.1526 & -0.1183 & -0.0875 & -0.0642 & -0.0562 & -0.0635 \\ -0.0597 & -0.1121 & -0.1214 & -0.1180 & -0.1025 & -0.0834 & -0.0642 & -0.0464 & -0.0391 & -0.0441 \\ -0.0267 & -0.0504 & -0.0551 & -0.0543 & -0.0480 & -0.0400 & -0.0318 & -0.0233 & -0.0191 & -0.0214 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{49}$$

<u>External-store effects</u>.- The presence of an external store requires the modification of the matrix equation $\left[S_3\right] = \left[\left[S_2\right]\left[S_1\right]^{-1}\left[4\ \frac{m_o}{m_R}\right]\right]$ as described in the "Method" section. The external store at the 38.2-percent-semispan station is estimated, from wind-tunnel data, to carry about 2 percent of the rigid-wing airload $\left(\frac{m_{es}}{m_R} = 0.02\right)$ from which the wing twist due to store load per unit of store angle of attack is calculated as

$$\left\{\frac{d\alpha_s}{d\alpha_{es}}\right\} = \begin{Bmatrix} 0.0005851 \\ 0.0005851 \\ 0.0005851 \\ 0.0005851 \\ 0.0005851 \\ 0.0005851 \\ 0.0004710 \\ 0.0002181 \\ 0.0000791 \\ 0 \end{Bmatrix} \tag{50}$$

The store is located between vortex stations 6 and 7 $\left(\frac{y}{b/2} = 0.45\right.$ and $\left.0.35\right)$. The store angle of attack, which is assumed to be the same as that of the wing at the store station, is by linear interpolation

$$\alpha_{es} = 0.32\alpha_6 + 0.68\alpha_7$$

If 32 percent of $\left\{\frac{d\alpha_s}{d\alpha_{es}}\right\}$ is added to column 6 of the $\left[S_3\right]$ matrix (eq. (49)) and the remainder is added to column 7, the external-store effect on wing twist will be included when the adjusted matrix $\left[S_3\right]_{adj}$ which is given as

$$\left[S_3\right]_{adj} = \begin{bmatrix} -0.3435 & -0.5167 & -0.4529 & -0.3612 & -0.2652 & -0.1745 & -0.1071 & -0.1166 & -0.1072 & -0.1222 \\ -0.3326 & -0.5138 & -0.4507 & -0.3597 & -0.2640 & -0.1736 & -0.1063 & -0.1158 & -0.1064 & -0.1214 \\ -0.2973 & -0.4865 & -0.4392 & -0.3517 & -0.2583 & -0.1691 & -0.1025 & -0.1123 & -0.1030 & -0.1174 \\ -0.2468 & -0.4241 & -0.4050 & -0.3339 & -0.2458 & -0.1596 & -0.0945 & -0.1051 & -0.0958 & -0.1091 \\ -0.1921 & -0.3416 & -0.3426 & -0.2986 & -0.2258 & -0.1445 & -0.0819 & -0.0941 & -0.0850 & -0.0966 \\ -0.1407 & -0.2564 & -0.2664 & -0.2441 & -0.1949 & -0.1250 & -0.0658 & -0.0800 & -0.0714 & -0.0809 \\ -0.0969 & -0.1797 & -0.1914 & -0.1817 & -0.1526 & -0.1032 & -0.0555 & -0.0642 & -0.0562 & -0.0635 \\ -0.0597 & -0.1121 & -0.1214 & -0.1180 & -0.1025 & -0.0764 & -0.0493 & -0.0464 & -0.0391 & -0.0441 \\ -0.0267 & -0.0504 & -0.0551 & -0.0543 & -0.0480 & -0.0374 & -0.0265 & -0.0233 & -0.0191 & -0.0214 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (51)$$

is postmultiplied by the column matrix of wing angle of attack (eq. (50)).

Load-Distribution Coefficients

The next series of calculations evaluates the coefficients required to determine the distribution of final angle of attack, airload, shear, bending moment, and pitching moment resulting from (a) a unit change in wing angle of attack and (b) the initial wing twist due to wing inertia loading per unit of change in load factor. Both final-angle-of-attack distributions are calculated by the use of equation (25), which requires an iteration procedure to determine the number of terms necessary.

Equations for final-angle-of-attack distribution.- The coefficients in the equation for the final-angle-of-attack distribution (eq. (25)) resulting from a unit symmetrical initial-angle-of-attack distribution are evaluated in table IV. The α_i column in table IV is, in this case, unity at each station. The values under $\left\{C_{\theta,1}\right\}$ are by definition

$$\left\{C_{\theta,1}\right\} = \left[S_3\right]_{adj}\left\{\alpha_i\right\}$$

and the succeeding columns are defined as

$$\left\{C_{\theta,N}\right\} = \left[S_3\right]_{adj}\left\{C_{\theta,N-1}\right\}$$

Each $\{C_\theta\}$ column in table IV, therefore, is obtained by premultiplying the previous column by $[S_3]_{adj}$ from equation (51). After three $\{C_\theta\}$ columns were calculated, a check was made to see whether the errors resulting from using only the first two $\{C_\theta\}$ columns would be excessive. In order to perform the check, a value of $\frac{1}{1 - \lambda qm_R}$ was estimated from the values of $\{C_{\theta,2}\}$ and $\{C_{\theta,3}\}$ appearing at the 65-percent-semispan station (station 4) with qm_R taken to be 100 lb/ft^2-deg and with the use of equation (23). The value of $\frac{1}{1 - \lambda qm_R}$ thus obtained was recorded at the foot of the $\{C_{\theta,2}\}$ column and was used to obtain the errors in the $\{E\}_{N=2}$ column by applying equation (24). These errors were considered too large. Therefore, another column $\{C_{\theta,4}\}$ was computed and the check was repeated. This time the errors, listed in the $\{E\}_{N=3}$ column, were considered satisfactorily small. A sufficiently accurate and convenient form for the angle-of-attack equations was, therefore, considered to be

$$\left\{\alpha_{f,\alpha}\right\} = \left\{\alpha_1\right\} + \left\{C_{\theta,1}\right\}qm_R + \left\{C_{\theta,2}\right\}(qm_R)^2 + \left\{C_{\theta,3}\right\}(qm_R)^3 \frac{1}{1 - \lambda qm_R} \tag{52}$$

Equation (52) may be rewritten with the numerical values of the $\{C_\theta\}$ coefficients from table IV as follows:

$$\left\{\alpha_{f,\alpha}\right\} = \begin{Vmatrix} 1.0 & -0.025671 & 0.051241 \times 10^{-2} & -0.101164 \times 10^{-4} \\ 1.0 & -0.025442 & 0.050743 \times 10^{-2} & -0.100176 \times 10^{-4} \\ 1.0 & -0.024374 & 0.048428 \times 10^{-2} & -0.095584 \times 10^{-4} \\ 1.0 & -0.022198 & 0.043760 \times 10^{-2} & -0.086333 \times 10^{-4} \\ 1.0 & -0.019028 & 0.037071 \times 10^{-2} & -0.073092 \times 10^{-4} \\ 1.0 & -0.015256 & 0.029291 \times 10^{-2} & -0.057710 \times 10^{-4} \\ 1.0 & -0.011447 & 0.021565 \times 10^{-2} & -0.042449 \times 10^{-4} \\ 1.0 & -0.007690 & 0.014141 \times 10^{-2} & -0.027801 \times 10^{-4} \\ 1.0 & -0.003622 & 0.006550 \times 10^{-2} & -0.012868 \times 10^{-4} \\ 1.0 & 0 & 0 & 0 \end{Vmatrix} \begin{Bmatrix} 1 \\ qm_R \\ (qm_R)^2 \\ \dfrac{(qm_R)^3}{1 - \lambda qm_R} \end{Bmatrix} \tag{53}$$

Equation (53) gives the final angle of attack $\left\{\alpha_{f,\alpha}\right\}$ resulting from a symmetrical angle-of-attack change of 1°. An equation of similar form which gives the final-angle-of-attack distribution $\left\{\alpha_{f,n}\right\}$ caused by the wing twisting under an inertia loading of 1 g is as follows:

$$\left\{\alpha_{f,n}\right\} = \begin{Vmatrix} 0.5208 & -0.0098535 & 0.019398 \times 10^{-2} & -0.038266 \times 10^{-4} \\ 0.5094 & -0.0097554 & 0.019209 \times 10^{-2} & -0.037892 \times 10^{-4} \\ 0.4859 & -0.0093007 & 0.018327 \times 10^{-2} & -0.036155 \times 10^{-4} \\ 0.4315 & -0.0083871 & 0.016541 \times 10^{-2} & -0.032654 \times 10^{-4} \\ 0.3358 & -0.0070838 & 0.014011 \times 10^{-2} & -0.027645 \times 10^{-4} \\ 0.2246 & -0.0055770 & 0.011060 \times 10^{-2} & -0.021826 \times 10^{-4} \\ 0.1507 & -0.0040856 & 0.008134 \times 10^{-2} & -0.016053 \times 10^{-4} \\ 0.1329 & -0.0026597 & 0.005325 \times 10^{-2} & -0.010513 \times 10^{-4} \\ 0.0693 & -0.0012269 & 0.002464 \times 10^{-2} & -0.004866 \times 10^{-4} \\ 0 & 0 & 0 & 0 \end{Vmatrix} \left\{ \begin{matrix} 1 \\ qm_R \\ (qm_R)^2 \\ \dfrac{(qm_R)^3}{1 - \lambda qm_R} \end{matrix} \right\} \tag{54}$$

The first column of the matrix of coefficients in equation (54) is the wing angle-of-attack distribution due to dead-weight loading. Each succeeding column is the product of the previous column premultiplied by $\left[S_3\right]_{adj}$. It is not necessary to repeat the iteration procedure, because the number of coefficient columns required to obtain acceptable accuracy is only slightly dependent on the shape of the initial-angle-of-attack distribution.

<u>External-store angle of attack</u>.- In order to calculate the airload carried on the external store, the angle of attack at the store location must be known. The final angle of attack of the store resulting from a unit change in wing-root angle of attack is obtained by interpolation between the rows of equation (53) which correspond to stations 6 and 7. Thus,

$$\alpha_{es,\alpha} = \left\lfloor 1.0 \quad -0.012666 \quad 0.024037 \times 10^{-2} \quad -0.047333 \times 10^{-4} \right\rfloor \left\{ \begin{matrix} 1 \\ qm_R \\ (qm_R)^2 \\ \dfrac{(qm_R)^3}{1 - \lambda qm_R} \end{matrix} \right\} \tag{55}$$

Similarly, the following equation for store angle of attack resulting from the unit-load-factor inertia loading is obtained by interpolation in the rows of equation (54):

$$\alpha_{es,n} = \lfloor 0.1743 \quad -0.004563 \quad 0.009070 \times 10^{-2} \quad -0.01790 \times 10^{-4} \rfloor \left\{ \frac{\begin{matrix} 1 \\ qm_R \\ (qm_R)^2 \\ (qm_R)^3 \end{matrix}}{1 - \lambda qm_R} \right\} \tag{56}$$

Running-load-distribution equations.- The equation for running-load distribution $\{l_{f,\alpha}\}$ resulting from a unit change in wing-root angle of attack is obtained by premultiplying the coefficients of equation (53) by $qm_R [S_1]^{-1} \left\lfloor 4 \frac{m_o}{m_R} \right\rfloor$. Thus,

$$\{l_{f,\alpha}\} = \begin{Vmatrix} 6.17868 & -0.13389 & 0.26554 \times 10^{-2} & -0.52407 \times 10^{-4} \\ 9.15804 & -0.19511 & 0.38646 \times 10^{-2} & -0.76266 \times 10^{-4} \\ 10.74948 & -0.21411 & 0.42228 \times 10^{-2} & -0.83316 \times 10^{-4} \\ 11.91720 & -0.21281 & 0.41795 \times 10^{-2} & -0.82211 \times 10^{-4} \\ 12.79740 & -0.19548 & 0.37936 \times 10^{-2} & -0.74784 \times 10^{-4} \\ 13.54236 & -0.16930 & 0.32506 \times 10^{-2} & -0.64046 \times 10^{-4} \\ 14.25372 & -0.13989 & 0.26550 \times 10^{-2} & -0.52281 \times 10^{-4} \\ 14.79540 & -0.10807 & 0.20308 \times 10^{-2} & -0.39942 \times 10^{-4} \\ 15.23520 & -0.07651 & 0.14320 \times 10^{-2} & -0.28179 \times 10^{-4} \\ 15.62484 & -0.05195 & 0.09783 \times 10^{-2} & -0.19257 \times 10^{-4} \end{Vmatrix} \left\{ \frac{\begin{matrix} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ (qm_R)^4 \end{matrix}}{1 - \lambda qm_R} \right\} \tag{57}$$

The equation for running-load distribution $\{l_{f,n}\}$ resulting from the twist due to an inertia loading of 1 g is obtained by premultiplying the coefficients of equation (54) by $qm_R [S_1]^{-1} \left\lfloor 4 \frac{m_o}{m_R} \right\rfloor$. Thus,

$$\left\{ l_{f,n} \right\} = \left\| \begin{array}{llll} 1.39791 & -0.05098 & 0.10048 \times 10^{-2} & -0.19824 \times 10^{-4} \\ 3.80110 & -0.07416 & 0.14621 \times 10^{-2} & -0.28848 \times 10^{-4} \\ 4.10930 & -0.08095 & 0.15920 \times 10^{-2} & -0.31514 \times 10^{-4} \\ 3.95352 & -0.07984 & 0.15755 \times 10^{-2} & -0.31094 \times 10^{-4} \\ 3.39820 & -0.07242 & 0.14333 \times 10^{-2} & -0.28284 \times 10^{-4} \\ 2.70251 & -0.06188 & 0.12274 \times 10^{-2} & -0.24222 \times 10^{-4} \\ 2.15264 & -0.05039 & 0.10018 \times 10^{-2} & -0.19772 \times 10^{-4} \\ 1.79402 & -0.03840 & 0.07653 \times 10^{-2} & -0.15105 \times 10^{-4} \\ 1.31759 & -0.02707 & 0.05398 \times 10^{-2} & -0.10666 \times 10^{-4} \\ 0.88601 & -0.01853 & 0.03689 \times 10^{-2} & -0.07282 \times 10^{-4} \end{array} \right\| \left\{ \begin{array}{c} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ \dfrac{(qm_R)^4}{1 - \lambda qm_R} \end{array} \right\} \quad (58)$$

Shear-distribution equations.- The matrix of coefficients for the shear-distribution equations is calculated by premultiplying the matrix of running-load coefficients by the integrating matrix $\left\| I_{int,p} \right\|$, which is given in table III. This integrating matrix is based on Simpson's parabolic method and is given in nondimensional form for 10 stations, with an extra row added to give the integral to the center line. The results are made dimensionally correct by multiplying them by the wing semispan (in feet, for the sample problem).

The complete shear equation must also include the concentrated load acting on the nacelle. This load which is obtained by substituting $\alpha_{es,\alpha}$ from equation (55) into equation (34) is given by

$$L_{es,\alpha} = \frac{m_{es}}{m_R} \frac{S}{2} \left\lfloor 1.0 \quad -0.012666 \quad 0.024037 \times 10^{-2} \quad -0.047333 \times 10^{-4} \right\rfloor \left\{ \begin{array}{c} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ \dfrac{(qm_R)^4}{1 - \lambda qm_R} \end{array} \right\} \quad (59)$$

Since $\frac{m_{es}}{m_R} = 0.02$ and $\frac{S}{2} = 714$ square feet, equation (59) can be written as

$$L_{es,\alpha} = \left\lfloor 14.3 \quad -0.18 \quad 0.34 \times 10^{-2} \quad -0.68 \times 10^{-4} \right\rfloor \left\{ \begin{array}{c} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ \dfrac{(qm_R)^4}{1 - \lambda qm_R} \end{array} \right\} \quad (60)$$

Equation (60) gives the external-store lift per unit of wing-root angle of attack. The store airload due to inertia twist per unit of load factor is calculated similarly by substituting $\alpha_{es,n}$ from equation (56) into equation (34) to obtain

$$L_{es,n} = \lfloor 2.5 \quad -0.07 \quad 0.13 \times 10^{-2} \quad -0.26 \times 10^{-4} \rfloor \left\{ \begin{matrix} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ (qm_R)^4 \end{matrix} \right\} \frac{}{1 - \lambda qm_R} \tag{61}$$

Since the external-store lift acts as a concentrated load on the wing, it is to be added to the shear at each station inboard of the store. For the unit symmetrical angle-of-attack distribution, integration of equation (57) by using table III and the addition of the external-store coefficients from equation (60) (with the qm_R terms factored out) result in the following equation for shear distribution:

$$\{L_\alpha\} = \left\| \underbrace{\begin{Vmatrix} 9.7 & -0.21 & 0.0042 & -0.000083 \\ 54.9 & -1.19 & 0.0235 & -0.000464 \\ 112.8 & -2.38 & 0.0471 & -0.000931 \\ 178.7 & -3.63 & 0.0717 & -0.001414 \\ 250.4 & -4.82 & 0.0949 & -0.001870 \\ 326.8 & -5.88 & 0.1153 & -0.002274 \\ 407.5 & -6.77 & 0.1325 & -0.002611 \\ 491.8 & -7.49 & 0.1460 & -0.002878 \\ 578.9 & -8.02 & 0.1560 & -0.003075 \\ 668.4 & -8.39 & 0.1629 & -0.003211 \\ 714.0 & -8.53 & 0.1655 & -0.003262 \end{Vmatrix}}_{\text{Load on wing:}} + \underbrace{\begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 14.3 & -0.18 & 0.0034 & -0.000068 \\ 14.3 & -0.18 & 0.0034 & -0.000068 \\ 14.3 & -0.18 & 0.0034 & -0.000068 \\ 14.3 & -0.18 & 0.0034 & -0.000068 \\ 14.3 & -0.18 & 0.0034 & -0.000068 \end{Vmatrix}}_{\text{Load on external store:}} \right\| \left\{ \begin{matrix} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ (qm_R)^4 \end{matrix} \right\} \frac{}{1 - \lambda qm_R} \tag{62}$$

Similarly, the equation for shear distribution due to wing-inertia twist per unit of load factor is obtained from equations (58) and (61) as

$$\{L_n\} = \left\| \begin{array}{c} \text{Load on wing:} \\ \left\| \begin{matrix} 2.1 & -0.08 & 0.0016 & -0.000031 \\ 18.1 & -0.45 & 0.0089 & -0.000175 \\ 41.3 & -0.90 & 0.0178 & -0.000352 \\ 64.9 & -1.37 & 0.0271 & -0.000535 \\ 86.3 & -1.82 & 0.0358 & -0.000707 \\ 103.9 & -2.21 & 0.0436 & -0.000860 \\ 117.9 & -2.53 & 0.0500 & -0.000988 \\ 129.4 & -2.79 & 0.0551 & -0.001089 \\ 138.4 & -2.98 & 0.0589 & -0.001163 \\ 144.8 & -3.11 & 0.0615 & -0.001214 \\ 147.0 & -3.16 & 0.0625 & -0.001234 \end{matrix} \right\| \end{array} + \begin{array}{c} \text{Load on external store:} \\ \left\| \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2.5 & -0.07 & 0.0013 & -0.000026 \\ 2.5 & -0.07 & 0.0013 & -0.000026 \\ 2.5 & -0.07 & 0.0013 & -0.000026 \\ 2.5 & -0.07 & 0.0013 & -0.000026 \\ 2.5 & -0.07 & 0.0013 & -0.000026 \end{matrix} \right\| \end{array} \right\| \left\{ \frac{\begin{matrix} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ (qm_R)^4 \end{matrix}}{1 - \lambda qm_R} \right\} \qquad (63)$$

The shear coefficients due to distributed wing lift and concentrated store lift are to be kept as two separate matrices for ease in obtaining the bending-moment and pitching-moment equations.

Bending-moment-distribution equations.- The matrix of coefficients for the component of the shear equation due to distributed wing lift is integrated to obtain the matrix of coefficients for the component of bending moment due to distributed wing lift. For this purpose the integrating matrix $\left\|I_{int,p}\right\|$ in table III multiplied by $b/2$ may be used. However, shear-distribution curves are usually straight enough that the much simpler trapezoidal type of integrating matrix can be used. This integrating matrix $\left\|I_{int,T}\right\|$ is given in table V and is nondimensionalized with respect to semispan. For use in obtaining bending moments in inch-pounds, the elements of table V are multiplied by the semispan in inches.

The coefficients for calculating the bending moments due to the concentrated load imposed by the external store are obtained by premultiplying each row of the matrix of shear coefficients due to concentrated store load by a diagonal matrix whose elements are $y_{es} - y_{sta} = \Delta y_{es}$. For the sample problem,

$$\left\lfloor \Delta y_{es} \right\rceil = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 22.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 91.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 161.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 231.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 266.0 \end{bmatrix} \qquad (64)$$

By using equation (62), table V, and $\left\lfloor \Delta y_{es} \right\rceil$ from equation (64) in the prescribed manner, the complete equation for bending moment per unit of wing-root angle of attack is given as follows:

$$\left\{M_{b,\alpha}\right\} = \left\| \begin{array}{c} \text{Load on wing:} \\ \left\| \begin{matrix} 338 & -7.3 & 0.145 & -0.00287 \\ 2,585 & -55.9 & 1.109 & -0.02188 \\ 8,420 & -180.0 & 3.567 & -0.07040 \\ 18,563 & -389.2 & 7.702 & -0.15198 \\ 33,495 & -683.0 & 13.498 & -0.26627 \\ 53,582 & -1,055.0 & 20.814 & -0.41048 \\ 79,136 & -1,495.2 & 29.437 & -0.58047 \\ 110,432 & -1,991.7 & 39.130 & -0.77151 \\ 147,692 & -2,531.6 & 49.642 & -0.97867 \\ 191,098 & -3,103.0 & 60.742 & -1.19740 \\ 214,359 & -3,395.0 & 66.412 & -1.30913 \end{matrix} \right\| \end{array} + \begin{array}{c} \text{Load on external store:} \\ \left\| \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 318 & -4.0 & 0.076 & -0.00151 \\ 1,312 & -16.6 & 0.315 & -0.00621 \\ 2,306 & -29.2 & 0.554 & -0.01091 \\ 3,300 & -41.8 & 0.793 & -0.01562 \\ 3,798 & -48.1 & 0.913 & -0.01798 \end{matrix} \right\| \end{array} \right\| \left\{ \frac{\begin{matrix} \bar{q}m_R \\ (\bar{q}m_R)^2 \\ (\bar{q}m_R)^3 \\ (\bar{q}m_R)^4 \end{matrix}}{1 - \lambda \bar{q}m_R} \right\} \qquad (65)$$

The equation for bending moment due to inertia twist per unit of load factor, obtained from equation (63) in a fashion similar to that of equation (65), is as follows:

$$
\left\{M_{b,n}\right\} = \left\|\overset{\text{Load on wing:}}{\begin{Vmatrix} 72 & -2.8 & 0.055 & -0.00109 \\ 775 & -21.3 & 0.420 & -0.00828 \\ 2{,}844 & -68.5 & 1.349 & -0.02663 \\ 6{,}540 & -147.8 & 2.912 & -0.05749 \\ 11{,}800 & -258.8 & 5.100 & -0.10071 \\ 18{,}418 & -398.9 & 7.862 & -0.15526 \\ 26{,}136 & -563.8 & 11.119 & -0.21956 \\ 34{,}741 & -749.0 & 14.778 & -0.29181 \\ 44{,}060 & -949.8 & 18.747 & -0.37017 \\ 53{,}914 & -1{,}161.7 & 22.937 & -0.45290 \\ 58{,}951 & -1{,}269.9 & 25.077 & -0.49516 \end{Vmatrix}} + \overset{\text{Load on external store:}}{\begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 55 & -1.5 & 0.029 & -0.00057 \\ 229 & -6.0 & 0.119 & -0.00235 \\ 402 & -10.5 & 0.209 & -0.00413 \\ 575 & -15.1 & 0.299 & -0.00591 \\ 662 & -17.3 & 0.345 & -0.00680 \end{Vmatrix}}\right\| \frac{\left\{\begin{matrix} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ (qm_R)^4 \end{matrix}\right\}}{1 - \lambda qm_R} \tag{66}
$$

Pitching-moment-distribution equations.- In the general case, the coefficients in the equation for pitching moment due to distributed wing lift are obtained by integrating the shear coefficients due to distributed lift from wing tip to center line in the x-direction. However, when the wing-quarter-chord locus is a straight line, this integration is equivalent to multiplying the corresponding bending-moment coefficients by $-\tan \Lambda_{c/4}$.

The pitching-moment-equation coefficients due to external-store load are calculated by premultiplying the external-store load-coefficient matrix by a diagonal matrix whose elements are the chordwise distances from the store center of pressure to the vortex stations inboard of the store. For the sample problem, this moment-arm matrix is

$$
\left\lfloor \Delta x_{es} \right\rfloor = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 141.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 93.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 44.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -28.8 \end{bmatrix} \tag{67}
$$

The resulting equation for pitching moment per unit of wing-root angle of attack is as follows:

Load on wing: Load on external store:

$$\left\{M_{Y,\alpha}\right\} = \left\| \begin{Vmatrix} -237 & 5.1 & -0.102 & 0.00201 \\ -1,810 & 39.1 & -0.776 & 0.01532 \\ -5,894 & 126.0 & -2.497 & 0.04928 \\ -12,994 & 272.4 & -5.392 & 0.10639 \\ -23,447 & 478.1 & -9.449 & 0.18639 \\ -37,505 & 738.6 & -14.570 & 0.28734 \\ -55,395 & 1,046.7 & -20.606 & 0.40633 \\ -77,302 & 1,394.2 & -27.391 & 0.54006 \\ -103,384 & 1,772.1 & -34.749 & 0.68507 \\ -133,769 & 2,172.1 & -42.519 & 0.83818 \\ -150,051 & 2,351.3 & -46.488 & 0.91639 \end{Vmatrix} + \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2,024 & -25.6 & 0.486 & -0.00958 \\ 1,328 & -16.8 & 0.319 & -0.00629 \\ 633 & -8.0 & 0.152 & -0.00299 \\ 63 & -0.8 & 0.015 & -0.00030 \\ -411 & 5.2 & -0.099 & 0.00195 \end{Vmatrix} \right\| \left\{ \begin{matrix} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ \dfrac{(qm_R)^4}{1 - \lambda qm_R} \end{matrix} \right\} \quad (68)$$

The pitching moment due to inertia twist per unit of load factor is given by the following equation:

Load on wing: Load on external store:

$$\left\{M_{Y,n}\right\} = \left\| \begin{Vmatrix} -50 & 2.0 & -0.039 & 0.00076 \\ -542 & 14.9 & -0.294 & 0.00579 \\ -1,991 & 47.9 & -0.945 & 0.01864 \\ -4,578 & 103.4 & -2.038 & 0.04024 \\ -8,260 & 181.2 & -3.570 & 0.07050 \\ -12,893 & 279.2 & -5.504 & 0.10868 \\ -18,296 & 394.7 & -7.783 & 0.15369 \\ -24,319 & 524.3 & -10.345 & 0.20427 \\ -30,842 & 664.9 & -13.123 & 0.25912 \\ -37,740 & 813.2 & -16.056 & 0.31703 \\ -41,266 & 888.9 & -17.554 & 0.34661 \end{Vmatrix} + \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 353 & -9.2 & 0.184 & -0.00362 \\ 232 & -6.1 & 0.121 & -0.00238 \\ 110 & -2.9 & 0.057 & -0.00113 \\ 11 & -0.3 & 0.006 & -0.00011 \\ -72 & 1.9 & -0.037 & 0.00074 \end{Vmatrix} \right\| \left\{ \begin{matrix} qm_R \\ (qm_R)^2 \\ (qm_R)^3 \\ \dfrac{(qm_R)^4}{1 - \lambda qm_R} \end{matrix} \right\} \quad (69)$$

Evaluation of $\dfrac{1}{1 - \lambda qm_R}$.- Equations have been written for shear, bending moment, and pitching moment in terms of qm_R and $\dfrac{1}{1 - \lambda qm_R}$. In the evaluation of $\dfrac{1}{1 - \lambda qm_R}$ given in equation (23), the quantity λ is seen to be

$$\lambda = \frac{C_{\theta,N+1}}{C_{\theta,N}} \tag{70}$$

As recommended in the "Method" section, values of $C_{\theta,N+1}$ and $C_{\theta,N}$ corresponding to the 65-percent-semispan station are used. From table IV which is used for the calculation of the $\left\{C_\theta\right\}$ coefficients, it can be seen that

$$C_{\theta,N+1} = \left\{C_{\theta,4}\right\} = 0.000000170340$$

and

$$C_{\theta,N} = \left\{C_{\theta,3}\right\} = -0.0000086333$$

from which

$$\lambda = -0.0197306$$

Additional Airloads

The calculations just completed may now be used to determine the total additional airload shear, the bending moment and pitching moment, the horizontal-tail load per unit of normal load factor, the wing center of pressure, and the aerodynamic-center position. The numerical relationship must be established between the wing-root angle of attack $\Delta\alpha_r$ and the airplane load factor Δn by use of equation (31). The first step in the procedure is the calculation of wing-root loading components for selected values of the parameter qm_R.

<u>Calculation of wing-root loading components</u>.- The values of $L_{\alpha,r}$, $M_{b,\alpha,r}$, $M_{Y,\alpha,r}$, $L_{n,r}$, $M_{b,n,r}$, and $M_{Y,n,r}$ (wing-root values) to be used in the succeeding calculations are evaluated from equations (62), (65), (68), (63), (66), and (69), respectively, by using only the bottom row of coefficients from each equation. These quantities, with the value of qm_R for which they are calculated, are presented in table VI.

<u>Additional wing and tail airloads and wing centers of pressure</u>.- The quantities listed in table VI are used in the sample calculations presented in table VII. The first seven columns of table VII represent the steps involved in calculating $\frac{\Delta\alpha_r}{\Delta n}$ by use of equation (31). Column 8

lists the resulting values of $\frac{\Delta\alpha_r}{\Delta n}$. Column 9 represents the evaluation of wing-root shear per unit of load factor from equation (33). Column 10 lists the horizontal-tail load per unit of load factor from equation (32).

The wing-root pitching moment and bending moment per unit of load factor are listed in table VII in columns 13 and 14, respectively, as evaluated from equations of the same form as that of equation (33). The chordwise and spanwise centers of pressure x_{cp} and y_{cp} are evaluated in columns 15 and 16, respectively, by dividing the proper moment per unit of load factor by the shear per unit of load factor. Although the centers of pressure at $qm_R = 0$ appear indeterminate, they are equivalent to the centers of pressure for the theoretical rigid wing which may be calculated from the first coefficients in the rows of equations (62), (65), and (68) corresponding to the wing-root position for both wing and external-store parts. For the sample problem,

$$\left(x_{cp}\right)_{qm_R=0} = \frac{-150051 - 411}{714.0 + 14.3} = -206.6$$

and

$$\left(y_{cp}\right)_{qm_R=0} = \frac{214359 + 3798}{714.0 + 14.3} = 299.5$$

Column 17 shows the incremental change in chordwise center of pressure from that at $qm_R = 0$ and is expressed as a percentage of the wing mean aerodynamic chord. Thus, column 17 represents the change in wing-aerodynamic-center location due to flexibility.

Although the actual location of the aerodynamic center of the airplane may be different from that of the wing because of fuselage moments which were unaccounted for in the analysis, the change in aerodynamic-center location with change in qm_R should be practically unaffected by fuselage moments. If the aerodynamic-center location on a rigid wind-tunnel model is known, that location may be taken to be the actual value at $qm_R = 0$. The aerodynamic-center location at any other value of qm_R can be determined with good accuracy by adding the calculated change in aerodynamic-center location to the measured rigid-airplane position.

Although only the shears and bending moments per unit of load factor at the center-line station were listed, those at other wing stations might have been calculated to obtain distributions along the semispan and variations of the distribution with qm_R. Figure 3(a) shows the shear distributions along the semispan per unit of load factor at values of qm_R of 10 and 50 lb/ft^2-deg. Each distribution was divided by the center-line shear. These ratios were plotted to show more clearly the fraction of the total aerodynamic lift carried outboard of any wing station as it varies with qm_R. The corresponding bending moments per unit of shear at the center line are plotted in figure 3(b).

A plot such as that shown in figure 4 is useful in applying the calculations to design conditions of dynamic pressure and Mach number. The parameter qm_R is plotted against q for several constant Mach numbers. The values of m_R used in this plot were obtained in wind-tunnel tests of the particular airplane used as an example.

DISCUSSION

The matrix equation expressing the change in angle-of-attack distribution in terms of the applied angle-of-attack distribution is well suited to solution by matrix iteration. The sample problem included in this report illustrates that only three or four premultiplications of a column by a square matrix will give the elements of a simple expression for final angle of attack resulting from any given initial angle of attack. This expression is readily evaluated for any value of the parameter qm_R. The final spanwise distributions of lift, shear, bending moment, and pitching moment are also easily calculated. The solution can be carried to any desired degree of accuracy, and the degree of accuracy can be easily checked.

Combining the iterative solution with the use of the downwash-factor tables to obtain the matrix of aerodynamic-influence coefficients allowed the results of the sample problem to be calculated in about one-tenth of the estimated time required to obtain the solutions by calculating the elements of $\left[S_1\right]$ and by using matrix inversion.

The conditions under which the method of this report may be used are that the inverse of $\left[\left[S_2\right]\left[S_1\right]^{-1}\left\lfloor 4\,\frac{m_o}{m_R}\right\rfloor\right]$ must exist and that the dominant latent root of this matrix must be real and distinct from its other latent roots. These conditions will be met in any practical problem in aeroelasticity. The convergence of an infinite series of $\left\{C_{\theta,N}\right\}(qm_R)^N$ terms is not a necessary condition, as should be evident from the derivation of the equations pertinent to the iteration procedure and from the fact that solutions were obtained in the sample problem for values of qm_R above the value at which divergence of an infinite series would begin $\left(qm_R \approx 50 \text{ lb/ft}^2\text{-deg}\right)$. Actual wing divergence is indicated if the value of $\frac{1}{1 - \lambda qm_R}$ becomes infinite at some positive value of qm_R. This condition will occur only when the latent root λ is positive, as it would be for a sweptforward wing.

The methods of this report are designed to save time and effort in solving steady-state aeroelastic problems without degrading the accuracy of the results. There is, however, an added flexibility in that the various quantities are obtained as functions of the parameter qm_R and are readily applied to any flight condition of dynamic pressure and Mach number. This flexibility may make the iterative solution desirable even when automatic computing machines are available for matrix inversion.

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National Aeronautics and Space Administration,
Langley Field, Va., January 30, 1959.

APPENDIX A

AERODYNAMIC-INFLUENCE COEFFICIENTS FROM TABLES OF DOWNWASH FACTOR

The derivation of a system of aerodynamic-influence coefficients has been presented in reference 1. The computations involved in determining any one element of the aerodynamic-influence-coefficient matrix $\left[S_1\right]$ are rather lengthy, especially for manually operated desk calculators. Tables and charts for obtaining downwash factors, based on lifting-line theory, have been in existence for some time in reference 2. An element of the $\left[S_1\right]$ matrix is obtainable from these tables by finding the downwash factor at the three-quarter-chord station in question and multiplying this factor by the reciprocal of the semispan of the horseshoe vortex which causes the downwash. The resulting element is identical, within the accuracy of the tables, with that which would be calculated by the method of reference 1.

The tables of downwash factor in reference 2 have been included as tables I and II of this report with certain changes: the "corrector" vortex columns have been replaced by interpolating factors designated as K. Also, the values of downwash factor included in this report have been recalculated and are believed to be correct. The intervals used in the tables are such that linear interpolation will preserve the accuracy to four decimal places. A column for $\frac{\Delta y}{h} = 38$ has been added in table I so that 10 equally spaced vortex stations on each semispan may be used.

The arguments for tables I and II are $\frac{\Delta x}{h}$ and $\frac{\Delta y}{h}$ which are, respectively, the streamwise and spanwise distances from any vortex station to any downwash station, nondimensionalized with respect to the vortex semispan. The argument $\frac{\Delta x}{h}$ is negative when the downwash station is ahead of the vortex station.

It will be a help in using the tables if the values of $\frac{\Delta y}{h}$ and $\frac{\Delta x}{h}$ are arranged in matrix form. A systematic method for determining these $\left[\frac{\Delta y}{h}\right]$ and $\left[\frac{\Delta x}{h}\right]$ matrices for 10 vortex semispan stations is given as follows: With the use of a coordinate system having the origin at the point of intersection of the wing quarter-chord line with the airplane center line, the nondimensional lateral coordinate of any vortex center

$\left(\frac{y}{h}\right)_v$ is given by

$$\left(\frac{y}{h}\right)_v = \frac{\eta_v \frac{b}{2}}{\frac{b}{2N_v}} = N_v \eta_v \tag{A1}$$

where N_v is the number of vortices on both wings and η_v is the vortex spanwise station expressed as a fraction of the semispan. For example, at station 1 in the sample problem, $\eta_v = 0.95$, $N_v = 20$, and $\left(\frac{y}{h}\right)_v = 19$.

The coordinates of all the vortex stations expressed as a row matrix $\left\lfloor\left(\frac{y}{h}\right)_v\right\rfloor$ are, for 10 equally spaced vortex stations on each semispan,

$$\begin{aligned}\left\lfloor\left(\frac{y}{h}\right)_v\right\rfloor &= \lfloor 19 \quad 17 \quad 15 \quad 13 \quad 11 \quad 9 \quad 7 \quad 5 \quad 3 \quad 1 \rfloor \\ &= \left\lfloor \left(\frac{y}{h}\right)_1 \quad \left(\frac{y}{h}\right)_2 \quad \left(\frac{y}{h}\right)_3 \quad \cdots \quad \left(\frac{y}{h}\right)_{10} \right\rfloor\end{aligned} \tag{A2}$$

The nondimensional lateral coordinates of the downwash control points are the same as those of the vortex centers.

Two $\left[\frac{\Delta y}{h}\right]$ matrices will be needed. The first, $\left[\frac{\Delta y}{h}\right]_S$, will contain the nondimensional spanwise distances from each vortex center to every downwash control point on the same wing. The second, $\left[\frac{\Delta y}{h}\right]_O$, will contain the nondimensional spanwise distances from each vortex station to every downwash control point on the opposite wing. The first row of the matrix $\left[\frac{\Delta y}{h}\right]_S$ may be obtained by subtracting $\frac{y}{h}$ of downwash control point 1 from each element of the row matrix $\left\lfloor\left(\frac{y}{h}\right)_v\right\rfloor$. For 10 equally spaced vortex stations on each semispan, the first row of $\left[\frac{\Delta y}{h}\right]_S$ will be

$$\begin{aligned}&\lfloor 19\ 17\ 15\ 13\ 11\ 9\ 7\ 5\ 3\ 1 \rfloor - \lfloor 19\ 19\ 19\ 19\ 19\ 19\ 19\ 19\ 19\ 19 \rfloor \\ &\qquad = \lfloor 0\ 2\ 4\ 6\ 8\ 10\ 12\ 14\ 16\ 18 \rfloor\end{aligned} \tag{A3}$$

Absolute values are used since the downwash factor is independent of the sign of $\frac{\Delta y}{h}$.

The first row of $\left[\frac{\Delta y}{h}\right]_O$ is obtained by adding the two rows to obtain

$$\lfloor 38 \quad 36 \quad 34 \quad 32 \quad 30 \quad 28 \quad 26 \quad 24 \quad 22 \quad 20 \rfloor$$

It will be seen that the matrices $\left[\frac{\Delta y}{h}\right]_S$ and $\left[\frac{\Delta y}{h}\right]_O$ can be obtained by finding the difference and the sum, respectively, of the two matrices

$$\left[\left(\frac{y}{h}\right)_v\right] = \begin{bmatrix} 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \\ 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \\ 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \\ 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \\ 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \\ 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \\ 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \\ 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \\ 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \\ 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \end{bmatrix} \tag{A4}$$

and

$$\left[\left(\frac{y}{h}\right)_d\right] = \begin{bmatrix} 19 & 19 & 19 & 19 & 19 & 19 & 19 & 19 & 19 & 19 \\ 17 & 17 & 17 & 17 & 17 & 17 & 17 & 17 & 17 & 17 \\ 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 \\ 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 \\ 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 \\ 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \\ 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{A5}$$

where $\left[\left(\frac{y}{h}\right)_v\right]$ consists of 10 rows identical to the first row and $\left[\left(\frac{y}{h}\right)_d\right]$ consists of 10 columns identical to the first column. Thus,

$$\left[\frac{\Delta y}{h}\right]_S = \left[\left(\frac{y}{h}\right)_v\right] - \left[\left(\frac{y}{h}\right)_d\right] \tag{A6}$$

which can be expressed as

$$\left[\frac{\Delta y}{h}\right]_S = \begin{bmatrix} 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \\ 2 & 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 \\ 4 & 2 & 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 \\ 6 & 4 & 2 & 0 & 2 & 4 & 6 & 8 & 10 & 12 \\ 8 & 6 & 4 & 2 & 0 & 2 & 4 & 6 & 8 & 10 \\ 10 & 8 & 6 & 4 & 2 & 0 & 2 & 4 & 6 & 8 \\ 12 & 10 & 8 & 6 & 4 & 2 & 0 & 2 & 4 & 6 \\ 14 & 12 & 10 & 8 & 6 & 4 & 2 & 0 & 2 & 4 \\ 16 & 14 & 12 & 10 & 8 & 6 & 4 & 2 & 0 & 2 \\ 18 & 16 & 14 & 12 & 10 & 8 & 6 & 4 & 2 & 0 \end{bmatrix}$$

and

$$\left[\frac{\Delta y}{h}\right]_0 = \left[\left(\frac{y}{h}\right)_v\right] + \left[\left(\frac{y}{h}\right)_d\right] \tag{A7}$$

which can be expressed as

$$\left[\frac{\Delta y}{h}\right]_0 = \begin{bmatrix} 38 & 36 & 34 & 32 & 30 & 28 & 26 & 24 & 22 & 20 \\ 36 & 34 & 32 & 30 & 28 & 26 & 24 & 22 & 20 & 18 \\ 34 & 32 & 30 & 28 & 26 & 24 & 22 & 20 & 18 & 16 \\ 32 & 30 & 28 & 26 & 24 & 22 & 20 & 18 & 16 & 14 \\ 30 & 28 & 26 & 24 & 22 & 20 & 18 & 16 & 14 & 12 \\ 28 & 26 & 24 & 22 & 20 & 18 & 16 & 14 & 12 & 10 \\ 26 & 24 & 22 & 20 & 18 & 16 & 14 & 12 & 10 & 8 \\ 24 & 22 & 20 & 18 & 16 & 14 & 12 & 10 & 8 & 6 \\ 22 & 20 & 18 & 16 & 14 & 12 & 10 & 8 & 6 & 4 \\ 20 & 18 & 16 & 14 & 12 & 10 & 8 & 6 & 4 & 2 \end{bmatrix}$$

If using fewer than 10 equally spaced vortex stations on each semispan is desired, the matrices $\left[\left(\frac{y}{h}\right)_v\right]$ and $\left[\left(\frac{y}{h}\right)_d\right]$ can be modified to suit by omitting the required number of rows and columns from the top and left.

By using the same coordinate origin as before, a matrix of the nondimensional longitudinal coordinates of the vortex centers $\left[\left(\frac{x}{h}\right)_v\right]$ when the quarter-chord locus is a straight line is

$$\left[\left(\frac{x}{h}\right)_v\right] = \left[\left(\frac{y}{h}\right)_v\right]\tan \Lambda_{c/4} \tag{A8}$$

For the downwash control points on the three-quarter-chord line,

$$\left[\left(\frac{x}{h}\right)_d\right] = \left[\left(\frac{y}{h}\right)_d\right]\tan \Lambda_{3c/4} + \left[\frac{c_r}{2h}\right] \tag{A9}$$

where each element in the matrix $\left[\frac{c_r}{2h}\right]$ is half the wing-root chord divided by the vortex semispan h.

The correct matrix of nondimensional longitudinal distances between vortex and control points $\left[\frac{\Delta x}{h}\right]$ is given by the equation

$$\left[\frac{\Delta x}{h}\right] = \left[\left(\frac{x}{h}\right)_d\right] - \left[\left(\frac{x}{h}\right)_v\right] \tag{A10}$$

or

$$\left[\frac{\Delta x}{h}\right] = \left[\left(\frac{y}{h}\right)_d\right] \tan \Lambda_{3c/4} + \left[\frac{c_r}{2h}\right] - \left[\left(\frac{y}{h}\right)_v\right] \tan \Lambda_{c/4} \tag{A11}$$

Only one $\left[\frac{\Delta x}{h}\right]$ matrix is required since chordwise distances from a vortex to any control point are the same whether the vortex and control point are on the same wing or on opposite wings.

Two complete matrices of downwash factors will be obtained: the first by using $\left[\frac{\Delta y}{h}\right]_S$ and $\left[\frac{\Delta x}{h}\right]$ and the second by using $\left[\frac{\Delta y}{h}\right]_O$ and $\left[\frac{\Delta x}{h}\right]$ as arguments in tables I and II. The symmetrical-load matrix $\left[S_1\right]$ is calculated by adding these two matrices of downwash factors and multiplying each element by the reciprocal of the vortex semispan $\frac{1}{h}$. For antisymmetric loadings, the value of $\left[S_1\right]$ is the difference between the two downwash-factor matrices multiplied by $\frac{1}{h}$.

Tables I and II are useful only for equally spaced horseshoe vortices. Unequal spacing would require interpolation in both $\frac{\Delta y}{h}$ and $\frac{\Delta x}{h}$, which is not practical.

The columns in tables I(a) and II entitled "K" contain interpolating factors which were calculated by dividing each increment in downwash factor by the corresponding increment in $\frac{\Delta x}{h}$. Each factor is listed in the same row with the smaller of the two $\frac{\Delta x}{h}$ values used to calculate it. In order to obtain the downwash factor F corresponding to any value of $\frac{\Delta x}{h}$, find the value of downwash factor F' corresponding to the next smaller value of $\frac{\Delta x'}{h}$ in the table (in absolute value). Add to F' the product of $\frac{\Delta x}{h}$ and the interpolating factor and subtract the product of $\frac{\Delta x'}{h}$ and the interpolating factor. Thus,

$$F = F' + \frac{\Delta x}{h} K - \frac{\Delta x'}{h} K \tag{A12}$$

As an example, suppose it is required that a value of F be obtained that corresponds to $\frac{\Delta x}{h} = 1.3421$ and $\frac{\Delta y}{h} = 0$. By using table II, the next smaller value of $\frac{\Delta x}{h}$ is 1.34 which equals $\frac{\Delta x'}{h}$ for which the value of F' is 4.4956 and the interpolating factor is -0.660. The required value of F is

$$F = 4.4956 + 1.3421(-0.660) - 1.3400(-0.660)$$

$$F = 4.4942$$

With the inclusion of the interpolating factors, all information necessary to obtain the downwash factors corresponding to a given value of $\frac{\Delta x}{h}$ lies in one row of the tables, the row corresponding to the value of $\frac{\Delta x}{h}$ that is next smaller than the given value. Interpolation can be done without writing or remembering any intermediate steps with any calculating machine capable of accumulative multiplication. Where the interpolating factors have been omitted $\left(\frac{\Delta y}{h} > 10\right)$, interpolation by inspection will be sufficiently accurate.

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TABLE I.- DOWNWASH FACTOR F[1]

(a) Values of F for $\Delta y/h = 0$ to ± 10

$\Delta x/h$	$\Delta y/h = 0$		$\Delta y/h = \pm 2$		$\Delta y/h = \pm 4$		$\Delta y/h = \pm 6$		$\Delta y/h = \pm 8$		$\Delta y/h = \pm 10$	
	F	K	F	K	F	K	F	K	F	K	F	K
-30			-0.0011	--------	-0.0011	--------	-0.0011	--------	-0.0011	--------	-0.0010	--------
-25			-.0016	--------	-.0016	--------	-.0015	--------	-.0015	--------	-.0014	--------
-20			-0.0025	-0.00018	-0.0024	-0.00018	-0.0023	-0.00018	-0.0022	--------	-0.0021	--------
-18			-.0031	-.00030	-.0030	-.00030	-.0029	-.00030	-.0027	-0.00025	-.0025	-0.00020
-16			-.0039	-.00040	-.0037	-.00040	-.0035	-.00030	-.0033	-.00030	-.0030	-.00025
-14			-.0050	-.00055	-.0048	-.00055	-.0045	-.00050	-.0041	-.00040	-.0037	-.00035
-12			-.0068	-.00090	-.0064	-.00080	-.0059	-.00070	-.0053	-.00060	-.0046	-.00040
-10			-0.0097	-0.00145	-0.0089	-0.00125	-0.0079	-0.00100	-0.0069	-0.00080	-0.0059	-0.00060
-9			-.0119	-.00220	-.0108	-.00190	-.0093	-.00140	-.0079	-.00100	-.0066	-.00070
-8			-.0149	-.00300	-.0132	-.00240	-.0111	-.00180	-.0092	-.00130	-.0075	-.00090
-7			-.0192	-.00430	-.0165	-.00330	-.0134	-.00230	-.0107	-.00150	-.0086	-.00110
-6			-.0256	-.00640	-.0210	-.00450	-.0163	-.00290	-.0125	-.00180	-.0097	-.00110
-5			-0.0356	-0.01000	-0.0274	-0.00640	-0.0201	-0.00380	-0.0148	-0.00230	-0.0111	-0.00140
-4.5			-.0429	-.01460	-.0317	-.00860	-.0224	-.00460	-.0160	-.00240	-.0119	-.00160
-4.0					-.0368	-.01020	-.0249	-.00500	-.0174	-.00280	-.0126	-.00140
-3.5					-.0431	-.01260	-.0278	-.00580	-.0189	-.00300	-.0135	-.00180
-3.0					-.0507	-.01520	-.0311	-.00660	-.0205	-.00320	-.0143	-.00160
-2.5					-.0599	-.01841	-.0347	-.00720	-.0222	-.00340	-.0153	-.00200
-2.0	(2)				-0.0709	-0.02200	-0.0386	-0.00780	-0.0239	-0.00340	-0.0162	-0.00180
-1.8					-.0759	-.02500	-.0403	-.00850	-.0247	-.00400	-.0166	-.00200
-1.6					-.0812	-.02650	-.0420	-.00850	-.0254	-.00350	-.0170	-.00200
-1.4					-.0869	-.02850	-.0438	-.00900	-.0262	-.00400	-.0174	-.00200
-1.2					-.0928	-.02950	-.0456	-.00900	-.0270	-.00400	-.0178	-.00200
-1.0					-.0990	-.03100	-.0475	-.00950	-.0278	-.00400	-.0182	-.00200
-.8					-.1056	-.03300	-.0494	-.00950	-.0285	-.00350	-.0186	-.00200
-0.6					-0.1123	-0.03350	-0.0513	-0.00950	-0.0293	-0.00400	-0.0190	-0.00200
-.5					-.1156	-.03300	-.0523	-.01000	-.0298	-.00500	-.0192	-.00200
-.4					-.1192	-.03600	-.0532	-.00900	-.0302	-.00400	-.0194	-.00200
-0.3					-0.1227	-0.03500	-0.0542	-0.01000	-0.0305	-0.00300	-0.0196	-0.00200
-.25					-.1245	-.03600	-.0547	-.01000	-.0308	-.00600	-.0197	-.00200
-.20					-.1262	-.03400	-.0552	-.01000	-.0310	-.00400	-.0198	-.00200
-.15					-.1280	-.03600	-.0557	-.01000	-.0312	-.00400	-.0199	-.00200
-.10					-.1298	-.03600	-.0561	-.00800	-.0314	-.00400	-.0200	-.00200
-.05					-.1315	-.03400	-.0565	-.00800	-.0316	-.00400	-.0201	-.00200
0			(3)		-0.1333	-0.03600	-0.0571	-0.01200	-0.0318	-0.00400	-0.0202	-0.00200
0.05					-0.1351	-0.03400	-0.0577	-0.00800	-0.0320	-0.00400	-0.0203	-0.00200
.10					-.1368	-.03800	-.0581	-.01000	-.0322	-.00400	-.0204	-.00200
.15					-.1387	-.03400	-.0586	-.01000	-.0324	-.00400	-.0205	-.00200
.20					-.1404	-.03600	-.0591	-.01000	-.0326	-.00400	-.0206	-.00200
.25					-.1422	-.03600	-.0596	-.01000	-.0328	-.00400	-.0207	-.00200
.3					-.1440	-.03500	-.0601	-.00900	-.0330	-.00400	-.0208	-.00200
0.4					-0.1475	-0.03500	-0.0610	-0.01000	-0.0334	-0.00400	-0.0210	-0.00200
.5					-.1510	-.03400	-.0620	-.01000	-.0338	-.00400	-.0212	-.00200
.6					-.1544	-.03350	-.0630	-.00950	-.0342	-.00400	-.0214	-.00200
0.8					-0.1611	-0.03250	-0.0649	-0.00950	-0.0350	-0.00350	-0.0218	-0.00200
1.0	(3)				-.1676	-.03150	-.0668	-.00950	-.0357	-.00400	-.0222	-.00200
1.2					-.1739	-.02950	-.0687	-.00900	-.0365	-.00400	-.0226	-.00200
1.4					-.1798	-.02800	-.0705	-.00850	-.0373	-.00400	-.0230	-.00200
1.6					-.1854	-.02700	-.0722	-.00900	-.0381	-.00350	-.0234	-.00200
1.8					-.1908	-.02450	-.0740	-.00850	-.0388	-.00400	-.0238	-.00200
2.0					-.1957	-.02220	-.0757	-.00780	-.0396	-.00360	-.0242	-.00180
2.5					-0.2068	-0.01840	-0.0796	-0.00720	-0.0414	-0.00320	-0.0251	-0.00200
3.0					-.2160	-.01520	-.0832	-.00660	-.0430	-.00320	-.0261	-.00160
3.5					-.2236	-.01240	-.0865	-.00580	-.0446	-.00300	-.0269	-.00180
4.0					-.2298	-.01040	-.0894	-.00500	-.0461	-.00280	-.0278	-.00160
4.5	4.0488	0.01840	-1.2905	-0.01440	-.2350	-.00840	-.0919	-.00460	-.0475	-.00240	-.0286	-.00140
5	4.0396	.01200	-1.2977	-.01010	-.2392	-.00650	-.0942	-.00380	-.0487	-.00230	-.0293	-.00140
6	4.0276	0.00730	-1.3078	-0.00640	-0.2457	-0.00450	-0.0980	-0.00290	-0.0510	-0.00180	-0.0307	-0.00120
7	4.0203	.00470	-1.3142	-.00430	-.2502	-.00330	-.1009	-.00230	-.0528	-.00150	-.0319	-.00100
8	4.0156	.00330	-1.3185	-.00300	-.2535	-.00240	-.1032	-.00180	-.0543	-.00130	-.0329	-.00090
9	4.0123	.00230	-1.3215	-.00220	-.2559	-.00180	-.1050	-.00140	-.0556	-.00100	-.0338	-.00070
10	4.0100	.00155	-1.3237	-.00140	-.2577	-.00130	-.1064	-.00100	-.0566	-.00085	-.0345	-.00065
12	4.0069	0.00095	-1.3265	-0.00090	-0.2603	-0.00080	-0.1084	-0.00070	-0.0583	-0.00050	-0.0358	-0.00045
14	4.0050	.00055	-1.3283	-.00060	-.2619	-.00050	-.1098	-.00050	-.0594	-.00040	-.0367	-.00035
16	4.0039	.00040	-1.3295	-.00040	-.2629	-.00040	-.1108	-.00030	-.0602	-.00030	-.0374	-.00025
18	4.0031	.00030	-1.3303	-.00030	-.2637	-.00025	-.1114	-.00025	-.0608	-.00025	-.0379	-.00020
20	4.0025	.00018	-1.3309	-.00018	-.2642	-.00018	-.1119	-.00018	-.0613	--------	-.0383	--------
25	4.0016	-------	-1.3318	--------	-0.2651	--------	-0.1128	--------	-0.0620	--------	-0.0390	--------
30	4.0011	-------	-1.3322	--------	-.2656	--------	-.1132	--------	-.0624	--------	-.0394	--------

[1]Values of F are obtained from reference 2 with slight modification.

[2]Values in this range are not used.

[3]Values in this range are given in table II.

TABLE I.- DOWNWASH FACTOR F - Concluded

(b) Values of F for $\Delta y/h = \pm 12$ to ± 38

$\Delta x/h$	F for values of $\Delta y/h$ of -													
	±12	±14	±16	±18	±20	±22	±24	±26	±28	±30	±32	±34	±36	±38
-30	-0.0010	-0.0010	-0.0009	-0.0009	-0.0008	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006
-25	-.0014	-.0013	-.0012	-.0012	-.0011	-.0010	-.0010	-.0009	-.0009	-.0008	-.0007	-.0007	-.0007	-.0007
-20	-0.0020	-0.0019	-0.0017	-0.0016	-0.0015	-0.0013	-0.0012	-0.0012	-0.0011	-0.0010	-0.0009	-0.0009	-0.0008	-0.0008
-18	-.0023	-.0022	-.0020	-.0018	-.0017	-.0015	-.0014	-.0013	-.0012	-.0011	-.0010	-.0009	-.0009	-.0008
-16	-.0028	-.0025	-.0023	-.0021	-.0019	-.0017	-.0015	-.0014	-.0013	-.0012	-.0011	-.0010	-.0009	-.0008
-14	-.0034	-.0030	-.0027	-.0024	-.0021	-.0019	-.0017	-.0016	-.0014	-.0013	-.0012	-.0011	-.0010	-.0009
-12	-.0041	-.0036	-.0031	-.0028	-.0024	-.0022	-.0019	-.0017	-.0015	-.0014	-.0013	-.0012	-.0011	-.0010
-10	-0.0050	-0.0043	-0.0037	-0.0032	-0.0028	-0.0024	-0.0021	-0.0019	-0.0017	-0.0015	-0.0014	-0.0012	-0.0011	-0.0010
-9	-.0056	-.0047	-.0040	-.0034	-.0030	-.0026	-.0023	-.0020	-.0018	-.0016	-.0014	-.0013	-.0012	-.0010
-8	-.0062	-.0052	-.0043	-.0037	-.0031	-.0027	-.0024	-.0021	-.0019	-.0017	-.0015	-.0013	-.0012	-.0012
-7	-.0069	-.0057	-.0047	-.0039	-.0034	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014	-.0012	-.0012
-6	-.0077	-.0062	-.0051	-.0042	-.0036	-.0031	-.0026	-.0023	-.0020	-.0018	-.0016	-.0014	-.0013	-.0012
-5	-0.0086	-0.0068	-0.0055	-0.0045	-0.0038	-0.0032	-0.0028	-0.0024	-0.0021	-0.0019	-0.0017	-0.0015	-0.0013	-0.0012
-4.5	-.0091	-.0071	-.0057	-.0047	-.0039	-.0033	-.0028	-.0025	-.0021	-.0019	-.0017	-.0015	-.0014	-.0012
-4.0	-.0095	-.0074	-.0059	-.0048	-.0040	-.0034	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014	-.0013
-3.5	-.0101	-.0078	-.0062	-.0050	-.0041	-.0035	-.0030	-.0026	-.0022	-.0020	-.0017	-.0016	-.0014	-.0013
-3.0	-.0106	-.0081	-.0064	-.0052	-.0043	-.0036	-.0030	-.0026	-.0023	-.0020	-.0018	-.0016	-.0014	-.0014
-2.5	-.0111	-.0085	-.0066	-.0053	-.0044	-.0037	-.0031	-.0027	-.0023	-.0020	-.0018	-.0016	-.0014	-.0014
-2.0	-0.0117	-0.0088	-0.0069	-0.0055	-0.0045	-0.0038	-0.0032	-0.0027	-0.0024	-0.0021	-0.0018	-0.0016	-0.0015	-0.0014
-1.8	-.0119	-.0089	-.0070	-.0056	-.0046	-.0038	-.0032	-.0028	-.0024	-.0021	-.0018	-.0016	-.0015	-.0014
-1.6	-.0121	-.0091	-.0071	-.0056	-.0046	-.0038	-.0032	-.0028	-.0024	-.0021	-.0019	-.0017	-.0015	-.0014
-1.4	-.0124	-.0092	-.0072	-.0057	-.0047	-.0039	-.0033	-.0028	-.0024	-.0021	-.0019	-.0017	-.0015	-.0014
-1.2	-.0126	-.0094	-.0073	-.0058	-.0047	-.0039	-.0033	-.0028	-.0024	-.0021	-.0019	-.0017	-.0015	-.0014
-1.0	-.0128	-.0095	-.0074	-.0058	-.0048	-.0040	-.0033	-.0028	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014
-.8	-.0131	-.0097	-.0075	-.0059	-.0048	-.0040	-.0034	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014
-0.6	-0.0133	-0.0098	-0.0075	-0.0060	-0.0049	-0.0040	-0.0034	-0.0029	-0.0025	-0.0022	-0.0019	-0.0017	-0.0015	-0.0014
-.5	-.0134	-.0099	-.0076	-.0060	-.0049	-.0040	-.0034	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014
-.4	-.0135	-.0100	-.0076	-.0061	-.0049	-.0041	-.0034	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014
-0.3	-0.0136	-0.0100	-0.0077	-0.0061	-0.0049	-0.0041	-0.0034	-0.0029	-0.0025	-0.0022	-0.0019	-0.0017	-0.0015	-0.0014
-.25	-.0137	-.0101	-.0077	-.0061	-.0050	-.0041	-.0034	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014
-.20	-.0137	-.0101	-.0077	-.0061	-.0050	-.0041	-.0034	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014
-.15	-.0138	-.0102	-.0078	-.0061	-.0050	-.0041	-.0035	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014
-.10	-.0139	-.0102	-.0078	-.0062	-.0050	-.0041	-.0035	-.0030	-.0025	-.0022	-.0019	-.0017	-.0015	-.0014
-.05	-.0140	-.0102	-.0078	-.0062	-.0050	-.0041	-.0035	-.0030	-.0025	-.0022	-.0020	-.0017	-.0015	-.0014
0	-0.0140	-0.0103	-0.0078	-0.0062	-0.0050	-0.0041	-0.0035	-0.0030	-0.0026	-0.0022	-0.0020	-0.0017	-0.0015	-0.0014
0.05	-0.0140	-0.0103	-0.0079	-0.0062	-0.0050	-0.0042	-0.0035	-0.0030	-0.0026	-0.0022	-0.0020	-0.0017	-0.0015	-0.0014
.10	-.0141	-.0103	-.0079	-.0062	-.0050	-.0042	-.0035	-.0030	-.0026	-.0022	-.0020	-.0017	-.0015	-.0014
.15	-.0142	-.0104	-.0079	-.0062	-.0050	-.0042	-.0035	-.0030	-.0026	-.0022	-.0020	-.0017	-.0016	-.0014
.20	-.0142	-.0104	-.0079	-.0063	-.0051	-.0042	-.0035	-.0030	-.0026	-.0022	-.0020	-.0017	-.0016	-.0014
.25	-.0143	-.0104	-.0080	-.0063	-.0051	-.0042	-.0035	-.0030	-.0026	-.0022	-.0020	-.0017	-.0016	-.0014
.3	-.0144	-.0105	-.0080	-.0063	-.0051	-.0042	-.0035	-.0030	-.0026	-.0022	-.0020	-.0017	-.0016	-.0014
0.4	-0.0145	-0.0105	-0.0080	-0.0063	-0.0051	-0.0042	-0.0035	-0.0030	-0.0026	-0.0023	-0.0020	-0.0018	-0.0016	-0.0014
.5	-.0146	-.0106	-.0081	-.0064	-.0051	-.0042	-.0036	-.0030	-.0026	-.0023	-.0020	-.0018	-.0016	-.0014
.6	-.0147	-.0107	-.0081	-.0064	-.0052	-.0043	-.0036	-.0030	-.0026	-.0023	-.0020	-.0018	-.0016	-.0014
0.8	-0.0149	-0.0109	-0.0082	-0.0065	-0.0052	-0.0043	-0.0036	-0.0031	-0.0026	-0.0023	-0.0020	-0.0018	-0.0016	-0.0014
1.0	-.0152	-.0110	-.0083	-.0065	-.0053	-.0043	-.0036	-.0031	-.0026	-.0023	-.0020	-.0018	-.0016	-.0014
1.2	-.0154	-.0111	-.0084	-.0066	-.0053	-.0044	-.0037	-.0031	-.0027	-.0023	-.0020	-.0018	-.0016	-.0014
1.4	-.0156	-.0113	-.0085	-.0067	-.0054	-.0044	-.0037	-.0031	-.0027	-.0023	-.0020	-.0018	-.0016	-.0014
1.6	-.0159	-.0114	-.0086	-.0067	-.0054	-.0044	-.0037	-.0031	-.0027	-.0023	-.0021	-.0018	-.0016	-.0014
1.8	-.0161	-.0116	-.0087	-.0068	-.0055	-.0045	-.0037	-.0032	-.0027	-.0024	-.0021	-.0018	-.0016	-.0014
2.0	-.0163	-.0117	-.0088	-.0069	-.0055	-.0045	-.0038	-.0032	-.0027	-.0024	-.0021	-.0018	-.0016	-.0014
2.5	-0.0169	-0.0121	-0.0091	-0.0070	-0.0056	-0.0046	-0.0038	-0.0032	-0.0028	-0.0024	-0.0021	-0.0019	-0.0017	-0.0014
3.0	-.0174	-.0124	-.0093	-.0072	-.0058	-.0047	-.0039	-.0033	-.0028	-.0024	-.0021	-.0019	-.0017	-.0014
3.5	-.0179	-.0128	-.0095	-.0074	-.0059	-.0048	-.0040	-.0034	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015
4.0	-.0184	-.0131	-.0098	-.0075	-.0060	-.0049	-.0041	-.0034	-.0029	-.0025	-.0022	-.0019	-.0017	-.0015
4.5	-.0189	-.0134	-.0100	-.0077	-.0061	-.0050	-.0041	-.0035	-.0030	-.0026	-.0022	-.0020	-.0017	-.0016
5	-.0194	-.0137	-.0102	-.0079	-.0062	-.0051	-.0042	-.0035	-.0030	-.0026	-.0023	-.0020	-.0018	-.0016
6	-0.0203	-0.0143	-0.0106	-0.0082	-0.0065	-0.0052	-0.0043	-0.0036	-0.0031	-0.0027	-0.0023	-0.0020	-0.0018	-0.0016
7	-.0211	-.0149	-.0110	-.0084	-.0067	-.0054	-.0045	-.0037	-.0032	-.0027	-.0024	-.0021	-.0018	-.0016
8	-.0218	-.0154	-.0114	-.0087	-.0069	-.0056	-.0046	-.0038	-.0033	-.0028	-.0024	-.0021	-.0019	-.0016
9	-.0224	-.0158	-.0117	-.0090	-.0071	-.0057	-.0047	-.0039	-.0033	-.0029	-.0025	-.0022	-.0019	-.0017
10	-.0230	-.0162	-.0120	-.0092	-.0073	-.0059	-.0048	-.0040	-.0034	-.0029	-.0025	-.0022	-.0020	-.0017
12	-0.0239	-0.0170	-0.0126	-0.0096	-0.0076	-0.0061	-0.0050	-0.0042	-0.0036	-0.0031	-0.0026	-0.0023	-0.0020	-0.0018
14	-.0246	-.0175	-.0130	-.0100	-.0079	-.0064	-.0052	-.0044	-.0037	-.0032	-.0027	-.0024	-.0021	-.0019
16	-.0252	-.0180	-.0134	-.0103	-.0081	-.0066	-.0054	-.0045	-.0038	-.0033	-.0028	-.0025	-.0022	-.0020
18	-.0256	-.0184	-.0137	-.0106	-.0084	-.0068	-.0056	-.0047	-.0039	-.0034	-.0029	-.0025	-.0022	-.0020
20	-.0260	-.0187	-.0140	-.0108	-.0086	-.0069	-.0057	-.0048	-.0040	-.0035	-.0030	-.0026	-.0023	-.0020
25	-0.0266	-0.0192	-0.0145	-0.0112	-0.0089	-0.0073	-0.0060	-0.0050	-0.0043	-0.0037	-0.0032	-0.0028	-0.0024	-0.0021
30	-.0270	-.0196	-.0148	-.0115	-.0092	-.0075	-.0062	-.0052	-.0044	-.0038	-.0033	-.0029	-.0025	-.0022

TABLE II.- DOWNWASH FACTOR F FOR $\Delta y/h$ = 0 and ±2

[Expansion of sections (3) of table I(a)]

$\Delta y/h$ = 0					
$\Delta x/h$	F	K	$\Delta x/h$	F	K
0.20	12.1978	-46.580	0.85	5.0883	-2.060
.21	11.7321	-42.420	.86	5.0677	-2.050
.22	11.3079	-38.570	.87	5.0472	-1.960
.23	10.9222	-35.230	.88	5.0276	-1.930
.24	10.5699	-32.420	.89	5.0083	-1.870
.25	10.2457	-29.820	.90	4.9896	-1.800
.26	9.9475	-27.460	.91	4.9716	-1.740
.27	9.6729	-25.490	.92	4.9542	-1.740
.28	9.4180	-23.780	.93	4.9368	-1.670
.29	9.1802	-21.990	.94	4.9201	-1.640
.30	8.9603	-20.610	.95	4.9037	-1.570
.31	8.7542	-19.180	.96	4.8880	-1.560
.32	8.5624	-18.010	.97	4.8724	-1.490
.33	8.3823	-16.870	.98	4.8575	-1.500
.34	8.2136	-15.980	.99	4.8425	-1.400
.35	8.0538	-14.920	1.00	4.8285	-1.390
.36	7.9046	-14.130	1.02	4.8007	-1.310
.37	7.7633	-13.300	1.04	4.7745	-1.245
.38	7.6303	-12.620	1.06	4.7496	-1.200
.39	7.5041	-11.910	1.08	4.7256	-1.140
.40	7.3850	-11.260	1.10	4.7028	-1.070
.41	7.2724	-10.750	1.12	4.6814	-1.060
.42	7.1649	-10.210	1.14	4.6602	-.990
.43	7.0628	-9.680	1.16	4.6404	-.950
.44	6.9660	-9.220	1.18	4.6214	-.885
.45	6.8738	-8.810	1.20	4.6037	-.875
.46	6.7857	-8.410	1.22	4.5862	-.845
.47	6.7016	-8.030	1.24	4.5693	-.800
.48	6.6213	-7.610	1.26	4.5533	-.770
.49	6.5452	-7.290	1.28	4.5379	-.740
.50	6.4723	-7.000	1.30	4.5232	-.710
.51	6.4023	-6.760	1.32	4.5090	-.670
.52	6.3347	-6.400	1.34	4.4956	-.660
.53	6.2707	-6.140	1.36	4.4824	-.615
.54	6.2093	-5.910	1.38	4.4701	-.605
.55	6.1502	-5.690	1.40	4.4580	-.600
.56	6.0933	-5.460	1.42	4.4460	-.560
.57	6.0387	-5.330	1.44	4.4348	-.540
.58	5.9864	-5.070	1.46	4.4240	-.520
.59	5.9357	-4.840	1.48	4.4136	-.490
.60	5.8873	-4.680	1.50	4.4038	-.478
.61	5.8405	-4.500	1.55	4.3799	-.430
.62	5.7955	-4.350	1.60	4.3585	-.394
.63	5.7520	-4.190	1.65	4.3388	-.370
.64	5.7101	-4.050	1.70	4.3203	-.334
.65	5.6696	-3.860	1.75	4.3036	-.316
.66	5.6310	-3.810	1.80	4.2878	-.288
.67	5.5929	-3.600	1.85	4.2734	-.262
.68	5.5569	-3.530	1.90	4.2603	-.254
.69	5.5216	-3.420	1.95	4.2476	-.230
.70	5.4874	-3.270	2.0	4.2361	-.212
.71	5.4547	-3.170	2.1	4.2149	-.177
.72	5.4230	-3.090	2.2	4.1972	-.167
.73	5.3921	-3.000	2.3	4.1805	-.136
.74	5.3621	-2.880	2.4	4.1669	-.129
.75	5.3333	-2.810	2.5	4.1540	-.114
.76	5.3052	-2.690	2.6	4.1426	-.096
.77	5.2783	-2.670	2.7	4.1330	-.093
.78	5.2516	-2.540	2.8	4.1237	-.084
.79	5.2262	-2.480	2.9	4.1153	-.071
.80	5.2014	-2.390	3.0	4.1082	-.065
.81	5.1175	-2.350	3.2	4.0952	-.054
.82	5.1540	-2.260	3.4	4.0843	-.044
.83	5.1314	-2.180	3.6	4.0755	-.036
.84	5.1096	-2.130	3.8	4.0684	-.034
			4.0	4.0616	------

$\Delta y/h$ = ±2		
$\Delta x/h$	F	K
-4.0	-0.0526	-------
-3.8	-.0571	-0.0230
-3.6	-.0629	-.0290
-3.4	-.0690	-.0305
-3.2	-.0759	-.0345
-3.0	-.0839	-.0400
-2.8	-.0933	-.0470
-2.6	-.1042	-.0545
-2.4	-.1167	-.0625
-2.2	-.1316	-.0745
-2.0	-.1494	-.0890
-1.9	-.1595	-.1010
-1.8	-.1706	-.1110
-1.7	-.1825	-.1190
-1.6	-.1956	-.1310
-1.5	-.2101	-.1450
-1.4	-.2255	-.1540
-1.3	-.2434	-.1790
-1.2	-.2622	-.1880
-1.1	-.2834	-.2120
-1.0	-.3064	-.2300
-.95	-.3188	-.2480
-.90	-.3318	-.2600
-.85	-.3453	-.2700
-.80	-.3596	-.2860
-.75	-.3742	-.2920
-.70	-.3899	-.3140
-.65	-.4058	-.3180
-.60	-.4226	-.3360
-.55	-.4400	-.3480
-.50	-.4580	-.3600
-.45	-.4768	-.3760
-.40	-.4963	-.3900
-.35	-.5161	-.3960
-.30	-.5365	-.4080
-.25	-.5574	-.4180
-.20	-.5787	-.4260
-.15	-.6004	-.4340
-.10	-.6223	-.4380
-.05	-.6445	-.4440
0	-.6667	-.4440
.05	-.6889	-.4420
.10	-.7110	-.4380
.15	-.7329	-.4340
.20	-.7546	-.4260
.25	-.7759	-.4180
.30	-.7968	-.4080
.35	-.8172	-.3960
.40	-.8370	-.3900
.45	-.8565	-.3760
.50	-.8753	-.3600
.55	-.8933	-.3480
.60	-.9107	-.3360
.65	-.9275	-.3180
.70	-.9434	-.3140
.75	-.9591	-.2920
.80	-.9737	-.2860
.85	-.9880	-.2700
.90	-1.0015	-.2600
.95	-1.0145	-.2480
1.0	-1.0269	-.2300
1.1	-1.0499	-.2120
1.2	-1.0711	-.1880
1.3	-1.0899	-.1790
1.4	-1.1078	-.1540
1.5	-1.1232	-.1450
1.6	-1.1377	-.1310
1.7	-1.1508	-.1190
1.8	-1.1627	-.1110
1.9	-1.1738	-.1010
2.0	-1.1839	-.0890
2.2	-1.2017	-.0745
2.4	-1.2166	-.0625
2.6	-1.2291	-.0545
2.8	-1.2400	-.0470
3.0	-1.2494	-.0400
3.2	-1.2574	-.0345
3.4	-1.2643	-.0305
3.6	-1.2704	-.0290
3.8	-1.2762	-.0230
4.0	-1.2808	-------

TABLE III.- PARABOLIC INTEGRATING MATRIX $\|I_{int,p}\|$

$\frac{y}{b/2}$	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05
0.95	0.029167	-0.001389	0	0	0	0	0	0	0	0
.85	.070833	.065278	-.008333	0	0	0	0	0	0	0
.75	.070833	.106944	.058333	-.008333	0	0	0	0	0	0
.65	.070833	.106944	.100000	.058333	-.008333	0	0	0	0	0
.55	.070833	.106944	.100000	.100000	.058333	-.008333	0	0	0	0
.45	.070833	.106944	.100000	.100000	.100000	.058333	-.008333	0	0	0
.35	.070833	.106944	.100000	.100000	.100000	.100000	.058333	-.008333	0	0
.25	.070833	.106944	.100000	.100000	.100000	.100000	.100000	.058333	-.008333	0
.15	.070833	.106944	.100000	.100000	.100000	.100000	.100000	.100000	.058333	-.008333
.05	.070833	.106944	.100000	.100000	.100000	.100000	.100000	.091667	.125000	.033333
0	.070833	.106944	.100000	.100000	.100000	.100000	.100000	.100000	.095833	.104167

TABLE IV.- CALCULATION OF $\{C_\theta\}$ COEFFICIENTS AND $\frac{1}{1 - \lambda q m_R}$ FOR $q m_R = 100 \ \frac{lb}{ft^2\text{-deg}}$

FOR A UNIT SYMMETRICAL ADDITIONAL ANGLE OF ATTACK

Span station		α_i, deg	$\{C_{\theta,1}\}$	$\{C_{\theta,2}\}$	$\{C_{\theta,3}\}$	$\{C_{\theta,4}\}$	$\{E\}_{N=2}$	$\{E\}_{N=3}$
Number	Percent of semispan						$q m_R = 100 \ \frac{lb}{ft^2\text{-deg}}$	
1	95	1	-0.025671	0.00051241	-0.0000101164	0.000000199609	0.0024	0.00022
2	85	1	-.025442	.00050743	-.0000100176	.000000197659	.0022	.00020
3	75	1	-.024374	.00048428	-.0000095584	.000000188597	.0014	.00015
4	65	1	-.022198	.00043760	-.0000086333	.000000170340	0	0
5	55	1	-.019028	.00037071	-.0000073092	.000000144209	-.0015	-.00019
6	45	1	-.015256	.00029291	-.0000057710	.000000113856	-.0026	-.00031
7	35	1	-.011447	.00021565	-.0000042449	.000000083745	-.0032	-.00031
8	25	1	-.007690	.00014141	-.0000027801	.000000054843	-.0033	-.00033
9	15	1	-.003622	.00006550	-.0000012868	.000000025384	-.0018	-.00018
10	5	1	0	0	0	0	0	0
$\frac{1}{1 - \lambda q m_R}$				[a]0.336375	[a]0.336354			

[a]Trial values at $q m_R = 100 \ \frac{lb}{ft^2\text{-deg}}$.

TABLE V.- TRAPEZOIDAL INTEGRATING MATRIX $\left\|I_{int,T}\right\|$

$\frac{y}{b/2}$	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05
0.95	0.05	0	0	0	0	0	0	0	0	0
.85	.1	.05	0	0	0	0	0	0	0	0
.75	.1	.1	.05	0	0	0	0	0	0	0
.65	.1	.1	.1	.05	0	0	0	0	0	0
.55	.1	.1	.1	.1	.05	0	0	0	0	0
.45	.1	.1	.1	.1	.1	.05	0	0	0	0
.35	.1	.1	.1	.1	.1	.1	.05	0	0	0
.25	.1	.1	.1	.1	.1	.1	.1	.05	0	0
.15	.1	.1	.1	.1	.1	.1	.1	.1	.05	0
.05	.1	.1	.1	.1	.1	.1	.1	.1	.1	.05
0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1

TABLE VI.- WING-ROOT SHEAR, BENDING-MOMENT, AND PITCHING-MOMENT COMPONENTS

qm_R, $\frac{lb}{ft^2\text{-deg}}$	$L_{\alpha,r}$, lb/deg	$M_{b,\alpha,r}$, in-lb/deg	$M_{Y,\alpha,r}$, in-lb/deg	$L_{n,r}$, lb/g	$M_{b,n,r}$, in-lb/g	$M_{Y,n,r}$, in-lb/g
	Wing-root values derived from equations -					
	(62)	(65)	(68)	(63)	(66)	(69)
0	0	0	0	0	0	0
10	6,553	1,893,500	-1,307,900	1,226	548,200	-338,990
20	12,052	3,372,200	-2,334,000	2,067	942,300	-571,300
30	16,878	4,588,400	-3,183,600	2,664	1,239,700	-736,600
40	21,244	5,627,300	-3,915,700	3,103	1,472,200	-857,100
50	25,283	6,540,600	-4,565,900	3,428	1,659,200	-945,900
60	29,076	7,360,700	-5,157,200	3,670	1,812,700	-1,011,700
70	32,680	8,110,500	-5,704,800	3,850	1,941,000	-1,059,980
80	36,134	8,805,300	-6,219,600	3,982	2,049,850	-1,094,700
90	39,467	9,456,300	-6,709,200	4,076	2,143,100	-1,118,500
100	42,699	10,071,800	-7,179,400	4,139	2,223,600	-1,133,500

TABLE VII.- SAMPLE CALCULATIONS PER UNIT OF CHANGE IN LOAD FACTOR

[W = 110,000 pounds; x_{cg} = -211 inches; x_t = -768.3 inches]

(a) Wing-root angle of attack, wing load, and tail load

1	2	3	4	5	6	7	8	9	10
qm_R, $\frac{\text{lb}}{\text{ft}^2\text{-deg}}$	$L_{\alpha,r}$, lb/deg	$M_{Y,\alpha,r}$, in-lb/deg	$(2) - \frac{(3)}{x_t}$	$L_{n,r}$, lb/g	$M_{Y,n,r}$, in-lb/g	$\frac{W}{2}\left(1 - \frac{x_{cg}}{x_t}\right) - (5) + \frac{(6)}{x_t}$	$\frac{\Delta\alpha_r}{\Delta n} = \frac{(7)}{(4)}$, deg/g	$(2)(8) + (5) = \frac{\Delta L_r}{\Delta n}$, lb/g	$110,000 - 2(9) = \frac{\Delta L_t}{\Delta n}$, lb/g
0	0	0	0	0	0	0	∞	(a)	(a)
10	6,553	-1,307,900	4,851	1,226	-338,990	39,112	8.0627	54,061	1,878
20	12,052	-2,334,000	9,014	2,067	-571,300	38,574	4.2793	53,642	2,716
30	16,878	-3,183,600	12,734	2,664	-736,600	38,192	2.9992	53,285	3,430
40	21,244	-3,915,700	16,147	3,103	-857,100	37,910	2.3478	52,980	4,040
50	25,283	-4,565,900	19,340	3,428	-945,900	37,700	1.9493	52,712	4,576
60	29,076	-5,157,200	22,363	3,670	-1,011,700	37,544	1.6788	52,484	5,032
70	32,680	-5,704,800	25,255	3,850	-1,059,980	37,427	1.4820	52,280	5,440
80	36,134	-6,219,600	28,039	3,982	-1,094,700	37,340	1.3317	52,102	5,796
90	39,467	-6,709,200	30,734	4,076	-1,118,500	37,277	1.2129	51,945	6,110
100	42,699	-7,179,400	33,354	4,139	-1,133,500	37,233	1.1163	51,804	6,392

(b) Pitching and bending moments

11	12	13	14	15	16	17
$M_{b,\alpha,r}$, in-lb/deg	$M_{b,n,r}$, in-lb/g	$(3)(8) + (6) = \frac{\Delta M_{Y,r}}{\Delta n}$	$(11)(8) + (12) = \frac{\Delta M_{b,r}}{\Delta n}$	$x_{cp} = \frac{(13)}{(9)}$, in.	$y_{cp} = \frac{(14)}{(9)}$, in.	$\frac{\Delta x_{ac}}{\bar{c}} \times 100$, percent
0	0	0	0	[b]-206.6	[b]299.5	0
1,893,500	548,200	-10,884,200	15,814,900	-201.3	292.5	3.399
3,372,200	942,300	-10,559,200	15,373,000	-196.8	286.6	6.286
4,588,400	1,239,700	-10,284,900	15,001,200	-193.0	281.5	8.723
5,627,300	1,472,200	-10,050,400	14,684,000	-189.7	277.2	10.840
6,540,600	1,659,200	-9,846,200	14,408,800	-186.8	273.3	12.700
7,360,700	1,812,700	-9,669,600	14,169,800	-184.2	270.0	14.368
8,110,500	1,941,000	-9,514,500	13,960,800	-182.0	267.0	15.779
8,805,300	2,049,850	-9,377,300	13,775,900	-180.0	264.4	17.062
9,456,300	2,143,100	-9,256,100	13,612,600	-178.2	262.1	18.217
10,071,800	2,223,600	-9,147,900	13,466,800	-176.6	260.0	19.243

[a]Indeterminate value.

[b]Rigid-wing value.

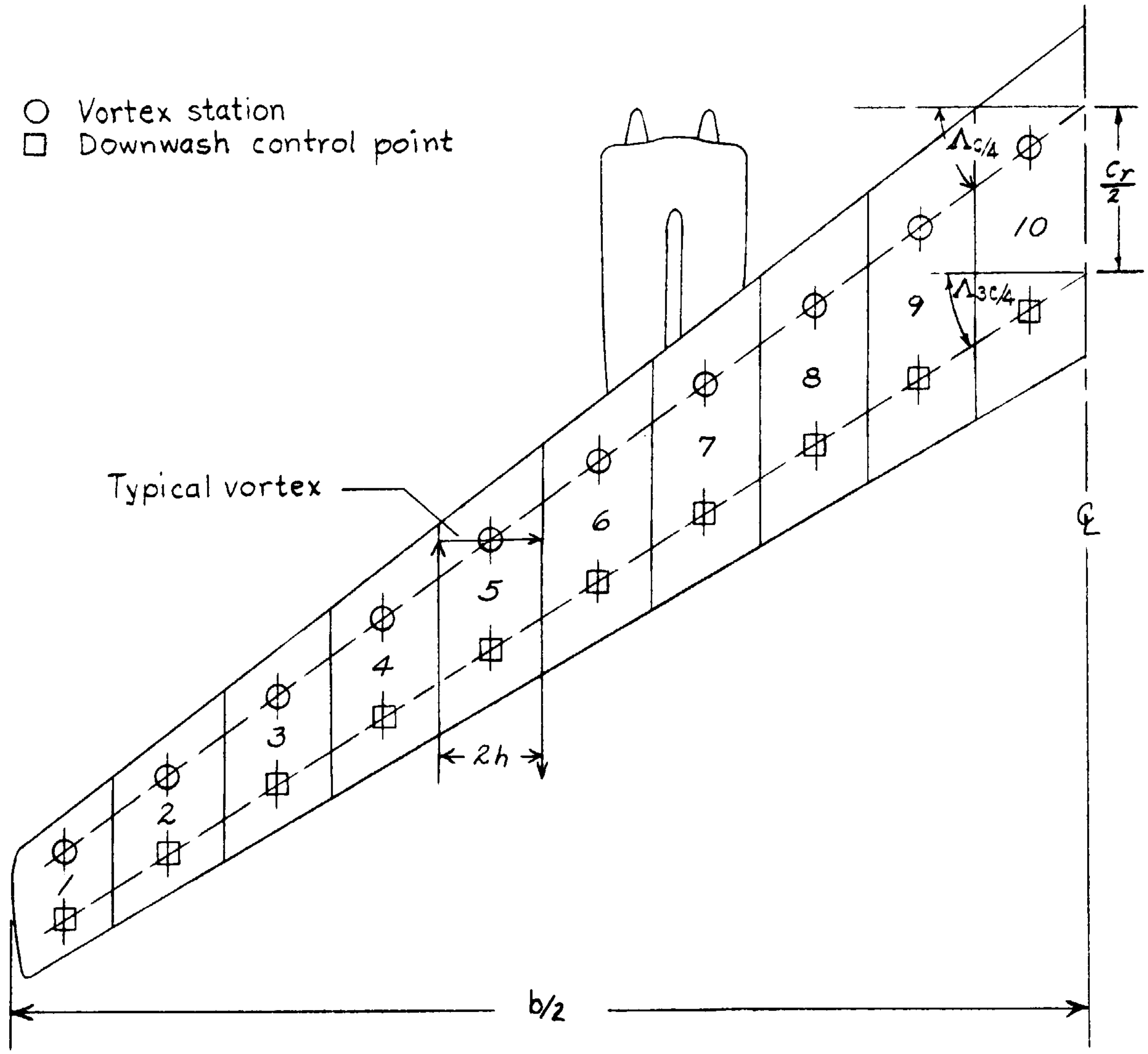


Figure 1.- Plan form of wing-nacelle combination used in sample problem.

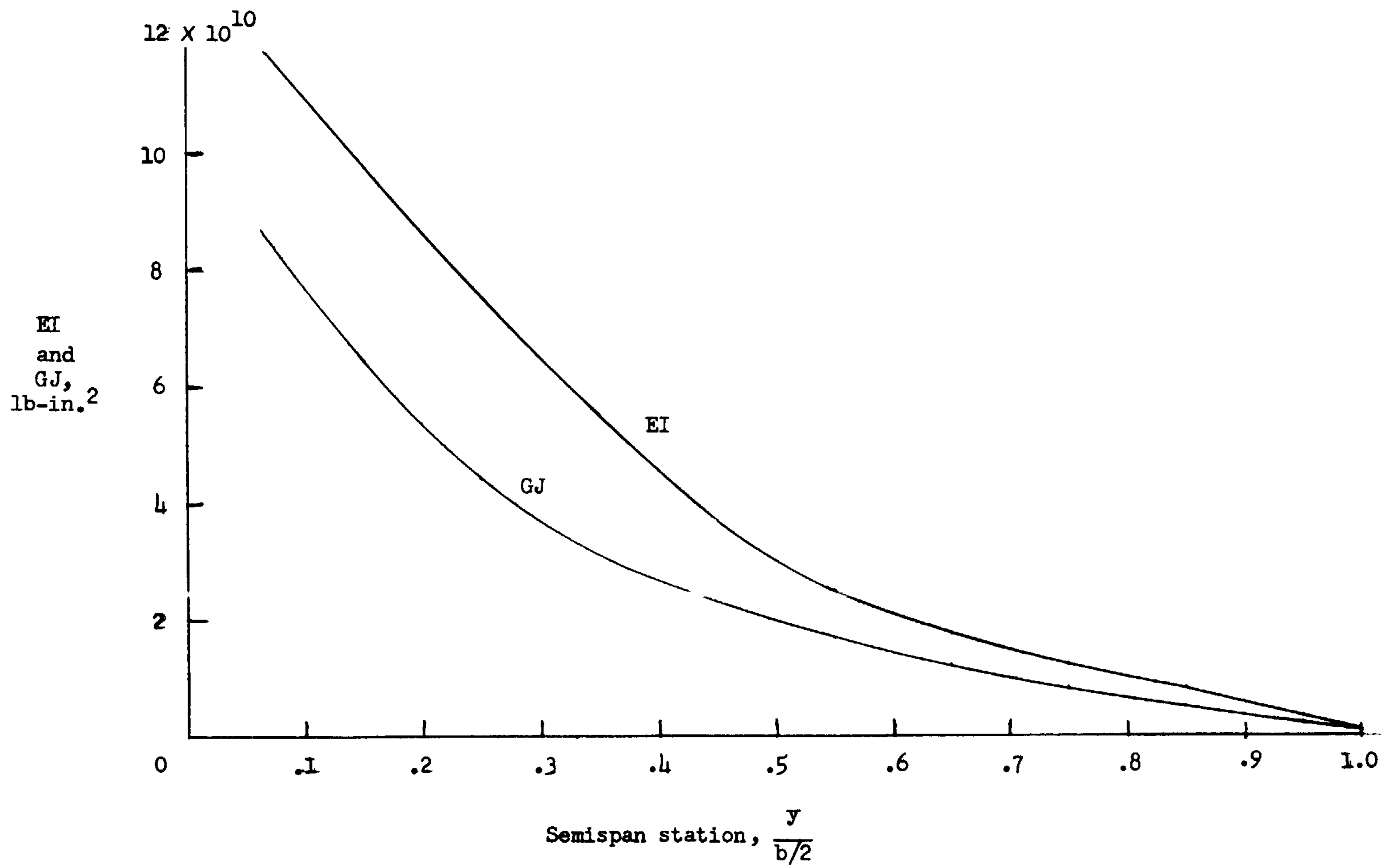
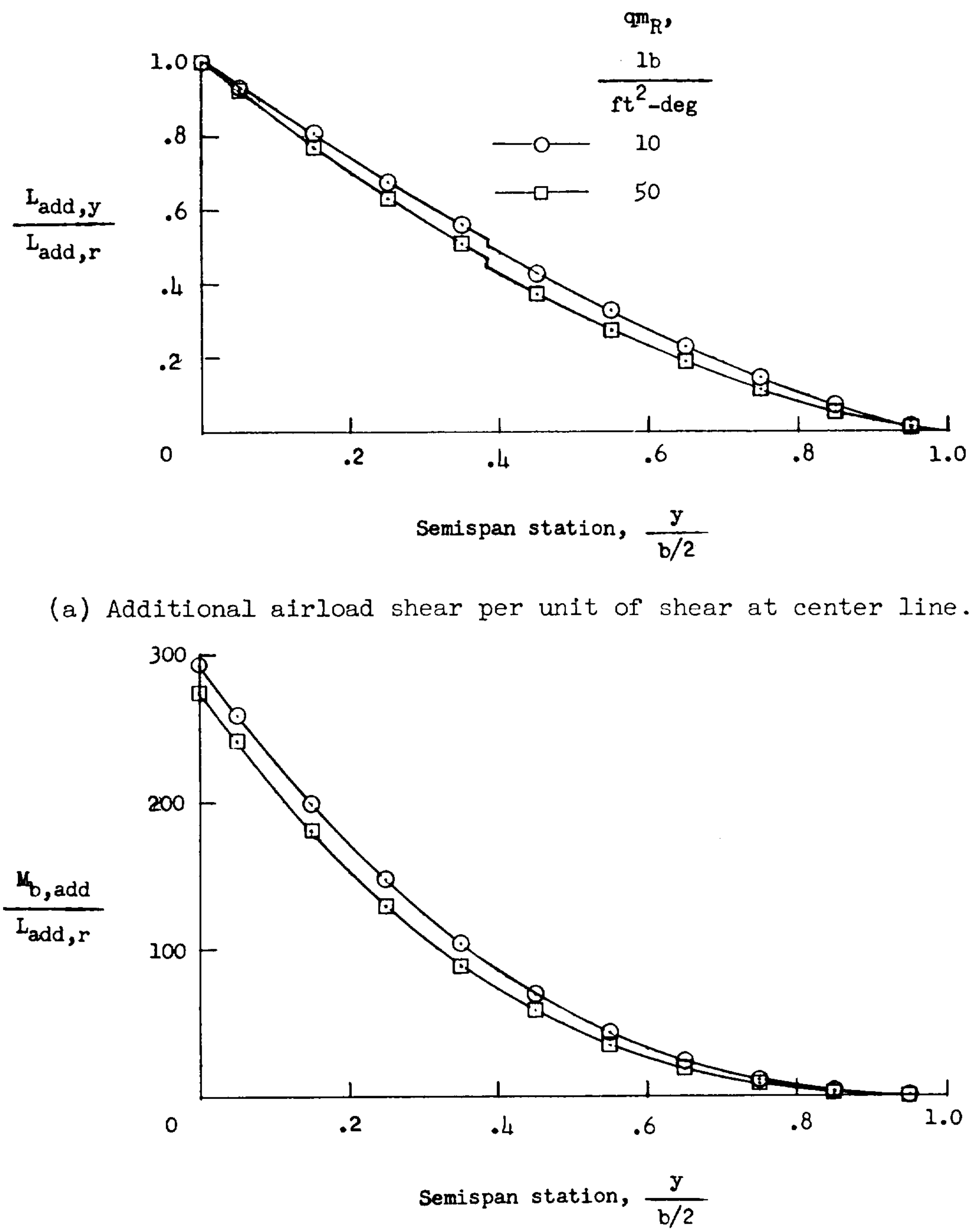


Figure 2.- Spanwise variation of EI and GJ for sample wing.



(a) Additional airload shear per unit of shear at center line.

(b) Additional airload bending moment per unit of shear at center line.

Figure 3.- Spanwise variation of additional airload shear and bending moment for sample wing at two values of qm_R.

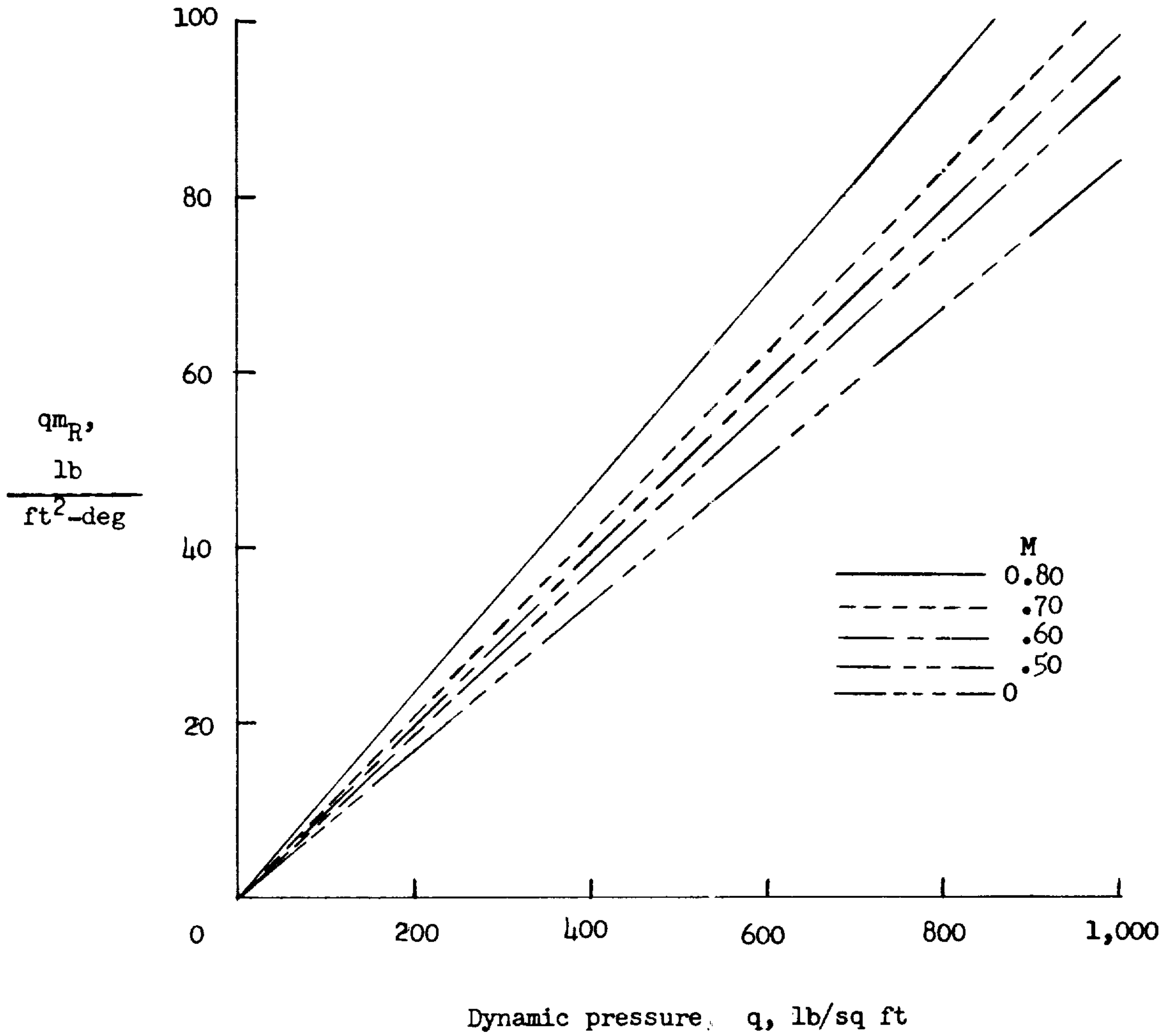


Figure 4.- Variation of qm_R with dynamic pressure for several Mach numbers for sample airplane.