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TECHNICAL NOTE

D-462

REAL-GAS CORRECTION FACTORS FOR HYPERSONIC
FLOW PARAMETERS IN HELIUM

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SUMMARY

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The real-gas hypersonic flow parameters for helium have been calculated for stagnation temperatures from 0° F to 600° F and stagnation pressures up to 6,000 pounds per square inch absolute. The results of these calculations are presented in the form of simple correction factors which must be applied to the tabulated ideal-gas parameters. It has been shown that the deviations from the ideal-gas law which exist at high pressures may cause a corresponding significant error in the hypersonic flow parameters when calculated as an ideal gas. For example, the ratio of the free-stream static to stagnation pressure as calculated from the thermodynamic properties of helium for a stagnation temperature of 80° F and pressure of 4,000 pounds per square inch absolute was found to be approximately 13 percent greater than that determined from the ideal-gas tabulation with a specific heat ratio of 5/3.

INTRODUCTION

Hypersonic helium tunnels are presently being used to study fluid-dynamic problems. The main advantage of using helium over other test fluids is the extremely low condensation temperature of helium which eliminates the need for a stagnation heater for Mach numbers up to the mid-twenties. Higher Mach numbers can be obtained by slightly increasing the stagnation temperature.

Helium obeys the ideal-gas law over a broad range of temperature and pressure but exhibits deviations at high densities due to intermolecular-force effects. The stagnation conditions for which a hypersonic helium tunnel operates may extend well into this non-ideal-gas region.

The purpose of this paper is to calculate the real-gas hypersonic flow parameters for helium from the thermodynamic properties of helium and present the results in the form of corrections to be applied to the ideal-gas parameters presented by Mueller in reference 1. The calculation procedure used and the calculated results are presented in an easily usable graphical form.

SYMBOLS

a	velocity of sound
c_p	specific heat at constant pressure
F	correction factor (The correction factor is a particular flow parameter calculated for a real gas divided by the calculated ideal-gas value for $\gamma = 5/3$ at the same M_1 . For example, $F_{p,1} = (p_1/p_{t,1}) / (p_1/p_{t,1})_i$)
h	specific enthalpy
M	Mach number
p	pressure
q	dynamic pressure
R	gas constant
s	specific entropy
t	temperature
T	absolute temperature
T'	static temperature obtained from ref. 2 by isentropically expanding from $p_{t,1}$ and $T_{t,1}$ to 14.7 lb/sq in. abs
V	velocity
Z	compressibility factor
γ	ratio of specific heats
ρ	density
Subscripts:	
i	ideal
t,1	stagnation conditions upstream of shock

t,2 stagnation conditions downstream of shock
 1 static conditions upstream of shock

METHOD OF CALCULATION

L The following calculations are based on the tabulated thermodynamic
 1 properties of helium presented by Akin. (See ref. 2.) Figure 1 shows
 1 a Mollier diagram for helium prepared from Akin's data and covers the
 3 limited range of interest. The selected stagnation temperatures for
 5 which the hypersonic flow parameters were calculated are 0° F, 100° F,
 300° F, and 600° F and the selected stagnation pressures are 1,500, 2,500,
 4,000, and 6,000 pounds per square inch absolute. This region of selected
 stagnation conditions is shown in figure 1 and is clearly in the region
 where real-gas effects are important as indicated by the rather large
 variation of enthalpy with pressure for a given temperature.

Consider the flow of helium through a hypersonic nozzle. In the
 region of flow from the stagnation chamber to some free-stream station,
 the flow is assumed to expand at constant entropy, the value of which
 is determined by the stagnation temperature and pressure. Since the
 entropy of the moving gas remains constant during the expansion from the
 stagnation chamber, specifying a pressure less than $p_{t,1}$ at a point
 along the expansion will fix all state properties at that point. The
 lowest pressure which can be specified and still make use of Akin's data
 is 14.7 pounds per square inch absolute. By choosing $T_{t,1}$ and $p_{t,1}$
 and specifying an isentropic expansion to 14.7 pounds per square inch
 absolute, a static temperature T' is determined from Akin's data. Now,
 by assuming that helium is an ideal gas with constant γ at 14.7 pounds
 per square inch absolute and below, the relation between static tempera-
 ture and static pressure is determined from the ideal-gas equation for
 isentropic flow

$$T_1/T' = \left(\frac{p_1}{14.7} \right)^{\frac{\gamma-1}{\gamma}} \quad (1)$$

where $\gamma = 5/3$ and T' is the static temperature obtained from Akin's
 data by isentropically expanding from $p_{t,1}$ and $T_{t,1}$ to 14.7 pounds
 per square inch absolute. The expansion is continued to an arbitrary
 static pressure at which it is assumed a normal shock occurs and T_1
 is computed from equation (1).

The enthalpy in the ideal-gas region is then calculated from the expression

$$h_1 = c_p T_1 \quad (2)$$

where $c_p = \frac{5}{2} R$ for a monatomic ideal gas.

The velocity is determined from the first-law flow equation for an adiabatic nozzle which is

$$h_{t,1} = h_1 + \frac{1}{2} V_1^2 \quad (3)$$

where $h_{t,1}$ is fixed from the chosen stagnation conditions. The free-stream static density is calculated from the equation of state for an ideal gas

$$\rho_1 = p_1 / RT_1 \quad (4)$$

The velocity of sound is calculated from the ideal expression

$$a_1 = \sqrt{\gamma RT_1} \quad (5)$$

For a given set of stagnation conditions and a chosen value of p_1 , a Mach number as well as T_1 and ρ_1 is calculated from these equations. These values of T_1 , p_1 , and ρ_1 for a given stagnation condition and Mach number are then divided by the ideal-gas parameters as determined from reference 1 for the same stagnation conditions and Mach number. These ratios of real-gas to ideal-gas parameters, that is, the correction factors, are, of course, independent of Mach number and are a function only of the stagnation conditions as long as the expansion is taken into the ideal-gas region. It can be shown that the correction factor for the free-stream dynamic pressure $F_{q,1}$ must be equal to the correction factor for the free-stream static pressure $F_{p,1}$ for the same stagnation conditions when the free-stream properties are in the ideal-gas region.

It is also assumed that the ideal-gas law applies downstream of the normal shock. This assumption requires that the various correction factors for the static ratios across the shock be unity. The correction factors for the stagnation conditions behind a normal shock, however, are not unity. It can be shown that the correction factor for the stagnation pressure behind a normal shock $F_{p,t,2}$ is also equal to $F_{p,1}$ for a given set of stagnation conditions. The correction factor for the stagnation temperature behind a shock $F_{T,t,2}$ is simply the product of c_p and $T_{t,1}$ divided into $h_{t,1}$.

The ideal-gas law was assumed to apply at 14.7 pounds per square inch absolute and below and downstream of a normal shock which leads to a Mach number independency for the various correction factors. It is now necessary to define the minimum Mach number for which this assumption is valid for the various stagnation conditions. The transition from the ideal-gas region to the real-gas region is, of course, gradual. For the purpose of defining the minimum Mach number for the various stagnation conditions, the ideal-gas law was assumed to apply only up to $Z = 1.005$. The approximate line of $Z = 1.005$ is shown in figure 1. It was found by calculation that the stagnation conditions behind the normal shock define the minimum Mach numbers for the selected upstream stagnation conditions. The region of free-stream static conditions is well into the ideal-gas region when the stagnation conditions behind a normal shock are such that Z does not exceed 1.005 as shown in figure 1.

The calculated minimum Mach numbers for which Z behind a normal shock does not exceed 1.005 for various stagnation conditions are presented in figure 2. The symbols indicate the calculated points.

RESULTS AND DISCUSSION

The results of the real-gas calculations for helium are presented in figures 3 to 6 as correction factors to be used in conjunction with the ideal-gas parameters presented by Mueller in reference 1. The approximate minimum Mach numbers for various stagnation conditions for which these correction factors should be used are given in figure 2. The accuracy of any particular flow parameter as calculated from the correction factors presented herein is probably within 1 or 2 percent.

The use of the correction-factor charts is most simply shown by an example. Assume a hypersonic helium tunnel operating with a stagnation temperature of 80° F and a pressure of 4,000 pounds per square inch absolute. Also assume that the pitot-impact tube measures a pressure $p_{t,2}$ of 6 pounds per square inch absolute. The measured total

pressure ratio $p_{t,2}/p_{t,1}$ is therefore 0.0015. The correction factor $F_{p,t,2}$ for the assumed stagnation conditions from figure 4 is 1.131. It follows then that the ideal-gas value for $p_{t,2}/p_{t,1}$ is $(0.0015)/(1.131)$ or 0.001326. From this ratio, the actual Mach number is found in reference 1 to be 25.75. If the uncorrected total pressure ratio $p_{t,2}/p_{t,1} = 0.0015$ had been used, the calculated value for M_1 would have been 24.71 and would have given an error in M_1 of about -4.0 percent. It is also interesting to look at the effect on $T_{t,2}$. The correction factor $F_{T,t,2}$ from figure 6 for the same stagnation conditions is 1.064. Since the ideal-gas ratio is always unity for $T_{t,2}/T_{t,1}$, the actual value of $T_{t,2}$ should be calculated as $(539.69)(1.064) = 574.2^\circ \text{R}$. In other words, $T_{t,2}$ will be 34.5° greater than $T_{t,1}$.

CONCLUDING REMARKS

The real-gas hypersonic flow parameters for helium have been calculated for stagnation temperatures from 0°F to 600°F and stagnation pressures up to 6,000 pounds per square inch absolute. It was found that the error incurred by using the uncorrected ideal-gas parameters may be significant for high stagnation pressures and moderate temperatures. The results are presented as simple correction factors which must be applied to the tabulated ideal-gas parameters presented in NACA Technical Note 4063. For example, the free-stream static pressure is approximately 13 percent greater than that calculated for an ideal gas when the stagnation temperature is 80°F and stagnation pressure is 4,000 pounds per square inch absolute.

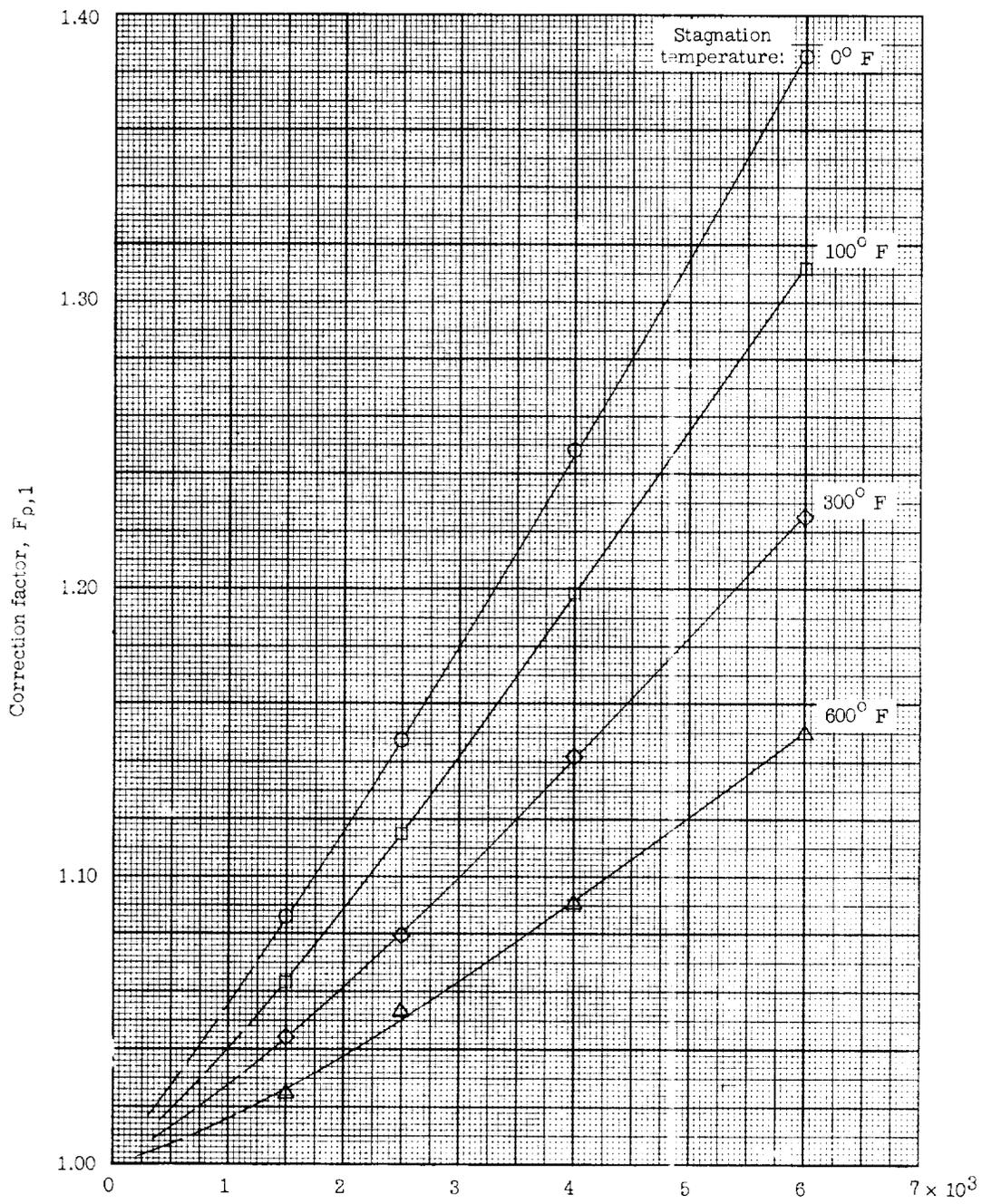
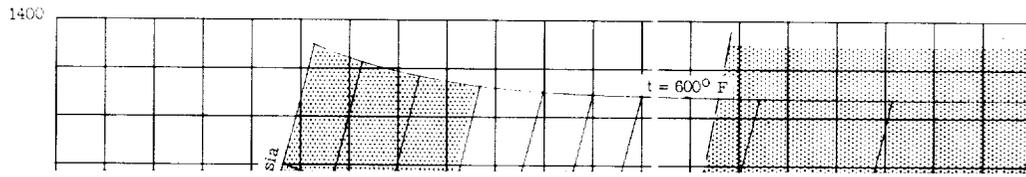
Langley Research Center,
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Langley Field, Va., June 9, 1960.

REFERENCES

1. Mueller, James N.: Equations, Tables, and Figures for Use in the Analysis of Helium Flow at Supersonic and Hypersonic Speeds. NACA TN 4063, 1957.
2. Akin, S. W.: The Thermodynamic Properties of Helium. Trans. A.S.M.E., vol. 72, no. 6, Aug. 1950, pp. 751-757.

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not exceed 1.00) against stagnation pressure for various stagnation temperatures.



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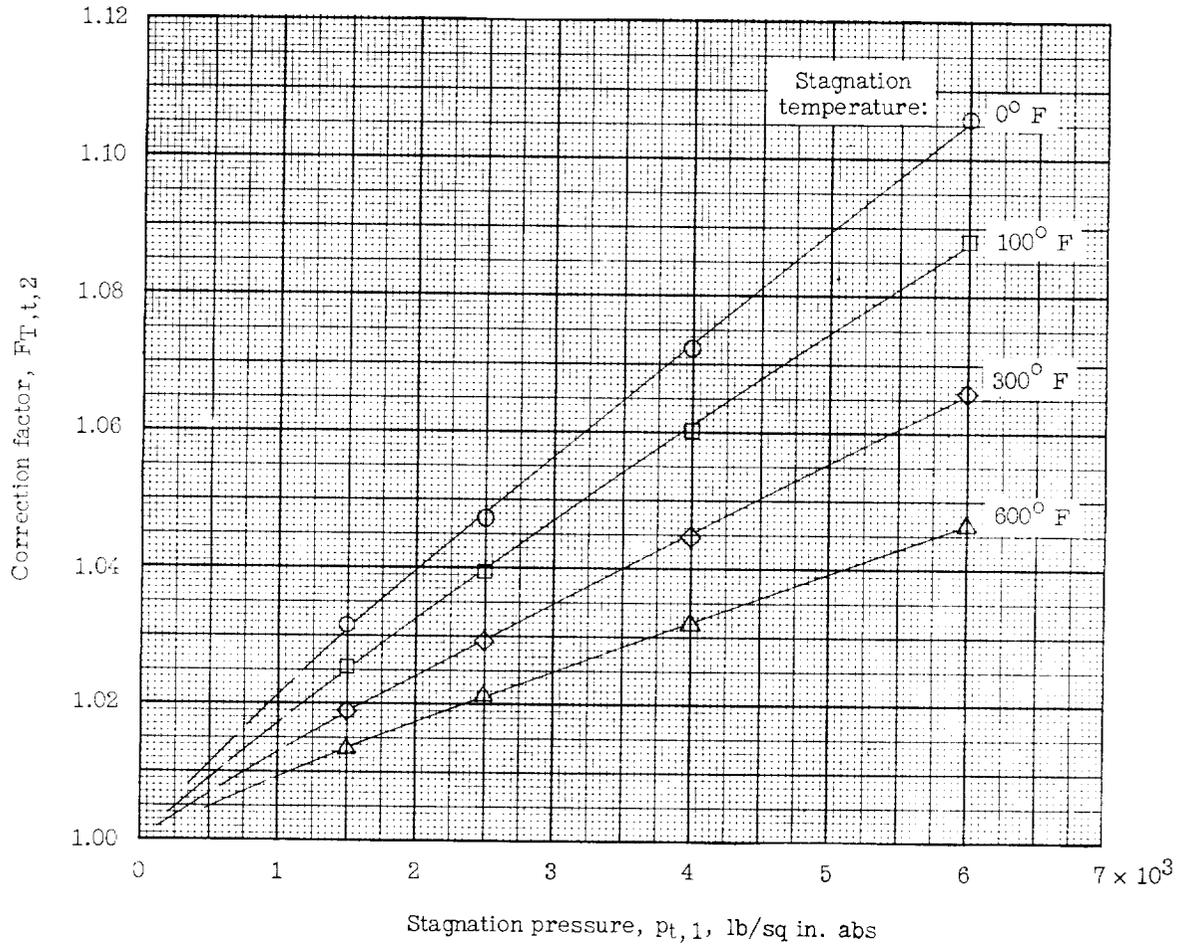


Figure 6.- Variation of correction factor $F_{T,t,2}$ with stagnation pressure for various stagnation temperatures.

