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TO VIBRATIONS OF A 45° DELTA WING

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SUMMARY

The Levy method which deals with an idealized structure was used to obtain the natural modes and frequencies of a large-scale built-up 45° delta wing. The results from this approach, both with and without the effects of transverse shear, were compared with the results obtained experimentally and also with those calculated by the Stein-Sanders method. From these comparisons it was concluded that the method as proposed by Levy gives excellent results for thin-skin delta wings, provided that corrections are made for the effect of transverse shear.

INTRODUCTION

The literature contains many methods for obtaining the deflectional characteristics of low-aspect-ratio and delta wings. (See, for example, refs. 1 to 5.) Although these methods use a variety of approaches and assumptions, they can be classified into two categories: the method either deals with the actual structure and restricts the allowable deflection shape or deals with a simplified structure and allows arbitrary deflections. One analysis from the first category, the Stein-Sanders method, is described in reference 1. In this analysis, the actual structure was analyzed by assuming that its neutral surface was strain free, the effects of transverse shear were negligible, and its chordwise deformation was parabolic. An analysis from the second category, namely the Levy method, is described in reference 2. In this method an idealized structure consisting of interconnected beams and torque boxes whose deflections are unrestrained is analyzed.

Although methods of calculating the deflectional characteristics of low-aspect-ratio and delta wings do exist, there is available very little information concerning the application of the methods and the reliability of their results.

An experimental investigation of the stiffness and vibration characteristics of a large-scale built-up 45° delta wing has been discussed in reference 6. Since the detailed stiffness and weight distributions of the specimen are presented therein, the results of the investigation can serve as a reliable basis for the evaluation of the analytical methods. These results have been used in reference 7 to evaluate the Stein-Sanders method. In the present paper the experimental results are used to evaluate the Levy method. A summary of some of the results of this investigation was presented in references 8 and 9.

The purpose of the present paper is threefold: First, to describe in detail the application of the Levy method to a 45° delta wing; second, to show how the Levy method can be easily modified to include approximately the influence of transverse shear; and third, to evaluate the method in the light of the results of the Stein-Sanders method and experimental results.

SYMBOLS

A_sG	shear stiffness of beam
D	constant defined in equation (A8)
E	modulus of elasticity
EI	bending stiffness of beam
GJ	torsional stiffness of torque box
h	depth of beam
i,j,n,N	integers
J	torsional constant
K_{ij}	constants defined in equation (A7)
l	length of torque box
M,M_x,M_y; M_{xx},M_{yy},M_{xy}	constants defined in equation (A9)
P_i	concentrated load at station i
V	shear in beam web

w_i	deflection of ith station of free wing
w_i^{3P}	deflection of ith station of wing on three-point support
w_o	rigid-body translation
x,y	coordinates of station
x_o	distance of force from support
α,β	a rigid body tipping about y- and x-axis, respectively
δ	influence coefficient of cantilevered beam
Δ	stiffness coefficient of wing
Δ^{3P}	stiffness coefficient of wing on three-point support
$\Delta_S^i,\Delta_R^i,\Delta_T^i$	stiffness coefficient of ith spar, rib, and torque box, respectively
ω	circular natural frequency
$[F]$	square matrix defined in equations (A12) and (A24)
$[H]$	square matrix defined in equations (A11) and (A21)
$[I]$	unit matrix
$\left[M^s \right]$	diagonal mass matrix for half-span
$\lfloor 1 \rfloor$	row matrix of ones
$\lvert 1 \rvert$	column matrix of ones
$[\;]$	rectangular matrix
$\left[\diagdown\right]$	diagonal matrix
$\lfloor\;\rfloor$	row matrix
$\lvert\;\rvert$	column matrix

Subscripts:

C	stations on center line
R	stations on right side of center line
L	stations on left side of center line
i,n	integers

Superscripts:

s	symmetrical
a	antisymmetrical

ANALYSES

Specimen

The specimen used in the investigation discussed in reference 6 is a large-scale built-up 45° delta wing shown in figure 1. It has a span of 18 feet $11\frac{7}{8}$ inches, a midchord of 8 feet $1\frac{5}{8}$ inches and a uniform carrythrough bay of 2 feet 8 inches. The wing is uniform in depth in the chordwise direction but varies linearly in depth in the spanwise direction from $5\frac{1}{2}$ inches at the carrythrough section to $1\frac{3}{4}$ inches at the tip.

The top and bottom covers of the delta wing are of skin stringer construction with four light stringers between each spar. The interior construction consists of four straight spars spaced 24 inches apart, a bent leading-edge spar, and light streamwise bulkhead spaced 8 inches on centers. Detailed dimensions, section properties, and weight distribution of the specimen are given in reference 6. All parts were constructed of 2024-T6 aluminum alloy.

Idealization

In order to apply the Levy method, the actual structure in figure 1 was idealized as shown in figure 2 into an orthogonal set of crisscrossing beams with torque boxes attached at their four corners to the intersection of the beams. The locations of the idealized spars were chosen to coincide with the center line of the actual spars. The spacing of the ribs in the idealized structure, however, was increased over that of the specimen in order to decrease the number of redundants in the analysis from 53 (if the actual rib locations are used) to 34.

All the spanwise normal-stress-carrying material of the spars, cover sheets, and stringers was concentrated into the spars of the idealized wing whereas all the chordwise bending ability of the actual ribs and covers was accounted for in the idealized ribs. The condition suggested by Levy (see ref. 2) of limiting the effectiveness of the sheets in the chordwise direction to 0.181 of the rib length to either side of the rib governed only in the last two outboard ribs of the actual structure. The stiffnesses of the idealized ribs were obtained by first distributing the moments of inertia of the actual ribs and then reconcentrating the inertias at the new stations. The moments of inertias of the idealized spars and ribs are given in tables I and II.

The shear-carrying capacity of the cover sheets is accounted for by the torque boxes in the spar-rib cells of the idealized structure. In the calculation of the torsional stiffness GJ of these boxes, the axis of twist was assumed to be in the spanwise direction. The values of J at the center section of each torque box are given in table III. Note that, when these values were obtained, the side walls of the torque boxes were considered to be rigid in shear as suggested by Levy in reference 2.

Application of Levy Method

The first step in the analysis of the idealized wing is to determine the loads carried by the individual components in terms of the deflection at the junctions of the spars and ribs. These loads can be expressed as follows:

$$\left|P\right| = E\left[\Delta_S^n\right]\left|w\right| \qquad (n = 1,2,\ldots 5) \qquad (1a)$$

$$\left|P\right| = E\left[\Delta_R^n\right]\left|w\right| \qquad (n = 1,2,\ldots 10) \qquad (1b)$$

$$\left|P\right| = E\left[\Delta_T^n\right]\left|w\right| \qquad (n = 1,2,\ldots 20) \qquad (1c)$$

where Δ_S^n, Δ_R^n, and Δ_T^n are the stiffness coefficients of the nth spar, rib, and torque box, respectively. In equation (1b), $n = 10$ refers to the swept portion of the leading-edge spar.

In these calculations the influence of shear deformation in the spar and rib webs along with the torque-carrying capacity of the triangular cells was neglected. Furthermore, no moment transfer was permitted to take place between the spars and ribs and between the straight and swept portion of the leading-edge spar. The stiffness coefficients of the nonuniform spars were obtained as described in reference 2 by inversion of the influence coefficients of cantilevered beams. These influence coefficients were calculated by an approximate procedure described in reference 10, which was based on an assumption of a linear $1/EI$ variation between stations. An example of the resulting influence-coefficient matrix $[\delta]$ is shown in table IV(a) for the trailing-edge spar. When the stiffness coefficients of the spars were calculated, cognizance of the type of loading was taken. For the case of symmetrical loading the stiffness coefficients of the spars were obtained for the condition of zero slope at the center line, whereas for antisymmetrical loading the condition of zero deflection at the center line was maintained. The resulting stiffness coefficients for symmetrical loading for the trailing-edge spar $\left[\Delta_S^1\right]$ are shown in table V.

Inasmuch as the ribs and torque boxes were uniform, there were no complications involved in the calculations of their stiffness matrices. Typical examples of the stiffness coefficients are shown in table VI for rib number 4 and in table VII for torque boxes 15 and 16. The stiffness coefficients of the swept portion of the leading-edge spar were obtained by considering that the swept portion of the spar acts as a rib and that no moment is transferred at any point of attachment including the junction of the unswept and swept portion of the spar.

The loads carried by the idealized structure are considered to be the sum of the loads carried by the idealized spars in spanwise bending, by the ribs in chordwise bending, and by the torque boxes in torsion. Thus the stiffness coefficients of the composite structure were obtained by summing the stiffness coefficients of the components:

$$\left|P\right| = E\left[\Delta\right]\left|w\right| \tag{2}$$

where

$$\left[\Delta\right] = \sum_{n=1}^{5}\left[\Delta_S^n\right] + \sum_{n=1}^{10}\left[\Delta_R^n\right] + \sum_{n=1}^{20}\left[\Delta_T^n\right] \tag{3}$$

The synthesis of a typical row of $\left[\Delta\right]$ for symmetrical loading is illustrated in table VIII for row 24. The elements of this row represent the contribution of the deflections at each station of the wing to the load at station 24. As can be seen, elements associated with stations not on the spar, rib, or torque boxes common to station 24 are zero. The remaining elements of row 24 are the summations of the rows of $\left[\Delta_S^1\right]$, $\left[\Delta_R^4\right]$, $\left[\Delta_T^{15}\right]$ and $\left[\Delta_T^{16}\right]$ associated with P_{24} and are shown in tables V to VII.

As yet there have not been any restraining or boundary conditions placed on the stiffness matrix $[\Delta]$. Thus the structure represented by this matrix is free to move with a rigid-body displacement. Obviously, the deflections of such a structure are not uniquely related to the loads and therefore the inverse of its stiffness matrix cannot exist, that is, the matrix $[\Delta]$ is singular. In order to obtain a structure whose stiffness matrix can be inverted, the wing was assumed to be simply supported at three points (stations 1 and 22) in figure 2. This particular support condition was used because the results from a three-point support can be converted to influence coefficients for most other support and loading conditions. The particular stations used were chosen to conform to the supporting condition used in the static tests of the delta wing described in reference 6.

The stiffness matrix of the wing on a three-point support was obtained by omitting from the $[\Delta]$ matrix the rows and columns associated with stations 1 and 22. The resulting stiffness matrix $\left[\Delta^{3P}\right]$ for a delta wing on a three-point support is shown in table IX for both symmetrical and antisymmetrical cases. The influence coefficients of the idealized structure on a three-point support were obtained by inverting the $\left[\Delta^{3P}\right]$ matrix

$$\left|w\right| = \frac{1}{E}\left[\Delta^{3P}\right]^{-1}\left|P\right| \tag{4}$$

The influence coefficient matrices $\left[\Delta^{3P}\right]^{-1}$ for symmetrical and antisymmetrical loading conditions are shown in table X.

Since the influence coefficients of the delta wing are known for a three-point support, the load deflection characteristics of the wing can be calculated for other support conditions. (See ref. 1.) Furthermore, the frequency equations necessary to determine the natural modes and frequencies can readily be obtained. A method for "freeing" a wing is discussed in the appendix. In this method the displacements of a free-free wing vibrating in a natural mode are described in terms of the influence coefficients of the wing on a three-point support. With the use of the results of the appendix, the frequency equation for a free-free wing can be written as follows (see eqs. (A23) and (A27)):

For symmetrical vibrations:

$$\left\{w\right\} = \frac{\omega^2}{E}\left[\left[I\right] + \left[F^s\right]\left[\left\lceil M^s \right\rfloor\right]\right]\left[\Delta^s\right]^{-1}\left[\left\lceil M^s \right\rfloor\right]\left\{w\right\} \tag{5}$$

For antisymmetrical vibrations:

$$\left\{w\right\} = \frac{\omega^2}{E}\left[\left[I\right] + 2K_{22}\left\{x\right\}\lfloor x \rfloor\right]\left[\Delta^a\right]^{-1}\left[\left\lceil M^s \right\rfloor\right]\left\{w\right\} \tag{6}$$

where

$\left[F^s\right]$ matrix defined in eq. (A24)

K_{22} constant defined in eq (A7)

$\left[I\right]$ unit matrix

x spanwise coordinate

$\left[\Delta^s\right],\left[\Delta^a\right]$ stiffness matrix for wing on three-point support for symmetrical and antisymmetrical loading conditions, respectively

$\left[\left\lceil M^s \right\rfloor\right]$ diagonal mass matrix for half-span

The elements of the diagonal mass matrix represent the mass that is considered to be concentrated at each station. In order to obtain these elements the components of the wing tabulated in reference 6 were divided into two groups. One group contained the cover sheets, stringers, spars, and spar-to-cover and stringer-to-cover rivets and the second group contained the weights of the ribs and the concentrated weights (such as those of the filler blocks, splice plates, pickup, and the moving elements of vibrators). The contribution of the components of the first group to the elements of the mass matrix was obtained by dividing the wing into regions (shown in fig. 3) and then allotting the weights of the portion of the components included in each region to the station associated with the region. The contribution of the components of the second group was obtained in such a way that the total and first and second moments about the wing center line of these contributions were the same as the total and first and second moments of the weight of the actual components in the second group. The sum of all the weights associated with the stations shown in figure 3 was within 0.1 percent of the actual weight of the wing.

Modification of the Levy Method to Include Transverse Shear

In the previous calculations the effects of transverse shear were neglected as suggested in reference 2. On the other hand in reference 9 it was shown that the influence of transverse shear could be of importance especially in the higher modes of vibration.

If the effects of transverse shear were to be included exactly in a consistent deformation analysis, such as that of reference 2, the slopes in both the spanwise and chordwise direction in addition to the deflections at each spar-rib intersection must be treated as unknowns. This requirement would, of course, cause a threefold increase in the number of redundants in the solution. The influence of transverse shear, however, can be included in the Levy method approximately with no increase in the number of redundants and with little additional labor.

In the previous calculations the stiffness coefficients of the spars and ribs were obtained by inversion of the influence coefficients of cantilever beams. These influence coefficients, however, contained only the deflections due to bending. The effects of shear deformation on the spars and the ribs can be included in the influence coefficients by super-imposing the deflections due to shear onto those due to bending. The influence coefficients including shear deformation can be obtained from the equation

$$w = \int_0^x \frac{P}{EI}(x_o - \eta)(x - \eta)d\eta + \int_0^x \frac{P}{A_sG}\, d\eta + \int_0^x \frac{P}{A_sG}\frac{h'}{h}(x_o - \eta)d\eta +$$

$$\int_0^x \frac{P}{A_sG}\frac{h'}{h}(x - \eta)d\eta + \int_0^x \frac{P}{A_sG}\left(\frac{h'}{h}\right)^2(x_o - \eta)(x - \eta)d\eta \tag{7}$$

where w is the deflection of a cantilever beam at any point x (distance from the root) due to a load P at x_o, h' is the derivative of h with respect to η, and EI and A_sG are the bending stiffness and effective shear stiffness, respectively. The first term on the right-hand side of equation (7) is the portion of the deflection due to bending stresses. The second term is the shear deformation that would occur if the beam was nontapered. The third term represents the deflection due to the effect of the normal stresses in the flanges of the tapered beams on the shear in the webs. The last two terms represent the deflections due to the effects of taper on the shear strain.

As an example, the influence coefficients with transverse-shear deformations included are shown in table IV(b) for the trailing-edge spar. Comparisons of these coefficients with those in table IV(a) will give an indication of the magnitude of the transverse-shear deformation. In these calculations the effective shear areas of the spar and rib webs were taken to be the product of the web thickness and the depth of channel.

The set of influence coefficients for all spars and ribs resulting from the use of equation (7) was inverted to obtain the stiffness coefficients of the spars and ribs. The stiffness coefficients of the torque boxes were left unchanged.

The influence coefficients of the idealized delta wing were then obtained in the same manner as described in the previous section. The numerical values of the resulting influence coefficients including transverse shear are shown in table XI for the wing simply supported at three points and loaded both symmetrically and antisymmetrically.

RESULTS AND DISCUSSION

The first nine free-free modes (5 symmetrical and 4 antisymmetrical) of the delta wing were calculated with the use of equations (5) and (6) for both the case where transverse shear was neglected and the case where the influence of transverse shear was included.

In figure 4 the node lines and frequencies as obtained by the Levy method with transverse shear neglected are compared with the node lines and frequencies obtained by the Stein-Sanders method (ref. 7) and with the experimental node lines and frequencies (ref. 6).

Note that the frequencies given in figure 4 for the Levy method are smaller than those given in reference 8. This discrepancy was due to the fact that, in the calculations for the frequencies in reference 8, 12 inches of the cover sheet were included in the moments of inertia of the leading-edge spar whereas in the present calculation only 6.14 inches were included as suggested by the criteria of reference 2. Furthermore, in the calculations of the results in reference 8, moment transfer was allowed between the unswept and swept portions of the leading-edge spar whereas in the calculations of the present paper no moment transfer was allowed.

As can be seen in figure 4, the node-line patterns of both the Stein-Sanders and Levy methods agree fairly well with the ones obtained experimentally. The node lines obtained by the Levy method, however, are not as good as those obtained by the Stein-Sanders method, especially in the vicinity of the leading edge. Examination of the figure seems to indicate that the stiffness of the leading edge in the idealized structure is too great.

Although the Stein-Sanders method predicts the experimental node-line pattern fairly well, the frequency agreement is poor. The errors range from 7 percent in the first mode to 38 percent in the fifth symmetrical mode. On the other hand, the frequency agreement in the Levy method is much better. The largest error in the first 8 modes occurs in the third antisymmetrical mode and is only $8\frac{1}{2}$ percent; the error in the fifth symmetrical mode is only 20 percent.

One of the principal sources of error in the Stein-Sanders method is the assumption of a parabolic chordwise variation of deformation. As this particular specimen had no extra chordwise stiffening in the center section such as would be furnished by a fuselage, for example, the errors due to this assumption may be large. Another source of error which is in both the Stein-Sanders and the Levy methods is that the results shown in figure 4 do not include the effects of transverse shear.

The results of the calculations of the frequencies of the first nine free-free modes of the delta wing by various methods are summarized in table XII. The frequencies that were obtained experimentally are given in the first row. The corresponding frequencies as calculated by the Stein-Sanders method and by the Levy method without shear are tabulated in the second and third rows, respectively. The frequencies obtained by the modified Levy method that includes transverse shear are given in the fourth row. The last row contains frequencies that were calculated from the experimentally determined influence coefficients of reference 6 by the method discussed in the appendix. This calculation was included because a popular method of obtaining frequencies is to measure influence coefficients on a model or full-scale structure and then use them in a vibrational analysis.

A comparison of the results tabulated in rows 1 and 4 of table XII shows that the frequencies calculated by the Levy method with shear are in excellent agreement with the experimental frequencies. The largest error occurs in the seventh (fourth symmetrical) mode and is slightly less than 4 percent. The effect of transverse shear on the calculated nodal-line patterns was slight. The changes that did occur, however, tended to improve the agreement between the calculated and experimental node lines.

Comparison of rows 3 and 4 of table XII indicates that the effect of transverse shear can be important. For instance, the inclusion of transverse shear caused an 18-percent reduction in the calculated frequencies of the fifth symmetrical mode. Also, a comparison of frequencies shown in rows 1, 4, and 5 shows that, for this particular specimen, the modified Levy method gave results which were as good as those obtained from experimental influence coefficients.

Although a comparison of experimental and calculated frequencies provides a test of the accuracy of calculated influence coefficients, a comparison of calculated to experimental deflections of a cantilever delta wing under static loading is of some interest. Therefore the deflections of a delta-wing specimen clamped along the center line under a uniform load of one pound per square inch were obtained from the influence coefficients shown in table XI and were compared with deflections obtained from the experimental influence coefficients shown in reference 6.

The results of these calculations are shown in figure 5. The deflections of the five spars as calculated by the Levy method with transverse shear are shown by the solid lines whereas the deflections as obtained from the experimental influence coefficients are shown as points. From figure 5 it can be seen that, with the exception of the tip, the deflections as given by the modified Levy method agree well with those obtained from experimental influence coefficients. The large discrepancy in the tip deflections can be attributed to the neglect of the torsional stiffness of triangular boxes in the analysis. As can be seen from figure 2, such an assumption in the idealized beam leaves only the leading- and trailing-edge spars to transfer the tip load to the inboard stations. In the actual structure, however, the triangular box contributed a large amount of the torsional stiffness.

CONCLUDING REMARKS

From a comparison of calculated and experimental frequencies it has been shown that a method which deals with an idealized structure, such as the method proposed by Levy, gives excellent results for thin-skin wings, such as the 45° delta-wing specimen investigated, provided that corrections are made for the effects of transverse shear. Furthermore, the Stein-Sanders type of approach seems to be inapplicable to low-aspect-ratio wings with center sections which have not been stiffened against chordwise bending.

Langley Research Center,
National Aeronautics and Space Administration,
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APPENDIX A

FREEING OF INFLUENCE-COEFFICIENT MATRIX FOR GENERAL THREE-POINT SUPPORT

Asymmetrical Structure

The problem of obtaining influence coefficients for other loading and support conditions from the influence coefficients for a three-point support was discussed in reference 1. This appendix is concerned with the problem of obtaining a frequency determinant for a structure from its influence coefficients on an arbitrarily located three-point support.

It is assumed that a structure is simply supported at three arbitrary points and that the influence coefficients of this structure at N points (including the supports) are known. The coordinate system is chosen so that the x-axis goes through two of the supports and the y-axis through the third. The deflections of this structure at any of the points in terms of loading at the points $i = 1$ to N are given by the following matrix:

$$\left|w^{3P}\right| = \left[\delta\right]\left|P\right| \tag{A1}$$

where the elements of the matrices are

w_i^{3P} deflection of point i when $i = 1, 2, 3, \ldots N$

P_i load at station i when $i = 1, 2, 3, \ldots N$

δ_{ij} deflection at point i due to a load at point j when $i, j = 1, 2, 3, \ldots N$

If the system is permitted to be completely unrestrained, the deflection at any point can be written as

$$\left|w\right| = \left|w^{3P}\right| + w_o\left|1\right| + \alpha\left|x\right| + \beta\left|y\right| \tag{A2}$$

where

w_o rigid-body translation

α rigid-body rotation about y-axis

β rigid-body rotation about x-axis

x_i, y_i coordinates of point i

$|1|$ column matrix of ones

The loadings on this structure must then satisfy the following equilibrium conditions:

$$\left.\begin{aligned} \lfloor 1 \rfloor |P| &= 0 \\ \lfloor x \rfloor |P| &= 0 \\ \lfloor y \rfloor |P| &= 0 \end{aligned}\right\} \tag{A3}$$

When a structure is vibrating in its natural mode, the inertial loading can be written as:

$$|P| = \omega^2 \left[M\right] |w| \tag{A4}$$

where ω is the natural circular frequency and M_i is the effective concentrated mass of the structure at station i. With the use of equation (A2), equation (A4) can be written as:

$$|P| = \omega^2 \left[M\right] \Big| |w^{3P}| + w_o |1| + \alpha |x| + \beta |y| \Big| \tag{A5}$$

The values of α, β, and w_o can be obtained in terms of $|w^{3P}|$ by substituting equation (A5) into equation (A3) and solving the resulting equations to yield

$$\left.\begin{aligned} w_o &= \left\lfloor K_{11}\lfloor 1 \rfloor + K_{12}\lfloor x \rfloor + K_{13}\lfloor y \rfloor \right\rfloor \left[M\right] |w^{3P}| \\ \alpha &= \left\lfloor K_{12}\lfloor 1 \rfloor + K_{22}\lfloor x \rfloor + K_{23}\lfloor y \rfloor \right\rfloor \left[M\right] |w^{3P}| \\ \beta &= \left\lfloor K_{13}\lfloor 1 \rfloor + K_{23}\lfloor x \rfloor + K_{33}\lfloor y \rfloor \right\rfloor \left[M\right] |w^{3P}| \end{aligned}\right\} \tag{A6}$$

where

$$\left.\begin{aligned}
K_{11} &= \frac{1}{D}\left(M_{xy}{}^2 - M_{xx}M_{yy}\right) \\
K_{12} &= \frac{1}{D}\left(M_xM_{yy} - M_yM_{xy}\right) \\
K_{13} &= \frac{1}{D}\left(M_{xx}M_y - M_xM_{xy}\right) \\
K_{22} &= \frac{1}{D}\left(M_y{}^2 - MM_{yy}\right) \\
K_{23} &= \frac{1}{D}\left(MM_{xy} - M_xM_y\right) \\
K_{33} &= \frac{1}{D}\left(M_x{}^2 - MM_{xx}\right)
\end{aligned}\right\} \tag{A7}$$

$$D = MM_{xx}M_{yy} + 2M_xM_yM_{xy} - M_{xx}M_y{}^2 - M_{yy}M_x{}^2 - MM_{xy}{}^2 \tag{A8}$$

and

$$\left.\begin{aligned}
M &= \lfloor 1 \rfloor \left[M \right] \left| 1 \right| \\
M_x &= \lfloor x \rfloor \left[M \right] \left| 1 \right| \\
M_y &= \lfloor y \rfloor \left[M \right] \left| 1 \right| \\
M_{xy} &= \lfloor x \rfloor \left[M \right] \left| y \right| = \lfloor y \rfloor \left[M \right] \left| x \right| \\
M_{xx} &= \lfloor x \rfloor \left[M \right] \left| x \right| \\
M_{yy} &= \lfloor y \rfloor \left[M \right] \left| y \right|
\end{aligned}\right\} \tag{A9}$$

With equation (A6), equation (A2) becomes

$$\left|w\right| = \left[H\right]\left|w^{3P}\right| \tag{A10}$$

where

$$\left[H\right] = \left[\left[I\right] + \left[F\right]\left\lceil M \right\rfloor\right] \tag{A11}$$

$$\left[I\right] = \text{unit matrix}$$

and

$$\begin{aligned}\left[F\right] = {} & K_{11}\left|1\right|\left\lfloor 1 \right\rfloor + K_{12}\left|1\right|\left\lfloor x \right\rfloor + K_{13}\left|1\right|\left\lfloor y \right\rfloor + \\ & K_{12}\left|x\right|\left\lfloor 1 \right\rfloor + K_{22}\left|x\right|\left\lfloor x \right\rfloor + K_{23}\left|x\right|\left\lfloor y \right\rfloor + \\ & K_{13}\left|y\right|\left\lfloor 1 \right\rfloor + K_{23}\left|y\right|\left\lfloor x \right\rfloor + K_{33}\left|y\right|\left\lfloor y \right\rfloor\end{aligned} \tag{A12}$$

Substitution of equations (A1) and (A4) into equation (A10) yields the frequency equation:

$$\left|w\right| = \omega^2\left[H\right]\left[\delta\right]\left\lceil M \right\rfloor\left|w\right| \tag{A13}$$

From this frequency or characteristic equation, all modes and frequencies of the free-free asymmetrical structure can be calculated. However, much simplification of the calculation is possible if the structure is symmetrical.

Symmetrical Structure

For a symmetrical structure that is symmetrically supported and whose stations are symmetrically located, the stations can be arranged in three groups: The first group has stations on the center line $x_{C,i}$, $y_{C,i}$, the second group has stations on the right-hand side of the center line $x_{R,i}$,$y_{R,i}$, and the third group has stations on the left-hand side $x_{L,i}$,$y_{L,i}$. Furthermore, the stations of the last group should be numbered so that the ith station on the left is symmetrical with the ith station on the right. Thus,

$$\lfloor x_C \rfloor = 0$$

$$\lfloor x_L \rfloor = -\lfloor x_R \rfloor \qquad \text{(A14)}$$

$$\lfloor y_L \rfloor = \lfloor y_R \rfloor$$

The characteristic or frequency equations (A10) can now be partitioned as follows:

$$\begin{vmatrix} |w_C| \\ |w_R| \\ |w_L| \end{vmatrix} = \omega^2 \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix} \begin{bmatrix} \lceil M_C \rfloor & 0 & 0 \\ 0 & \lceil M_R \rfloor & 0 \\ 0 & 0 & \lceil M_L \rfloor \end{bmatrix} \begin{vmatrix} |w_C| \\ |w_R| \\ |w_L| \end{vmatrix}$$

(A15)

From consideration of the symmetry of the structure and the symmetry of the station location, the following relationships exist:

$$\lceil M_R \rfloor = \lceil M_L \rfloor \qquad \text{(A16)}$$

$$\delta_{12} = \delta_{21} = \delta_{13} = \delta_{31} \qquad \text{(A17)}$$

$$\delta_{23} = \delta_{32} \qquad \text{(A18)}$$

From equations (A9) and (A14) it can be seen that for symmetrical structures

$$M_x = M_{xy} = 0 \qquad \text{(A19)}$$

and therefore

$$K_{12} = K_{23} = 0 \tag{A20}$$

Thus, the elements of $[H]$ in equation (A11) can be defined in terms of the locations of the stations on the center line and right-hand side of the structure as

$$\left.\begin{aligned}
H_{11} &= [I] + \Big[K_{11}\left|1\right|\left\lfloor 1\right\rfloor + K_{13}\left|1\right|\left\lfloor y_C\right\rfloor + K_{13}\left|y_C\right|\left\lfloor 1\right\rfloor + K_{33}\left|y_C\right|\left\lfloor y_C\right\rfloor\Big]\left\lceil M_C \right\rfloor \\
H_{12} = H_{13} &= \Big[K_{11}\left|1\right|\left\lfloor 1\right\rfloor + K_{13}\left|1\right|\left\lfloor y_R\right\rfloor + K_{13}\left|y_C\right|\left\lfloor 1\right\rfloor + K_{33}\left|y_C\right|\left\lfloor y_R\right\rfloor\Big]\left\lceil M_R \right\rfloor \\
H_{21} = H_{31} &= \Big[K_{11}\left|1\right|\left\lfloor 1\right\rfloor + K_{13}\left|1\right|\left\lfloor y_C\right\rfloor + K_{13}\left|y_R\right|\left\lfloor 1\right\rfloor + K_{33}\left|y_R\right|\left\lfloor y_C\right\rfloor\Big]\left\lceil M_C \right\rfloor \\
H_{22} = H_{33} &= [I] + \Big[K_{11}\left|1\right|\left\lfloor 1\right\rfloor + K_{13}\left|1\right|\left\lfloor y_R\right\rfloor + K_{22}\left|x_R\right|\left\lfloor x_R\right\rfloor + \\
&\qquad K_{13}\left|y_R\right|\left\lfloor 1\right\rfloor + K_{33}\left|y_R\right|\left\lfloor y_R\right\rfloor\Big]\left\lceil M_R \right\rfloor \\
H_{23} = H_{32} &= \Big[K_{11}\left|1\right|\left\lfloor 1\right\rfloor + K_{13}\left|1\right|\left\lfloor y_R\right\rfloor - K_{22}\left|x_R\right|\left\lfloor x_R\right\rfloor + \\
&\qquad K_{13}\left|y_R\right|\left\lfloor 1\right\rfloor + K_{33}\left|y_R\right|\left\lfloor y_R\right\rfloor\Big]\left\lceil M_R \right\rfloor
\end{aligned}\right\} \tag{A21}$$

If the frequency equation (eq. (A13)) is used, both symmetrical and antisymmetrical modes are obtained. However, if the symmetrical and antisymmetrical vibrations are considered separately, the order of the frequency matrix can be considerably reduced.

Symmetrical modes.- For the symmetrical structure vibrating in a symmetrical mode,

$$\left|w_R\right| = \left|w_L\right|$$

Thus, only the deflections at the center line and on the right-hand side of the structure need to be considered and the frequency equation (eq. (A13)) reduces to

$$\begin{vmatrix} \lvert w_C \rvert \\ \lvert w_R \rvert \end{vmatrix} = \omega^2 \begin{bmatrix} H_{11} & H_{12} + H_{13} \\ H_{21} & H_{22} + H_{23} \end{bmatrix} \begin{bmatrix} 2\delta_{11} & \delta_{12} + \delta_{13} \\ 2\delta_{21} & \delta_{22} + \delta_{23} \end{bmatrix} \begin{bmatrix} \left\lceil \dfrac{M_C}{2} \right\rfloor & 0 \\ 0 & \left\lceil M_R \right\rfloor \end{bmatrix} \begin{vmatrix} \lvert w_C \rvert \\ \lvert w_R \rvert \end{vmatrix} \tag{A22}$$

or

$$\lvert w \rvert = \omega^2 \Big[[I] + [F^s] \left[\lceil M^s \rfloor\right] \Big] [\delta^s] \left[\lceil M^s \rfloor\right] \lvert w \rvert \tag{A23}$$

where

$$[F^s] = 2\Big[K_{11} \lvert 1 \rvert \lfloor 1 \rfloor + K_{13} \lvert 1 \rvert \lfloor y \rfloor + K_{13} \lvert y \rvert \lfloor 1 \rfloor + K_{33} \lvert y \rvert \lfloor y \rfloor \Big] \tag{A24}$$

Note that in equation (A21) only the properties of the stations on the center line and on the right-hand side of the structure are involved. Also note that the mass associated with the center-line stations in the $\left[\lceil M^s \rfloor\right]$ matrix is one-half of the total assigned mass. The matrix $[\delta^s]$ is the influence coefficient of the structure on a three-point support under a symmetrical loading. When the coefficients K_{11}, K_{13}, and K_{33} as shown in equations (A7), (A8), and (A9) are calculated, the $\left[\lceil M^s \rfloor\right]$ matrix can be used instead of the total $\left[\lceil M \rfloor\right]$ matrix. In this case,

$$\left.\begin{aligned} M &= 2\lfloor 1 \rfloor \left[\lceil M^s \rfloor\right] \lvert 1 \rvert \\ M_y &= 2\lfloor y \rfloor \left[\lceil M^s \rfloor\right] \lvert 1 \rvert \\ M_{yy} &= 2\lfloor y \rfloor \left[\lceil M^s \rfloor\right] \lvert y \rvert \\ M_{xx} &= 2\lfloor x \rfloor \left[\lceil M^s \rfloor\right] \lvert x \rvert \\ M_x &= M_{xy} = 0 \end{aligned}\right\} \tag{A25}$$

Antisymmetrical modes.- For a symmetrical structure vibrating in an antisymmetrical mode,

$$\left|w_C\right| = 0$$

and

$$\left|w_R\right| = -\left|w_L\right|$$

Thus, only the deflections on one side need to be considered. For this case, the frequency equation (A15) reduces to

$$\left|w_R\right| = \omega^2\left[H_{22} - H_{23}\right]\left[\delta_{22} - \delta_{23}\right]\left[\diagdown M_R \diagdown\right]\left|w_R\right| \tag{A26}$$

or

$$\left|w\right| = \omega^2\left[\left[I\right] + 2K_{22}\left|x\right|\left\lfloor x\right\rfloor\right]\left[\delta^a\right]\left[\diagdown M^s \diagdown\right]\left|w\right| \tag{A27}$$

Note that in this equation only the properties of the stations on one side of the center line are involved. The influence-coefficient matrix $\left[\delta^a\right]$ is the influence coefficient matrix of the structure on a three-point support under an antisymmetrical loading.

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TABLE I.- MOMENTS OF INERTIA OF IDEALIZED SPARS

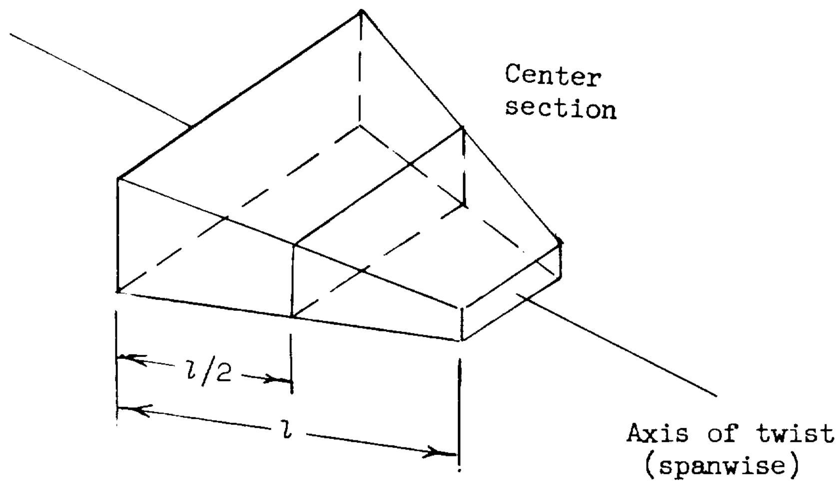
x	Moments of inertia of -					
	Spar 1	Spar 2	Spar 3	Spar 4	Spar 5	Swept leading edge
0	26.123	43.546	45.643	40.300	20.635	
16	26.123	43.546	45.643	40.300	20.635	8.624
28	21.748	36.374	31.131	33.720		7.119
40	17.785	29.858	31.306	27.738		5.785
52	14.230	23.995	25.163			4.599
64	11.080	18.781	19.700			3.559
76	8.331	14.213				2.659
88	5.980	10.287				1.897
100	4.022					1.268
112	2.456					.770

TABLE II.- MOMENT OF INERTIAS OF IDEALIZED RIBS

Rib	I (*)
1	11.588
2	15.647
3	11.550
4	9.402
5	7.509
6	5.816
7	4.369
8	3.115
9	1.255

*Ribs are assumed to be uniform.

TABLE III.- TORSIONAL CONSTANT OF TORQUE BOXES



Torque box	J
1, 3, 7, and 13	98.517
2	67.674
4, 8, and 14	90.232
5, 9, and 15	74.750
6	45.544
10 and 16	60.726
11 and 17	48.154
12	27.782
18	37.043
19	27.385
20	14.387

TABLE IV.- INFLUENCE COEFFICIENTS FOR SPAR 1 AS CANTILEVER BEAM

$$\left[\left| w \right| = \frac{1}{E} \left[\delta_S \right] \left| P \right| \right]$$

(a) Neglecting the effects of transverse shear

$$\left[\delta_S \right]$$

Station	22	23	24	25	26	27	28	29	30
22	52.266	111.064	169.863	228.662	287.461	346.259	405.058	463.857	522.656
23	111.064	281.220	463.508	645.797	828.086	1,010.375	1,192.664	1,374.953	1,557.242
24	169.863	463.508	834.758	1,220.725	1,606.692	1,992.659	2,378.626	2,764.593	3,150.560
25	228.662	645.797	1,220.725	1,890.308	2,578.308	3,266.207	3,954.107	4,642.006	5,329.905
26	287.461	828.086	1,606.692	2,578.308	3,668.163	4,781.134	5,894.105	7,007.076	8,120.047
27	346.259	1,010.375	1,992.659	3,266.207	4,781.134	6,447.672	8,144.491	9,841.309	11,538.128
28	405.058	1,192.664	2,378.626	3,954.107	5,894.105	8,144.491	10,596.220	13,089.314	15,582.408
29	463.857	1,374.953	2,764.593	4,642.006	7,007.076	9,841.309	13,089.314	16,617.617	20,205.800
30	522.656	1,557.242	3,150.560	5,329.905	8,120.047	11,538.128	15,582.408	20,205.800	25,246.469

(b) Including the effects of transverse shear

$$\left[\delta_S \right]$$

Station	22	23	24	25	26	27	28	29	30
22	175.545	234.344	293.143	351.942	410.740	469.539	528.338	587.137	645.935
23	234.344	492.616	665.770	838.925	1,012.080	1,185.235	1,358.389	1,531.544	1,704.699
24	293.143	665.770	1,125.587	1,492.177	1,858.767	2,225.358	2,591.978	2,958.538	3,325.128
25	351.942	838.925	1,492.177	2,251.988	2,908.724	3,565.461	4,222.197	4,878.933	5,535.670
26	410.740	1,012.080	1,858.767	2,908.724	4,091.808	5,159.590	6,227.372	7,295.155	9,362.937
27	469.539	1,185.235	2,225.358	3,565.461	5,159.590	6,924.698	8,558.872	10,193.046	11,827.220
28	528.338	1,358.389	2,591.978	4,222.197	6,227.372	8,558.872	11,117.942	13,525.292	15,932.642
29	587.137	1,531.544	2,958.538	4,878.933	7,295.155	10,193.046	13,525.292	17,175.353	20,644.405
30	645.935	1,704.699	3,325.128	5,535.670	9,362.937	11,827.220	15,932.642	20,644.405	25,831.532

TABLE V.- STIFFNESS COEFFICIENTS FOR SPAR 1 UNDER SYMMETRICAL LOADING

$\left[\Delta_S^1 \right]$

Station	21	22	23	24	25	26	27	28	29	30
21	0.04908861	-0.0822049	0.0411729	0.00999641	0.0024012	-0.00056916	0.00013281	-0.00003010	0.000005979	-0.000000933
22	-.0822049	.172020	-.131713	.0519875	.012489	.0029607	.0006896	.00015617	.000032568	.000004646
23	.0411729	-.131713	.173902	-.119798	.045101	-.010692	.00249059	-.0056472	.00011823	.000016929
24	-.00999641	.0519875	-.119798	.146262	-.09788	.036196	-.0084332	.0019138	-.0040124	.000057546
25	.0024012	-.012489	.045101	-.097788	.117039	-.0776837	.0280409	-.0063648	.00013349	-.00019150
26	-.00056916	.0029607	-.010692	.036196	-.0770837	.0908740	-.0587813	.0208393	-.0043710	.00062716
27	.00013281	-.0006896	.00249059	-.0084332	.0280409	-.0587813	.0679500	-.0427307	.0140343	-.0020138
28	-.00003010	.00015617	-.00056472	.0019138	-.0063648	.0208393	-.0427307	.0476624	-.0271777	.0062964
29	.000005979	-.000032568	.00011823	-.00040124	.0013349	-.0043710	.0140343	-.0271777	.0248476	-.0083585
30	-.000000933	.000004646	-.000016929	.000057546	-.00019150	.00062716	-.0020138	.0062964	-.0083585	.00359591

TABLE VI.- STIFFNESS COEFFICIENTS FOR RIB 4

$$\left[\Delta_R^4\right]$$

Station	6	10	16	24
6	0.00108821	-0.0024847	0.00163231	-0.00027205
10	-.00244847	.00652924	-.00571307	.00163231
16	.00163231	-.00571308	.00652924	-.00244847
24	-.00027205	.00163231	-.00244847	.00108821

TABLE VII.- STIFFNESS COEFFICIENTS FOR TORQUE BOXES 15 AND 16

$\left[\Delta_T^{15}\right]$

Station	15	16	23	24
15	0.004055431	0.004055431	0.004055431	0.004055431
16	0.004055431	0.004055431	0.004055431	0.004055431
23	0.004055431	0.004055431	0.004055431	0.004055431
24	0.004055431	0.004055431	0.004055431	0.004055431

$\left[\Delta_T^{16}\right]$

Station	16	17	24	25
16	0.003294499	0.003294499	0.003294499	0.003294499
17	0.003294499	0.003294499	0.003294499	0.003294499
24	0.003294499	0.003294499	0.003294499	0.003294499
25	0.003294499	0.003294499	0.003294499	0.003294499

TABLE VIII.- ELEMENTS OF ROW 24 OF STIFFNESS COEFFICIENT

OF DELTA WING UNDER SYMMETRICAL LOADING

$$P_{24} = E\lfloor\Delta_{24,n}\rfloor\left|w_i\right|$$

$$\lfloor\Delta_{24,n}\rfloor$$

n	$\Delta_{24,n}$	n	$\Delta_{24,n}$
1	0	18	0
2	0	19	0
3	0	20	0
4	0	21	-.00999641
5	0	22	.0519875
6	-.00027205	23	-.1238534
7	0	24	.15470014
8	0	25	-.10108250
9	0	26	.036196
10	.00163231	27	-.0084332
11	0	28	.019138
12	0	29	-.00040124
13	0	30	.00005746
14	0	31	0
15	.004055431	32	0
16	-.009798400	33	0
17	.003294499	34	0

TABLE IX.- STIFFNESS MATRIX FOR

(a) Symmetrical deflections

Station	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	0.02682143	0.0040087	-0.01180341	0.00734302	0.00363035	0	0.002910508	0	0	0	0.000202307	0	-0.000727625	0	0	0
3	.0040087	.09182983	-.1338191	.0595852	-.00933340	-.01191208	.00400870	0	0	0	0	.00323320	0	0	0	0
4	-.01180341	-.1338191	.28773245	-.1950308	.04860282	.00400870	-.01957586	.004895355	0	0	0	0	.00436574	0	0	0
5	.00734302	.0595852	-.1950308	.2379946	-.07429793	0	.004895355	-.02154475	.004055431	0	0	0	0	.00440182	0	0
6	.00363035	-.00933340	.04860282	-.07429793	.05502343	0	0	.004055431	-.008974781	.004941755	.004113883	0	0	0	.00163231	0
7	0	-.01191208	.00400870	0	0	.10530996	-.1517220	.0720054	-.0174432	.0039563	-.00061043	-.01191208	.00400870	0	0	0
8	.002910508	.00400870	-.01957586	.004895355	0	-.1517220	.3341983	-.24033251	.0907950	-.0205936	.00317793	.00400870	-.01957586	.004895355	0	0
9	0	0	.004895355	-.02154475	.004055431	.0720054	-.24033251	.3551829	-.2171329	.0744876	-.0114950	0	.004895355	-.01714303	.004055431	0
10	0	0	0	.004055431	-.008974781	-.0174432	.0907950	-.2171329	.2728925	-.1640481	.0389947	0	0	.004055431	-.01306301	.003294499
11	0	0	0	0	.004941755	.0039563	-.0205936	.0744876	-.1640481	.18031633	-.05584437	0	0	0	.003294499	-.01331399
12	.000202307	0	0	0	.004113883	-.00061043	.00317793	-.0114950	.0389947	-.05584437	.03823392	0	0	0	0	.002612571
13	0	.00323320	0	0	0	-.01191208	.00400870	0	0	0	0	.09813152	-.1451224	.0687170	-.0167129	.00402300
14	-.000727625	0	.00436574	0	0	.00400870	-.01957586	.004895355	0	0	0	-.1451224	.3160310	-.2298377	.0869980	-.0209390
15	0	0	0	.00440182	0	0	.004895355	-.01714303	.004055431	0	0	.0687170	-.2298377	.31716106	-.2088919	.07573110
16	0	0	0	0	.00163231	0	0	.004055431	-.01306301	.003294499	0	-.0167129	.0869980	-.2088919	.2668131	-.1710800
17	0	0	0	0	0	0	0	0	.003294499	-.01331399	.002612571	.00402300	-.0209390	.0757311	-.1710800	.21488768
18	0	0	0	0	0	0	0	0	0	.002612571	-.005381887	-.00095210	.0049575	-.0179288	.0608116	-.13471734
19	0	0	0	0	0	0	0	0	0	0	.003014435	.00021018	-.00109551	.00396083	-.0134340	.0447692
20	.000010606	0	0	0	.000216011	0	0	0	0	0	.00234956	-.00003167	.000165125	-.000597105	.00202525	-.006743[illegible]
21	0	-.000538868	0	0	0	.00215547	0	0	0	0	0	-.00706228	.00400870	0	0	0
23	0	0	0	-.000733628	0	0	0	.00220089	0	0	0	0	.004895355	-.01200758	.004055431	0
24	0	0	0	0	-.000272050	0	0	0	.00163231	0	0	0	0	.004055431	-.009798400	.003294499
25	0	0	0	0	0	0	0	0	0	.00177766	0	0	0	0	.003294499	-.007981009
26	0	0	0	0	0	0	0	0	0	0	.00063105	0	0	0	0	.002612571
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	.000000312	0	0	0	.000006387	0	0	0	0	0	.000069580	0	0	0	0	0
31	-.000002182	0	0	0	-.000044994	0	0	0	0	0	-.000489792	0	0	0	0	0
32	-.000046755	0	0	0	-.000955418	0	0	0	0	0	-.00966743	0	0	0	0	0
33	-.000856076	0	0	0	-.01605958	0	0	0	.004941755	-.01758671	-.008775619	0	0	0	0	.00355532
34	-.01248979	0	.00734302	-.02666847	-.01228425	0	0	.005868998	0	0	-.001135430	0	0	-.00146728	0	0

WING ON THREE-POINT SUPPORT

transverse shear neglected

18	19	20	21	23	24	25	26	27	28	29	30	31	32	33	34
0	0	0.000010606	0	0	0	0	0	0	0	0	0.000000312	-0.000002182	-0.000046755	-0.000856076	-0.01248979
0	0	0	-.000538868	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.00734302
0	0	0	0	-.000733628	0	0	0	0	0	0	0	0	0	0	-.02666847
0	0	.000216011	0	0	-.000272050	0	0	0	0	0	.000006387	-.000044994	-.000955418	-.01605958	-.01228425
0	0	0	.00215547	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	.00220089	0	0	0	0	0	0	0	0	0	0	.005868998
0	0	0	0	0	.00163251	0	0	0	0	0	0	0	0	.004941755	0
.002612571	0	0	0	0	0	.00177766	0	0	0	0	0	0	0	-.01758671	0
-.005381887	.003014435	.00234956	0	0	0	0	.00063105	0	0	0	.000069580	-.000489792	-.00966743	-.008775619	-.00113543
-.00095210	.00021018	-.00005167	-.00706228	0	0	0	0	0	0	0	0	0	0	0	0
.0049575	-.00109531	.000169125	.00400870	.004895355	0	0	0	0	0	0	0	0	0	0	0
-.0179288	.00596083	-.000597105	0	-.01200758	.004055431	0	0	0	0	0	0	0	0	0	-.00146728
.0608116	-.0154340	.00202525	0	.004055431	-.00979840	.003294499	0	0	0	0	0	0	0	0	0
-.15471714	.0447695	-.0067496	0	0	.003294499	-.007981009	.002612571	0	0	0	0	0	0	.00355532	0
.16251218	-.09582706	.0221608	0	0	0	.002612571	-.005884293	.002009622	0	0	0	0	.003014435	0	0
-.09582706	.1015649	-.03144626	0	0	0	0	.002009622	-.005391406	.001485664	0	0	0	-.009821085	0	0
.0221608	-.03144626	.02058634	0	0	0	0	0	.001485664	-.002266188	.001561045	.000694609	-.004582569	-.00480243	-.000719573	-.000059597
0	0	0	.05444448	.0411729	-.00999641	.0024012	-.00056916	.00013281	-.00003010	.000005979	-.000000933	0	0	0	0
0	0	0	.04117290	.184197771	-.12385343	.0451010	-.0106920	.00249059	-.00056477	.00011823	-.000016929	0	0	0	.000244542
0	0	0	-.00999641	-.12385343	.15470014	-.10108250	.0361960	-.00843320	.00191380	-.00040124	.000057546	0	0	0	0
.002612571	0	0	.0024012	.0451010	-.10108250	.1238349	-.07969627	.0280409	-.0063648	.0013349	-.00019150	0	0	-.000592552	0
-.005884293	.002009622	0	-.00056916	-.0106920	.0361960	-.07969627	.09612724	-.06079092	.0208393	-.0045710	.00062716	0	0	0	0
.002009622	-.005391406	.001485664	.00013281	.00249059	-.00843320	.0280409	-.06079092	.07207732	-.04421636	.0140343	-.00201380	0	.00126408	0	0
0	.001485664	-.002266188	-.00003010	-.00056477	.00191380	-.0063648	.0208393	-.04421636	.04992859	-.02873875	.00629640	.001561045	0	0	0
0	0	.001561045	.000005979	.00011823	-.00040124	.0013349	-.0045710	.0140343	-.02873875	.02796970	-.00835850	-.003122095	0	0	0
0	0	.000694609	-.000000933	-.000016929	.000057546	-.00019150	.00062716	-.00201380	.00629640	-.00835850	.00399202	-.000921125	-.00022282	-.0000212887	-.0000017616
0	0	-.004582569	0	0	0	0	0	0	.001561045	-.003122095	-.000921125	.005871415	.001567973	.000149948	.0000123806
.003014435	-.009821085	-.00480243	0	0	0	0	0	.00126408	0	0	-.000222820	.001567973	.01622378	.00318224	.000263681
0	0	-.000719573	0	0	0	-.000592552	0	0	0	0	-.0000212887	.000149948	.00318224	.02797735	.004804766
0	0	-.000059597	0	.000244542	0	0	0	0	0	0	-.0000017616	.0000123806	.000263681	.004804766	.035573975

TABLE IX.- STIFFNESS MATRIX FOR WING

(b) Antisymmetrical deflections;

Station	2	4	5	6	8	9	10	11
2	0.01170746	-0.01180341	0.00734302	0.00363035	0.002910508	0	0	0
4	-.01180341	.17812116	-.15969538	.0430679	-.019575855	.004895355	0	0
5	.00734302	-.15969538	.22660361	-.072513631	.004895355	-.021544726	.004055431	0
6	.00363035	.0430679	-.072513631	.05474396	0	.004055431	-.008974781	.004941755
8	.002910508	-.019575855	.004895355	0	.20868271	-.19741071	.0803964	-.01823517
9	0	.004895355	-.021544726	.004055431	-.19741071	.32050486	-.21357686	.0736813
10	0	0	.004055431	-.008974781	.0803964	-.21357686	.272031481	-.16385325
11	0	0	0	.004941755	-.01823517	.0736813	-.16385325	.180272034
12	.0002023067	0	0	.0041138834	.00281407	-.01137046	.0389646	-.055837571
14	-.000727625	.00436574	0	0	-.019575855	.004895355	0	0
15	0	0	.00440182	0	.004895355	-.017143026	.004055431	0
16	0	0	0	.00163231	0	.004055431	-.013063010	.003294499
17	0	0	0	0	0	0	.003294499	-.0133159894
18	0	0	0	0	0	0	0	.0026125709
19	0	0	0	0	0	0	0	0
20	0.000010608	0	0	.0002160107	0	0	0	0
23	0	0	-.000733628	0	0	.00220089	0	0
24	0	0	0	-.000272050	0	0	.00163231	0
25	0	0	0	0	0	0	0	.00177766
26	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0
30	.000000312	0	0	.000006387	0	0	0	0
31	-.000002182	0	0	-.0000449943	0	0	0	0
32	-.000046755	0	0	-.000955418	0	0	0	0
33	-.000856076	0	0	-.016059575	0	0	.004941755	-.01758671
34	-.01248979	.00734502	-.02666848	-.01228425	0	.005868998	0	0

Station	12	14	15	16	17	18
2	0.0002023067	-0.000727625	0	0	0	0
4	0	.00436574	0	0	0	0
5	0	0	.00440182	0	0	0
6	.0041138834	0	0	.00163231	0	0
8	.00281407	-.019575855	.004895355	0	0	0
9	-.01137046	.004895355	-.017143026	.004055431	0	0
10	.0389646	0	.004055431	-.01306301	.003294499	0
11	-.055837571	0	0	.003294499	-.01331[illegible]	.0026125709
12	.03823282	0	0	0	.0026125709	-.0053818866
14	0	.1962260	-.18886471	.0770356	-.0185388	.0043917
15	0	-.18886471	.303146062	-.20548386	-.0749101	-.0177353
16	0	.0770356	-.205483862	.26598510	-.17088000	.0607646
17	.0026125709	-.0185388	.0749101	-.17088000	.21483968	-.134706142
18	-.0053818866	.0043917	-.0177353	.0607646	-.134706142	.16250918
19	.0030144350	-.000971203	.0039184	-.0134237	.0447670	-.095826457
20	.002349564	.000146573	-.00059076	.00202371	-.00674923	.0221607
23	0	.004895355	-.012007576	.004055431	0	0
24	0	0	.004055431	-.009798400	.003294499	0
25	0	0	0	.003294499	-.0079810094	.002612571
26	.00063105	0	0	0	.002612571	-.0058842927
27	0	0	0	0	0	.0020096217
28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	.000069580	0	0	0	0	0
31	-.0004897922	0	0	0	0	0
32	-.00966743	0	0	0	0	.003014435
33	-.008775619	0	0	0	.00355532	0
34	-.001135430	0	-.00146728	0	0	0

ON THREE-POINT SUPPORT - Concluded

transverse shear neglected

19	20	23	24	25	26	27	28	29	30	31	32	33	34
0	0.000010608	0	0	0	0	0	0	0	0.000000312	-0.000002182	-0.000046755	-0.000856076	-0.01248979
0	0	0	0	0	0	0	0	0	0	0	0	0	.00734302
0	0	-.000733628	0	0	0	0	0	0	0	0	0	0	-.02666848
0	.0002160107	0	-.00027205	0	0	0	0	0	.000006387	-.0000449943	-.000955418	-.01605958	-.01228425
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	.00220089	0	0	0	0	0	0	0	0	0	0	.005868998
0	0	0	.00163231	0	0	0	0	0	0	0	0	.004941755	0
0	0	0	0	.00177766	0	0	0	0	0	0	0	-.017586705	0
.003014435	.002349564	0	0	0	.00063105	0	0	0	.000069580	-.0004897922	-.00966743	-.008775619	-.001135430
-.000971203	.000146573	.004895355	0	0	0	0	0	0	0	0	0	0	0
.0039184	-.00059076	-.012007576	.004055431	0	0	0	0	0	0	0	0	0	-.00146728
-.0134237	.00202371	.004055431	-.009798400	.003294499	0	0	0	0	0	0	0	0	0
.0447670	-.00674923	0	.003294499	-.0079810094	.002612571	0	0	0	0	0	0	.00355532	0
-.09582646	.0221607	0	0	.002612571	-.005884292	.0020096217	0	0	0	0	.003014435	0	0
.10156481	-.031446264	0	0	0	.0020096217	-.0053914059	.001485664	0	0	0	-.009821085	0	0
-.031446264	.020586341	0	0	0	0	.001485664	-.002266188	.001561045	.000694609	-.004582568	-.004802426	-.000719573	-.0000595973
0	0	.17581577	-.12181843	.0446126	-.0105760	.00246316	-.000558374	.00011776	-.000016596	0	0	0	.000244542
0	0	-.12181843	.15420614	-.10096400	.0361678	-.00842654	.00191066	-.00040113	.000057465	0	0	0	0
0	0	.0446126	-.10096400	.12380690	-.079689471	.0280393	-.0063644	.00133487	-.00019148	0	0	-.000592552	0
.0020096217	0	-.0105760	.0361678	-.079689471	.09612564	-.06079052	.0208392	-.00437099	.00062716	0	0	0	0
-.005391406	.001485664	.00246316	-.00842654	.0280393	-.06079052	.072077226	-.044216369	.0140343	-.0020138	0	.00126408	0	0
.001485664	-.002266188	-.000558374	.00191066	-.0063644	.0208392	-.044216369	.049928588	-.028738745	.00629641	.001561045	0	0	0
0	.0015610450	.00011776	-.00040113	.00133487	-.00437099	.0140343	-.028738745	.027969695	-.0083585	-.0031220951	0	0	0
0	.000694609	-.000016596	.000057465	-.00019148	.00062716	-.0020138	.00629641	-.0083585	.003992017	-.0009211247	-.000222820	-.0000212887	-.0000017616
0	-.004582569	0	0	0	0	0	.001561045	-.0031220951	-.0009211247	.00587142	.001567973	.000149948	.0000123806
-.009821085	-.004802426	0	0	0	0	.00126408	0	0	-.000222820	.001567973	.016223776	.00318224	.000263681
0	-.000719573	0	0	-.000592552	0	0	0	0	-.000021288	.000149948	.00318224	.02797735	.004804766
0	-.0000595973	.000244542	0	0	0	0	0	0	-.0000017616	.0000123806	.000263681	.004804766	.03557398

TABLE X.- INFLUENCE COEFFICIENT MATRIX

(a) Symmetrical loading;

Station	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	51.840556	23.292260	29.565920	35.316710	39.978140	19.903370	20.465700	21.979080	24.066920	26.079420	27.874650	10.371690	10.662470	11.151180	11.880520	12.859990
3	23.292260	247.57966	235.62897	225.04320	218.53306	274.60063	267.11635	254.96158	241.53798	232.01163	225.45891	164.56462	163.44822	161.46636	158.78607	155.24781
4	29.565920	235.62897	244.74328	253.15837	262.86324	275.27832	274.16204	271.45005	268.34342	268.80281	271.56951	167.60093	168.29999	170.05828	172.81753	175.78873
5	35.316710	225.04320	253.15837	301.02180	349.99520	278.08520	286.89350	301.63130	319.61840	341.12420	363.51070	173.12440	177.60530	186.76790	200.24900	216.25470
6	39.978140	218.53306	262.86324	349.99570	464.82890	282.61720	302.58830	338.78200	386.00540	438.00640	488.87480	179.68910	189.38920	208.71050	236.80540	270.80980
7	19.903370	274.60063	275.27832	278.08520	282.61720	400.21242	380.98263	356.70920	333.69741	316.57739	303.34514	262.04986	255.15322	244.46439	232.30749	219.90623
8	20.465700	267.11635	274.16204	286.89350	302.58830	380.98263	379.13397	368.82967	356.48657	349.05079	345.38343	254.06752	252.97341	250.97131	248.46662	245.69783
9	21.979080	254.96158	271.45005	301.63130	338.78200	356.70920	368.82967	386.51508	399.42266	413.65496	430.96817	240.75359	248.39250	262.01677	278.84214	296.30163
10	24.066920	241.53798	268.34342	319.61840	386.00540	333.69741	356.48657	399.42266	455.56540	511.50330	567.69530	225.14606	242.68510	276.02460	321.20280	371.41560
11	26.079420	232.01163	268.80281	341.12420	438.00640	316.57739	349.05079	413.65496	511.50330	635.76580	761.92350	210.82960	238.66860	293.53450	372.83110	467.89410
12	27.874650	225.45891	271.56951	363.51070	488.87480	303.34514	345.38343	430.96817	567.69530	761.92350	996.95490	197.80460	236.17010	313.35490	429.32630	576.26070
13	10.371690	164.56462	167.60093	173.12440	179.68910	262.04986	254.06752	240.75359	225.14600	210.82960	197.80460	229.36349	209.72454	186.17612	164.32654	145.25174
14	10.662470	163.44822	168.29999	177.60530	189.38920	255.15322	252.97341	248.39250	242.68510	238.66860	236.17010	209.72454	208.83389	200.62860	190.55103	182.10886
15	11.151180	161.46636	170.05828	186.76790	208.71050	244.46439	250.97131	262.01677	276.02460	293.53450	313.35490	186.17612	200.62860	222.92021	240.43412	255.52490
16	11.880520	158.78607	172.81753	200.24900	236.80540	232.30749	248.46662	278.84214	321.20280	372.83110	429.32630	164.32654	190.55103	240.43412	304.29198	362.89558
17	12.859990	155.24781	175.78873	216.25470	270.80980	219.90623	245.69783	296.30163	371.41560	467.89410	576.26070	145.25174	182.10886	255.52490	362.89558	488.94366
18	13.982530	151.74494	179.24462	233.83320	308.14390	208.69561	243.89594	314.55785	423.54160	570.41270	742.74980	128.92425	176.28969	272.30171	419.03029	610.42877
19	15.076700	150.32964	184.68959	253.22980	347.11230	201.29282	245.85556	336.51075	479.52110	678.14670	918.40940	116.22396	174.33366	293.22524	478.86119	731.90409
20	16.109680	150.46747	191.46258	273.48250	386.25640	196.49111	250.38445	360.90139	537.54060	787.05970	1,094.0155	105.59449	174.67950	316.92398	542.12197	857.10929
21	-.07296640	-3.1444147	-4.0419050	-5.8257790	-8.3152540	-3.8222471	-5.8823531	-10.127744	-16.442398	-24.396833	-33.371936	4.3631906	-3.5299065	-15.510915	-28.981237	-42.554224
23	.17780080	8.8867806	10.690521	14.348473	19.454219	12.940574	17.179794	25.535828	37.950270	53.822627	71.960922	3.1766129	15.1207210	36.603710	63.293266	91.419690
24	.47206800	20.378383	24.953327	34.227550	47.194640	29.228680	39.685375	60.297604	91.042460	130.83680	176.85771	9.0331190	34.336940	82.388967	147.29465	219.76164
25	.88695100	31.571625	39.857275	56.602570	79.98220	43.866486	61.949256	97.859930	152.01413	223.05953	306.26422	13.190036	51.792825	127.53538	236.35265	367.11114
26	1.4306880	41.110207	53.922929	79.759030	115.77058	54.995079	81.566313	134.81631	216.23918	324.71777	453.24553	14.441825	65.851401	168.74845	322.59439	518.77975
27	2.1018470	48.470065	66.449152	102.67839	153.14557	62.193320	97.686169	169.40080	280.51503	430.88413	611.19202	12.676594	76.394282	205.55581	403.71972	667.05769
28	2.8628750	54.163450	77.711786	125.16849	191.27610	66.530017	111.13500	201.85722	343.96616	538.88300	775.10536	8.773488	84.456357	239.19491	480.78051	811.04487
29	3.6482160	59.464670	88.706978	147.64494	229.74819	70.153652	123.95841	233.91545	407.50780	647.87559	941.33495	4.3319400	91.881388	272.02239	556.89418	954.43266
30	4.4353400	64.753420	99.703880	170.15070	268.28680	73.740310	136.76250	265.99583	471.14950	757.09070	1,107.9447	-.13972000	99.266390	304.79918	632.97939	1,097.9474
31	10.281080	107.56165	145.60106	221.96440	327.59470	134.95946	193.50242	313.57655	504.84210	773.14900	1,102.7935	52.59278	136.80602	310.67067	587.49599	978.21725
32	21.931720	192.34443	235.99242	323.11560	442.33220	256.92773	305.52020	405.02772	564.54620	791.76740	1,071.9111	157.87029	211.86575	322.19852	493.88594	726.32478
33	33.878620	280.05895	285.01765	378.63170	505.83910	316.53644	348.76665	412.07144	507.40530	631.45210	762.8350	206.31744	229.08259	274.29645	340.48261	421.36746
34	45.386280	142.07974	171.27049	222.96610	279.80200	172.68259	181.55129	197.26304	217.33680	238.39220	258.73280	106.25322	110.01762	117.61472	128.65921	142.13633

L-153

FOR WING ON THREE-POINT SUPPORT

transverse shear neglected

18	19	20	21	23	24	25	26	27	28	29	30	31	32	33	34
13.982530	15.076700	16.109680	-0.07296640	0.17780080	0.47206800	0.88695100	1.4306880	2.1018470	2.8628750	3.6482160	4.4353400	10.281080	21.931720	33.878620	45.386280
151.74494	150.32964	150.46747	-3.1444147	8.8867806	20.378383	31.571625	41.110207	48.470065	54.163450	59.464670	64.753420	107.56165	192.34443	280.05895	142.07974
179.24462	184.68959	191.46258	-4.0419050	10.690521	24.953327	39.857275	53.922929	66.449152	77.711786	88.706978	99.703880	145.60106	235.99242	285.01765	171.27049
233.83320	253.22980	273.48250	-5.8257790	14.348473	34.227550	56.602570	79.759030	102.67839	125.16849	147.64494	170.15070	221.96440	323.11560	378.63170	222.96610
308.14390	347.11230	386.25640	-8.3152540	19.454219	47.194640	79.98220	115.77058	153.14557	191.27610	229.74819	268.28680	327.59470	442.33220	505.83910	279.80200
208.69561	201.29282	196.49111	-3.8222471	12.940574	29.228680	43.866486	54.995079	62.193320	66.530017	70.153652	73.740310	134.95946	256.92773	316.53644	172.68259
243.89594	245.85556	250.38445	-5.8823531	17.179794	39.685375	61.949256	81.566313	97.686169	111.13500	123.95841	136.76250	193.50242	305.52020	348.76665	181.55129
314.55785	336.51075	360.90139	-10.127744	25.535828	60.297604	97.859930	134.81631	169.40080	201.85722	233.91545	265.99583	313.57655	405.02772	412.07144	197.26304
423.54160	479.52110	537.54060	-16.442398	37.950270	91.042460	152.01413	216.23918	280.51503	343.96616	407.50780	471.14950	504.84210	564.54620	507.40530	217.33680
570.41270	678.14670	787.05970	-24.396833	53.822627	130.83680	223.05953	324.71777	430.88413	538.88300	647.87559	757.09070	773.14900	791.76740	631.45210	238.39220
742.74980	918.40940	1,094.0155	-33.371936	71.960922	176.85771	306.26422	453.24553	611.19202	775.10536	941.33495	1,107.9447	1,102.7935	1,071.9111	762.8350	258.73280
128.92425	116.22396	105.59449	4.3631906	3.1766129	9.0331190	13.190036	14.441825	12.676594	8.773488	4.3319400	-.13972000	52.59278	157.87029	206.31744	106.25322
176.28969	174.33366	174.67950	-3.5299065	15.1207210	34.336940	51.792825	65.851401	76.394282	84.456357	91.881388	99.266390	136.80602	211.86575	229.08259	110.01762
272.30171	293.22524	316.92398	-15.510915	36.603710	82.388967	127.53538	168.74845	205.55581	239.19491	272.02239	304.79918	310.67067	322.19852	274.29645	117.61472
419.03029	478.86119	542.12197	-28.981237	63.293266	147.29465	236.35265	322.59439	403.71972	480.78051	556.89418	632.97939	587.49599	493.88594	340.48261	128.65921
610.42877	731.90409	857.10929	-42.554224	91.419690	219.76164	367.11114	518.77975	667.05769	811.04487	954.43266	1,097.9474	978.21725	726.32478	421.36746	142.13633
829.85701	1,050.7351	1,273.2953	-55.921498	119.61066	294.36837	508.42847	744.88680	687.92661	1,230.3223	1,474.2653	1,718.8660	1,499.2098	1,018.6292	510.42373	157.03819
1,050.7351	1,422.4527	1,803.8809	-69.144242	147.71831	369.65915	654.54554	988.98284	1,354.9392	1,735.5302	2,125.0120	2,516.6172	2,170.0146	1,365.9733	601.87258	172.72000
1,273.2953	1,803.8809	2,421.3313	-82.424203	176.03564	445.86586	803.82472	1,242.9649	1,749.6830	2,303.5498	2,882.7581	3,467.1308	2,967.7070	1,732.3414	692.86161	188.54721
-55.921498	-69.144242	-82.424203	34.101482	-28.646445	-49.925225	-67.311928	-82.651004	-96.965158	-110.80212	-124.41642	-137.98376	-110.01505	-56.296255	-18.326739	-3.4812120
119.61066	147.71831	176.03564	-28.646445	55.286058	104.33753	143.24589	176.85787	207.83517	237.59495	266.79144	295.87043	235.46637	120.33467	40.653402	8.3884820
294.36837	369.65915	445.86586	-49.925225	104.33753	233.84507	345.32982	439.21412	524.35111	605.45558	684.71425	763.59274	603.12241	300.77097	99.250677	20.289094
508.42847	654.54554	803.82472	-67.311928	143.24589	345.32982	568.29663	764.81350	938.36308	1,101.2755	1,259.4146	1,416.5901	1,106.0620	532.42950	170.15335	34.145660
744.88680	988.98284	1,242.9649	-82.651004	176.85787	439.21412	764.81350	1,114.4713	1,432.2505	1,723.4281	2,003.0316	2,280.3699	1,751.7830	805.87976	249.08669	49.079710
687.92661	1,354.9392	1,749.6830	-96.965158	207.83517	524.35111	938.36308	1,432.2505	1,963.1403	2,460.5436	2,930.4702	3,395.2474	2,549.7758	1,108.7179	332.82553	64.530150
1,230.3223	1,735.5302	2,303.5498	-110.80212	237.59495	605.45558	1,101.2755	1,723.4281	2,460.5436	3,266.4371	4,052.6611	4,827.6153	3,515.6047	1,428.3372	419.09673	80.200630
1,474.2653	2,125.0120	2,882.7581	-124.41642	266.79144	684.71425	1,259.4146	2,003.0316	2,930.4702	4,052.6611	5,341.8541	6,664.9163	4,657.7507	1,752.4211	506.36398	95.986690
1,718.8660	2,516.6172	3,467.1308	-137.98376	295.87043	763.59274	1,416.5901	2,280.3699	3,395.2474	4,827.6153	6,664.9163	8,856.8365	5,880.4965	2,076.8729	593.79682	111.79743
1,499.2098	2,170.0146	2,967.7070	-110.01505	235.46637	603.12241	1,106.0620	1,751.7830	2,549.7758	3,515.6047	4,657.7507	5,880.4965	4,519.7728	1,906.4817	644.13873	150.29541
1,018.6292	1,365.9733	1,732.3414	-56.296255	120.33467	300.77097	532.42950	805.87976	1,108.7179	1,428.3372	1,752.4211	2,076.8729	1,906.4817	1,486.7911	735.8056	225.7527
510.42373	601.87258	692.86161	-18.326739	40.653402	99.250677	170.15335	249.08669	332.82553	419.09673	506.36398	593.79682	644.13873	735.8056	706.99680	279.00310
157.03819	172.72000	188.54721	-3.4812120	8.3884820	20.289094	34.145660	49.079710	64.530150	80.200630	95.986690	111.79743	150.29541	225.7527	279.00310	213.88123

TABLE X.- INFLUENCE COEFFICIENT MATRIX FOR

(b) Antisymmetrical loading;

Station	2	4	5	6	8	9	10	11	12	14	15	16	17	18
2	575.7224	360.5361	601.7281	823.7979	232.9755	404.9876	571.0150	727.6955	876.5052	124.9455	226.5982	337.1404	454.6444	575.4706
4	360.5361	309.3417	518.1413	713.5195	220.3955	379.0422	529.5642	672.5078	809.4466	120.0355	217.4012	322.5558	433.2395	546.4655
5	601.7281	518.1413	894.3876	1250.551	379.7971	657.7621	924.7133	1179.133	1422.640	209.3141	379.4903	563.9921	758.9652	958.9030
6	823.7979	713.5195	1250.551	1788.111	533.7412	929.9671	1316.661	1687.412	2041.955	297.5370	540.2849	804.5489	1085.073	1373.601
8	232.9755	220.3955	379.7971	533.7412	194.2970	323.1545	441.2496	553.6178	662.5724	111.8606	198.2547	287.5461	379.1012	471.9543
9	404.9896	379.0422	657.7621	929.9671	323.1545	557.2611	773.0448	976.0771	1172.523	190.7794	342.7637	502.9228	668.1963	835.8900
10	571.0150	529.5642	924.7133	1316.661	441.2496	773.0448	1099.712	1408.627	1705.653	264.5714	481.5123	716.9574	693.9362	1215.762
11	727.6955	672.5078	1179.133	1687.412	553.6178	976.0771	1408.627	1846.530	2271.723	334.4817	614.9803	928.2693	1265.833	1615.077
12	876.5052	809.4466	1422.640	2041.955	662.5724	1172.523	1705.653	2271.723	2861.562	402.0729	744.8913	1136.470	1569.478	2027.530
14	124.9455	120.0355	209.3141	297.5370	111.8606	190.7794	264.5714	334.4817	402.0729	88.21691	143.8514	196.3179	249.9188	503.3133
15	226.5982	217.4012	379.4903	540.2849	198.2547	342.7637	481.5123	614.9803	744.8913	143.8514	256.1005	363.5947	470.0499	578.6888
16	337.1404	322.5558	563.9921	804.5489	287.5461	502.9228	716.9574	928.2693	1136.470	196.3179	363.5947	546.3199	727.0367	907.0376
17	454.6444	433.2395	758.9652	1085.073	379.1012	668.1963	693.9362	1265.833	1569.478	249.9188	470.0499	727.0367	1007.767	1286.898
18	575.4706	546.4655	958.9030	1373.601	471.9543	835.8900	1215.762	1615.077	2027.530	503.3133	578.6888	907.0376	1286.898	1698.932
19	695.8834	659.7765	1159.022	1662.447	565.7222	1005.146	1469.988	1968.729	2494.570	362.3366	690.0625	1090.039	1565.981	2113.005
20	814.5225	771.9030	1356.955	1947.981	659.3562	1174.044	1723.354	2320.541	2958.426	419.9036	802.5235	1274.774	1847.050	2526.946
23	38.35874	38.24327	67.09824	96.04945	36.60115	64.49765	93.29599	123.1929	153.8919	28.58607	56.24736	88.30693	122.2403	156.7459
24	105.4341	104.7382	183.7961	263.2276	99.01192	175.0705	254.2079	337.1822	423.1287	74.43128	147.0713	235.4857	332.7353	433.3575
25	191.0006	188.9669	331.7017	475.2769	176.1990	312.5988	455.8918	607.7461	766.5208	128.0436	253.2921	410.7206	592.4898	786.9185
26	288.6308	284.2863	499.2316	715.7050	261.5968	465.4293	681.5948	913.2406	1157.603	185.0113	365.7954	597.1513	874.5669	1184.964
27	393.8827	386.1788	678.5074	973.3144	351.1174	626.0849	920.0474	1238.396	1577.201	243.1567	480.2726	786.7557	1163.579	1602.011
28	503.3678	491.5430	864.0397	1240.165	442.5008	790.3360	1164.566	1573.427	2011.937	301.6006	595.1023	976.6871	1453.633	2024.719
29	614.3577	598.2163	1051.910	1510.433	534.7234	956.1780	1411.678	1912.448	2452.473	360.3014	710.3637	1167.306	1745.137	2451.449
30	725.5679	705.0981	1240.148	1781.231	627.1087	1122.324	1659.256	2252.161	2893.949	419.0725	825.7551	1358.153	2037.103	2879.251
31	771.1284	739.5294	1300.366	1867.218	644.0469	1149.697	1693.665	2289.752	2930.782	419.8628	814.8397	1317.677	1944.427	2708.292
32	851.0952	797.3192	1401.382	2011.412	668.3934	1186.848	1735.363	2326.621	2953.807	416.5422	783.7281	1220.577	1728.534	2302.308
33	866.6755	782.3820	1374.774	1972.989	617.3092	1085.215	1560.231	2040.695	2512.023	359.8214	658.7492	990.2826	1347.053	1717.064
34	711.2854	559.0436	964.5270	1351.587	393.6721	683.6136	963.9637	1229.640	1482.730	214.7360	389.4617	579.0498	780.0244	986.4768

WING ON THREE-POINT SUPPORT - Concluded

transverse shear neglected

19	20	23	24	25	26	27	28	29	30	31	32	33	34
695.8834	814.5225	38.35874	105.4341	191.0006	288.6308	393.8827	503.3678	614.3577	725.5679	771.1284	851.0952	866.6755	711.2854
659.7765	771.9030	38.24327	104.7382	188.9669	284.2863	386.1788	491.5430	598.2163	705.0981	739.5294	797.3192	782.3820	559.0436
1159.022	1356.955	67.09824	183.7961	331.7017	499.2316	678.5074	864.0397	1051.910	1240.148	1300.366	1401.382	1374.774	964.5270
1662.447	1947.981	96.04945	263.2276	475.2769	715.7050	973.3144	1240.165	1510.433	1781.231	1867.218	2011.412	1972.989	1351.587
565.7222	659.3562	36.60115	99.01192	176.1990	261.5968	351.1174	442.5008	534.7234	627.1087	644.0469	668.3964	617.3092	393.6721
1005.146	1174.044	64.49765	175.0705	312.5988	465.4293	626.0849	790.3360	956.1780	1122.324	1149.697	1186.848	1085.215	683.6136
1469.988	1723.354	93.29599	254.2079	455.8918	681.5948	920.0474	1164.566	1411.678	1659.256	1693.665	1735.363	1560.231	963.9637
1968.729	2320.541	123.1929	337.1822	607.7461	913.2406	1238.396	1573.427	1912.448	2252.161	2289.752	2326.621	2040.695	1229.640
2494.570	2958.426	153.8919	423.1287	766.5208	1157.603	1577.201	2011.937	2452.473	2893.949	2930.782	2953.807	2512.023	1482.730
362.3366	419.9036	28.58607	74.43128	128.0436	185.0113	243.1567	301.6006	360.3014	419.0725	419.8628	416.5422	359.8214	214.7360
690.0625	802.5235	56.24736	147.0713	253.2921	365.7954	480.2736	595.1023	710.3637	825.7551	814.8397	783.7281	658.7492	389.4617
1090.039	1274.774	88.30693	235.4857	410.7206	597.1513	786.7557	976.6871	1167.306	1358.153	1317.677	1220.577	990.2826	579.0498
1565.981	1847.050	122.2403	332.7353	592.4898	874.5669	1163.579	1453.633	1745.137	2037.103	1944.427	1728.534	1347.053	780.0244
2113.005	2526.946	156.7459	433.3575	786.9185	1184.964	1602.011	2024.719	2451.449	2879.251	2708.292	2302.308	1717.064	986.4768
2714.100	3323.006	191.4536	535.3350	987.2658	1514.956	2088.708	2684.388	3291.864	3901.957	3624.717	2931.885	2088.546	1192.692
3323.006	4203.940	226.3014	638.0764	1190.529	1854.487	2602.685	3406.315	4238.620	5076.664	4666.654	3577.888	2456.087	1396.264
191.4536	226.3014	41.53388	86.37670	126.1979	163.4775	199.5231	235.0022	270.2040	305.3432	265.5966	188.7385	124.2525	67.46783
535.3350	638.0764	86.37670	218.7270	344.2830	458.3183	566.7128	672.5222	777.1148	881.4462	758.7331	525.1686	340.5937	185.1267
987.2658	1190.529	126.1979	344.2830	601.1648	841.3429	1063.548	1277.445	1487.578	1696.936	1440.519	964.2659	615.0777	334.6642
1514.956	1854.487	163.4775	458.3183	841.3429	1262.039	1657.844	2030.309	2392.611	2752.909	2295.124	1475.596	926.1097	504.6002
2088.708	2602.685	199.5231	566.7128	1063.548	1657.844	2298.178	2909.246	3494.658	4075.260	3317.985	2033.552	1259.034	687.0963
2684.388	3406.315	235.0022	672.5222	1277.445	2030.309	2909.246	3862.151	4797.624	5722.244	4516.403	2617.092	1603.616	876.4354
3291.864	4238.620	270.2040	777.1148	1487.578	2392.611	3494.658	4797.624	6270.249	7777.237	5894.344	3208.527	1952.572	1068.264
3901.957	5076.664	305.3432	881.4462	1696.936	2752.909	4075.260	5722.244	7777.237	10187.43	7353.460	3800.904	2302.291	1260.470
3624.717	4666.654	265.5966	758.7331	1440.519	2295.124	3317.985	4516.403	5894.344	7353.460	6108.619	3694.117	2382.563	1330.225
2931.885	3577.888	188.7385	525.1686	964.2659	1475.596	2033.552	2617.092	3208.527	3800.904	3694.117	3370.296	2506.623	1450.230
2088.546	2456.087	124.2525	340.5937	615.0777	926.1097	1259.034	1603.616	1952.572	2302.291	2382.563	2506.623	2349.878	1449.599
1192.692	1396.264	67.46783	185.1267	334.6642	504.6002	687.0963	876.4354	1068.264	1260.470	1330.225	1450.230	1449.599	1097.784

TABLE XI.- INFLUENCE COEFFICIENT

(a) Symmetrical loading;

Station	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	120.35205	36.734840	76.193610	83.271920	89.270480	37.010830	47.999140	51.984090	56.100670	58.890890	61.451610	20.759760	23.944300	25.726630	27.867660	29.916430
3	36.734840	299.37711	265.03733	253.70413	249.02304	309.26130	289.69646	275.00126	259.89854	258.69112	257.37209	184.41614	176.35587	173.08019	169.12306	167.80952
4	76.193610	265.03733	307.53218	306.90593	313.17751	302.30288	313.11302	308.10967	301.75963	308.89947	314.96415	185.34736	187.06303	189.50129	191.70571	196.78247
5	83.271920	253.70413	306.90593	394.37370	432.09930	303.14420	325.09880	358.14680	374.36200	401.45120	424.62490	189.39650	197.72210	215.60830	231.52290	250.83510
6	89.270480	249.02304	313.17751	432.09930	578.44860	305.92510	338.42500	393.76650	460.05440	521.76270	572.98700	193.20400	208.69710	239.95970	279.66130	321.53950
7	37.010830	309.26130	302.30288	303.14420	305.92510	464.77454	410.34276	381.07269	355.56917	345.87803	337.51787	299.32651	271.67894	258.11004	245.030306	235.39509
8	47.999140	289.69646	313.11302	325.09880	338.42500	410.34276	427.94241	406.63526	389.61244	389.002688	390.08433	273.99946	276.81015	273.07936	269.48204	269.14111
9	51.984090	275.00126	308.10967	358.14680	393.76650	381.07269	406.63526	453.16613	452.46764	469.15185	488.49009	256.61782	269.41681	399.01494	314.45366	332.43914
10	56.100670	259.89854	301.75963	374.36200	460.05440	355.56917	389.61244	452.46764	540.29083	587.96176	460.17002	239.25271	261.87873	310.76958	373.70774	423.98431
11	58.890890	258.69112	308.89947	401.45120	521.76270	345.87803	389.002688	469.15185	587.96176	767.88230	884.38938	229.50153	262.21265	330.61334	427.75825	549.15978
12	61.451610	257.37209	314.96415	424.62490	572.98700	337.51787	390.08433	488.49009	460.17002	884.38938	1175.8515	219.93570	263.04370	351.35800	483.27980	660.57080
13	20.759760	184.41614	185.34736	189.39650	193.20400	299.32651	273.99946	256.61782	239.25271	229.50153	219.93570	292.99692	227.80503	200.52437	178.28642	160.93131
14	23.944300	176.35587	187.06303	197.72210	208.69710	271.67894	276.81015	269.41681	261.87873	262.21265	263.04370	227.80503	239.09940	221.19739	208.63229	201.33023
15	25.726630	173.08019	189.50129	215.60830	239.95970	258.11004	273.07936	399.01494	310.76958	330.61334	351.35800	200.52437	221.19739	274.54093	279.64538	290.97712
16	27.867660	169.12306	191.70571	231.52290	279.66130	245.030306	269.48204	314.45366	373.70774	427.75825	483.27980	178.28642	208.63229	279.64538	377.53620	421.55907
17	29.916430	167.80952	196.78247	250.83510	321.53950	235.39509	269.14111	332.43914	423.98431	549.15978	660.57080	160.93131	201.33023	290.97712	421.55907	588.36288
18	32.033350	165.66927	200.92146	269.06370	361.97740	225.27392	268.07614	349.04786	471.59689	651.17249	858.45170	144.85740	195.69748	304.92037	469.82717	691.44124
19	33.810090	169.23542	210.23057	290.86860	403.16620	224.94650	277.117727	376.33156	530.67243	763.52273	1051.8467	136.15633	198.01456	327.97103	529.40613	809.24602
20	35.537950	172.92590	219.46150	312.13320	443.03620	225.10725	286.45244	403.39847	588.29190	871.33104	1232.6685	127.92834	201.11335	352.50732	591.93696	932.49011
21	-.9904708	4.8940853	1.3998475	-1.881605	-5.9751180	15.859302	4.4048441	-2.5574321	-10.500781	-19.325624	-28.865574	44.872860	6.5459042	-7.4569525	-20.852491	-33.87989
23	.7265680	11.933484	14.858489	21.941970	29.528450	17.631738	24.638295	41.030962	55.662777	72.207728	90.238040	8.3538420	22.558272	65.753774	91.864732	117.28292
24	1.7270130	23.198359	29.807105	44.680310	64.846700	33.400701	48.286027	79.137810	120.74555	164.33844	212.26541	14.801656	42.562286	111.76345	199.44808	268.75058
25	2.890104	33.649807	44.552239	68.53420	102.741310	46.600737	69.893969	136.41280	183.79211	269.96626	361.52391	18.704249	59.458217	153.58342	284.71994	439.08304
26	4.242289	42.141734	57.738189	91.765180	141.44839	56.099550	88.079581	150.95999	245.69146	374.42338	526.89599	19.418336	72.718818	190.88218	364.25910	585.00631
27	5.656023	50.296616	70.771349	115.20675	180.88158	64.657478	105.56544	185.40205	308.75269	482.45680	697.72878	18.865657	84.437318	226.05044	440.81864	727.14957
28	7.159676	57.189570	82.697917	137.92692	220.20388	70.886545	120.75216	217.66388	369.94624	589.54891	869.13423	16.193032	93.815174	258.41299	513.81372	864.78480
29	8.622690	65.197020	95.642390	161.39040	259.80150	79.112510	137.96675	251.95140	433.03974	697.81623	1039.9317	15.274590	105.15660	293.22073	590.28029	1,007.8795
30	10.085890	73.116620	108.49461	184.76460	299.31630	87.195980	155.02138	286.04680	495.88755	805.78433	1210.3917	14.240820	116.37356	327.87581	666.49690	1,150.4553
31	22.81697	123.32579	164.29627	248.76880	371.48290	156.64091	221.28888	345.38947	542.94566	839.60562	1222.7202	71.475320	159.17132	340.72594	360.09286	1,043.4195
32	48.35806	219.73846	271.79719	372.75020	512.05020	288.99371	346.49413	455.27216	625.77449	893.28144	1230.6576	180.63418	238.98590	359.30560	545.91874	806.62975
33	75.02467	267.97473	350.01652	447.06030	601.96600	342.36183	385.91400	464.93350	578.11752	757.76814	888.47490	221.15924	249.77595	308.03854	390.80202	498.91569
34	104.10897	160.71208	215.01073	304.24530	354.26080	189.35261	210.88883	244.39359	269.39819	293.81508	314.93210	116.96461	125.33342	141.28909	157.37906	174.60403

MATRIX FOR WING ON THREE-POINT SUPPORT

transverse shear included

18	19	20	21	23	24	25	26	27	28	29	30	31	32	33	34
32.033350	33.810090	35.537950	-0.9904708	0.7265680	1.7270130	2.890104	4.242289	5.656023	7.159676	8.622690	10.085890	22.81697	48.35806	75.02467	104.10897
165.66927	169.23542	172.92590	4.8940853	11.933484	23.198359	42.141734	50.296616	57.189570	65.197020	73.116620	123.32579	219.73846	267.97473	160.71208	160.71208
200.92146	210.23057	219.46150	1.3998475	14.858489	29.807105	44.552239	57.738189	70.771349	82.697917	95.642390	108.49461	164.29627	271.79719	350.01652	215.01073
269.06370	290.86860	312.13320	-1.881605	21.941970	44.680310	68.53420	91.765180	115.20675	137.92692	161.39040	184.76460	248.76880	372.75020	447.06030	304.24530
361.97740	403.16620	443.03620	-5.9751180	29.528450	64.846700	102.741310	141.44839	180.88158	220.20388	259.80150	299.31630	371.48290	512.05020	601.96600	354.26080
225.27392	224.94650	225.10725	15.859302	17.631738	33.400701	46.600737	56.099550	64.657478	70.886545	79.112510	87.195980	156.64091	288.99371	342.36183	189.35261
268.07614	277.117727	286.45244	4.4048441	24.638295	48.286027	69.893969	88.079581	105.56544	120.75216	137.96675	155.02138	221.28888	346.49413	385.91400	210.88883
349.04786	376.33156	403.39847	-2.5574321	41.030962	79.137810	136.41280	150.95999	185.40205	217.66388	251.95140	286.04680	345.38947	455.27216	464.93350	244.39359
471.59689	530.67243	588.29190	-10.500781	55.662777	120.74555	183.79211	245.69146	308.75269	369.94624	433.03974	495.88755	542.94566	625.77449	578.11752	269.39819
651.17249	763.52273	871.33104	-19.325624	72.207728	164.33844	269.96626	374.42338	482.45680	589.54891	697.81623	805.78433	839.60562	893.28144	757.76814	293.81508
858.45170	1,051.8467	1,232.6685	-28.865574	90.238040	212.26541	361.52391	526.89599	697.72878	869.13423	1,039.9317	1,210.3917	1,222.7202	1,230.6576	888.47490	314.93210
144.85740	156.15633	127.92834	44.872860	8.3538420	14.801656	18.704249	19.418336	18.865657	16.193032	15.274590	14.240820	71.475320	180.63418	221.15924	116.96461
195.69748	198.01456	201.11335	6.5459042	22.558272	42.562286	59.458217	72.718818	84.437318	93.815174	105.15660	116.37356	159.17132	238.98590	249.77595	125.33342
304.92037	327.97103	352.50732	-7.4569525	65.753774	111.76345	153.58342	190.88218	226.05044	258.41299	293.22073	327.87581	340.72594	359.30560	308.03854	141.28909
469.82717	529.40613	591.93696	-20.852491	91.864732	199.44808	284.71994	364.25910	440.81864	513.81372	590.28029	666.49690	360.09286	545.91874	390.80202	157.37906
691.44124	809.24602	932.49011	-33.87989	117.28292	268.75058	439.08304	585.00631	727.14957	864.78480	1,007.8795	1,150.4553	1,043.4195	806.62975	498.91569	174.60403
958.26547	1,164.2587	1,381.0311	-46.858128	142.58515	337.86839	575.82010	837.88244	1,077.8267	1,312.5158	1,556.4907	1,799.2554	1,594.2682	1,132.9822	599.01743	191.42752
1,164.2587	1,620.6646	1,997.1294	-59.78820	167.83147	407.17793	713.94670	1,077.4731	1,485.9834	1,872.3459	2,272.9878	2,670.8295	2,343.4404	1,568.9615	697.69605	208.84382
1,381.0311	1,997.1294	2,722.5282	-73.012755	193.66160	478.18059	855.70115	1,324.0896	1,887.2107	2,509.2269	3,143.4850	3,769.4370	3,273.8650	1,948.4244	792.27889	225.77026
-46.858128	-59.78820	-73.012755	111.14219	-21.025117	-38.818408	-54.628194	-69.304289	-83.378962	-97.292852	-111.02123	-124.83234	-98.656715	-49.648739	-15.536155	-2.6479184
142.58515	167.83147	193.66160	-21.025117	107.97039	143.98901	175.60547	204.68997	232.44035	259.81703	286.77487	313.91773	253.19317	139.23862	56.280553	14.032684
337.86839	407.17793	478.18059	-38.818408	143.98901	319.78886	409.49785	491.16514	568.60192	644.73105	719.63171	795.10279	634.78426	337.42009	128.99924	29.388196
575.82010	713.94670	855.70115	-54.628194	175.60547	409.49785	678.95479	847.49798	1,005.7842	1,160.5526	1,312.6409	1,466.0645	1,156.5015	592.08790	215.19818	45.775940
837.88244	1,077.4731	1,324.0896	-69.304289	204.68997	491.16514	847.49798	1,249.2974	1,535.2540	1,812.4621	2,084.5686	2,359.5150	1,832.4263	894.86976	303.66028	62.457000
1,077.8267	1,485.9834	1,887.2107	-83.378962	232.44035	568.60192	1,005.7842	1,535.2540	2,134.1227	2,604.4168	3,065.6182	3,532.6383	2,690.6336	1,244.9141	394.30871	79.424060
1,312.5158	1,872.3459	2,509.2269	-97.292852	259.81703	644.73105	1,160.5526	1,812.4621	2,604.4168	3,510.4859	4,276.8773	5,054.7197	3,743.9088	1,593.0313	485.32786	96.300630
1,556.4907	2,272.9878	3,143.4850	-111.02123	286.77487	719.63171	1,312.6409	2,084.5686	3,065.6182	4,276.8773	5,760.4159	7,117.5355	5,095.3811	1,939.9281	576.26305	113.36913
1,799.2554	2,670.8295	3,769.4370	-124.83234	313.91773	795.10279	1,466.0645	2,359.5150	3,532.6383	5,054.7197	7,117.5355	9,625.7003	6,412.0561	2,285.4766	667.04292	130.39714
1,594.2682	2,343.4404	3,273.8650	-98.656715	253.19317	634.78426	1,156.5015	1,832.4263	2,690.6336	3,743.9088	5,095.3811	6,412.0561	5,021.9441	2,121.5448	730.22235	178.23725
1,132.9822	1,568.9615	1,948.4244	-49.648739	139.23862	337.42009	592.08790	894.86976	1,244.9141	1,593.0313	1,939.9281	2,285.4766	2,121.5448	1,722.8558	849.23198	272.16214
599.01743	697.69605	792.27889	-15.536155	56.280553	128.99924	215.19818	303.66028	394.30871	485.32786	576.26305	667.04292	730.22235	849.23198	849.73590	343.92730
191.42752	208.84382	225.77026	-2.6479184	14.032684	29.388196	45.775940	62.457000	79.424060	96.300630	113.36913	130.39714	178.23725	272.16214	343.92730	503.16737

TABLE XI.- INFLUENCE COEFFICIENT MATRIX FOR

(b) Antisymmetrical loading;

Station	2	4	5	6	8	9	10	11	12	14	15	16	17	18
2	664.5865	435.6276	680.6178	612.6123	278.6191	472.3982	660.6795	833.0287	997.6331	147.2090	281.3849	422.8304	565.6384	708.4203
4	435.6276	380.6261	592.7688	798.8377	264.0352	440.5289	610.0023	772.0952	926.6229	141.6636	268.9312	401.7767	536.9514	671.5259
5	680.6178	592.7688	1,032.748	1,405.865	435.3010	766.0688	1,070.319	1,360.526	1,636.103	241.5507	470.6019	706.8619	947.5047	1,187.079
6	612.6123	798.8377	1,405.865	2,019.019	600.2362	1,065.821	1,528.929	1,958.564	2,363.037	338.7714	664.9336	1,011.996	1,363.699	1,713.716
8	278.6191	264.0352	435.3010	600.2362	241.1957	376.0705	508.6251	638.9806	766.0272	135.8592	243.7442	354.7006	467.2502	579.1817
9	472.3982	440.5289	766.0688	1,065.821	376.0705	673.0784	913.7638	1,151.125	1,383.316	222.2218	435.5921	640.4300	847.6122	1,053.354
10	660.6795	610.0023	1,070.319	1,528.929	508.6251	913.7638	1,332.223	1,687.743	2,037.232	305.3881	607.9971	932.9471	1,246.503	1,557.214
11	833.0287	772.0952	1,360.526	1,958.564	638.9806	1,151.125	1,687.743	2,260.407	2,756.579	386.6501	777.0480	1,207.087	1,664.049	2,101.407
12	997.6331	926.6229	1,636.103	2,363.037	766.0272	1,383.316	2,037.232	2,756.579	3,507.773	465.9746	942.6906	1,476.400	2,059.530	2,663.030
14	147.2090	141.6636	241.5507	338.7714	135.8592	222.2218	305.3881	386.6501	465.9746	114.9871	174.8473	238.9946	305.9231	373.9022
15	281.3849	268.9312	470.6019	664.9336	243.7442	435.5921	607.9971	777.0480	942.6906	174.8473	359.3019	497.0443	640.1399	785.4992
16	422.8304	401.7767	706.8619	1,011.996	354.7006	640.4300	932.9471	1,207.087	1,476.400	238.9946	497.0443	786.0877	1,021.877	1,261.737
17	565.6384	536.9514	947.5047	1,363.699	467.2502	847.6122	1,246.503	1,664.049	2,059.530	305.9231	640.1399	1,021.877	1,441.862	1,798.077
18	708.4203	671.5259	1,187.079	1,713.716	579.1817	1,053.354	1,557.214	2,101.407	2,663.030	373.9022	785.4992	1,261.737	1,798.077	2,379.692
19	846.5773	803.4266	1,421.402	2,054.819	692.0864	1,261.016	1,870.972	2,540.949	3,254.237	443.8083	234.6744	1,508.200	2,164.806	2,896.768
20	981.9168	932.6754	1,650.973	2,388.839	803.0764	1,465.055	2,178.733	2,970.331	3,826.232	513.2624	1,083.383	1,754.864	2,533.696	3,420.225
23	86.12594	84.65226	151.4270	217.4603	78.40340	148.5744	215.6364	283.4477	351.7224	56.47173	143.4252	214.2897	285.0582	355.7912
24	190.0611	185.8989	331.1825	478.8885	170.2997	317.6516	472.7408	627.2312	783.7252	119.8828	282.8905	466.7043	633.5189	800.3203
25	305.4440	297.9933	530.1599	767.7709	270.4453	501.6768	748.8850	1,010.724	1,274.112	187.0861	428.7939	711.4122	1,019.566	1,310.366
26	528.3692	146.6691	740.8234	1,073.765	375.1425	693.9846	1,037.821	1,409.856	1,800.193	256.0191	577.5021	959.8497	1,392.906	1,858.470
27	554.9558	538.7776	957.5168	1,388.443	482.5512	891.3225	1,334.706	1,820.948	2,341.644	325.8487	727.6012	1,210.016	1,768.420	2,391.666
28	683.4830	662.3809	1,176.914	1,707.203	590.5572	1,089.779	1,633.408	2,235.168	2,888.089	395.4240	877.0961	1,459.004	2,142.037	2,922.448
29	811.8107	786.0634	1,396.354	2,025.800	699.2492	1,289.392	1,933.606	2,650.610	3,434.433	466.0090	1,028.603	1,711.512	2,521.628	3,463.496
30	939.8441	909.4441	1,615.272	2,343.665	807.6335	1,488.458	2,233.002	3,065.000	3,979.494	536.3698	1,179.693	1,963.338	2,900.094	4,002.557
31	961.9050	922.1010	1,634.921	2,368.778	806.4063	1,478.626	2,208.607	3,021.258	3,907.244	525.5784	1,132.961	1,861.426	2,720.708	3,718.099
32	994.1112	934.9840	1,652.729	2,388.806	791.0607	1,436.001	2,126.053	2,889.253	3,707.556	494.5318	1,021.882	1,628.866	2,314.906	3,066.618
33	965.7547	877.6160	1,548.466	2,235.464	697.1640	1,249.908	1,821.355	2,430.348	2,975.027	509.7573	816.1166	1,260.108	1,732.656	2,196.925
34	795.1994	632.0434	1,092.738	1,496.815	447.8560	787.8441	1,109.591	1,407.850	1,691.279	246.0945	479.4174	722.8857	969.3915	1,215.342

WING ON THREE-POINT SUPPORT - Concluded

transverse shear included

19	20	23	24	25	26	27	28	29	30	31	32	33	34
846.5773	981.9168	86.12594	190.0611	305.4440	528.3692	554.9558	683.4830	811.8107	939.8441	961.9050	994.1112	965.7547	795.1994
803.4266	932.6754	84.65226	185.8989	297.9933	146.6691	538.7776	662.3809	786.0634	909.4441	922.1010	934.9840	877.6160	632.0434
1,421.402	1,650.973	151.4270	331.1825	530.1599	740.8234	957.5168	1,176.914	1,396.354	1,615.272	1,634.921	1,652.729	1,548.466	1,092.738
2,054.819	2,388.839	217.4603	478.8885	767.7709	1,073.765	1,388.443	1,707.203	2,025.800	2,343.665	2,368.778	2,388.806	2,235.464	1,496.815
692.0864	803.0764	78.40340	170.2997	270.4453	375.1425	482.5512	590.5572	699.2492	807.6335	806.4063	791.0607	697.1640	447.8560
1,261.016	1,465.055	148.5744	317.6516	501.6768	693.9846	891.3225	1,089.779	1,289.392	1,488.458	1,478.626	1,436.001	1,249.908	787.8441
1,870.972	2,178.733	215.6364	472.7408	748.8850	1,037.821	1,334.706	1,633.408	1,933.606	2,233.002	2,208.607	2,126.053	1,821.355	1,109.591
2,540.949	2,970.331	283.4477	627.2312	1,010.724	1,409.856	1,820.948	2,235.168	2,650.610	3,065.000	3,021.258	2,889.253	2,430.348	1,407.850
3,254.237	3,826.232	351.7224	783.7252	1,274.112	1,800.193	2,341.644	2,888.089	3,434.433	3,979.494	3,907.244	3,707.556	2,975.027	1,691.279
443.8083	513.2624	56.47173	119.8828	187.0861	256.0191	325.8487	395.4240	466.0090	536.3698	525.5784	494.5318	509.7573	246.0945
234.6744	1,083.383	143.4252	282.8905	428.7939	577.5021	727.6012	877.0961	1,028.603	1,179.693	1,132.961	1,021.882	816.1166	479.4174
1,508.200	1,754.864	214.2897	466.7043	711.4122	959.8497	1,210.016	1,459.004	1,711.512	1,963.338	1,861.426	1,628.866	1,260.108	722.8857
2,164.806	2,533.696	285.0582	633.5189	1,019.566	1,392.906	1,768.420	2,142.037	2,521.628	2,900.094	2,720.708	2,314.906	1,732.656	969.3915
2,896.768	3,420.225	355.7912	800.3203	1,310.366	1,858.470	2,391.666	2,922.448	3,463.496	4,002.557	3,718.099	3,066.618	2,196.925	1,215.342
3,726.823	4,471.743	426.6003	967.6372	1,603.128	2,311.630	3,073.603	3,816.534	4,575.176	5,330.070	4,913.520	3,922.500	2,650.546	1,454.438
4,471.743	5,626.605	497.5801	1,135.697	1,897.947	2,769.574	3,745.672	4,784.020	5,836.504	6,879.561	6,284.682	4,715.509	3,093.696	1,688.660
426.6003	497.5801	142.6733	217.7445	291.0805	363.3977	435.2005	506.8428	578.3484	649.9258	573.5230	423.7588	283.8784	152.1718
967.6372	1,135.697	217.7445	477.0185	656.1563	830.6203	1,002.684	1,173.906	1,344.529	1,515.471	1,324.543	956.0355	628.4669	334.0873
1,603.128	1,897.947	291.0805	656.1563	1,066.413	1,381.178	1,688.643	1,993.326	2,296.348	2,600.298	2,246.057	1,576.275	1,015.220	535.5112
2,311.630	2,769.574	363.3977	830.6203	1,381.178	1,984.781	2,476.684	2,960.843	3,441.338	3,924.104	3,339.307	2,263.881	1,422.653	749.3836
3,073.603	3,745.672	435.2005	1,002.684	1,688.643	2,476.684	3,339.489	4,074.996	4,803.281	5,536.649	4,624.256	3,008.688	1,841.434	969.4498
3,816.534	4,784.020	506.8428	1,173.906	1,993.326	2,960.843	4,074.996	5,304.802	6,397.244	7,500.234	6,106.995	3,755.516	2,265.058	1,192.574
4,575.176	5,836.504	578.3484	1,344.529	2,296.348	3,441.338	4,803.281	6,397.244	8,266.250	10,007.76	7,890.489	4,502.317	2,688.553	1,415.512
5,330.070	6,879.561	649.9258	1,515.471	2,600.298	3,924.104	5,536.649	7,500.234	10,007.76	12,959.40	9,638.002	5,246.729	3,111.087	1,637.936
4,913.520	6,284.682	573.5230	1,324.543	2,246.057	3,339.307	4,624.256	6,106.995	7,890.489	9,638.002	8,144.210	4,989.119	3,105.691	1,665.079
3,922.500	4,715.509	423.7588	956.0355	1,576.275	2,263.881	3,008.688	3,755.516	4,502.317	5,246.729	4,989.119	4,361.136	3,054.714	1,698.327
2,650.546	3,093.696	283.8784	628.4669	1,015.220	1,422.653	1,841.434	2,265.058	2,688.553	3,111.087	3,105.691	3,054.714	2,731.569	1,617.700
1,454.438	1,688.660	152.1718	334.0873	535.5112	749.3836	969.4498	1,192.574	1,415.512	1,637.936	1,665.079	1,698.327	1,617.700	1,228.548

TABLE XII.- COMPARISON OF EXPERIMENTAL AND CALCULATED

FREQUENCIES FOR FREE-FREE VIBRATION

Row	Frequency determined by -	Frequency, cps, for -								
		Symmetrical					Antisymmetrical			
		1st mode	2d mode	3d mode	4th mode	5th mode	1st mode	2d mode	3d mode	4th mode
1	Experiment	43.3	88.8	122.8	164.2	179.7	52.2	91.7	131.1	169.2
2	Stein-Sanders method	46.4	105.3	150.0	202.0	248.0	56.70	103.4	166.6	216.5
3	Levy method (without shear)	44.6	94.7	132.0	172.0	216.0	52.20	96.29	142.26	200.66
4	Levy method (with shear)	42.8	88.9	120.1	158.0	184.0	50.52	90.25	126.83	174.26
5	Experimental influence coefficient	43.1	83.0	118.0	146.0	172.0	51.1	89.0	124.1	166.7

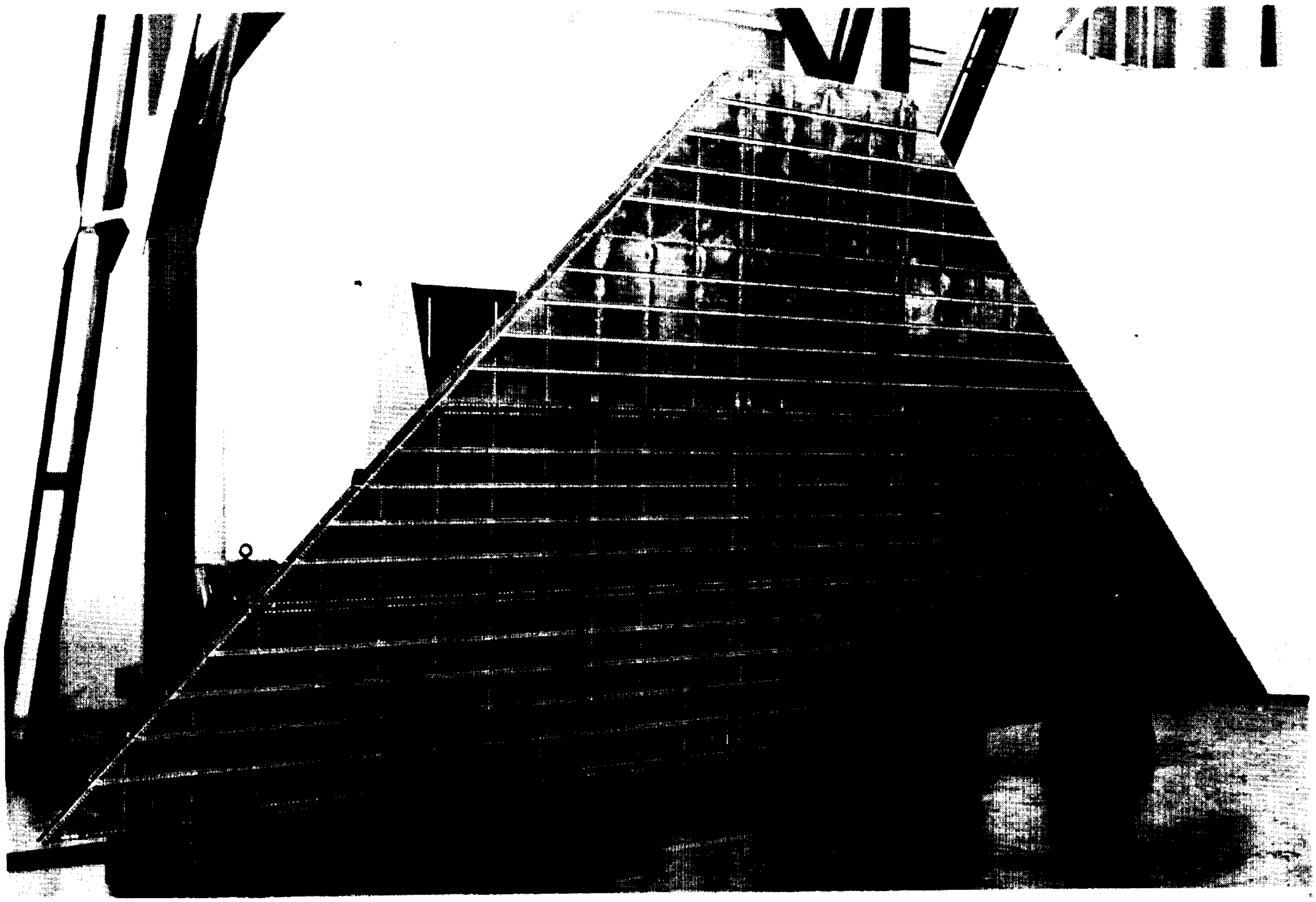


Figure 1.- Delta-wing specimen. L-88269

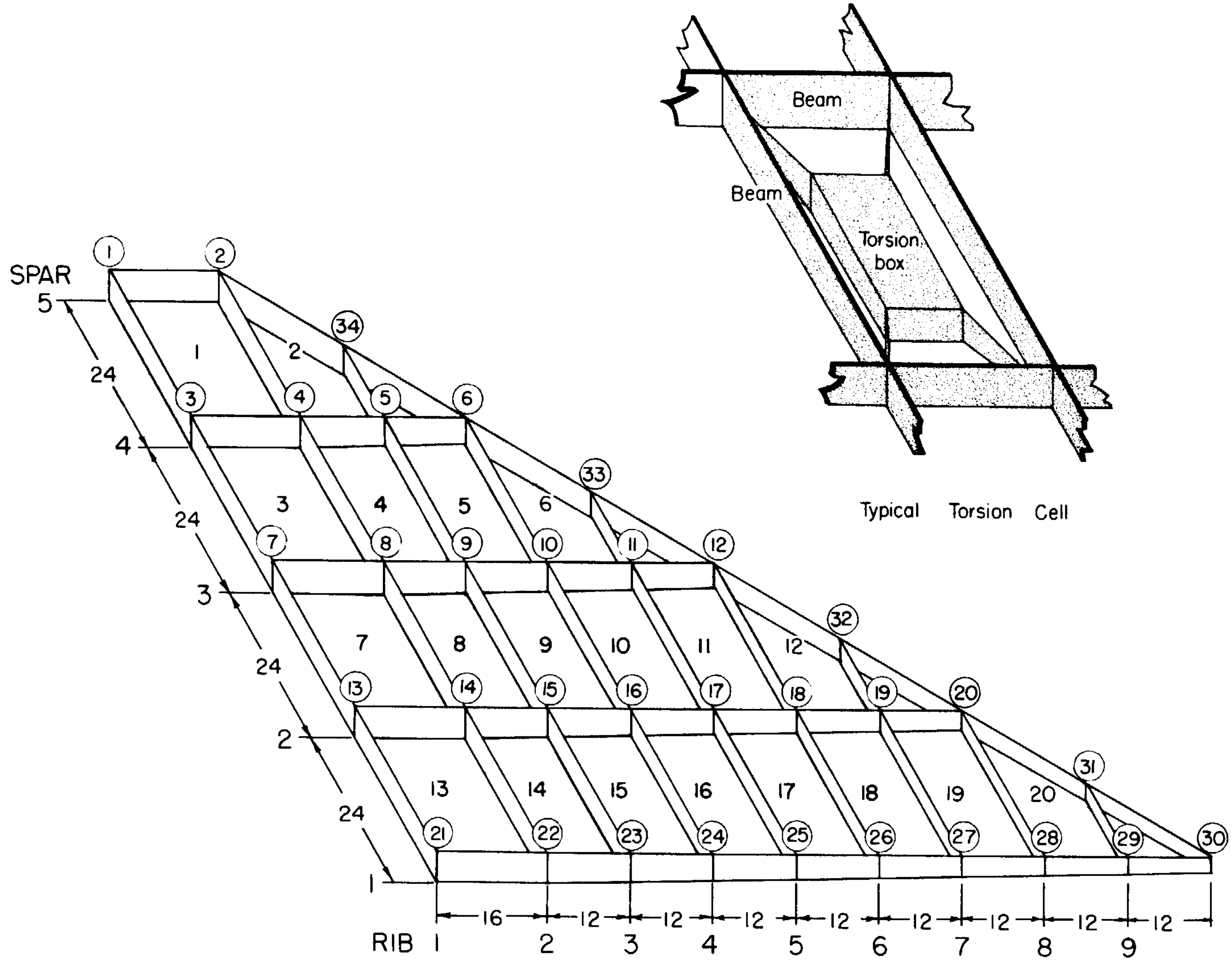
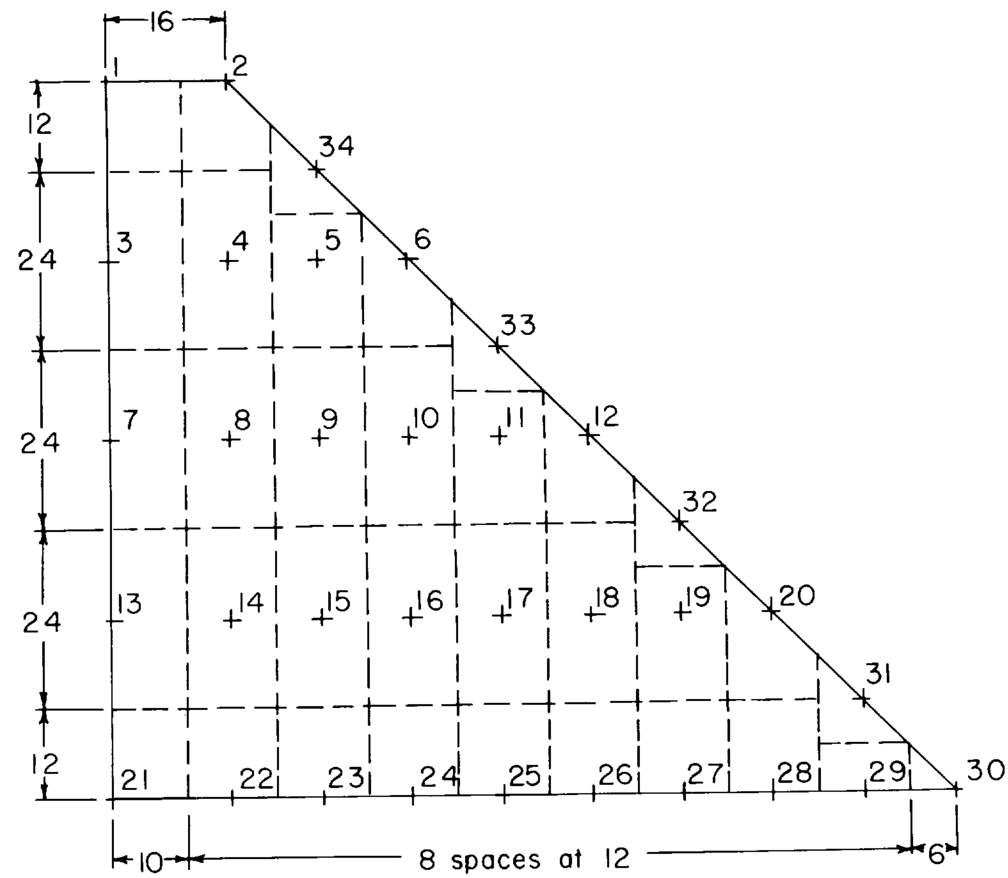


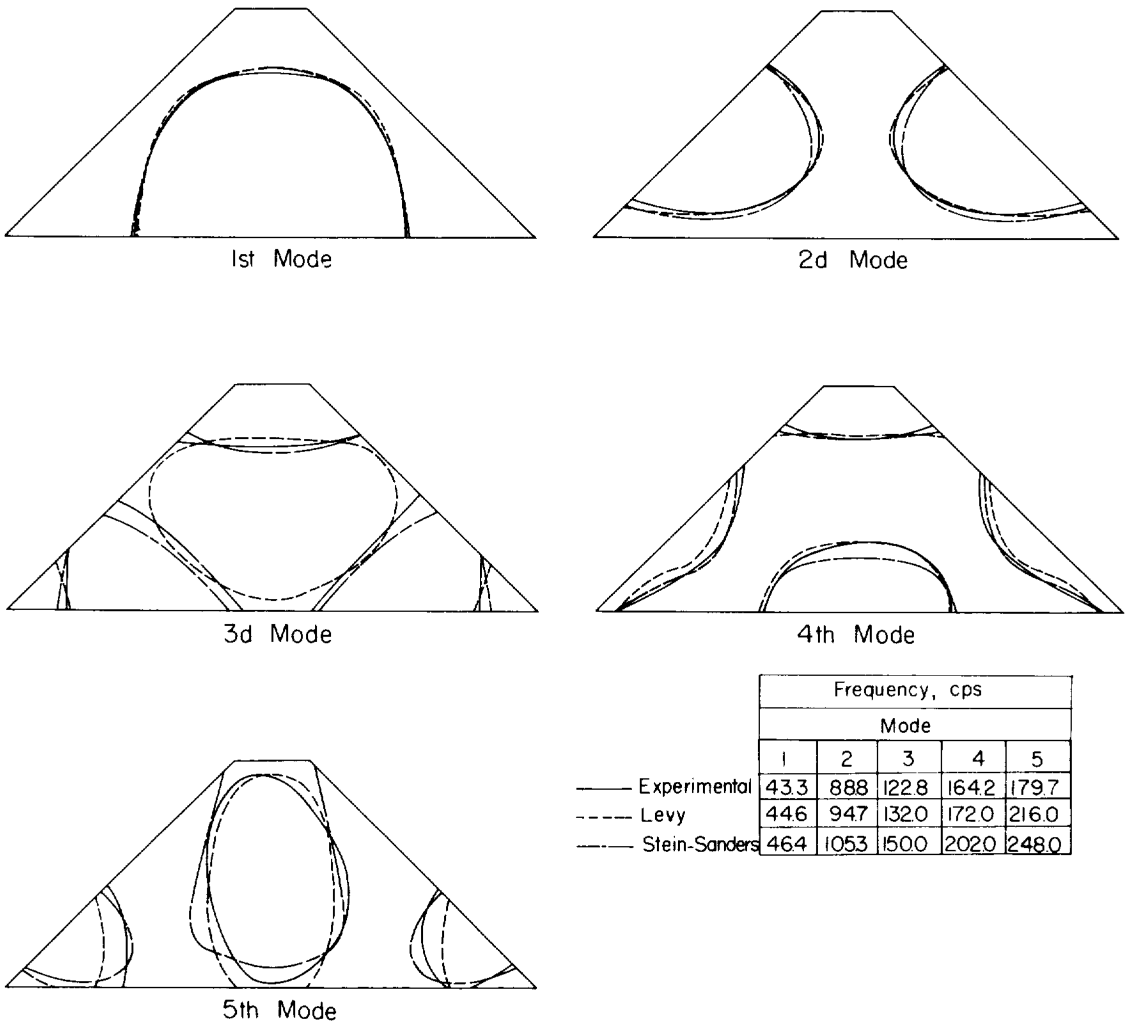
Figure 2.- Idealized delta wing.



i	W_i	i	W_i	i	W_i	i	W_i
1	3.702	10	10.726	19	6.649	28	6.488
2	5.975	11	7.232	20	5.458	29	2.486
3	6.575	12	5.840	21	4.457	30	1.717
4	10.538	13	7.200	22	7.058	31	2.358
5	6.404	14	11.316	23	5.496	32	2.535
6	5.553	15	8.963	24	5.884	33	2.705
7	7.477	16	9.095	25	5.294	34	2.877
8	11.649	17	8.652	26	5.729		
9	9.295	18	8.874	27	5.095		

W_i = Weight concentrated at i th station in pounds

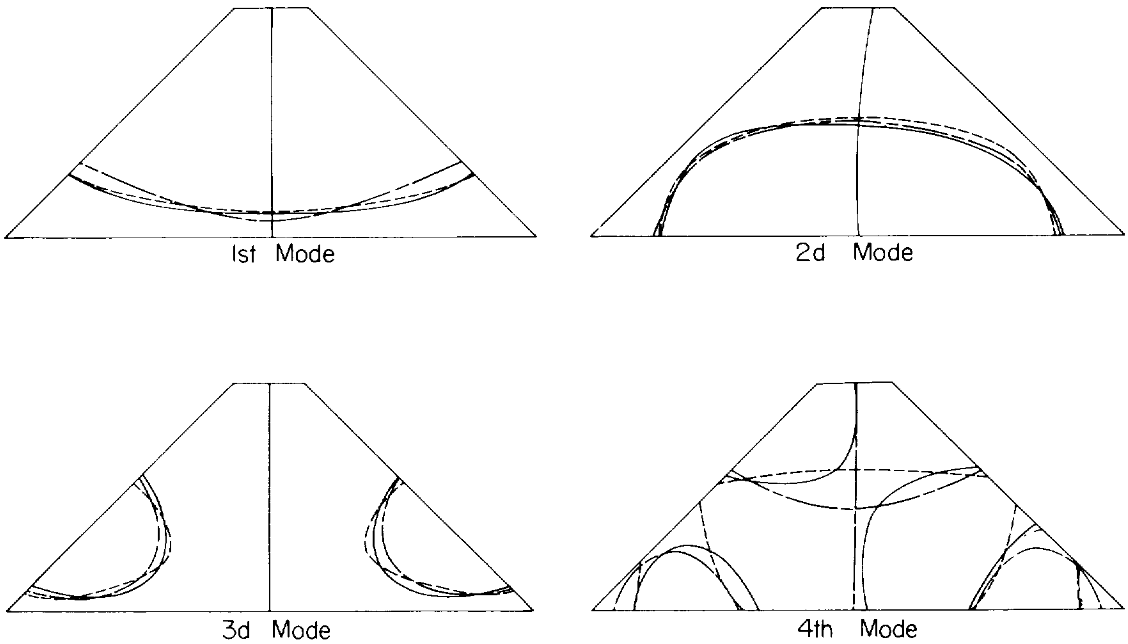
Figure 3.- Mass distribution.



	Frequency, cps				
	Mode				
	1	2	3	4	5
——— Experimental	43.3	88.8	122.8	164.2	179.7
- - - - Levy	44.6	94.7	132.0	172.0	216.0
—·— Stein-Sanders	46.4	105.3	150.0	202.0	248.0

(a) Symmetrical modes.

Figure 4.- Calculated and experimental node lines and frequencies.



	Frequency, cps			
	Mode			
	1	2	3	4
——— Experimental	52.2	91.7	131.1	169.2
– – – Levy	52.2	96.3	142.3	200.7
—-— Stein-Sanders	56.7	103.4	166.6	216.5

(b) Antisymmetrical modes.

Figure 4.- Concluded.

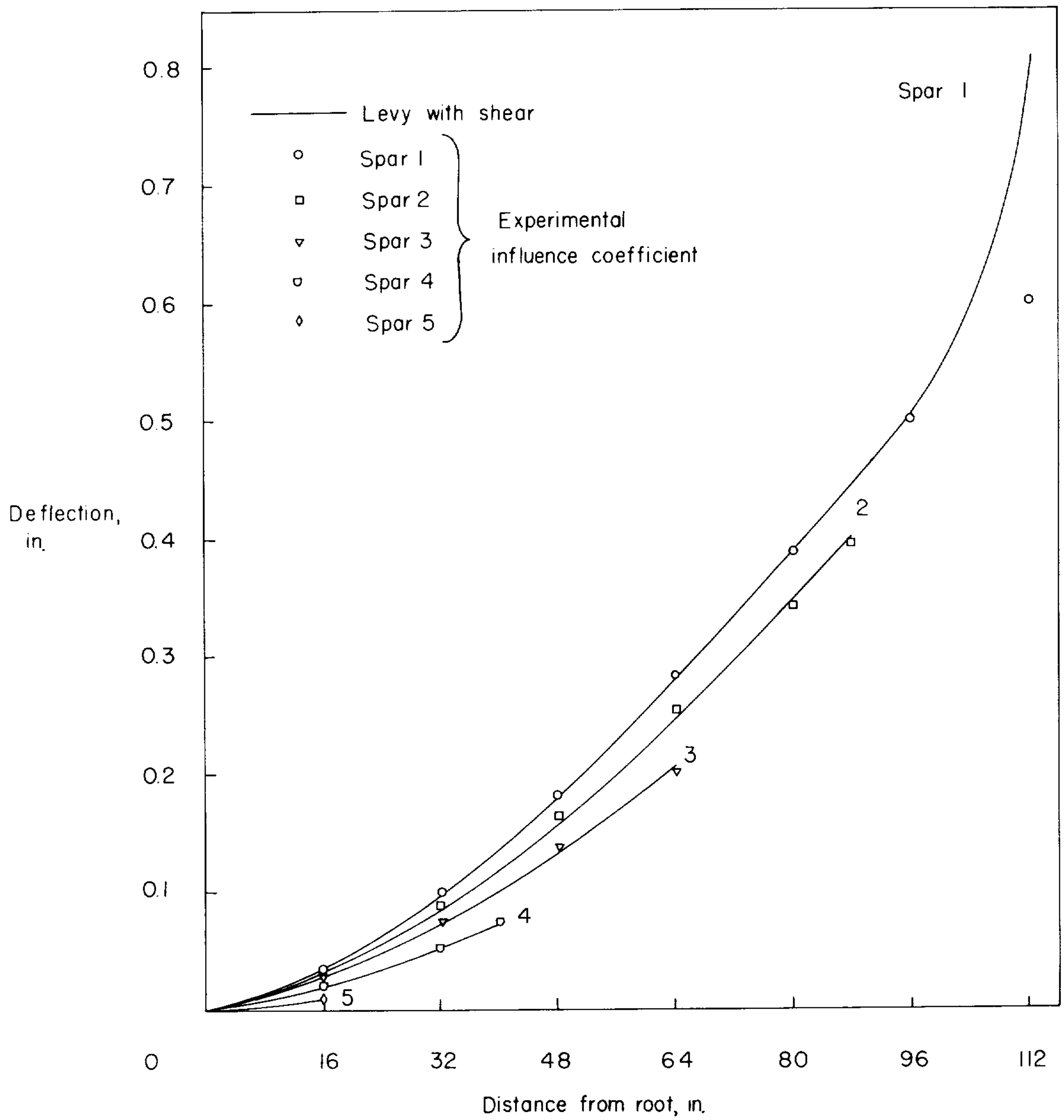


Figure 5.- Deflection of cantilevered wing under uniform load.

NASA MEMO 2-2-59L
National Aeronautics and Space Administration.
EVALUATION OF THE LEVY METHOD AS APPLIED TO VIBRATIONS OF A 45° DELTA WING. Edwin T. Kruszewski and Paul G. Waner, Jr. February 1959. 48p. diagrs., photo., tabs.
(NASA MEMORANDUM 2-2-59L)

The Levy method which deals with an idealized structure was used to obtain the natural modes and frequencies of a large-scale built-up 45° delta wing. The results from this approach, both with and without the effects of transverse shear, were compared with the results obtained experimentally and also with those calculated by the Stein-Sanders method. From these comparisons it was concluded that the method as proposed by Levy gives excellent results for thin-skin delta wings, provided that corrections are made for the effect of transverse shear.

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