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MEMORANDUM

COMPUTER PROGRAM FOR SOLVING NINE-GROUP DIFFUSION EQUATIONS FOR CYLINDRICAL REACTORS

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SUMMARY

This report presents a method for determining the critical size of a cylindrical reactor by a one-dimensional group-diffusion solution in the radial plane. The effect of leakage in the axial direction is taken into account by prescribing values of axial leakage based on assumed flux levels. The method is based on dividing the neutron energy spectrum into nine groups and the reactor into four concentric cylinders (or regions). The nuclear cross sections used for each group are the average values in each energy group.

A computing machine program for an IBM 650 computer (plus attachments specified herein), which performs the solution of the diffusion equations given, and a method for using the program for less than nine groups and four, or less, regions are included.

INTRODUCTION

The subject program for the IBM type 650 magnetic drum data-processing machine provides a method for determining the critical size of a cylindrical reactor on the basis of age-diffusion theory. The age-diffusion equation is approximated by group-diffusion equations wherein the entire neutron energy spectrum is divided into nine energy groups selected on the basis of neutron cross-section characteristics. The neutron cross sections and diffusion coefficients for each energy group are obtained by a suitable averaging procedure performed prior to the subject program. The program is written in such a way that fissions can occur in any, or all, of the energy groups and the neutrons may be born into any, or all, of the energy groups.

The reactor consists of four concentric cylinders, herein called regions. Any, or all, of the inner three regions may contain fissionable material, but the outer region may not. In the axial direction, each region is assumed to be uniform in composition.

By proper adjustments of the input information, the number of energy groups and/or regions can be reduced.

The subject program was written in the language of IBM SOAP II (Symbolic Optimal Assembly Program) and then converted by the IBM 650 computer into basic machine language. The machine time required for the solution of one case is approximately 6 minutes. Thus, in approximately 18 minutes three values of the effective multiplication factor can be plotted as a function of one geometric variable to determine the approximate critical size of the reactor in question.

This report presents the method of analysis and a discussion of the calculating procedure used to determine the critical size of a reactor. Information is included on the use of the machine program given and the use of the program for less than nine energy groups for four, or less, regions, as well as an outline and tabulation of the machine program. The method of obtaining the necessary input data is similar to that of reference 1, but substitutions of other methods for obtaining the required input can be made at the discretion of the reader. References 2 to 5 may be of use in understanding the programming and operating procedures used with this program.

METHOD OF ANALYSIS

The program calculates the effective multiplication factor k_{eff} of a reactor by means of the equations and procedure given herein. On the basis of three or four such solutions for various values of one of the geometric or physical variables, the critical size of the reactor can be obtained.

The solution for k_{eff} is begun by first assuming a radial distribution of neutrons born and then dividing the neutrons into energy groups on the basis of the division of the fission spectrum. From this assumed radial and energywise flux distribution of neutrons born, a new radial and energywise flux distribution can be calculated. On the basis of the new flux distribution, a calculation of the neutrons born as a result of the fissions in each energy group is made. The reactor power is proportional to the number of neutrons born. The new value of reactor power is divided by the previous one to determine a corresponding value of the effective multiplication factor k_{eff}. The new radial distribution of neutrons born then replaces the originally assumed distribution, and this cycle of calculations is continued until the value of k_{eff} is stabilized; or, as considered in the subject program, Δk_{eff} approaches zero.

The equations used to make these calculations are developed in the following sequence:

(1) The steady-state group-diffusion equation is written for each energy group employing group-averaged cross sections.

(2) The group-diffusion equation is then written in finite difference form for selected radial points in the reactor.

(3) A procedure for solving equations at each radial point is described for determining the neutron flux at each point for the energy group in question.

(4) For the radial flux distribution just calculated, the equation is given for computing the radial distribution of neutrons born out of a particular energy group.

(5) Using the summation of neutrons born at each radial point, equations are given for calculating the reactor power and k_{eff}.

The value of the results obtained from the group-diffusion calculations may in some cases be improved with an increase in the number of energy groups. For this reason, the number of energy groups selected is nine, which is the largest number of groups that could be accommodated by the computing machine for which the program was designed. These nine groups are numbered a to i, and any one group generally will be referred to as the n^{th} group. Group a designates the group of highest neutron energy, and group i designates the group of neutrons at thermal energy.

At any point within the reactor, the steady-state equation can be written in the form:

$$\text{Neutron loss} = \text{Neutron production} \tag{1}$$

When the case of a unit volume within the reactor and neutrons of a particular energy group n is considered, the terms of the equation can be subdivided as follows (all symbols are defined in appendix A):

<u>Neutron loss per unit time</u>:

(1) Diffusion of neutrons from volume element = $-D_n\nabla^2\varphi_n$

(2) Neutron absorption = $\Sigma_{a,n}\varphi_n$

(3) Slowing out of group n = $\Sigma_{q,n}\varphi_n$

(4) Neutron leakage from reactor in axial direction = $D_nB_z^2\varphi_n$

Neutron production per unit time:

(1) Fission neutrons born into group $n = \alpha_n\beta_t$ where

$$\beta_t = \nu \sum_{n=a}^{i} \Sigma_{f,n}\varphi_n$$

(2) Neutrons slowing down into group $n = \Sigma_{q,n-1}\varphi_{n-1}$

If the previous terms are substituted in equation (1), the general diffusion equation becomes

$$-D_n\nabla^2\varphi_n + (\Sigma_{a,n} + \Sigma_{q,n} + D_nB_z^2)\varphi_n = \alpha_n\beta_t + \Sigma_{q,n-1}\varphi_{n-1} \tag{2}$$

Grouping the terms within the parentheses into one term, $\Sigma_n\varphi_n$ reduces equation (2) to the form

$$-D_n\nabla^2\varphi_n + \Sigma_n\varphi_n = \alpha_n\beta_t + \Sigma_{q,n-1}\varphi_{n-1} \tag{3}$$

For a given unit volume in the reactor, the diffusion equations for the nine energy groups can be written

$$\left.\begin{aligned}
(-D\nabla^2\varphi + \Sigma\varphi)_a &= \alpha_a\beta_t + 0 \\
(-D\nabla^2\varphi + \Sigma\varphi)_b &= \alpha_b\beta_t + \Sigma_{q,a}\varphi_a \\
(-D\nabla^2\varphi + \Sigma\varphi)_c &= \alpha_c\beta_t + \Sigma_{q,b}\varphi_b \\
&\cdots\cdots\cdots \\
(-D\nabla^2\varphi + \Sigma\varphi)_i &= 0 + \Sigma_{q,h}\varphi_h
\end{aligned}\right\} \tag{4}$$

For the cylindrical geometry selected, the flux φ is assumed to be a function of the radius r only; therefore,

$$\nabla^2\varphi = \frac{d^2\varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} \tag{5}$$

The terms on the right side of equation (5) may be approximated by expressing them in terms of finite differences based on values at adjacent points within a region. For this reason, each region is subdivided into cylinders of equal thickness h except for the cylinder at the center of the reactor, the two cylinders on either side of an interface, and the outermost cylinder (see fig. 1). Figure 1 presents the assumed configuration of the reactor and indicates the location of the numbered points. The radial dimensions of the four regions (I, II, III, and IV)

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may vary independently; in other words, the distance h between two adjacent radial points in a region may vary from region to region. Thus, the value of h is specified by means of the general subscript m.

At a point k the derivatives of equation (5) can be written in terms of finite differences as

$$\left(\frac{d\varphi}{dr}\right)_k \approx \frac{\varphi_{k+1} - \varphi_{k-1}}{2h_m} \tag{6}$$

$$\left(\frac{d^2\varphi}{dr^2}\right)_k \approx \frac{\varphi_{k+1} - 2\varphi_k + \varphi_{k-1}}{h_m^2} \tag{7}$$

Substituting equations (6) and (7) into equation (5) and simplifying result in the following equation:

$$\nabla^2\varphi \approx \frac{E_{k-1}\varphi_{k-1} - 2r_k\varphi_k + E_k\varphi_{k+1}}{r_k h_m^2} \tag{8}$$

where

$$E_{k-1} = r_k - \frac{h_m}{2}$$

$$E_k = r_k + \frac{h_m}{2}$$

Substituting equation (8) into equation (3) and grouping the coefficients of φ_k give

$$-\frac{D_{n,m}}{r_k h_m^2}\left[E_{k-1}\varphi_{k-1,n} + \left(-2r_k - \frac{r_k h_m^2 \Sigma_{n,m}}{D_{n,m}}\right)\varphi_{k,n} + E_k\varphi_{k+1,n}\right]$$

$$= \alpha_n\beta_{t,k} + \Sigma_{q,n-1,m}\varphi_{k,n-1}$$

Further simplification gives

$$E_{k-1}\varphi_{k-1,n} + r_k G_{n,m}\varphi_{k,n} + E_k\varphi_{k+1,n} = W_{k,n} \tag{9}$$

where

$$G_n = -2 - \frac{h_m^2 \Sigma_{n,m}}{D_{n,m}}$$

$$W_{k,n} = - \frac{r_k h_m^2}{D_{n,m}} (\alpha_n \beta_{t,k} + \Sigma_{q,n-1,m} \varphi_{k,n-1})$$

Equation (9) for the n^{th} group is then of the form

$$C_{1,k}\varphi_{k-1} + C_{2,k}\varphi_k + C_{3,k}\varphi_{k+1} = W_k \tag{10}$$

Equation (9) is applicable at all points in the reactor except where special conditions are imposed, such as points 1, 11, 21, 31, and 38 in figure 1. Points 1 and 38 can be treated as special cases of equation (9); however, points 11, 21, and 31 at the interfaces of adjacent regions cannot be solved by equation (9) because the value of G_n is dependent on the cross sections of a particular region.

In the case of point 1, which is a distance of $h_I/2$ from the center of the reactor, it is assumed that $\varphi_1 = \varphi_0$; therefore, by substituting $h_I/2$ for h_m and φ_1 for φ_0, equation (9) becomes

$$\left(\frac{h_I}{4}\right) \varphi_{1,n} + \frac{h_I}{2} \left(-2 - \frac{h_I^2}{4} \frac{\Sigma_{n,I}}{D_{n,I}}\right) \varphi_{1,n} + \left(\frac{3h_I}{4}\right) \varphi_{2,n}$$

$$= - \frac{h_I^3}{8D_{n,I}} (\alpha_n \beta_{t,1} + \Sigma_{q,n-1,I} \varphi_{1,n-1})$$

(The subscript I refers to the central region (fig. 1(a)).) Dividing this equation by $h_I/4$ and simplifying gives

$$\left(-3 - \frac{h_I^2}{2} \frac{\Sigma_{n,I}}{D_{n,I}}\right) \varphi_{1,n} + 3\varphi_{2,n} = - \frac{h_I^2}{2D_{n,I}} (\alpha_n \beta_{t,1} + \Sigma_{q,n-1,I} \varphi_{1,n-1}) \tag{11}$$

which, in the form of equation (10) for the n^{th} group, is

$$C_{2,1}\varphi_1 + C_{3,1}\varphi_2 = W_1$$

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For the solution of equation (9) at point 38, it is assumed that $\varphi_{39} = 0$ and that $\beta_{t,38} = 0$; therefore, equation (9) can be written as follows:

$$E_{37}\varphi_{37,n} + r_{38}G_{38,n}\varphi_{38,n} = -\frac{r_{38}h_{IV}^2}{D_{n,IV}}(\Sigma_{q,n-1,IV}\varphi_{38,n-1}) \tag{12}$$

or, after the general form of equation (10) for the n^{th} group,

$$C_{1,38}\varphi_{37} + C_{2,38}\varphi_{38} = W_{38}$$

In order to determine an equation that is suitable for computing the flux at the interface of two regions (i.e., points 11, 21, and 31), the requirement that the net neutron current (-D grad φ) be the same for each region at the interface is considered. At point 11, for example,

$$-D_{I,n}(\text{grad } \varphi)_{I,n,11} = -D_{II,n}(\text{grad } \varphi)_{II,n,11}$$

This can be expressed as

$$-D_{I,n}\left(\frac{d\varphi}{dr}\right)_{I,n,11} = -D_{II,n}\left(\frac{d\varphi}{dr}\right)_{II,n,11} \tag{13}$$

The value of $(d\varphi/dr)_{I,n,11}$ can be determined approximately by the following equation, representing the first two terms in a Taylor's series expansion of $d\varphi/dr$ at point 10:

$$\left(\frac{d\varphi}{dr}\right)_{I,n,11} \approx \left(\frac{d\varphi}{dr}\right)_{n,10} + h_I\left(\frac{d^2\varphi}{dr^2}\right)_{n,10}$$

Substituting for the derivatives their equivalents as written in equations (6) and (7) gives

$$\left(\frac{d\varphi}{dr}\right)_{I,n,11} \approx \frac{\varphi_{11,n} - \varphi_{9,n}}{2h_I} + h_I\left(\frac{\varphi_{11,n} - 2\varphi_{10,n} + \varphi_{9,n}}{h_I^2}\right)$$

which, simplified, is

$$\left(\frac{d\varphi}{dr}\right)_{I,n,11} \approx \frac{3\varphi_{11,n} - 4\varphi_{10,n} + \varphi_{9,n}}{2h_I} \tag{14}$$

By a similar procedure,

$$\left(\frac{d\varphi}{dr}\right)_{II,n,11} \approx -\frac{3\varphi_{11,n} - 4\varphi_{12,n} + \varphi_{13,n}}{2h_{II}} \tag{15}$$

Rearranging equation (13) in the form

$$\left(\frac{d\varphi}{dr}\right)_{I,n,11} - \frac{D_{II,n}}{D_{I,n}}\left(\frac{d\varphi}{dr}\right)_{II,n,11} = 0$$

and substituting equations (14) and (15) give

$$\frac{3\varphi_{11,n} - 4\varphi_{10,n} + \varphi_{9,n}}{2h_I} - \frac{D_{II,n}}{D_{I,n}}\left(-\frac{3\varphi_{11,n} - 4\varphi_{12,n} + \varphi_{13,n}}{2h_{II}}\right) = 0$$

Then, simplifying this equation gives

$$\varphi_{9,n} - 4\varphi_{10,n} + 3(1 + a_{I,n})\varphi_{11,n} - 4a_{I,n}\varphi_{12,n} + a_{I,n}\varphi_{13,n} = 0 \tag{16}$$

where

$$a_{I,n} = \frac{h_I D_{II,n}}{h_{II} D_{I,n}}$$

At each interface (or $k = 11, 21, 31$), equation (16) can be written in general form for the n^{th} group as follows:

$$\varphi_{k-2} + d_{1,k}\varphi_{k-1} + d_{2,k}\varphi_k + d_{3,k}\varphi_{k+1} + d_{4,k}\varphi_{k+2} = 0 \tag{17}$$

As a result of this development, equations (9), (11), (12), and (16) make it possible to write one equation at each of the 38 radial points in the reactor in terms of the flux with coefficients based on the nuclear cross-section data for each energy group. The flux coefficients on the left side of each equation for the n^{th} group form a matrix (designated in the machine program as the left matrix) in which each row represents an equation at a particular radial point and each column represents the flux at a particular point as indicated:

$$
\begin{array}{cccccccccc}
c_{2,1} & c_{3,1} & & & & & & & & \\
c_{1,2} & c_{2,2} & c_{3,2} & & & & & & & \\
 & c_{1,3} & c_{2,3} & c_{3,3} & & & & & & \\
 & & & \cdots & \cdots & \cdots & & & & \\
 & & & c_{1,10} & c_{2,10} & c_{3,10} & & & & \\
 & & & 1 & d_{1,11} & d_{2,11} & d_{3,11} & d_{4,11} & & \\
 & & & & & c_{1,12} & c_{2,12} & c_{3,12} & & \\
 & & & & & & \cdots & \cdots & \cdots & \\
 & & & & & & c_{1,36} & c_{2,36} & c_{3,36} & \\
 & & & & & & & c_{1,37} & c_{2,37} & c_{3,37} \\
 & & & & & & & & c_{1,38} & c_{2,38}
\end{array}
$$

The two simultaneous equations represented by the first two rows of coefficients in the preceding matrix can be reduced to an equation of two unknowns, namely $\varphi_{2,n}$ and $\varphi_{3,n}$, represented by the primed coefficients in the second row of the following primed matrix. This new equation and the equation represented by the third row of coefficients in the original matrix can then be reduced to an equation of two unknowns, namely $\varphi_{3,n}$ and $\varphi_{4,n}$, represented by the primed coefficients in the third row of the primed matrix. By continuing this process from the upper left corner to the lower right corner, the original matrix can be reduced to a new, primed matrix of the following form:

$$
\begin{array}{ccccccccccc}
C'_{2,1} & C'_{3,1} & & & & & & & & & \\
 & C'_{2,2} & C'_{3,2} & & & & & & & & \\
 & & C'_{2,3} & C'_{3,3} & & & & & & & \\
 & & & \cdots & \cdots & & & & & & \\
 & & & & C'_{2,10} & C'_{3,10} & & & & & \\
 & & & & d'_{1,11} & d'_{2,11} & d'_{3,11} & d'_{4,11} & & & \\
 & & & & & & C'_{2,12} & C'_{3,12} & & & \\
 & & & & & & & \cdots & \cdots & & \\
 & & & & & & & & C'_{2,36} & C'_{3,36} & \\
 & & & & & & & & & C'_{2,37} & C'_{3,37} \\
 & & & & & & & & & & C'_{2,38}
\end{array}
$$

The right member of each of the 38 equations is altered, also, as the progression through the original matrix occurs. Therefore, the right side of each equation is treated as a separate matrix (called the right matrix in the program).

Equating the bottom row of the left, primed matrix with that of the right, primed matrix gives

$$C'_{2,38}\varphi_{38} = W'_{38}$$

which can be written as

$$\varphi_{38} = \frac{W'_{38}}{C'_{2,38}}$$

After φ_{38} is thus determined, the flux at point 37 can be calculated by moving up one row in the primed matrix and writing

$$C'_{2,37}\varphi_{37} + C'_{3,37}\varphi_{38} = W'_{37}$$

which can be rearranged in the form

$$\varphi_{37} = \frac{W'_{37} - C'_{3,37}\varphi_{38}}{C'_{2,37}}$$

This process continues to the upper left corner of the primed matrix, and thus the flux at each of the 38 points can be calculated. The preceding procedure is a modified matrix reduction technique that is a modification of the Crout matrix reduction technique.

After a value of the flux at each of the 38 radial points in the reactor is thus computed for an assumed value of β_t at each radial position in the core (i.e., the three inner regions), the radial distribution of neutrons born out of a particular energy group, designated as $\beta_{n,k}$, is calculated by

$$\beta_{n,k} = \nu\Sigma_{f,n,m}\varphi_{n,k}$$

It should be noted that at points 11 and 21 two values of $\beta_{n,k}$ can be calculated, one value based on the Σ_f of one region and the other value based on the Σ_f of the adjacent region. For a calculation of the power of the reactor by the method outlined herein, both values must be computed at an interface within the core. Thus, there are 33 values of $\beta_{n,k}$ for each energy group. It so happens that a running total of $\beta_{n,k}$ at each radial point can be made as the problem proceeds; therefore, a large amount of machine storage is not required to store the values of $\beta_{n,k}$ for each energy group. During the entire cycle of computing through all energy groups, the previously assumed values of $\beta_{t,k}$, or those calculated from the previous cycle, are used.

After the flux distribution and corresponding values of $\beta_{n,k}$ are computed for the first, or highest, energy group, the program proceeds to the solution of the matrix of the second energy group to determine the radial flux distribution and the 33 values of $\beta_{n,k}$ that are added to the $\beta_{n,k}$ of the next higher group. This procedure continues until the solution of all energy groups is completed.

Then, after the summations of $\beta_{n,k}$, designated as $\beta_{t,k}$, are computed for each radial point in the core, a value of the reactor power P is computed by the following equation:

$$P = A \sum_{r_0}^{r_{31}} 2\pi r_k \Delta r \beta_{t,k}$$

$$= A' \sum_{r_0}^{r_{31}} r_k \Delta r \beta_{t,k} \tag{18}$$

where A' is an arbitrary constant that cancels itself in computing the ratio of powers to determine k_{eff}. In computing the power, a point k is assumed to be at the mean radius of a cylinder of thickness h_m. Thus, figure 1 shows that special equations must be derived for computing the power generated between points 0 and 1, which are a distance $h_I/2$ apart, and for the cylinders of thickness $h_m/2$ on either side of an interface within the core. These special equations are listed below:

Between r_0 and r_1:

$$r_k \Delta r \beta_{t,k} = \frac{h_I}{4} \frac{h_I}{2} \beta_{t,1} = \frac{h_I^2}{8} \beta_{t,1} \tag{19}$$

(Note: $\beta_{t,1}$ is assumed to represent the average β_t from the center of the reactor to point 1.)

Between $r_{10\frac{1}{2}}$ and r_{11}:

$$r_k \Delta r \beta_{t,k} = \int_{r_{10\frac{1}{2}}}^{r_{11}} \left(r_{10} + \frac{3h_I}{4}\right) \beta_t dr = \int \frac{39h_I}{4} \beta_t dr \tag{20}$$

where it is assumed by numerical integration that

$$\int_{r_{10\frac{1}{2}}}^{r_{11}} \beta_t dr = \frac{h_I}{24} (-\beta_{t,9} + 5\beta_{t,10} + 8\beta_{t,11}) \tag{21}$$

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(Note: $\beta_{t,I,11}$ corresponds to the value calculated from a value of Σ_f for region I. A similar regional subscript notation is used on β_t in the following equations.)

Substituting equation (21) into equation (20) gives

$$r_k \Delta r \beta_{t,k} = \frac{13h_I^2}{32} (-\beta_{t,9} + 5\beta_{t,10} + 8\beta_{t,I,11}) \tag{22}$$

Between r_{11} and $r_{11\frac{1}{2}}$:

$$r_k \Delta r \beta_{t,k} = \int_{r_{11}}^{r_{11\frac{1}{2}}} \left(r_{11} + \frac{h_{II}}{4}\right)\beta_t dr = \int \left(10h_I + \frac{h_{II}}{4}\right)\beta_t dr \tag{23}$$

where

$$\int_{r_{11}}^{r_{11\frac{1}{2}}} \beta_t dr = \frac{h_{II}}{24} (8\beta_{t,II,11} + 5\beta_{t,12} - \beta_{t,13}) \tag{24}$$

Then, substitution of equation (24) into equation (23) gives

$$r_k \Delta r \beta_{t,k} = \frac{h_{II}}{96} (40h_I + h_{II})(8\beta_{t,II,11} + 5\beta_{t,12} - \beta_{t,13}) \tag{25}$$

At points 21 and 31 equations representing $r_k \Delta r \beta_{t,k}$ can be derived in a manner similar to that used at point 11.

Then, substituting into equation (18) the right members of equations (19), (22), (25), and the equations for points 21 and 31 and the interior points of each region results in the following equation:

$$\frac{P}{A'} = h_I\left(\sum_{k=2}^{10} r_k\beta_{t,I,k} + Y_1(Z_1 + Y_2Z_2)\right) +$$

$$h_{II}\left(\sum_{k=12}^{20} r_k\beta_{t,II,k} + Y_3Z_3 + Y_4Z_4\right) +$$

$$h_{III}\left(\sum_{k=22}^{30} r_k\beta_{t,III,k} + Y_5Z_5 + Y_6Z_6\right) \tag{26}$$

where

$$Y_1 = \frac{h_I}{8} \qquad Z_1 = \beta_{t,1}$$

$$Y_2 = \frac{39}{12} \qquad Z_2 = -\beta_{t,9} + 5\beta_{t,10} + 8\beta_{t,I,11}$$

$$Y_3 = \frac{40h_I + h_{II}}{96} \qquad Z_3 = 8\beta_{t,II,11} + 5\beta_{t,12} - \beta_{t,13}$$

$$Y_4 = \frac{40h_I + 39h_{II}}{96} \qquad Z_4 = -\beta_{t,19} + 5\beta_{t,20} + 8\beta_{t,II,21}$$

$$Y_5 = \frac{40h_I + 40h_{II} + h_{III}}{96} \qquad Z_5 = 8\beta_{t,III,21} + 5\beta_{t,22} - \beta_{t,23}$$

$$Y_6 = \frac{40h_I + 40h_{II} + 39h_{III}}{96} \qquad Z_6 = -\beta_{t,29} + 5\beta_{t,30} + 8\beta_{t,III,31}$$

With the value of P/A' calculated for a previous set of values of $\beta_{t,k}$ and a new value of P/A' for the values of $\beta_{t,k}$ obtained from the flux values in the reduced matrix solution, a value of k_{eff} is defined as

$$k_{eff,new} = \frac{(P/A')_{new}}{(P/A')_{previous}} \tag{27}$$

This value of k_{eff} is then compared with a previous value of k_{eff} to determine whether or not the change in k_{eff}, noted herein as Δk_{eff}, is less than an assumed value (e.g., 10^{-4}) that specifies a satisfactory limit on the convergence. If the limit on Δk_{eff} is not met, the new values of $\beta_{t,k}$ and P/A' are normalized by dividing each one by k_{eff} (eq. (27)). The normalized values of $\beta_{t,k}$ are then used to compute a new flux distribution that results in a new P/A'. A new k_{eff} is determined by equation (27), and the Δk_{eff} is inspected for being less than, for example, 10^{-4}. This inspection and recalculation of the problem continues until the limits on Δk_{eff} are met. A discussion of the values punched out during, and at the end of, the problem are discussed subsequently.

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CALCULATION PROCEDURE

The solution for the effective multiplication factor k_{eff} of a reactor by means of the equations and procedure indicated in the previous section is begun by assuming an initial radial distribution of neutrons born and then dividing the neutrons into energy groups on the basis of the division of the fission spectrum. On the basis of this distribution of neutrons born, a radial and energywise flux distribution is calculated. This new flux distribution results in a new radial distribution of neutrons born and new values of reactor power and k_{eff}. During each subsequent iteration the radial distribution of neutrons born resulting from the previous cycle of flux calculations is used to compute new values of reactor power and k_{eff}. This iteration procedure continues until the value of k_{eff} has converged within the limits specified. Then, one more time through the problem causes the program to punch out: (1) the radial distribution of neutron production as a summation of neutrons born out of each energy group plus those born out of all higher groups, (2) a normalized flux distribution, and (3) the radius of each radial point and the values of k_{eff} and Δk_{eff}.

The details of the program to perform the solution described are included in the appendixes, and an outline of the machine program is given in the form of a block diagram in figure 2. Appendix B lists the computing machine components used and gives instructions with regard to use of the machine program. Appendix C discusses the use of the subject program for less than nine energy groups and four, or less, concentric regions. Appendix D presents an outline and discussion of the various sections of the program and gives a complete listing of the machine program.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, October 1, 1958

APPENDIX A

SYMBOLS

A, A'	coefficients relating power to total number of neutrons born (see eq. (18))
a	coefficient defined by equation (16)
B_z^2	geometric buckling in axial direction
C	flux coefficient defined by equations (9) and (10) used in forming matrix
C'	flux coefficient in reduced matrix
D	diffusion coefficient, cm
d	flux coefficient defined by equations (16) and (17)
d'	flux coefficient in reduced matrix
E	distance from axis of reactor to the halfway point between adjacent numbered points k and $k+1$ (fig. 1(b)), cm
G	flux coefficient defined in equation (9)
h	thickness of concentric cylinders within a region of the reactor (fig. 1(b)), cm
k_{eff}	effective multiplication factor
Δk_{eff}	change in effective multiplication factor
n	neutron energy group (a, b, c, ... i, in order of decreasing energy to thermal group i)
P	reactor total power defined by equation (18), neutrons/sec
r	radius, cm
W	term in matrix equations representing the neutrons entering a group as defined by equation (9)
W'	term representing neutrons entering a group in the reduced, or primed, matrix
$Y_1, Y_2, \ldots, Y_6$	coefficients defined by equation (26), cm

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$Z_1, Z_2, \ldots, Z_6$	terms representing neutrons born in equation (26), neutrons/(cm^3)(sec)
α	ratio of neutrons born into an energy group to the total number born
β	neutrons born into any one energy group per unit volume per unit time, neutrons/(cm^3)(sec)
β_t	total number of neutrons born into all energy groups per unit volume per unit time, neutrons/(cm^3)(sec)
ν	average number of neutrons produced per fission
Σ	sum of neutron cross sections representing neutron loss from a given volume element other than by diffusion, cm^{-1}
Σ_a	neutron absorption cross section, cm^{-1}
Σ_f	neutron fission cross section, cm^{-1}
Σ_q	neutron slowing-down cross section specifying the number of neutrons slowing out of one energy group into the next lower group, cm^{-1}
φ	neutron flux, neutrons/(cm^2)(sec)

Subscripts:

a,b,c...i	nine neutron energy groups
k	numbered radial point in reactor (0, 1, 2, 3, ... as shown in fig. 1)
m	numbered region (I, II, III, and IV as shown in fig. 1)
0,1,2,3,...39	radial points
I,II,III,IV	radial regions

APPENDIX B

INFORMATION ON USE OF PROGRAM

The machine for which the subject program was developed is basically an IBM 650 magnetic drum data-processing machine (ref. 2) with an IBM 653 high-speed storage unit (ref. 3) and additional features including indexing accumulators A, B, and C and a floating decimal arithmetic unit (ref. 4). The language in which the program of appendix D is written is an optimal programming method called SOAP II (Symbolic Optimal Assembly Program II) developed for the IBM 650 computer (ref. 5). The results of processing the original program by the method of SOAP II into the basic machine language is shown in the appendix D tabulation, where the original coded program is shown on the left and the program in basic machine language is given in the four columns of numbers on the right. These four columns of numbers represent, column by column from left to right,

(a) Location of command

(b) Machine operation code

(c) Address of data used with machine operation

(d) Address of next instruction

The plus signs associated with each command, or the data, are not indicated; however, the sign, either plus or minus, should be punched in the cards to be used as the program. The manner in which the machine program is to be punched into cards, of course, depends on the wiring of the control panel in the card reader unit, which varies from one computing machine to another. Therefore, general card punching instructions cannot be given and are not included in this report.

The order of the punched cards to be read into the machine is

1. Suitable load - punch subroutine

2. Program of appendix D

3. Set of input data as specified in table I

4. Basic load card to transfer control to the first instruction of program, which is 0500

When these four items are read in, the machine will compute and punch out answers until the end of the program is reached, where there is a HALT command.

When the first run is completed, a second run with only the changes desired can be performed by reading the changes into the computer and transferring control to address 0500.

The input locations are specified in table I. The location of each input value is dependent on both the physical composition of the region and/or the region location; therefore, each storage location has been specified in detail in table I.

The output locations are specified by the set of punch constants (program instructions 837 to 844) used in conjunction with an output (or PUNCH) subroutine. The instructions in the program that cause punching of the output are shown in table II in the order of occurrence during the program. Only the values of k_{eff} and Δk_{eff} are punched out during each iteration to indicate the progress of the convergence of k_{eff} to the final value punched out at the end of the program.

The means used for loading and punching out data can be adapted to the requirements of a similar IBM 650 machine by writing loading and punching commands in the drum storages reserved for this purpose (namely, 1871 to 1999) and utilizing high-speed storages 9055 to 9059 during the punching operation. This would mean a possible revision of the program instructions used for this purpose, which are listed in table II, and the punch constants (837 to 844) in appendix D. The method used by the authors for reading the input and punching the output is not discussed herein, because the subject program is adaptable to any method presently in use by the reader. The authors' method is obtainable from the authors or the Machine Computing Branch at the NASA Lewis Research Center. The system used by the authors is designed to suit many other projects and cannot be recommended as a specific means of performing the functions required by this problem.

The only addresses not used that could be used for other purposes are addresses 0000, 0037, and 0332. It is understood that locations 1871 to 1999 are also available for a load-punch routine, as previously discussed.

The program is written in such a way that 1.0 is the original guess for each of the 33 values of β in the inner three regions. In some cases that have been run, this β distribution leads to negative fluxes in the first iteration. If this difficulty should arise, steps 373 to 377 can be bypassed by loading the following correction to replace step 372:

0330 21 0200 0390

Then it will be necessary to load with the input data the 33 original values of β stored in addresses 9019 to 9051.

APPENDIX C

USE OF PROGRAM FOR LESS THAN NINE ENERGY GROUPS

AND FOUR OR LESS REGIONS

The subject program can be used for any number of neutron energy groups up to nine and any number of regions up to four. If fewer than nine energy groups are used, input values for the unused energy groups must still be read into the machine as discussed in the next paragraph. The reason for this is that the machine program generates input for nine matrices regardless of the number of groups specified. However, after program step 155 (see appendix D), the program computes only the number of groups desired.

In a solution of fewer than nine groups, the valid cross-section values should be located toward the end of each storage region in which the input data are stored (see table I). For those groups not used, the cross-section values of Σ_q, Σ, and $\nu\Sigma_f$ should be zero. Also, the values of α for the energy groups not used should be zero. The value of Σ_q for the first group for each region should also be zero, and these zeros have to be read in with the other input; whereas, in the case of nine groups, steps 832 to 835 provide the necessary zeros for the four regions. The values of D for the energy groups not used should be equal to 1 because the D values appear in the denominator of expressions used in the program, and zeros in this case would cause the computing machine to stop.

With this storage arrangement, in the case of nine energy groups it is necessary to change the basic machine language of program steps 158 and 519, which control the input to the matrix calculations. An example for six groups would be as follows:

$$\text{Step } 158 = 0407\ 83\ \underline{0005}\ 0463$$

$$\text{Step } 519 = 0257\ 83\ \underline{0005}\ 0363$$

The underlined numbers are equal to 1 less than the number of groups being used.

If fewer than four regions are desired, it is necessary to duplicate the cross-section values and diffusion coefficients in adjacent regions and reduce the value of h in the adjacent regions. In other words, one region of the reactor can be treated as two identical regions insofar as the machine program is concerned. Thus, a three-region reactor can be treated as having four regions, and the machine program can proceed with its solution for four regions.

APPENDIX D

COMPUTING MACHINE PROGRAM INSTRUCTIONS

The subject program was written for the computing machine described in appendix B. In order that the organization of the machine program can be better understood, an outline of the program is given in figure 2. The numbers in the parentheses in each block of figure 2 specify the machine instructions that perform the operations indicated. In the tabulation of the instructions included in this appendix, these instruction numbers appear at the extreme left. (In the discussion in this appendix, the numbers in parentheses refer to these same instruction numbers.)

To utilize the indexing features of the computing machine, data are stored in groups of consecutive storage locations known as regions (3 to 18). The machine loading and punching subroutines are contained in a separate group of instructions that are read into the machine ahead of the subject program and stored in locations 1871 to 1999 (reserved by instruction 22).

As indicated in figure 2, there are parts of the program (3 to 379) that are performed at the beginning of the problem and are not repeated during each iteration to stabilize k_{eff}. The remainder of the program (383 to 811) is repeated during each iteration, as shown by the arrow indicating the return of the program to instruction (385). Inside of this outer loop is an inner loop in the program that computes the neutrons born out of each energy group (518 to 811). As soon as k_{eff} has been stabilized, the program is set for the last iteration, and punching takes place during this iteration (746 to 749, 764, 777, 790, 796 to 805, 494 to 498). After the last iteration, the program is stopped, and restarting the problem to compute the next case can be done as outlined in appendix B.

Instruction numbers that are not indicated on the outline in figure 2 are instructions that are not machine commands, such as heading and spacing instructions or constants used in the program. The constants used are as follows:

(815 to 823) Numbers 1 to 9 used for code designation of each energy group and as constants

(825 to 830) Other constants that appear in the equations

(832 to 835) Zero values for the slowing-in cross section of the first energy group in each region of the reactor

(837 to 844) Code numbers used with punch subroutine to prescribe a certain format for the output

(846 to 847) Values used in the left matrix

(849) Limit of convergence of k_{eff} (in this case, 10^{-4})


```
1                 NINE GROUP REACTOR PROBLEM
1
           REG A0753      0761         ALPHAS
           REG B1767      1799         SUM BETAS
           REG C1720      1801         BDRY D
           REG D0762      0797         D COEFFS
           REG E0798      0835         E VALUES
           REG F1729      1766         FLUX
           REG G1840      1866         F OF ALPHA
           REG H0717      0752         F OF SIG Q
           REG J0681      0716         G VALUES
           REG M0836      1573         MATRICES
           REG N1802      1839         NORM FLUX
           REG Q1574      1609         SIGMA Q
           REG R1610      1647         PT RADII
           REG S1648      1683         SIGMA
           REG V1684      1710         NU SIGMA F
           REG W1711      1719         H SQ A
           SYN BEGIN      0500         START HERE
           EQU PUNCH      1930         RKT PUNCH
           BLR  0000      0000         BLR 0000
           BLR  1871      1999         LOAD PUNCH
           EQU KCHEK      9018         TEST FOR
1                                       LAST
1                                       ITERATION
1
1               FUNCTIONS OF INPUT VALUES
1
   BEGIN   RSA   0003    GO 1                          0500   81 0003 0556
   GO 1    RAU   9217                                  0556   60 9217 0471
           FMP   8003               H SQUARE           0471   39 8003 0325
           STU   9213                                  0325   21 9213 0267
           NZA           GO 2                          0267   40 0320 0521
           RAU   9217                                  0320   60 9217 0571
           FDV   9218               H RATIO            0571   34 9218 0118
           STU   9260                                  0118   21 9260 0464
           AXA   0001    GO 1                          0464   50 0001 0556
1
   GO 2    RSB   0008               H SQ A             0521   83 0008 0177
           RSC   0035    GO 4                          0177   89 0035 0434
   GO 4    RAU S0036 C                                 0434   60 7683 0437
           FDV D0036 C                                 0437   34 6797 0447
```

```
43          FMP  9210                            0447  39 9210 0210
44          FMP MIN 1                            0210  39 0466 0317
45          STU J0036 C                          0317  21 6716 0519
46          NZB          GO 5                    0519  42 0172 0673
47          AXB  0001    GO 3                    0172  52 0001 0178
48   GO 5   AXA  0001                            0673  50 0001 0429
49          SXB  0008                            0429  53 0008 0285
50          NZC GO 3     GO 6                    0285  48 0178 0439
51   GO 3   AXC  0001    GO 4                    0178  58 0001 0434
52 1
53   GO 6   RSB  0026    GO 7                    0439  83 0026 0446
54   GO 7   RAU D0036 B                          0446  60 4797 0552
55          FDV D0027 B           D RATIO        0552  34 4788 0038
56          STU  9456                            0038  21 9456 0359
57          NZB          GO 8                    0359  42 0412 0514
58          AXB  0001    GO 7                    0412  52 0001 0446
59 1
60   GO 8   RSC  0000             BDRY COEFS     0514  89 0000 0370
61          RSA  0002              FOR LEFT      0370  81 0002 0226
62          SXB  0008    GO 10     MATRIX        0226  53 0008 0132
63   GO 10  RAU  9630                            0132  60 9630 0335
64          FMP  9259                            0335  39 9259 0409
65          STU C0001 C           D4 D8 D12      0409  21 7720 0174
66          NZB          GO 11                   0174  42 0227 0228
67          AXB  0001    GO 9                    0227  52 0001 0484
68   GO 11  NZA          GO 12                   0228  40 0331 0182
69          AXA  0001                            0331  50 0001 0487
70          SXB  0008    GO 9                    0487  53 0008 0484
71   GO 9   AXC  0001    GO 10                   0484  58 0001 0132
72 1
73   GO 12  RSC  0026    GO 13                   0182  89 0026 0088
74   GO 13  RAU C0027 C                          0088  60 7746 0602
75          FMP C0082             D3 D7 D11      0602  39 1801 0652
76          STU C0054 C                          0652  21 7773 0276
77          RAU C0027 C                          0276  60 7746 0304
78          FAD  9001                            0304  32 9001 0534
79          FMP  9003             D2 D6 D10      0534  39 9003 0537
80          STU C0081 C                          0537  21 7800 0354
81          NZC          GO 14                   0354  48 0557 0158
82          AXC  0001    GO 13                   0557  58 0001 0088
83 1
84   GO 14  RSA  0003             H SQ SIG Q     0158  81 0003 0564
85          RSB  0008    GO 16     OVER D        0564  83 0008 0420
86   GO 16  RAU  9213                            0420  60 9213 0367
87          FMP Q0001 C                          0367  39 7574 0224
88          FDV D0001 C                          0224  34 6762 0462
89          FMP MIN 1                            0462  39 0466 0417
90          STU H0001 C                          0417  21 6717 0470
91          NZB          GO 17                   0470  42 0274 0324
92          AXB  0001    GO 15                   0274  52 0001 0180
93   GO 17  NZA          GO 18                   0324  40 0277 0278
94          AXA  0001                            0277  50 0001 0584
95          SXB  0008    GO 15                   0584  53 0008 0180
```

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```
 96    GO 15   AXC  0001    GO 16                   0180  58 0001 0420
 97 1
 98    GO 18   RSA  0002            H SQ ALPHA      0278  81 0002 0634
 99            RSB  0008             OVER D         0634  83 0008 0090
100            RSC  0026    GO 20                   0090  89 0026 0496
101    GO 20   RAU A0009 B                          0496  60 4761 0467
102            NZU          GO 21                   0467  44 0621 0222
103            FDV D0027 C                          0621  34 3788 0138
104            FMP MIN 1                            0138  39 0466 0517
105            FMP  9212    GO 21                   0517  39 9212 0222
106    GO 21   STU G0027 C                          0222  21 7866 0569
107            NZB          GO 22                   0569  42 0272 0374
108            AXB  0001    GO 19                   0272  52 0001 0328
109    GO 22   NZA          GO 23                   0374  40 0327 0378
110            AXA  0001                            0327  50 0001 0385
111            SXB  0008    GO 19                   0385  53 0008 0328
112    GO 19   AXC  0001    GO 20                   0328  58 0001 0496
113 1
114    GO 23   RSA  0003            R AND E         0378  81 0003 0435
115            RSB  0008             CALC           0435  83 0008 0641
116            RSC  0028                            0641  89 0028 0497
117            RAU  9014                            0497  60 9014 0505
118            FDV  9002                            0505  34 9002 0208
119            STU R0001                            0208  21 1610 0614
120            STU  9000                            0614  21 9000 0671
121            STU E0001                            0671  21 0798 0404
122            FAD  9000                            0404  32 9000 0485
123            STU R0002    GO 24                   0485  21 1611 0664
124    GO 24   FAD  9000                            0664  32 9000 0444
125            STU E0030 C                          0444  21 6827 0230
126            FAD  9000                            0230  32 9000 0459
127            STU R0031 C                          0459  21 7640 0494
128            NZB          GO 25                   0494  42 0547 0498
129            AXB  0001    GO 26                   0547  52 0001 0454
130    GO 26   AXC  0001    GO 24                   0454  58 0001 0664
131    GO 25   NZA          JCALC                   0498  40 0504 0554
132            AXA  0001                            0504  50 0001 0260
133            RAU  9217                            0260  60 9217 0322
134            FDV  9002                            0322  34 9002 0375
135            STU  9000                            0375  21 9000 0535
136            RAU R0031 C                          0535  60 7640 0546
137            NZC          GO 27                   0546  48 0349 0604
138            SXB  0009    GO 26                   0349  53 0009 0454
139    GO 27   SXB  0006    GO 26                   0604  53 0006 0454
140 1
141    JCALC   RSA  0008    GO 90   J VALUES        0554  81 0008 0310
142    GO 90   RAU J0009 A                          0310  60 2689 0544
143            STU W0009 A                          0544  21 3719 0372
144            FSB  9002                            0372  33 9002 0654
145            STU J0009 A                          0654  21 2689 0292
146            NZA          GO 91                   0292  40 0596 0646
147            AXA  0001    GO 90                   0596  50 0001 0310
148    GO 91   RSA  0026    GO 92                   0646  81 0026 0555
```

```
    GO 92    RAU J0036 A                               0555   60 2716 0422
             FSB  9002                                 0422   33 9002 0605
             STU J0036 A                               0605   21 2716 0619
             NZA           CROUT                       0619   40 0472 0251
             AXA  0001     GO 92                       0472   50 0001 0555
1
1                 LEFT MATRIX POINTS 1-10
1
    CROUT    RAC  0000               MATRIX            0251   88 0000 0257
             RSB  0008     GO 34     GROUP             0257   83 0008 0363
    GO 34    RSA  0008               NET POINT         0363   81 0008 0369
             RAU W0009 B                               0369   60 5719 0373
             FDV  9002                                 0373   34 9002 0176
             FSB  9003                                 0176   33 9003 0305
             STU M0001 C                               0305   21 6836 0189
             RAU  9003                                 0189   60 9003 0297
             FDV M0001 C                               0297   34 6836 0136
             STU M0041 C GO 28                         0136   21 6876 0279
    GO 28    FMP E0009 A                               0279   39 2806 0356
             STU  9000                                 0356   21 9000 0413
             RAU R0010 A                               0413   60 3619 0423
             FMP J0009 B                               0423   39 4689 0239
             FSB  9000                                 0239   33 9000 0419
             STU M0002 C                               0419   21 6837 0040
             RAU E0010 A                               0040   60 2807 0311
             FDV M0002 C                               0311   34 6837 0287
             STU M0042 C                               0287   21 6877 0130
             NZA           BDRY1                       0130   40 0383 0284
             AXA  0001                                 0383   50 0001 0289
             AXC  0001     GO 28                       0289   58 0001 0279
1
1                 LEFT MATRIX POINTS 11 AND 12
1
    BDRY1    SXC  0008               POINT 11          0284   59 0008 0140
             RAU C0082                                 0140   60 1801 0655
             FSB M0049 C                               0655   33 6884 0511
             STU M0081 C                               0511   21 6916 0669
             FMP M0050 C                               0669   39 6885 0585
             STU  9000                                 0585   21 9000 0594
             RAU C0063 B                               0594   60 5782 0587
             FSB  9000                                 0587   33 9000 0567
             STU M0011 C                               0567   21 6846 0399
             RAU C0036 B                               0399   60 5755 0509
             FDV M0011 C                               0509   34 6846 0597
             STU M0051 C                               0597   21 6886 0489
             RAU C0009 B                               0489   60 5728 0635
             FDV M0011 C                               0635   34 6846 0647
             STU M0082 C                               0647   21 6917 0520
1
             RAU E0011               POINT 12          0520   60 0808 0617
             FMP M0051 C                               0617   39 6886 0336
             STU  9000                                 0336   21 9000 0644
             RAU R0012                                 0644   60 1621 0425
```

```
             FMP J0018 B                        0425  39 4698 0548
             FSB  9000                          0548  33 9000 0377
             STU M0012 C                        0377  21 6847 0606
             RAU E0011                          0606  60 0808 0667
             FMP M0082 C                        0667  39 6917 1867
             STU  9000                          1867  21 9000 0475
             RAU E0012                          0475  60 0809 0168
             FSB  9000                          0168  33 9000 0598
             FDV M0012 C                        0598  34 6847 0648
             STU M0052 C                        0648  21 6887 0190
             SXA  0007   GO 29                  0190  51 0007 0301
    1
    1           LEFT MATRIX POINTS 13-20
    1
      GO 29  RAU E0019 A                        0301  60 2816 0221
             FMP M0052 C                        0221  39 6887 0337
             STU  9000                          0337  21 9000 0595
             RAU R0020 A                        0595  60 3629 0433
             FMP J0018 B                        0433  39 4698 0348
             FSB  9000                          0348  33 9000 0127
             STU M0013 C                        0127  21 6848 0351
             RAU E0020 A                        0351  60 2817 0271
             FDV M0013 C                        0271  34 6848 0398
             STU M0053 C                        0398  21 6888 0541
             NZA         BDRY2                  0541  40 0394 0645
             AXA  0001                          0394  50 0001 0401
             AXC  0001   GO 29                  0401  58 0001 0301
    1
    1           LEFT MATRIX POINTS 21 AND 22
    1
      BDRY2  SXC  0007              POINT 21    0645  59 0007 0656
             RAU C0082                          0656  60 1801 0607
             FSB M0059 C                        0607  33 6894 0522
             STU M0079 C                        0522  21 6914 0218
             FMP M0060 C                        0218  39 6895 0449
             STU  9000                          0449  21 9000 0657
             RAU C0072 B                        0657  60 5791 0499
             FSB  9000                          0499  33 9000 0479
             STU M0021 C                        0479  21 6856 0559
             RAU C0045 B                        0559  60 5764 1869
             FDV M0021 C                        1869  34 6856 0258
             STU M0061 C                        0258  21 6896 0549
             RAU C0018 B                        0549  60 5737 0342
             FDV M0021 C                        0342  34 6856 0308
             STU M0039 C                        0308  21 6874 0427
    1
             RAU E0021              POINT 22    0427  60 0818 0424
             FMP M0061 C                        0424  39 6896 0599
             STU  9000                          0599  21 9000 0358
             RAU R0022                          0358  60 1631 0386
             FMP J0027 B                        0386  39 4707 0408
             FSB  9000                          0408  33 9000 0637
             STU M0022 C                        0637  21 6857 0360
```

```
255              RAU E0021                             0360  60 0818 0474
256              FMP M0039 C                           0474  39 6874 0524
257              STU  9000                             0524  21 9000 0381
258              RAU E0022                             0381  60 0819 0574
259              FSB  9000                             0574  33 9000 0458
260              FDV M0022 C                           0458  34 6857 0508
261              STU M0062 C                           0508  21 6897 0558
262              SXA  0007   GO 30                     0558  51 0007 0451
263 1
264 1               LEFT MATRIX POINTS 23-30
265 1
266    GO 30     RAU E0029 A                           0451  60 2826 0231
267              FMP M0062 C                           0231  39 6897 0347
268              STU  9000                             0347  21 9000 0355
269              RAU R0030 A                           0355  60 3639 0393
270              FMP J0027 B                           0393  39 4707 0307
271              FSB  9000                             0307  33 9000 0387
272              STU M0023 C                           0387  21 6858 0361
273              RAU E0030 A                           0361  60 2827 0281
274              FDV M0023 C                           0281  34 6858 0058
275              STU M0063 C                           0058  21 6898 0501
276              NZA         GO 31                     0501  40 0054 0405
277              AXA  0001                             0054  50 0001 0110
278              AXC  0001   GO 30                     0110  58 0001 0451
279 1
280 1               LEFT MATRIX POINTS 31 AND 32
281 1
282    GO 31     SXC  0007              POINT 31       0405  59 0007 0561
283              RAU C0082                             0561  60 1801 0608
284              FSB M0069 C                           0608  33 6904 0431
285              STU M0080 C                           0431  21 6915 0268
286              FMP M0070 C                           0268  39 6905 0658
287              STU  9000                             0658  21 9000 0318
288              RAU C0081 B                           0318  60 5800 0609
289              FSB  9000                             0609  33 9000 0539
290              STU M0031 C                           0539  21 6866 0570
291              RAU C0054 B                           0570  60 5773 0477
292              FDV M0031 C                           0477  34 6866 0368
293              STU M0071 C                           0368  21 6906 0659
294              RAU C0027 B                           0659  60 5746 0410
295              FDV M0031 C                           0410  34 6866 0418
296              STU M0040 C                           0418  21 6875 0428
297 1
298              RAU E0031              POINT 32       0428  60 0828 0436
299              FMP M0071 C                           0436  39 6906 0460
300              STU  9000                             0460  21 9000 0468
301              RAU R0032                             0468  60 1641 0649
302              FMP J0036 B                           0649  39 4716 0518
303              FSB  9000                             0518  33 9000 0510
304              STU M0032 C                           0510  21 6867 0620
305              RAU E0031                             0620  60 0828 0486
306              FMP M0040 C                           0486  39 6875 0525
307              STU  9000                             0525  21 9000 0536
```

```
              RAU E0032                            0536  60 0829 0586
              FSB  9000                            0586  33 9000 0568
              FDV M0032 C                          0568  34 6867 0618
              STU M0072 C                          0618  21 6907 0560
              SXA  0005   GO 32                    0560  51 0005 0551
    1
    1            LEFT MATRIX POINTS 33-38
    1
     GO 32    RAU E0037 A                          0551  60 2834 0339
              FMP M0072 C                          0339  39 6907 0357
              STU  9000                            0357  21 9000 0415
              RAU R0038 A                          0415  60 3647 0601
              FMP J0036 B                          0601  39 4716 0366
              FSB  9000                            0366  33 9000 0246
              STU M0033 C                          0246  21 6868 0321
              RAU E0038 A                          0321  60 2835 0389
              FDV M0033 C                          0389  34 6868 0068
              STU M0073 C                          0068  21 6908 0411
              NZA         GO 33                    0411  40 0214 0465
              AXA  0001                            0214  50 0001 0270
              AXC  0001   GO 32                    0270  58 0001 0551
    1
    1            SET-UP MATRIX FOR NEXT GROUP
    1
     GO 33    NZB         GO 35                    0465  42 0668 0670
              AXB  0001                            0668  52 0001 0624
              AXC  0077   GO 34                    0624  58 0077 0363
    1
    1            STORE ORIGINAL KEFF AND POWER
    1
     GO 35    RAU  9001                            0670  60 9001 0527
              STU  9011              ORIG KEFF     0527  21 9011 0288
              STU  9010              ORIG POWER    0288  21 9010 0376
    1
    1            COEFFS FOR COMPUTING POWER
    1
              RAU  9014                            0376  60 9014 0338
              FDV  9008                            0338  34 9008 0442
              STU Y1                               0442  21 0028 0531
              RAU CT39                             0531  60 0388 0426
              FDV CT12                             0426  34 0579 0629
              STU Y2                               0629  21 0046 0476
              RAU CT40                             0476  60 0679 0438
              FMP  9014                            0438  39 9014 0492
              FAD  9015                            0492  32 9015 0526
              STU  9000                            0526  21 9000 0488
              FDV CT96                             0488  34 0542 0592
              STU Y3                               0592  21 0066 0576
              RAU CT38                             0576  60 0280 0538
              FMP  9015                            0538  39 9015 0642
              FAD  9000                            0642  32 9000 0626
              STU  9000                            0626  21 9000 0588
              FDV CT96                             0588  34 0542 0676
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                STU  Y4                                   0676  21 0064 0577
                RAU  9015                                 0577  60 9015 0638
                FAD  9000                                 0638  32 9000 0627
                FAD  9016                                 0627  32 9016 0677
                STU  9000                                 0677  21 9000 0589
                FDV  CT96                                 0589  34 0542 0528
                STU  Y5                                   0528  21 0102 0578
                RAU  CT38                                 0578  60 0280 0639
                FMP  9016                                 0639  39 9016 0628
                FAD  9000                                 0628  32 9000 0678
                FDV  CT96                                 0678  34 0542 0330
                STU  Y6                                   0330  21 0200 0380
                RSA  0032                                 0380  81 0032 0290
                RAU  9001      MBETA                      0290  60 9001 0430
        MBETA   STD  9251                                 0430  24 9251 0480
                NZA            MKCHK                      0480  40 0340 0390
                AXA  0001      MBETA                      0340  50 0001 0430
        MKCHK   RAU  MIN 1                                0390  60 0466 0530
                STU  KCHEK     ZEROB                      0530  21 9018 0572
    1
    1              MOVE BETAS   ZERO BETA SUMS
    1
        MOVEB   SET  9019                                 0448  27 9019 0610
                LDB  B0001     ZEROB                      0610  09 1767 0572
        ZEROB   RSA  0032                                 0572  81 0032 0478
                RAU  H0001     RESET                      0478  60 0717 0622
        RESET   STU  B0033 A                              0622  21 3799 0660
                NZA            POWER                      0660  40 1868 0150
                AXA  0001      RESET                      1868  50 0001 0622
    1
    1              POWER COMPUTATION
    1
        POWER   SXA  0008                  REGION I       0150  51 0008 0156
                RAU  9029                                 0156  60 9029 0063
                FMP  9008                                 0063  39 9008 0016
                FSB  9027                                 0016  33 9027 0295
                STU  9000                                 0295  21 9000 0253
                RAU  9028                                 0253  60 9028 0011
    1
                FMP  9005                                 0011  39 9005 0014
                FAD  9000                                 0014  32 9000 0093
                FMP  Y2                                   0093  39 0046 0096
                FAD  9019                                 0096  32 9019 0075
                FMP  Y1                                   0075  39 0028 0078
                STU  9000      GO 38                      0078  21 9000 0135
        GO 38   RAU  R0010 A                              0135  60 3619 0023
                FMP  9228                                 0023  39 9228 0026
                FAD  9000                                 0026  32 9000 0105
                NZA            GO 39                      0105  40 0008 0109
                STU  9000                                 0008  21 9000 0115
                AXA  0001      GO 38                      0115  50 0001 0135
        GO 39   FMP  9014                                 0109  39 9014 0012
                STU  9000                                 0012  21 9000 0069
```

414 1									
415		SXA	0008		REGION II	0069	51	0008	0125
416		RAU	9030			0125	60	9030	0133
417		FMP	9008			0133	39	9008	0086
418		FSB	9032			0086	33	9032	0165
419		STU	9052			0165	21	9052	0073
420		RAU	9031			0073	60	9031	0031
421		FMP	9005			0031	39	9005	0084
422		FAD	9052			0084	32	9052	0113
423		FMP	Y3			0113	39	0066	0116
424		STU	9052			0116	21	9052	0123
425		RAU	9040			0123	60	9040	0081
426		FMP	9008			0081	39	9008	0134
427		FSB	9038			0134	33	9038	0163
428		STU	9053			0163	21	9053	0021
429		RAU	9039			0021	60	9039	0029
430		FMP	9005			0029	39	9005	0082
431		FAD	9053			0082	32	9053	0061
432		FMP	Y4			0061	39	0064	0114
433		FAD	9052			0114	32	9052	0143
434		STU	9052	GO 40		0143	21	9052	0051
435	GO 40	RAU	R0020 A			0051	60	3629	0183
436		FMP	9239			0183	39	9239	0087
437		FAD	9052			0087	32	9052	0017
438		NZA		GO 41		0017	40	0070	0071
439		STU	9052			0070	21	9052	0027
440		AXA	0001	GO 40		0027	50	0001	0051
441	GO 41	FMP	9015			0071	39	9015	0024
442		FAD	9000			0024	32	9000	0303
443		STU	9000			0303	21	9000	0111
444 1									
445		RAU	9041		REGION III	0111	60	9041	0119
446		FMP	9008			0119	39	9008	0022
447		FSB	9043			0022	33	9043	0101
448		STU	9052			0101	21	9052	0159
449		RAU	9042			0159	60	9042	0067
450		FMP	9005			0067	39	9005	0120
451		FAD	9052			0120	32	9052	0099
452		FMP	Y5			0099	39	0102	0152
453		STU	9052			0152	21	9052	0209
454		RAU	9051			0209	60	9051	0117
455		FMP	9008			0117	39	9008	0170
456		FSB	9049			0170	33	9049	0149
457		STU	9053			0149	21	9053	0007
458		RAU	9050			0007	60	9050	0215
459		FMP	9005			0215	39	9005	0018
460		FAD	9053			0018	32	9053	0097
461		FMP	Y6			0097	39	0200	0250
462		FAD	9052			0250	32	9052	0079
463		STU	9052			0079	21	9052	0137
464		SXA	0008	GO 42		0137	51	0008	0193
465	GO 42	RAU	R0030 A			0193	60	3639	0243
466		FMP	9250			0243	39	9250	0298

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                FAD  9052                                 0298   32 9052 0077
                NZA             GO 43                     0077   40 0080 0131
                STU  9052                                 0080   21 9052 0187
                AXA  0001       GO 42                     0187   50 0001 0193
        GO 43   FMP  9016                                 0131   39 9016 0636
                FAD  9000                   NEW POWER     0636   32 9000 0282
                STU  9000                                 0282   21 9000 0672
    1
    1              NEW KEFF AND DEL KEFF PLUS TST
    1
                FDV  9010                                 0672   34 9010 0575
                STU  9012                   NEW KEFF      0575   21 9012 0188
                FSB  9011                                 0188   33 9011 0674
                STU  9013                   DELTA KEFF    0674   21 9013 0481
                RAU KCHEK                                 0481   60 9018 0392
                BMI             R PCH                     0392   46 0611 0661
                RAU DEL K                                 0611   60 0625 0529
                FSM  9013                                 0529   38 9013 0512
                STU KCHEK       K PCH                     0512   21 9018 0240
    1
    1              PUNCH KEFF AND DEL KEFF
    1
        K PCH   RAL PCKEF                                 0240   65 0662 0326
                LDD MKEFF       PUNCH                     0326   69 0562 1930
    1
    1              PUNCH R  KEFF AND DEL KEFF
    1
        R PCH   RAL PCR                     PCH RADII     0661   65 0581 0590
                LDD             PUNCH                     0590   69 0631 1930
                RAL PCKEF                   PUNCH KEFF    0631   65 0662 0232
                LDD             PUNCH        AND DEL K    0232   69 0640 1930
                HLT  9999                                 0640   01 9999 9999
    1
    1              END OF CALCULATION
    1
    1              MOVE KEFF AND NORMALIZE POWER
    1
        MKEFF   RAU  9012                                 0562   60 9012 0675
                STU  9011                                 0675   21 9011 0238
                RSA  0032       NORMB                     0238   81 0032 0382
        NORMB   RAU  9251                                 0382   60 9251 0432
                FDV  9012                                 0432   34 9012 0482
                STU  9251                                 0482   21 9251 0532
                NZA             NORMP                     0532   40 0582 0632
                AXA  0001       NORMB                     0582   50 0001 0382
        NORMP   RAU  9000                                 0632   60 9000 0612
                FDV  9012                                 0612   34 9012 1870
                STU  9010       COMPW                     1870   21 9010 0651
    1
    1              RIGHT MATRIX POINT 1
    1
        COMPW   RSC  0000                   MATRIX        0651   89 0000 0407
                RSB  0008       GO 78       GROUP NO      0407   83 0008 0463
```

```
   GO 78    RSA  0008              NET POINT    0463  81 0008 0469
            RAU G0009 B                         0469  60 5848 0553
            NZU            GO 44                0553  44 0457 0108
            FDV  9002                           0457  34 9002 0160
            FMP  9019      GO 44                0160  39 9019 0108
   GO 44    STU  9000                           0108  21 9000 0515
            RAU H0009 B                         0515  60 4725 0329
            NZU            GO 45                0329  44 0483 0334
            FDV  9002                           0483  34 9002 0186
            FMP F0001      GO 45                0186  39 1729 0334
   GO 45    FAD  9000                           0334  32 9000 0513
            FDV M0001 C                         0513  34 6836 0236
            STU F0001      GO 48                0236  21 1729 0206
   GO 48    FMP E0009 A                         0206  39 2806 0406
            STU  9054                           0406  21 9054 0563
            RAU G0009 B                         0563  60 5848 0603
            NZU            GO 46                0603  44 0507 0300
            FMP  9228      GO 46                0507  39 9228 0300
1
1               RIGHT MATRIX POINTS 2-10
1
   GO 46    STU  9000                           0300  21 9000 0057
            RAU H0009 B                         0057  60 4725 0129
            NZU            GO 47                0129  44 0233 0184
            FMP F0010 A GO 47                   0233  39 3738 0184
   GO 47    FAD  9000                           0184  32 9000 0213
            FMP R0010 A                         0213  39 3619 0169
            FSB  9054                           0169  33 9054 0199
            FDV M0002 C                         0199  34 6837 0237
            STU F0010 A                         0237  21 3738 0391
            NZA            GO 49                0391  40 0294 0345
            AXC  0001                           0294  58 0001 0350
            AXA  0001      GO 48                0350  50 0001 0206
1
1               RIGHT MATRIX POINT 11
1
   GO 49    SXC  0008                           0345  59 0008 0302
            FMP M0081 C                         0302  39 6916 0416
            FAD F0009                           0416  32 1737 0613
            FMP MIN 1                           0613  39 0466 0516
            FDV M0011 C                         0516  34 6846 0296
            STU F0011                           0296  21 1739 0142
            SXA  0008      GO 52                0142  51 0008 0400
1
1               RIGHT MATRIX POINTS 12-20
1
   GO 52    FMP E0019 A                         0400  39 2816 0166
            STU  9054                           0166  21 9054 0173
            RAU G0018 B                         0173  60 5857 0161
            NZU            GO 50                0161  44 0265 0216
            FMP  9239      GO 50                0265  39 9239 0216
   GO 50    STU  9000                           0216  21 9000 0223
            RAU H0018 B                         0223  60 4734 0089
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                NZU          GO 51      0089  44 0293 0344
                FMP F0020 A  GO 51      0293  39 3748 0344
      GO 51     FAD  9000               0344  32 9000 0273
                FMP R0020 A             0273  39 3629 0179
                FSB  9054               0179  33 9054 0259
                FDV M0012 C             0259  34 6847 0147
                STU F0020 A             0147  21 3748 0151
                NZA          GO 53      0151  40 0004 0155
                AXC  0001               0004  58 0001 0060
                AXA  0001    GO 52      0060  50 0001 0400
    1
    1             RIGHT MATRIX POINT 21
    1
      GO 53     SXC  0008               0155  59 0008 0461
                FMP M0079 C             0461  39 6914 0264
                FAD F0019               0264  32 1747 0473
                FMP MIN 1               0473  39 0466 0566
                FDV M0021 C             0566  34 6856 0456
                STU F0021               0456  21 1749 0352
                SXA  0008    GO 54      0352  51 0008 0450
    1
    1             RIGHT MATRIX POINTS 22-30
    1
      GO 54     FMP E0029 A             0450  39 2826 0076
                STU  9054               0076  21 9054 0283
                RAU G0027 B             0283  60 5866 0121
                NZU          GO 55      0121  44 0175 0126
                FMP  9250    GO 55      0175  39 9250 0126
      GO 55     STU  9000               0126  21 9000 0333
                RAU H0027 B             0333  60 4743 0197
                NZU          GO 56      0197  44 0201 0202
                FMP F0030 A  GO 56      0201  39 3758 0202
      GO 56     FAD  9000               0202  32 9000 0181
                FMP R0030 A             0181  39 3639 0139
                FSB  9054               0139  33 9054 0219
                FDV M0022 C             0219  34 6857 0107
                STU F0030 A             0107  21 3758 0211
                NZA          GO 57      0211  40 0164 0315
                AXC  0001               0164  58 0001 0220
                AXA  0001    GO 54      0220  50 0001 0450
    1
    1             RIGHT MATRIX POINT 31
    1
      GO 57     SXC  0008               0315  59 0008 0371
                FMP M0080 C             0371  39 6915 0565
                FAD F0029               0565  32 1757 0533
                FMP MIN 1               0533  39 0466 0616
                FDV M0031 C             0616  34 6866 0666
                STU F0031               0666  21 1759 0262
                SXA  0006    GO 58      0262  51 0006 0650
    1
    1             RIGHT MATRIX POINTS 32-38
    1
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    GO 58   FMP E0037 A           0650  39 2834 0234
            STU  9054             0234  21 9054 0441
            RAU H0036 B           0441  60 4752 0207
            NZU           GO 59   0207  44 0261 0162
            FMP F0038 A           0261  39 3766 0266
            FMP R0038 A GO 59     0266  39 3647 0162
    GO 59   FSB  9054             0162  33 9054 0491
            FDV M0032 C           0491  34 6867 0167
            STU F0038 A           0167  21 3766 0269
            NZA           GO 60   0269  40 0072 0323
            AXC  0001             0072  58 0001 0128
            AXA  0001     GO 58   0128  50 0001 0650
1
1              FLUX CALC POINTS 37-32
1
    GO 60   AXA  0005             0323  50 0005 0229
            SXC  0006     GO 61   0229  59 0006 0235
    GO 61   FMP M0077 C           0235  39 6912 0212
            STU  9000             0212  21 9000 0319
            RAU F0032 A           0319  60 3760 0365
            FSB  9000             0365  33 9000 0545
            STU F0032 A           0545  21 3760 0313
            NZA           GO 62   0313  40 0316 0217
            SXC  0001             0316  59 0001 0122
            SXA  0001     GO 61   0122  51 0001 0235
1
1              FLUX CALC POINT 31
1
    GO 62   AXC  0005             0217  58 0005 0523
            RAU F0033             0523  60 1761 0615
            FMP M0040 C           0615  39 6875 0275
            STU  9000             0275  21 9000 0583
            RAU F0031             0583  60 1759 0663
            FSB  9000             0663  33 9000 0443
            STU F0031             0443  21 1759 0312
            RAU F0032             0312  60 1760 0665
            FMP M0071 C           0665  39 6906 0506
            STU  9000             0506  21 9000 0314
            RAU F0031             0314  60 1759 0364
            FSB  9000             0364  33 9000 0493
            STU F0031             0493  21 1759 0362
            AXA  0008     GO 63   0362  50 0008 0550
1
1              FLUX CALC POINTS 30-22
1
    GO 63   FMP M0070 C           0550  39 6905 0205
            STU  9000             0205  21 9000 0263
            RAU F0022 A           0263  60 3750 0255
            FSB  9000             0255  33 9000 0185
            STU F0022 A           0185  21 3750 0353
            NZA           GO 64   0353  40 0256 0157
            SXC  0001             0256  59 0001 0062
            SXA  0001     GO 63   0062  51 0001 0550
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CY-5 back

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 1
 1            FLUX CALC POINT 21
 1
   GO 64   AXC  0008                       0157  58 0008 0414
           RAU M0039 C                     0414  60 6874 0379
           FMP F0023                       0379  39 1751 0402
           STU  9000                       0402  21 9000 0309
           RAU F0021                       0309  60 1749 0653
           FSB  9000                       0653  33 9000 0633
           STU F0021                       0633  21 1749 0452
           RAU F0022                       0452  60 1750 0455
           FMP M0061 C                     0455  39 6896 0346
           STU  9000                       0346  21 9000 0104
           RAU F0021                       0104  60 1749 0154
           FSB  9000                       0154  33 9000 0384
           STU F0021                       0384  21 1749 0502
           AXA  0008   GO 65               0502  50 0008 0600
 1
 1            FLUX CALC POINTS 20-12
 1
   GO 65   FMP M0060 C                     0600  39 6895 0395
           STU  9000                       0395  21 9000 0403
           RAU F0012 A                     0403  60 3740 0445
           FSB  9000                       0445  33 9000 0225
           STU F0012 A                     0225  21 3740 0343
           NZA         GO 66               0343  40 0146 0247
           SXC  0001                       0146  59 0001 0252
           SXA  0001   GO 65               0252  51 0001 0600
 1
 1            FLUX CALC POINT 11
 1
   GO 66   AXC  0008                       0247  58 0008 0204
           RAU M0082 C                     0204  60 6917 0421
           FMP F0013                       0421  39 1741 0591
           STU  9000                       0591  21 9000 0299
           RAU F0011                       0299  60 1739 0543
           FSB  9000                       0543  33 9000 0573
           STU F0011                       0573  21 1739 0192
           RAU F0012                       0192  60 1740 0396
           FMP M0051 C                     0396  39 6886 0286
           STU  9000                       0286  21 9000 0593
           RAU F0011                       0593  60 1739 0643
           FSB  9000                       0643  33 9000 0623
           STU F0011                       0623  21 1739 0242
           AXA  0009   GO 67               0242  50 0009 0100
 1
 1            FLUX CALC POINTS 10-1
 1
   GO 67   FMP M0050 C                     0100  39 6885 0035
           STU  9000                       0035  21 9000 0043
           RAU F0001 A                     0043  60 3729 0083
           FSB  9000                       0083  33 9000 0013
           STU F0001 A                     0013  21 3729 0032
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          NZA            GO 68                   0032  40 0085 0036
          SXC  0001                              0085  59 0001 0291
          SXA  0001      GO 67                   0291  51 0001 0100
   GO 68  AXC  0009                              0036  58 0009 0042
          RAU KCHEK                              0042  60 9018 0341
          BMI GO 88                              0341  46 0244 0245
          RAL CODE                               0245  65 0198 0203
          LDD GO 88      PUNCH                   0203  69 0244 1930
   GO 88  NZC GO 80      GO 79                   0244  48 0050 0248
1
1            STORE + PUNCH NORMAL FLUX
1
   GO 79  RAU KCHEK                              0248  60 9018 0630
          BMI GO 80                              0630  46 0050 0580
          RAU F0001                              0580  60 1729 0440
          STU NORMF                              0440  21 0074 0490
          RAL PNORM                              0490  65 0680 0540
          LDD GO 80      PUNCH                   0540  69 0050 1930
1
1                 NEUTRON REGENERATION
1
   GO 80  SXA  0010      GO 69     REGION I      0050  51 0010 0006
   GO 69  RAU V0009 B                            0006  60 5692 0047
          NZU            GO 71                   0047  44 0001 0002
          FMP F0011 A                            0001  39 3739 0039
          FAD B0011 A                            0039  32 3777 0003
          STU B0011 A                            0003  21 3777 0030
          NZA            GO 81                   0030  40 0033 0034
          AXA  0001      GO 69                   0033  50 0001 0006
   GO 81  RAU KCHEK                              0034  60 9018 0041
          BMI GO 82                              0041  46 0044 0045
          RAL PCB1                               0045  65 0048 0053
          LDD GO 82      PUNCH                   0053  69 0044 1930
1
   GO 82  SXA  0010      GO 71     REGION II     0044  51 0010 0002
   GO 71  RAU V0018 B                            0002  60 5701 0005
          NZU            GO 73                   0005  44 0009 0010
          FMP F0021 A                            0009  39 3749 0049
          FAD B0022 A                            0049  32 3788 0015
          STU B0022 A                            0015  21 3788 0091
          NZA            GO 83                   0091  40 0094 0095
          AXA  0001      GO 71                   0094  50 0001 0002
   GO 83  RAU KCHEK                              0095  60 9018 0141
          BMI GO 84                              0141  46 0144 0145
          RAL PCB2                               0145  65 0098 0103
          LDD GO 84      PUNCH                   0103  69 0144 1930
1
   GO 84  SXA  0010      GO 73     REGION III    0144  51 0010 0010
   GO 73  RAU V0027 B                            0010  60 5710 0065
          NZU            GO 93                   0065  44 0019 0020
          FMP F0031 A                            0019  39 3759 0059
          FAD B0033 A                            0059  32 3799 0025
          STU B0033 A                            0025  21 3799 0052
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                NZA            GO 85                                0052  40 0055 0056
                AXA   0001     GO 73                                0055  50 0001 0010
      GO 85     RAU KCHEK                                           0056  60 9018 0191
                BMI GO 86                                           0191  46 0194 0195
                RAL PCB3                                            0195  65 0148 0153
                LDD GO 75      PUNCH                                0153  69 0106 1930
      GO 93     RAU KCHEK                                           0020  60 9018 0241
                BMI GO 86      GO 75                                0241  46 0194 0106
    1
    1               NORMALIZE FLUX
    1
      GO 75     RAL CODE                                            0106  65 0198 0453
                LDD            PUNCH                                0453  69 0306 1930
                RSA   0037     GO 76                                0306  81 0037 0112
      GO 76     RAU F0038 A                                         0112  60 3766 0171
                FDV NORMF                                           0171  34 0074 0124
                STU N0038 A                                         0124  21 3839 0092
                NZA            GO 77                                0092  40 0495 0196
                AXA   0001     GO 76                                0495  50 0001 0112
      GO 77     RAL PCNF                                            0196  65 0249 0503
                LDD GO 86      PUNCH                                0503  69 0194 1930
    1
    1               GO TO NEXT GROUP OR CALC POWER
    1
      GO 86     NZB            MOVEB                                0194  42 0397 0448
                AXB   0001                                          0397  52 0001 0254
                AXC   0082     GO 78                                0254  58 0082 0463
    1
    1                          CONSTANTS
    1
        9001     10   0000       0051        ENERGY                 9001  10 0000 0051
        9002     20   0000       0051        GROUP CODE             9002  20 0000 0051
        9003     30   0000       0051                               9003  30 0000 0051
        9004     40   0000       0051                               9004  40 0000 0051
        9005     50   0000       0051                               9005  50 0000 0051
        9006     60   0000       0051                               9006  60 0000 0051
        9007     70   0000       0051                               9007  70 0000 0051
        9008     80   0000       0051                               9008  80 0000 0051
        9009     90   0000       0051                               9009  90 0000 0051
    1
      MIN 1 -    10   0000       0051        OTHER                  0466 -10 0000 0051
      CT12       12   0000       0052        CONSTANTS              0579  12 0000 0052
      CT38       38   0000       0052                               0280  38 0000 0052
      CT39       39   0000       0052                               0388  39 0000 0052
      CT40       40   0000       0052                               0679  40 0000 0052
      CT96       96   0000       0052                               0542  96 0000 0052
    1
      Q0001      00   000         000        SIGMA Q                1574  00 0000 0000
      Q0010      00   000         000         FOR FIRST             1583  00 0000 0000
      Q0019      00   000         000         GROUP                 1592  00 0000 0000
      Q0028      00   000         000                               1601  00 0000 0000
    1
      CODE       06   9409       0001        PUNCH                  0198  06 9409 0001
```

838	PCB1	06	B0001	0011	CONSTANTS	0048	06	1767	0011
839	PCB2	06	B0012	0011		0098	06	1778	0011
840	PCB3	06	B0023	0011		0148	06	1789	0011
841	PCNF	06	N0001	0038		0249	06	1802	0038
842	PCR	06	R0001	0038		0581	06	1610	0038
843	PCKEF	06	9012	0002		0662	06	9012	0002
844	PNORM	06	NORMF	0001		0680	06	0074	0001
845 1									
846	C0082 -	40	0000	0051	D1 D5 D9	1801	-40	0000	0051
847	E0038	00	000	000	E38	0835	00	0000	0000
848 1									
849	DEL K	10	0000	0047	DEL K TEST	0625	10	0000	0047

REFERENCES

1. Glasstone, Samuel, and Edlund, Milton C.: The Elements of Nuclear Reactor Theory. D. Van Nostrand Co., Inc., 1952, pp. 227-229.

2. Anon.: 650 Magnetic Drum Data-Processing Machine - Manual of Operation. Form 22-6060-1, international Business Machines Corp., June 1955.

3. Anon.: IBM Data Processing - 650 Magnetic Drum Data-Processing Machine - Manual of Additional Features: Magnetic Tapes, High-Speed Storage, Printer. Form 22-6265-1, International Business Machines Corp., 1955.

4. Anon.: IBM - 650 MDDPM Additional Features - Indexing Accumulators, Floating-Decimal Arithmetic. Form 22-6258-0, International Business Machines Corp., 1955.

5. Poley, Stan.: SOAP II - Programmers' Reference Manual. Service Bureau Corp., International Business Machines Corp., 1957.

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TABLE I. - STORAGE LOCATIONS OF INPUT

Storage region	Storage location	Input value
A	0753 to 0761	$\alpha_a, \alpha_b, \cdots \alpha_i$
D	0762 to 0770	$D_{I,a}, D_{I,b}, \cdots D_{I,i}$
	0771 to 0779	$D_{II,a}, D_{II,b}, \cdots D_{II,i}$
	0780 to 0788	$D_{III,a}, D_{III,b}, \cdots D_{III,i}$
	0789 to 0797	$D_{IV,a}, D_{IV,b}, \cdots D_{IV,i}$
Q	1575 to 1582	$\Sigma_{q,I,a}, \Sigma_{q,I,b}, \cdots \Sigma_{q,I,h}$
	1584 to 1591	$\Sigma_{q,II,a}, \Sigma_{q,II,b}, \cdots \Sigma_{q,II,h}$
	1593 to 1600	$\Sigma_{q,III,a}, \Sigma_{q,III,b}, \cdots \Sigma_{q,III,h}$
	1602 to 1609	$\Sigma_{q,IV,a}, \Sigma_{q,IV,b}, \cdots \Sigma_{q,IV,h}$
S	1648 to 1656	$\Sigma_{I,a}, \Sigma_{I,b}, \cdots \Sigma_{I,i}$
	1657 to 1665	$\Sigma_{II,a}, \Sigma_{II,b}, \cdots \Sigma_{II,i}$
	1666 to 1674	$\Sigma_{III,a}, \Sigma_{III,b}, \cdots \Sigma_{III,i}$
	1675 to 1683	$\Sigma_{IV,a}, \Sigma_{IV,b}, \cdots \Sigma_{IV,i}$
V	1684 to 1692	$\nu\Sigma_{f,I,a}, \nu\Sigma_{f,I,b}, \cdots \nu\Sigma_{f,I,i}$
	1693 to 1701	$\nu\Sigma_{f,II,a}, \nu\Sigma_{f,II,b}, \cdots \nu\Sigma_{f,II,i}$
	1702 to 1710	$\nu\Sigma_{f,III,a}, \nu\Sigma_{f,III,b}, \cdots \nu\Sigma_{f,III,i}$
None	9014 to 9017	$h_I, h_{II}, h_{III}, h_{IV}$

TABLE II. - ORDER OF PUNCHED OUTPUT

Program instruction	Output value	Location punched out	Remarks
489,490	$k_{eff}, \Delta k_{eff}$	9012,9013	(Punched for every iteration)
739,740	Group code number	(9009+IAB)*	
748,749	Normal flux, $\varphi_{1,a}$	0680	
763,764	$\sum_{n=a}^{i} \beta_{I,n,k}$	1767 to 1777	For points 1 to 11, when $\nu\Sigma_{f,I,n} \neq 0$
776,777	$\sum_{n=a}^{i} \beta_{II,n,k}$	1778 to 1788	For points 11 to 21, when $\nu\Sigma_{f,II,n} \neq 0$
789,790	$\sum_{n=a}^{i} \beta_{III,n,k}$	1789 to 1799	For points 21 to 31, when $\nu\Sigma_{f,III,n} \neq 0$
796,797	Group code number	(9009+IAB)*	
804,805	$\frac{\varphi_{1,n}}{\varphi_{1,a}}, \ldots \frac{\varphi_{38,n}}{\varphi_{1,a}}$	1802 to 1839	
494,495	$r_1, \ldots r_{38}$	1610 to 1647	
496,497	$k_{eff}, \Delta k_{eff}$	9012,9013	Last values punched out

*When the first neutron energy group is being considered in the program, the value of IAB equals -0008; this results in the punching of the contents of location 9001, which equals 10 0000 0051. Here, the last two digits indicate the position of the decimal to be 1.0 (ref. 4).

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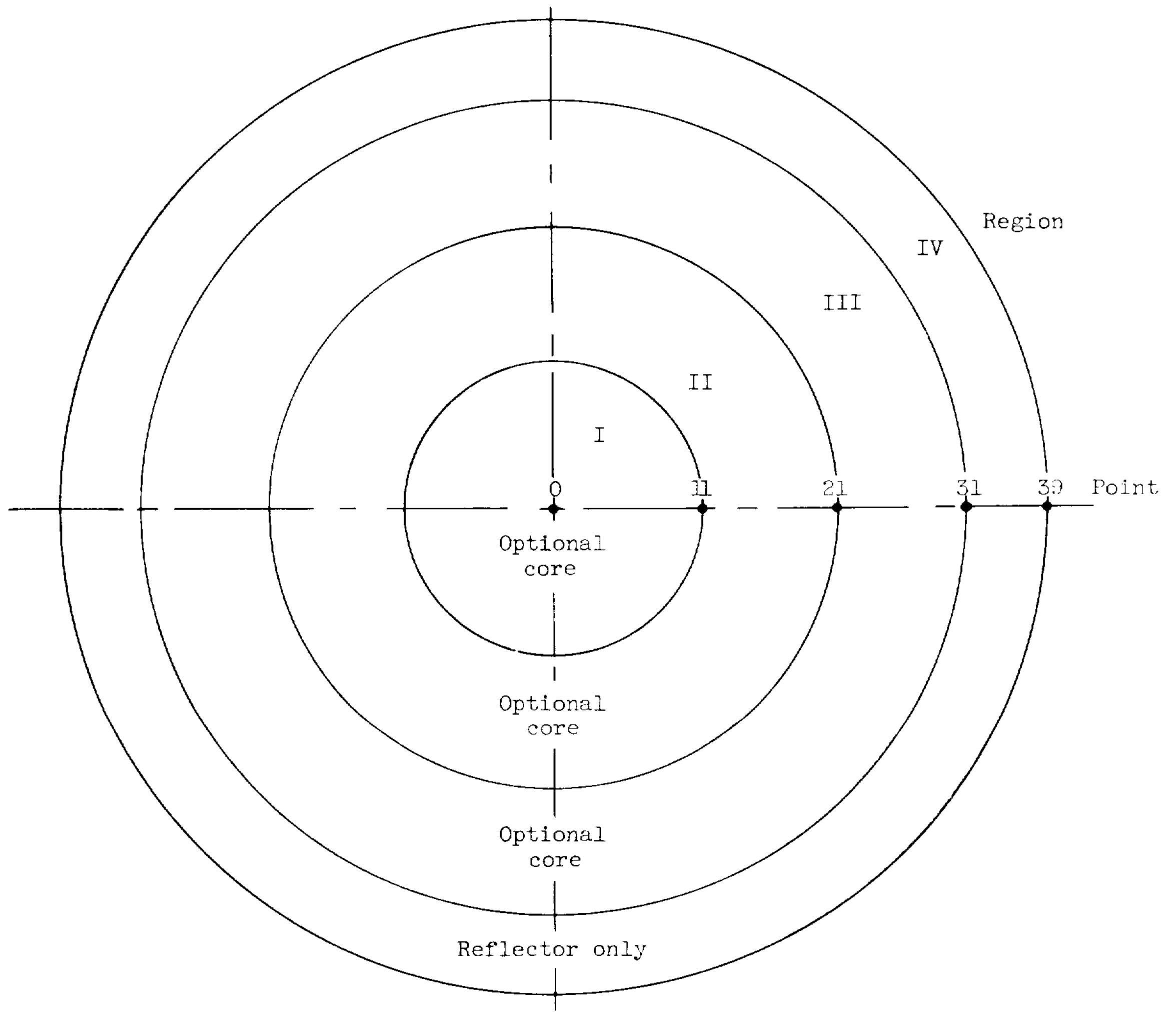
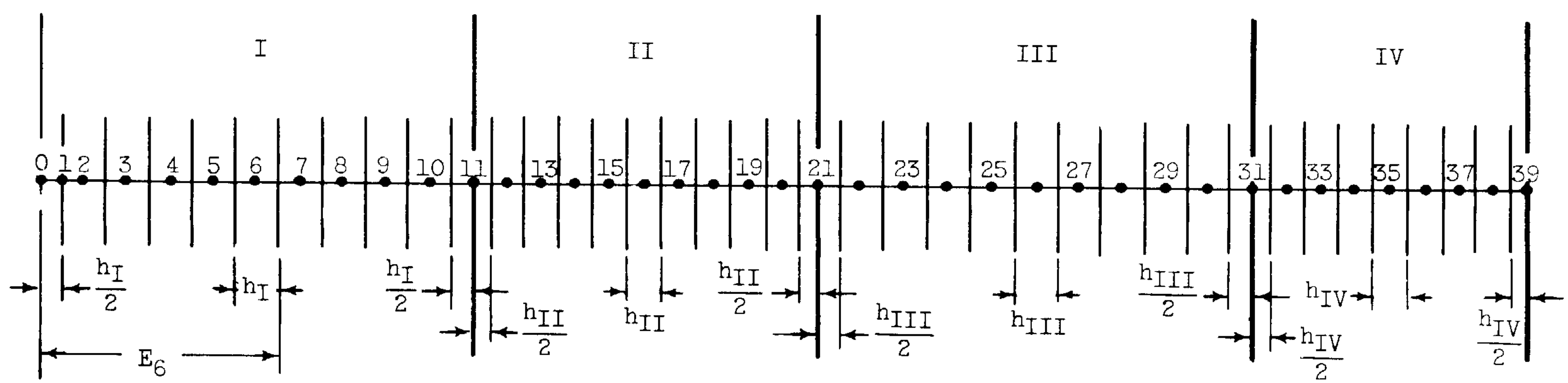


(a) End view.

Figure 1. - Nuclear reactor composed of four concentric cylinders.



(b) Longitudinal cross-sectional view.

Figure 1. - Concluded. Nuclear reactor composed of four concentric cylinders.

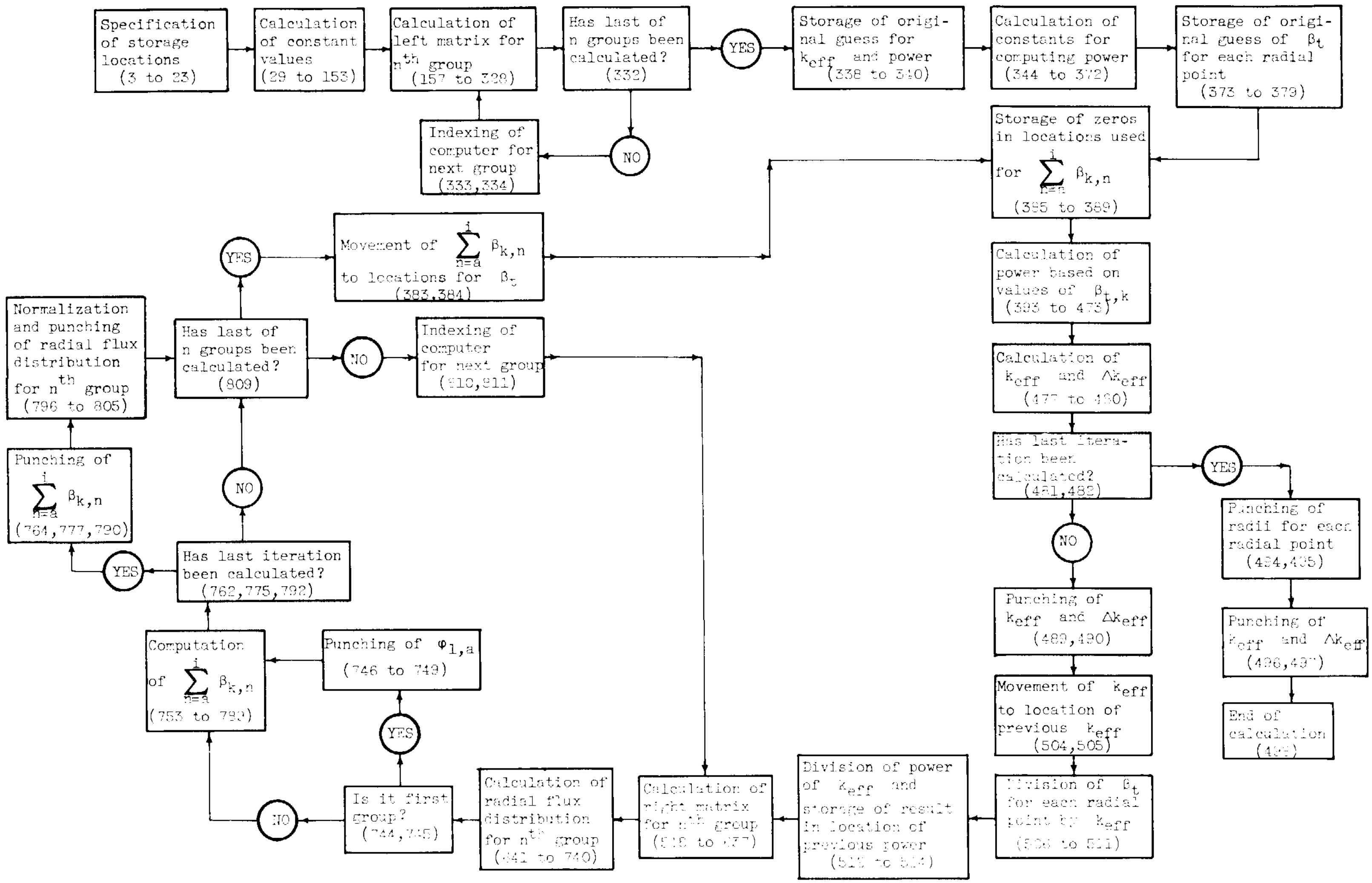


Figure 2. - Outline of machine program. (Numbers in parentheses refer to stop numbers in program.)

NASA MEMO 12-24-58E
National Aeronautics and Space Administration.
COMPUTER PROGRAM FOR SOLVING NINE-GROUP DIFFUSION EQUATIONS FOR CYLINDRICAL REACTORS. James W. Miser, Robert E. Hyland, and Daniel Fieno. January 1959. 43p. diagrs., tabs. (NASA MEMORANDUM 12-24-58E)

A method is presented for determining the critical size of a cylindrical reactor by a one-dimensional group-diffusion method extended to a two-dimensional solution by prescribing values of axial leakage based on assumed flux levels. The neutron energy spectrum is divided into nine groups, and the reactor into four concentric cylinders. A computing machine program for an IBM 650 computer with attachments and a method for using the program with nine, or less, groups and four, or less, regions are given.

Copies obtainable from NASA, Washington

1. Nuclear-Energy Systems (3.1.10)
I. Miser, James W.
II. Hyland, Robert E.
III. Fieno, Daniel
IV. NASA MEMO 12-24-58E

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