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MEMORANDUM

INFLUENCE OF LARGE POSITIVE DIHEDRAL ON HEAT TRANSFER

TO LEADING EDGES OF HIGHLY SWEPT WINGS

AT VERY HIGH MACH NUMBERS

By Morton Cooper and P. Calvin Stainback

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SUMMARY

A geometric study has been made of some of the effects of dihedral on the heat transfer to swept delta wings. The results of this study show that the incorporation of large positive dihedral on highly swept wings can shift, even at moderately low angles of attack, the stagnation-line heat-transfer problem from the leading edges to the axis of symmetry (ridge line). An order-of-magnitude analysis (assuming laminar flow) indicates conditions for which it may be possible to reduce the heating at the ridge line (except in the vicinity of the wing apex) to a small fraction of the leading-edge heat transfer of a flat wing at the same lift. Furthermore, conditions are indicated where dihedral reduces the leading-edge heat transfer for angles of attack less than those required to shift the stagnation line from the leading edge to the ridge line.

INTRODUCTION

An intensive effort is now being directed to develop configurations suitable for long-range hypersonic gliders. For such configurations, the wing leading-edge region presents one of the areas of major heating and, hence, a region for which reductions in heat transfer would yield significant gains. Inasmuch as positive dihedral can have a significant influence on leading-edge heat transfer, it is the purpose of the present paper to discuss this influence from geometric considerations and, furthermore, to discuss the interrelation between heat transfer at the leading edge and at the axis of symmetry (ridge line).

No explicit consideration has been given in this investigation to the effects of positive dihedral on other aerodynamic parameters, but in view of the reductions indicated in leading-edge heat transfer, further studies are in order.

SYMBOLS

The symbols are defined with the aid of figure 1 which presents a schematic picture of a delta wing with dihedral and at an angle of attack. The complete wing with dihedral OABG is shown on the right in figure 1. L
The wing is symmetrical about the line OB which is in the plane of the 1
X and Z axes. The sweepback of the wing is defined as the complement 8
of the semiapex angle. In the present analysis two separate semiapex 2
angles are used: the panel semiapex angle ϵ_o and the plan-form semiapex angle ϵ_p. Also shown with the dihedral wing is a reference plane OA'BG' which passes through OB and is perpendicular to the plane of the X and Z axes. Dihedral is measured from the reference plane in a plane perpendicular to OB. On the left in figure 1, half of the wing with dihedral OAB and a portion of the reference plane OA'B are shown together with some of the angles used in the discussion.

M free-stream Mach number in direction of positive X-axis

V free-stream velocity in direction of positive X-axis

V_N component of free-stream velocity normal to leading edge of wing and located in plane formed by wing leading edge and free-stream velocity

V_P component of free-stream velocity along leading edge of wing

X,Y,Z rectangular coordinate axes

α angle of attack of ridge line OB

α_{ϵ_e} angle of attack at which effective sweeps of leading edge OA and ridge line OB are equal

α' angle of attack of plane AOG of leading edges, angle XOH, plan-form angle of attack

α'_{min} minimum value of angle of attack of plane AOG of the leading edges

Γ dihedral angle

δ angle between plane of velocity vectors EFODAC and plane of wing OAB, turning angle

ϵ_e angle between leading edge OA and free-stream direction (X-axis), effective semiapex angle

ϵ_n angle between ridge line OB and plane of leading edges AOG, angle BOH

ϵ_o angle between leading edge OA and ridge line OB of wing, panel semiapex angle

ϵ_p half of angle between leading edges OA and OG, angle AOH, plan-form semiapex angle

Λ_e complement of ϵ_e, effective sweep

Λ_o complement of ϵ_o, panel sweep

Λ_p complement of ϵ_p, plan-form sweep

Subscripts:

Γ value at dihedral

$\Gamma=0$ value for zero dihedral (flat-wing value)

DISCUSSION

Effective Sweep

In order to develop the geometry for a delta wing with dihedral and at an angle of attack, it is convenient to resolve the free-stream velocity into two components (fig. 1): one parallel to the leading edge (V_P) and one normal to the leading edge (V_N). By analogy with the flat wing at 0° angle of attack, the effective semiapex angle, which is the angle between the leading edge and the free-stream direction, is designated by ϵ_e in the velocity-vector diagram. This effective semiapex angle ϵ_e or its complement, the effective sweep Λ_e, can be computed from the geometry of figure 1. The present analysis treats the effects of dihedral for two cases:

(1) Constant plan-form semiapex angle $(\epsilon_p = \text{Constant})$

(2) Constant panel semiapex angle $(\epsilon_o = \text{Constant})$

The values of ϵ_e or Λ_e are given in forms convenient for either case as

$$\cos \epsilon_e = \sin \Lambda_e = \cos \epsilon_p \cos \alpha' \tag{1a}$$

for ϵ_p = Constant or

$$\cos \epsilon_e = \sin \Lambda_e = \cos \epsilon_o \cos \alpha + \sin \epsilon_o \sin \alpha \sin \Gamma \tag{1b}$$

for ϵ_o = Constant where

$$\alpha' = \alpha - \epsilon_n$$

and

$$\sin \epsilon_p = \sin \epsilon_o \cos \Gamma \tag{2a}$$

$$\tan \epsilon_n = \tan \epsilon_o \sin \Gamma \tag{2b}$$

For the flat wing ($\Gamma = 0^{\circ}$) the effective sweep has been considered previously (ref. 1).

Leading-Edge Heat Transfer

The evaluation of the effects of dihedral on the leading-edge heat transfer of highly swept wings is made in this investigation for a constant wing lift and laminar flow. Initially, the approximation is made that the leading edge can be treated as an isolated swept cylinder and, therefore, the leading-edge heat transfer is proportional to the cosine of the effective sweep. In this study, two methods of introducing dihedral are considered.

In the first method the plan-form semiapex angle is maintained constant as dihedral is introduced. If the angle of attack is referenced to the plane of the leading edges (eq. (1a)) the effective sweep is clearly independent of dihedral and is a function only of the plan-form semiapex angle and plan-form angle of attack (exactly as in the case of the flat delta wing). For a given plan form, however, the lift[1] is, according to Newtonian theory, a function of dihedral at a given angle of attack and is given by

$$\text{Lift} \propto \alpha^2 \cos^2\Gamma \left(\cos \epsilon_n\right)^m \tag{3}$$

The parameter m equals 1 if the plan-form area is maintained constant by passing a plane through AG (fig. 1) perpendicular to the ridge line

[1]The lift and normal force are used interchangeably since this discussion is limited to small angles of attack.

OB, and m equals -1 if the plane passes through AG perpendicular to the plane of the leading edges OAG. Throughout the present paper only the case of $m = 1$ is treated because it parallels the constant-panel-sweep case and because it is conservative in that it predicts a lower lift than the $m = -1$ case. The differences between the results for $m = 1$ and $m = -1$ are slight when the sweep is large or the dihedral is small. For a given lift, the Newtonian pressure and wing loading are independent of dihedral. The panel geometry, that is, the panel semiapex angle ϵ_o, varies with dihedral as specified by equation (2a).

In the second method the panel semiapex angle ϵ_o is maintained constant (independent of dihedral). The effective sweep will increase, equation (1b), with the addition of positive dihedral at a given angle of attack. Dihedral introduced in this fashion results in an increased wing loading for a given lift because of the corresponding decrease in plan-form area for a given length. Since the pressure is uniform over the wing lower surface, according to Newtonian theory, the pressure increases with dihedral by the factor $(\cos \Gamma)^{-1}$ for a given lift at low angles of attack.

The effects of dihedral on the stagnation-line heat transfer at the leading edges of 45° and 75° swept delta wings[2] at a given lift are presented in figure 2. (No curve has been presented for a wing having 45° plan-form sweep and 45° dihedral because for this case the panel size vanishes.) In this figure, the parameter $\dfrac{(\cos \Lambda_e)_\Gamma}{(\cos \Lambda_e)_{\Gamma=0}}$ which, according to the cosine relation, is equal to the heat-transfer ratio with and without dihedral is presented as a function of angle of attack of the flat wing. Lines of constant lift are vertical. Since the lift is maintained constant as the dihedral is increased, the angle of attack of the wing with dihedral, α_Γ (which is measured from the ridge line) is greater than the angle of attack of the flat wing. The lift was estimated from Newtonian theory, and for small angles of attack the relationships between the angles of attack for a given lift with and without dihedral are for ϵ_p constant,

$$\frac{\alpha_\Gamma}{\alpha_{\Gamma=0}} = \frac{\left(\cos \Gamma \cos \epsilon_p\right)^{m/2}}{\cos \Gamma\left(\cos^2\Gamma - \sin^2\epsilon_p\right)^{m/4}} \tag{4a}$$

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[2]The notation 75° swept delta wings means that values of both Λ_o and Λ_p (which are complements of ϵ_o and ϵ_p) are being considered. The same interpretation applies to 45° swept delta wings.

and for ϵ_o constant,

$$\frac{\alpha_\Gamma}{\alpha_{\Gamma=0}} = \frac{1}{\cos^{3/2}\Gamma} \tag{4b}$$

where m equals either 1 or -1 as defined for relation 3.

From figure 2(a) it can be seen from the lower limit of the solid curve that for the 75° panel sweep and 45° dihedral the leading-edge heat transfer of the wing with dihedral is about 0.67 of the flat wing having the same lift. In all cases shown for the constant panel sweep there is a reduction in leading-edge heat transfer due to dihedral. For the case of the constant plan-form sweep of 75° (fig. 2(b)) the wing with 45° dihedral has approximately 0.83 (limit of the solid curve) of the leading-edge heat transfer of the flat wing of the same sweep. In the low angle-of-attack range the leading-edge heat transfer of this wing is higher than that of the flat wing. For the results presented in figure 2, portions of the curves have been dashed to indicate the region in which isolated-swept-cylinder analysis breaks down for the prediction of stagnation-line heat transfer for very high Mach numbers. The dashed sections of the curves probably underestimate the reduction in leading-edge heat transfer as will be discussed more fully subsequently.

Stagnation-Line Location

Up to this point in the discussion it has been tacitly assumed that the stagnation-line location is unaffected by dihedral. It is reasonable to assume that small shifts in location would not affect the stagnation heat-transfer rate significantly but that very large shifts surely would. Hence, in order to establish the effect of dihedral on the stagnation-line location, the angle between the plane of the velocity vectors and the wing panel was determined with the aid of figure 1 as

$$\cos \delta = \frac{\cos \alpha - \cos \epsilon_o \cos \epsilon_e}{\sin \epsilon_o \sin \epsilon_e} \tag{5}$$

This angle δ, designated as the turning angle, indicates the angular shift of the stagnation line from the plane of the wing panel. Values of the turning angle are presented for the 45° and 75° swept delta wings in figures 3 and 4. Again, the results are plotted as a function of angle of attack of the flat wing and lines of constant lift are vertical. Constant-angle-of-attack lines are superposed on these figures. (Some α curves have been omitted for reasons of clarity in figs. 3(b) and 4(b).) In the interpretation of these figures, it should be noted that for the lower values of δ, equal values of δ correspond to approximately the same stagnation-line location. (If the wing leading edges were replaced by small swept cylinders and if the presence of the remainder of the wing panels were neglected, then equal values of δ would correspond exactly to the same stagnation-line location for all values of δ.)

For both the 45° and 75° swept wings (figs. 3 and 4, respectively) the effect of dihedral on turning angle is either small or in the direction of increasing turning angle. Increasing the turning angle with dihedral means that there is a larger shift of the stagnation line towards the under surface for the wing with dihedral than for the flat wing. It is of interest to note that the turning angle may exceed 90° for a wing with dihedral, a fact which can be verified by consideration of a wing with dihedral at 90° angle of attack. If the wing is again replaced by cylinders at the leading edges, then when the turning angle equals 90°, the upper forward half of the leading edge, in the conventional sense, becomes a rear quadrant of a swept cylinder in the aerodynamic sense. Hence, it would have very low heat transfer, perhaps 0.1 of the stagnation value. When the turning angle is greater than 90°, even a portion of the lower forward half of the leading edge becomes a sector of the rear half of the cylinder in the aerodynamic sense.

The presence of the wing panel modifies this discussion of flows with large turning angles. Two sources for this modification are considered. First, when the angle between the normal Mach number component ($M \sin \epsilon_e$) and the wing panel, the angle δ, exceeds the maximum value for attached flow the presence of the wing panel is manifested at the leading edge. Illustrative values of this maximum turning angle as a function of normal Mach number (specific-heat ratio, 1.4) are given in the following table:

$M \sin \epsilon_e$	Maximum value of δ for attached flow, deg
2	23
4	39
6	42
10	44
∞	45

When the values of δ indicated in figures 3 and 4 exceed these maximum values, the presence of the wing panel probably will reduce the stagnation-line velocity gradient and, hence, reduce the heat transfer. The second consideration (which is really related to the first) pertains to the sonic-line location on a circular cylinder. If it is assumed that the sonic line on a cylinder occurs at a radial position of 45°, the values of δ indicated in figures 3 and 4 should be restricted to about 45°. At this value the sonic point occurs at the tangency point of the rounded leading edge and the wing panel. For larger values of the turning angle the influence of the wing panel is manifested at the stagnation line. Hence, for the low normal Mach number condition, the maximum value of δ for attached flow establishes the limiting conditions for which an isolated-cylinder analysis can be used for stagnation-line heat transfer. For the higher Mach numbers either criterion will indicate a limit of about 45°. It should be noted, of course, that real gas effects will increase the maximum angle for shock detachment. (See ref. 1.)

Ridge-Line Heat Transfer

When the presence of the wing plane is considered further, the question arises as to whether the wing leading edges (OA and OG in fig. 1) are stagnation lines which can be treated by swept-cylinder analyses or whether the ridge line (OB in fig. 1) is the effective stagnation line. A complete answer to this problem can be developed only from a solution to the inviscid flow about the entire wing. A plausible criterion, however, can be established by considering whether the effective sweep of the leading edges or of the ridge line is less and by assuming the stagnation line to be located at the edge which has the least sweep. This criterion would be exact if at α_{ϵ_e} the wing were replaced by a portion of a circular cone (X-axis coincident with cone axis) passing through the leading edges and the ridge line and if the Mach number were sufficiently high so that the absence of the upper portion of the cone could be neglected.[3] The actual case of the flat-paneled wing of the present analysis is complicated by the fact that, though the flow is conical, it is not radial and, hence, even at an angle of attack of α_{ϵ_e} cross components of velocity exist. This means that there is a range of angle of attack for certain limited conditions for which both the leading edge and ridge line may be treated as stagnation lines.

When the complement of the effective sweep equals the angle of attack $\left(\alpha = \epsilon_e = \alpha_{\epsilon_e}\right)$, the effective sweep of the leading edges OA and OG and the ridge line OB is the same. (See fig. 1.) The angle of attack at which this occurs is given by

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[3]For low values of dihedral, the semiapex angle of the cone α_{ϵ_e} will exceed the maximum value for attached flow and, hence, it will not be possible to neglect the absence of the upper portion of the cone at any Mach number.

$$\alpha_{\epsilon_e} = \tan^{-1} \frac{1 - \cos \epsilon_o}{\sin \epsilon_o \sin \Gamma} \tag{6}$$

For this condition the free-stream velocity effectively "sees" the leading edges and ridge line at the same time in a fashion very similar to the zero-angle-of-attack flow about a cone of semiapex angle α_{ϵ_e}; the leading edges and ridge line are elements of the cone. For angles of attack α greater than the complement of the effective sweep, the ridge line has less sweep than the leading edges and, hence, sees the flow first. In order to illustrate this in detail consider the 75° swept wings with 45° dihedral. For a panel sweep of 75° the effective sweep of the leading edges and ridge line is the same at 10.5° angle of attack. For a plan-form sweep of 75° this equality occurs at 15.1° angle of attack. For angles of attack greater than these values the ridge line is less swept than the leading edge and, hence, becomes the effective leading edge within the criterion assumed. Of course, as previously mentioned, there is a narrow range of angle of attack beyond α_{ϵ_e} for which both the leading edge and ridge line may be treated as stagnation lines.

It is interesting to speculate that when the angle of attack becomes considerably greater than the complement of the effective sweep, the stagnation-line heat-transfer problem becomes similar to the yawed-cone problem (ref. 2); then, the wing can be treated in the same fashion as a cone at very large angles of attack. Then, except possibly in the region of the apex, an order-of-magnitude estimate of the ratio of the stagnation-line heat transfer on the wing at a reasonably large angle of attack to the stagnation-line heat transfer at 0° angle of attack is, for fixed stream conditions, given by[4]

$$C^{1/2} \left(\frac{\text{Local leading-edge diameter}}{\text{Local span}}\right)^{1/2} \frac{\sin\left(\alpha - \alpha_{\epsilon_e}\right)}{\cos \Lambda_o} \qquad \left(\alpha > 2\alpha_{\epsilon_e}\right) \tag{7}$$

[4]In relation (7) the approximation is made that the cone can be treated as an isolated swept cylinder at an angle of attack equal to the semiapex angle of the cone rather than twice the semiapex angle as suggested in reference 2. This less stringent requirement is imposed because the swept-cylinder analysis is still a good approximation (ref. 2) for this condition and, furthermore, it extends the angle-of-attack range for which the heat transfer can be roughly estimated.

The factor C is a velocity-gradient correction accounting for the difference in shape between the leading-edge profile and the local span profile. If the leading edges were round and the wing flat ($\Gamma = 0^\circ$), C would be the ratio of the stagnation-point velocity gradient on a flat body to the stagnation-point velocity gradient on a round body of the same diameter, a numerical value of about 0.31. Relation (7) should be restricted to dihedral values less than approximately 45° for which the local span is the characteristic dimension in determining the cross-flow velocity gradient. A large reduction in stagnation-line heat transfer, as evidenced by the change in characteristic dimension in expression (7), would occur if the radius at the ridge line were sufficiently large. This reduction in stagnation-line heat transfer would occur at lower angles of attack for the highly swept dihedral wing than for the flat wing. As a point of emphasis it should be noted that the heat-transfer estimate presented as relation (7) has been restricted to an angle of attack at least twice the value of α_{ϵ_e}. For this condition the heat-transfer rate would be expected to decrease with distance from the ridge line.

Illustration

There are several possibilities for incorporating dihedral into hypersonic glide configurations. The straightforward addition of wing dihedral by inclining the wing panels as illustrated in figure 1 might be one means. An alternate procedure might be to contemplate a glide configuration such as the one depicted in figure 5. For volume and structural considerations the top (plane of the leading edges) is enclosed.[5] When the top plane is added to the configuration[6] an additional requirement should be imposed on the configuration. In order to minimize the heating to this top plane and not seriously penalize the lift-drag ratio, it is reasonable that the configuration probably would not fly with this plane as a compression surface. Furthermore, from the previous discussion, it would be desirable to establish less sweep at the ridge line than at the leading edge for all angles of attack of practical interest. These two requirements can be satisfied simultaneously if

$$\alpha \geqq \alpha_{\epsilon_e} \tag{8}$$

where

$$\alpha = \alpha' + \epsilon_n$$

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[5]The lift calculations presented in figure 2 were made for the open-top configuration. They apply to the closed-top configuration only when the top plane is an expansion surface.

[6]It should be noted that similar concepts apply for the case of a highly swept flat wing having a deep fuselage located on the undersurface.

and

$$\alpha' \geqq 0$$

From the heat-transfer estimate (relation (7)), it would be desirable to impose a more stringent requirement, namely,

$$\alpha \geqq 2\alpha_{\epsilon_e} \tag{9}$$

Though this would be the preferable condition, equation (9) imposes exceedingly severe restrictions as will be demonstrated. However, it is reasonably certain that reductions in the leading-edge heat transfer would occur somewhere in the range

$$\alpha_{\epsilon_e} < \alpha < 2\alpha_{\epsilon_e}$$

The minimum values of dihedral required to establish less sweep at the ridge line than at the leading edge (eq. 8) for all angles of attack of the top plane greater than a specified minimum value α'_{min} are presented in figure 6 for values of α'_{min} of 0°, 5°, and 10°. The proper value of α'_{min} is assumed to be dictated by lift-drag ratio or by upper-surface heat-transfer considerations. The minimum values of dihedral required to satisfy the heat-transfer restriction, equation (9), are also presented in figure 6 for α'_{min} of 10°. (The curve ceases to exist beyond an ϵ_p value of 33° because for this condition ϵ_o becomes 90° and the panels cease to exist.) For lower values of α'_{min} the restrictions imposed by equation (9) would require considerably larger dihedrals. If it is assumed that hypersonic configurations will have plan-form sweeps between 60° and 80° (ϵ_p from 10° to 30°), then, based on the $\alpha \geqq \alpha_{\epsilon_e}$ criterion, dihedral values of at least 20° to 30° would be required if α'_{min} is 10°. Higher values of dihedral would be required for lower values of α'_{min}. A maximum value of dihedral of about 45° appears reasonable from crossflow heat-transfer considerations as previously indicated.

The configuration shown in figure 5 ($\Gamma = 45^{\circ}$, $\epsilon_p = 10.5^{\circ}$) is noted in figure 6. This configuration satisfies the low heat-transfer requirement, $\alpha \geqq 2\alpha_{\epsilon_e}$ at $\alpha'_{min} = 10.3^{\circ}$. Hence, at least for $\alpha' \geqq 10.3^{\circ}$, it would be expected to have the low heating rate (except in the vicinity of the apex) associated with the local span as the characteristic dimension. For comparative purposes, a flat wing at 15° angle of attack and having the same plan-form sweep would have, at very high Mach numbers, the same lift but would still have the leading-edge heat-transfer problem associated with the leading-edge diameter as the characteristic dimension.

CONCLUDING REMARKS

A geometric study has been made of some of the effects of dihedral on the heat transfer to swept delta wings. The results of this study show that the incorporation of large positive dihedral on highly swept wings can shift, even at moderately low angles of attack, the stagnation-line heat-transfer problem from the leading edges to the axis of symmetry (ridge line). An order-of-magnitude analysis (assuming laminar flow) indicates conditions for which it may be possible to reduce the heating at the ridge line (except in the vicinity of the wing apex) to a small fraction of the leading-edge heat transfer of a flat wing at the same lift. Furthermore, conditions are indicated where dihedral reduces the leading-edge heat transfer for angles of attack less than those required to shift the stagnation line from the leading edge to the ridge line.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., December 29, 1958.

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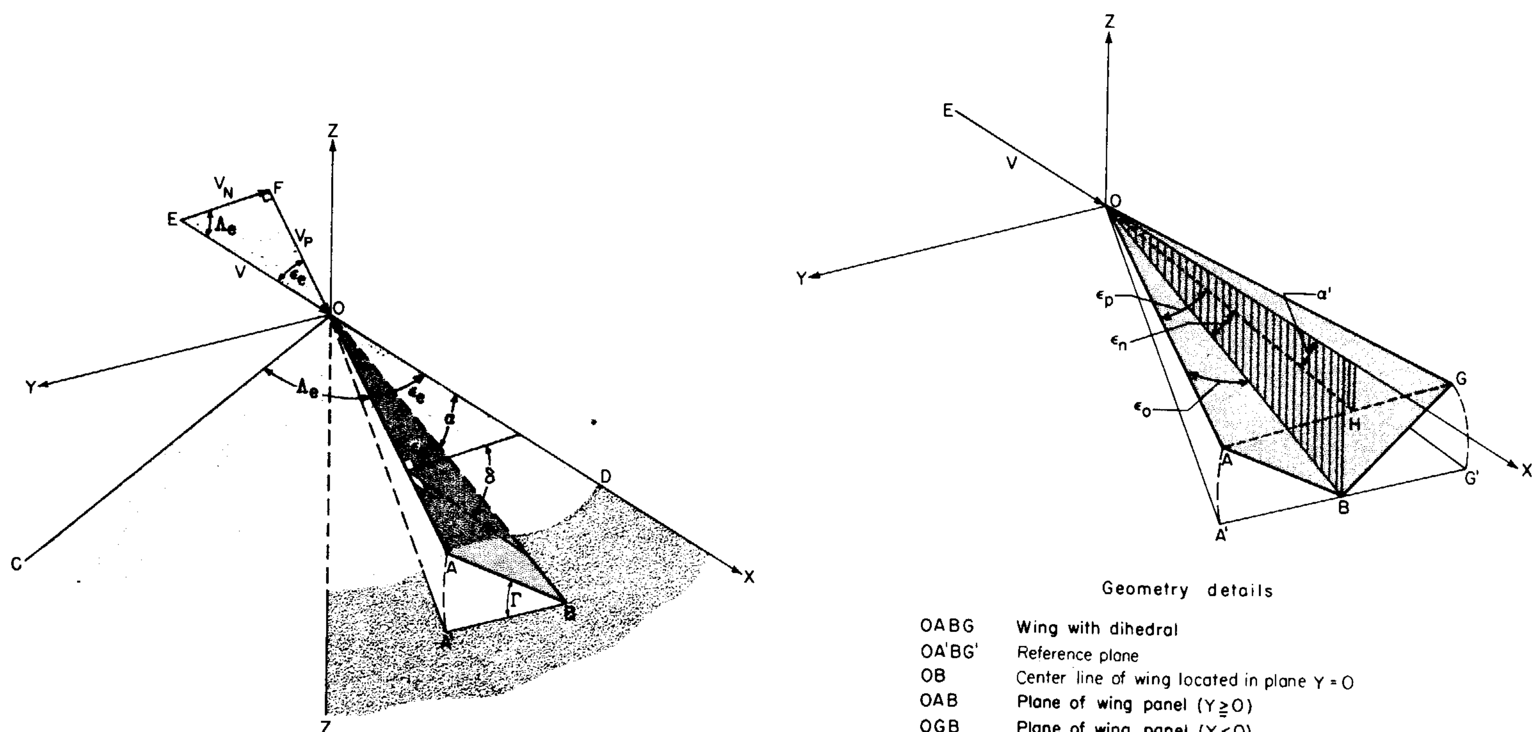

Geometry details

OABG	Wing with dihedral
OA'BG'	Reference plane
OB	Center line of wing located in plane $Y = 0$
OAB	Plane of wing panel ($Y \geqq 0$)
OGB	Plane of wing panel ($Y \leqq 0$)
EFODAC	Plane of resultant velocity vector and components V_N and V_P; also plane formed by leading edge of wing (OA) and x axis
	Dihedral angle Γ in plane perpendicular to OB

Figure 1.- Wing geometry.

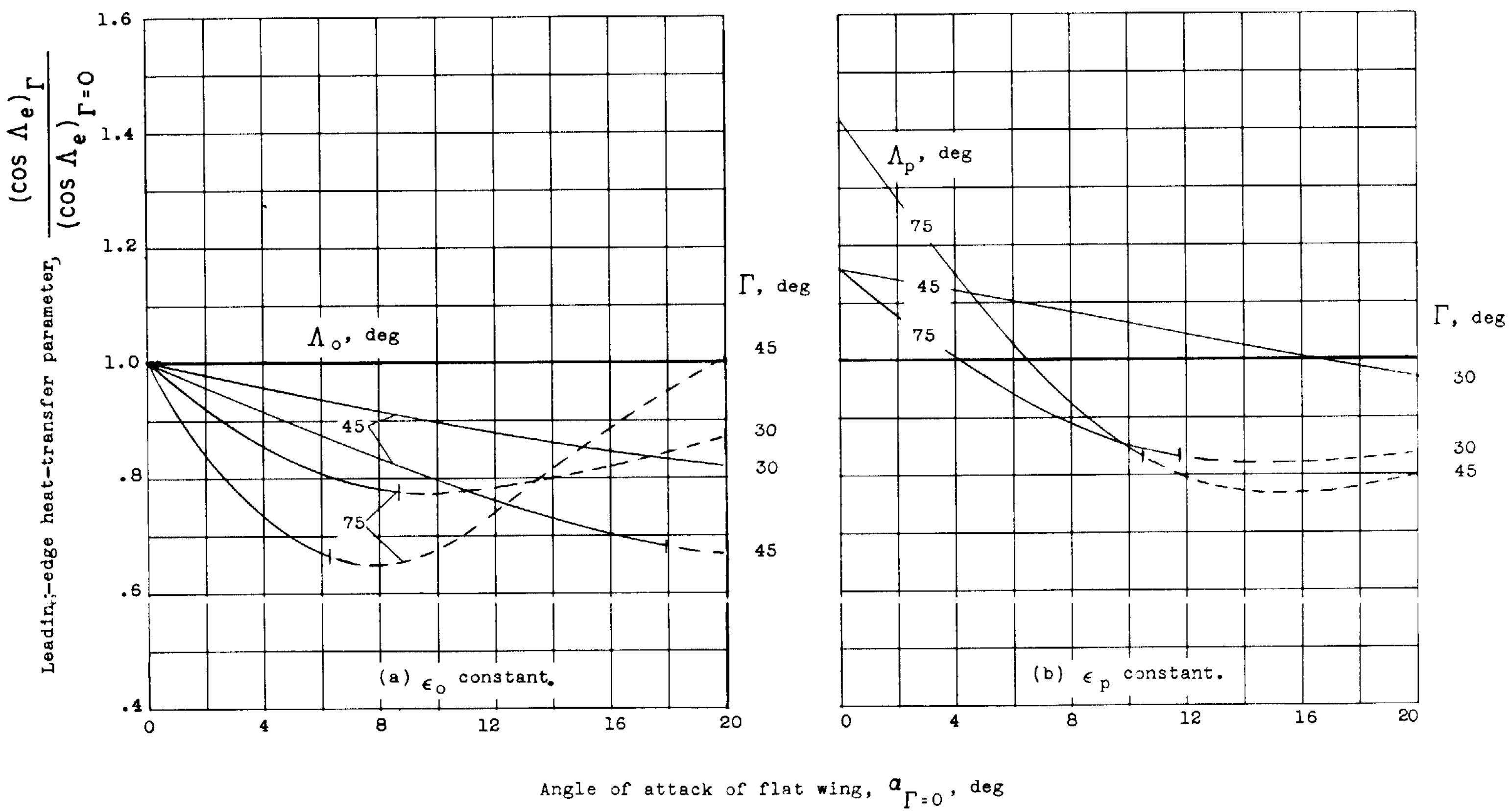

Figure 2.- Effect of dihedral and sweep on leading-edge heat transfer. Lines of constant lift are vertical. Dashed lines indicate region in which isolated swept-cylinder analysis breaks down at very high Mach numbers.

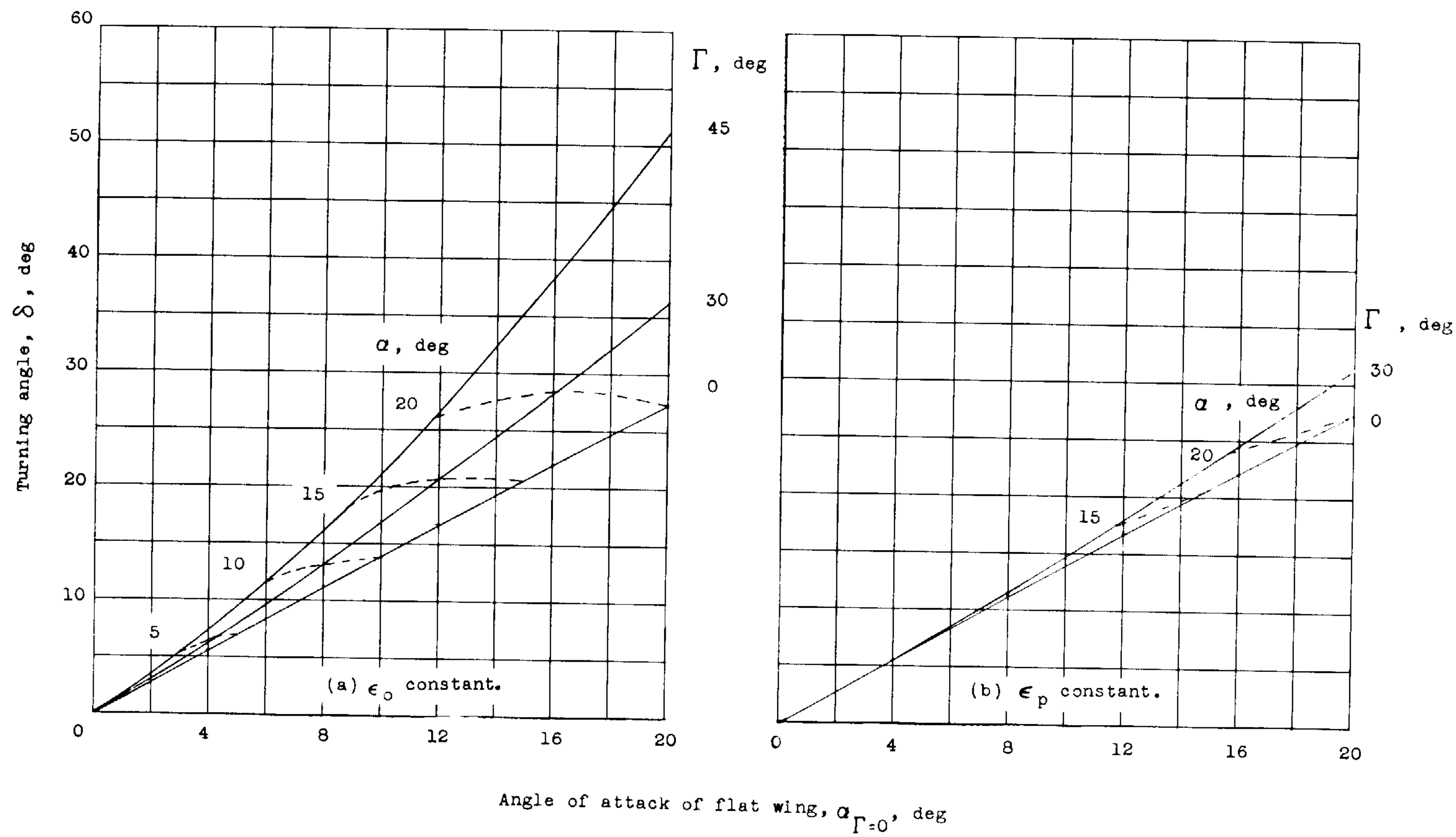

Figure 3.- Turning angles for various angles of attack and dihedral for 45° swept wings. Lines of constant lift are vertical.

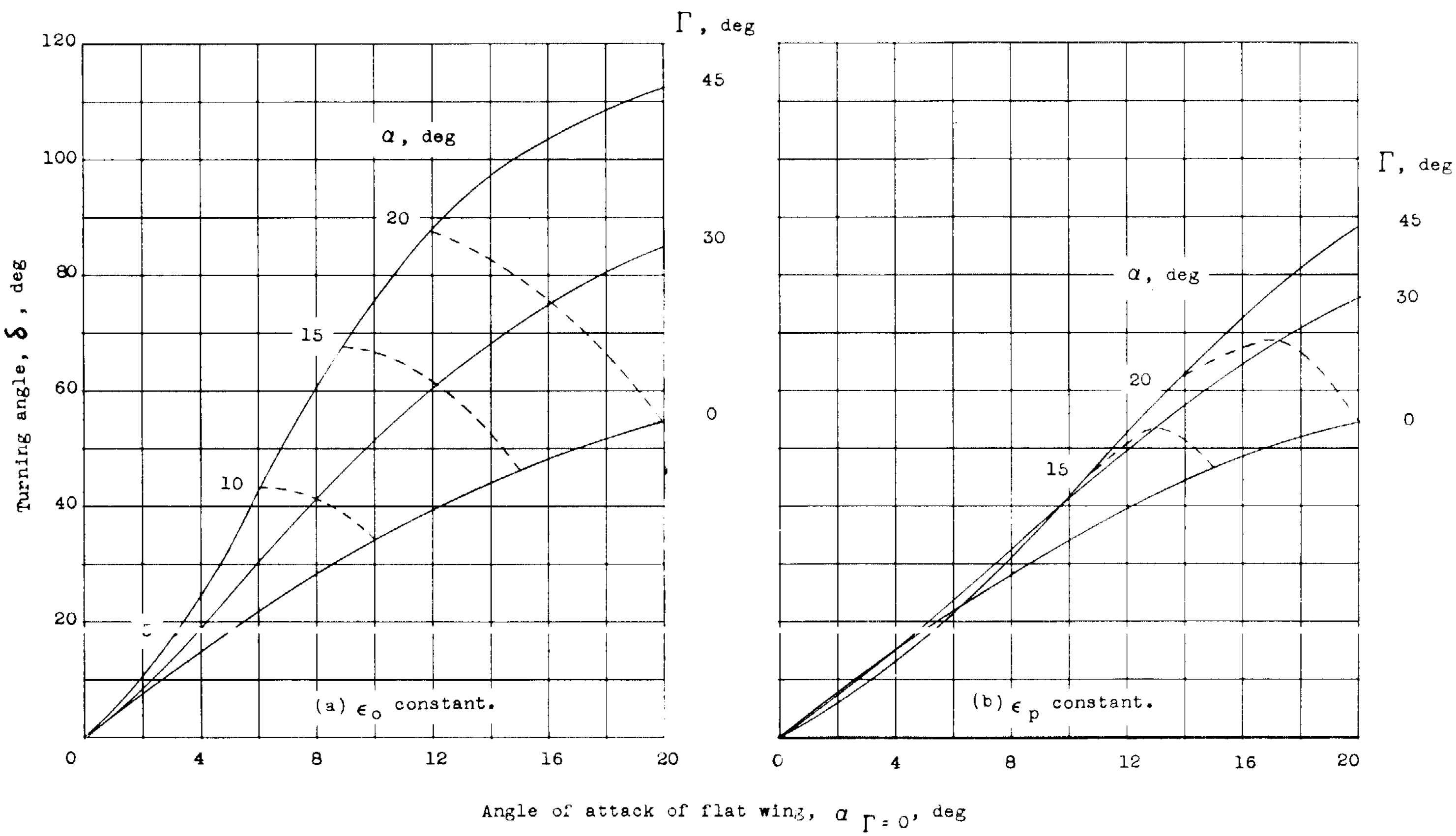

Figure 4.- Turning angles for various angles of attack and dihedral for 75° swept wings. Lines of constant lift are vertical.

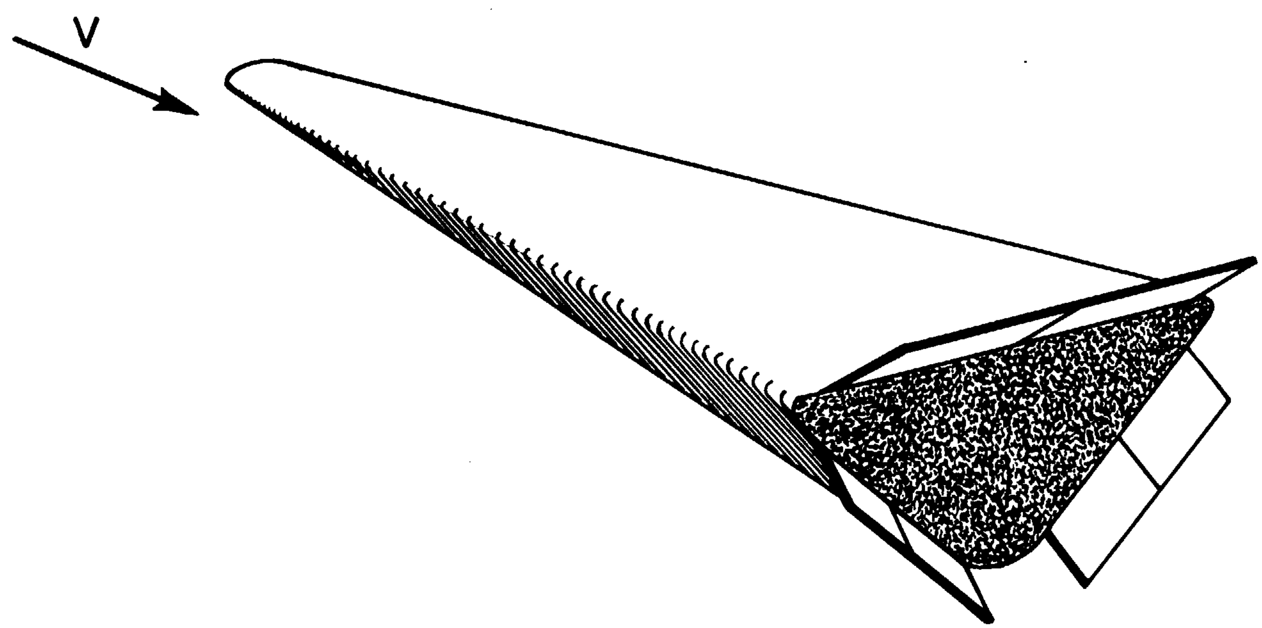

Figure 5.- Illustrative configuration.

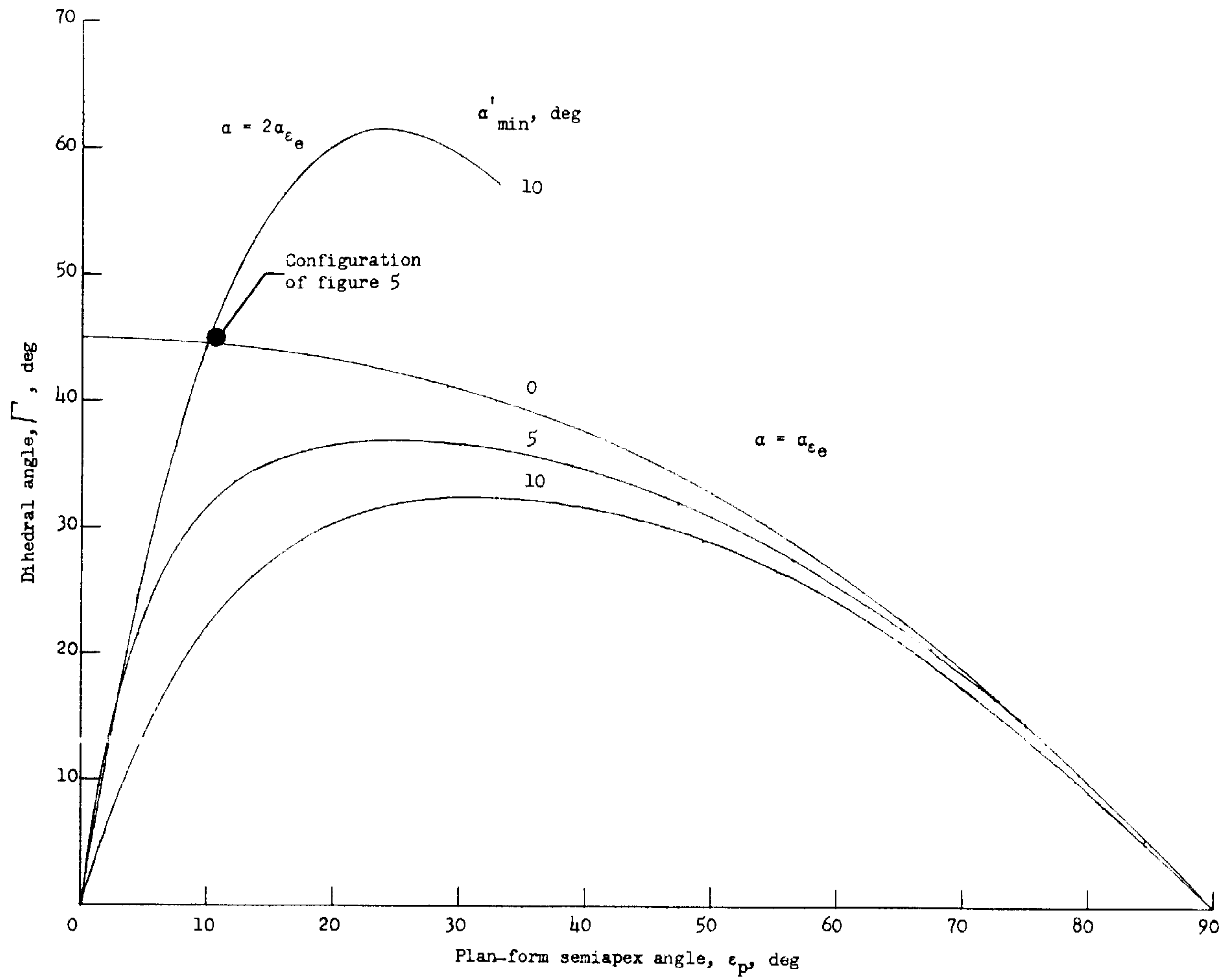

Figure 6.- Minimum dihedral angle required to establish $\alpha = \alpha_{\epsilon_e}$ or $\alpha = 2\alpha_{\epsilon_e}$ for several values of α'_{min}.