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SOME NUCLEAR CALCULATIONS OF U^{235}-D_2O

GASEOUS-CORE CAVITY REACTORS

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SOME NUCLEAR CALCULATIONS OF U^{235}-D_2O GASEOUS-CORE CAVITY REACTORS

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SUMMARY

The results of a multigroup, diffusion theory study of spherical gaseous-core cavity reactors are presented in this report. The reactor cavity of gaseous U^{235} is enclosed by a region of hydrogen gas and is separated from an external D_2O moderator-reflector by a zirconium structural shell. Some cylindrical reactors are also investigated.

A parametric study of spherical reactors indicates that, for the range of variables studied, critical mass increases as: (1) Fuel region is compressed within the reactor cavity, (2) moderator thickness is decreased, (3) structural shell thickness is increased, and (4) moderator temperature is increased.

A buckling analogy is used to estimate the critical mass of fully reflected cylindrical reactors from spherical results without fuel compression. For a reactor cavity of a 120-centimeter radius uniformly filled with fuel, no structural shell, a moderator temperature of 70° F, and a moderator thickness of 100 centimeters, the critical mass of a spherical reactor is 3.1 kilograms while that of a cylinder with a length-to-diameter ratio of 1.0 (L/D = 1) is approximately 3.8 kilograms and, with L/D = 2, 5.9 kilograms.

For the range of variables considered for U^{235}-D_2O gaseous-core cavity reactors, the systems are characterized by 95 to 99 percent thermal absorptions, with the flux reaching a maximum in the moderator about 10 to 15 centimeters from the reactor cavity.

Results obtained with a two-dimensional, four-group diffusion code indicate that the increase of critical mass of a fully reflected cylindrical reactor due to radial fuel compression is much less severe than that found in spherical reactors.

INTRODUCTION

The concept of an externally moderated reactor as a nuclear rocket motor has received attention during the past decade. Interest in such a concept is motivated by the possibility of achieving propellant temperatures beyond materials limitations imposed by heat-transfer-type solid-core reactors. The main obstacle to any gaseous reactor scheme is the attainment of criticality with an acceptable uranium-to-propellant flow-rate ratio. An understanding of the flow conditions in a nuclear rocket motor is therefore essential. While research is underway in an attempt to solve the flow problems within a gaseous reactor, it is desirable to know the requirements imposed on uranium concentrations as determined by criticality.

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Though the terms "gaseous reactor" and "cavity reactor" appear to be used interchangeably in the published literature to date, it would seem that a distinction should be made. The term "gaseous reactor" implies that the fuel is in the gaseous state but not that the neutron moderation is accomplished external to the fuel region. The implication of "cavity reactor" is that the moderator-reflector is external to the fuel region but not that the fuel is necessarily in the gaseous state.

Safonov (ref. 1) and Bell(ref. 2) have analyzed a gaseous-core cavity reactor. In an extensive study, Safonov has computed criticality curves of spherical reactors with an external moderator-reflector surrounding a cavity completely and uniformly filled with a gaseous fissile material. Also included is the case where the fuel region is a spherical shell within the reactor cavity. For moderator-reflector thicknesses from 0.3 meter to infinity, he has evaluated reactor systems utilizing U^{233}, U^{235}, and Pu^{239} as fuel with C, Be, and D_2O as reflector materials.

Reference 3 describes in detail a Russian zero-power, gaseous reactor that has been successfully operated. This reactor utilized UF_6 as the gaseous fuel. The moderation was accomplished by containing the UF_6 within a matrix of beryllium tubes. This assembly was surrounded by a graphite reflector. The direct use of UF_6 as a fuel, since it is the feed material for a diffusion plant, would eliminate further processing. In addition, it provides the interesting possibility of a circulating gaseous fuel reactor that could be coupled directly with a turbine.

Recent studies (refs. 4 to 6) have considered the use of a gaseous-core cavity reactor as a rocket motor. In such a reactor, the gaseous fuel would impart the fission energy directly to the propellant gas. A gaseous nuclear rocket motor employing coaxial flow (fuel within propellant) of the gas through the reactor has been proposed in reference 6.

Regardless of what scheme is used to provide a rocket motor of this type, certain nuclear and geometrical characteristics will necessarily exist. Such a reactor obviously must be externally moderated, and a structural wall will be necessary between the cavity and the moderator. The fuel will not fill the entire reactor cavity; that is, a gaseous fuel core will be surrounded by a region of propellant gas that will greatly reduce the thermal radiation to the cavity walls through absorption. There will be some wall heating, of course, and any resulting increase in moderator temperatures must be accounted for in criticality calculations. These factors will affect the flux distribution and criticality requirements of the reactor.

It is the purpose of the study reported herein to extend some of the work of Safonov and Bell on spherical cavity reactors to include the effects of structural materials, moderator heating, and compression of the fuel region within the reactor cavity for a U^{235}-D_2O system. For reasonable ranges of these variables, critical masses have been calculated from a one-dimensional, six-group diffusion analysis (ref. 7). Some calculations were also made with a two-dimensional, four-group diffusion code (ref. 8), and an approximate method is suggested to estimate the criticality requirements of cylindrical gaseous-core cavity reactors from the one-dimensional results. For all the calculations reported herein, the moderator was considered to be 100 percent D_2O.

SYMBOLS

a	numerical constant
B^2	buckling
b	numerical constant
D	neutron diffusion coefficient, or reactor cavity diameter
H	reactor height
k_{eff}	effective multiplication constant
L	reactor cavity length
l	transport mean free path
R	radius of equal buckling
r	radial coordinate
r^*	dimensionless radius, $\sqrt{\frac{r_o^2 - r_i^2}{r_i^2 - r_{min}^2}}$
r_{min}	minimum radius for critical sphere

S	neutron source strength
T_M	moderator temperature
V	volume of fuel region
W	critical mass
W^*	spherical critical mass ratio, compression to no-compression
$\alpha_n \beta_t$	fraction of fission neutrons born in energy group n
γ	interior greyness factor
δ_M	moderator thickness
δ_s	structural shell thickness
ρ	density
$\nu\Sigma_f$	macroscopic neutron production cross section
Σ_a	macroscopic neutron absorption cross section
Σ_q	macroscopic neutron slowing-down cross section
φ	neutron flux
$\overline{\varphi}$	average neutron flux in fuel region

Subscripts:

c	cylinder
i	inner, or fuel
max	maximum
min	minimum
n	neutron group
o	outer, or cavity
s	sphere
t	total number of groups
U	uranium

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REACTOR ANALYSIS

The basic reactor configuration used in the analysis was a D_2O moderator-reflector shell with a low-density U^{235} gaseous cavity. The variations applied to this configuration, shown in figure 1, were (1) variation of the moderator thickness δ_M, (2) variation of the fuel-region radius r_i within a reactor cavity of radius r_o (the void formed is filled with hydrogen gas), (3) addition of a structural material of thickness δ_s on the inner moderator radius, and (4) variation of the moderator temperature T_M.

For the first part of the report, a spherical geometry was used to assist in determining how these variations change the critical mass of the system. Then a fully reflected cylindrical model, with the same type arrangement of the D_2O and U^{235} as shown in figure 1, was used for analysis with a two-dimensional code.

Spherical Reactors

In an externally moderated reactor the thermal neutron source is supplied by the moderator-reflector region, and in the cases presented in this report the temperature of this region governs the thermal energy of the neutrons. The thermal cross sections for all regions were obtained at this energy regardless of whether these regions would physically be at the same or much higher temperatures. The temperature of the D_2O moderator-reflector region was varied from 70° to 180° F, and the results are discussed in a later section.

In a cavity reactor using an external moderator such as D_2O, the neutron energy spectrum from the D_2O is predominantly thermal. Since the only high thermal neutron absorber, U^{235}, is in the cavity, the ratio of the average thermal flux in the fuel region $\overline{\varphi}$ to flux at the boundary of the cavity will vary inversely with the density of the fuel. Therefore, for very low densities in the cavity region, the thermal flux will not be reduced significantly and a diffusion approximation to the Boltzmann equation was assumed to be valid for the cases considered herein.

For the one-dimensional solutions of the spherical reactors presented in this report, the diffusion equation shown in equation (1) was applied. The equation is applied in group form for n groups:

$$-D_n \nabla^2\varphi_n + (\Sigma_{a,n} + \Sigma_{q,n} + D_nB^2)\varphi_n = \alpha_n\beta_t + \Sigma_{q,n-1}\varphi_{n-1} \tag{1}$$

The first term on the left represents the rate of diffusion of neutrons from a volume element. The $\nabla^2\varphi$ for spherical geometry, for radial variation only, is $\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr}$. The second term, enclosed in parentheses, represents the rate of neutrons lost through absorption, slowing-out of group n, and the transverse leakage of neutrons which, for a spherical reactor, was equal to zero. The source terms on the right side of the equation are the steady-state sources. The first term $\alpha_n\beta_t$ is the fraction of fission neutrons born in group n, and the second term is the slowing-in source from a higher group, n-1.

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Six groups were used in the spherical calculations for the one-dimensional results. Three of these groups were in the fission spectrum range, lethargy of 0.0 to 7.75. Two groups were used between the fission spectrum and thermal, with the cutoff energy of the fifth group taken as 18.2 lethargy units. Zero lethargy was taken as 10 million electron volts. The method used to solve the diffusion equation for the six groups was patterned after the one reported in reference 7. The code in reference 7 was revised to handle more groups, regions, and mesh points for cylindrical, spherical, and slab geometries and was programed for the IBM 704. The derivatives are approximated by use of finite differences based on adjacent mesh-point values.

Inasmuch as the diffusion analysis, as used in this report, cannot accurately enough predict critical masses when the ratio of neutron current to flux becomes large, there is an area in which the results would be doubtful. In a report by G. Safonov (ref. 1) a discussion of when diffusion theory should be used is given. The factor used as a basis of determining whether diffusion or transport theory would apply was called the "interior greyness factor" and was defined as:

$$\gamma \equiv \frac{l}{3} \times \frac{d\varphi/dr}{\varphi} \qquad (2)$$

The gradient of the flux and the value of φ are taken at the boundaries. The equation states that the "greyness factor" is equal to the thermal neutron current into the interior divided by the thermal flux at the boundary. In reference 1, if γ was less than 1/3, diffusion theory was considered adequate. This greyness factor has been applied to a range of the calculations discussed in this report, and the results indicated that for the most part the factor was less than 1/3 and went over 1/3 only when the critical mass began to increase rapidly as the fuel was compressed.

Cylindrical Reactors

The geometry that is of most practical use is a cylindrical one. Since the initial calculations were done on a spherical model for first estimations as to critical mass, it would be helpful to have an indication as to what would happen to the critical mass if a cylindrical geometry were used. Since the reactor would be fully reflected by D_2O and the usual buckling assumptions are not valid, a two-dimensional analysis was selected. Again, a diffusion analysis was used. The two-dimensional diffusion program used was the Westinghouse PDQ 02 code (ref. 8). The code was written for the IBM 704 and, for the size of the 704 that was available, would handle up to 2500 mesh points. The program was operated with four energy groups and for an (r, z) geometry. As in the one-dimensional analysis, nuclear parameters were specified for each material for each of the four energy groups. All neutron data used are listed in table I.

When the fuel was compressed, the region between the fuel cavity and the D_2O region was filled with hydrogen at a density of 0.8×10^{21} atoms per cubic centimeter; absorption and slowing-down cross sections were assumed to be zero. The only process allowed was one of diffusion.

The machine time required to obtain solutions with the PDQ 02 program became prohibitively large for the low fuel densities encountered in cylindrical cavities larger than 40 centimeters in radius. An analogy based on a comparison of the geometric buckling of a sphere and a cylinder was developed to provide an estimate of the critical mass of the larger cylinders. A few runs with the PDQ 02 code were made to determine if the analogy seemed valid. The analogy used was to equate the geometric buckling of a spherical reactor with the geometric buckling of a cylindrical reactor. This approach seemed to be reasonable if the material densities and the moderator thickness were held constant. In equation (3) the geometric bucklings of the sphere and the cylinder are equated:

$$\left(\frac{\pi}{R_S}\right)^2 = \left(\frac{2.405}{R_C}\right)^2 + \left(\frac{\pi}{H_C}\right)^2 \tag{3}$$

These bucklings usually refer to bare reactors, and therefore the dimensions would apply to an equivalent bare reactor. Since the cavity reactor cannot be considered as a bare case, some limits were applied to determine the range of dimensions. The limits were that the radius used be either the radius of the cavity or the radius of the reactor. The same condition applied to the length; that is,

$$R = r_o \tag{4}$$

$$R = r_o + \delta_M \tag{4a}$$

$$H = 2r_o \frac{L}{D} \tag{5}$$

$$H = 2\left(r_o \frac{L}{D} + \delta_M\right) \tag{5a}$$

The assumption was made then that, if a radius existed such that the geometric bucklings could be equated to give dimensions for a cylinder, these dimensions should lie between the limits imposed in equations (4), (4a), (5), and (5a). That is, for thick reflectors the effective bare-core radius should be somewhere between the cavity radius and the reactor radius (including reflector). With these dimensions and the assumption of equal densities in the fuel region, the critical mass of the fuel region can be obtained for the cylindrical reactor from the following equation:

$$\left(\frac{W}{\frac{4}{3}\pi r_o^3}\right)_s = \left(\frac{W}{2\pi r_o^3 \frac{L}{D}}\right)_c \tag{6}$$

where L/D is the length-to-diameter ratio of the fuel region. The L/D values of 1/2, 1, and 2 were used in the analysis.

RESULTS AND DISCUSSION

Spherical Reactors

Critical masses of the spherical reactor configuration shown in figure 1 were computed with an IBM 704 code, using the one-dimensional, six-group diffusion theory equations given in the section REACTOR ANALYSIS. By a systematic variation of parameters, a study was made of the effects on critical mass of moderator thickness, fuel compression within the reactor cavity, structural shell thickness, and moderator temperature. All the calculations were made for a U^{235}-D_2O (0-percent H_2O) reactor system; for the cases where the fuel-region radius was less than that of the reactor cavity, the remaining volume was assumed to contain hydrogen gas (0.8×10^{21} atoms/cc).

Effect of moderator thickness. - Figure 2 shows critical mass as a function of reactor cavity radius for moderator thicknesses of 50, 100, and 200 centimeters. These curves are for the case where the fuel uniformly fills the reactor cavity, no structural shell exists between the reactor cavity and the moderator, and the moderator temperature is 70° F. Included in figure 2 are the curves from reference 1. As can be seen, the analytical technique used by Safonov gives results that are in good agreement with those obtained with the six-group diffusion code.

As would be expected, the required critical mass increases as the moderator-reflector thickness decreases, because of increased leakage of thermal neutrons. A 200-centimeter thickness provides what is essentially an infinite moderator, while a 50-centimeter thickness allows an appreciable leakage. The remainder of the spherical results - showing the effects of fuel compression, a structural shell, and moderator temperature - are all for a moderator-reflector thickness of 100 centimeters.

Effect of fuel compression. - When the fuel region is compressed, or restricted, to a radius r_i within the reactor cavity of radius r_o, several effects combine to alter reactor criticality. One obvious effect is that all the neutrons entering the reactor cavity from the moderator no longer pass through the fuel region. This results in a decrease of the neutron flux at the boundary of the fuel region.

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When the fuel region is compressed to a radius ratio r_i/r_o, certain effects result from the change in fuel density. First, an increase of the fuel density causes self-shielding, which decreases the utilization of the central portion of the fuel. An increase in fuel density also causes a slight shift in the neutron energy spectrum; that is, there is a change in the average neutron energy at a given fuel-region radius.

A consideration of all these factors cannot, a priori, suggest with surety whether fuel compression would cause an increase or decrease in either critical density or critical mass.[1] The computed effect of compressing, or restricting, the fuel region to a radius r_i within a spherical reactor cavity of radius r_o is shown in figure 3. These results are for a moderator thickness of 100 centimeters, no structural shell, and a moderator temperature of 70° F.[2] Again, the results of Safonov are shown for comparison with the basic curve (i.e., no "compression" of the fuel region).

The results show that there is a continuous increase in critical mass as the fuel region is restricted to a smaller and smaller radius r_i within a reactor cavity of constant radius r_o. For a reactor cavity of 102 centimeters, 2.5 kilograms fuel is required for criticality if the fuel uniformly fills the entire reactor volume. If the fuel is compressed to a radius of 40 centimeters, the critical mass increases to around 3 kilograms; a fuel region of 25-centimeter radius requires approximately 6 kilograms for criticality. Another compression curve is shown, for a reactor cavity radius of 155 centimeters, to illustrate the general nature of the family of curves.

Effect of structural materials. - In any actual reactor assembly, the use of certain structural materials is, of course, a necessity. Since the fuel densities in this type of reactor are low (10^{18} to 10^{20} atoms/cc), a neutron may make as many as ten or twenty traverses through the structural shell and reactor core before capture; thus, the effect of captures in the structural material becomes important.

The increase of critical mass due to a zirconium shell between the moderator region and the reactor cavity can be seen in figure 4. These calculations are for a moderator thickness of 100 centimeters and a moderator temperature of 70° F. The critical mass for a cavity radius of 120 centimeters increases from 3.1 kilograms to 3.4 kilograms because of a 1/8-inch zirconium shell, and further increases to 4.1 kilograms for a shell thickness of 1/4 inch. The compression curves with a 1/4-inch shell

[1] Safonov has estimated in reference 1 that compression of the fuel region within the reactor cavity at constant mass would decrease k_{eff}. This is in agreement with the results reported herein.

[2] Hydrogen gas (0.8×10^{21} atoms/cc) fills the void created by compressing the fuel region.

exhibit the same trends seen in figure 3. Although specifically for zirconium, with a microscopic absorption cross section σ_a of 0.18 barn, the same qualitative characteristics should apply for Al ($\sigma_a = 0.23$) or Mg ($\sigma_a = 0.063$).

Effect of moderator temperature. - The effect on critical mass of the moderator temperature is also shown in figure 4. As the moderator temperature is increased, there is a corresponding increase in the required critical mass. These calculations were made for the case of no fuel compression, no structural shell, and a moderator thickness of 100 centimeters. The critical mass required for a reactor cavity of a 120-centimeter radius and a moderator temperature of 70° F is 3.1 kilograms. The required critical mass increases to 3.4 kilograms for a moderator temperature of 122° F, and an additional increase to 4.2 kilograms is required for a moderator temperature of 180° F.

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Generalized criticality map. - To facilitate parametric performance calculations, an empirical relation was developed to extend the range of the basic compression curves shown in figure 3. It was found that these curves can be represented by the following equation:

$$W_s^* = a e^{b r^{*2}} \tag{7}$$

where W_s^* is the ratio of critical mass for the compressed condition to that for the uncompressed. The dimensionless radius r^{*2} is a measure of the degree of fuel compression and is given by:

$$r^{*2} \equiv \frac{r_o^2 - r_i^2}{r_i^2 - r_{min}^2}$$

where r_{min} is the radius to which the critical mass appears asymptotic in figure 5; this was taken to be 18 centimeters. The numerical constants a and b are as follows:

$$a = 1,\ b = 0.031 \quad \text{for} \quad 0 \le r^{*2} \le 12$$

$$a = 0.642,\ b = 0.068 \quad \text{for} \quad 12 < r^{*2} \le 21$$

This correlation was used to extend the two computed compression curves of figure 3 to a generalized criticality map. Figure 5 shows this map for a spherical cavity reactor with no structural shell, a moderator thickness of 100 centimeters, and a moderator temperature of 70° F. Equation (7) is intended only to extend the criticality curves for the U^{235}-D_2O system. Although the use of r_{min} in the definition of r^* and the use of W_s^* as a dimensionless multiplying factor are suggestive of a general correlation, this has not been proven. Since Safonov's results indicate a general similarity, it is reasonable to expect that the same general type of function would also correlate compression curves for other reactor systems such as Pu-C or U^{233}-BeO.

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Thermal flux distribution. - Figure 6(a) shows the thermal neutron flux distribution for a critical spherical reactor with no structural shell, a moderator thickness of 100 centimeters, and a moderator temperature of 70° F. The radius of the reactor cavity is 155 centimeters, and the fuel-region radius is 55 centimeters.

The remaining cavity volume is filled with hydrogen gas having a density of 0.8×10^{21} atoms per cubic centimeter. Essentially the same results would have been obtained if the region between the fuel and moderator had been considered to be a vacuum. It was felt, however, that the presence of the hydrogen gas would more closely represent realistic operating conditions for this type of reactor, that is, the propellant gas flow in a gaseous nuclear rocket motor.

The flux behavior in the gas and moderator regions is typical for this class of reactors. The thermal flux reaches a maximum just inside (10 to 15 cm) the moderator region and then decreases in a near-linear fashion to zero at the outer edge. This same characteristic distribution was also obtained for moderator thicknesses of 50 and 200 centimeters. The thermal flux in the hydrogen gas decreases from the moderator boundary to the fuel region. Again, a similar pattern was found for all cases where the fuel region was separated from the moderator.

The thermal flux ratio in the fuel region decreases from 0.71 at the outer boundary to 0.56 at the center - a decrease of 21 percent, based on the flux level at the boundary of the fuel region. The flux gradients in the fuel region are, of course, a function of the atom density. The critical fuel density for the conditions shown in figure 6(a) is 0.2×10^{20} atoms per cubic centimeter.

Figure 6(b) illustrates a different kind of reactor condition. The moderator thickness is the same as shown in figure 6(a), but there is now a 1/4-inch zirconium shell between the hydrogen and moderator regions, and the moderator temperature is 180° F rather than 70° F. Although the thermal flux distribution in the moderator and hydrogen regions is very similar to that in figure 6(a), there is a marked change in the fuel region. Both the increase in moderator temperature and the presence of the zirconium shell contribute to a further increase in the required critical mass. The reactor size is somewhat smaller than that shown in 6(a); this results in a decrease, though slight, of critical mass. The fuel density for the reactor of figure 6(b) is 1.3×10^{20} atoms per cubic centimeter, six and one-half times greater than that shown in figure 6(a). Here the thermal flux falls off sharply in the fuel region. This results in self-shielding of the U^{235} in the fuel region.

Since this type of reactor has negligible fast leakage and is almost completely thermal (95 to 99 percent), no fast fluxes are shown. For the same reason, no power distribution curves are given, since they have the same shape as the thermal flux in the fuel region.

Neutron balance. - To supplement the thermal flux plots, an analysis was made of the neutron balance throughout a spherical reactor in terms of production, leakage, and regional absorption for each energy group. In table II, parts (a) and (b) give this information for a reactor cavity radius of 104 centimeters, a 1/4-inch zirconium structural shell, a 100-centimeter moderator thickness, and a moderator temperature of 70° F. Table II(a) is for a reactor with the fuel filling the entire reactor cavity, and table II(b) is for the fuel region compressed to a 44-centimeter radius. The neutron balance is based on a source of 1000 neutrons born in the first three energy groups (labeled "Fast" in the tables).

The fuel compression from 104 to 44 centimeters has no effect on the total neutron distribution. Although there is a slight shift away from thermal energy with fuel compression, both cases show that 60.3 percent of the neutrons are lost by absorptions, and 39.7 percent by leakage from the reactor. Thus, from the standpoint of neutron economy, compression of the fuel carries no penalty; however, there is an investment penalty, since the critical mass has increased from 3.5 to 4.8 kilograms.

Table II(c) summarizes the effects of fuel compression in spherical reactors. This example is the same 104-centimeter cavity as was used for tables II(a) and (b). Included in table II(c) is the subcritical case, where the original critical mass is compressed from 104 to 44 centimeters. For constant mass, compression results in:

(1) An increase in reactor leakage

(2) A decrease in effective multiplication constant

(3) A decrease in thermal flux in the fuel region

(4) An increase in thermal flux in the moderator

The third line of table II(c) shows the reactor conditions when the fuel mass is increased enough to regain criticality. Now, the leakage and moderator flux are the same as before fuel compression, but the average flux in the fuel itself is considerably less. In other words, the moderator region feels no effects of fuel compression if sufficient fuel is added to maintain criticality. This suggests that the effects of fuel compression can be expressed in terms of fuel-region and cavity parameters alone. This is an important conclusion, and will be used later in conjunction with self-shielding to give a consistent explanation of fuel compression effects in both cylinders and spheres.

Cylindrical Reactors

A fully reflected, right circular cylinder is the reactor geometry that best lends itself to practical applications, such as the Russian test reactor of reference 3 or the nuclear rocket motor described in reference 6. It is also a reactor configuration that is notably inconvenient to analyze. Some two-dimensional, four-group calculations were made with the PDQ 02 diffusion code. Because of the large diffusion coefficients and flux gradients that exist in cavity reactors, the required machine time prohibited the calculation of reactors with radii larger than 60 centimeters. The buckling analogy described in ANALYSIS was used to estimate the critical mass of cylindrical reactors with uniformly filled cavities of length-to-diameter ratios of 1/2, 1, and 2.

Buckling analogy. - Equations obtained by means of a buckling analogy were used to estimate the critical mass of fully reflected cylindrical reactors from the spherical reactor results. This technique was applied only to reactors with the cavity region uniformly filled with fuel. It should be emphasized that the buckling analogy provides what is simply an "educated guess." The value of this approach is that it does furnish an estimate of cylindrical reactor critical masses for conditions (reactor cavity radii greater than 60 cm) which render the PDQ 02 two-dimensional diffusion code unusable.

Figure 7 shows the results obtained from the buckling analogy for cylindrical reactors of $L/D = 1/2$, 1, and 2. The cases computed with the PDQ 02 program are included for comparison. Also shown is the basic curve for a spherical reactor. All results are for reactors with no structural shell, moderator thickness of 100 centimeters, and moderator temperature of 70° F. Two curves are shown for each L/D value. One curve results from equating the spherical and cylindrical reactor buckling at the cavity radius; the other curve represents equal buckling at a radius that includes all of the moderator-reflector. While it is not clear what fraction of a very thick moderator should be included with the "reactor" in order that the buckling should be the same, it is felt that the two extremes chosen should bound the correct value.

Because of the limited range of reactor sizes that were feasible to compute with PDQ 02, no positive conclusions can be drawn as to the validity of the buckling analogy. It can be said, however, that the trends indicated by the buckling analogy are in general agreement with results of the two-dimensional diffusion code for small reactors.

Cylindrical reactors with fuel compression. - A brief study of the effect of fuel compression in a cylindrical reactor was made by means of some four-group calculations with the PDQ 02 code. The results are shown graphically in figure 8. These results are for reactors with no structural shell, moderator thickness of 100 centimeters, and moderator temperature of 70° F. As for the spherical case, the volume between the fuel and moderator regions is filled with hydrogen gas ($\rho = 0.8\times10^{21}$ atoms/cc).

The reactor shown in figure 8(a) has a cavity radius of 40 centimeters, uniformly filled with fuel. The reactor cavity length-to-diameter ratio is 2. The same reactor is shown in figure 8(b), except that the fuel-region radius has been compressed to 20 centimeters within the 40-centimeter cavity. The effect of this compression is slight; the critical mass increased from 1.29 to 1.31 kilograms.

Figure 8(c) shows a reactor cavity of 150-centimeter radius and L/D = 1, with a fuel-region radius of 15 centimeters. The critical mass of this reactor is 7.7 kilograms. Figure 7 indicates that the critical mass of this reactor with no fuel compression would be approximately 5.4 kilograms. The results shown in figure 8 indicate that critical mass increases with fuel compression in cylinders as well as spheres but that the effect is much less severe.

It is worthwhile to note two factors which suggest at least a partial explanation of this difference. First is the obvious fact that, as the fuel region is radially compressed, the volume varies as the square of the radius in a cylinder and as the cube of the radius in a sphere. Secondly, for the same cavity radius and no fuel compression, the critical density of a cylindrical reactor is less than that of a spherical one. For the reactor of figure 8(c), the density ratio, sphere-to-cylinder, is approximately 1.2. The result of both of these factors is that a critical mass increase due to self-shielding first occurs in spherical reactors.

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Of course, self-shielding is not the only factor to be considered. As the fuel region is compressed within the reactor cavity, it presents a more inaccessible target for neutrons returning from the moderator region. This geometrical effect results in a decrease of average thermal flux in the fuel region with compression; again, the increase in critical mass due to this factor should be greater in the spherical case.

Although a rigorous consideration of self-shielding and geometric factors for spheres and cylinders is beyond the scope of this study, a simplified approach can be used to relate the parameters affecting cavity reactor criticality. Since nearly all the fissions are caused by thermal neutron capture, two reactors can be compared if they both have the same source strength. This source, in neutrons per second, is given by:

$$S = \rho \nu \sigma_f \overline{\varphi} V$$

Here, V is the volume of the fuel region, ρ the fuel density, $\overline{\varphi}$ the average thermal flux in the fuel region, and $\nu\sigma_f$ the microscopic neutron production cross section. For a given fuel material, this expression can be simplified to:

$$W\overline{\varphi} = \text{Constant}$$

Thus, the increase in critical mass due to fuel compression is inversely proportional to the change in average thermal flux in the fuel region. For the spherical reactor shown in table II(a), the critical mass is 3.5 kilograms and the average flux is 0.75. When the fuel is compressed from 104 to 44 centimeters, the average flux decreases to 0.54, as seen in table II(b). The expected critical mass would then be

$$W_s = 3.5 \frac{0.75}{0.54} = 4.9 \text{ kg}$$

This agrees closely with the value of 4.8 kilograms obtained from the six-group diffusion code. For the cylindrical case shown in figures 8(a) and 8(b), the critical mass with fuel compression calculated from average flux values is 1.30 kilograms; the PDQ 02 program gave 1.31 kilograms.

The use of this procedure to predict the effect of fuel compression on critical mass requires a knowledge of flux behavior in the fuel region. A detailed two-dimensional study of the total neutron distribution throughout the reactor would be necessary to reveal the more subtle effects of fuel compression in cavity reactors. The simplified approach presented here is only intended to provide a consistent physical picture of the interrelation of the parameters affecting cavity reactor criticality.

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Thermal flux distribution. - Thermal flux distributions are shown in figures 9(a) and (b) for the two cylindrical reactor configurations of figures 8(a) and (b), respectively. Only radial fluxes are shown; the axial flux distribution is essentially the same except for the slight effect of the hydrogen gas in the compressed case. The flux change through the fuel region is small; for the reactor with no fuel compression, figure 9(a), the flux through the fuel is nearly constant. The flux behavior in the hydrogen and moderator regions has the same characteristic trends as were found in spherical reactors.

SUMMARY OF RESULTS

A study has been made of the nuclear characteristics of gaseous-core cavity reactors. The reactor cavity is composed of a spherical fuel region of gaseous U^{235} enclosed by a region of hydrogen gas. This cavity is separated from a D_2O moderator-reflector by a shell of structural material such as zirconium. Critical masses were calculated by one-dimensional, six-group diffusion theory for the following conditions:

(1) Cavity radii from 18 to 200 centimeters

(2) Fuel-region to cavity radius ratios from 0.2 to 1.0

(3) Moderator thicknesses of 50, 100, and 200 centimeters

(4) Structural shell thicknesses of 0, 1/8, and 1/4 inch

(5) Moderator temperatures of 70°, 122°, and 180° F

The critical mass of the reactor was found to increase as: (1) the fuel region is reduced within the reactor cavity, (2) moderator thickness is decreased, (3) structural shell thickness is increased, and (4) the moderator temperature is increased.

The conditions listed below exemplify these effects:

1. A 200-centimeter thickness provides what is essentially an infinite moderator, while a 50-centimeter thickness allows an appreciable leakage of thermal neutrons.

2. For a reactor cavity of 102-centimeter radius, no structural shell, a moderator temperature of 70° F, and a moderator thickness of 100 centimeters, the critical mass increases from 2.5 to 6 kilograms as the fuel-region radius is reduced from 102 to 25 centimeters.

3. For a reactor cavity of 120-centimeter radius uniformly filled with U^{235}, a moderator temperature of 70° F, and a moderator thickness of 100 centimeters, the critical mass increases from 3.1 to 4.1 kilograms because of the presence of a 1/4-inch structural shell between the reactor cavity and moderator region.

4. For a reactor cavity of 120-centimeter radius uniformly filled with fuel, no structural shell, and a moderator thickness of 100 centimeters, the critical mass increases from 3.1 to 4.2 kilograms for a moderator increase from 70° to 180° F.

An estimate of the critical mass of cylindrical reactors was obtained by equating the geometrical buckling of a cylinder to that of a sphere. For a reactor cavity of 120 centimeters uniformly filled with fuel, no structural shell, a moderator temperature of 70° F, and a moderator thickness of 100 centimeters, the critical mass of a spherical reactor is 3.1 kilograms while that of a cylindrical reactor with $L/D = 1$ is between 3.5 and 4.1 kilograms, and that of a cylindrical reactor with $L/D = 2$ is between 5.5 and 6.4 kilograms.

To extend the usefulness of the spherical calculations, the effects of the fuel-region to reactor-cavity radius ratio were generalized by an empirical equation. For no structural shell, a moderator temperature of 70° F, and a moderator thickness of 100 centimeters, the ratio of critical mass for a given radius ratio to that for a uniformly filled cavity is given by

$$W_S^* = ae^{br^{*2}}$$

where a and b are constants and r^{*2} is a dimensionless function of the reactor cavity radius, fuel-region radius, and r_{min} - the reactor cavity radius at which critical mass becomes infinite.

The U^{235}-D_2O gaseous-core cavity reactor is 95 to 99 percent thermal. Typically for such a reactor, the thermal flux in the moderator reaches a maximum at a distance of 10 to 15 centimeters from the reactor cavity and then decreases in a near-linear manner to zero at the outer boundary. The flux depression is slight in the central core for fuel densities less than 0.2×10^{20} atoms per cubic centimeter; densities of 1.3×10^{20} atoms per cubic centimeter or greater result in poor fuel utilization because of self-shielding.

Based on 1000 neutrons born in the first three energy groups, the total neutron consumption in a spherical reactor with no fuel compression, a 1/4-inch zirconium shell, and a cavity radius of 104 centimeters is as follows: 488 neutrons are absorbed by the fuel, 68 by the zirconium, 47 by the moderator; and the remaining 397 neutrons are lost from the reactor by leakage.

The critical mass of a fully reflected cylindrical reactor with a 100-centimeter moderator thickness, no structural material, and a reactor cavity radius of 40 centimeters (cavity $L/D = 2$) was calculated to be 1.29 kilograms for no fuel compression. When the fuel region is compressed to 20 centimeters, the critical mass was found to increase to 1.31 kilograms. This diminished effect of fuel compression in cylindrical reactors as compared with spherical ones is attributed in part to the fact that self-shielding occurs to a greater degree in a spherical geometry than in a cylindrical one because of higher fuel densities required for criticality. The effect of fuel compression in both cylinders and spheres is expressed as a function of critical mass and average thermal flow in the fuel region.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, June 15, 1961

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TABLE I. - NEUTRON CROSS-SECTION DATA

(a) Six-group

Energy group	D	Σ_a	Σ_q	$\nu\Sigma_f$	Lethargy
U^{235}; $\rho = 0.019 \times 10^{20}$					
1	24,370	0.27×10^{-5}	0	0.857×10^{-5}	0 - 1.5
2	25,930	$.26 \times 10^{-5}$		$.657 \times 10^{-5}$	1.5 - 2.4
3	13,800	$.572 \times 10^{-5}$		$.114 \times 10^{-5}$	2.4 - 7.75
4	5,030	$.412 \times 10^{-4}$		$.661 \times 10^{-4}$	7.75 - 12.00
5	1,540	$.189 \times 10^{-3}$		$.361 \times 10^{-3}$	12.00 - 18.2
Thermal	241	$.116 \times 10^{-2}$		$.241 \times 10^{-2}$	19.8
H_2; $\rho = 0.8 \times 10^{21}$					
1	32.0	0	0	0	
2	32.0				
3	32.0				
4	32.5				
5	32.5				
Thermal	32.5				
D_2O					
1	3.52	0.535×10^{-3}	0.647×10^{-1}	0	
2	1.55	0	.179		
3	1.25	0	$.343 \times 10^{-1}$		
4	1.22	0	$.423 \times 10^{-1}$		
5	1.22	$.4 \times 10^{-5}$	$.29 \times 10^{-1}$		
Thermal	.84	$.29 \times 10^{-4}$	0		
Zr					
1	1.284	0	0	0	
2	1.284	0			
3	1.284	0			
4	1.284	0			
5	1.284	$.212 \times 10^{-2}$			
Thermal	1.284	$.846 \times 10^{-2}$			

(b) Four-group

U^{235}; $\rho = 0.1 \times 10^{20}$					
1	5480	0.14×10^{-4}	0	0.39×10^{-4}	0 - 1.9
2	5240	$.17 \times 10^{-4}$		$.38 \times 10^{-4}$	1.9 - 7.75
3	1120	$.224 \times 10^{-3}$		$.397 \times 10^{-3}$	725 - 18.2
Thermal	45.9	$.632 \times 10^{-2}$		$.131 \times 10^{-1}$	19.8
H_2; $\rho = 0.8 \times 10^{21}$					
1	32	0	0	0	
2	32	0	0	0	
3	32	0	0	0	
Thermal					
D_2O					
1	2.83	0.301×10^{-3}	0.93×10^{-1}	0	
2	1.25	0	$.344 \times 10^{-1}$		
3	1.22	$.3 \times 10^{-5}$	$.2 \times 10^{-1}$		
Thermal	.840	$.29 \times 10^{-4}$	0		

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TABLE II. - OVERALL NEUTRON BALANCE FOR SPHERICAL REACTOR

[1/4-in. zirconium structural shell; moderator thickness, 100 cm; moderator temperature, 70° F; cavity radius, r_o, 104 cm.]

(a) Neutron distribution with no fuel compression (r_o = 104 cm; r_i = 104 cm; W_s = 3.5 kg; $\bar{\varphi}/\varphi_{max}$ = 0.75)

Energy group	Losses				Production
	Absorptions			Leakage	
	Fuel	Zr	D_2O		
Fast Epithermal Thermal	1 12 475	0 2 66	3 0 44	0 0 397	2 22 976
Loss totals	488	68	47	397	1000
	603				
Balance	1000				

(b) Neutron distribution with fuel compression (r_o = 104 cm; r_i = 44 cm; W_s = 4.8 kg; $\bar{\varphi}/\varphi_{max}$ = 0.54)

Energy group	Losses					Production
	Absorptions				Leakage	
	Fuel	H_2	Zr	D_2O		
Fast Epithermal Thermal	2 16 468	0 0 0	0 2 68	3 0 44	0 0 397	4 29 967
Loss totals	486	0	70	47	397	1000
	603					
Balance	1000					

(c) Summary of effects on neutron distribution of fuel compression in spherical cavity (r_o, 104 cm)

Fuel radius, r_i, cm	Fuel loading, W_s, kg	k_{eff}	Neutron loss, percent		Thermal flux, φ/φ_{max}			
			Captures	Leakage	r = 0	r_i	r_o	$r_o + \delta_s$
104	3.5	1.0	60.3	39.7	0.75	0.76	0.76	0.77
44	3.5	.92	57.6	42.4	.54	.70	.80	.82
44	4.8	1.0	60.3	39.7	.39	.64	.76	.77

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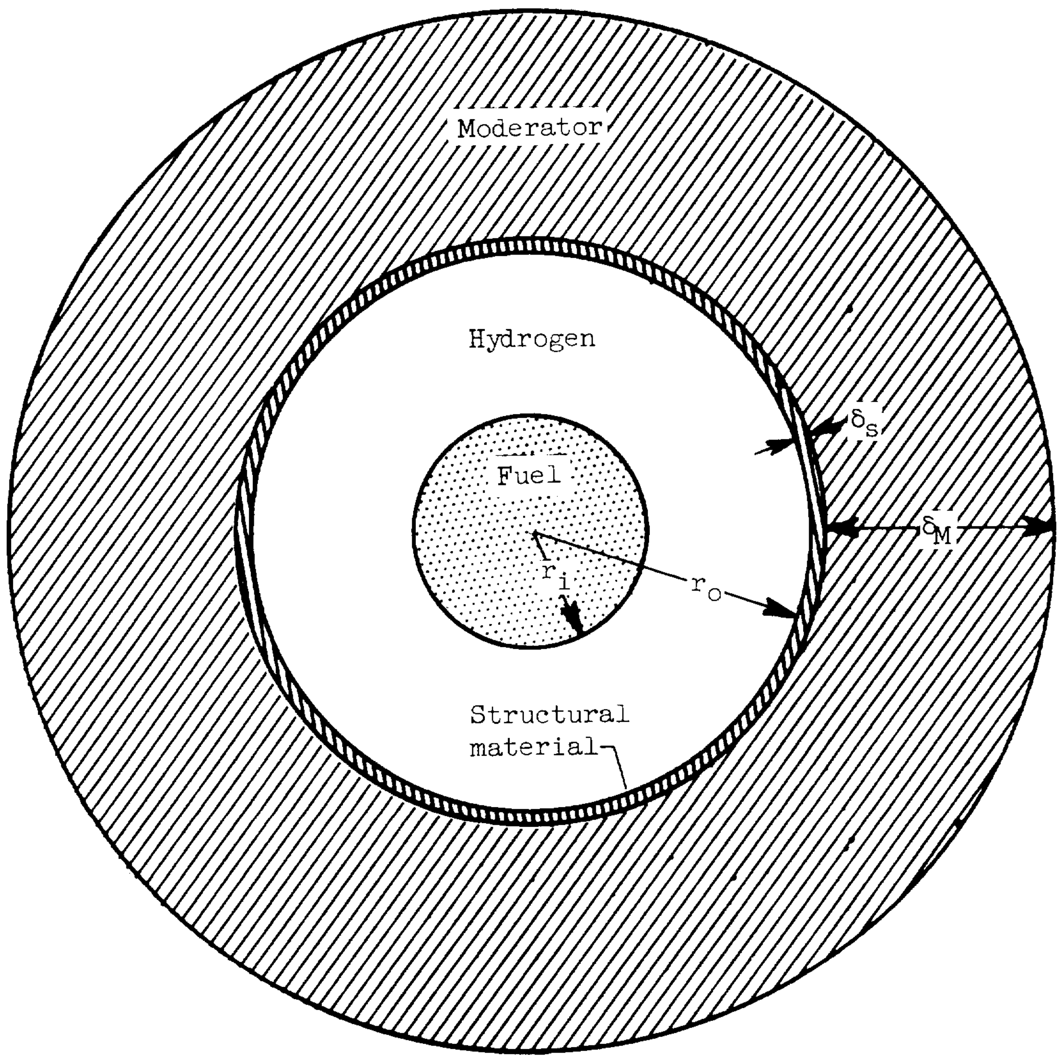

Figure 1. - Reactor model.

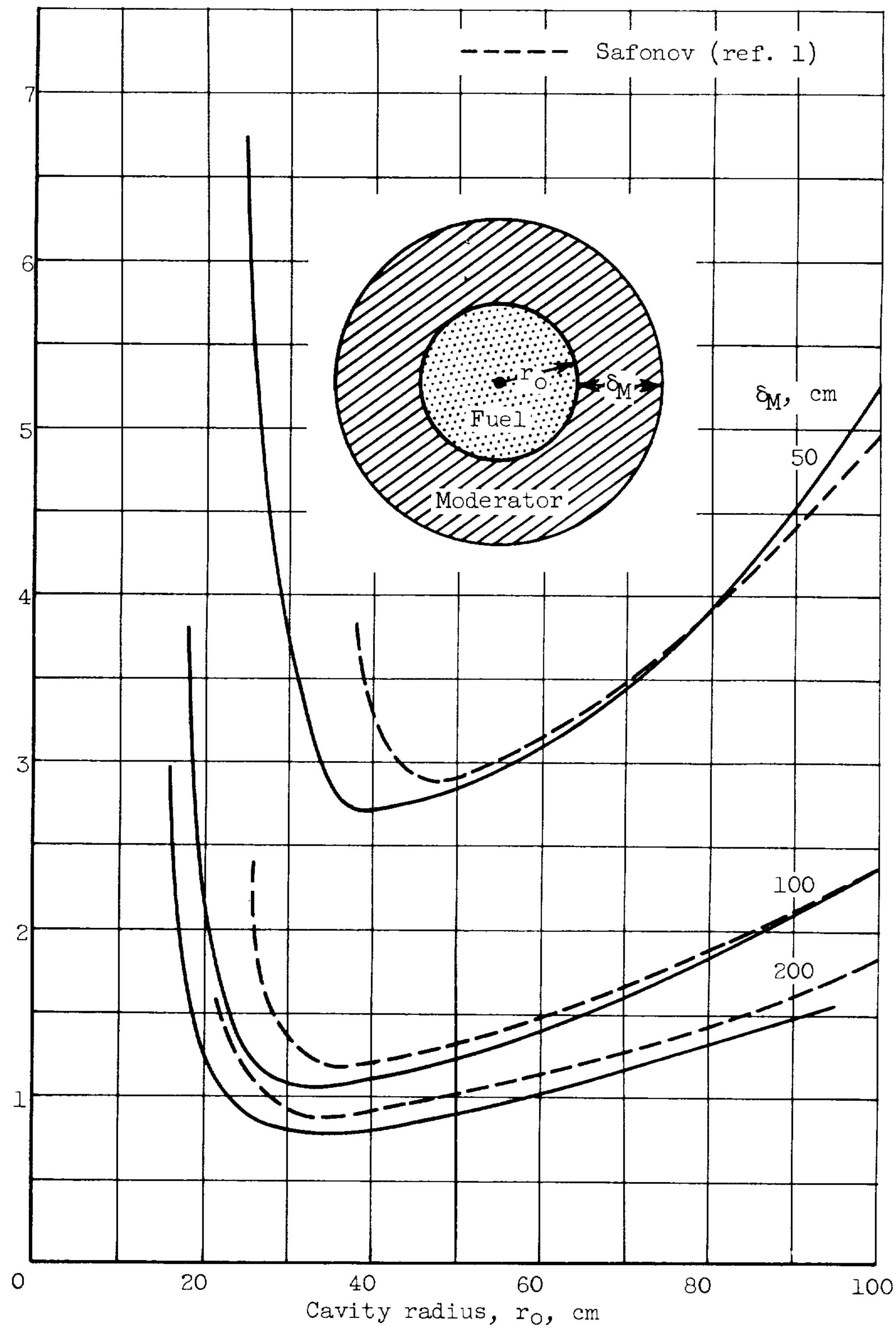

Figure 2. - Effect of moderator thickness on critical mass. Moderator temperature, 70° F; no structural material.

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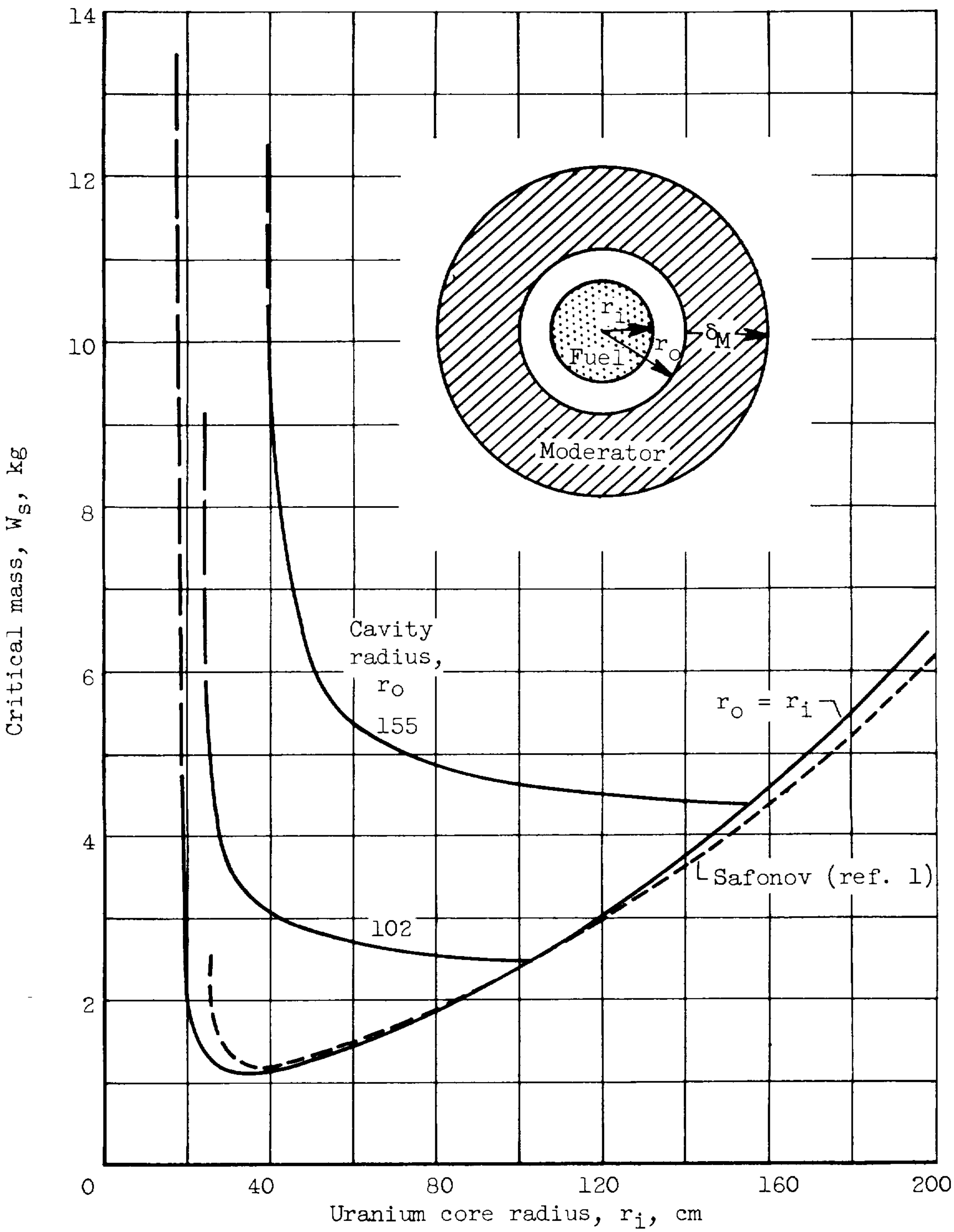

Figure 3. - Criticality map for spherical U^{235}-D_2O cavity reactor. Moderator thickness, 100 centimeters; moderator temperature, 70° F; no structural material.

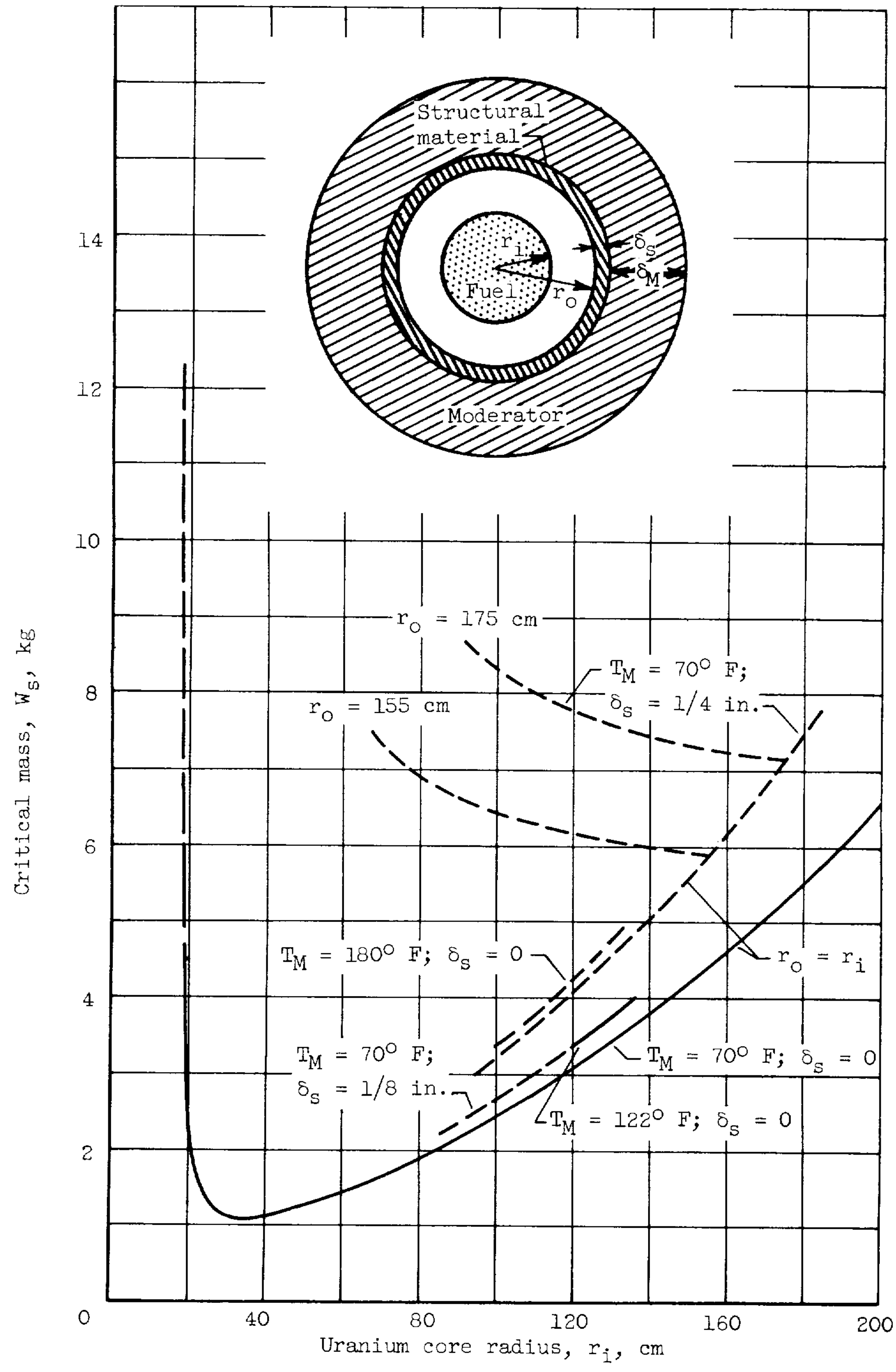

Figure 4. - Effect of structural materials and moderator temperature on reactor critical mass. Moderator thickness, 100 centimeters.

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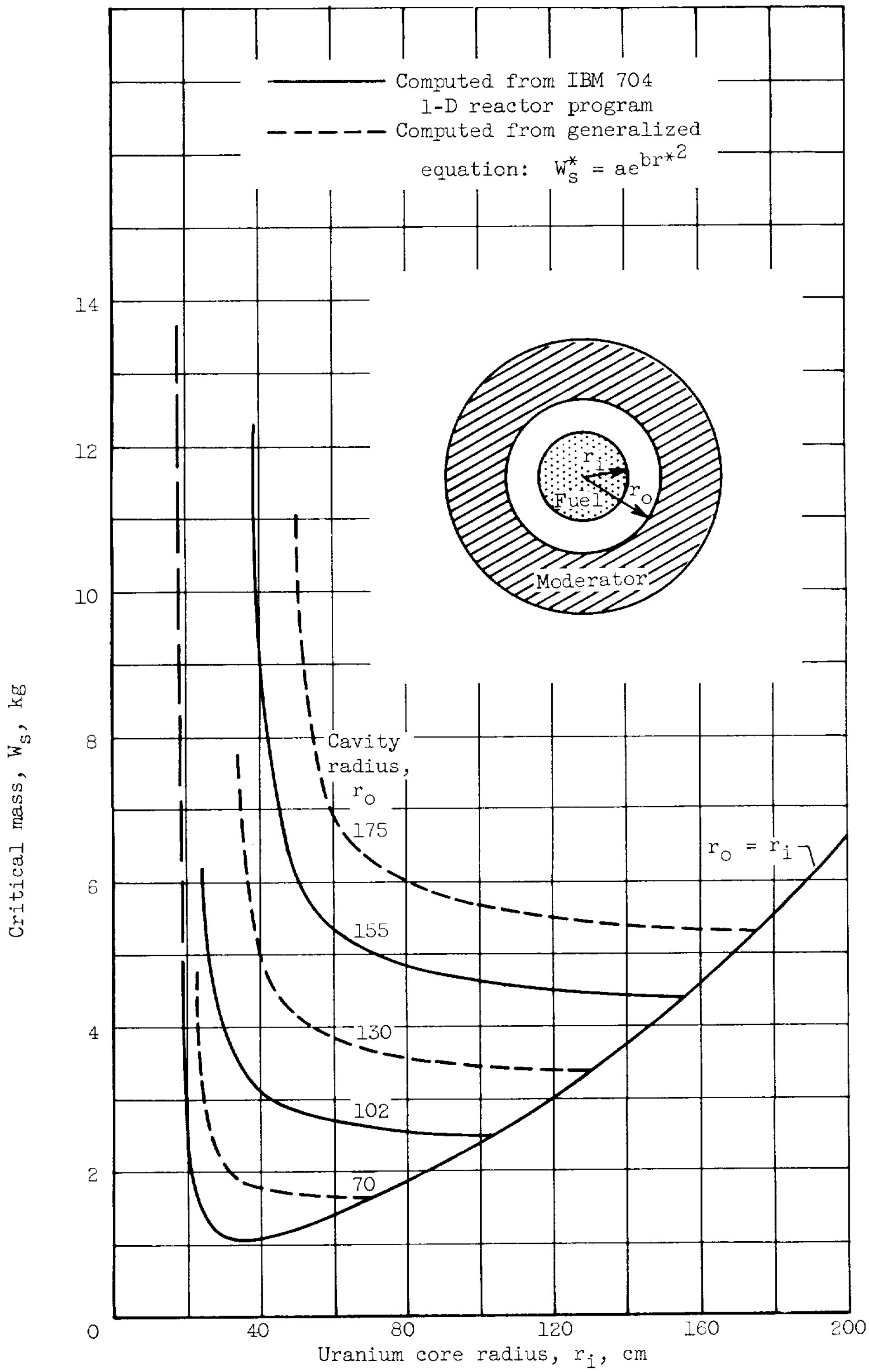
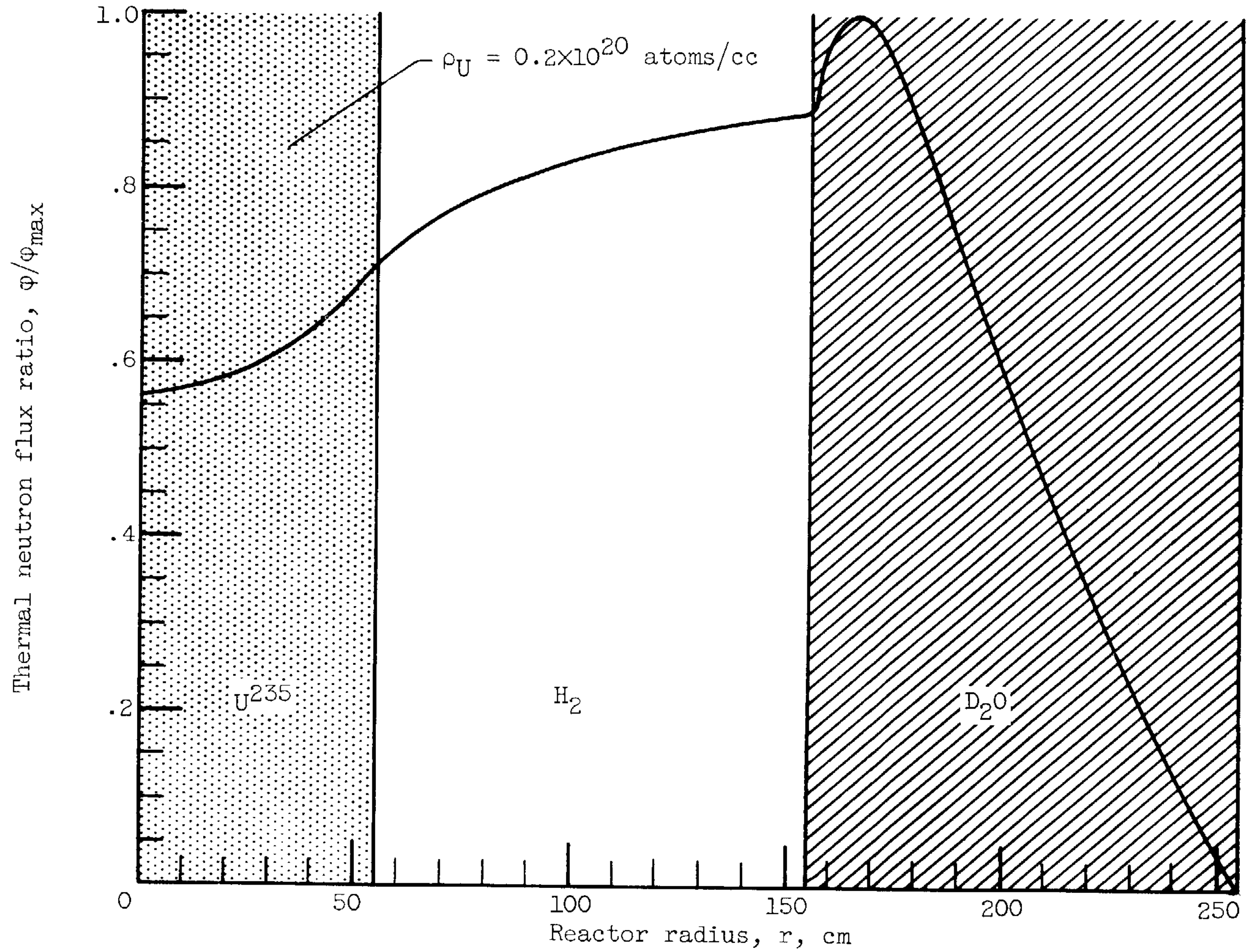
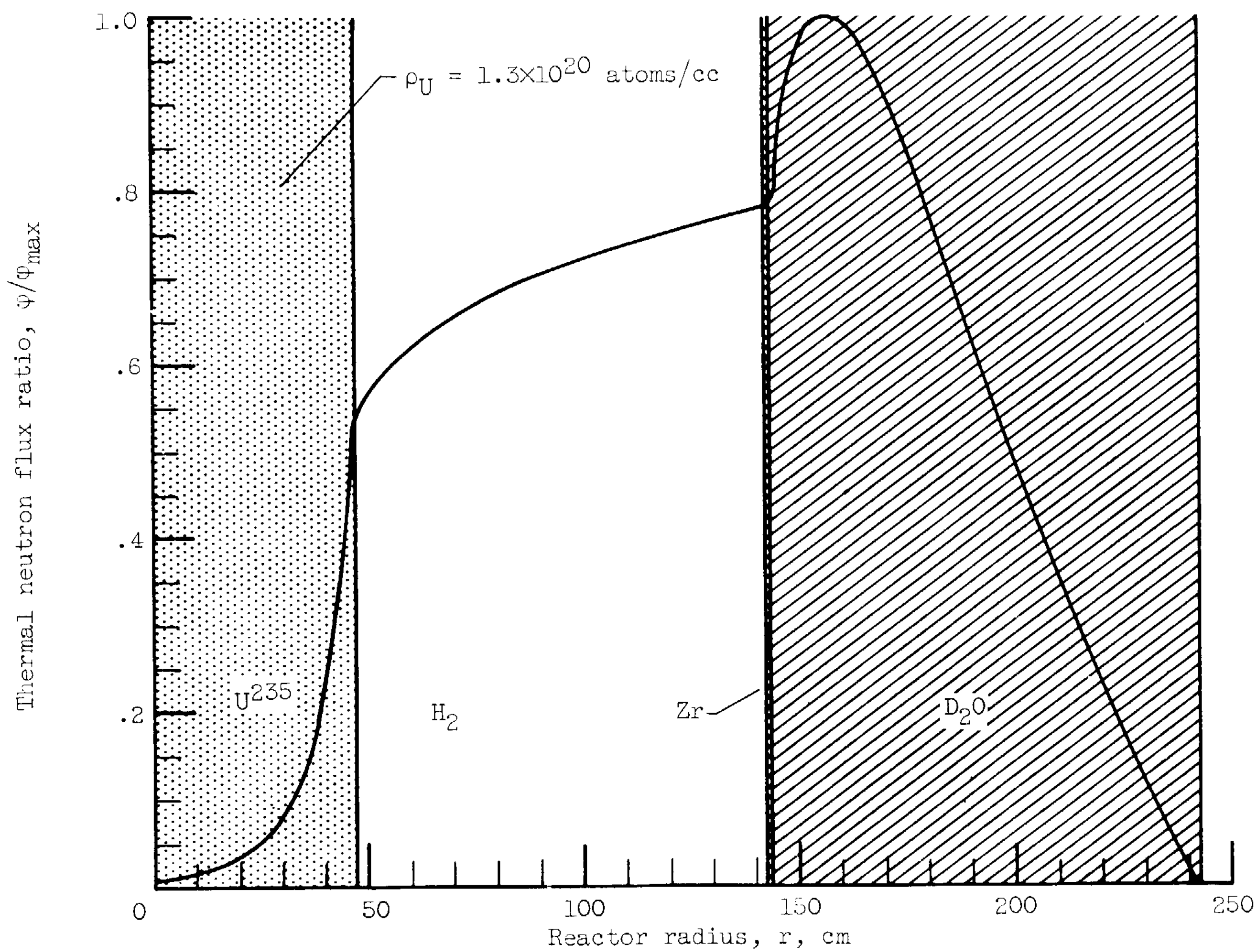


Figure 5. - Generalized criticality map for spherical cavity reactor. Moderator thickness, 100 centimeters; moderator temperature, 70° F; no structural material.



(a) Moderator temperature, 70° F; no structural material.

Figure 6. - Thermal flux distribution for spherical reactor. Moderator thickness, 100 centimeters.



(b) Moderator temperature, 180° F; 1/4-inch zirconium structural shell.

Figure 6. - Concluded. Thermal flux distribution for spherical reactor. Moderator thickness, 100 centimeters.

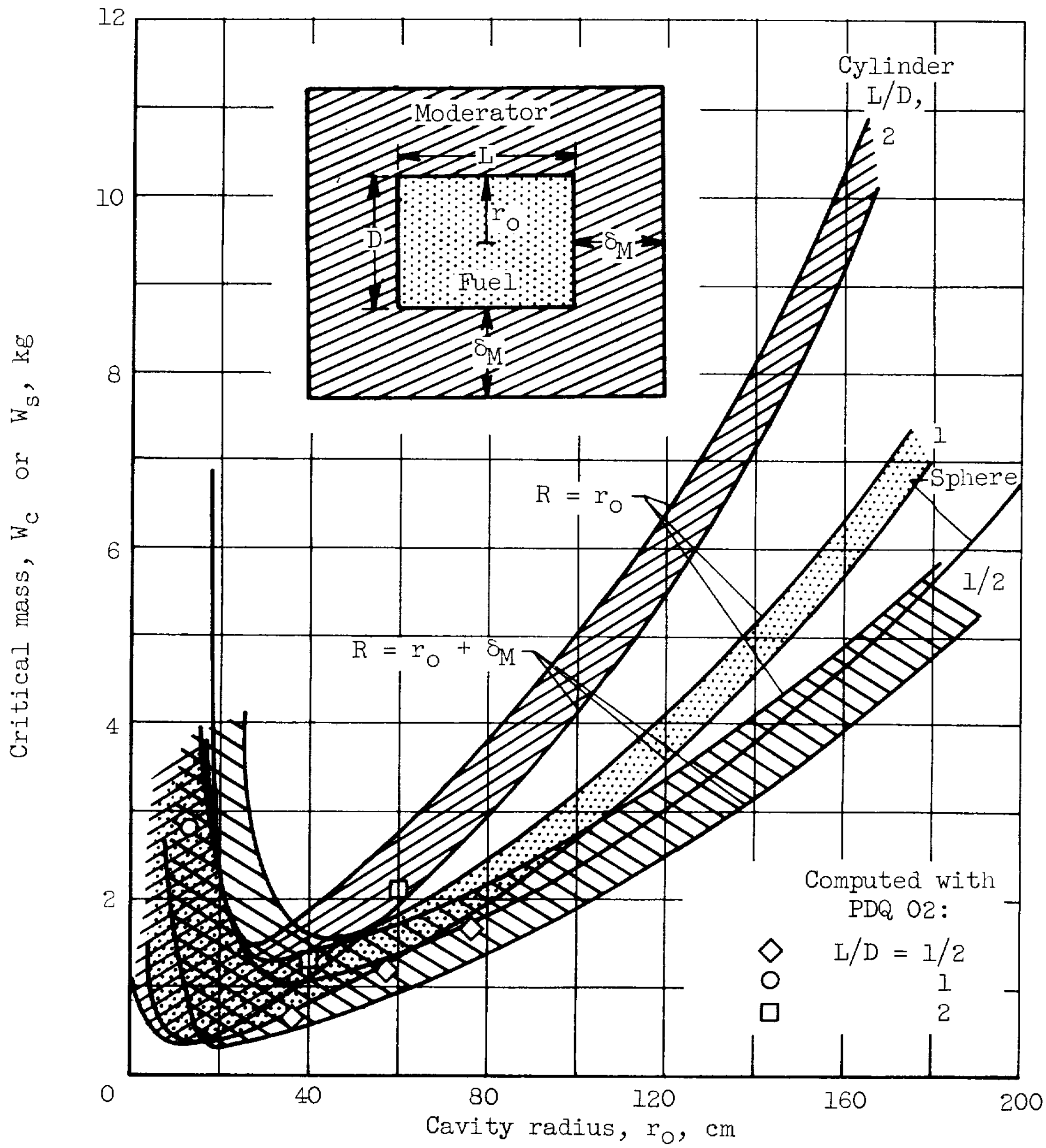


Figure 7. - Two-dimensional critical mass. Moderator thickness, 100 centimeters; moderator temperature, 70° F; no structural material; curves computed from buckling analogy: $B_s^2 = B_c^2$ at R, and $\rho_s = \rho_c$.

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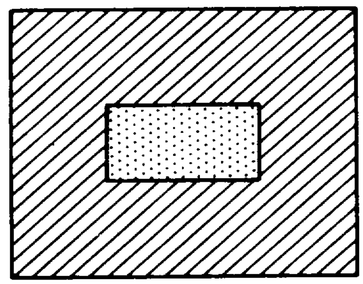
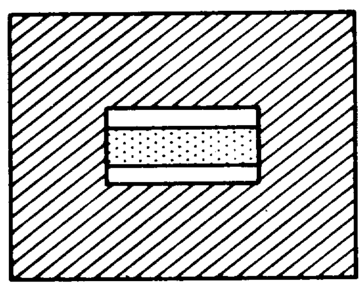
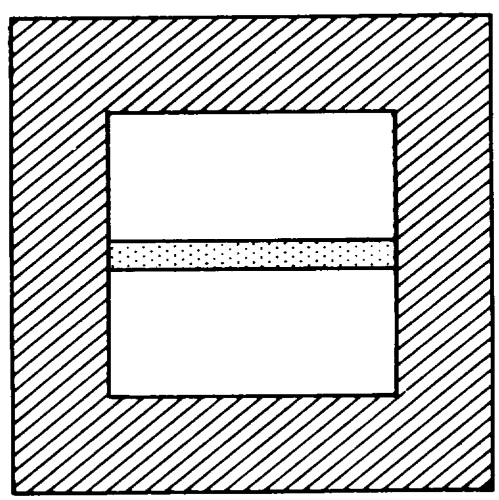


(a) r_o = 40 cm; r_i = 40 cm; L/D = 2;
W_c = 1.29 kg; ρ_U = 0.041×10^{20} atoms/cc.

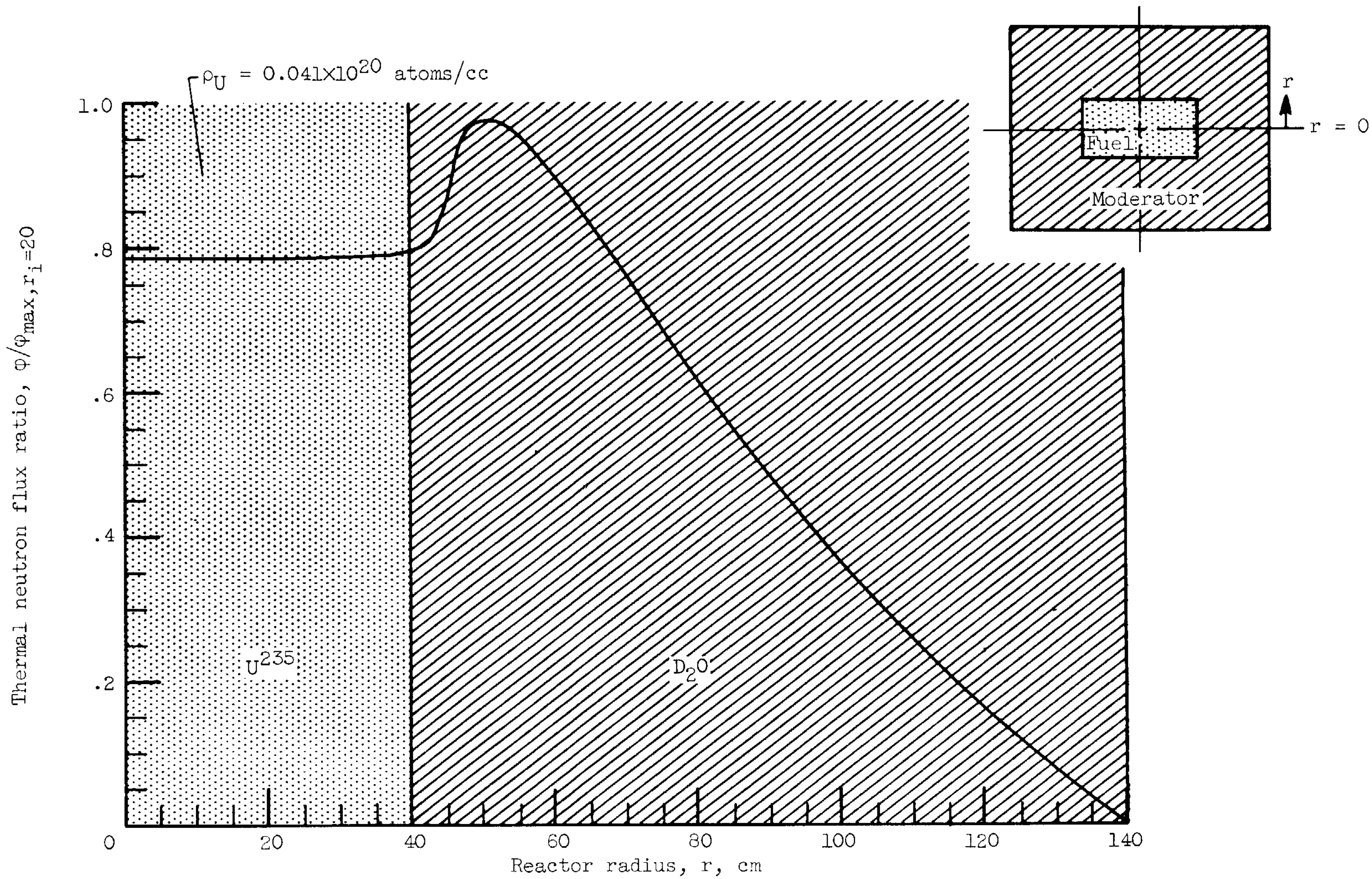


(b) r_o = 40 cm; r_i = 20 cm; L/D = 2;
W_c = 1.31 kg; ρ_U = 0.167×10^{20} atoms/cc.



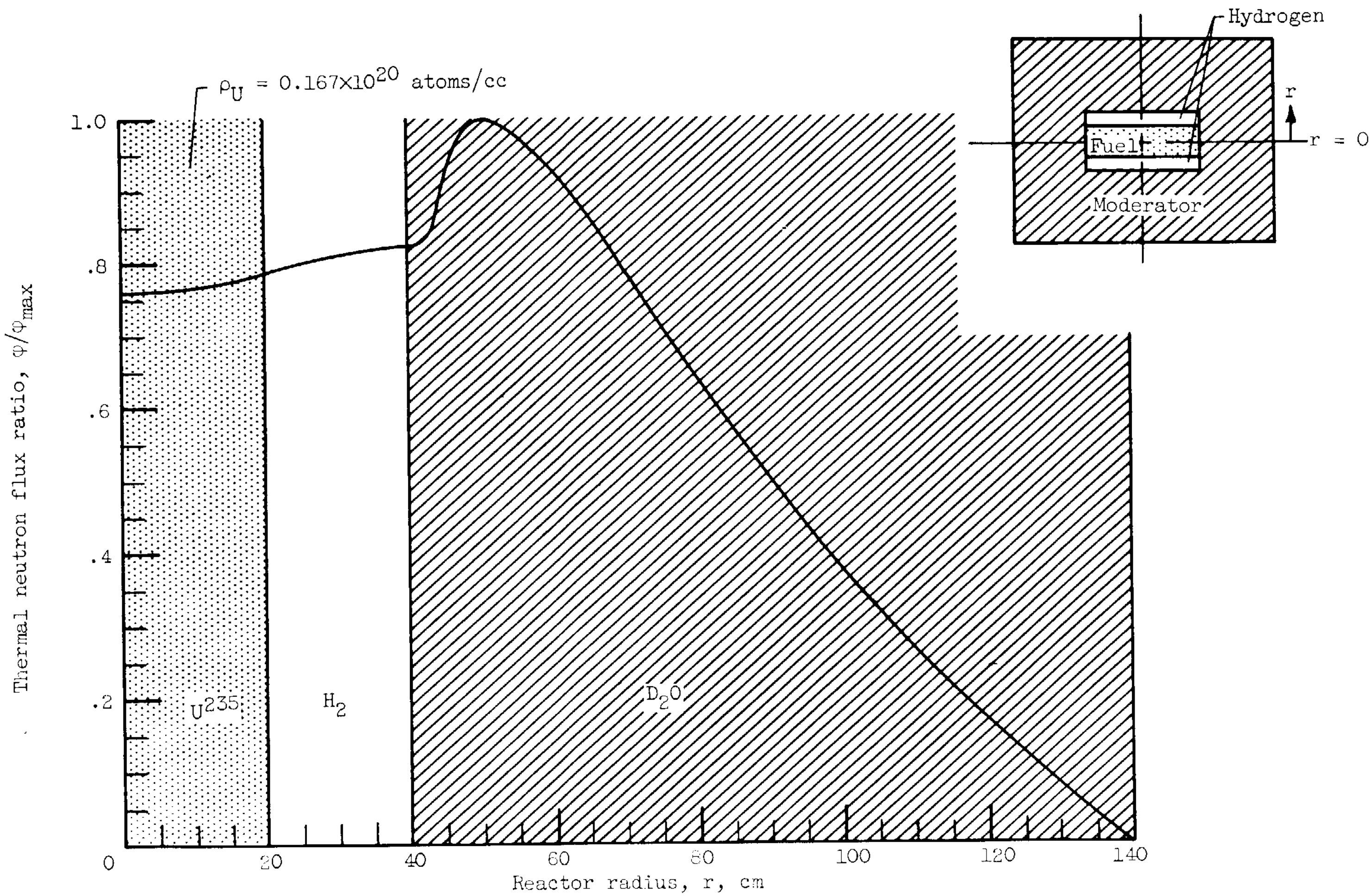
(c) r_o = 150 cm; r_i = 15 cm; L/D = 1;
W_c = 7.7 kg; ρ_U = 0.93×10^{20} atoms/cc.

Figure 8. - Some critical loadings of cylindrical reactors. Moderator thickness, 100 centimeters; moderator temperature, 70° F; no structural shell.



(a) r_o = 40 centimeters; r_i = 40 centimeters.

Figure 9. - Radial thermal flux distribution for cylindrical reactor. Moderator thickness, 100 centimeters; moderator temperature, 70° F; no structural shell; reactor cavity L/D = 2.



(b) r_o = 40 centimeters; r_i = 20 centimeters.

Figure 9. - Concluded. Radial thermal flux distribution for cylindrical reactor. Moderator thickness, 100 centimeters; moderator temperature, 70° F; no structural shell; reactor cavity L/D = 2.

NASA TN D-475
National Aeronautics and Space Administration.
SOME NUCLEAR CALCULATIONS OF U^{235}-D_2O GASEOUS-CORE CAVITY REACTORS. Robert G. Ragsdale and Robert E. Hyland. October 1961. 31p. OTS price, $1.00. (NASA TECHNICAL NOTE D-475)

Nuclear characteristics are analyzed for a spherical cavity reactor with a gaseous U^{235} core surrounded by a region of hydrogen gas and enclosed by an external D_2O moderator reflector. Critical masses for moderator thicknesses of 50, 100, and 200 centimeters are obtained from a one-dimensional, six-group diffusion analysis. Curves are presented to show the effects on critical mass of: shrinking the fuel region within the cavity, adding a zirconium structural wall between moderator and fuel regions, and moderator heating from 70° to 180° F. Thermal flux distributions are shown. Fully reflected cylindrical models are analyzed with a two-dimensional code.

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