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EARTH OBLATENESS AND RELATIVE SUN MOTION CONSIDERATIONS IN THE DETERMINATION OF AN IDEAL ORBIT FOR THE NIMBUS METEOROLOGICAL SATELLITE

William R. Bandeen

Goddard Space Flight Center
Greenbelt, Maryland

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

It is desired that the Nimbus meteorological satellite always cross the equator around local noon and, half-an-orbit later, cross the equator in the other direction around local midnight. The application of the phenomenon of nodal regression toward this end is discussed, and an analysis of the parameters angles of inclination, periods, and heights of such "ideal" circular orbits is presented. Also, the relative motion of the apparent versus the fictitious mean sun is briefly discussed.

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INTRODUCTION

The Nimbus meteorological satellite will carry improved television, radiation, and possibly other types of experiments in a 600 nautical mile circular orbit about the earth for a nominal instrumental lifetime of six months. Because it will be earth oriented, Nimbus will rotate exactly once about its pitch axis during each orbit.

For maximum earth coverage, a near-polar orbit is planned. In order to collect data under conditions of high solar elevation angles in the sunlit portion of the orbit and conditions of maximum diurnal change over the entire orbit, it is desired that the satellite always cross the equator near local noon and, half-an-orbit later, cross in the other direction near local midnight. If the orbital nodes did not regress, the relative motion of the sun would change such a "noon-midnight" orbit into a "sunrise-sunset" orbit (at the equator) in three months (Figure 1).

However, the phenomenon of nodal regression can be used to advantage by prescribing an orbit with characteristics such that the line of nodes advances (eastward) in right ascension (R.A.) along the celestial equator at the same rate as the mean sun, i.e., 0.9856 degrees per day. Under these conditions a noon-midnight orbit would continue indefinitely. Throughout this paper the term "ideal orbit" will refer to a retrograde circular orbit whose nodal regression is 0.9856 degrees per day, exactly that of the mean sun's relative motion.

THEORY OF THE REGRESSION OF THE LINE OF NODES

The earth has the shape of an oblate spheroid with an equatorial diameter some 43 kilometers greater than the polar diameter. Hence, the earth's gravitational field is not a central force field but is distorted by this oblateness. For purposes of illustration the satellite orbit can be considered a huge wheel. The gravitational attraction between the equatorial bulge and the rim of the orbit wheel exerts forces directed outside the orbital plane which create a torque about the line of nodes, tending to turn the orbital plane into the earth's equatorial plane. Considering the dynamics of precession, torque is the rate of change of angular momentum; hence the angular momentum vector of the orbit (which is perpendicular to the orbital plane) moves in the direction of the torque vector. However, after the angular momentum vector moves an infinitesimal distance in the direction of the line of nodes, this line rotates through an infinitesimal angle, resulting in a conical motion or precession of the orbital angular momentum vector about a fixed axis — in this case the earth's axis. Hence, the inclination angle of the orbital plane remains constant (within the scope of this discussion) but the line of nodes regresses (i.e., it moves around the equator in a direction opposite to the motion of the satellite in orbit). For a satellite revolving in orbit in the direction of the earth's rotation, the nodal regression is from east to west; and for a satellite revolving in orbit against the direction of rotation of the earth (a retrograde orbit), the nodal regression is from west to east (Figure 1).

EQUATION FOR NODAL REGRESSION

Only circular orbits will be considered here because it can be shown that, with reasonably good injection guidance, the slightly elliptical departure from a circular orbit resulting from injection errors is negligible compared to other effects. The equation for the regression of the nodes of a circular orbit about the earth is

$$\frac{\Delta\Omega}{t} = -6\mu \left(\frac{a}{r}\right)^2 \sqrt{\frac{KM}{r^3}} \cos i \, , \tag{1}$$

where

$\Delta\Omega$ = change in "absolute longitude" (R.A.)

t = unit of time,

μ = constant specifying the quadrupole strength of the earth's gravitational field (due to the earth's oblateness),

a = earth's equatorial radius,

r = magnitude of the radius vector from the earth's center to the satellite,

K = gravitational constant,

M = mass of the earth,

i = angle between the equatorial plane and the orbital plane (more specifically, the angle between the earth's spin vector and the orbital spin vector, e.g., for a retrograde orbit $i > 90°$).

Substituting the numerical values

$$6\mu = 1.637 \times 10^{-3},$$

$$a = 6{,}378.388 \text{ km},$$

$$KM = 3.986329 \times 10^{5} \text{ km}^3/\text{sec}^2,$$

$$t = 1 \text{ day} = 86{,}400 \text{ sec},$$

into Equation 1 and multiplying by 57.29578 degrees/radian, we have

$$\dot{\Omega} = -\frac{20.8158 \times 10^{13}}{r^{\frac{7}{2}}} \cos i \,, \tag{2}$$

where r is in kilometers and $\dot{\Omega}$ is the rate of regression of the orbital nodes in degrees/day (positive indicates eastward and negative westward motion).

ORBITAL PERIOD

Equating the acceleration due to gravity with the centripetal acceleration of a circular orbit, we have

$$\frac{KM}{r^2} = \left(\frac{2\pi}{P}\right)^2 r \,, \tag{3}$$

where P is the nominal period for one orbit. Substituting numerical values into Equation 3, we have

$$P = 1.6586 \times 10^{-4} \, r^{\frac{3}{2}} \,, \tag{4}$$

where P is in minutes and r in kilometers.

Equations 2 and 4 were used to calculate the values of i and P for various values of r of a circular orbit. In Equation 2, $\dot{\Omega}$ was set at 0.9856 degrees/day, the same as the rate of advance in the R.A. of the mean sun. In Figure 2 the height of a circular orbit, H, is $r - 6371.2$ kilometers, where 6371.2 kilometers is the radius of a spherical earth having the same volume as the actual earth. The number of orbits per day equals $1440/P$. For example, it is seen in Figure 2 that an ideal circular orbit, having a height above the surface of the earth of 600 nautical miles (691 statute miles or 1112 kilometers), has an inclination of 99.89 degrees and a period of 107.4 minutes, and that there are 13.41 orbits per day.

EFFECT OF INJECTION ERRORS ON NODAL REGRESSION

Differentiating Equation 2, we have

$$d\dot{\Omega} = -\dot{\Omega}\left[\frac{\tan i}{57.2958}\,di + \frac{7}{2}\,\frac{dr}{r}\right], \tag{5}$$

where di is in degrees. Using Equation 5, the deviation of the line of nodes of a 600 nautical mile circular orbit from the R.A. of the mean sun versus days after launch was calculated for partial errors in i and H (Figures 3 and 4 respectively).

The 3σ errors expected in the Nimbus orbit are ± 1 degree in i and ± 40 nautical miles in H using the Thor-Agena B launch vehicle. From Figure 3 it is seen that with an injection error of 1.0 degree in i, the deviation after six months is about 18 degrees; and from Figure 4 it is seen that with an injection error in H of 40 nautical miles the deviation after six months is only about 6.2 degrees.

DEVIATION OF THE APPARENT SUN FROM THE MEAN SUN

During the course of a year the earth-sun vector will sweep an arc of about 47 degrees in astronomical declination (earth latitude), ranging from N23-1/2 degrees at the summer solstice (June 22) to S23-1/2 degrees at the winter solstice (December 22). Moreover, even though an ideal orbit is achieved whose line of nodes advances at the rate of 0.9856 degrees per day (the same as the rate of advance in R.A. of the mean sun), the R.A. of the apparent sun will differ at times from that of the mean sun by more than 4 degrees. These deviations are due mainly to two causes: (1) the variable revolution of the earth around the sun, owing to the eccentricity of its orbit and (2) the obliquity (or inclination to the celestial equator by about 23-1/2 degrees) of the ecliptic. The "equation of time" in terms of the amount by which the R.A. of the apparent sun differs from that of the mean sun, versus calendar date is shown in Figure 5. It is seen that the two are the same only four times throughout the entire year. The greatest deviation occurs shortly after November 1 when the R.A. of the apparent sun is more than 4 degrees behind that of the mean sun.

LOCAL TIME OF SATELLITE PASSAGE AS A FUNCTION OF LATITUDE

If an ideal orbit is achieved so that the ascending node occurs always at local noon, it is evident from Figure 1 that passage of the satellite at any other latitude will always occur at some time other than local noon. By solving the applicable right spherical triangle we have:

$$\Delta t = 4 \cdot \arcsin \left[\tan (i - 90°) \tan \phi \right], \tag{6}$$

where the arc sin and i are in degrees, ϕ is the latitude (+ for north, - for south), and Δt is the number of minutes, local time, by which satellite passage over latitude ϕ differs from the local time of the ascending node. For example, if for a 600 nautical mile ideal orbit (i = 99.89 degrees from Figure 2) the ascending node occurs at local high noon, the local time of passage at 40 degrees north latitude will always be

$$12{:}00 - 4 \arcsin (\tan 9.89° \tan 40°) = 11{:}26 \text{ a.m. local time},$$

from Equation 6.

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CONCLUDING REMARKS

The brief and simplified application of orbital theory and the associated analyses presented herein are intended for use in planning the Nimbus program. Terms of higher order than the second in the earth's gravity potential, and the perturbations caused by them, were neglected. However, the important criteria for determining the Nimbus orbital characteristics are included. A more sophisticated approach will, of course, be used in the final launch program and possibly in other specialized subsystem investigations.

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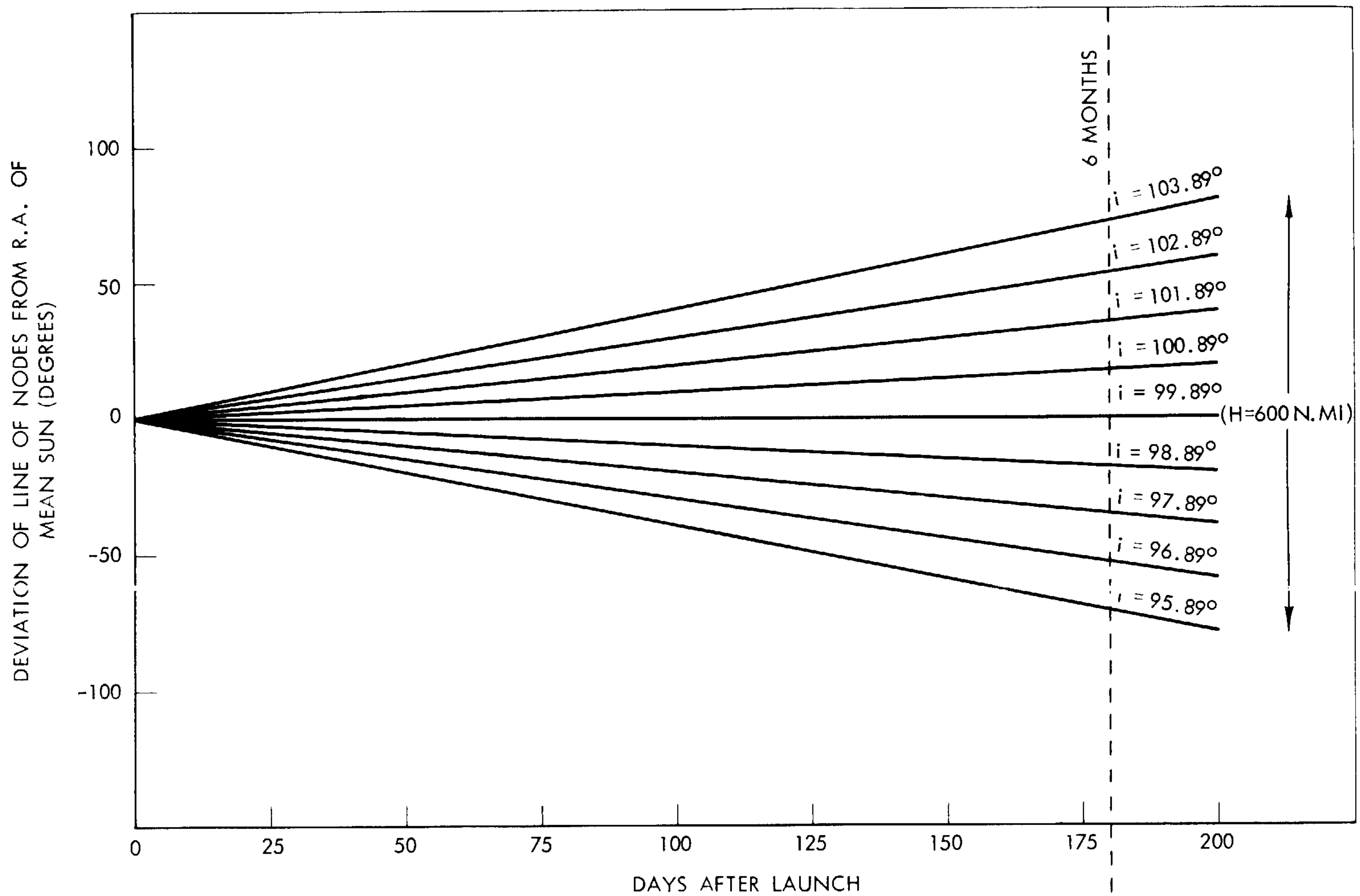

Figure 3 – Deviation of the line of nodes from the R.A. of the mean sun due to an injection error in the angle of inclination i of an "ideal" 600 nautical mile circular orbit, versus days after launch

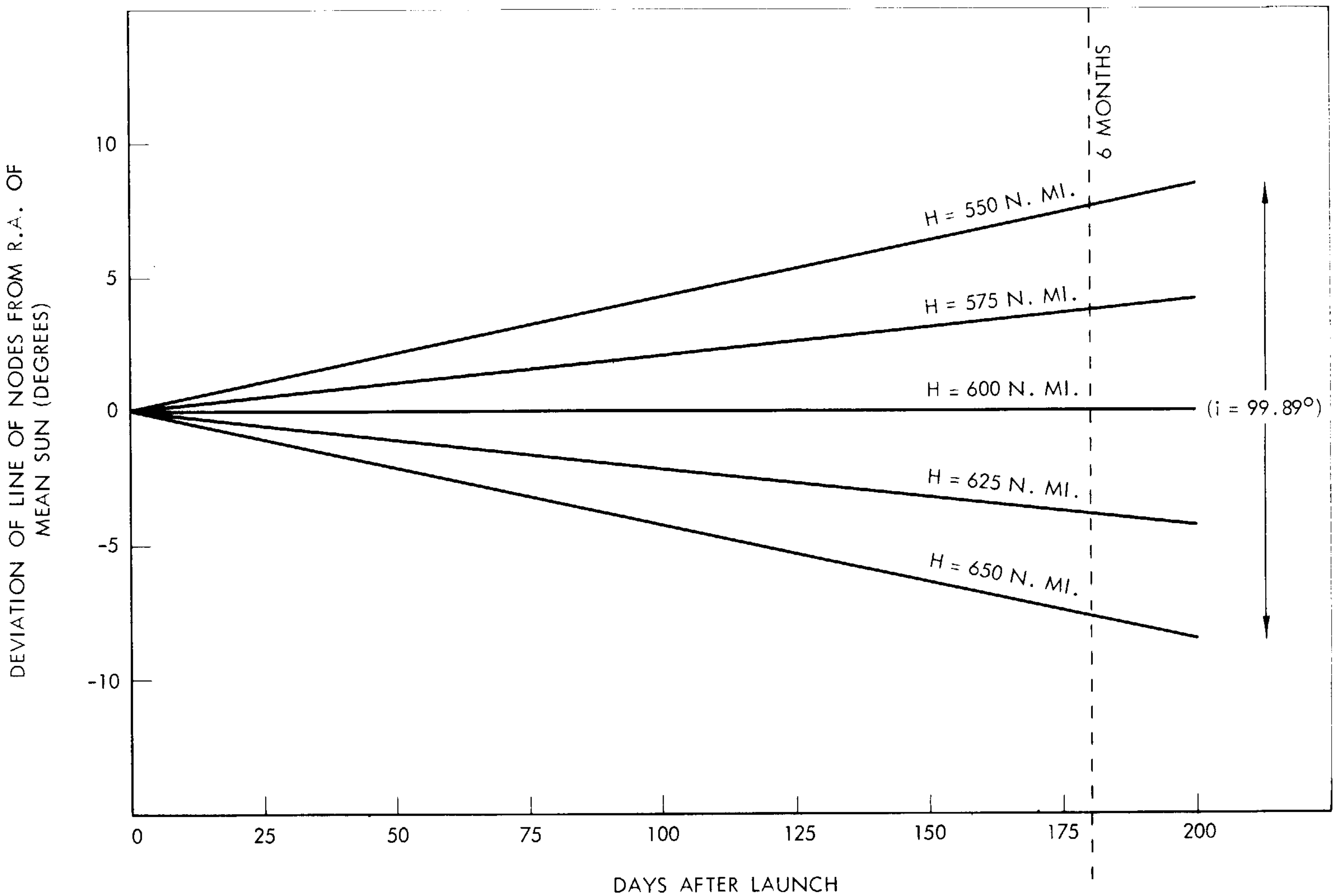

Figure 4 – Deviation of the line of nodes from the R.A. of the mean sun due to an injection error in the height H of an "ideal" 600 nautical mile circular orbit, versus days after launch

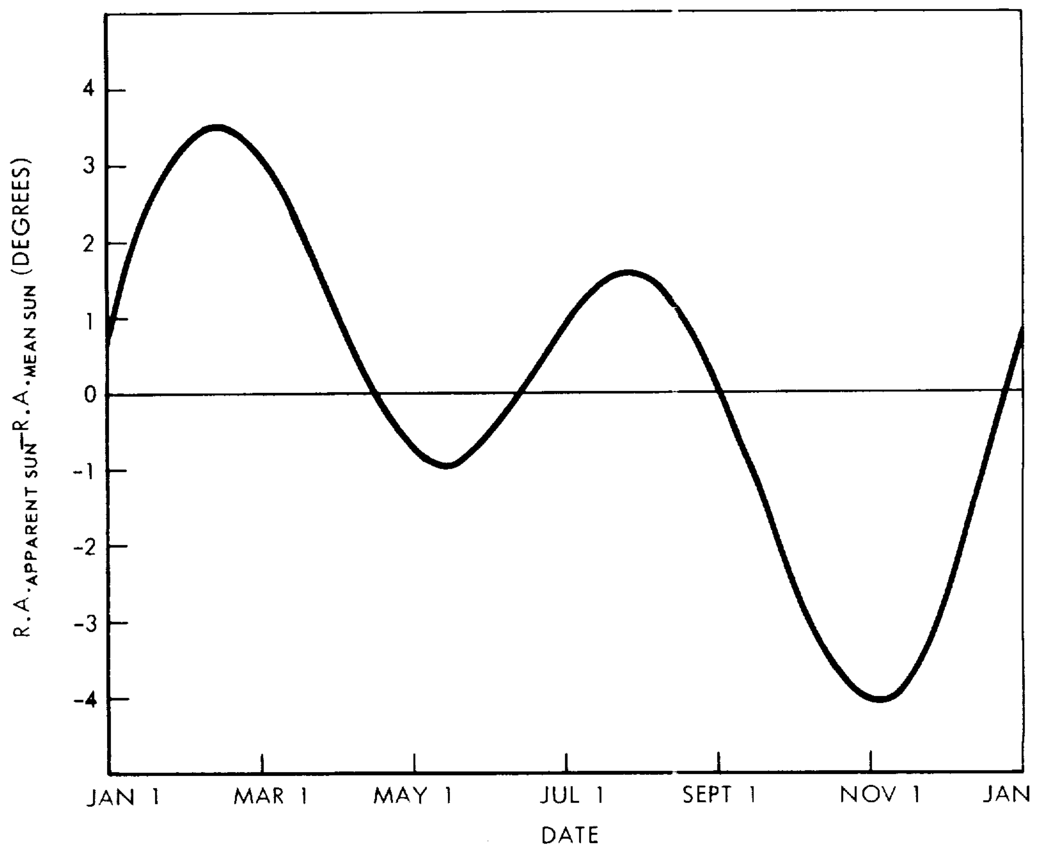

Figure 5 – Difference in the R.A. of the apparent sun and the mean sun, versus calendar date