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ANALYSIS OF HEAT TRANSFER AND PRESSURE DROP FOR A GAS
FLOWING THROUGH A SET OF MULTIPLE PARALLEL
FLAT PLATES AT HIGH TEMPERATURES

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SUMMARY

Equations were derived representing heat transfer and pressure drop for a gas flowing in the passages of a heater composed of a series of parallel flat plates. The plates generated heat which was transferred to the flowing gas by convection. The relatively high temperature level of this system necessitated the consideration of heat transfer between the plates by radiation.

The equations were solved on an IBM 704 computer, and results were obtained for hydrogen as the working fluid for a series of cases with a gas inlet temperature of 200°R , an exit temperature of 5000°R , and exit Mach numbers ranging from 0.2 to 0.8. The length of the heater composed of the plates ranged from 2 to 4 feet, and the spacing between the plates was varied from 0.003 to 0.01 foot. Most of the results were for a five-plate heater, but results are also given for nine plates to show the effect of increasing the number of plates. The heat generation was assumed to be identical for each plate but was varied along the length of the plates. The axial variation of power used to obtain the results presented is the so-called "2/3-cosine variation." The boundaries surrounding the set of plates, and parallel to it, were assumed adiabatic, so that all the power generated in the plates went into heating the gas.

The results are presented in plots of maximum plate and maximum adiabatic wall temperatures as functions of parameters proportional to $f(L/D)$, for the case of both laminar and turbulent flow. Here f is the Fanning friction factor and (L/D) is the length to equivalent diameter ratio of the passages in the heater. The pressure drop through the heater is presented as a function of these same parameters, the exit Mach number, and the pressure at the exit of the heater.

One of the more interesting effects associated with this heater geometry was the nonuniform distribution of weight flow between the passages, despite the fact that the heat generation for all plates was the

same and that the spacings between the plates were identical. It was found that the magnitude of this nonuniformity of flow distribution increased with decreasing $f(L/D)$. Another unusual effect was that, when the $f(L/D)$ ratio of the heater was decreased, although the maximum plate temperature became higher as expected, the maximum adiabatic wall temperature actually decreased substantially.

INTRODUCTION

The analysis of heat transfer by forced convection to a fluid flowing in a single passage between a pair of parallel flat plates is well known for the cases of both turbulent and laminar flow, and is thoroughly treated in the literature. However, little work has been done on treating the problem of heat transfer to flow in a series of parallel passages formed by a set of multiple parallel flat plates. The basic relations governing heat transfer from the plates to the fluid in the passages remain unchanged from those in a single passage; however, the problem is now complicated because there will be interaction between the heat transfer to one passage and that to adjacent passages, resulting in nonuniform plate temperatures and nonuniform flow distribution between the passages. In addition to the foregoing complications that arise in the multipassage case, if the heater formed by the set of parallel plates is operated at a high temperature, then radiation between the plates becomes significant. This effect must also be considered. It is therefore the purpose of this report to present an analysis of this heat-transfer geometry, and to obtain some quantitative results at high temperatures, which reflect the effect of interaction between the passages and take into consideration the effect of radiation between the plates.

Before proceeding, one might justifiably raise the question of where this heat-transfer geometry would find application. In the general case, the plates would be heated either electrically, or by nuclear fission, the latter alternative implying the use of this geometry as a fuel element in a nuclear reactor. Many of the stationary nuclear reactor powerplants in use today use this scheme for a fuel element, and the application of this geometry for a fuel element is of interest for nuclear rocket reactors.

This report, then, describes an analysis of heat transfer and pressure drop in a high-temperature heater consisting of parallel flat plates. The analysis considers the effects of radiation between the plates and allows for both laminar and turbulent flow of the working fluid. Provisions are also made for adjusting the division of fluid flow between the passages such that the static-pressure drops in all the passages will be equalized.

Specific results will be presented using hydrogen as the gas flowing through the heater. Although hydrogen partially dissociates, resulting in marked increases in heat-transfer coefficient at the higher temperatures for which the results are presented, and is thus in this respect not typical of the behavior of most gases at more moderate temperatures, this gas was chosen for the presentation of results because of the high level of current interest in it as a propellant for space vehicles.

SYMBOLS

C_p	specific heat of gas, Btu/lb-°R
D	equivalent diameter of passages, ft
$\mathcal{F}$	radiation form factor, Btu/ft ² -sec-°R ⁴
f	Fanning friction factor, dimensionless
G	mean mass flow of gas, lb/(sec-ft ²)
g	acceleration of gravity, 32.2 ft/sec ²
h	convective heat-transfer coefficient, Btu/sec-ft ² -°R
J	total number of plates in heater
k	thermal conductivity of gas, Btu/sec-ft-°R
L	total length of plates, ft
M	Mach number, dimensionless
N	number of passages in one symmetrical half of heater
Nu	Nusselt number, dimensionless
P	overall static pressure drop, lb/ft ²
Pr	Prandtl number, dimensionless
p	local static pressure, lb/ft ²
q	heat flux, Btu/sec-ft ²
R	gas constant, ft/°R
Re	Reynolds number, dimensionless

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s plate spacing, ft

T surface or plate temperature, $^{\circ}\text{R}$

t total gas temperature, $^{\circ}\text{R}$

t_s static gas temperature, $^{\circ}\text{R}$

V gas velocity, ft/sec

w passage weight flow, lb/sec

x distance from inlet, ft

γ ratio of specific heats, dimensionless

ϵ radiation emissivity, dimensionless

μ viscosity of gas, lb/ft-sec

ρ density of gas, lb/ft³

σ Stefan-Boltzmann constant, 4.75×10^{-13} Btu/sec-ft²- $^{\circ}\text{R}^4$

Subscripts:

b bulk conditions

f film conditions

g generated

i axial increment number

in inlet conditions

j trial number in iteration for pressure drop equalization

l trial number in iterative solution of plate heat balance equations

n plate or passage number

o exit conditions

x axial location

Superscripts:

— mean value

' from interior side of a plate

ANALYSIS

A sketch of the parallel plate heater under consideration is shown in figure 1. The analysis of its operation will be divided into three distinct sections. First, the analysis of combined forced convection and radiation heat transfer from and between the plates will be presented, and the many simplifying assumptions that are made in this area will be discussed. The second phase of the analysis to be covered is the calculation of the combined friction and momentum pressure drop in each passage. Finally, the iterative method of readjusting the assumed weight flows in each passage to cause equalization of the pressure drops in all the passages is illustrated.

Heat Transfer

Because of the allowance in the analysis for variable fluid properties, variable heat-transfer coefficient, arbitrary variations in power distribution, and the inclusion of radiation in this heat-transfer problem, a closed-form solution is out of the question. Therefore, the solution must be obtained numerically by breaking the heater up into finite segments and solving the heat-balance equations separately for each segment. The heater was therefore broken up into 100 axial segments, and the continuous variation of plate surface temperature and gas temperature along the length of the heater is then approximated by the temperature distribution of these 100 segments. The segments are considered small enough that the plate and gas temperatures of each segment may be considered to be constant over the length of that segment. The number 100 was chosen arbitrarily as large enough to ensure a sufficiently accurate approximation to the actual temperature distribution over the length of the heater, and yet not so large as to require an excessively long time for the computation of the solution. Each segment of length is then composed of J plates and $J + 1$ passages. In order to simplify the computations associated with the analysis, the number of plates J was arbitrarily taken to be odd, and the arrangement of plate spacings and power generations, though not necessarily all identical, was assumed to be symmetrical about the center plate.

Therefore, it becomes necessary to apply the analysis to only one-half of the segment, from the center plate to the outer wall. The arrangement of plates in each segment that is considered in the analysis is illustrated in figure 2. The heat transfer in each segment is by radiation between the plates, and by convection from the plates to the gas flowing in the adjacent passages.

For radiation:

$$q_{\text{rad}} = \mathcal{F}(T_n^4 - T_{n-1}^4) \quad (1)$$

The coefficient $\mathcal{F}$ is a radiation form factor and is a function of the plate emissivity and the geometry under consideration. Its application is subject to several assumptions that will be discussed in a later paragraph.

For convection:

$$\left. \begin{aligned} q_{\text{conv}} &= h_n(T'_n - t_n) && \text{heat transfer to outer passage} \\ q_{\text{conv}} &= h'_n(T_n - t_{n-1}) && \text{heat transfer to inner passage} \end{aligned} \right\} \quad (2)$$

Here the coefficient h is the convection heat-transfer coefficient. It is a function of the gas properties, the spacing between plates, gas flow rate, and whether or not the flow is turbulent.

Since a given plate transfers heat by radiation to the two adjacent plates on either side of it, and by convection to the gas in the two passages adjacent to it, the heat balance on a single plate may be written by combining equations of the form (1) and (2) and referring to figure 2. Such a heat balance for the n^{th} plate in one of the axial increments is then:

$$\begin{aligned} q_{g_n} + \mathcal{F}_{n-1}(T_{n-1}^4 - T_n^4) - \mathcal{F}_n(T_n^4 - T_{n+1}^4) - h'_{n-1}(T_n - \bar{t}_{n-1}) \\ - h_n(T_n - \bar{t}_n) = 0, \quad n = 1, 2, \dots, N+1 \end{aligned} \quad (3)$$

Here N is the number of plates in the symmetrical half under consideration including the center plate, and q_{g_n} is the heat generated in the n^{th} plate of the segment. Since the centerline of symmetry splits the center plate in half, heat transfer from only one side of the center plate is considered in equation (3), and the value of q_{g_1} , the heat generation term for the center plate in equation (3), is only one-half the total heat actually generated in the center plate. Since heat transfer from only one side of the center plate is considered, all variables having zero for a subscript (those whose subscripts are $n-1$, for the case $n = 1$) should be considered to be identically zero. The subscript $N+1$ refers to the outer wall, and here similarly all variables having a subscript of $N+2$ should be considered to vanish identically.

It is assumed in finding the solution to the set of equations (3) that the only unknowns are the plate temperatures T_n ; the other variables in equation (3) are either known explicitly or are computable from other relations. The problem of finding the T_n is then one of solving the $N+1$ nonlinear algebraic equations (eqs. (3)) simultaneously for $T_1, T_2, \dots, T_{N+1}$. The method of solution employed is presented in the appendix.

To obtain the gas temperature rise over each increment, a set of heat balance equations may be written for the gas flowing in each passage

of the axial segment:

$$WC_p \Delta t_n = h_n(T_n - \bar{t}_n) + h'_{n+1}(T_{n+1} - \bar{t}_n) \quad n = 1, 2, \dots, N \quad (4)$$

Thus, the temperature of the gas flowing in each passage may be found at each increment, once the plate temperatures are computed from equation (3). A few words of clarification are appropriate here. The gas temperature in each passage of an increment will of course continually increase over the length of that increment. Therefore, the $\bar{t}_n$ used in equations (3) and (4) actually represent mean values of t_n over the increment, whereas the Δt_n in equation (4) represent the temperature increases over that increment; in other words, $\Delta t_{n,i} = t_{n,i+1} - t_{n,i}$, where $t_{n,i}$ is now the gas temperature in the n^{th} passage entering the i^{th} increment. To simplify the use of equation (4) in determining the mean value $\bar{t}_n$ in equation (3), for the next increment, the following assumption is made:

$$\Delta t_{n,i} = t_{n,i+1} - t_{n,i} = \bar{t}_{n,i+1} - \bar{t}_{n,i} \quad (5)$$

This is a good assumption if the size of the increments is relatively small (heater broken into a large number of increments). To summarize the computational procedure, at each increment the set of equations (3) is solved simultaneously for the T_n ; then the mean gas temperatures in the next increment are found using equations (4) and (5), and the process is repeated. A discussion of the convection and radiation coefficients, and assumption regarding them, is in order.

Convection heat-transfer coefficients. - The convection coefficients are determined from one of two relations for Nusselt number, depending upon whether the flow in a particular passage is laminar or turbulent. The relation for laminar flow is obtained from reference 1:

$$\frac{hs}{k_f} = 3.75$$

For parallel plates of large aspect ratio:

$$D \approx 2s$$

Thus,

$$h = 7.50 \frac{k_f}{D} \quad (6)$$

For turbulent flow the equation for the modified Boelter equation given in reference 2 is used:

$$\frac{hD}{k_f} = 0.023 \left(\frac{GD}{\mu_f} \frac{\rho_f}{\rho_b} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)_f^{0.4}$$

which upon rearrangement becomes, for gases:

$$h = 0.023 \frac{G^{0.8}}{D^{0.2}} \left(\frac{t_b}{t_f} \right)^{0.8} \left(\frac{C_p}{\mu} \right)_f^{0.4} k_f^{0.6} \quad (7)$$

In both equations (6) and (7) the subscript f denotes film conditions. That is, the properties are evaluated at temperature t_f , halfway between the bulk temperature of the gas and the surface temperature of the plate:

$$t_f = \frac{t_b + T}{2}$$

The validity of equation (7) has been experimentally verified for high surface temperatures (2000° to 5000° R) and high surface to gas temperature ratio (from 2 to 4) in reference 3. Unfortunately, similar verification of the film correlation does not exist for laminar flow (eq. (6)). As stated earlier, the use of equation (6) or (7) depends upon whether the flow in a passage at a particular increment is laminar or turbulent. It is entirely possible that the flow in a given passage can change from laminar to turbulent, or vice versa, along the length of the passage. In reality such transitions in flow are usually neither well defined nor smooth. To avoid analytical and computational difficulties associated with determining the actual flow transition, a model has been postulated, whereby the Nusselt number varies continuously, as a function of Reynolds number from laminar flow, through transition, to turbulent flow. This model is illustrated in figure 3(a).

Another assumption made regarding convection heat transfer is that, while equations (6) and (7) apply strictly to the case where the surface bounding the flow is at a uniform temperature, the results may also be applied to the present case where the two plates bounding any passage are not generally at the same temperature.

The last item of importance relating convection heat transfer concerns the entrance effect on heat-transfer coefficient. This effect results in the heat-transfer coefficient being increased over its normal value for approximately the first 10 diameters of length along each passage from the inlet of the heater. This effect is described for the case of turbulent flow in reference 4. For laminar flow the effect is similar qualitatively but is somewhat more pronounced. This increase in heat-transfer coefficient is predominantly a function of the number of diameters, x/D , from the inlet, but it is also a weak function of Reynolds

number. For the present purpose, the dependence of the increase in heat-transfer coefficient on Reynolds number is neglected, and an empirical equation representing the average results from reference 4 has been used:

$$\frac{h_x}{h_\infty} = 1 + \left(\frac{x}{D}\right)^{-\frac{x+2}{5}} \quad (8)$$

Equation (8) is assumed to be valid for all Reynolds numbers, even for laminar flow, and, although this approximation for the case of laminar flow is perhaps not very good, the effect of this discrepancy on the overall performance of the heater is negligible.

Radiation form factor. - The radiation form factor $\mathcal{F}$ is a function of the geometry of the enclosure in which the radiant energy is transferred and of the emissivity and absorptivity of the radiating surfaces. The first assumption that is made in this respect is that the radiating surfaces constitute "gray bodies," that is to say that the absorptivity and emissivity of the body are equal, which is a good assumption for most of the refractory metals. The second assumption is that the gas flowing between the plates is transparent to radiation. This assumption is valid for gases such as hydrogen whose molecules are symmetrical (p. 82 of ref. 5).

For the purposes of simplification, it is assumed that the form factor between two adjacent parallel surfaces in an axial segment of the heater may be approximated by the form factor for a pair of infinite parallel flat plates. At first glance this assumption appears rather ludicrous, since the length of each segment is far from infinite; however, on closer examination, this assumption is not too bad if the aspect ratio of each segment is fairly large (10 or greater), and if the surface temperatures do not vary too radically from one axial segment to the next. Aspect ratio here is defined as the ratio of segment length to spacing between the plates. The last assumption is really an extension of the previous one. It is assumed that the losses of radiation from the ends of the heater are negligible. Reference 6 shows that this is a good assumption, especially when the temperature of the environment surrounding an end of the heater is nearly the same as the surface temperature at the end of the heater. On the basis of the infinite parallel plate assumption, the form factor is derived by McAdams (p. 63 of ref. 5) as

$$\mathcal{F}_n = \frac{\sigma}{\frac{1}{\epsilon_n} + \frac{1}{\epsilon_{n+1}} - 1} \quad (9)$$

where $\epsilon_n, \epsilon_{n+1}$ are the emissivities of the two adjacent surfaces and σ is the Boltzmann constant $\sigma = 4.75 \times 10^{-13}$ Btu/sec-ft²-°R⁴

Effect of plate conduction. - To further simplify the solution, it is assumed that the conduction of heat through the plate material does not significantly alter the problem. Specifically, this infers that the plates are sufficiently thin so that the resistance to heat flow across the width of each plate is negligible. On the basis of this stipulation, the temperatures on both sides of a plate may be considered identical. In addition, if the plates are thin, the resistance to heat flow along the length of each plate will be very high, and therefore it may be assumed that no heat is conducted axially along the length of the plate.

Heat-transfer summary. - The heat-transfer analysis is now briefly summarized. The radiation form factor from equation (9) is substituted in equations (3). The Reynolds number in each passage is computed at each increment, and on this basis the heat-transfer coefficients for each passage are computed from equation (6) or (7) in accordance with figure 3. The system of equations (3) is then solved simultaneously as discussed previously.

Pressure Drop

When the heat-transfer problem for the entire length of the heater has been solved, one may proceed to determine the pressure drop through each passage. Since the pressure drop in each passage is dependent upon the variation of gas temperature in that passage, it will be necessary to refer to the results of the heat-transfer calculations to obtain the gas temperatures upon which the pressure drops are dependent. As in the case of the heat-transfer calculations, the heater is divided into 100 axial segments. The pressure drop is then computed recursively over each segment and the total pressure drop in a passage is then the sum of the pressure drops of all the segments of that passage.

The pressure drop of gas flowing in a passage is due to two separate components. The first component is a friction component and represents the pressure drop caused by the shear stress between the surfaces bounding the passage and the flowing gas. It is given by the following relation:

$$\Delta p_{fr} = \frac{-G^2}{g_p} 2f \frac{\Delta x}{D} \quad (10)$$

where f is the well-known Fanning friction factor and Δx is the length of each incremental segment. The second component contributing to the pressure drop is the momentum component and represents the pressure drop required to cause an increase of the fluid velocity in the passage. In the heater under consideration the cross-sectional area of a

passage is a constant over its entire length. The reason the fluid accelerates down the passage causing a momentum pressure drop is that, since it is a gas, its density is decreased by the drop in pressure due to friction and also by the increase in temperature due to heating. In order to satisfy continuity, for constant passage cross-sectional area, as the density decreases the velocity of the gas must increase, resulting in a pressure drop due to the momentum increase. The equation for momentum pressure drop Δp_{mom} is:

$$\Delta p_{\text{mom}} = - \frac{G}{g} (V_{i+1} - V_i)$$

since

$$V = \frac{G}{\rho}$$

$$\Delta p_{\text{mom}} = - \frac{G^2}{g} \left(\frac{1}{\rho_{i+1}} - \frac{1}{\rho_i} \right) \quad (11)$$

Combining equations (10) and (11) to obtain the overall pressure drop over the i^{th} increment, one obtains

$$\Delta p_{n,i} = - \frac{G^2}{g} \left[\frac{2f \Delta x}{D \left(\frac{\rho_{n,i} + \rho_{n,i+1}}{2} \right)} + \left(\frac{1}{\rho_{n,i+1}} - \frac{1}{\rho_{n,i}} \right) \right] \quad (12)$$

for each passage $n = 1, 2, \dots, N$; all increments $i = 1, \dots, 100$.

The density ρ is computed from the perfect gas relation

$$\rho = \frac{p}{R t_s} \quad (13)$$

where t_s is the static temperature and is related to the total gas temperature obtained from the heat-transfer calculations by the following relation:

$$t_s = \frac{t}{1 + \frac{\gamma - 1}{2} M^2} \quad (14)$$

and the Mach number M is given by

$$M = \frac{G}{p} \sqrt{\frac{R t_s}{\gamma g}} \quad (15)$$

In solving equations (12) to (15) for the overall pressure drop in a passage, it was found advisable, from a computational standpoint, to start the calculations at the heater exit with a known value of exit pressure ($P_{n,101}$) and to progressively calculate the incremental pressure drops toward the heater inlet ($P_{n,100}$, $P_{n,99}$, . . . , $P_{n,1}$). The overall passage pressure drop in a passage is then found from:

$$P_n = P_{n,1} - P_{n,101}$$

The reason for choosing this method of computing the pressure drops, rather than starting with a known inlet pressure and working towards the exit, is that in the latter case the computed pressures are continually decreasing; and, unless the pressure at the heater inlet is chosen with extreme care, it is possible in the process of computation to generate negative pressures, clearly an unrealistic and undesirable result. It was found that this condition was quite probable for high values of exit Mach numbers, where overall pressure drop is extremely sensitive to changes in inlet conditions. This difficulty is clearly avoided if one starts the computation at the heater exit and computes the pressure drop in the direction of increasing pressure. Another advantage of working with a specified exit pressure is that it allows one to solve the problem for a specified value of exit Mach number, an important design parameter.

Proceeding in the manner described previously, equations (12) to (15) are solved iteratively for each increment of passage length in conjunction with equation (16):

$$P_{n,i} = P_{n,i+1} - \Delta p_{n,i} \quad (16)$$

The solution is an iterative one since at each increment only $P_{n,i+1}$ and thus $\rho_{n,i+1}$ are known. The density $\rho_{n,i}$, which appears on the right side of (12) and is a function of $P_{n,i}$, is unknown. Briefly, a guess is made at $P_{n,i}$, on the basis of which $\rho_{n,i}$ is calculated from (13); this value of $\rho_{n,i}$ is then used in (12) to compute $\Delta p_{n,i}$. Using this value of $\Delta p_{n,i}$ a new value of $P_{n,i}$ is computed from (16), and the iteration continues until the changes in the computed values of $\Delta p_{n,i}$ become arbitrarily small.

The overall pressure drops in each of the passages were computed individually in the previous manner. The last item to be discussed in this area is the calculation of the friction factor f appearing in equations (10) and (12). As in the case of heat-transfer coefficient, the friction factor was also assumed to be a function of the film Reynolds number. For the case of turbulent flow, this assumption is verified by existing experimental evidence (ref. 2). Again the validity of this

assumption for laminar flow is not as clearly defined. For laminar flow the friction factor for flow between infinite parallel plates is (eq. 4-85 of ref. 7):

$$f = \frac{24}{Re_f} \quad (17)$$

Though the parallel plates of this heater are by no means infinite, the aspect ratio of the cross section of a heater passage is assumed large enough that equation (17) is a good approximation.

For turbulent flow a popular equation for friction factor is the simple Blasius relation (eq. 7-64 of ref. 7):

$$f = \frac{0.046}{Re^{0.2}}$$

Unfortunately, this equation is only approximate over the range of Reynolds numbers $10^4 < Re < 2 \times 10^5$. For $Re < 10^4$ the approximation of the previous equation to experimental results is particularly bad.

A more exact relation between friction factor and Reynolds number for circular pipes is the so-called Von Kármán - Nikuradse equation (eq. 7-60, ref. 7):

$$\frac{1}{\sqrt{f}} = 4.0 \log_{10} (Re\sqrt{f}) - 0.40$$

A similar relation valid for flat plates may be obtained using the same methods (p. 169, ref. 7) as those used in the derivation for the circular pipe equation above. The Von Kármán - Nikuradse equation for flat plates becomes:

$$\frac{1}{\sqrt{f}} = 4.0 \log_{10}(Re\sqrt{f}) - 0.98$$

Obviously, because the friction factor appears on both sides of the Von Kármán - Nikuradse equation, this relation is not generally a convenient way of obtaining friction factor as a function of Reynolds number. Therefore, it was decided to develop an empirical equation that would closely approximate the Von Kármán - Nikuradse equation for flat plates and would give friction factor explicitly as a function of Reynolds number. Such a relation is given by the following equation:

$$f = \frac{0.84}{(\ln Re_f - 1.11)^{2.22}} \quad (18)$$

Equation (18) yields results substantially more accurate than those given by the Blasius relation, particularly at low turbulent Reynolds numbers.

As in the case of heat transfer, to avoid computational difficulties it was assumed that the variation in friction factor from the laminar case (eq. (17)) to turbulent (eq. (18)) was continuous as shown in figure 3(b). As in the case of heat-transfer coefficient, the friction factor is computed on the basis of the local film Reynolds number computed at each segment of each passage.

Pressure Drop Equalization

In obtaining the solution to the heat-transfer and pressure drop problems, if the mass flow is initially assumed to be uniformly distributed between the passages, the resulting pressure drops in all the passages will not generally be equal. This is the case even when the dimensions of the passages are identical and the heat generation in each plate is the same, because the outer passages adjacent to the walls bounding the heater element effectively receive heat from only one side, since the wall is usually adiabatic or at least nearly so. Since the heat balance equations for each plate are coupled, the fact that the heat transfer to the outer passages is not the same as that to the remaining passages affects the heat transfer and resulting temperature distribution in every passage of the heater. This then results in unequal pressure drops in the passages despite the apparent symmetry of mass flow, dimensions, and so forth between the passages.

However, it is expected that, in actual operation, the passage mass flows will be distributed such that the pressure drops in all the passages will be identical. The present problem then is to determine a distribution between the passages of mass flow such that the resulting pressure drops will be equal. The pressure drop in a given passage is primarily a function of the mass flow through that passage; because of the coupling between the passages, it is also a weak function of the mass flows in other passages, but this effect is second order and is neglected in order to simplify the analysis. Thus,

$$P_n = f(w_n)$$

where P_n is the overall passage pressure drop and w_n is the flow rate in that passage, and thus,

$$\Delta w_n = \left(\frac{\delta w_n}{\delta P_n} \right) \Delta P_n \quad (19)$$

where $\delta w_n / \delta P_n$ are influence coefficients defined in equation (23).

The ΔP_n represent the changes in pressure drop for each passage required to equalize all the passage pressure drops, and the Δw_n are the estimated changes in passage weight flows required to yield the necessary values of ΔP_n . At the end of each sequence of heat transfer and pressure drop calculations, the ΔP_n required to equalize the P_n are computed. From this one obtains the required values of Δw_n from equation (19), and new values of passage weight flow are computed by:

$$w_{n,j+1} = w_{n,j} + \Delta w_{n,j}$$

Using these new values of passage weight flows, the heat-transfer and pressure drop problems are again solved, and this iterative procedure continues until the differences among any of the passage pressure drops become less than some arbitrary limit.

Since it is desired to correct all the passage pressure drops to the same value $\bar{P}$, ΔP in equation (19) is defined as

$$\Delta P_n = \bar{P} - P_n \quad (20)$$

where $\bar{P}$ is a mean pressure drop to which it is desired to correct the P_n . The next consideration is by what criterion should one compute $\bar{P}$? At first glance a simple arithmetic average of the P_n would appear adequate; however, this is not convenient because of the following additional consideration. In finding the solution to the overall heater problem for a particular case, the mass flow through the entire heater is given either directly or indirectly, and naturally this total mass flow must remain fixed during all phases of solution of the problem. This imposes a constraint on our pressure drop equalization scheme. The perturbation of the passage weight flows to equalize the passage pressure drops is subject to the constraint that the sum of the passage weight flows remains constant. This can also be expressed by saying that the sum of perturbations in passage weight flows must be equal to zero. Thus,

$$\sum_{n=1}^N \Delta w_n = 0 \quad (21)$$

Substituting equations (19) and (20) into (21) gives

$$\sum_{n=1}^N \left(\frac{\delta w_n}{\delta P_n} \right) P_n - \bar{P} \sum_{n=1}^N \frac{\delta w_n}{\delta P_n} = 0$$

$$\bar{P} = \frac{\sum_{n=1}^N \frac{\delta w_n}{\delta P_n} P_n}{\sum_{n=1}^N \frac{\delta w_n}{\delta P_n}}$$

Thus,

$$\Delta w_n = \frac{\delta w_n}{\delta P_n} \left(\frac{\sum_{n=1}^N \frac{\delta w_n}{\delta P_n} P_n}{\sum_{n=1}^N \frac{\delta w_n}{\delta P_n}} - P_n \right) \quad (22)$$

The influence coefficients used in equation (22) are obtained from numerical results. That is,

$$\left(\frac{\delta w_n}{\delta P_n} \right)_j = \frac{w_{n,j} - w_{n,j-1}}{P_{n,j} - P_{n,j-1}} \quad (23)$$

where j designates the number of the trial. Originally, an arbitrary perturbation is made in the values w_n , then both the heat transfer and pressure drop problems are recomputed, and the new set of P_n values is obtained. Then the first set of influence coefficients $(\delta w_n / \delta P_n)_1$ is computed. For the second and subsequent trials the Δw_n are computed from equation (22), the problem is recomputed using the new mass flows, and this process is continued until the maximum difference between any of the pressure drops becomes arbitrarily small. The problem is then considered completed. It is important to note that, in the process of iteration to equalize the pressure drops, both the heat-transfer and pressure drop problems are recomputed, not just pressure drop alone.

RESULTS AND DISCUSSION

Results are presented for the solution of the heat-transfer and pressure drop equations of the heater using hydrogen as the gas with a fixed heater inlet temperature of 200°R and a mixed-mean exit temperature of 5000°R . It was necessary to define a mixed-mean exit temperature since the exit gas temperatures in the individual passages were not generally equal. The hydrogen properties used in the computation were

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taken from reference 8. Incipient dissociation of the hydrogen caused an attendant sharp increase in specific heat and thermal conductivity at temperatures above 4000°R . The effect of this on the results is to increase the coefficient of heat transfer near the exit of the heater. At temperatures up to 5000°R , the effect of dissociation on the density of the dissociated hydrogen mixture is negligible. The results presented here were computed under the additional assumptions that the power generation in the plates varied axially according to a $2/3$ -cosine power distribution (described in fig. 4), that power generation was equally divided among the plates, that the spacings between the plates were identical, and that the outer wall bounding the plate bundle was adiabatic. In addition, most of the results are for the case of five plates, although results are presented for nine plates for the purpose of illustrating the effect of changing the number of plates on heater performance.

It was felt that the most significant parameters resulting from the computations were maximum plate temperature and overall pressure drop. Maximum plate temperature was selected as a parameter because, at the high operating temperatures under consideration, the performance of the heater is limited by the maximum allowable temperature for the materials from which the plates are fabricated. Thus, the effect of number of plates, plate emissivity, and L/D ratio on the maximum plate temperature was investigated. The results are indicated on plots of maximum temperature against $(L/D)/(GD)^{0.2}$ for turbulent flow and $(L/D)/GD$ for laminar flow. These two parameters are proportional to $f(L/D)$ for turbulent and laminar flow, respectively, where f is the turbulent or laminar friction factor, D is the equivalent diameter of all the equally spaced passages, and G is the mean value, over the passages, of the mass flow through the heater. Thus, one would expect pressure drop to be a function of these parameters, and, because of the Reynolds analogy, one might also expect the heat-transfer coefficient (and thus the maximum plate temperatures) to also be a function of these parameters. Because of the different flow conditions, the results for laminar flow are presented separately. It was felt that the most probable region of operation of this heater would be turbulent flow, and therefore most of the results presented here are for that case. The results presented in this report are of course only valid for the case of hydrogen as the working fluid.

All the results for maximum plate temperature are derived from plots of the form of figure 5, which show the variation of center plate temperature and adiabatic wall temperature with dimensionless distance from the inlet. As expected, the maximum temperature for the entire heater always occurred on the center plate; since the power generation in each plate was the same, the passages were of equal cross section, and the outer wall was adiabatic. In most cases the maximum plate temperature occurred in the vicinity of the heater exit. For certain cases the maximum point was inward from the exit because of the combined effect of reduced heat

flux at the exit given by the $2/3$ -cosine axial power distribution and the increased heat-transfer coefficient due to incipient dissociation of the high-temperature hydrogen. The maximum temperatures from plots of temperature against length such as figure 5 were obtained for a wide range of values of L/D and GD and are plotted in the form of figure 6 for turbulent flow and in figure 11 for laminar flow.

In figure 6 it is seen that the parameter $(L/D)/(GD)^{0.2}$, which is proportional to $f(L/D)$ for turbulent flow, does indeed represent the heat-transfer results for turbulent flow quite well. There are a few points, however, that warrant discussion. It is seen that the maximum adiabatic wall temperature decreases as the maximum plate temperature increases and is a function not only of $f(L/D)$, but also of mass flow G . The reason for this, as will be discussed in more detail later, is that the flow distribution between the passages is a function of the total average mass flow G . As the mass flow is increased, more flow is shunted to the passage adjacent to the wall, resulting in a drop of maximum temperature there. Another interesting point is the sharp corner in the curve of maximum plate temperature. This corner results because the absolute maximum plate temperature shifts from a region near the exit to the vicinity of the inlet of the heater. That the maximum plate temperature should ever occur in the vicinity of the inlet of the heater, where the cold gas is entering, does at first glance seem surprising. This phenomenon will be the subject of the next paragraph.

The reason the plate temperature near the inlet of the heater may rise enough to become the highest in the entire heater may be understood by examining equation (7). The right side of (7) contains the factor $(t_b/t_f)^{0.8}$. From the definition of t_f it is seen that this factor becomes quite small when the fluid bulk temperature is low and the surface temperature is high. Because of this, the value of h computed from equation (7) is decreased. From equation (2) it is also seen that $q = h(T - t_b)$, so that, as the heat-transfer coefficient is decreased, the surface temperature must increase to transfer the same heat flux. This effect is also somewhat self-perpetuating since, as the surface temperature is increased, $(t_b/t_f)^{0.8}$ decreases even more, resulting in further reduction of heat-transfer coefficient. This effect is only pronounced, however, at high heat fluxes and/or low inlet temperatures, which result in low values of t_b/t_f , on the order of 0.5 or less.

Returning to figure 6, it is seen that these high plate temperatures in the inlet region occur at comparatively low values of $(L/D)/(GD)^{0.2}$. Since the inlet and exit temperatures for the gas being heated are fixed, low values of L/D and/or high values of GD correspond to high heat fluxes resulting in the phenomenon described above. In figure 7 one may see directly the effect of t_b/t_f caused by a change in inlet temperature on the maximum plate temperature. The two cases illustrated are identical except for inlet temperature, and the results away from the

inlet are nearly the same. However, near the inlet, the plate temperature is much higher for the case of lower inlet temperature. However, it should be mentioned that this effect at very low values of t_b/t_f has never received experimental verification.

Figure 8 shows the effect of the parameter $(L/D)/(GD)^{0.2}$ and mass flow on the flow distribution between the plates, which is represented by the ratio of the flow in the passage adjacent to the adiabatic wall to that in the innermost passage. It is seen that the effect of mass flow is only important at conditions corresponding to low values of $f(L/D)$. Also, the weight flow ratio increases with decreasing $f(L/D)$ and increasing mass flow. These results are consistent with the variation of adiabatic wall temperature with mass flow and $f(L/D)$ illustrated in figure 6. The reasons for this nonuniform distribution of flow over the passages are interesting and warrant some discussion. Since the outer passage, being heated on only one side, runs cooler than the inner passage, there is less pressure drop there for the same flow than in the inner passages. As a result, more flow is shifted towards the outer passage from the others to equalize the pressure drop. As this is done, the inner passages run even hotter because of their reduced flow. The situation is finally equalized by the heat transfer from the hot inner plates across the heater to the cooler outer wall by means of both convection and radiation. The increase of the passage weight flow ratio with decreasing $(L/D)/(GD)^{0.2}$ is believed to be due to the higher plate temperatures associated with low values of $(L/D)/(GD)^{0.2}$. As the plate temperatures are increased, the exit gas temperatures in the inner passages also rise, increasing the pressure drops relative to those in the outer passages. Thus, more flow is shifted from the inner to the outer passage to again equalize the pressure drops. The reason that the passage flow ratio also rises with increased mass flow appears to be caused by the fact that, since the gas temperature at the exit of the outer passage is much less than at the exit of the inner passages, the rate of increase of pressure drop with increasing flow rate is less for the outer passage than for the others. Therefore, as additional flow is passed through the heater, proportionately more and more of it is passed through the outer passage.

Effect of Plate Emissivity and Number of Plates

The effect of changing plate emissivity on the maximum temperature is seen in figure 9. The results indicate that as emissivity is decreased the maximum plate temperature is increased, while the maximum wall temperature decreases. In short, the temperature difference between the center plate and the wall is increased. The reason for this is that decreasing the emissivity increases the resistance to radiation heat transfer between the plates, and thus the temperature distribution from the center plate to the wall, across the heater, becomes more nonuniform.

Increasing the number of plates has an effect similar to decreasing the plate emissivity, in that resistance to heat flow across the plates is increased; this is seen in figure 10. The important difference here is that, while causing an increase in the temperature difference between the center plate and the wall, increasing the number of plates also causes the maximum plate temperature to drop. This is in contrast to the results obtained when the emissivity was decreased, which resulted in an increase in maximum plate temperature. The reason for this latter effect is that, with a larger number of plates at the same plate spacing, a larger percentage of the total flow passes through the passages between the interior plates, mitigating the temperature increases caused by the passages along the outer walls taking flow away from the interior passages. As a result the maximum plate temperatures are decreased.

Effect of Laminar Flow

For purpose of this discussion, laminar flow operation is arbitrarily defined to exist whenever the flow in any of the heater passages becomes laminar. Therefore the case of mixed flow conditions for the different passages is included in this definition.

The results for laminar flow are presented as a function of the parameter $(L/D)/(GD)$, which, as for $(L/D)/(GD)^{0.2}$ for turbulent flow, corresponds to $f(L/D)$. The plot of maximum temperature is seen in figure 11. Since in the case of laminar flow the heat fluxes transferred in the heater are much lower than in the turbulent case, the region of maximum plate temperature remains in the vicinity of the heater exit, and no abrupt shifts of maximum temperature to the region near the inlet of the heater occur, as in the turbulent case. Another point of interest is that, except for very low values of GD (corresponding to distinctly laminar flow), the variation of maximum temperature with the parameter $(L/D)/GD$ is not as well defined as in the turbulent case. The primary reason for this is that at higher values of GD the flow through the heater is no longer purely laminar. In figure 12 it is seen that the relatively large amount of flow is shifted towards the outer passage in the laminar cases, with the result that, while the flow in the inner passages may be purely laminar, the flow in the outer passage may be turbulent. With the heater operating in this hybrid flow regime the parameter $(L/D)/GD$ no longer can be used satisfactorily to represent the operating characteristics of the heater, and the divergence of the curves in figures 11 and 12 for low values of $(L/D)/GD$ is the result.

Overall Pressure Drop

For the case of turbulent flow the pressure drop results are illustrated in figure 13. Similarly, the results for laminar flow are plotted

in figure 14. Again the parameters $(L/D)/(GD)^{0.2}$ and $(L/D)/GD$, respectively, were used successfully in presenting the results. It was found that the results were best presented in the manner shown, the pressure drop being divided by $M_O^2 p_O$, and the exit Mach number M_O also being used as a parameter. The general trend of results for both laminar and turbulent flow is quite similar. In both cases the pressure drop parameter increases nearly linearly with increasing $f(L/D)$ at constant exit Mach number and the slope of the pressure parameter curves decreases with increasing Mach number. It should be pointed out that, for the cases presented, $M_O^2 p_O$ is a parameter that is almost directly proportional to momentum pressure drop.

Momentum pressure drop is given by equation (11a):

$$P_{\text{mom}} = - \frac{G^2}{g} \left(\frac{1}{\rho_O} - \frac{1}{\rho_{\text{in}}} \right) \quad (11a)$$

For the cases presented, since $t_O = 5000^\circ \text{ R}$, $t_{\text{in}} = 200^\circ \text{ R}$, $p_O < p_{\text{in}}$, and therefore $1/\rho_O \gg 1/\rho_{\text{in}}$. Thus equation (11a) may be written approximately:

$$P_{\text{mom}} \approx - \frac{G^2}{g} \frac{1}{\rho_O} = - \frac{G^2}{g} \frac{R t_{s_O}}{p_O} \quad (24)$$

By definition,

$$M_O^2 = \frac{G^2}{p_O^2} \frac{R t_{s_O}}{\gamma g} \quad (25)$$

Substituting equation (25) into (24) one obtains

$$P_{\text{mom}} \approx - M_O^2 p_O \gamma$$

Otherwise the results for pressure drop are not unusual.

SUMMARY OF RESULTS

The following results were obtained from the analytical investigation of a high-temperature equally spaced parallel flat plate heater:

1. Heat transfer and pressure drop may be presented as functions of a parameter corresponding to $f(L/D)$ for the cases of both laminar and turbulent flow, where f is the Fanning friction factor and L/D is the length-to-equivalent-diameter ratio of the passages in the heater.

2. When the outer wall bounding the plate bundle is adiabatic, the outer passage receives more flow than the interior passages in order to equalize the pressure drops in all passages. The proportion of the flow passing through the outer passage increases as $f(L/D)$ decreases.

3. As the parameter proportional to $f(L/D)$ decreases, the maximum plate temperature increases as expected. However, at the same time the maximum adiabatic wall temperature decreases because of result 2.

4. The analysis indicates that, at high heat fluxes (low values of the parameters corresponding to $f(L/D)$) and/or low inlet gas temperatures, it is possible for the maximum plate temperature to occur at a point near the inlet of the heater because of the effect of the temperature ratio t_b/t_f appearing in the expression for the turbulent-flow heat-transfer coefficient. However, this result has never been experimentally verified.

5. Increasing the number of plates or decreasing the emissivity of the plates has the effect of increasing the resistance to the transverse flow of heat from the inner plates to the outer wall, resulting in an increase in the difference between the maximum plate and maximum wall temperatures. As a corollary, the effect of radiation between the plates is to equalize the plate temperatures.

6. Increasing the number of plates lowers the maximum plate temperatures slightly because the proportion of the flow passing through the interior passages is increased.

7. The overall pressure drop through the heater increases nearly linearly with increasing $f(L/D)$ at constant exit Mach number and pressure, and for a given $f(L/D)$ and M_0 is roughly proportional to the approximate momentum pressure drop $M_0^2 p_0$.

Lewis Research Center

National Aeronautics and Space Administration

Cleveland, Ohio, September 20, 1961

APPENDIX - SOLUTION OF PLATE HEAT BALANCE

EQUATIONS FOR PLATE TEMPERATURE

The set of equations for the plate temperatures to be solved is:

$$q_{g_n} + \mathcal{F}_{n-1}(T_{n-1}^4 - T_n^4) - \mathcal{F}_n(T_n^4 - T_{n+1}^4) - h'_{n-1}(T_n - \bar{t}_{n-1}) - h_n(T_n - \bar{t}_n) = 0, \\ n = 1, 2, \dots, N + 1 \quad (3)$$

This system constitutes a set of $N + 1$ polynomial equations in the $N + 1$ unknowns $T_1, T_2, \dots, T_{N+1}$. The solution to this set of equations will be obtained using the Newton-Raphson method, described in reference 9. An initial set of trial values of T_n is substituted into the set (3). Since these trial values do not represent the solution exactly, substitution into the left side of (3) will result in the right sides becoming nonzero. Designate the nonzero residuals on the right sides by R_n . Then upon slight rearrangement, (3) is rewritten to give:

$$q_{g_n} + \mathcal{F}_{n-1}T_{n-1}^4 - (\mathcal{F}_{n-1} + \mathcal{F}_n)T_n^4 - (h'_{n-1} + h_n)T_n + \mathcal{F}_nT_{n+1}^4 \\ + h'_{n-1}t_{n-1} + h_nt_n = R_n \quad n = 1, 2, \dots, N + 1 \quad (A1)$$

The Newton-Raphson method involves the solution of the following set of $N + 1$ linear equations for ΔT_n , the changes to be applied to the trial values of T_n to give the new trial values of T_n . This set of equations is described by:

$$\frac{\partial R_n}{\partial T_{n-1}} \Delta T_{n-1} + \frac{\partial R_n}{\partial T_n} \Delta T_n + \frac{\partial R_n}{\partial T_{n+1}} \Delta T_{n+1} = - R_n \\ n = 1, 2, \dots, N + 1 \quad (A2)$$

The other partial derivatives $\partial R_n / \partial T_{n-2}$, $\partial R_n / \partial T_{n+2}$, and so forth do not appear because of the form of equation (A1). Partial differentiation of (A1) and substitution into (A2) give:

$$\left(4\mathcal{F}_{n-1}T_{n-1}^3 \right) \Delta T_{n-1} - \left[4(\mathcal{F}_{n-1} + \mathcal{F}_n)T_n^3 - (h'_{n-1} + h_n) - (T_n - t_n) \frac{\partial h_n}{\partial T_n} \right. \\ \left. - (T_n - t_{n-1}) \frac{\partial h'_{n-1}}{\partial T_n} \right] \Delta T_n + 4\mathcal{F}_nT_{n+1}^3 \Delta T_{n+1} = - R_n \quad (A3)$$

The terms of the form $\partial h_n / \partial T_n$ are due to the dependence of h_n on the film temperature $t_{f,n} = (t_{b,n} + T_n)/2$, making h_n functionally dependent upon T_n .

As in the main text, all terms having subscripts of 0 or $N + 2$ in the first or last equations, respectively, are identically zero. For simplicity the following terms are now defined:

$$\begin{aligned} a_n &= 4 \mathcal{F}_{n-1} T_{n-1}^3 \\ b_n &= - \left[4(\mathcal{F}_{n-1} + \mathcal{F}_n) T_n^3 - (h_{n-1}' + h_n) - (T_n - t_n) \frac{\partial h_n}{\partial T_n} - (T_n - t_{n-1}) \frac{\partial h_{n-1}'}{\partial T_n} \right] \\ c_n &= 4 \mathcal{F}_n T_{n+1}^3 \end{aligned}$$

Then the system of equations (A3) is more simply described by:

$$\begin{aligned} b_1 \Delta T_1 + c_1 \Delta T_2 &= -R_1 \\ a_n \Delta T_{n-1} + b_n \Delta T_n + c_n \Delta T_{n+1} &= -R_n \quad n = 2, 3, \dots, N \\ c_{N+1} \Delta T_N + b_{N+1} \Delta T_{N+1} &= -R_{N+1} \end{aligned} \tag{A4}$$

The solution of this system of linear equations for ΔT_n presents no problem when performed by matrix methods on a digital computer. However, the use of general matrix inversion routines for the solution K linear simultaneous equations on a digital computer in this case, though entirely possible, is wasteful both of computer time and storage. Because of the particular arrangement of elements about the main diagonal of the coefficient matrix for (A3) and the resulting large number of zero elements in that matrix, a direct elimination method may be used to advantage over the more general technique in obtaining the solution to this particular set of equations. The direct elimination method may also be programmed for a digital computer, and it is much simpler, faster, and shorter than a matrix inversion routine. A brief description of this method has recently been given in reference 10.

When the ΔT_n are obtained from the solution of (A4), the new T_n for the next trial are computed as follows:

$$T_{n,l+1} = T_{n,l} + \Delta T_{n,l}$$

$$n = 1, 2, \dots, N + 1$$

$$l = \text{Trial number}$$

With the new values of T_n , the calculations are repeated, beginning with equation (A1), and this process is continued until all the ΔT_n become arbitrarily small, the resulting values of T_n being a good approximation to the exact solution of (3).

REFERENCES

1. Eckert, E. R. G., and Drake, Robert M., Jr.: Heat and Mass Transfer. Second ed., McGraw Hill Book Co., Inc., 1959, pp. 177; 195; 198.
2. Humble, Leroy V., Lowdermilk, Warren H., and Desmon, Leland G.: Measurements of Average Heat-Transfer and Friction Coefficients for Subsonic Flow of Air in Smooth Tubes at High Surface and Fluid Temperatures. NACA Rep. 1020, 1951. (Supersedes NACA RM's E7L31, E8L03, E5OE23, and E5OH23.)
3. Taylor, Maynard F., and Kirchgessner, Thomas: Measurements of Heat Transfer and Friction Coefficients for Helium Flowing in a Tube at Surface Temperatures up to 5900° R. NASA TN D-133, 1959.
4. Deissler, Robert G.: Turbulent Heat Transfer and Friction in the Entrance Regions of Smooth Passages. Trans. ASME, vol. 77, no. 8, Nov. 1955, pp. 1221-1232; discussion, p. 1233.
5. McAdams, William H.: Heat Transmission. Third ed., McGraw-Hill Book Co., Inc., 1954.
6. Siegel, R., and Perlmutter, M.: Convective and Radiant Heat Transfer for Flow of a Transparent Gas in a Tube with a Grey Wall. (To be publ. in Int. Jour. Heat and Mass Transfer.)
7. Knudsen, James G., and Katz, Donald L.: Fluid Dynamics and Heat Transfer. McGraw-Hill Book Co., Inc., 1958.
8. King, Charles R.: Compilation of Thermodynamic Properties, Transport Properties, and Theoretical Rocket Performance of Gaseous Hydrogen. NASA TN D-275, 1960.
9. Scarborough, James B.: Numerical Mathematical Analysis. Third ed., The Johns Hopkins Press, 1955, pp. 203-206.
10. Wenrick, R. C., and Houghton, A. V.: Solution of Tridiagonal Matrices. Communications of ACM, vol. 4, no. 7, July 1961, p. 314.

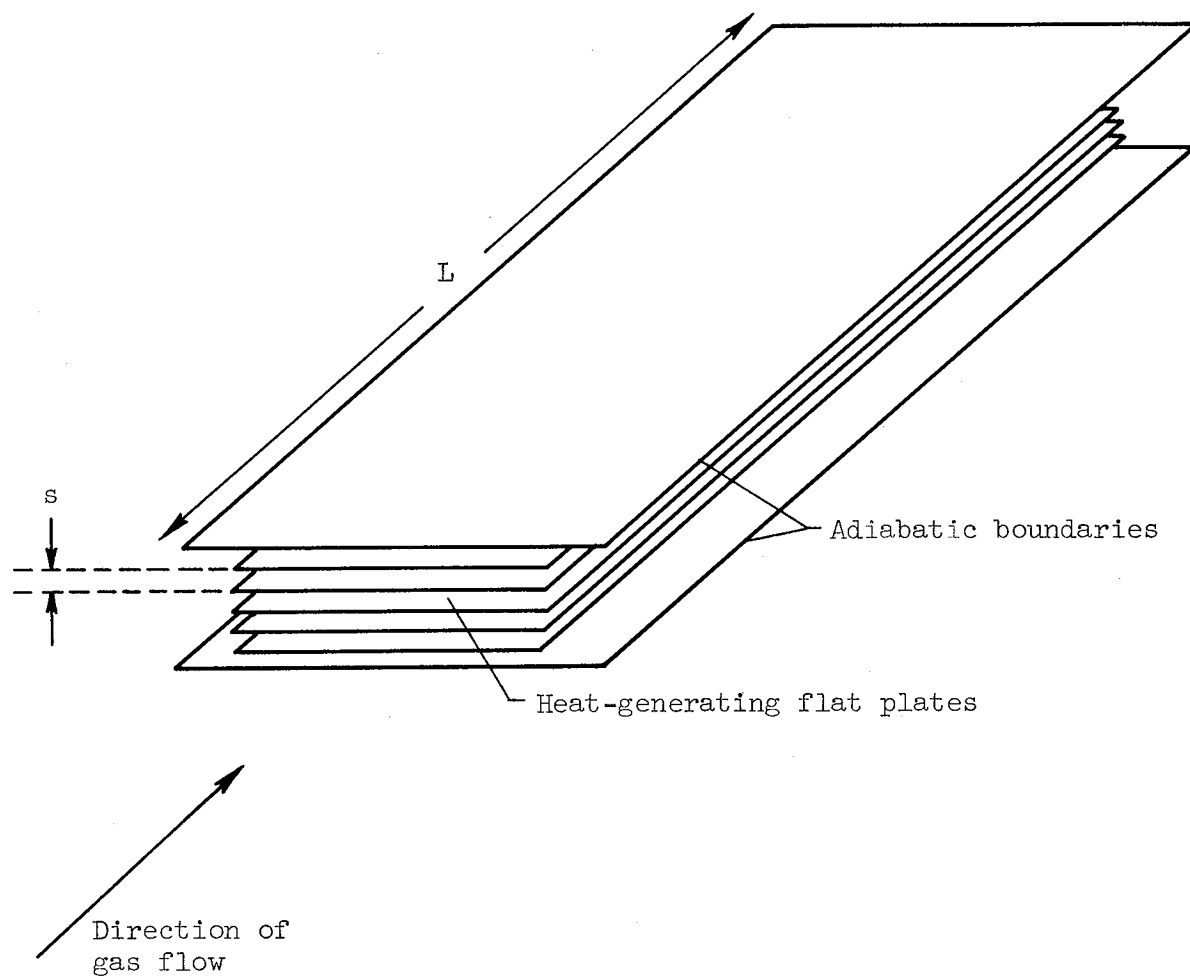


Figure 1. - Parallel plate heat geometry.

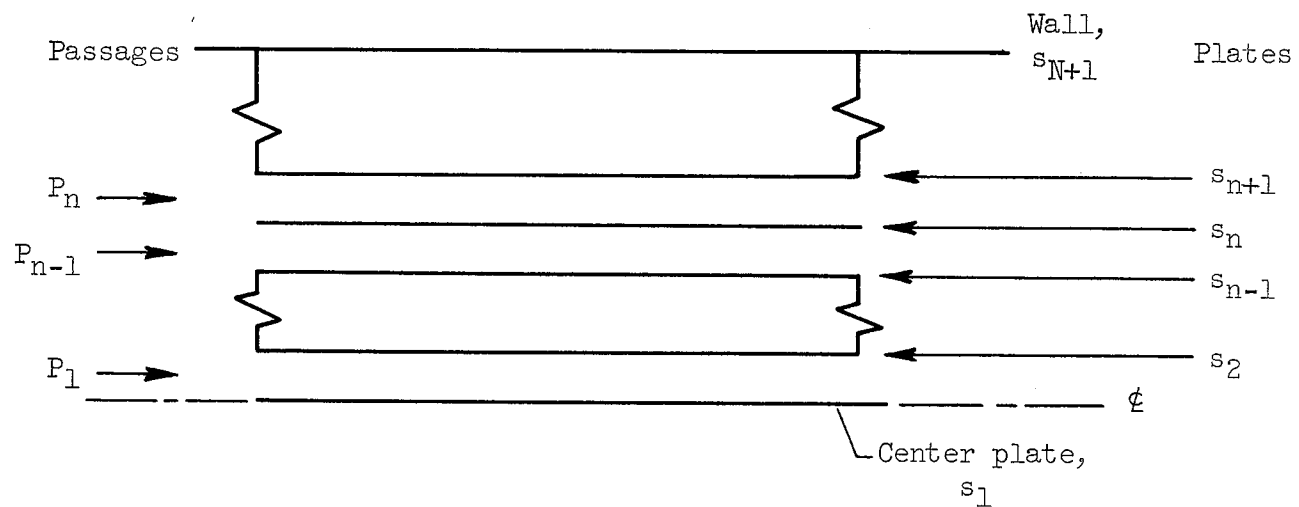
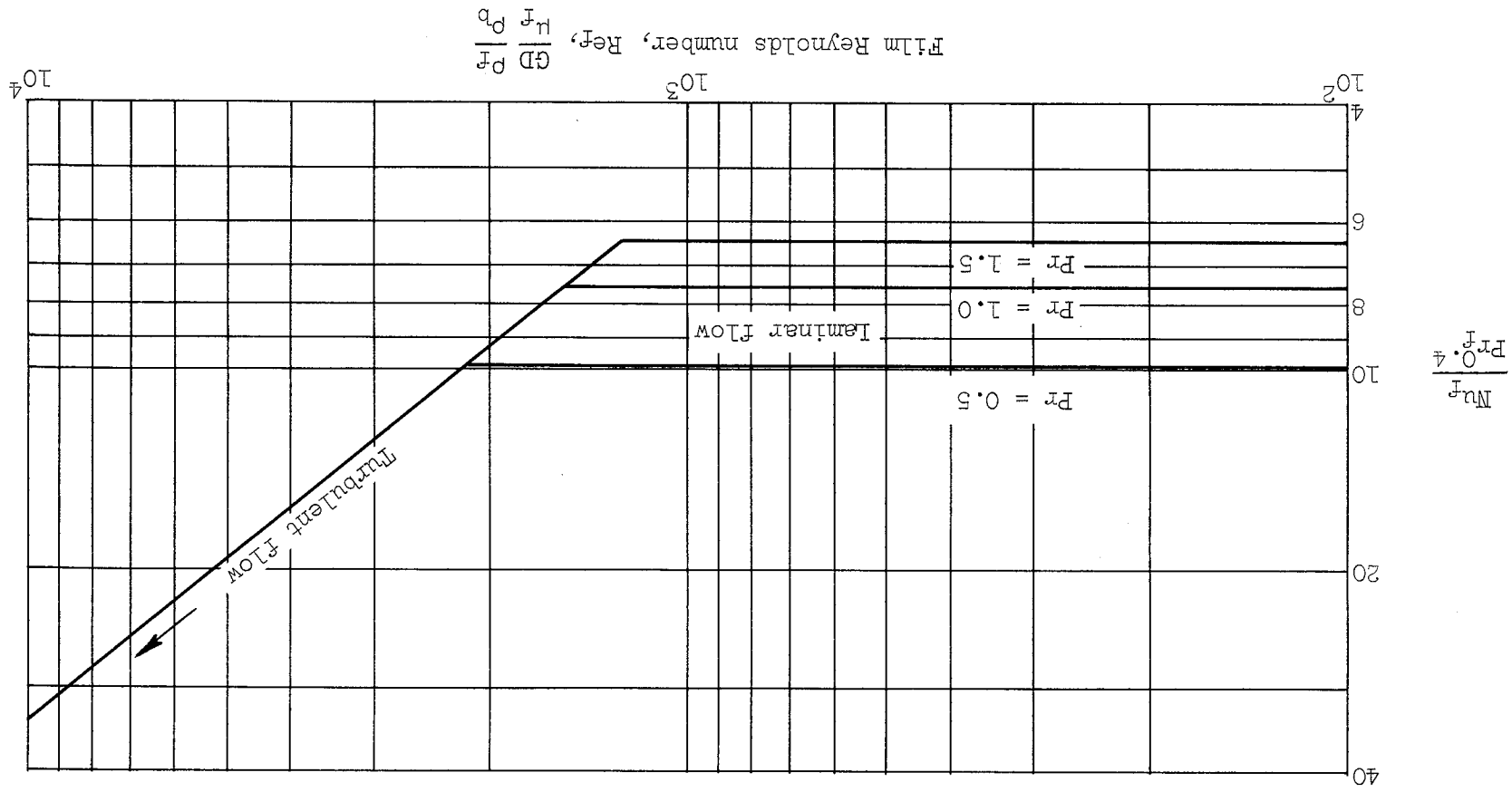
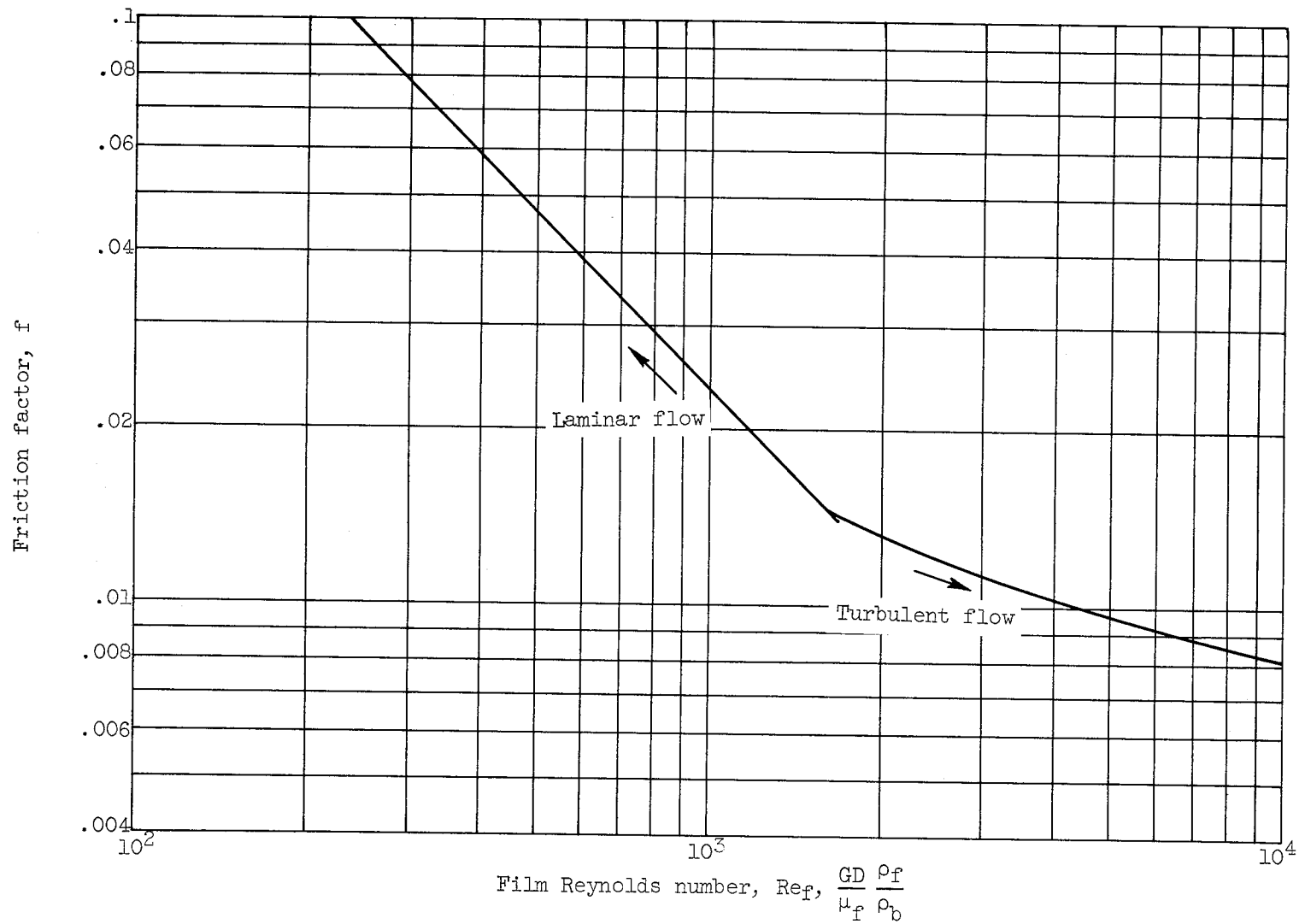


Figure 2. - Plate and passage labeling of parallel flat plate heater symmetrical about center plate.

Figure 3. - Assumed variation of heat transfer and friction parameters in the transition region.
 (a) Nusselt number.





(b) Friction factor.

Figure 3. - Concluded. Assumed variation of heat transfer and friction parameters in the transition region.

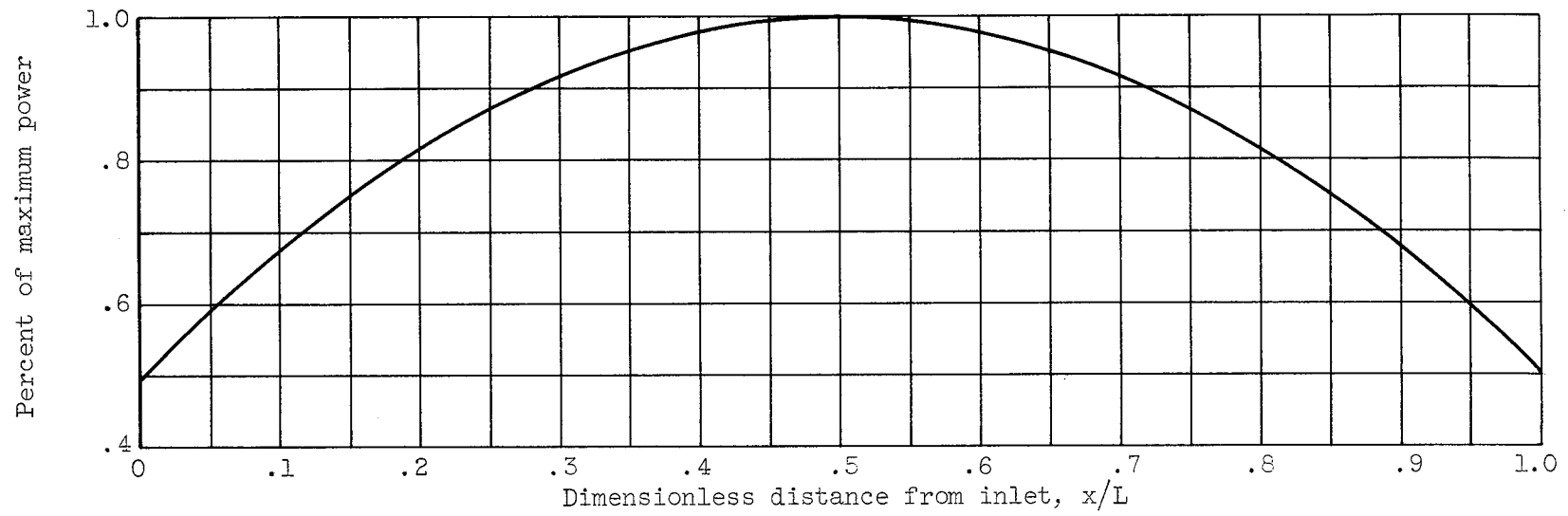


Figure 4. - $2/3$ -Cosine axial power distribution.

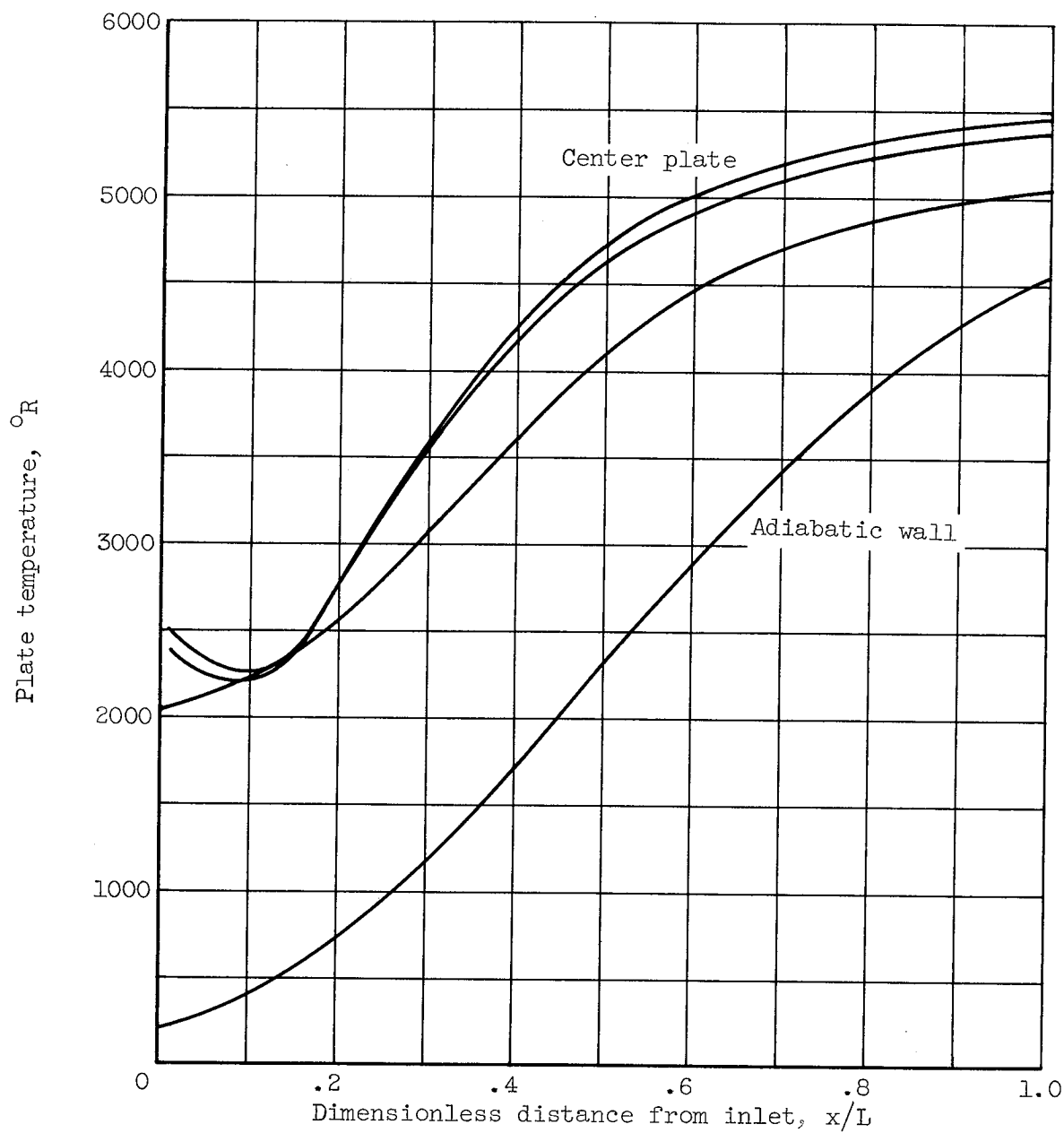


Figure 5. - Typical variation of plate and wall temperature along length of heater. Axial power distribution, $2/3$ cosine; exit static pressure, 40,000 pounds per square foot; plate length, 4.0 feet; exit Mach number, 0.4; exit total temperature, 5000°R ; inlet total temperature, 200°R ; plate spacing, 0.004 foot; number of plates, five; plate emissivity, 0.35.

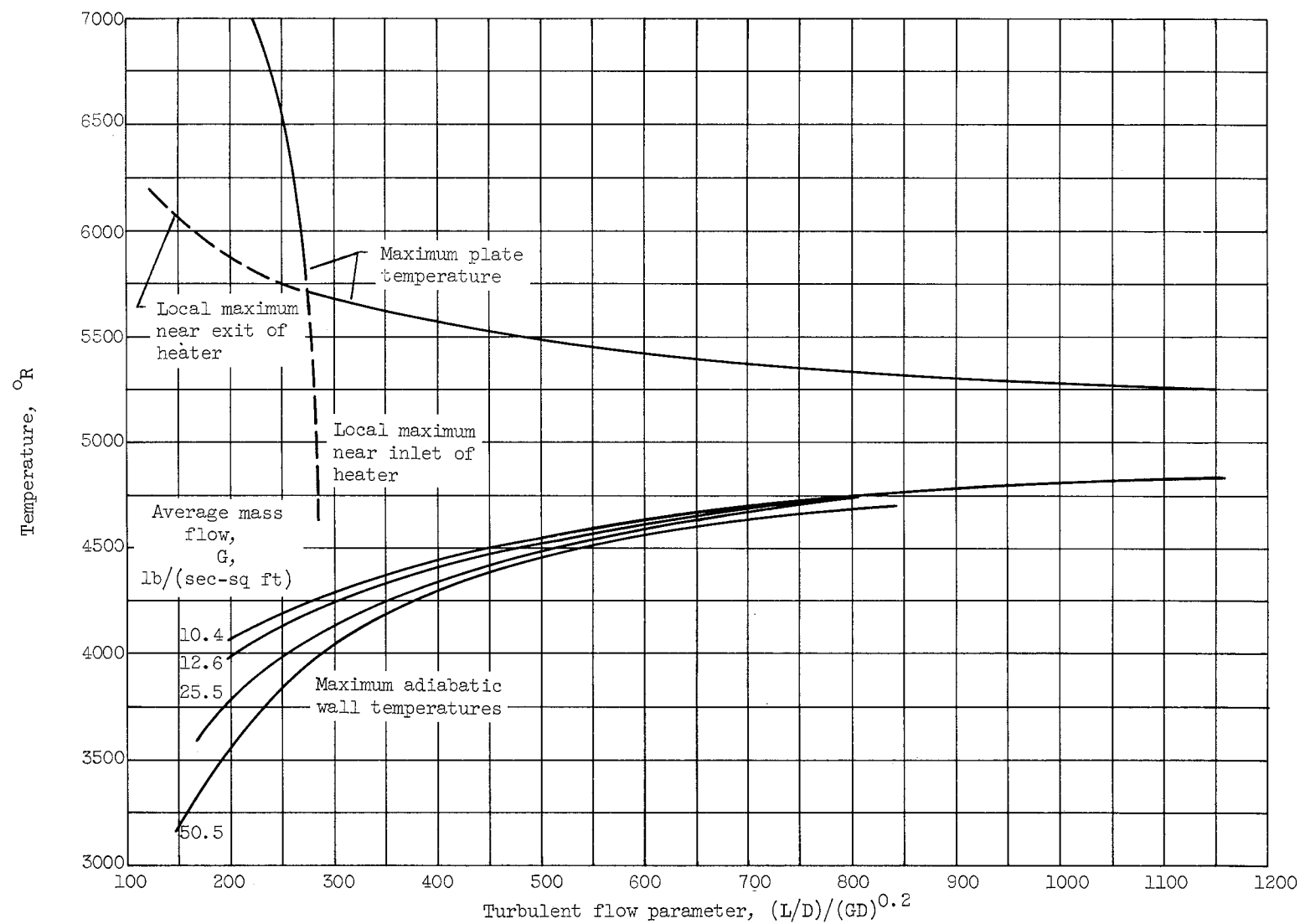


Figure 6. - Maximum plate and adiabatic wall temperatures in a multiple parallel plate heater for turbulent flow. Axial power distribution, $2/3$ cosine; exit total temperature, 5000°R ; inlet total temperature, 200°R ; number of plates, five; plate emissivity, 0.35.

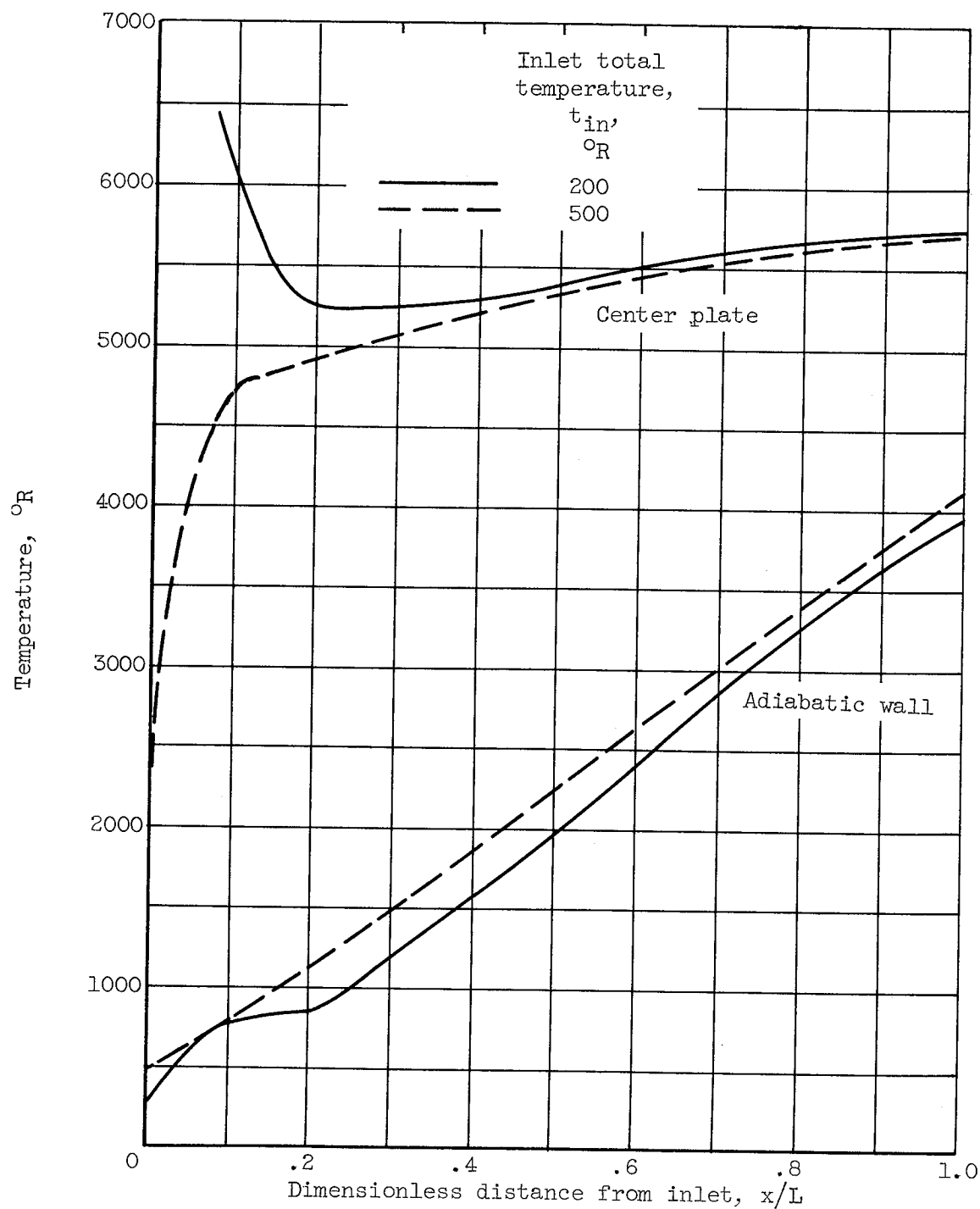


Figure 7. - Effect of high T_s/t_b at inlet (low inlet temperature) on axial temperature distribution along plates. Axial power distribution, $2/3$ cosine; exit static pressure, 40,000 pounds per square foot; exit Mach number, 0.4; total length of plates, 3.0 feet; plate spacing, 0.006 foot; exit total temperature, 5000°R ; plate emissivity, 0.35; number of plates, five.

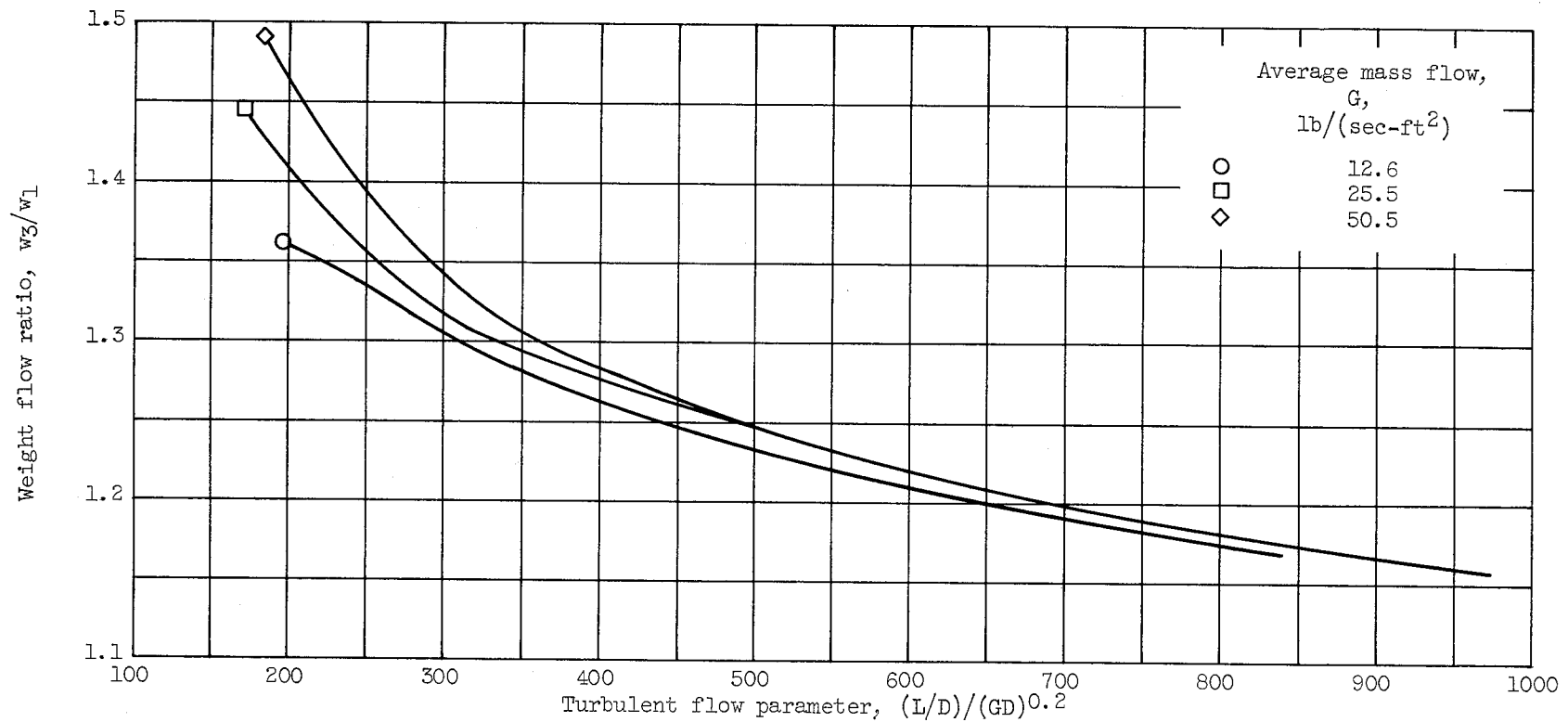


Figure 8. - Ratio of flow in outer passage to that in innermost passage as a function of $(L/D)/(GD)^{0.2}$ for turbulent flow. Exit total temperature, 5000°R ; inlet total temperature, 200°R ; number of plates, five; plate emissivity, 0.35. w_3 is flow in passage adjacent to adiabatic wall; w_1 is flow in innermost passage.

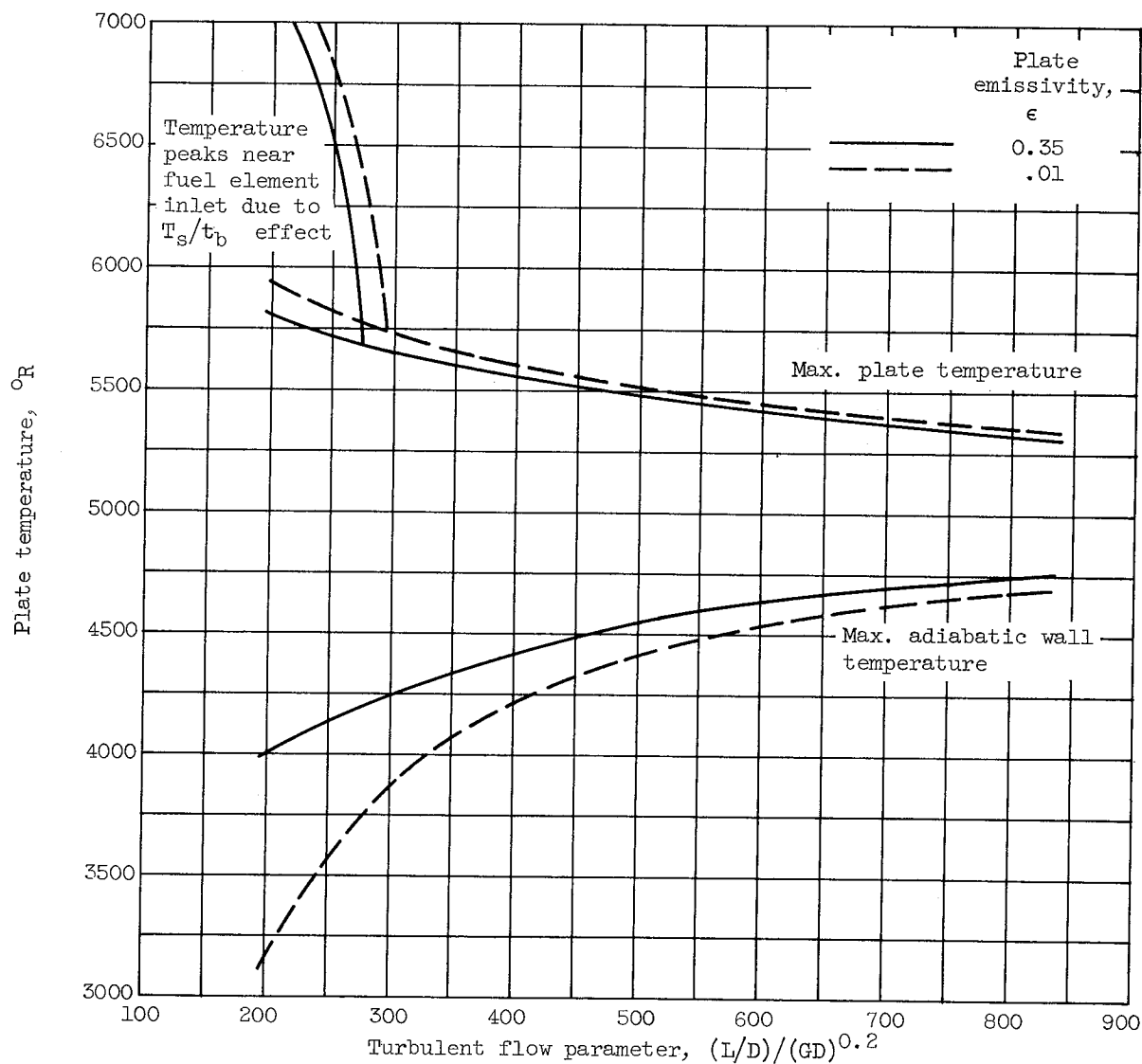


Figure 9. - Effect of radiation between plates on maximum plate and wall temperatures. Exit static pressure, 10,000 pounds per square foot; exit Mach number, 0.4; inlet total temperature, 200° R; outlet total temperature, 5000° R; number of plates, five.

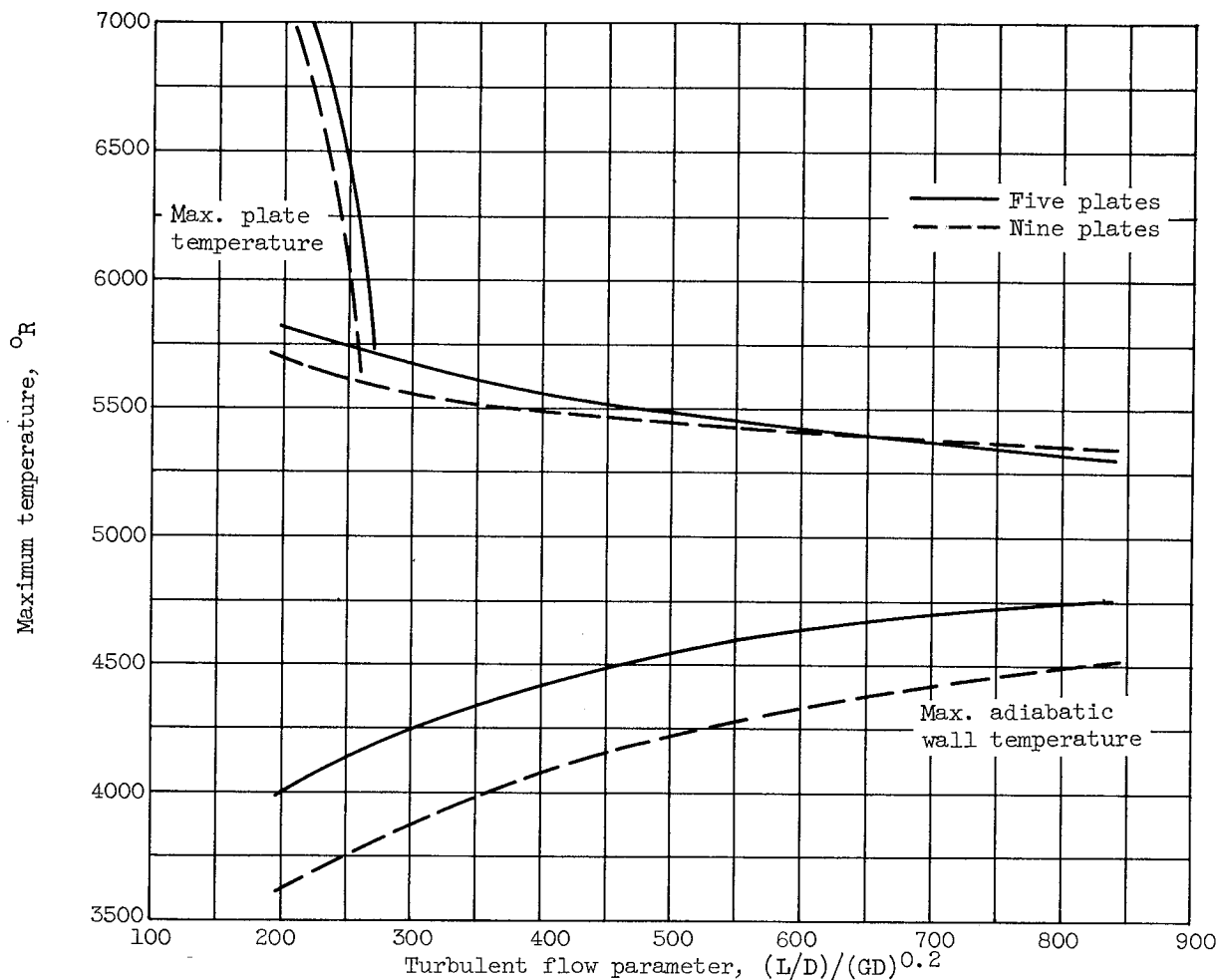


Figure 10. - Effect of number of plates in heater on maximum temperatures. Exit total temperature, 5000° R; inlet total temperature, 200° R; exit static pressure, 10,000 pounds per square foot; exit Mach number, 0.4; plate emissivity, 0.35.

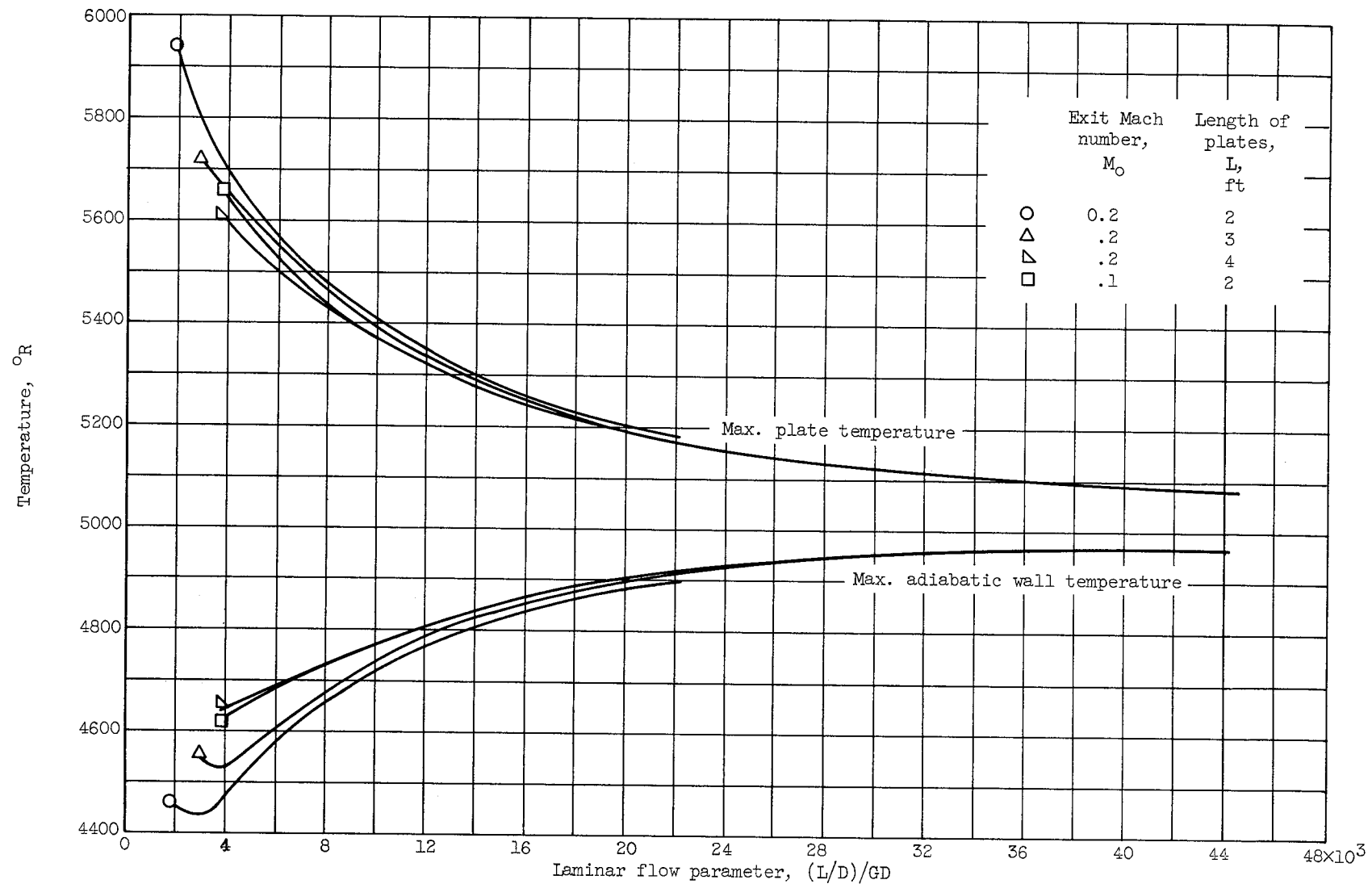


Figure 11. - Maximum plate and adiabatic wall temperatures as a function of $(L/D)/GD$ for laminar flow. Axial power distribution, $2/3$ cosine; exit total temperature, $5000^\circ R$; inlet total temperature, $200^\circ R$; number of plates, five; plate emissivity, 0.35.

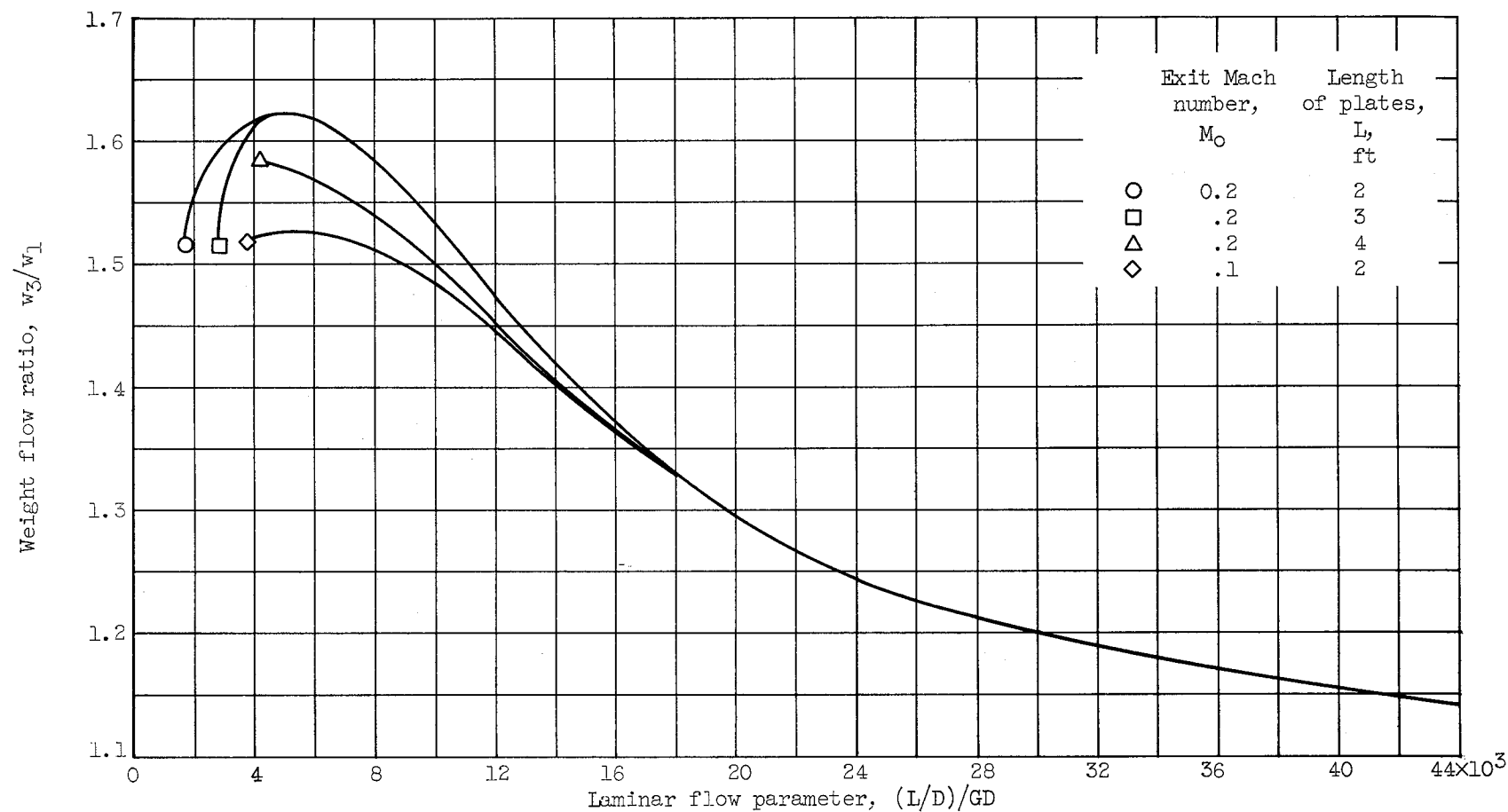


Figure 12. - Ratio of flow in outer passage to that in innermost passage as a function of $(L/D)/GD$ for laminar flow. Exit total temperature, 5000°R ; inlet total temperature, 200°R ; number of plates, five; plate emissivity, 0.35. w_3 is flow in passage adjacent to adiabatic wall; w_1 is flow in innermost passage.

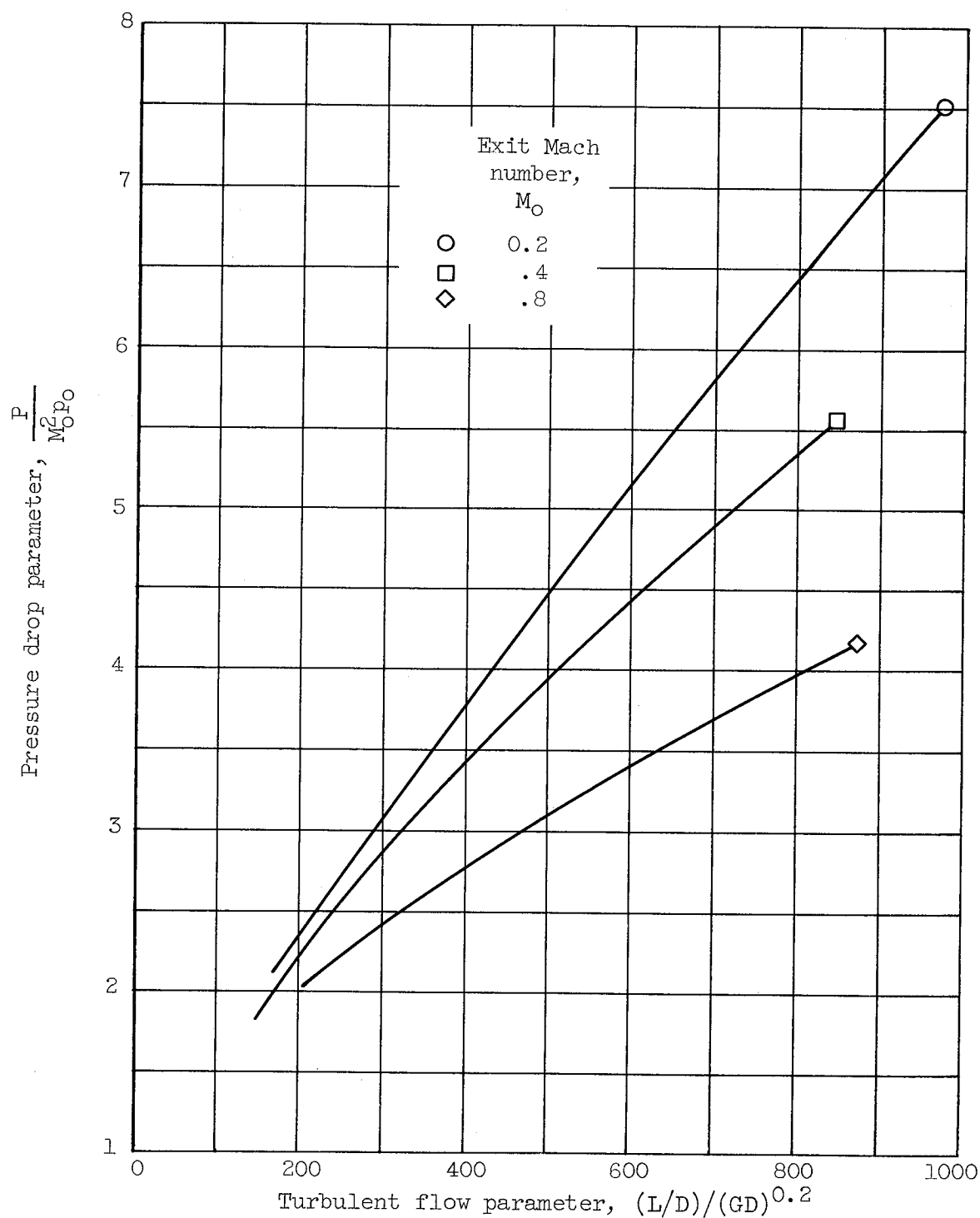


Figure 13. - Variation of pressure drop parameter with Mach number and turbulent flow correlating parameter for turbulent flow. Inlet total temperature, 200°R ; exit total temperature, 5000°R ; number of plates, five.

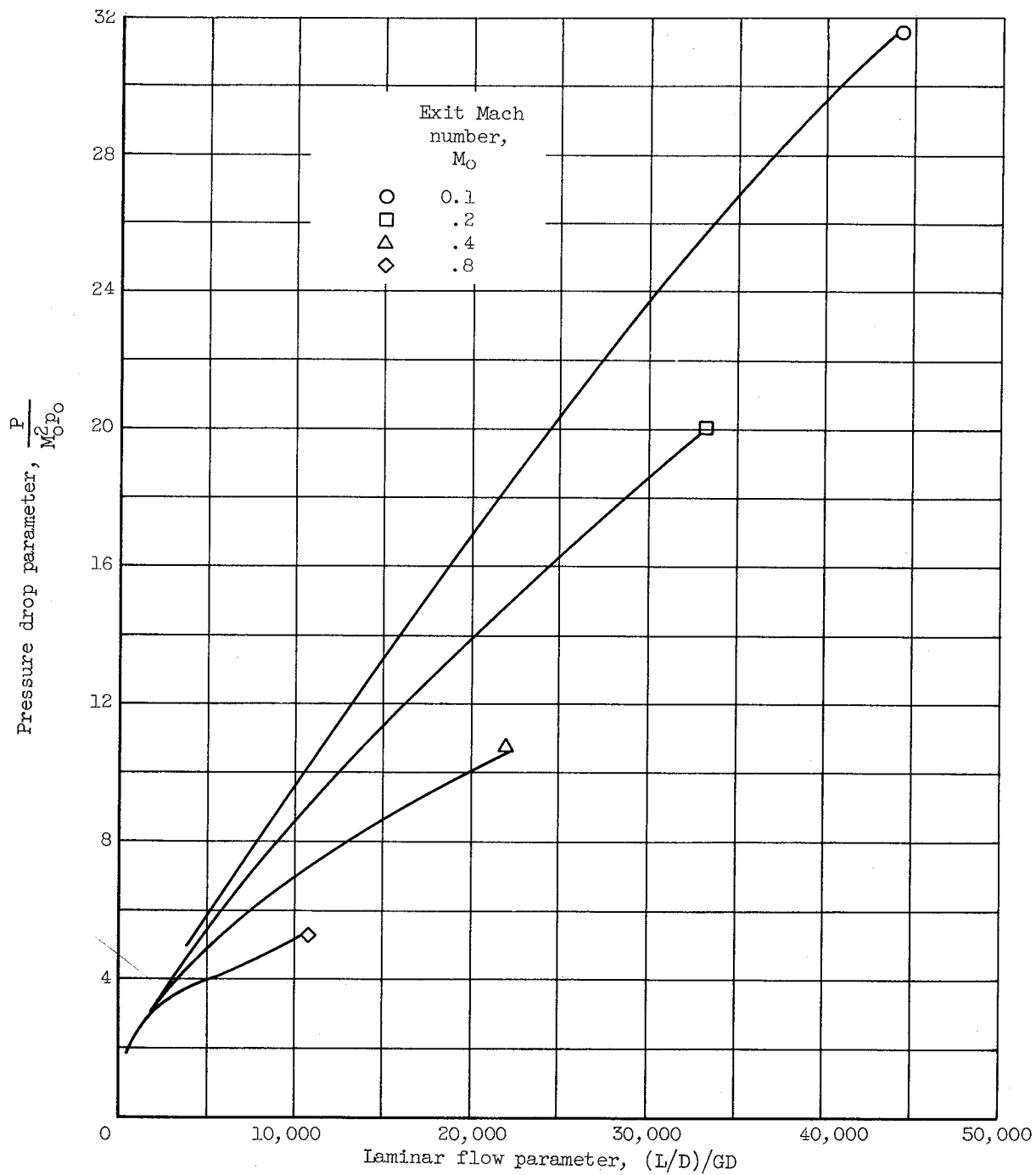


Figure 14. - Variation of pressure drop parameter with Mach number and laminar flow correlating parameter for laminar flow. Inlet total temperature, 200°R ; exit total temperature, 5000°R ; number of plates, five.

NASA TN D-1165

National Aeronautics and Space Administration.
ANALYSIS OF HEAT TRANSFER AND PRESSURE
DROP FOR A GAS FLOWING THROUGH A SET OF
MULTIPLE PARALLEL FLAT PLATES AT HIGH
TEMPERATURES. Thomas H. Einstein. December
1961. 40p. OTS price, \$1.00.
(NASA TECHNICAL NOTE D-1165)

Equations are derived for heat transfer and pressure drop in a gas flowing through a heater composed of parallel, heat generating, flat plates operating at high temperatures. The effect of radiation between the plates is considered. The resulting equations were solved on an IBM 704 computer for a series of cases in which hydrogen was the working fluid with gas inlet and exit temperatures of 200° and 5000° R. The results presented are maximum plate temperature, flow distribution between the plates, and over-all pressure drop. The results also show the effects of laminar and turbulent flow, of changing the length to

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20, Fluid mechanics;
37, Propulsion system
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NASA TN D-1165

National Aeronautics and Space Administration.
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MULTIPLE PARALLEL FLAT PLATES AT HIGH
TEMPERATURES. Thomas H. Einstein. December
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(NASA TECHNICAL NOTE D-1165)

Equations are derived for heat transfer and pressure drop in a gas flowing through a heater composed of parallel, heat generating, flat plates operating at high temperatures. The effect of radiation between the plates is considered. The resulting equations were solved on an IBM 704 computer for a series of cases in which hydrogen was the working fluid with gas inlet and exit temperatures of 200° and 5000° R. The results presented are maximum plate temperature, flow distribution between the plates, and over-all pressure drop. The results also show the effects of laminar and turbulent flow, of changing the length to

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