Displacement Compensation of Temperature Probe Data

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KEYWORDS

ABSTRACT

Analysis of temperature data from a probe in a vertical Bridgman furnace growing germanium crystals revealed a displacement of the temperature profile due to conduction error. A theoretical analysis shows that the displacement compensation is independent of local temperature gradient. A displacement compensation value should become a standard characteristic of temperature probes used for temperature profile measurements.

TEMPERATURE PROBE COMPENSATION

When temperature probes are used to measure the temperature of surrounding fluids, liquids, or gases, corrections are frequently made to the raw temperature data. These corrections are intended to account for rapid temporal and spatial changes of the environmental temperature, direct heating of the probe through absorption of radiation, and conductive heat loss through the structure supporting the sensitive element. For each of these corrections, formulas have been developed over the years
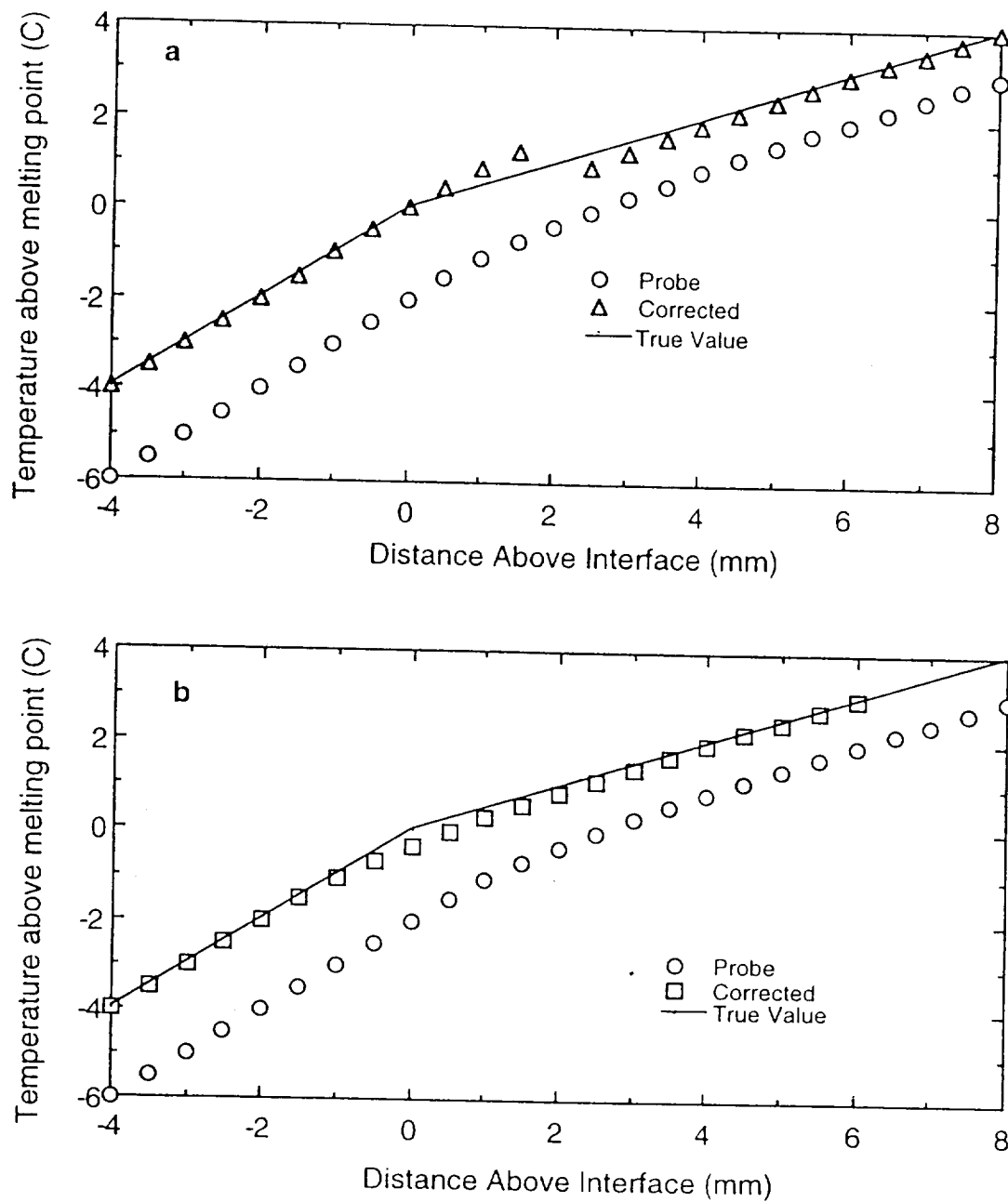
Figure 2. Example showing corrections to simulated probe data. True temperature is shown by the line segments and probe data are shown by circles. In 2a, the triangles indicate the temperature correction based on remote slopes and the apparent break in the temperature curve while in 2b, the squares indicate a displacement correction. The simulation uses idealized parameters resembling Bridgman furnace conditions.
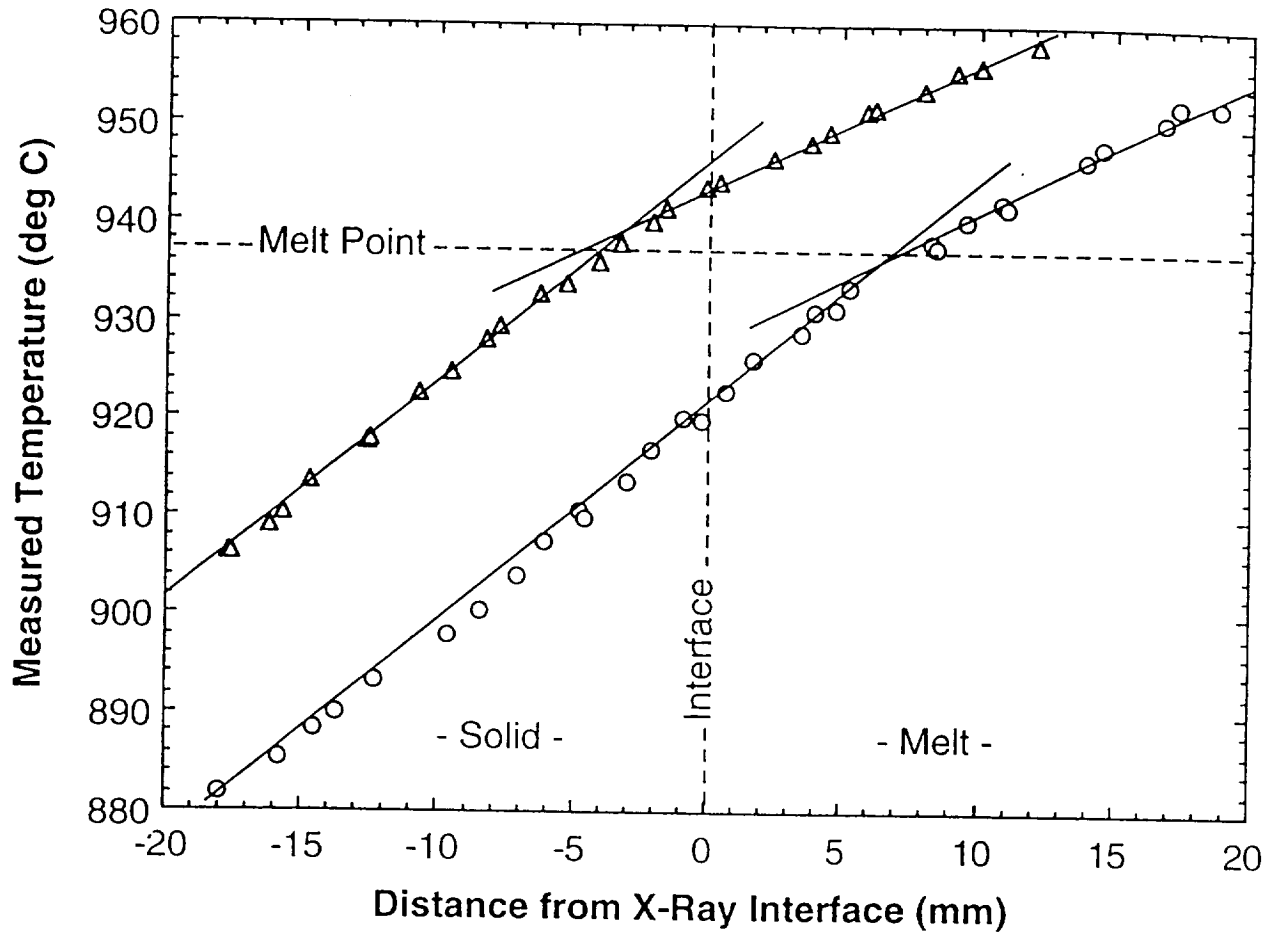
Figure 3. Temperature Profiles in a Bridgman Furnace obtained by thermocouple probes extending from the top (triangles) and bottom (circles) of a centerline capillary tube. Each measurement is made with respect to a melt-solid interface in a sample ampule containing germanium. (after Barber, *et al*, 1996)

and are available in handbooks so that practitioners can have access to ready guidance. For thermocouple measurements a reference such as Moffatt [1962] is among those available. In general, these formulas provide correction values of temperature to be applied at the position ascribed to the sensitive element of the probe. This paper reports another interpretation which applies a correction to the position, rather than the temperature, of the sensitive element. In some cases, a displacement correction is more intuitive and somewhat simpler than the temperature correction.

DISCREPANCY BETWEEN RADIOGRAPHIC AND TEMPERATURE PROBE MEASUREMENTS

The motivation for this work came from an experiment in which the position of the melt-solid interface in a vertical Bridgman furnace used for the growth of germanium crystals was estimated by two independent methods: x-ray radiography and temperature profile measurements from a thermocouple probe. The objective of the experiment was to verify whether thermocouple measurements could be used to monitor crystal growth.

The furnace consisted of two nearly isothermal sections maintained at approximately 1100 K in the lower section and 1270 K in the upper section. The temperatures of these two sections bracketed the 1210 K melting point of germanium. With this furnace configuration the melt-solid interface can be kept in the 3 cm region between the isothermal zones during most of the growth. A schematic diagram of the experimental setup is shown in Figure 1.

Because the solid and liquid phases of germanium differ in density by 4%, the melt-solid boundary could be measured using x-ray radiography. Measurements were made from radiographic images on film, and the boundary location was estimated by measuring the position of the interface with respect to fixed objects which provided fiducial locations on the film. One of the features visible on the film is the tip of the thermocouple probe, providing a verification of the position measurement of the probe.

The temperatures were measured using a thermocouple which moved inside a centerline capillary tube. The capillary tube was fabricated as part of the sample ampule [Hubert, et al, 1993]. Because the thermal conductivity of the liquid and crystalline phases of germanium differ, the interface location could be estimated using a plot of the temperature vs position data obtained by the thermocouple in the centerline capillary. The location of the interface was postulated as the point at which the slope of the temperature versus position data changes. This assumes that heat flux is primarily axial in the cylindrical germanium sample and continuous through the interface. The

difference in thermal conductivity requires a compensating difference in temperature gradient to maintain continuity in the heat flux. In the measured data, the sharp break in temperature gradient was smoothed by conduction effects, so the position was determined as the intersection of two straight lines fit by least squares to the data on either side of the approximate position.

The positions of the melt-solid interface determined radiographically and thermally differed systematically by 3 mm, The melt-solid interface determined from the thermal data was 3 mm into the liquid zone as determined by the x-ray measurements.

NUMERICAL MODEL OF PROBE TIP TEMPERATURE

Hubert [1992] constructed a series of numerical models to evaluate the radiation environment and thermal flow in the entire furnace, including the thermocouple probes. These models were designed and run to represent varying locations of the thermocouples found in the experimental data. The results verified the experimental observation that the temperature profile measured at the thermocouple tip is shifted 3 mm toward the liquid zone. The results also showed that the heat flow through the sample is generally axial near the insulating region of the furnace. This result along with the axial symmetry of the experiment allow a good approximation of the thermocouple temperature end-effects to be obtained through one-dimensional analysis.

MATHEMATICAL ANALYSIS LEADING TO DISPLACEMENT INTERPRETATION

The formula given by Moffatt [1962] for correction of conduction error in thermocouples used to measure temperatures in gas streams is

$$T_T - T_J = (T_T - T_M)/\cosh[L(4h_c/d\,k_s)^{1/2}] \tag{1}.$$

In this equation, T_T represents the true temperature of the flowing gas stream being sampled by the thermocouple, which is considered as extending into the stream from a wall of the experimental chamber or model surface. T_J is the temperature of the thermocouple junction, so the left side of the equation corresponds to the conduction error. T_M is the temperature of the mount for the thermocouple, or the chamber wall. L is the length of the thermocouple junction, h_c is the coefficient of thermal transfer between the junction and the surrounding gas, and k_s is the thermal conductivity of the junction along the length of the wires. One way to visualize the thermocouple

described by this equation is as a long cylinder of length, L, much greater than its diameter, d, extending from a flat mount with the junction temperature represented by the temperature at the terminal end of the cylinder. Equation (1) can be derived from a one-dimensional analysis of the conductive heat equation on such a structure if the heat flux through the top of the cylinder is neglected and the gas surrounding the cylinder is considered to be isothermal with a constant thermal transfer coefficient.

The argument of the hyperbolic cosine in Eq. 1 contains many of the physical parameters which determine the amount of conduction error. As the product of L, which has dimensions of $(length)^1$ and the factor within the square root, must be dimensionless, the factor within the square root must have dimensions of $(length)^{-2}$. The inverse of this factor can be interpreted as a product of two terms, each having the dimension of $(length)^1$. As noted by Moffatt [1962], one of these, $d/4$, represents the ratio of the area of a cross-section of the cylinder to its perimeter. The value of this factor can clearly be altered by choosing different junction cross-sections, so the factor forms a useful design parameter. The other factor, k_s/h_c, is the ratio of the axial thermal conductivity of the junction to its heat transfer coefficient. This factor has dimensions of $(length)^1$. The square root of the product of these factors occurs so often in the analysis that we choose to give it a separate symbol, Λ, and treat it as the major parameter of the analysis,

$$\Lambda = [(w/p)(k_s/h_c)]^{1/2}, \tag{2}$$

where w and p denote respectively the area and perimeter of the cross section. In this form, the argument of the hyperbolic cosine in Eq. 1 becomes (L/Λ), the ratio of the junction length to a characteristic length.

To adapt this analysis to the environment of the Bridgman furnace, the most important addition is a temperature gradient along the length of the probe. The one–dimensional heat equation is then

$$T''(x) - (1/\Lambda^2)T(x) = -(a/\Lambda^2)x, \tag{3}$$

as given in Carslaw and Jaeger [1959]. The new parameter, a, denotes the thermal gradient of the surroundings. If the cylindrical rod carrying the thermocouple is considered to extend from negative infinity to x=0 and the ambient temperature scale is set to zero at x=0, , the solution is given as

$$T(x) = ax - a\Lambda \exp(x/\Lambda), \quad (x<0). \tag{4}$$

At the tip (x=0), the temperature of the surroundings is zero, but that of the probe tip is -aΛ. This happens to be the temperature of the surroundings at location x=-Λ, so one can interpret the difference as a temperature difference at the probe tip or alternately as a displacement difference of the measurement location. If the displacement interpretation is chosen, the correction needed does not depend on the value of the thermal gradient, a value which is not available prior to the measurement.

CORRECTION TO DATA FROM THE BRIDGMAN FURNACE

The Bridgman furnace provides an example which illustrates the difference between adjusting for conduction in a thermocouple through a temperature correction and through a displacement. The data are simulated for a material with exactly a two-to-one ratio of thermal conductivities for the melt and solid phases respectively. Hubert [1992] solved the linear heat equation for the temperature profile in a probe passing through the interface in this case. For a probe entering from the cold region, the tip temperature is given by

$$T_j(x) = a (x - \Lambda), \text{ for } x < 0 \tag{5}$$

and

$$T_j(x) = (a x /2) - (a \Lambda /2) (1 + \exp (- x / \Lambda)), \text{ for } x > 0, \tag{6}$$

where $T_j(x)$ denotes the tip temperature for a probe with its tip at location x, the distance above the interface. Equations (5) and (6) are plotted as circles in Fig. 2a and 2b for an example probe having a value for Λ of 2 mm, and the heat flow is chosen to produce exactly a 1 degree/mm gradient in the solid phase and 0.5 degree/mm gradient in the liquid phase. The temperature of the material in this idealization is represented by the solid lines. The change in slope of the probe temperatures is seen to occur approximately 2 mm into the liquid phase of the material.

If a temperature correction is applied to these data, the correction is dependent of the slope of the data. The data on either side of the observed change in slope can be corrected by the corresponding amount and are represented by triangles in Figure 2a. As shown, the data between the actual interface and the observed change in slope cannot be properly corrected. Also, the data near the observed change in slope cannot easily be used due to the curvature present. If linear curve fits are applied to the remainder of the data, the approximate location of the actual interface can be found. However, this process requires estimation of the slope of the data twice, which induces additional error into the correction, and it cannot use all the available data.

The alternative is simply to apply the displacement correction Λ to the entire data set, thereby shifting the curve appropriately. Figure 2b shows the same material

temperature and measured values as figure 2a. The squares represent the displacement correction of Λ (2 mm) towards the solid region. This simple, uniform correction is independent of the particular data, and its accuracy is given by the thermocouple and environmental properties. The displacement correction automatically produces an appropriate correction for the conduction error associated with the measured temperature values.

EXPERIMENTAL VERIFICATION

Barber, *et al* [1996] performed an experiment in which a thermocouple was used to measure the temperature profile in a Bridgman furnace compared with a radiographically determined phase interface with the thermocouple inserted from both the cold zone and the hot zone of the furnace along a centerline capillary. The two temperature profiles, shown in Figure 3, were displaced from one another by about 11 mm. The temperature error at the melt point was 5 K for the probe coming from the hot zone and 16 K for the probe entering from the cold zone. The breaks in the two sets of lines differed in temperature by only 3 K. The midpoint between the two breaks in the the lines fell within one degree K of the melt point of germanium and within 2 mm of the interface as determined from x–radiography. This finding agrees substantially with the one-dimensional theoretical analysis. In this experiment, type R thermocouples were used which have a significantly larger displacement correction parameter, Λ, than the type K thermocouples used to obtain the data reported by Hubert [1992]. As a further test of the hypothesis that conduction error manifested in an end effect was responsible for the spatial offset between the radiographically determined interface and that determined by probe measurements, Barber, *et al.*[1996] constructed a type "R" (Pt - Pt/10%Rhodium) thermocouple with one leg extending from the hot region of the furnace and the other one extending from the cold region. The measured break in the curves occurred well within 1 K of the melt point and slightly more than 1 mm from the radiographically measured interface. The remaining spatial offset may be attributable to the difference in thermal conductivities of the two thermocouple materials, the alloyed platinum having slightly less than half the thermal conductivity of the pure platinum.

CONCLUSION AND RECOMMENDATION

We conclude that conductivity error leads to a spatial offset in temperature profile data taken with the thermocouple probe in the Bridgman furnace experiments. Because the error arose from causes unrelated to the specifics of the particular environment, we conclude that spatial offset is a general feature of temperature profile data obtained with

probes. We recommend that the offset parameter, Λ, become a standard, reported characteristic of temperature probes which are intended for use in temperature profiling measurements.

ACKNOWLEDGEMENTS

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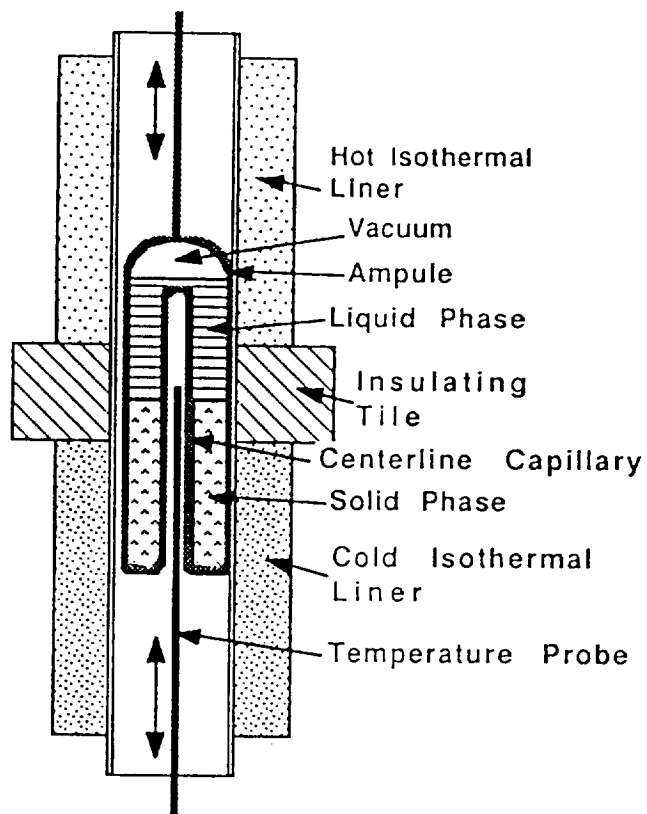
Figure 1. Schematic representation of the temperature measurement apparatus in a Bridgman furnace used to monitor crystal growth. The ampule can be moved to grow or melt the solid phase, and the temperature probe can be moved to sample different positions. The drawing is not to scale.