Using Sequential Earth Images to Determine the Orientation of a Satellite at L1

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Abstract. This paper introduces a method for determining the three-axis attitude (orientation) of a spacecraft from a sequence of Earth images taken while in orbit about the stationary Lagrange point between the Earth and the sun (L1). Two axes of information can be obtained simply by monitoring the Earth's position within the field-of-view of the camera. The third axis is obtained by correlating sequential Earth images in such a way as to detect the flow of the features within the Earth's disk over time. When taken correctly, this measurement is perpendicular to the Earth's spin axis and, consequently, locates the Earth's spin axis within the images. This is equivalent to determining the orientation of the spacecraft about the instrument's boresight. Actual data from the Galileo spacecraft is used to test the algorithms outlined in this paper.

Introduction

Typically, Earth-imaging spacecraft carry a myriad of independent attitude (orientation) determination hardware that is used to "blindly" point an Earth-imaging instrument at the Earth, without actually "looking" at the image of the Earth. Advances in on-board processors, as well as increases in on-board data flow rates have started to open the door to all kinds of possible uses for the actual science data in the attitude determination process. These advances could potentially reduce the amount of stand-alone hardware needed to perform some of these missions, subsequently reducing the size, weight, and cost of these missions. If nothing else, some of these Image-aided Attitude Determination (IAD) techniques could serve as back-ups to the primary attitude determination systems.

This paper examines a technique for doing three-axis spacecraft attitude determination from an orbit about the stationary Lagrange point located between the Earth and the sun (L1) with nothing more than a sequence of images of the mostly sun-lit (from that vantage point) Earth. This work was initiated as part of the Triana preliminary design effort currently underway at NASA's Goddard Space Flight Center. The Triana spacecraft is being designed to orbit L1 and provide images of a mostly sun-lit Earth, 24 hours-a-day. This paper makes use of an existing set of data taken from the Galileo spacecraft on December 11-12, 1990, as it swung by the Earth on its way to Jupiter. These particular images are very similar to those which Triana is expected to provide from its vantage point at L1. This similarity makes the analysis of these images directly applicable to the Triana effort.

The Approach

The location of the Earth within an instrument's field-of-view (FOV) provides two axes of spacecraft attitude information. The third axis can be obtained by determining the orientation of the Earth within the instrument's FOV. The Earth's spin axis can be detected by comparing sequential images of the Earth's lit disk. Features within the disk move from one frame to the next, relative to the disk's boundary, as the Earth spins. The features move in paths which follow the projections of the Earth's latitude lines onto the instrument's focal plane. Depending on the orientation of the Earth within the instrument's FOV, these paths may or may not be perpendicular to the Earth's spin axis at every point within the visible disk. At a minimum, these paths are perpendicular to the Earth's spin axis at the points where they intersect the projection of the spin axis onto the instrument's focal plane (see the representative orientations in Figure 1 given below). The Earth's spin axis passes through the center of the Earth's disk, regardless of its orientation. Therefore, the apparent motion of the features at the center of the Earth's disk will always be perpendicular to the Earth's spin axis.
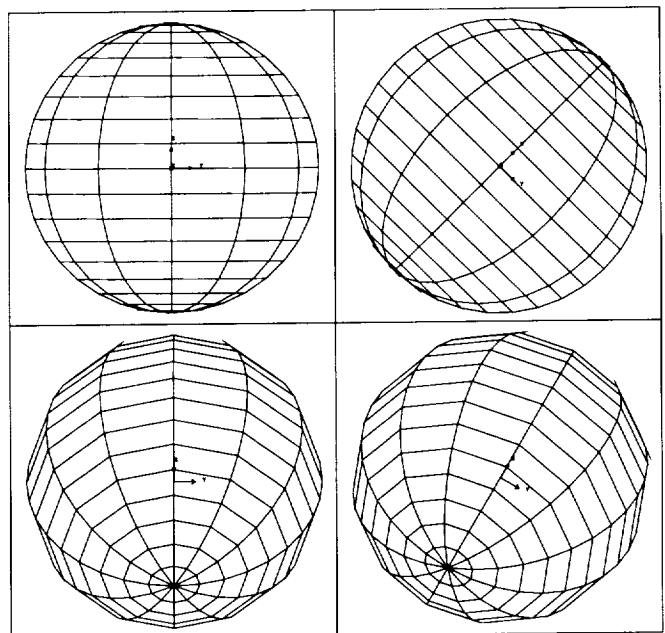

Figure 1: representative Earth orientations within the field-of-view

Satellites in orbit about L1 see an Earth-disk which is as much as 20% dark (like a less-than-full moon). Consequently, locating the center of the physical Earth-disk is not equivalent to finding the centroid of the partially lit Earth-image within the instrument's FOV. An approach for estimating the center of the Earth's actual disk from a partially lit disk is described in the following section.

Center Estimation

The Earth's idealized (circular) disk can be circumscribed by a square. This square has four points in common with the disk. If any neighboring pair of these points is known, and lines are drawn perpendicular to the sides (of the square) containing these points, through these points, then the intersection of these two lines locates the center of the Earth's disk. A partially lit Earth-image can be circumscribed, not by a square, but by a rectangle. Two sides of this rectangle share points with the actual Earth-disk. The other two sides are tangent to the terminator. Consequently, the rectangle has two sides in common with the larger square circumscribing the actual, not entirely visible Earth-disk. These sides can be determined and used to estimate the center of the actual disk.
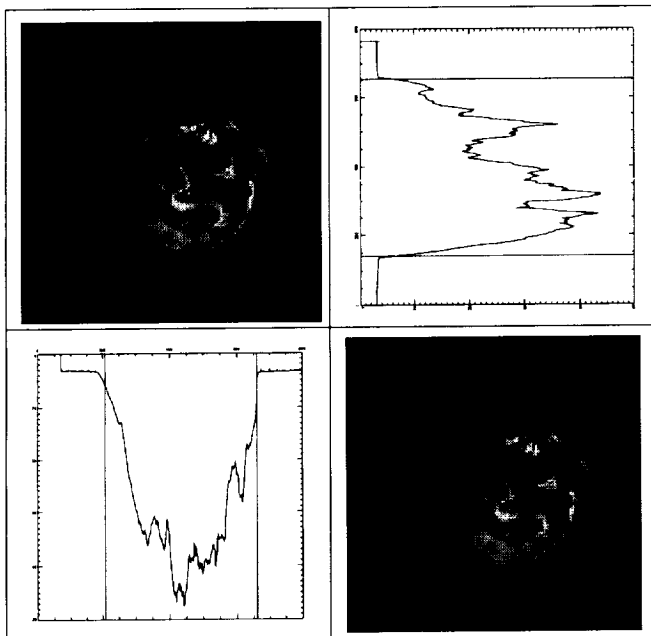

Figure 2: the rectangle circumscribing the partially lit Earth-image
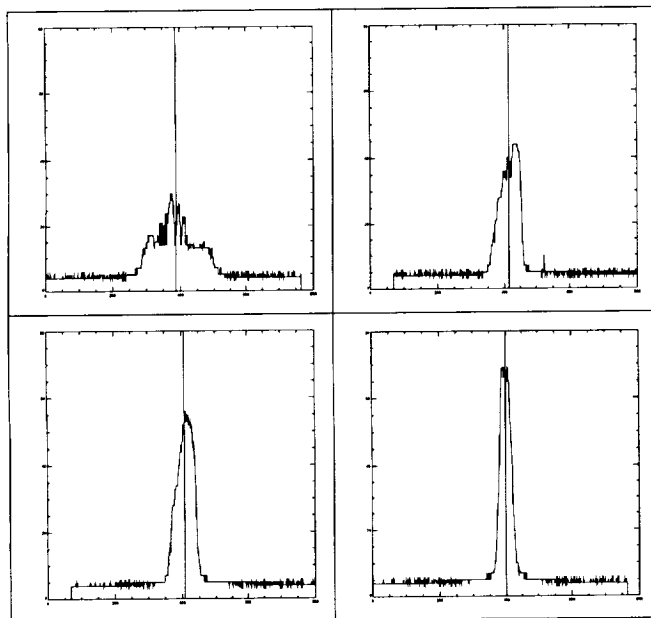

Figure 3: the tangent points along each edge of the rectangle
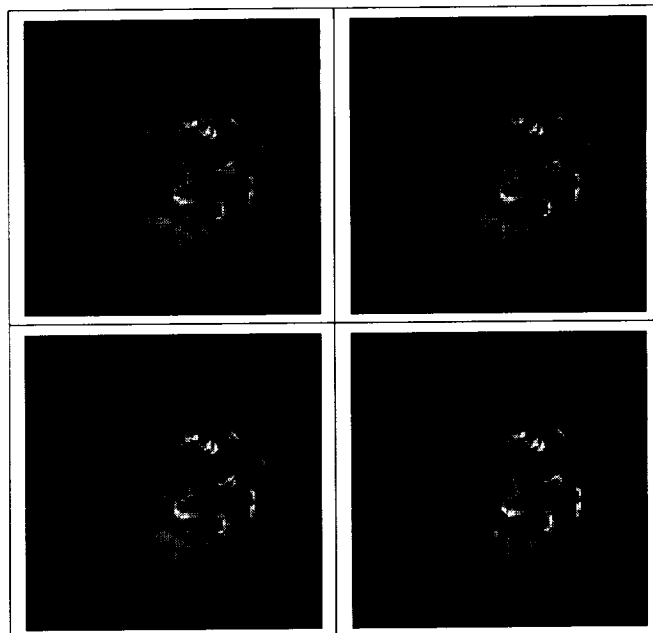

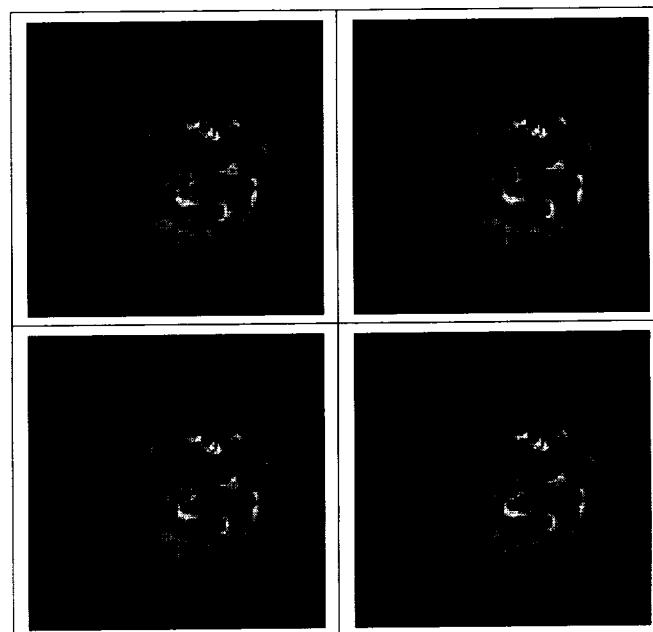Figure 4: edge/line pairs for the circumscribing rectangle

Figure 5: rectangles formed by edge/line pair combinations

Start by considering a typical Earth image like the one given in the first cell of Figure 2. The rectangle which circumscribes this image can be found by analyzing the row-sums and column-sums of the actual data (see Figure 2). The tangent points along each edge of this rectangle can be located by finding the center of the actual data along each of these lines (see Figure 3). Constructing a line perpendicular to each edge at its tangent point gives a set of four edge/line pairs (Figure 4). Each of these pairs, when combined with a neighboring pair, forms, at a minimum, a rectangle (Figure 5). The two pairs which correspond to points on the actual Earth-disk, when combined, form a perfect square which represents one-quarter of the square that circumscribes the full Earth-disk. This quarter-square locates the center of the full Earth-disk. The following equation can be used to deter-

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mine which combination of neighboring edge/line pairs most closely forms a square and, consequently, gives the best estimate of the Earth's center:

$$\hat{\mathbf{c}} = \arg\min_{c \in C} \left[\left| \left(\frac{c_4 - c_1}{c_3 - c_2} \right) - 1 \right| + \left| \left(\frac{c_3 - c_2}{c_4 - c_1} \right) - 1 \right| \right]$$

Where:
$$C = \left\{ \begin{array}{c} (B, B^{\perp}, L, L^{\perp}), (L, L^{\perp}, T, T^{\perp}), \\ (T, T^{\perp}, R, R^{\perp}), (R, R^{\perp}, B, B^{\perp}) \end{array} \right\}$$

B, L, T, R = bottom, left, top, and right edges

$B^{\perp}, L^{\perp}, T^{\perp}, R^{\perp}$ = lines $\perp$ to B, L, T, R

Motion Detection

An estimate for the location of the Earth's center gives no information about how the Earth is orientated within the image. This orientation information can be obtained by processing sequential images in such a way as to detect the flow of the features within the Earth's disk over time. The method used for doing this image correlation was the Mean-of-Squared Differences (MSD) algorithm, as it appears in [1]. This algorithm estimates the shift between two successive images by finding a window, in the second image, that best matches a predetermined test window in the first image. It does this by minimizing the following quadratic "similarity" function (cost function) over the permissible, two-dimensional shift space:

$$\hat{\mathbf{s}}_{MSD}(N) = \arg\min_{s \in S} \frac{1}{n_T} \sum_{\mathbf{x}_p \in X_T} \left[I_N(\mathbf{x}_p + \mathbf{s}) - I_{N-1}(\mathbf{x}_p) \right]^2$$

Where:
$I_{N, N-1}$ = current and previous images (respectively)

$\mathbf{x}_p$ = a given pixel location within I_N and I_{N-1}

X_T = the given test window within I_{N-1}

n_T = the number of pixels within the test window

S = the permissible shift space within I_N

A computationally efficient method of solving this minimization problem involves breaking it into two pieces; first, finding the best whole pixel shift estimate, and second, estimating the fractional pixel shift from the whole pixel cost "surface" in the neighborhood of the best whole pixel estimate. The cost "surface" is obtained by evaluating the cost function (given above) at every permissible whole pixel shift in the shift space (see Figure 6). The minimum value obtained in this manner corresponds to the best whole pixel shift estimate. This value, taken together with its closest neighbors, forms a 3x3 set of points that can be fitted with a parabolic surface whose minimum lies somewhere inside the 3x3 grid. The location of this minimum represents the fractional pixel shift portion of the overall shift estimate. This minimum can be found by evaluating the following equation:

$$\tilde{\mathbf{s}} = -\begin{bmatrix} a & b \\ b & c \end{bmatrix}^{-1} \begin{bmatrix} d \\ e \end{bmatrix}$$

Where:
$a = J(1,0) - 2J(0,0) + J(-1,0)$

$b = \frac{1}{4}(J(1,1) - J(1,-1) - J(-1,1) + J(-1,-1))$

$c = J(0,1) - 2J(0,0) + J(0,-1)$

$d = \frac{1}{2}(J(1,0) - J(-1,0))$

$e = \frac{1}{2}(J(0,1) - J(0,-1))$

$J(i,j)$ = cost value at $\hat{\mathbf{s}}$(pixel) $\pm$ (i,j) pixels

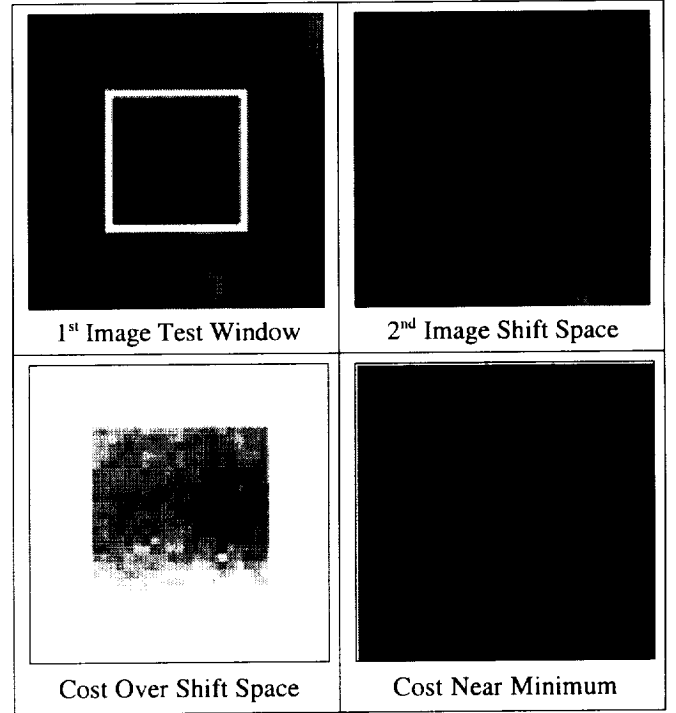| 1st Image Test Window	2nd Image Shift Space
Cost Over Shift Space	Cost Near Minimum

Figure 6: evaluating the cost function over the entire shift space

The Galileo Data

On December 11-12, 1990, the Earth measured approximately 500 pixels across in the 800x800-pixel Galileo imager. This gave a resolution of 25.5 km/pixel (two times the Earth's 6380 km radius over the 500 pixel image captured in the data). The Galileo images covered six different wavelengths (three of which were "visible") and came at a rate of one image every minute. The wavelengths were cycled, giving a resultant frame rate, in any given channel, of 1 frame every 6 minutes.

The Earth spins at a rate of 15°/hr (72.2 µrad/s). This results in a maximum ground speed (at the equator) of 0.46 km/s (the Earth's 6380 km radius multiplied by the given spin rate). Over a 6 minute interval, the features at the Earth's equator move a total of 166 km, which, taking into account the 25.5 km/pixel resolution of the Galileo images, gives a nominal image shift of 6.5 pixels/frame.

The Triana specification calls for a 1024x1024-pixel, 0.7° field-of-view image. From L1, the Earth's disk is roughly 0.5° across, which gives a resolution of roughly 15 km/pixel. The Triana images will come at a rate of 1 image every 3 minutes. This results in a nominal image shift (at the equator) of 5.5 pixels/frame. This number is similar to that given above for the Galileo data. This similarity suggests that the results obtained using the Galileo data reflect accuracies that can be achieved by employing the same IAD techniques on the Triana images.

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Results

Figure 7 represents roughly 6 hours worth of Galileo data processing. The first trace was obtained by processing every image (at a rate of one every 6 minutes). The second trace is the result of processing every other image (one every 12 minutes). The differences between the two plots can be attributed to the differences between the signal to noise ratios in each scenario. The shifts being detected between each pair of successive images is relatively small. Any error (e.g. in the determination of the center of the Earth from one frame to the next) has a large effect on the resulting angular calculations. By sampling images at a slower rate, the expected shift doubles, while the noise goes relatively unchanged.

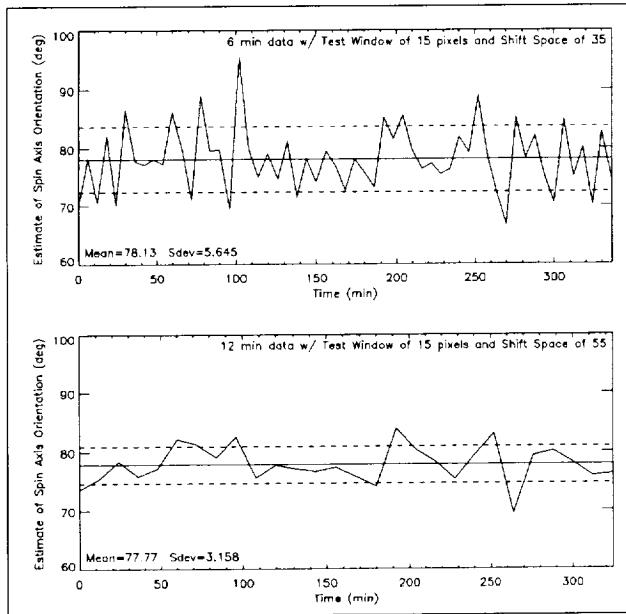Both of the runs shown above used 15x15-pixel test windows. A 35x35-pixel shift space was used with the 6-minute data, and a 55x55-pixel shift space was used with the 12-minute data (the larger shift space is necessitated by the larger nominal image shifts). Both runs gave a similar mean estimate for the Earth's orientation, namely 78° (see Figure 8). The standard deviations for the two data sets were 5.6° and 3.2° (6-minute and 12-minute data, respectively). These numbers were higher than expected, but the process has not been optimized. Some possible areas for improvement are given below.

Conclusion

The primary purpose of this paper was to introduce a method for determining three axes of spacecraft attitude information from a sequence of Earth images taken from L1. Simply monitoring the Earth's position within the instrument's FOV provides two axes worth of information that can be used within a spacecraft's attitude control system. The final piece of information can be obtained by correlating sequential images to determine the orientation of the Earth within the images, and, subsequently, the orientation of the spacecraft with respect to the Earth.

This analysis made use of existing data taken by the Galileo spacecraft in 1990. Only one of Galileo's imaging channels (green) was used in this analysis. This technique relies on strong similarities (content) between subsequent images. Images at different wavelengths would not be similar enough to be used together. Each channel could be used, independently, and the information from each averaged in some fashion, but this goes beyond the scope of this paper. This work focused on getting information out of a single channel. It lays the groundwork for more sophisticated endeavors. Possible avenues worth pursuing include a more sophisticated edge detection technique (the one presented here was meant to be quick) and filtering techniques, which would serve to smooth out some of the noise in the estimates. The primary error source appears to be the centering uncertainties. Because the shifts are small, any error in the determination of the Earth's position, from one frame to the next, will have a big affect on the subsequent spin axis calculations.

Figure 7: 6 hours worth of Galileo data at 6 & 12 minute intervals

Figure 8: mean orientation of the Earth within the Galileo images

References

[1] Y. Oshman and B. Menis. "Maximum a Posteriori Image Registration / Motion Estimation." *Journal of Guidance, Control, and Dynamics*, 17(5), 1994, 1115-1123.