A COMPARISON OF WOLF'S RECONSTRUCTED RECORD OF ANNUAL SUNSPOT NUMBER WITH SCHWABE'S OBSERVED RECORD OF 'CLUSTERS OF SPOTS' FOR THE INTERVAL OF 1826-1868

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Abstract. On the basis of a comparison of Wolf's reconstructed record of yearly averages of sunspot number against Schwabe's observations of yearly counts of 'clusters of spots' (i.e., the yearly number of *newly appearing* sunspot groups) during the interval of 1826-1868, one infers that Wolf probably *misplaced* and *underestimated* the maximum amplitude for cycle 7. In particular, Schwabe's data suggest that the maximum amplitude for cycle 7 occurred in 1828 rather than in 1830 and that it measured about 86.3 (±13.9; i.e., the 90% confidence level) rather than 70.4. If true, then, the ascent and descent durations for cycle 7 should be 5 years each instead of 7 and 3 years, respectively. Likewise, on the basis of the same comparison, one infers that the maximums for cycles 8 and 9, occurring, respectively, in 1837 and 1848, were of *comparable* size (~130), although, quite possibly, the one for cycle 8 may have been smaller. Lastly, presuming the continued action of the 'odd-even' effect (i.e., the odd-numbered following cycle of Hale even-odd cycle pairs having a maximum amplitude that is of *comparable or larger* size than the even-numbered leading cycle) during the earlier pre-modern era of cycles 6-9, one infers that Wolf's estimate for the size of cycle 6 probably is too low.

1. Introduction

In recent years, a reexamination of the pre-modern era of sunspot observations (i.e., the years before 1849) has led to the discovery of previously overlooked observations by Hevelius for the interval of 1653-1684 (Hoyt and Schatten, 1995a) and by Flamsteed for the interval of 1676-1700 and the years of 1703 and 1707 (Hoyt and Schatten, 1995b). Additionally, evidence has come to light that some of Rudolf Wolf's estimates of annual sunspot number, in particular, during the intervals of 1761-1777 and 1819-1833 may, in fact, be wrong (Hoyt and Schatten, 1995c, d). In this paper, Rudolf Wolf's re-constructions of annual sunspot number are compared against Samuel Heinrich Schwabe's observations of the number of 'clusters of spots' for the contemporaneous interval of 1826-1868.

2. A Brief Historical Overview

Recall that, although sunspots have been observed on occasion with the naked-eye for thousands of years and routinely with the telescope since the early 17th century, it was not until the mid 19th century that the quasi-periodic variation of the spottedness of the Sun was truly recognized (Bray and Loughhead, 1965; Schove, 1983; Hoyt and Schatten, 1997). Today, this cyclic variation of the number of spots on the Sun is referred to, simply, as the 'sunspot cycle.'

The individual who first publicly suggested the existence of the sunspot cycle was Samuel Heinrich Schwabe, a German apothecary and amateur astronomer, who diligently and meticulously observed the Sun for more than four decades between 1826 and 1868 from Dessau, Germany (Hoyt and Schatten, 1997; Schröder, 1997). On the basis of his sunspot observations (actually, the annual number of 'clusters of spots' – i.e., the yearly number of *newly appearing* sunspot groups – and the annual number of spotless days) spanning the interval of 1826-1843, Schwabe (1844) deduced that the spottedness of the Sun waxed and waned over a period of about 10 years. Following his announcement, he continued to make sunspot observations through 1868, reporting his tabulations annually as *monthly* counts in *Astronomiche Nachrichten.* (While Schwabe is given credit for being the first to *publicly* acknowledge the existence of the sunspot cycle, Hoyt and Schatten, 1995c, 1997, have shown that the basis for the sunspot cycle clearly exists in the record of observations made by Christian Horrebow and colleagues between 1761 and 1777.)

Following Schwabe's discovery, Rudolf Wolf, a Swiss astronomer from Zürich, set out to establish the validity of Schwabe's conclusion. In particular, he wanted to know whether or not the sunspot cycle was a *real and continuing* effect of the Sun and, furthermore, was there *historical* evidence for its existence (he also wanted to show a *causal* connection between terrestrial events, like aurora, and solar features, like sunspots; Schröder, 1997). To accomplish this, in 1848 he proposed his now famous method for estimating the relative strength of the sunspot cycle, using his 'relative sunspot number' (i.e., $r = k(f + 10g)$, where g is the number of sunspot groups, f is the number of individual spots, and k is a factor that is dependent upon the qualities of the observer, the observing site, the telescope, etc.; e.g., Kiepenheuer, 1953; Waldmeier, 1961). On the basis of values of relative sunspot number (also called Wolf number, Zürich sunspot number, and, now, International sunspot number), he was able to confirm Schwabe's contention that sunspots vary in number over a decadal time scale (although he found the period to be closer to 11 years in length rather than 10 years, as purported by Schwabe) and he was able to reconstruct the historical record of sunspot variation (from scattered and incomplete reports), in terms of *daily* values back to 1818, in terms of *monthly* estimates back to 1749, and in terms of *annual* estimates back to 1700. More importantly, he established an *international* collaboration that continues today, which has allowed for the determination of daily sunspot number (without gaps) since 1849.

While Wolf's record of daily sunspot number is complete (without gaps) since 1849, prior to this it varies considerably in its degree of completeness, with annual number of observing days ranging between a low of 150 days in 1837 (the epoch of maximum amplitude for cycle 8) to a high of 356 days in 1822 (the year before the epoch of minimum amplitude for cycle 7). Consequently, the record of sunspot observations is considered of highest quality only from 1849, of fair quality between 1818 and 1849, and of poor quality prior to 1818 (Waldmeier, 1961; Eddy, 1977, 1980). Additionally, there exists certain peculiarities in the early sunspot record (in particular, cycles 1-7) that, at least, suggest one use caution when examining or interpreting sunspot cycle relationships that are based, in part or in whole, on the early sunspot record (e.g., McNish and Lincoln, 1949; Sonett, 1983).

3. Results

Figure 1 displays the number of sunspot observing days during the interval of 1818-1868, where the solid line refers to Schwabe's data and the dashed line refers to Wolf's data. Two features of the records should be mentioned. First, for the 23-year contemporaneous interval of 1826-1848 (i.e., the time span immediately before full coverage begins), the number of observing days reported by Schwabe is found to *exceed* that reported by Wolf in 14 of those years. In particular, for the subinterval of 1834-1848, for *every* year except

1842 the number of observing days reported by Schwabe is greater than the number of observing days reported by Wolf. On average, Schwabe observed on 263.4 days per year during the interval of 1826-1848, while Wolf's reconstruction is based on 257.7 observing days per year. It is not known as to why Wolf did not include the additional observations of Schwabe in his reconstruction.

Second, several strong *dips* (or decreases) in the number of observing days per year, each persisting about 1-3 years or more in length, are clearly discernible in both data sets, especially, before 1849, the beginning of full-coverage of sunspot number based on daily values. The first of these dips (i.e., its *deepest* portion) is inferred to have occurred prior to 1818, while a second occurred about 1824, a third about 1830, a fourth about 1837, and a fifth about 1847. Previously, Waldmeier (1961) has suggested that these dips represent a *seasonal* effect due to bad weather conditions in central Europe, with the number of observing days being more numerous in summer and less in winter. However, because of the *persistent* nature of these dips (each lasting, typically, 1-3 years or more), their existence seems to suggest that each may be due to some sort of short-term, induced *climatic fluctuation*. Indeed, Wilson (1998b) has shown a propensity for each of the dips to be closely associated with the occurrence of a large, cataclysmic volcanic eruption located either in the tropics or in the extra-tropical northern hemisphere. For example, the increasing number of observing days that is seen from 1818 until about 1822 is thought to be related to the improving atmospheric conditions over Europe following the 1815 eruption of Tambora (Indonesia, 8° S), the strongest and deadliest eruption of the past several hundred years (Simkin and Siebert, 1994). Additionally, the other dips appear to be associated, respectively, with the eruptions of Galunggung (Indonesia, 7 ° S) in 1822, Kliuchevskoi (Russia, 56 ° N) in 1829, Cosiguina (Nicaragua, 13 ° N) in 1835, and Hekla (Iceland, 64 ° N) in 1845. Wilson also found that concurrent with these decreases in number of observing days per year were brief episodes of *cooler* clime, based on the annual mean temperatures as recorded at the Armagh Observatory in Northern Ireland (Butler and Johnston, 1996), thus, providing additional impetus that the dips may be linked to short-term climatic fluctuations (induced by the volcanic eruptions).

Large, cataclysmic volcanic eruptions (i.e., those with a volcanic explosivity index, or VEI, ≥4; Simkin and Siebert, 1994) can induce short-term climatic swings, certainly regionally and sometimes globally. The recent eruption of Mt. Pinatubo (Philippines, 15 ° N) in June 1991 provides a clear reminder of this (Bluth et al., 1992; Dutton and Christy, 1992; Trepte et al., 1993; McCormick et al., 1995; Hansen et al., 1996). Besides the strength (i.e., VEI), location, and time of year of the eruption, the most important aspect as to whether or not an eruption will induce a short-term climatic change (in particular, *cooling*) is the sulfur content of the emissions that reach into the stratosphere (e.g., Robock and Mao, 1995; Carroll, 1997). For the eruptions cited above, most are found to have a clearly defined elevated level of SO_4^{2-} in Greenlandic (e.g., Zielinski et al., 1994) and/or Antarctic (e.g., Cole-Dai et al., 1997) ice core deposits; so, one perceives that the dips as recorded in the observing record of Schwabe and the reconstructed record of Wolf very probably represent manifestations of short-term climatic fluctuations that were, indeed, induced by large, cataclysmic volcanic eruptions. (The amount of SO_4^{2-} associated with the Tambora blast is the largest recorded in the 19th and 20th centuries. Also, the length of deposition, according to ice cores, is typically 1-3 years for the previously cited eruptions, thus, supporting the contention that they can be genuine effectors of short-term climatic change for a year or more after the date of eruption.)

Figure 2 shows the variation of annual sunspot number (top) as reconstructed by Wolf and the annual number of clusters of spots (bottom) as actually recorded by Schwabe for the contemporaneous interval of 1826-1868, this interval corresponding to the rising phase of cycle 7 through the rising phase of cycle 11. Interestingly, dependent upon whose data set is regarded as sacrosanct, either cycles 7 and 10 and cycles 8 and 9 are inferred to be of *comparable* strengths (based on Schwabe's data) or cycle 8 is the strongest and cycle 7 the weakest (based on Wolf's data). Also, while the epochs of minimum and/or maximum amplitudes for cycles 8-11 are found to *always* agree in both Schwabe's and Wolf's data, in contrast, the maximum amplitude for cycle 7 is suggested to have occurred two years *earlier* (in 1828), based on Schwabe's data, as compared to that determined by Wolf, inferring that the ascent and descent durations for cycle 7 may be incorrect. Recall that cycle 7 is one of the *troublesome* cycles in the early sunspot record, chiefly because its ascent and descent durations are, respectively, the longest (7 years) and the shortest (3 years) on record (e.g., Wilson et al., 1996). (In Fig. 2, the plotted annual Wolf numbers are computed directly from *daily* values rather than using the published annual values – e.g., Waldmeier, 1961; McKinnon, 1987; Hoyt and Schatten, 1997 – which are computed from *monthly* values.)

Because of the strong similarity in their behaviors, one infers that Wolf's annual sunspot number and Schwabe's annual number of clusters of spots should be highly correlated. Figure 3 depicts the scatter plot of Wolf's reconstructed annual sunspot number versus

Schwabe's observed annual number of clusters of spots for the contemporaneous interval of 1826-1868. Clearly, the two different measures of solar activity are highly correlated, having a Pearson correlation coefficient $r = 0.969$ (inferring a coefficient of determination $r^2 = 0.939$, meaning that about 94% of the variance in Wolf's sunspot number can be explained by the variation in Schwabe's number of clusters of spots) and the regression is found to be highly statistically significant (at >>99.9% level of confidence). On the basis of the displayed 2 by 2 contingency table, one easily computes the probability of obtaining the observed result, or one more suggestive of a departure from independence (chance), by means of Fisher's exact test, to be $P = 4.4 \times 10^{-8}$ %.

Table I provides a summary of the yearly values that were used to construct Figs. 1-3. It also gives the inferred value of sunspot number ($\hat{y}$) based on Schwabe's annual number of clusters of spots. From Table I, one finds that, on occasion, there is wide disagreement between the reported and inferred Wolf numbers. As an example, in 1828 Wolf reports an annual sunspot number of only 65.1 (based on the daily values that he had accumulated), yet, according to the observed number of clusters of spots, one infers that the annual sunspot number should have been about 86.3. For 1830, Wolf reports an annual sunspot number of 70.4, which is found to be very close to that inferred for it based on the number of clusters of spots data (= 71.9). Thus, while Wolf places the maximum amplitude for cycle 7 in 1830, Schwabe's data suggest that it should have been placed two years earlier (in 1828).

Figure 4 displays the residual of Wolf's sunspot number minus the inferred sunspot number (based on $\hat{y}$). When the residual is *negative*, one perceives this as indicating that Wolf's estimate for annual sunspot number may be too *small* (i.e., underestimated), while a *positive* residual suggests that Wolf's estimate of annual sunspot number may be too *large* (i.e., overestimated). For convenience, the overall ±90% prediction interval is shown (as the dashed line; i.e., ±13.9). Each individual point (the filled circles) has associated with it a vertical line which represents the ±90% confidence limits based upon the statistics for that year's average value (i.e., Wolf's reconstructed annual sunspot number based on the published daily values; e.g., Waldmeier, 1961). The heavy line running through the field of annual residuals is the 4-year moving average (also called the 5-year running mean) of the residuals. The results of a runs test appears at the bottom, which suggest that the yearly residuals are *randomly* distributed (at the 95% confidence level). For the 43-year contemporaneous interval of 1826-1868, one finds that most of the residuals lie well within the ±90% prediction interval; however, for five specific years the residuals are found to lie outside this limit and for two other years the inferred residuals have estimated values that possibly could extend beyond the limit. The five anomalous years include 1828 (as previously mentioned), 1839, 1858, 1859, and 1860, and the two near anomalous years include 1835 and 1836. Residual values appear quite stable from 1861 onward.

4. Discussion and Conclusions

In studies of a phenomenon, it is imperative that one has reliable measurements over some lengthy interval of time to properly describe the behavioral aspects of the phenomenon in question. For the sunspot cycle, the conventional descriptor is Wolf's sunspot number (today, known as the International sunspot number), although, originally, the basis for the sunspot cycle was Schwabe's number of clusters of spots (i.e., newly appearing spot groups during each daily period of observation, summed over the entire year). The chief advantages of Schwabe's data set over the evaluations of annual sunspot number by Wolf (in particular, for the early portion of the sunspot record) is that it represents the results of a simple methodology based on observations by a single observer from a single site (in contrast to the more complicated reconstructed record of Wolf that is based on observations of the number of groups and the number of individual spots – assumed to be correctly reported and properly scaled – by several observers from several sites) and that, often, it was based on more observing days (than that of Wolf's annual sunspot number). Previously, Hoyt and Schatten (1995c, d) have wrestled with the thorny issue that, sometimes, a particular value of Wolf's reconstructed annual sunspot number may just be *wrong*. Such, apparently, is the case for the year 1769 when, based on a reexamination of Horrebow's drawings and those of other observers, one infers that the Wolf number may have been *overestimated* by about 25 units. (Undoubtedly, this also may have contributed to the problem encountered by McNish and Lincoln, 1949, when they attempted to predict future sunspot numbers on the basis of past sunspot numbers, especially, with regards to the statistics of cycles 1-7.)

Schwabe's data (see Figs. 2-4) clearly suggest that Wolf's evaluations of annual sunspot number may be unreliable for certain years. For example, on the basis of Schwabe's data, one perceives that the maximum amplitude for cycle 7 probably occurred in 1828 rather than 1830 (as deduced using Wolf's data). Thus, either cycle 7 had a rise time (ascent duration) of 5 years and a decay time (descent duration) of 5 years on the basis of Schwabe's data, or it had a rise time of 7 years and a decay time of 3 years on the basis of Wolf's data, where these latter values represent the *extremes* for the whole of cycles 1-22

(Wilson et al., 1996). On the basis of the modern era sunspot cycles 10-22 (see Fig. 5), the distribution of ascent durations is found to span only 3-5 years, with the greatest number of cycles having ascent durations in the range of 3-4 years. Similarly, the distribution of descent durations is found to span only 5-8 years, with the greatest number of cycles having descent durations in the range of 7-8 years. Clearly, the values of 7 and 3 years, respectively, of ascent and descent duration for cycle 7 as deduced by Wolf on the basis of his reconstructed sunspot number lies *outside* the regime of modern experience.

Likewise, a determination of the frequency distribution of maximum number of clusters of spots relative to the occurrence of sunspot number maximum for cycles 12-22 (using Royal Greenwich Observatory measurements and those from NOAA's Space Environment Center) indicate (see Fig. 6) that the two parameters, usually (7 out of 11 times), occur at the same time and *never* have they occurred more than 1 year apart. Thus, Schwabe's observation that the maximum number of clusters of spots for cycle 7 occurred in 1828 provides a very strong indication that its maximum amplitude (as adjudged using sunspot number) was *misplaced* by Wolf (i.e., more likely, it occurred in 1828 rather than 1830; certainly, it occurred no later than 1829). This implies that Wolf's reconstructed value for 1828 is *underestimated*, perhaps, by as much as 24 units of sunspot number (or as few as 18 units).

Schwabe's data also suggest that cycles 8 and 9 are much more *comparable* in size (~130) than Wolf's data would seem to indicate, and that, overall, cycle 9 is more *reliably known* than cycle 8. There is even a hint that the rising phase of cycle 10 may not be as reliably known as is generally accepted, because stability of the residuals does not appear to have been achieved until after 1861 (during the declining phase of cycle 10; see Fig. 4).

One of the most fascinating aspects of modern era sunspot cycles is the 'odd-even' effect, so-named because the odd-numbered following cycle has *always*, without fail (6 out of 6 times) been of comparable or larger size (based on sunspot number) than the even-numbered leading cycle in all modern era Hale cycle pairs (i.e., 10-11, 12-13, ..., 20-21). This aspect of the sunspot cycle has previously been used by Kopecký (1991) and Wilson (1992) to estimate the relative size of cycle 23 from the observed size of cycle 22. Hence, given that cycle 22 had a maximum amplitude of 157.6 (based on annual averages), statistically speaking, one expects cycle 23 to have a maximum amplitude of *comparable or larger size* in comparison. Such a finding compares reasonably well with that recently reported by Joselyn et al. (1997), who reports a consensus that cycle 23 should have a maximum amplitude of about 160 ±30, based on a variety of predictive schemes, and by Kane (1997) and Wilson et al. (1998), based upon various precursor techniques.

While some have described the odd-even effect as mere 'folklore' (e.g., Schatten et al., 1996), it should be noted that similar behavior exists in other parameters, as well, including the aa geomagnetic index and sunspot cycle length averages of annual mean surface air temperatures (Wilson, 1998a). So, one strongly suspects that the odd-even effect probably is an *inherent* property of the sunspot cycle (actually, the *Hale cycle*) and not a mere statistical quirk. Recall that Ohl (1971) was the first to advance the notion of the 'extended solar cycle,' suggesting that the true beginning of the solar cycle takes place several years before the epoch of sunspot minimum and noting that the size of a sunspot cycle is directly related to the strength of the *recurrent* storm component of the solar wind which maximizes in the vicinity of sunspot minimum (see also, Feynman, 1982; Kataja, 1986; Gonzalez and Schatten, 1987; Wilson, 1990; Thompson, 1993; Kane, 1997; Wilson et al., 1998).

Figure 7 shows the odd-even effect for cycles 10-21 (left; i.e., the *modern era* sunspot cycles) and for cycles 8-21 (right), using the inferred values of sunspot number maximum amplitudes for cycles 8 (130.5) and 9 (129.3), based on Schwabe's observations (see Table I). In each panel, the inferred regression line (the solid line, $\hat{y}$), as well as its inverse (i.e., using y as the independent variable; the dashed line, $\hat{x}$), is given. For the modern era cycles, one finds a very strong linear correlation between the two parameters, having r = 0.974 (implying that the correlation can explain about 95% of the variance of the odd-following cycle's maximum amplitude). The strength of the effect, however, is slightly reduced when one includes cycles 8 and 9 (i.e., r = 0.873, implying that the correlation can now explain only about 76% of the variance of the odd-following cycle's maximum amplitude).

Accepting the odd-even effect (to be true) and believing that cycle 9, indeed, is better determined than cycle 8, one can use the inverse relationship ($\hat{x}$) to ascertain another estimate of the maximum amplitude for cycle 8. Therefore, on the basis of cycle 9's estimated maximum amplitude (equal to 129.3, from Schwabe's data), one infers that the maximum amplitude for cycle 8 (given the modern era description of the odd-even effect) should have been about 91.2 ±18.3 (at the 90% confidence level, or 91.2 ±23.9 at the 95% confidence level). On the other hand, ignoring the odd-even effect and basing the estimate purely on Schwabe's data (Fig. 3; using a value of 333 for the number of clusters of spots for cycle 8), one infers cycle 8's maximum amplitude to be about 130.5 ±13.9 (at the 90% confidence level, or 130.5 ±16.7 at the 95% confidence level). Together, these inferences seem to suggest that cycle 8's maximum amplitude may have been *smaller* than that derived for it by Wolf, perhaps, considerably smaller (~115, based on the overlap of the 95% level of confidence predictions). (It is important to remember that the yearly value for 1837 is based only on 150 days of observations, which is the *smallest* number of observing days for the entire interval of 1818-1848, and numerous instances are found in 1837 when lapses in coverage occurred, some extending 6 or more days in length and one extending 21 days in length: the interval of July 23rd through August 12th.)

Another reason for doubting the veracity of the size of cycle 8's maximum amplitude as derived by Wolf is the frequency distribution of daily sunspot number values for 1837, especially as compared to that of cycle 9's (for 1848). Table II summarizes the frequency distributions for the years of 1837 (the epoch of maximum amplitude for cycle 8) and 1848 (the epoch of maximum amplitude for cycle 9). As previously noted, the value of annual sunspot number for 1837 is based on a mere 150 days of observation, with monthly averages computed from 7-19 observing days each. The range of daily values of sunspot numbers are found to extend from 261 (in February) to 45 (in November), and there is *no strong concentration* of sunspot numbers near either the monthly or yearly means. Instead, the values are *broadly* distributed, displaying a strong tendency to be *skewed* towards *higher* values. In contrast, the value of annual sunspot number for 1848 is based on 234 days of observation, with monthly averages computed from 13-27 observing days each. The range of daily values of sunspot numbers are found to extend from 212 (occurring twice, once each in July and December) to 50 (in September), and a fairly strong concentration of sunspot numbers is found, in particular, near the yearly mean. (In Table II, R refers to sunspot number; J, F, ..., D refer to the calendar months of the year; T refers to the total for the year; H refers to the *highest* daily value of sunspot number; M refers to the *mean* value of sunspot number; L refers to the *lowest* daily value of sunspot number; and n refers to the *number* of daily observations.)

Returning to cycle 7, recall that Schwabe's data suggest that its maximum amplitude should be about 86.3. This value could, likewise, be used to estimate values of maximum amplitude for cycle 6, on the basis of the regressions pictured in Fig. 7. Based on the modern era rendition of the odd-even effect (left panel), one estimates cycle 6 to have had a maximum amplitude of about 54.5, while based on the expanded rendition of the odd-even effect (right panel), one estimates cycle 6 to have had a maximum amplitude of about 60.5. Thus, it may be that Wolf slightly *underestimated* the maximum amplitude for cycle 6 (given by Wolf as 45.8).

In conclusion, this study has provided evidence (from Schwabe), albeit circumstantial evidence, which suggests that researchers exercise caution when using Wolf's sunspot numbers, especially those prior to 1849. Clearly, the indiscriminate use of the pre-modern era sunspot numbers yields results that, in comparison to those of Schwabe, sometimes, appear to be specious. As an example, Wolf appears to have *misplaced* (by 2 years) the epoch of maximum amplitude for cycle 7 and, probably, *underestimated* its size. Additionally, Wolf may have *overestimated* the relative size of cycle 8 and slightly *underestimated* the relative size of cycle 6.

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Figure Captions

Fig. 1. Number of observing days for Schwabe (solid line) and Wolf (dashed line) for the interval of 1818-1868. The modern era of sunspot observations begins in 1849. The epochs of minimum and maximum amplitude for cycles 6-11 are identified across the bottom, respectively, as unfilled and filled triangles. Two different epochs of maximum amplitudes are shown for cycle 7. The first, occurring in 1828, reflects the maximum according to Schwabe's data, and the second, occurring in 1830, reflects the maximum according to Wolf's reconstructed data.

Fig. 2. Wolf's annual mean sunspot number (top) and Schwabe's annual counts of 'clusters of spots' (bottom) for the contemporaneous interval of 1826-1868, spanning portions of cycles 7-11.

Fig. 3. Scatter plot of Wolf's annual sunspot number versus Schwabe's annual count of 'clusters of spots' for the contemporaneous interval of 1826-1868. The results of Fisher's exact test is shown for the observed 2x2 contingency table (determined from the median values of the two parameters), or one more suggestive of a departure from independence. The straight-line fit is the line of regression determined from linear regression analysis: $\hat{y}$ is the regression equation, r is the coefficient of regression, r^2 is the coefficient of determination, se is the standard error of estimate, t is a measure of the statistical significance of the inferred slope (as compared to the null slope) based on the Student t distribution, and cl is the inferred confidence level of the regression, based on t.

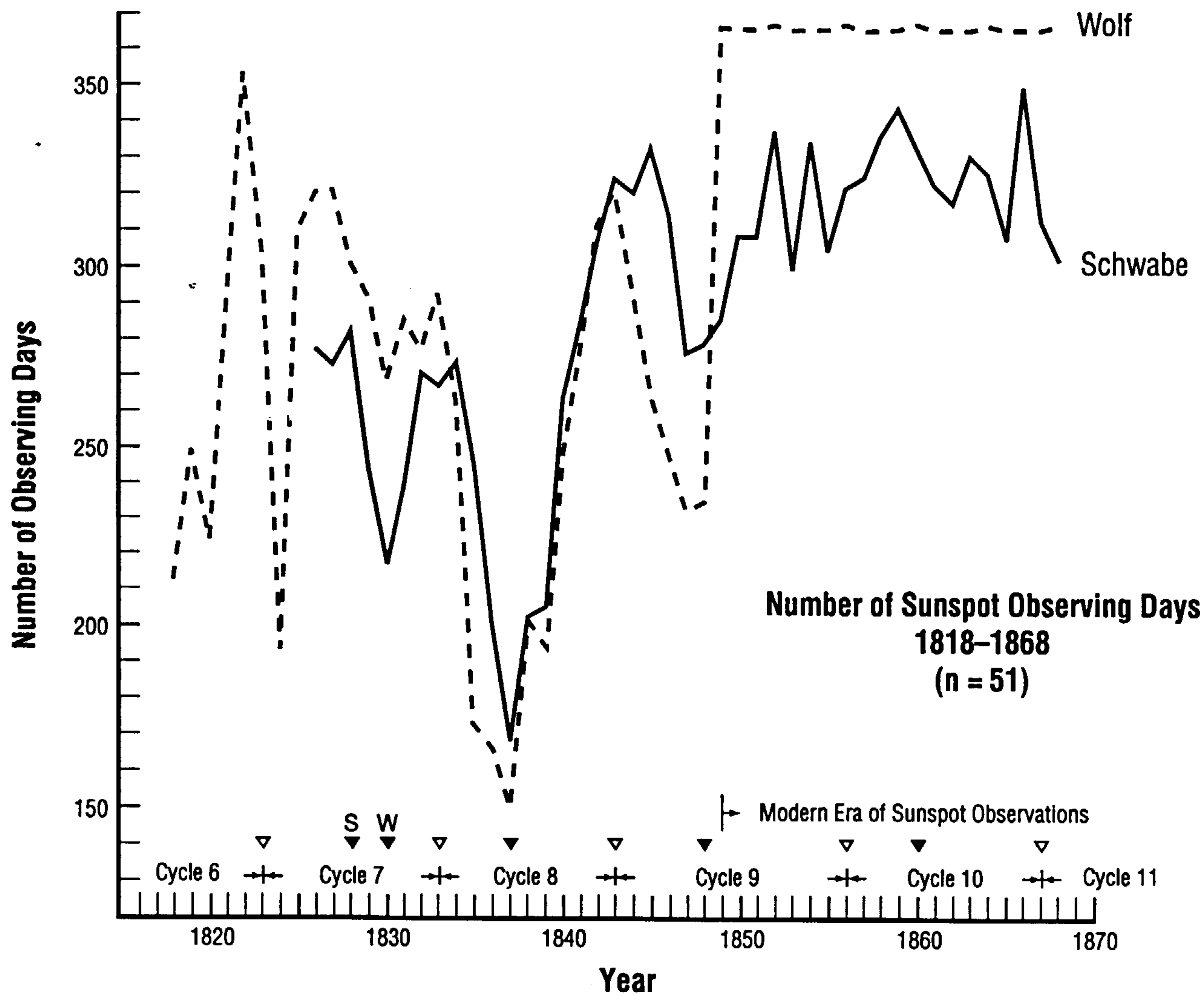
Fig. 4. A temporal display of the residual (observed Wolf number minus the inferred Wolf number) per year for the interval of 1826-1868. The dots represent the actual

residuals, the thin line merely connects the individual residuals, the heavy line is a 4-year moving average of the residuals, the dashed line is the ±90% prediction limits, and the thin vertical lines associated with each residual is the 90% prediction intervals based on the yearly statistics of observed Wolf numbers. As before, individual cycles are identified, including their maximums (filled triangles) and minimums (unfilled triangles). S refers to Schwabe and W refers to Wolf. The results of a runs test is given at the bottom, indicating that the yearly residuals are *randomly* distributed.

Fig. 5. The frequency distributions of ascent (asc), descent (des), and period (per) for the modern era sunspot cycles 10-22.

Fig. 6. The frequency distribution of maximum number of 'clusters of spots' (a la Schwabe) relative to E(max) occurrence, the epoch of sunspot maximum amplitude occurrence, for cycles 12-22 (based on Royal Greenwich Observatory and NOAA's Space Environment Center observations).

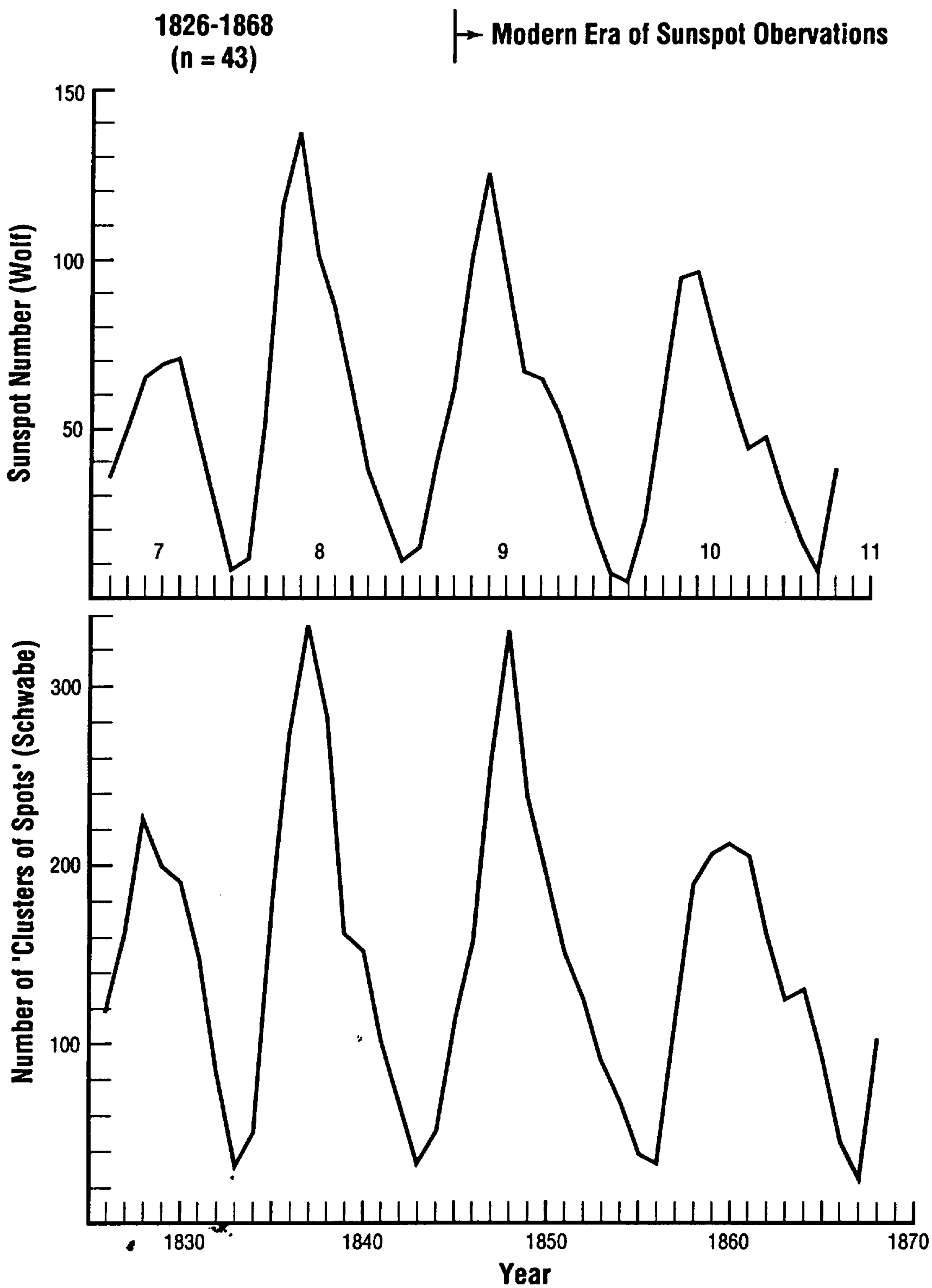
Fig. 7. The 'odd-even' effect for modern era cycles 10-21 only (left panel) and for cycles 8-21 (right panel). Individual Hale cycle pairs are identified within the parentheses. The solid line ($\hat{y}$) is the regression line using $R(max)_{el}$ as the independent variable; the dashed line ($\hat{x}$) is the regression line using $R(max)_{of}$ as the independent variable. The symbols r, r^2, and se have the same meanings as before (in Fig. 3).



Number of Sunspot Observing Days
1818–1868
(n = 51)
Wolf
Schwabe
Number of Observing Days
350
300
250
200
150
Year
1820
1830
1840
1850
1860
1870
S
W
Modern Era of Sunspot Observations
Cycle 6
Cycle 7
Cycle 8
Cycle 9
Cycle 10
Cycle 11

T1

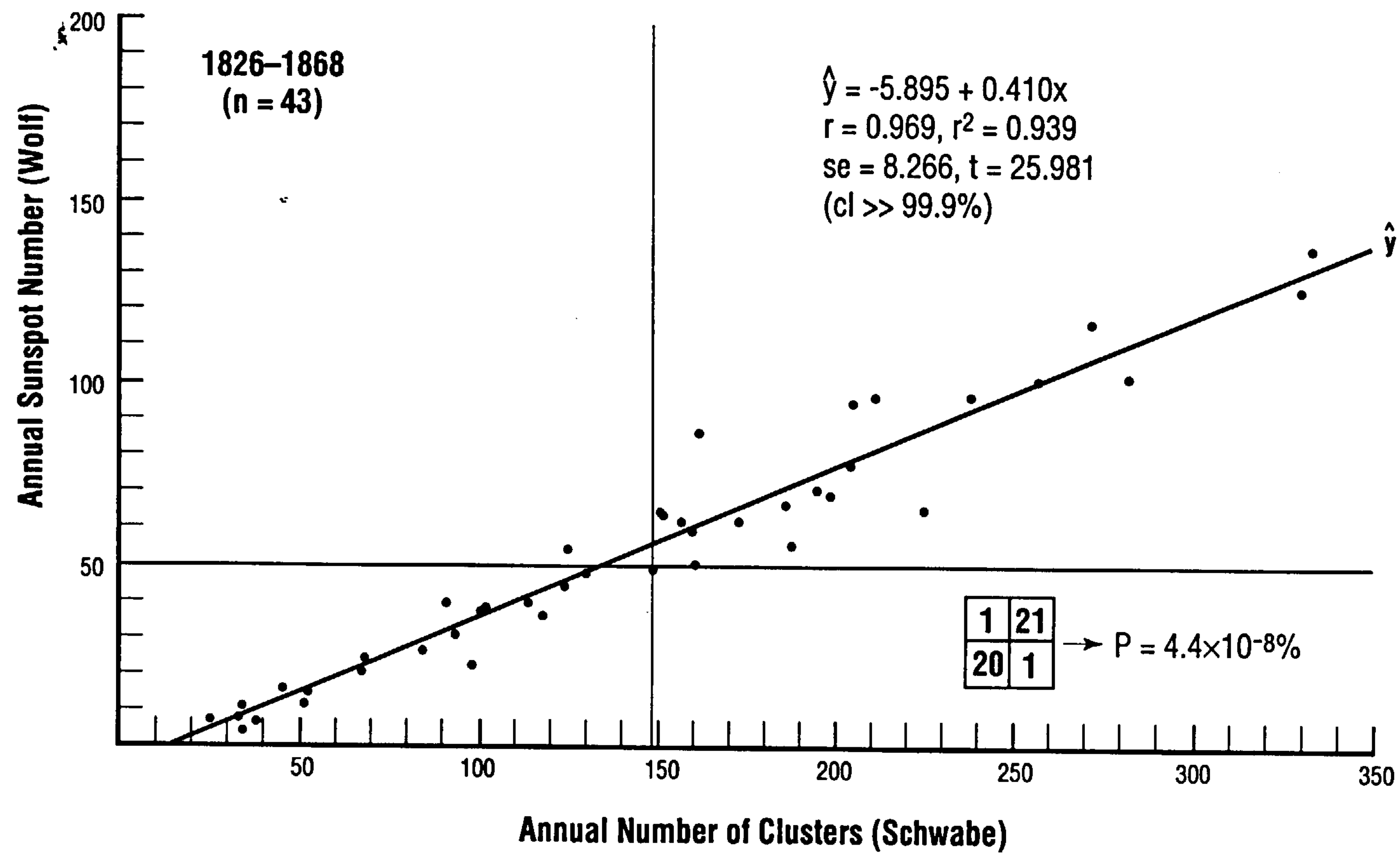
5-31244



1826-1868
(n = 43)
Modern Era of Sunspot Obervations
Sunspot Number (Wolf)
150
100
50
7
8
9
10
11
Number of 'Clusters of Spots' (Schwabe)
300
200
100
1830
1840
1850
1860
1870
Year

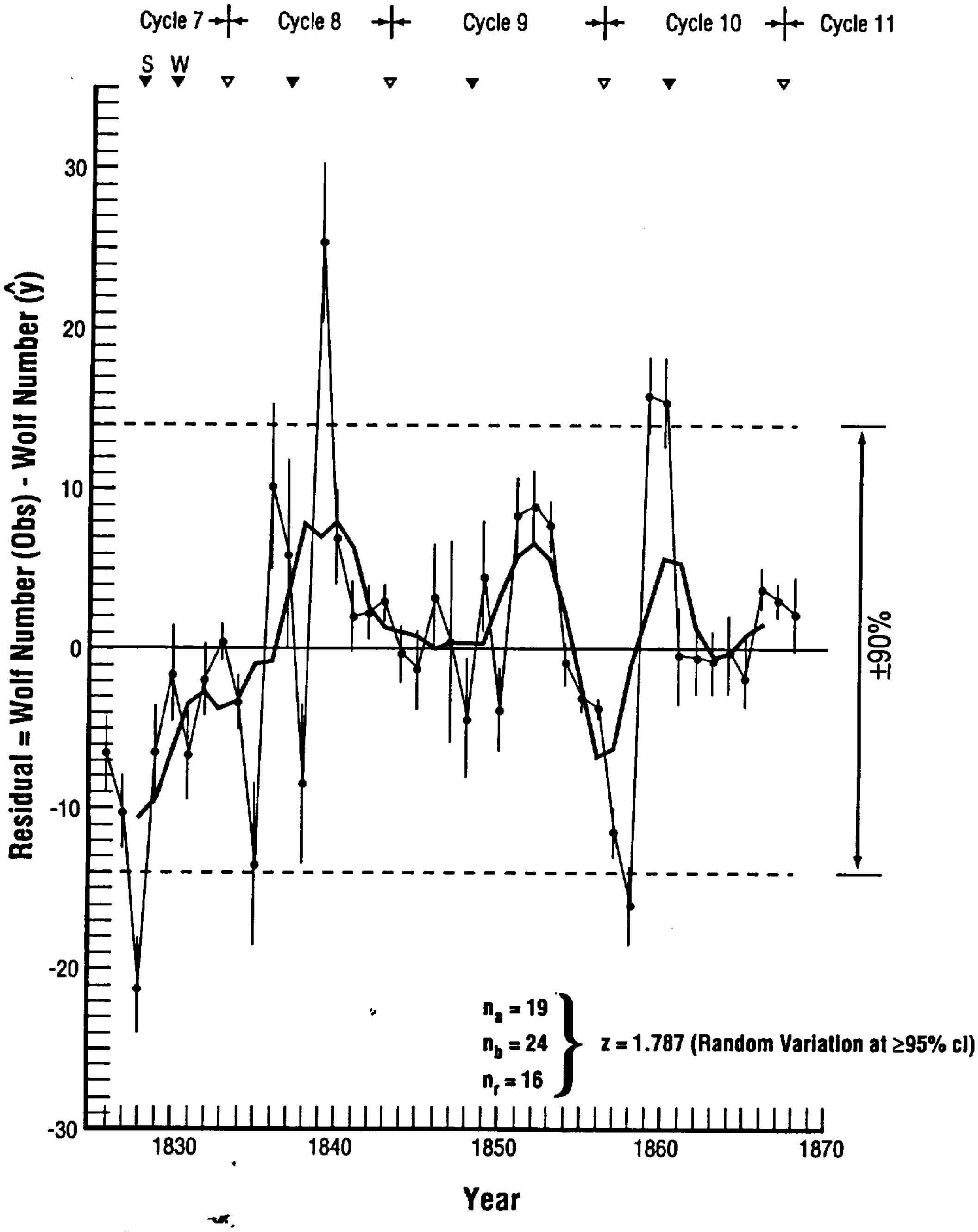
F2

5-31245



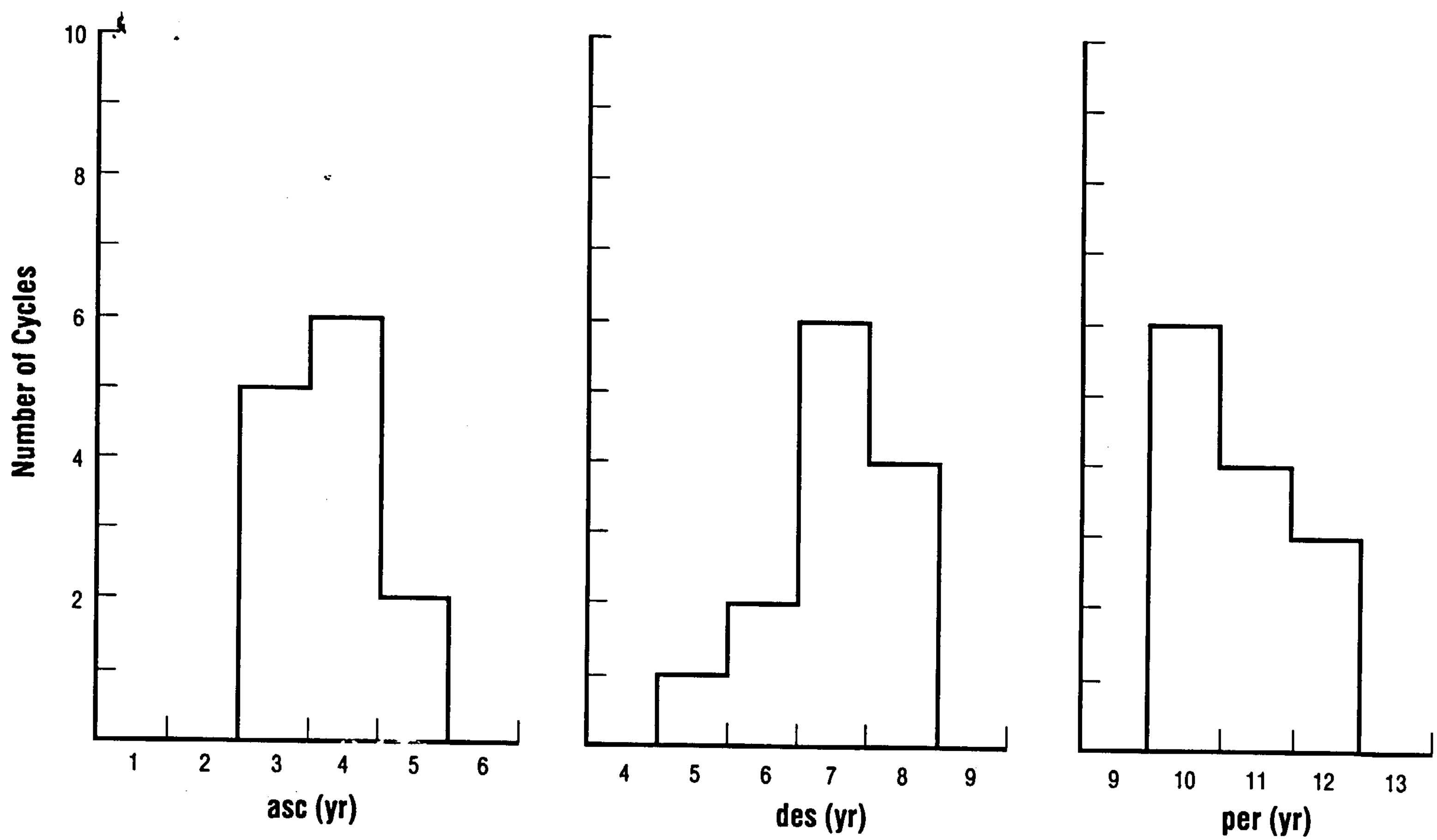
1826–1868
(n = 43)
ŷ = -5.895 + 0.410x
r = 0.969, r2 = 0.939
se = 8.266, t = 25.981
(cl >> 99.9%)
ŷ
1 21
20 1
P = 4.4×10-8%
Annual Sunspot Number (Wolf)
200
150
100
50
50
100
150
200
250
300
350
Annual Number of Clusters (Schwabe)

5-31246

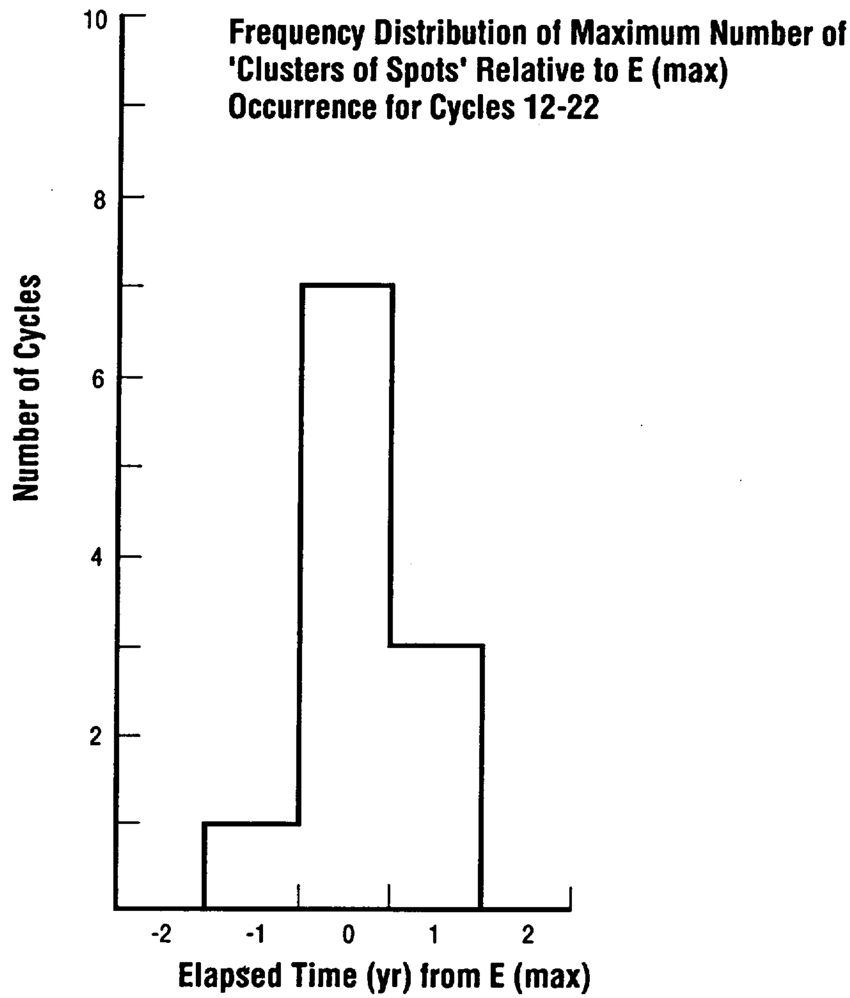

Cycle 7
Cycle 8
Cycle 9
Cycle 10
Cycle 11
S W
Residual = Wolf Number (Obs) - Wolf Number ($\hat{y}$)
30
20
10
0
-10
-20
-30
±90%
$n_a = 19$
$n_b = 24$
$n_r = 16$
z = 1.787 (Random Variation at ≥95% cl)
1830
1840
1850
1860
1870
Year

5-31247

Frequency Distributions of asc, des, and per for Modern Era Cycles 10-22

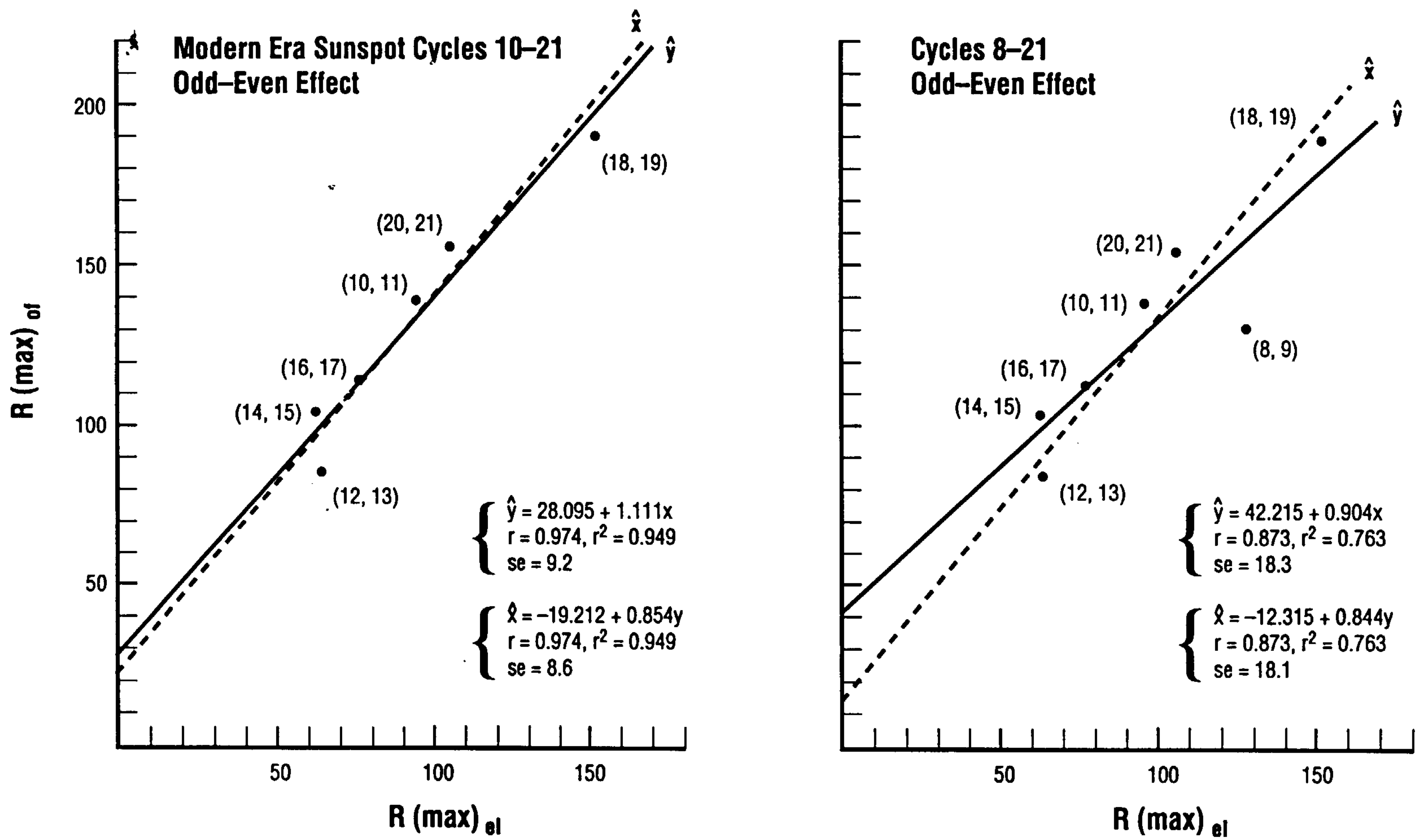


5-31248



Frequency Distribution of Maximum Number of 'Clusters of Spots' Relative to E (max) Occurrence for Cycles 12-22
10
8
6
4
2
Number of Cycles
-2
-1
0
1
2
Elapsed Time (yr) from E (max)

5-31249



Modern Era Sunspot Cycles 10–21
Odd–Even Effect
R (max) of
R (max) el
200
150
100
50
(18, 19)
(20, 21)
(10, 11)
(16, 17)
(14, 15)
(12, 13)
$\hat{y} = 28.095 + 1.111x$
$r = 0.974, r^2 = 0.949$
se = 9.2
$\hat{x} = -19.212 + 0.854y$
$r = 0.974, r^2 = 0.949$
se = 8.6
Cycles 8–21
Odd–Even Effect
(8, 9)
$\hat{y} = 42.215 + 0.904x$
$r = 0.873, r^2 = 0.763$
se = 18.3
$\hat{x} = -12.315 + 0.844y$
$r = 0.873, r^2 = 0.763$
se = 18.1

5-31250

F7

TABLE I.

Comparison of Schwabe's and Wolf's Data Sets: 1818-1868

	Schwabe's Data		Wolf's Reconstructed Data			
Year	No. Clusters	No. Obs. Days	Publ. Wolf No.	'Corr.' Wolf No.	No. Obs. Days	$\hat{Y}$
1818	–	–	30.1	31.7	213	–
19	–	–	23.9	23.1	249	–
20	–	–	15.6	14.3	224	–
21	–	–	6.6	5.5	304	–
22	–	–	4.0	3.8	353	–
23	–	–	1.8 min 7	1.3 min 7	302	–
24	–	–	8.5	6.9	194	–
25	–	–	16.6	16.9	310	–
26	118	277	36.3	35.9	320	42.4
27	161	273	49.6	49.8	321	60.0
28	225 max 7	282	64.2	65.1	301	86.3
29	199	244	67.0	69.1	291	75.6
30	190	217	70.9 max 7	70.4 max 7	268	71.9
31	149	239	47.8	48.5	285	55.1
32	84	270	27.5	26.6	277	28.5
33	33 min 8	267	8.5 min 8	8.0 min 8	292	7.6
34	51	273	13. 2	11.7	260	15.0
35	173	244	56.9	51.5	173	65.0
36	272	200	121.5	115.6	166	105.5
37	333 max 8	168	138.3 max 8	136.4 max 8	150	130.5
38	282	202	103.2	101.2	201	109.6
39	162	205	85.7	85.8	194	60.5
40	152	263	64.6	63.3	248	56.4
41	102	283	36.7	37.9	278	35.9
42	68	307	24.2	24.2	311	22.0
43	34 min 9	324	10.7 min 9	10.9 min 9	320	8.0
44	52	320	15.0	15.1	294	15.4
45	114	332	40.1	39.5	265	40.8
46	157	314	61.5	61.6	247	58.4
47	257	276	98.5	99.8	232	99.4
48	330 max 9	278	124.7 max 9	125.0 max 9	234	129.3
49	238	285	96.3	96.1	365	91.6
50	186	308	66.6	66.5	365	70.3
51	151	308	64.5	64.2	365	55.9
52	125	337	54.1	54.1	366	45.3
53	91	299	39.0	39.0	365	31.4
54	67	334	20.6	20.6	365	21.5
55	38	304	6.7	6.7	365	9.7
56	34 min 10	321	4.3 min 10	4.3 min 10	366	8.0
57	98	324	22.7	22.8	365	34.2
58	188	335	54.8	55.0	365	71.1
59	205	343	93.8	93.9	365	78.1
60	211 max 10	332	95.8 max 10	95.9 max 10	366	80.5
61	204	322	77.2	77.2	365	77.7
62	160	317	59.1	59.0	365	59.6
63	124	330	44.0	44.0	365	44.9
64	130	325	47.0	47.0	366	47.3
65	93	307	30.5	30.4	365	32.2
66	45	349	16.3	16.2	365	12.5
67	25 min 11	312	7.3 min 11	7.3 min 11	365	4.3
68	101	301	37.6	37.6	366	35.5

TABLE II.

Frequency Distributions of Daily Wolf Numbers for 1837 and 1848

1837

R	J	F	M	A	M	J	J	A	S	O	N	D	T
≥260		1											1
250 - 259													
240 - 249		1											1
230 - 239	2					2	1						5
220 - 229													
210 - 219		1										1	2
200 - 209	2	2											4
190 - 199	1												1
180 - 189	1					1	1			1			4
170 - 179		1	1	4	2	3	1	1		1		2	16
160 - 169	1	2	1	1		1	1			1			8
150 - 159	2				1	2	3	3		5			16
140 - 149		2	2	1	1	3		1	1		2		13
130 - 139		1	3		1	2	1	1				1	10
120 - 129	1		3		1	3		2	2		1		13
110 - 119		1	2	1		1		1	2	2		1	11
100 - 109		1		1	2		1	1	3	2	2	1	14
90 - 99					1				4	3			8
80 - 89				1	2			1	1	1			6
70 - 79				1	3				4	1	1		10
60 - 69					1				2	1		2	6
50 - 59													
40 - 49											1		1
30 - 39													
20 - 29													
10 - 19													
0 - 9													
H	237 (2)	261	170	176 (2)	179	236	236	175	149	184	146	212	261
M	188.0	175.6	134.6	138.2	111.7	158.0	162.8	134.0	96.3	123.7	107.0	129.8	136.4
L	124	107	111 (2)	77	69	115	106	81	61	69	45	62	45
n	10	13	12	10	15	18	9	11	19	18	7	8	150

1848

R	J	F	M	A	M	J	J	A	S	O	N	D	T
≥260													
250 - 259													
240 - 249													
230 - 239													
220 - 229													
210 - 219							1					2	3
200 - 209							1				1	2	4
190 - 199							3	2			1		6
180 - 189	2					1		2		1			6
170 - 179	2									1	1	3	7
160 - 169	5					2	3			1		6	17
150 - 159	3					2	3	2	1			2	13
140 - 149		1	1		2	5	1		3	2		3	18
130 - 139	1	2	1	1	1	2	1	3	1	2	1	2	18
120 - 129		2	4	4	4	3	4	1	2		1	2	27
110 - 119		4	2	3	4	3	3	7		2	1	1	30
100 - 109		2	2	5	4	2	3	2	1	3	3	1	28
90 - 99	1		6	2	6	3	3	2	4	1	2		30
80 - 89				2	3	1	1		3		2		12
70 - 79		1		1					2		2		6
60 - 69		1	1		2				1		1		6
50 - 59					1				2				3
40 - 49													
30 - 39													
20 - 29													
10 - 19													
0 - 9													
H	189	145	141	136	141 (2)	182	212	196	157	187	204	212	212(2)
M	159.1	111.8	108.6	107.1	102.2	129.0	139.2	132.6	100.3	132.4	114.6	159.5	125.0
L	97	61	67	79	52	84	87	95	50	92	67	104	50
n	14	13	17	18	27	24	27	21	20	13	16	24	234

5-31252