Lunar and solar torques on the oceanic tides

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Abstract. A general framework for calculating lunar and solar torques on the oceanic tides is developed in terms of harmonic constituents. Axial torques and their associated angular momentum and earth-rotation variations are deduced from recent satellite-altimeter and satellite-tracking tide solutions. Torques on the prograde components of the tide produce the familiar secular braking of the rotation rate. The estimated secular acceleration is approximately $-1300''/\text{century}^2$ (less 4% after including atmospheric tides); the implied rate of change in the length of day is 2.28 milliseconds/century. Torques on the retrograde components of the tide produce periodic rotation variations at twice the tidal frequency. Interaction torques, e.g. solar torques on lunar tides, generate a large suite of rotation-rate variations at sums and differences of the original tidal frequencies. These are estimated for periods from 18.6 years to quarter-diurnal. At subdaily periods the angular momentum variations are 5 to 6 orders of magnitude smaller than the variations caused by ocean tidal currents.

1. Introduction

Brosche and Seiler (1996) recently called attention to the interesting role that direct lunar and solar torques on the ocean tide play in the earth's short-period angular momentum balance ("short-period" here meaning daily and subdaily). Brosche and Seiler noted that such torques are a potential source of angular momentum in the earth-ocean system and that this source had been neglected in previous earth-rotation studies. The purpose of the present paper is to reexamine, clarify, and extend these ideas. We limit the discussion to the earth's spin rate and ignore the additional complications of wobble and nutation. It suffices therefore to examine only axial torques. We also limit the discussion to diurnal and semidiurnal tides of the second degree in the tidal potential, i.e. to the "major" short-period tides.

A qualitative understanding of the consequences of tidal torques can be obtained from the diagram in Figure 1, which is drawn for the principal semidiurnal tide M_2. Two spherical harmonic components of the ocean tide—the only two that induce nonzero torques—are displayed: prograde and retrograde components of degree 2, order 2. The prograde component is the classical tidal "bulge" that propagates westward under the moon; the retrograde component is a similar bulge, generally smaller, that propagates eastward. ("Prograde" here implies moving in the same direction as the tide-generating body; this prograde/retrograde terminology follows standard tidal usage, but it is opposite that used in polar-motion studies.) The retrograde bulge owes its existence, of course, to the highly nonequilibrium form of the oceanic tides; there is no comparable bulge in the body tide. The tidal force acting upon the prograde bulge produces the familiar secular braking torque (Munk and MacDonald, 1960). The tidal force acting upon the retrograde bulge produces a periodic torque at *twice* the tidal frequency. This latter phenomenon is quite similar to that of libration (Chao et al., 1991; Wünsch, 1991) in which the triaxial ellipsoid of the solid earth experiences a periodic torque at exactly the tidal frequency as it rotates beneath the moon. But because the ocean's retrograde bulge travels eastward at twice the earth's rotation (relative to the moon), the torques on it are twice as rapid.

In the same manner, there are interaction torques between tidal constituents—for example, solar tidal torques on lunar tidal bulges. These torques have periodicities at the sums and differences of the original constituent frequencies, the summed frequencies corresponding to torques on retrograde bulges and the differenced frequencies corresponding to torques on prograde bulges. These new frequencies are actually those of the familiar compound tides that occur in many shallow seas when nonlinear dynamical terms (e.g., quadratic bottom friction) cause interactions between tides. (The two phenomena are, of course, unrelated.) We thus find a whole new set of frequencies to explore in the earth's angular momentum budget, previous studies having been confined to only the tidal frequencies. Intriguing as they are, however, these new terms turn out to be quite insignificant.

In the following calculations, earth anelasticity is ignored. This allows us, among other things, to neglect the torques on the lagged body tide and to concentrate on the oceanic problem. Given the most recent estimates of the body-tide lag (e.g. Ray et al., 1996), neglecting anelasticity may induce errors of perhaps a few percent at most. There are, nonetheless, still nonzero torques on the solid-earth tide, since there is a fairly large radial-displacement load tide caused by the oceanic tide, which acts to reduce the overall torque by some 30%.

Finally, it is important to realize that the torques under discussion are on the tidal height fluctuations alone, not the entire ocean. Torques on the latter, integrated vertically from mean sea level to the ocean depths, are included in the solid-earth libration, which is dependent on the inertia tensor of the whole earth, including the ocean, and is determined from the gravitational Stokes coefficients of the whole earth. This point is indeed stressed by Brosche and Seiler (1996), but it is unclear from their paper whether they followed it, since they found all torques of the same tidal species to have precisely the same phase (see JT′ in their Table 1). A constant phase does occur for libration, but it should not be expected for torques on the ocean tides.

2. Torque Estimates

Much like the precession/nutation problem (Moritz and Mueller, 1987, p. 52), there are two ways to compute the required torques. One may compute the direct tidal torque on the ocean tide (and its load deformation), or one may compute the equal and opposite torque of the ocean tide's gravitational attraction (including its load deformation) on the moon (or sun). The advantage of the former method is that one may use directly the readily available harmonic developments of the tidal potential (e.g., Cartwright and Tayler, 1971) and thereby avoid dealing with the complications of the lunar and solar ephemerides; the disadvantage is that the torque on the load deformation, requiring volume integration, is difficult to calculate directly. We nonetheless prefer this method, because by using the Cartwright-Tayler expansion we find it easier to sort out the frequencies and phases of torque components and their underlying ocean constituents. In addition, the complicated torque on the earth's load deformation may be handled trivially by exploiting the symmetry of the two torque methods, as will be seen.

Note that for the problem at hand, torques internal to the ocean-earth system may be ignored. They would include, for example, the mechanical force of the body tide against the ocean tide and the Newtonian attraction of the ocean tide on the body tide. In the present analysis, such internal torques serve only to couple the ocean and solid earth into a mutually rotating solid body.

From the introductory discussion, it is evidently advantageous to express the ocean tidal height, for any given harmonic constituent of frequency σ_1, in the form (Lambeck, 1980, eq. 6.2.1—allowing for a misprint in 6.2.1c)

$$\zeta = \sum_n \sum_m \sum_{+}^{-} D_{nm}^{\pm} \cos(\sigma_1 t \pm m\phi - \psi_{nm}^{\pm})\, P_n^m(\cos\theta), \tag{1}$$

comprising prograde (+) and retrograde (−) waves of amplitude $D_{nm}^{\pm}$ and phase lag $\psi_{nm}^{\pm}$, where (θ, ϕ) are spherical polar coordinates and $P_n^m(\mu)$ an associated Legendre function. Lambeck's notation $\sum_{+}^{-} D^{\pm} \cos(\alpha \pm \beta)$ denotes $D^{+} \cos(\alpha + \beta) + D^{-} \cos(\alpha - \beta)$. This ζ is the ocean's tidal height fluctuation relative to the seabed. The $D_{nm}^{\pm}$ and $\psi_{nm}^{\pm}$ parameters are known either from numerical hydrodynamic models of the global tide or from space-geodetic measurements (e.g. Schrama and Ray, 1994).

Following Cartwright and Tayler (1971), the astronomical tidal potential at the earth's surface for a constituent of frequency σ_2 is written

$$U = g\tilde{H} \sqrt{(5/24\pi)}\, P_2^1(\cos\theta)\, \sin(\sigma_2 t + \phi) \tag{2a}$$

for diurnal tides and

$$U = g\tilde{H}\sqrt{(5/96\pi)}\, P_2^2(\cos\theta)\, \cos(\sigma_2 t + 2\phi) \tag{2b}$$

for semidiurnal tides, where g is the gravitational acceleration and the amplitudes $\tilde{H}$ (in length units) are as tabulated by Cartwright and Edden (1973). Note that frequency σ_1 refers to the tidal elevation (1), while σ_2 refers to the potential (2); each may be either diurnal or semidiurnal, and σ_1 may or may not equal σ_2.

The potential U acting on the ocean tide ζ produces an axial torque of general form

$$\Gamma_z^o = \int_V \rho(\mathbf{x} \times \nabla U)_z \, dV,$$

where ρ is the density of seawater and $\mathbf{x}$ is a vector from the geocenter to the differential mass element $\rho\, dV$. This torque can be simplified to a surface integral by the thin-shell approximation:

$$\Gamma_z^o = \int_S \rho r \sin\theta\, (\nabla U)_\phi \, dS = \rho \int_S \zeta(\partial U/\partial\phi)\, dS,$$

with dS a differential of area. The corresponding torque of U acting on the solid-earth load tide cannot be similarly simplified, but it can be trivially accounted for, as noted above, by considering the torque of the ocean tide's gravitational potential acting on the moon, which is equal to $-\Gamma_z^o$. For that torque, accounting for the load deformation requires only an additional factor of $(1 + k_2')$, where k_2' is a load Love number (Munk and MacDonald, 1960). The equivalence of the two torques implies that Γ_z must also include the torque on the deformation via the identical factor. Hence the combined torque is written

$$\Gamma_z = (1 + k_2')\rho \int_S \zeta\, (\partial U/\partial\phi)\, dS = (1 + k_2')\rho a_e^2 \int_\Omega \zeta\, (\partial U/\partial\phi)\, \sin\theta\, d\theta\, d\phi. \tag{3}$$

With the previous expressions for ζ and U, integration is straightforward. Owing to orthogonality of the spherical harmonics, only terms in ζ with $(n, m) =$ (2,1) for diurnal tides and (2,2) for semidiurnal tides are effective. (An identical simplification occurs for work and dissipation integrals; see, for example, Lambeck [1980] and Platzman [1984].) Inserting (1) and (2) into (3) and expanding terms, one finds intermediate integrals of two types (note that $m, \ell = 1$ or 2 and that $\delta_{m\ell}$ is Kronecker's

δ-function):

$$\int_0^{2\pi} \sin(\sigma_1 t + m\phi)\ \sin(\sigma_2 t + \ell\phi)\, d\phi = \pi\, \delta_{m\ell} \cos(\sigma_1 t - \sigma_2 t)$$

and

$$\int_0^{2\pi} \sin(\sigma_1 t + m\phi)\ \sin(\sigma_2 t - \ell\phi)\, d\phi = -\pi\, \delta_{m\ell}\ \cos(\sigma_1 t + \sigma_2 t),$$

and similar terms involving cosines and sine-cosine products, each of them yielding either sums or differences of tidal frequencies. The total resulting torques are

$$\Gamma_z = -\sqrt{(6\pi/5)}\,(1 + k_2')\,\rho\, GM\tilde{H} \sum_{+}^{-} D_{21}^{\pm} \cos(\sigma_1 t \mp \sigma_2 t \pm \psi_{21}^{\pm}) \tag{4a}$$

for diurnal tides and

$$\Gamma_z = -\sqrt{(96\pi/5)}\,(1 + k_2')\,\rho\, GM\tilde{H} \sum_{+}^{-} D_{22}^{\pm} \sin(\sigma_1 t \mp \sigma_2 t \pm \psi_{22}^{\pm}) \tag{4b}$$

for semidiurnals. The earth's gravitational constant GM has been used in place of ga_e^2. When a tidal constituent's potential is acting on its own tidal bulge, then $\sigma_1 = \sigma_2$ ($\equiv \sigma$) and the torque integrals simplify to

$$\Gamma_z = -\sqrt{(6\pi/5)}\,(1 + k_2')\rho GM\tilde{H}\left\{D_{21}^{+} \cos\psi_{21}^{+} + D_{21}^{-} \cos(2\sigma - \psi_{21}^{-})\right\}$$

for diurnal tides and

$$\Gamma_z = -\sqrt{(96\pi/5)}\,(1 + k_2')\rho GM\tilde{H}\left\{D_{22}^{+} \sin\psi_{22}^{+} + D_{22}^{-} \sin(2\sigma - \psi_{22}^{-})\right\}$$

for semidiurnals. In each expression the first term on the right is the secular braking torque.

The implications of Γ_z for the earth's angular momentum budget are considered below for three frequency regimes: secular ($\sigma_1 - \sigma_2 = 0$), long period ($\sigma_1 - \sigma_2 \neq 0$), and short period ($\sigma_1 + \sigma_2$). This requires estimates of the ocean-tide coefficients $D_{nm}^{\pm}, \psi_{nm}^{\pm}$, to which we now briefly turn.

3. Oceanic tide coefficients

For the oceanic tide coefficients we adopt (see Table 1) three sets of estimates determined from satellite geodetic measurements, one from Geosat altimetry, one from Topex/Poseidon (T/P) altimetry, and one from multiple-satellite tracking. In the case of altimetry, the $D^{\pm}_{2m}, \psi^{\pm}_{2m}$ coefficients are determined by numerical quadrature of deduced global oceanic cotidal charts. In the case of satellite tracking, they are determined directly from the tidally induced orbital perturbations (Lambeck, 1980), often as a part of a simultaneous, large-scale inversion for the earth's gravitational Stokes coefficients (e.g., Christodoulidis et al., 1987). The T/P and satellite-tracking solutions are more accurate than any available numerical hydrodynamic model of the tides; see Shum et al. (1997) and Figure 1 of Ray et al. (1996).

Because altimetry is a geometical measurement while tracking is a gravitational measurement, there are some important differences. For example, they differ significantly in estimates of the S_2 tide, since the tracking is also sensitive to the S_2 atmospheric tide (e.g., Cartwright and Ray, 1991); a smaller effect may also occur in K_1 owing to seasonal variations in the S_1 atmospheric tide. Both altimeter and tracking solutions also differ very slightly owing to the different ways that they are affected by earth anelasticity (Ray et al., 1996). These small anelastic effects may be ignored for present purposes.

The Geosat satellite altimeter determination is taken from Cartwright and Ray (1991). These tidal coefficients are less accurate than those from the later T/P mission (e.g., Shum et al., 1997), and they should now be considered obsolete. They are used here to compare deduced secular acceleration estimates to previously published estimates from the same model.

The T/P tidal coefficients are updates to those published by Schrama and Ray (1994). In addition to incorporating more altimeter observations and improving horizontal resolution, this solution uses a half-dozen supplemental hydrodynamic tidal models in various marginal and polar seas to ensure complete coverage of the global ocean. The hydrodynamic model of Le Provost et al. (1994) is used for all latitudes above the 66° turning latitude of the T/P satellite. The M_2 estimates of $D^{+}_{22}, \psi^{+}_{22}$ were previously published in Ray et al. (1996).

All satellite altimeter estimates of ocean-tide coefficients have been corrected for the geometrical effects of the body tide and the load tide. The latter is based on the iterative method outlined in Cartwright and Ray (1991; appendix A) and used loading Love numbers h'_n from Farrell (1973). For

T/P, the body tide was computed with Love number $h_2 = 0.609$ for all tides except K_1 and its nodal sidelines for which $h_2 = 0.52$. For Geosat, $h_2 = 0.619$ was used in the original mission data products, but our M_2 and S_2 coefficients were later adjusted to conform with $h_2 = 0.609$ (Cartwright and Ray, 1991).

The third set of estimates is taken from EGM96S (Lemoine et al., 1998), a satellite-only solution for the earth's gravitational field based on optical, doppler, laser, and GPS tracking of 40 artificial satellites. Only tides that cause significant long-period perturbations in satellite orbits were included in this solution, so they are limited to prograde terms only (Lambeck, 1980).

The EGM96S model provides well-calibrated estimates of tidal coefficient uncertainties, for which the reader may refer to Lemoine et al. (1998). For M_2 the EGM96S standard error for $D^+_{22} \sin \psi^+_{22}$ is 0.024 cm; other semidiurnal tides are comparable, while diurnal tides tend to be 3 to 4 times larger. Unfortunately, owing to the manner in which they were developed, standard errors for the two altimeter solutions are not available. However, M_2 errors have been estimated for two other T/P solutions—by Ray et al. (1994) and by Egbert et al. (1994)—and these should be comparable to the Schrama and Ray (1994) solution; for M_2 these uncertainties are 0.036 and 0.026 cm, respectively, which are comparable to EGM96S. Work is in progress (G. D. Egbert, personal comm., 1998) to establish a more comprehensive error model for the altimeter tide solutions. For our purposes, the given rough figures are adequate.

4. Secular braking of the earth's rotation

In this section, we examine only those terms in Γ_z having no explicit time dependence, taking $\sigma_1 = \sigma_2$. These terms, dependent only on the prograde oceanic tide coefficients, completely determine the present-day secular braking of the earth's rotation rate. That secular acceleration is given by

$$\dot{\Omega} = \Gamma_z / C$$

where Γ_z here represents the appropriate relevant terms in (4a,b). Here $C = 8.036 \times 10^{37}$ kg m^2 is the polar moment of inertia of the whole earth.

Estimates of the secular $\dot{\Omega}$ terms from the three tide models are listed in Table 2, along with standard error estimates from EGM96S. The potential amplitudes $\tilde{H}$ and the (slightly) frequency-dependent factor $(1 + k_2')$ are also tabulated; the former are from Cartwright and Edden (1973), the latter from Wahr (1980). Other required constants are:

$$\rho = 1035 \text{ kg m}^{-3}$$

$$GM = 3.986 \times 10^{14} \text{ m}^3 \text{ s}^{-2}.$$

As is well known, the dominant tide in $\dot{\Omega}$, responsible for about 70% of the total acceleration, is M_2. The next largest tide is S_2, but its oceanic effect is partly cancelled by the S_2 atmospheric tide, which tends to accelerate the earth's rotation rate by roughly $+55'' \text{cy}^{-2}$ (based on spherical harmonic coefficients listed in Haurwitz and Cowley, 1973). The totals listed in Table 2 are misleadingly close, a coincidence of favorable cancellations.

The largest tide neglected in Table 2 is probably ν_2. It can be estimated from the semidiurnal admittances; for the EGM96S model, we get $D_{22}^+ = 0.121$ cm, $\psi_{22}^+ = 117°$, which implies $\dot{\Omega} = -1.4'' \text{cy}^{-2}$. A handful of other minor tides would add a few more arcseconds.

In passing, we note that the long-period tides, with zonal potential

$$U = g\tilde{H}\sqrt{(5/4\pi)}\, P_2^0(\cos\theta)\, \cos\sigma t, \tag{2c}$$

add nothing to the earth's secular rotational braking. With perfect symmetry in the ϕ direction ($\partial U/\partial\phi = 0$), its axial torque is identically zero. This has not been sufficiently clear in some of the recent literature.

Allowing for an atmospheric contribution of $+55'' \text{cy}^{-2}$, we have a total secular acceleration of approximately $-1250'' \text{cy}^{-2}$. The corresponding rate of change in the length of day is

$$\dot{\Lambda} = -\dot{\Omega}/\Omega = 2.28 \,\text{ms/century}.$$

(Strictly, the units are ms/day/cy, but the day^{-1} is generally assumed.) The observed secular acceleration (Stephenson, 1997) includes, of course, nontidal effects as well, the most important being an additional $+300'' \text{cy}^{-2}$ usually attributed to melting of polar ice and/or the earth's viscous rebound (Yoder et al., 1983).

Finally, it is of interest to compare the $\dot{\Omega}$ estimates for the Cartwright-Ray model to previously published estimates for that model in Ray (1994). The previous values were computed, following Lambeck (1977), as follows: the expression for the gravitational attraction of the ocean tide on the moon is converted from spherical coordinates into lunar Keplerian elements and then inserted into the Lagrange planetary equations (Brouwer and Clemence, 1961), yielding secular rates for the moon's mean motion $\dot{n}$, eccentricity $\dot{e}$, and inclination $\dot{I}$. Invoking conservation of angular momentum then gives the earth's $\dot{\Omega}$. The two approaches to $\dot{\Omega}$ agree to within about 1% for M_2 and within $1''\,\mathrm{cy}^{-2}$ (the resolution of Table 2) for all other tides. These can likely be further reduced by closer attention to consistency in geophysical and astronomical constants and models; this will become more critical as observational accuracies continue to improve.

5. Long-period variations

The same prograde oceanic terms responsible for the secular acceleration also induce long-period oscillations in the earth's rotation rate when $\sigma_1 \neq \sigma_2$. We consider two cases: (1) a constituent acting upon its nodal sideline, which generates 18.6-year variations and (2) the case when σ_1, σ_2 correspond to the largest tides, M_2 and S_2, which generates variations at the frequency of the near-fortnightly MSf tide.

5.1 Variations at 18.6 years

All lunar tidal constituents undergo an 18.6-y nodal modulation as the lunar orbital plane precesses around the earth. As a consequence, any tide's contribution to the secular braking torque must undergo a corresponding periodic variation. O_1 has one of the largest nodal modulations, so we concentrate initially on it. The O_1 modulation in $\dot{\Omega}$ will be augmented by other lunar diurnal tides, all of which are largest when the lunar orbit is maximally inclined to the earth's equator. This occurs when the longitude of the moon's ascending node is aligned with the equinox (i.e., when the longitude of the node is zero). These diurnal-tide modulations are offset, however, by the semidiurnal tides, which are largest when the lunar orbit is closest to the equator (K_2 is an exception). As will be seen, the cancellation between diurnals and semidiurnals turns out to be nearly complete.

Consider Eq. (4a) when σ_1 is the frequency of the primary O_1 line, 545.555 in Doodson's nomenclature, and σ_2 is the frequency of the primary nodal line, 545.545. Then $\tilde{H} = 4.945$ cm (Cartwright and Edden, 1973). Adopting $D_{21}^{+}, \psi_{21}^{+}$ from the EGM96S solution yields

$$-\sqrt{(6\pi/5)}\,(1+k_2')\rho GM\tilde{H}D_{21}^{+}\cos(\sigma t+\psi_{21}^{+}) = -7.45\times10^{14}\cos(\sigma t+46^\circ)\ \mathrm{N\,m}$$

for the relevant component of the torque Γ_z. Here, $\sigma t = \sigma_1 t - \sigma_2 t = \dot{N}'t = N'$ where N' is the negative of the longitude of the lunar node (this being the argument associated with the fifth Doodson number) and $\dot{N}'$ the corresponding angular frequency, $1.070\times10^{-8}\ \mathrm{s}^{-1} = 1$ cycle/18.6 y.

Consider now the complementary torque obtained by switching σ_1 and σ_2. In principle, evaluation of this torque requires knowledge of the nodal line's D_{21}^{+} and ψ_{21}^{+}. But because the deep-ocean tidal admittance is known to be smooth (Munk and Cartwright, 1966), these parameters are equivalent to the primary line's parameters, but with amplitude scaled by the ratio in $\tilde{H}$; the known exceptions to this in tide-gauge data occur only in shallow, nearly resonant seas (e.g., Ku et al., 1985) and are unlikely to affect a global coefficient of spherical harmonic degree 2. Hence this second torque is

$$-\sqrt{(6\pi/5)}\,(1+k_2')\rho GM\tilde{H}D_{21}^{+}\cos(-\sigma t+\psi_{21}^{+}) = -7.45\times10^{14}\cos(-\sigma t+46^\circ)\ \mathrm{N\,m}$$

and the sum of the two torques is

$$-2\sqrt{(6\pi/5)}\,(1+k_2')\rho GM\tilde{H}D_{21}^{+}\cos\psi_{21}^{+}\ \cos\sigma t = -1.05\times10^{15}\cos N'\ \ \mathrm{N\,m}$$

At the 18.6-y period, we again assume that C represents the moment of inertia of the whole earth. The amplitude of the acceleration $\dot{\Omega}$ is thus $-1.31\times10^{-23}\ \mathrm{s}^{-2}$. The corresponding increment in the earth's rotation, expressed in terms of variations in Universal Time, is

$$\begin{aligned}\Delta\mathrm{UT} &= -\int\Delta\Lambda\,dt = (1/\Omega)\iint\dot{\Omega}\,dt^2\\ &= (\dot{N}^2\Omega)^{-1}\times1.31\times10^{-23}\cos N'\\ &= 1.57\cos N'\ \ \mathrm{ms.}\end{aligned}$$

In similar fashion, the major lunar tides generate the following terms:

$$\begin{aligned} \mathrm{Q_1}: &\quad 0.07 \cos N' \text{ ms} \\ \mathrm{O_1}: &\quad 1.57 \cos N' \text{ ms} \\ \mathrm{K_1}: &\quad 1.90 \cos N' \text{ ms} \\ \mathrm{N_2}: &\quad -0.17 \cos N' \text{ ms} \\ \mathrm{M_2}: &\quad -4.00 \cos N' \text{ ms} \\ \mathrm{K_2}: &\quad 0.36 \cos N' \text{ ms} \end{aligned}$$

The signs of these terms are consistent with the nodal modulations of these tides, which act to reduce the amplitudes of M_2 and N_2 when $N' = 0$ and to increase the other four (Doodson and Warburg, 1941, Table 7.3). Of the semidiurnals, K_2 is anomalous because it is a "declinational" tide: it arises from the modulation of the principal tide M_2 caused by the twice-monthly excursions of the moon away from the earth's equator. This modulation, and hence the amplitude of K_2, therefore increases, not decreases, with the lunar declination.

The large cancellations between diurnals and semidiurnals leave a residual of only 0.27 ms. This variation in UT is very small compared to the dominant tidal variation at the nodal frequency, which is due to the body tide's direct perturbation to the moment C. According to Yoder et al. (1981), the body tide generates oscillations in UT of amplitude 172.05 ms.

There is no dynamical reason for supposing that the above cancellations might in fact be perfect, since they depend on the tidal behavior of the oceans. One might imagine, for example, an ocean that somehow suppresses one tidal species, causing far less cancellation than now exists.

5.2 Variations at MSf

In a completely analogous manner, tidal interactions between constituents generate a large suite of periodic oscillations in the rotation rate with periods correspondingly shorter than 18.6 y. One of the largest is presumably the torque of M_2 on S_2, and vice-versa, which generates oscillations at the frequency of the near-fortnightly tide MSf (period 14.77 days). Because the potential amplitude of MSf is relatively weak (compared to, say, the nearby tide Mf), it exists in the ocean primarily as a nonlinear compound tide confined to shallow water. Its direct effect on UT through the body-tide variation in C is correspondingly weak. The ocean-torque mechanism is therefore more important for MSf than it is for most other long-period tides, and in fact it might conceivably explain the anomalously large observed variation in UT1 at this frequency (Chao et al., 1995). The question has been examined recently by Cartwright (1997), who finds the effect too small to explain the anomalous observations. The following calculation repeats Cartwright's in the context of the above formalism.

Unlike the nodal variation, the lag ψ_{22}^{+} for M_2 and S_2 are different, and the two torques cannot be trivially combined as in the nodal modulation case. The variation in UT1 is (with obvious subscripts)

$$\Delta\mathrm{UT} = (\sigma^2 \Omega C)^{-1} \sqrt{(96\pi/5)}\,(1+k_2')\,\rho G M \left\{ \tilde{H}_S D_M \sin(-\sigma t + \psi_M) + \tilde{H}_M D_S \sin(\sigma t + \psi_S) \right\}$$

Here σ is the frequency of MSf, and $\sigma t = 2D$ where D is the mean lunar elongation. At this frequency, the mantle is presumably uncoupled from the core and C should be decreased by 12% (Yoder et al., 1981). After some algebraic manipulation, with M_2 and S_2 coefficients from the Schrama-Ray model, this becomes

$$\Delta\mathrm{UT} = 0.23 \cos(2D + 3^\circ)\ \ \mu\mathrm{s}.$$

The excess length of day is

$$\Delta\Lambda = 0.098 \sin(2D + 3^\circ)\ \ \mu\mathrm{s},$$

very close to that estimated by Cartwright (1997). According to Yoder et al. (1981), the main body-tide effect at MSf causes a variation in UT of amplitude 78.1 μs. Hence, as Cartwright noted, the ocean-torque interaction is of no great significance.

6. Short-period variations

We come finally to the subject of Brosche and Seiler (1996) who suggested that torques on the ocean tide may be important at daily and sub-daily periods. The torques involving O_1, K_1, and M_2 are likely to be of comparable orders of magnitude. It suffices here to examine the torque of M_2 on its own retrograde oceanic bulge, which produces variations at twice the M_2 frequency. (Note that other combinations of tides will produce variations at precisely the M_2 frequency, for example $K_1 + O_1$, and $M_4 - M_2$. These torques are likely comparable or smaller than the $M_2 + M_2$ torque.)

The angular momentum variation associated with the torque of M_2 on its retrograde bulge is (with σ now the frequency of M_2)

$$
\begin{aligned}
J &= \int \Gamma_z \, dt \\
&= (2\sigma)^{-1} \sqrt{(96\pi/5)}\, (1 + k_2') \, \rho G M \tilde{H} D_{22}^{-} \cos(2\sigma t - \psi_{22}^{-}) \\
&= 3 \times 10^{19} \cos(2\sigma t - 26^\circ) \;\; \mathrm{kg\, m^2\, s^{-1}},
\end{aligned}
$$

based on the $D_{22}^{-}, \psi_{22}^{-}$ Schrama-Ray estimates. This is 5 to 6 orders of magnitude smaller than the angular momentum exchanges between the ocean and mantle that are due to tidal currents, according to estimates published by Seiler (1991) and Chao et al. (1996). We conclude that for the short-period angular momentum balance, direct torques on the ocean tide are of no significance.

This conclusion should be no surprise if we compare the oceanic torque mechanism with the closely related solid-earth libration. Libration is caused by a 70-meter difference between the semimajor and semiminor equatorial axes (Torge, 1980), and the torque on this solid-earth "bulge" causes UT1 variations of order $1\,\mu$s (Chao et al., 1991; Wünsch, 1991), which is an order of magnitude smaller than the variations induced by oceanic tidal currents. In contrast, the ocean tide is a bulge of only a few cm of water. The corresponding UT1 variations must therefore be 4 to 5 orders of magnitude smaller than the libration effect.

7. Summary

A general framework for computing lunar and solar tidal torques on the oceanic tides allows both the secular acceleration and a large suite of high-frequency periodic accelerations in the earth's rotation to be estimated. Estimates of the torques and accelerations require only the known amplitudes in a harmonic development of the tidal potential (e.g., Cartwright and Tayler, 1971) and the degree-2 spherical harmonic coefficients of the oceanic tidal elevations. The latter are available from numerical hydrodynamic models, from satellite altimeter analyses (e.g., Schrama and Ray, 1994), or from satellite orbit-perturbation analyses (e.g., Christodoulidis et al., 1987). For the secular acceleration we find (Table 2) a total oceanic contribution of approximately $-1300'' \mathrm{cy}^{-2}$. All the short-period variations turn out to be small. The torques on the retrograde bulges, which generate rotation variations at twice the original tidal frequencies, are especially small and can be ignored in studies of the earth's angular momentum balance.

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Figure 1. Geometry of the prograde and retrograde oceanic tidal bulges and the moon, seen from above the earth's north pole, in a frame rotating with the earth. The prograde bulge is shown lagging the moon by an angle $\psi_{22}^{+}/2 \approx 65°$ according to the estimate of Schrama and Ray (1994). The rotation rate is $\Omega = 7.2921 \times 10^{-5}$ s^{-1}; the moon's mean motion is $n = 2.6653 \times 10^{-6}$ s^{-1}.

Table 1. Estimates of Tidal Coefficients

Tide	D^+_{2m}, cm	ψ^+_{2m}, deg	D^-_{2m}, cm	ψ^-_{2m}, deg
	Geosat model – Cartwright & Ray (1991)			
Q_1	0.49	39.3	0.27	201.5
O_1	2.33	42.7	1.12	210.2
P_1	0.86	47.1	0.47	225.5
K_1	2.56	47.2	1.45	225.9
N_2	0.70	127.4	0.11	351.7
M_2	3.45	131.6	0.59	22.5
S_2	1.16	126.2	0.31	94.0
K_2	0.31	125.8	0.08	99.5
	T/P model – Schrama & Ray (1994, updated)			
O_1	2.592	46.09	1.160	209.58
K_1	3.001	42.43	1.528	224.92
N_2	0.702	119.51	0.117	10.74
M_2	3.231	129.38	0.585	26.07
S_2	1.241	131.91	0.217	80.04
	EGM96S model – Lemoine et al. (1998)			
Q_1	0.587	37.93		
O_1	2.732	45.09		
P_1	0.991	43.59		
K_1	2.828	39.45		
N_2	0.639	114.97		

Table 1. (continued)

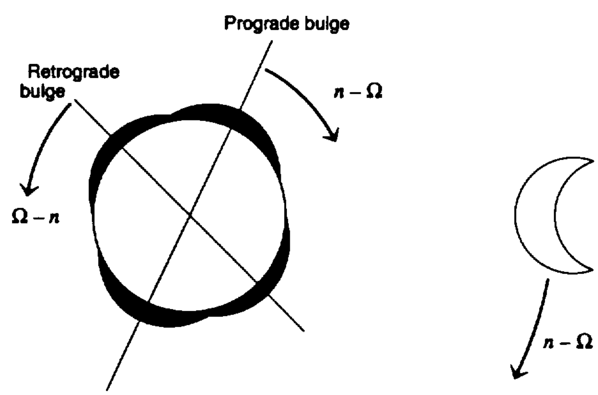
Tide	D^+_{2m}, cm	ψ^+_{2m}, deg	D^-_{2m}, cm	ψ^-_{2m}, deg
M_2	3.266	128.21		
S_2	0.785	145.90		
K_2	0.273	122.03		

$m = 1$ for diurnal tides, 2 for semidiurnal tides.

Table 2. Estimates of Secular Acceleration $\dot{\Omega}$ by Oceanic Tides

Tide	$\tilde{H}$, m	$(1+k_2')$	C-R	S-R	EGM96
Q_1	0.0502	0.689	−3	−3	−3 ± 1
O_1	0.2622	0.689	−63	−66	−71 ± 3
P_1	0.1220	0.700	−10	−13	−13 ± 2
K_1	0.3687	0.730	−96	−122	−120 ± 7
N_2	0.1210	0.692	−38	−42	−40 ± 2
M_2	0.6319	0.692	−924	−894	−919 ± 9
S_2	0.2940	0.692	−156	−154	−73 ± 5
K_2	0.0799	0.692	−11	−12	−10 ± 1
Total			−1305	−1306	−1304 ± 12

Units of $\dot{\Omega}$ are arcseconds per century^2, $1'' \mathrm{cy}^{-2} = 4.868 \times 10^{-25}\ \mathrm{s}^{-2}$. C-R = Cartwright and Ray (1991); S-R = Schrama and Ray (1994); EGM96 = Lemoine et al. (1998). S-R estimates of Q_1, P_1, K_2 are inferred from the admittance defined by its major tides. The EGM96 estimate of S_2 includes effects of the S_2 air tide, for which +55 has been added to its total.


Prograde bulge
Retrograde bulge
$n-\Omega$
$\Omega-n$
$n-\Omega$