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Evaluation of Methods for Multidisciplinary Design Optimization (MDO), Phase I

Srinivas Kodiyalam
Engineous Software, Inc., Morrisville, North Carolina

September 1998

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Table of Contents

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1.0 Objectives

Reference: Evaluation of Methods for MDO, Phase I, NASA Statement of Work by Natalia M. Alexandrov, Technical Project Monitor, MDOB, NASA Langley, 1997.

The general objective of the MDO Method Evaluation project is to collect numerical data on a number of promising MDO methods with the intent of providing some practical guidelines for their use.

The objective of Phase I was to collect data on Multidisciplinary Feasible Method (MDF), Individual Discipline Feasible Method (IDF), and Collaborative Optimization (CO).

The present intermediate report documents the numerical tests conducted in Phase I. This report does not report on other metrics, such as ease of implementation, nor does it analyze the data or draw conclusions in any way. Specifically, the report records the following:

1. A brief description of the methods under study.
2. A description of the work documented in the report.
3. Statement of the test problems.
4. Tables of data obtained during numerical tests.

The analysis of the tests, partial conclusions and recommendations, and the limitations of these conclusions, given the nature of the problems, implementation, tests, and problem formulations, will be presented in forthcoming publications (e.g., [1]).

2.0 Recorded Work

In this report, we record the work performed by each method during every optimization procedure. Here we define what is meant by "**work**" for each method.

For MDF, we report the total number of multidisciplinary analyses (MDA), including those necessary to compute the finite-difference derivatives. We also give the average number of fixed-point iterations taken to achieve each MDA. Thus, the average number of function evaluations for each run of MDF is equal to the number of MDA times the average number of fixed-point iterations per MDA times the number of disciplines.

For CO, we report the sum of the number of function evaluations in each subsystem, including those required for finite-difference evaluations, and the number of iterations taken by the system-level optimization problem.

For IDF, we report the total number of function evaluations, including those taken for finite-difference computation, times the number of disciplines. Note that the dimensions of the design space differ for IDF and CO.

Other metrics will be reported in [1].

3.0 MDO Methods

Phase I of the project collected numerical data on Multidisciplinary Feasible Method (MDF), Individual Discipline Feasible Method (IDF), and Collaborative Optimization (CO). MDF is a mathematical idealization of the conventional approach to MDO. The nomenclature was introduced in [5]. In this approach, multidisciplinary feasibility is achieved by iterating among the set of analyses to bring them into equilibrium. This method is implemented to serve as a baseline result. Methods of the type of CO ([4]) and IDF ([5]) have been known for a long time (see, for example, [16]). Both are intended for solving large, loosely coupled systems. All three methods were implemented in the iSIGHT framework, using MDOL, the iSIGHT MDO Language.

3.1 Multidisciplinary Feasible (MDF) Method:

The MDF formulation is a common way of approaching the solution of MDO problems. In this formulation, the vector of design variables $\mathbf{X_D}$ is provided to the coupled system of analysis disciplines and a complete multidisciplinary analysis (MDA) is performed via a fixed-point iteration with that value of $\mathbf{X_D}$ to obtain the system (MDA) output variable $\mathbf{U}(\mathbf{X_D})$ that is then used in evaluating the objective $F(\mathbf{X_D}, \mathbf{U}(\mathbf{X_D}))$ and the constraints $g(\mathbf{X_D}, \mathbf{U}(\mathbf{X_D}))$. The optimization problem is:

$$\text{Minimize: } F(\mathbf{X_D}, \mathbf{U}(\mathbf{X_D}))$$

$$\text{Subject to: } g(\mathbf{X_D}, \mathbf{U}(\mathbf{X_D})) \leq 0$$

$$\text{and bounds on design variable, } \mathbf{X_D}.$$

If a gradient-based method is used to solve the above problem, then a complete MDA is necessary not just at each iteration, but at every point where the derivatives are to be evaluated. Thus, attaining multidisciplinary feasibility can be prohibitively expensive in realistic application.

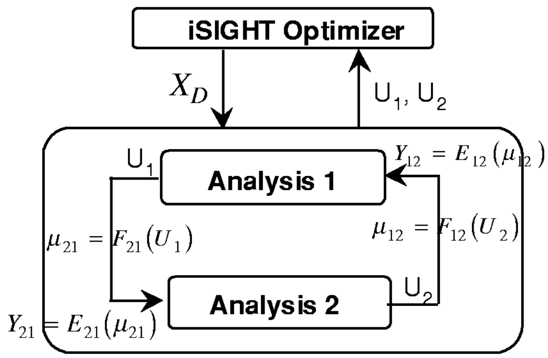

Figure 1. MDF Model

Figure 1 shows the data flow in a MDF analysis and optimization. In this figure, μ_{ij} is some spline coefficients obtained using a "fit" F_{ij} of the output of discipline j. F_{ij} may be either an interpolation or an approximation fit. The mapping E_{ij} is an evaluation of the spline representation from discipline j into a form suitable for use by discipline i (for example, calculating structural loads from aerodynamic pressures).

3.2 Individual Discipline Feasible (IDF) Method:

The IDF formulation provides a way to avoid a complete MDA at optimization. IDF maintains individual discipline feasibility, while allowing the optimizer to drive the individual disciplines to multidisciplinary feasibility and optimality by controlling the interdisciplinary coupling variables.

In IDF, the specific analysis variables that represent communication, or coupling, between analysis disciplines are treated as optimization variables and are in fact indistinguishable from design variables from the point of view of a single analysis discipline solver. The IDF formulation is:

$$\text{Minimize:} \quad F(\mathbf{X_D}, \mathbf{U}(\mathbf{X})) \text{ with respect to } \mathbf{X} = (\mathbf{X_D}, \mathbf{X_\mu})$$

$$\text{Subject to:} \quad g(\mathbf{X_D}, \mathbf{U}(\mathbf{X})) \leq 0$$

$$C(\mathbf{X}) = \mathbf{X}_\mu - \overline{\mu} = 0$$

and bounds on optimization variable, $\mathbf{X}$. $\mathbf{X_D}$ is the set of design variables and $\mathbf{X_\mu}$ is the set of interdisciplinary coupling variables. C is referred to as the interdisciplinary constraint. For implementation purposes, we use

$$J_j = C_j^2 \leq 0.0001, \ j = 1, \text{number of disciplines.}$$

It is important to note that an evaluation of U($\mathbf{X}$) involves executing all the single discipline analysis codes independently with simultaneously available multidisciplinary data $\mathbf{X}$. Therefore, the analysis computations can be performed concurrently.

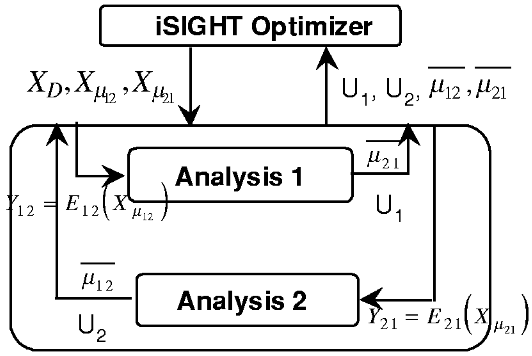

Figure 2: IDF Model

Figure 2 shows the data flow in an IDF analysis and optimization. The notations in Figure 2 are similar to those in Figure 1.

3.3 Collaborative Optimization (CO):

The CO formulation is a two-level hierarchical scheme for MDO, with the top level being the system optimizer that optimizes on the multidisciplinary variables (or, system level targets, **z**) to satisfy the interdisciplinary compatibility constraints (J^*) while minimizing the system objective (F). The objective of each subsystem optimizer is to minimize in a least squares sense the discrepancy between the subset of subspace design variables ($\mathbf{x}_i$) and subspace analysis computed responses ($\mathbf{y}_j$) that are common to more than one subspace analysis block and the system level values of these variables, **z**, while satisfying the subspace constraints ($\mathbf{g}_j$). The system level design variables, **z**, are considered to be fixed within a subspace problem. A distinction is made between the disciplinary design variables $\mathbf{x}_{sj}$, only of importance to subspace analysis j, and the interdisciplinary design variables $\mathbf{x}_j$, which are common to more than one subspace analysis block.

For implementation purposes, the interdisciplinary compatibility constraints (J's) were formulated as inequality constraints ($J \leq 0.0001$) as against strict equality constraints ($J = 0.0$). J is defined as:

$$J_j = | X_j - Z_j^s |**2 + | Y_j - Z_j^c |**2$$

where, $Z = \{Z^s, Z^c\}$; Z^s represents the system design variable and Z^c represents the system coupling variable.

The collaborative optimization formulation is intended for cases when the number of disciplinary variables $\mathbf{x}_{sj}$ is much larger than the number of interdisciplinary variables $\mathbf{x}_j$.. In other words, this formulation is intended for solving design problems with loosely coupled analyses of individually large dimension.

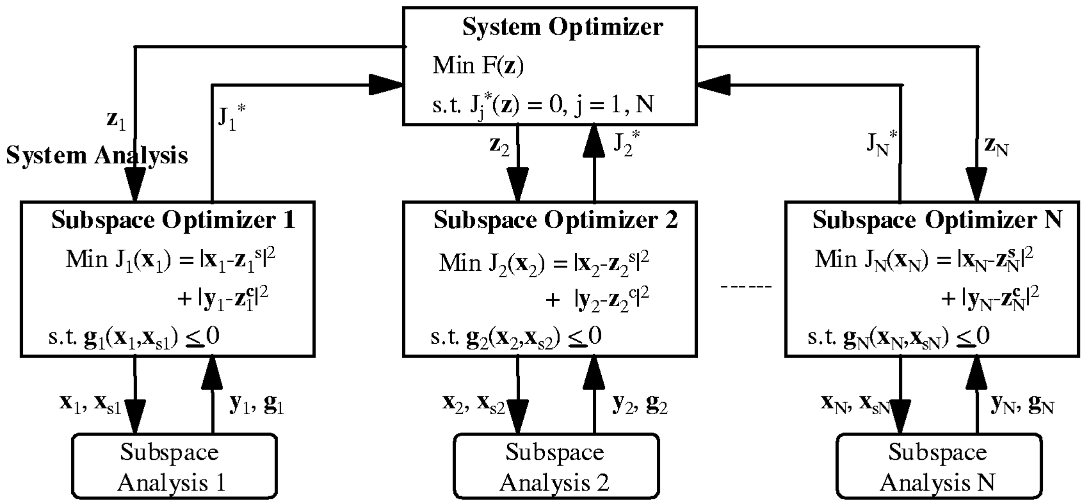

Figure 3: CO Model

Figure 3 shows the data flow in a CO analysis and optimization. The variables used in Figure 3 are defined in the CO method description provided under Section 3.3.

3.4 References:

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Problem 1: Conceptual Ship Design ([8])

In this problem, multidisciplinary design optimization of a conceptual design of an oil tanker ship is considered. The analysis disciplines involved are Propulsion, Hydrodynamics, Structures, and Cost and ROI (Return-on-Investment). The analyses of all these 5 disciplines involve simple methods (empirical relations) with a fidelity representative of conceptual design. A flow diagram of the concept-level analysis is provided in Figure 1.1.

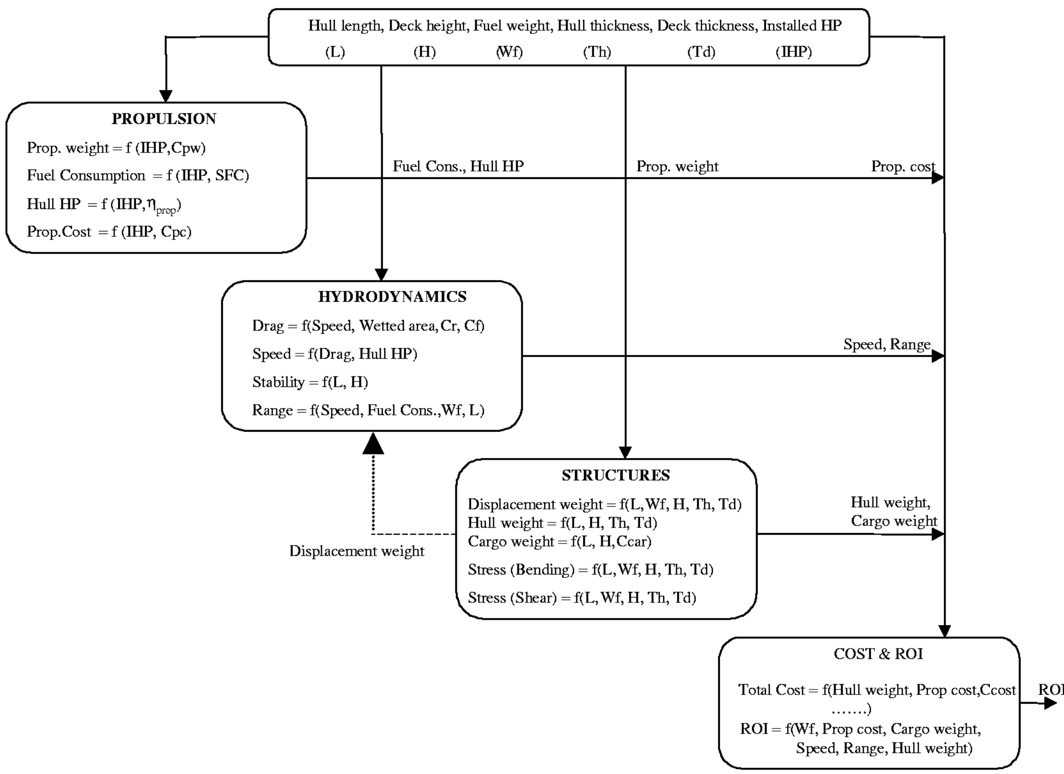

Figure 1.1: Conceptual Ship Design: Analysis Flow

The design objective is to maximize the Return-on-Investment (ROI) while satisfying design constraints on ship displacement weight, range (distance), stability, stresses (bending and shear) and bounds on design variables.

For the MDF approach, the optimization problem is stated as follows.

Find the set of design variables that:

Maximize: ROI

Subject to: ship displacement weight $= 2*10^8\ lbs$

Range = 10,000 Nm

Stability factor ≤ 0.0

Max (Bending and shear) stresses $\leq 30{,}000\,psi$

The MDF problem has a total of 6 design variables: Ship Length, Height, Hull Thickness, Deck Thickness, Engine HP, and Fuel Weight.

The MDF optimization problem is solved using SLP and Method of Feasible Directions techniques in iSIGHT for 12 different starting points.

For the IDF approach, the optimization problem is given by the following:

Find the set of design variables and coupling variables that:

Maximize: ROI

Subject to: Ship Displacement Weight $= 2 * 10^8 lbs$

Range = 10,000 Nm

Stability factor ≤ 0.0

Max (bending, shear) stresses $\leq 30{,}000 psi$

$Jprop \leq 0.0001$

$Jhydro \leq 0.0001$

$Jstruct \leq 0.0001$

$J\cos t \leq 0.0001$

The IDF optimization problem is solved using the Method of Feasible Directions and SQP techniques implemented in iSIGHT. All the required derivatives are computed by finite differences.

For the CO approach, the system-level optimization problem is stated as follows:

Find the set of system-level targets, Z_s, that:

Maximizes: ROI

Subject to: $Jprop \leq 0.0001$

$Jhydro \leq 0.0001$

$Jstruct \leq 0.0001$

$J\cos t \leq 0.0001$

$Jroi \leq 0.0001$

The CO approach has 11 system-level design variables $\{Z^s\}$.

{Z} = {Hull length (L), Fuel weight (Wf), Propulsion weight, Propulsion cost, Hull weight, Engine speed, Fuel consumption, Cargo weight, Hull HP, Ship cost, ROI}

J's are the interdisciplinary compatibility constraints at the system level as well as the subsystem objectives. The CO disciplinary analysis inputs and outputs are shown in Figure 1.2. The SLP and MFD (Method of Feasible Directions) implementations in iSIGHT are used to solve the system-level optimization problem. All the required derivatives are computed analytically.

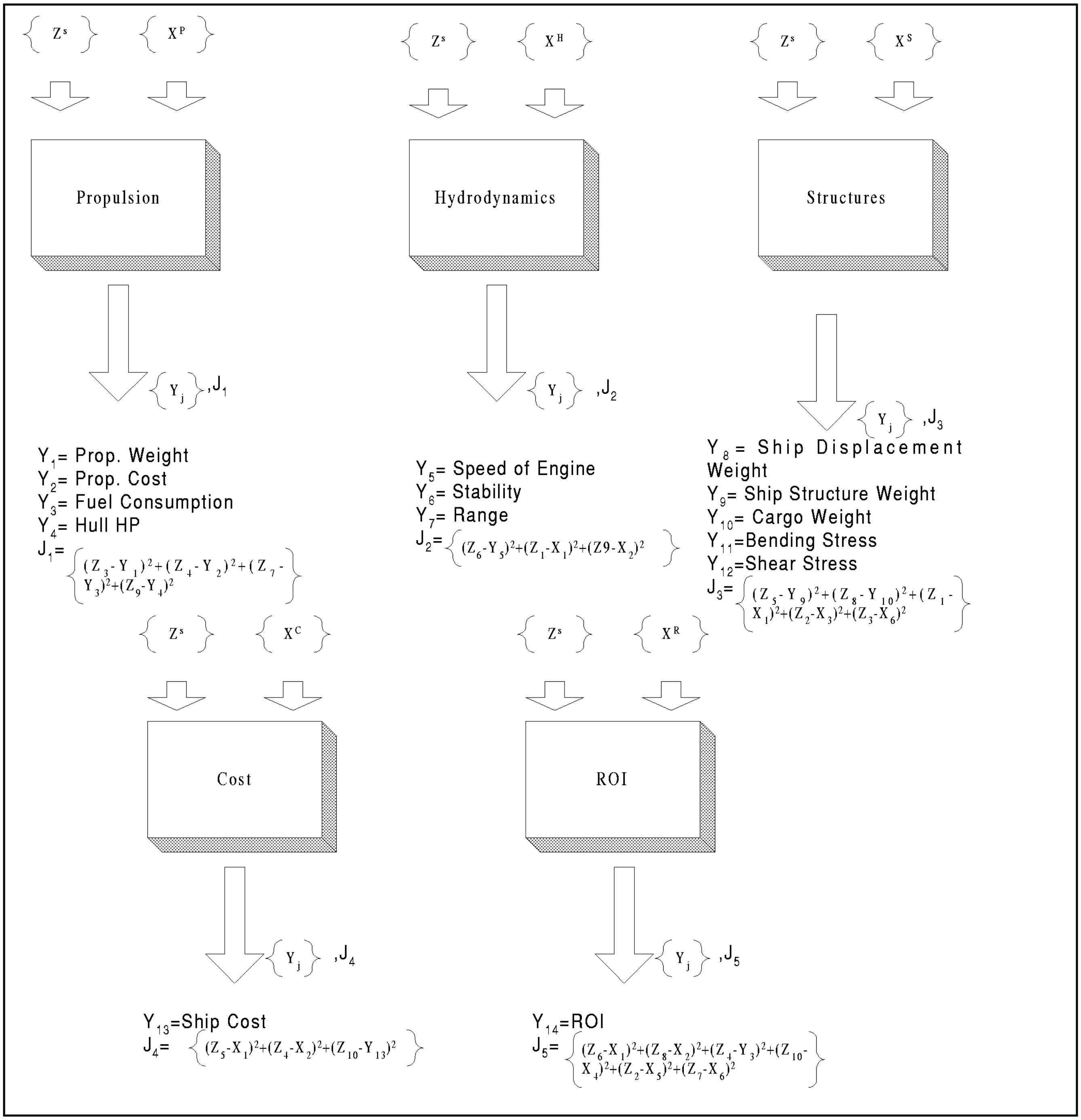

Figure 1.2: Disciplinary Analysis

The following states the subsystem optimization problems. All of the subsystem optimizations are done using the MFD technique and the required derivatives are computed using finite differences.

Propulsion Subsystem:
Find $\{X^P\}$ that
Minimizes J_1

Hydrodynamics Subsystem:
Find $\{X^h\}$ that
Minimizes J_2
Subject to: $Y_6 \le 0.0$

Structures Subsystem:
Find $\{X^s\}$ that
Minimizes J_3.
Subject to: $Y_8 = 2.0 * 10^8\, lbs\ (^+/_- 1\%)$
$Y_{11} \le 30{,}000 psi$
$Y_{12} \le 30{,}000 psi$

Cost Subsystem:
Find $\{X^c\}$ that
Minimizes J_4

ROI Subsystem:
Find $\{X^R\}$ that
Minimizes J_5
Subject to: $Y_7 = 10{,}000 Nm\ (^+/_- 1\%)$

The MDF approach results are shown in Table 1.1, and the IDF and CO results in Tables 1.2 and 1.3.

Table 1.1: MDF Solutions
(6 design variables, 9 constraints)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation	Work
1	2.48455D-01	+1.80338D+00(8)	2.78913D-01	+9.8600D-04(2)	122 x 1 x 5
2	4.16729D-02	+1.15759D+01(3)	2.78925D-01	+6.2400D-04(2)	103 x 1 x 5
3	0.00000D+00	+4.49422D+01(9)	2.78895D-01	+1.0000D-04(2)	154 x 1 x 5
4	1.92168D-02	+1.01019D+02(9)	2.78942D-01	+1.3433D-03(6)	144 x 1 x 5
5	6.53199D-02	+9.66009D+01(9)	2.78781D-01	+2.0000D-05(3)	103 x 1 x 5
6	0.00000D+00	+1.08266D+02(4)	3.36207D-03	+1.7500D-04(4)	104 x 1 x 5
7	5.87348D-02	+3.31992D+01(4)	2.78951D-01	+6.0670D-04(8)	201 x 1 x 5
8	2.65787D-02	+2.00189D+02(9)	2.79191D-01	+1.1833D-03(8)	116 x 1 x 5
9	1.19359D-01	+5.33065D+01(4)	2.79349D-01	+3.4650D-03(3)	142 x 1 x 5
10	4.83683D-02	+1.52135D+02(9)	2.78905D-01	+5.9000D-04(2)	99 x 1 x 5
11	9.74823D-03	+4.21540D+01(8)	2.79145D-01	+1.0533D-03(8)	159 x 1 x 5
12	3.74768D-02	+5.20867D+00(6)	2.78818D-01	+8.8500D-04(3)	153 x 1 x 5

Note: See page 1, Section 2.0 for definition of "**Work**"

Table 1.2: CO Solutions
(11 system variables)

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	0.249	+1.01 (Js)	0.277	+0.00009 (Jc)	189 system iter (2729,3029,3870,2975 ,4023) = 15626
2	0.249	+0.46 (Js)	0.2744	+0.00016 (Js)	158 system iter (2305,2529,3215,2485 ,3332) = 13866
3	0.1246	+0.143 (Jh)	0.247	+0.0001 (Jh)	159 system iter (2323,2446,3217,2440 ,3343) = 13769
4	0.1246	+0.735 (Js)	0.20	0.00009 (Js)	104 system iter (1476,1571,2135,1644 ,2175) = 9001

Note: See page 1, section 2.0 for definition of "**Work**"

Table 1.3: IDF Solutions
(14 system variables)

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	0.249	+1.80 (SigS)	0.237	+0.0001 (J's) +0.02 (Range)	1906 x 5
2	0.0951	+1.79 (SigS)	0.232	+0.0000 (J's)	1707 x 5
3	0.122	+1.0 (Jprop) +0.5 (Range)	0.27	+0.0001 (J's) +0.038 (Stability)	2170 x 5
4	0.280	+0.96 (Range) +0.75 (Jprop)	0.254	+0.0001 (J's)	1929 x 5

Note: See page 1, section 2.0 for definition of "**Work**"

Problem 2: Electronic Packaging ([12],[13])

The electronic packaging is a multidisciplinary problem with coupling between electrical and thermal subsystems. Component resistance is influenced by operating temperatures; the temperatures depend on resistance.

The objective of the problem is to maximize the watt density for the electronic package subject to constraints. The constraints require the operation temperatures for the resistors to be below a threshold temperature and the current through the two resistors to be equal.

For the MDF approach, the optimization problem is given as follows:

Maximize:	Y_1 (Watt Density)	
Subject to:	$h_1 = Y_4 - Y_5 = 0.0$	(branch current equality)
	$g_1 = Y_{11} - 85.0 \leq 0$	(component 1 reliability)
	$g_2 = Y_{12} - 85.0 \leq 0$	(component 2 reliability)

The MDF problem has 8 design variables that are the following:

$0.05 \leq$ heat sink width $(x_1) \leq 0.15$

$0.05 \leq$ heat sink length $(x_2) \leq 0.05$

$0.01 \leq$ fin length $(x_3) \leq 0.10$

$0005 \leq$ fin width $(x_4) \leq 0.05$

$10.0 \leq$ resistance #1 $(x_5) \leq 1000.0$

$0.004 \leq$ temp coefficient $(x_6) \leq 0.009$

$10.0 \leq$ resistance #2 $(x_7) \leq 1000.0$

$0004 \leq$ temp coefficient $(x_8) \leq 0.009$

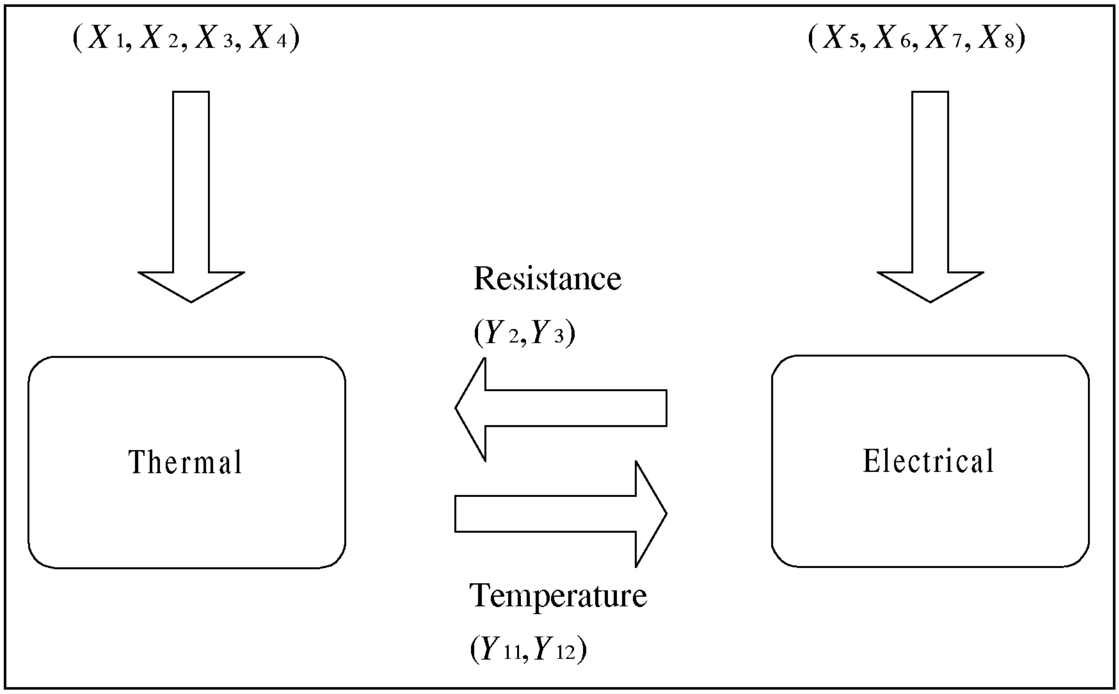

Figure 2.1: Interdisciplinary Interactions

For the IDF approach, the optimization problem is given by:

Maximize: Y_1

Subject to: $J_1, J_2 \leq 0.0001$

$h_1 = Y_4 - Y_5 = 0.0$

$g_1 = Z_{11} - 85.0 \leq 0$

$g_2 = Z_{12} - 85.0 \leq 0$

The IDF problem has 12 design variables, including 4 coupling variables that are the following:

$X_i; i = 1,8$

Z_2, Z_3, Z_{11}, Z_{12}

The Thermal subsystem evaluates Y_1, h_1 and J_1.

$$J_1 = (Y_{11} - Z_{11})^2 + (Y_{12} - Z_{12})^2$$

The Electrical subsystem evaluates J_2.

$$J_2 = (Y_2 - Z_2)^2 + (Y_3 - Z_3)^2$$

For the CO approach, the system-level optimization problem is given by:

Maximize: Z_1

Subject to: $J_1 \le 0.0001$
$J_2 \le 0.0001$

The system-level CO problem has 5 design variables that are coupling parameters:

$Z_1, Z_2, Z_3, Z_{11}, Z_{12}$

The system-level sensitivities are calculated analytically.

The thermal subsystem optimization task is given as:

Minimize: J_1

Subject to: $h_1 = 0.0$
$g_1 = Y_{11} - 85.0 \le 0$
$g_2 = Y_{12} - 85.0 \le 0$

and $J_1 = (Y_{11} - Z_{11})^2 + (Y_{12} - Z_{12})^2 + (Y_2 - Z_2)^2 + (Y_3 - Z_3)^2 + (Y_1 - Z_1)^2$

The thermal task has 6 design variables:

$X_i; i = 1,4$ & Y_2, Y_3

The Electrical subsystem optimization task is given as:

Minimize: J_2

Subject to: $g_1 = Y_{11} - 85.0 \le 0$
$g_2 = Y_{12} - 85.0 \le 0$

and $J_2 = (Y_2 - Z_2)^2 + (Y_3 - Z_3)^2 + (Y_{11} - Z_{11})^2 + (Y_{12} - Z_{12})^2$

The Electrical task has 6 design variables:

$X_i; i = 5,8$ & Y_{11}, Y_{12}

The MDF problem was solved for 12 different starting points using the feasible directions method in iSIGHT. The required derivatives were calculated using finite differences with the step size of 0.01 (1%). The results are provided in Table 2.1. The IDF and CO problems were solved using Exterior Penalty Function Method and Method of Feasible Directions for the system-level optimization and the Sequential Quadratic Programming - DONLP implementation in iSIGHT. The results are provided in Tables 2.2 and 2.3.

Table 2.1: MDF Solutions
(8 design variables, 3 constraints)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation	Work
1	7.79440D+01	+2.16630D-08(3)	6.39720D+05	+1.21880D-03(3)	83 x 3 x 2
2	6.83630D+03	-2.89560D-01(3)	6.39720D+05	+1.21880D-03(3)	44 x 3 x 2
3	1.51110D+03	-4.29240D-02(3)	6.36540D+05	+1.45140D-03(3)	44 x 3 x 2
4	1.46070D+03	-1.02490D-03(3)	6.36940D+05	+1.42110D-03(3)	35 x 3 x 2
5	2.61020D+02	-8.20230D-03(3)	3.16700D+05	-7.16410D-01(3)	33 x 3 x 2
6	5.59700D+02	-2.46210D-02(3)	6.39720D+05	+1.21880D-03(3)	50 x 3 x 2
7	1.35140D+03	-1.12180D-03(3)	6.39720D+05	+1.21880D-03(3)	49 x 3 x 2
8	1.08000D+04	-4.24340D-01(3)	6.39720D+05	+1.21880D-03(3)	40 x 3 x 2
9	1.74350D+03	-2.33980D-02(3)	6.39720D+05	+1.21880D-03(3)	52 x 3 x 2
10	2.84430D+02	-8.50890D-03(3)	6.36870D+05	+1.42660D-03(3)	41 x 3 x 2
11	1.21230D+03	+1.64300D-02(3)	3.24910D+05	-7.95220D-01(3)	32 x 3 x 2
12	6.75670D+02	+2.48320D-02(3)	3.26030D+05	-7.97960D-01(3)	46 x 3 x 2

Note: See page 1, section 2.0 for definition of "**Work**"

Table 2.2: CO Solutions (system-level sensitivities computed analytically)
(5 system variables, 6 Elec ss variables, 6 Thermal ss variables)

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	77.944	0.0 (Eq)	351968.0	0.0001 (J1)	110 system iter (4886,8899)=13785
2	6830.0	-0.289(Eq)	657162.9	+0.00023(J1)	123 system iter (6315,13557)=19872
3	1511.1	-0.042 (Eq)	65000.0^{F}	+0.0076(J1)	138 system iter (13414,12650)=26064
4	1460.7	-0.001 (Eq)	65000.0^{F}	+0.0048(J1)	94 system iter (10205,9396)=19701

Note: See page 1, section 2.0 for definition of "**Work**"

Note: The superscript "F" added to the value of the final objective indicates failure to converge to a Kuhn-Tucker point for the original problem.

Table 2.3: IDF Solutions

(12 system variables)

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	77.944	2.248e-3 (Eq)	681310.0	0.0006 (J1)	135 x 2
2	6836.3	-0.289	653670.0	+0.0001 (J's)	4488 x 2
3	1511.1	-0.042 (Eq)	677400.0	+0.0006 (J1)	2053 x 2
4	1460.76	-0.001 (Eq)	675767.7	+0.00017 (J1)	3437 x 2

Note: See page 1, section 2.0 for definition of "**Work**"

Problem 3: Power Converter ([9],[12])

The power converter is a multidisciplinary problem with couplings between an electrical subsystem and a loss subsystem. The power stage design dominates the overall efficiency, size and weight of the power converter.

The objective of the problem is to minimize the weight subject to several constraints. The constraints are on state variables, including fill window constraint, ripple specification, core saturation and minimum inductor size.

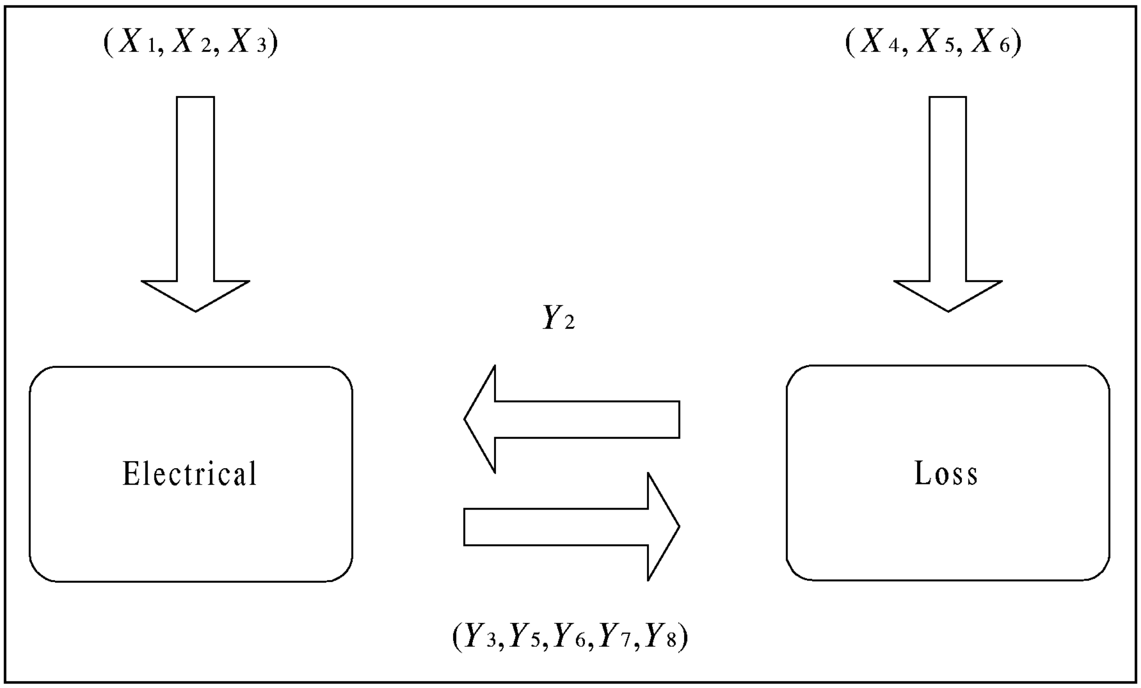

Figure 3.1: Interdisciplinary Interactions

For the MDF approach, the optimization problem is as follows:

Minimize: Y_1 (component weight)

Subject to: $g_1 = Y9 \leq 0.0$ (fill window constraint)

$g_2 = Y_{10} \leq 0.0$ (ripple specification)

$g_3 = Y_{11} \leq 0.0$ (core saturation)

$g_4 = Y_{12} \leq 0.0$ (minimum inductor size)

The MDF problem has 6 design variables:

Core center leg width $(X_1) \geq 0.001$

turns $(X_2) \geq 1$

copper size $(X_3) \geq 7.29e-08$

inductance $(X_4) \geq 1.0e-15$

capacitance $(X_5) \geq 0.1e-04$

Core Window width $(X_6) \geq 0.001$

For the IDF approach the optimization problem is as follows:

Minimize: Y_1 (Component weight)

Subject to: $J_1 \leq 0.0001$

$J_2 \leq 0.0001$

$g_1 = Y_9 \leq 0.0$

$g_2 = Y_{10} \leq 0.0$

$g_3 = Y_{11} \leq 0.0$

$g_4 = Y_{12} \leq 0.0$

The IDF problem has 12 design variables including the following:

$X_i; i = 1,6$

$Z_2, Z_3, Z_5, Z_6, Z_7, Z_8$ (coupling parameters)

The electrical subsystem evaluates J_1

$$J_1 = \sum_i (Y_i - Z_i)^2, i = 3,5,6,7,8$$

The Loss Subsystem evaluates J_2

$$J_2 = \sum_i (Y_i - Z_i)^2, i = 2$$

At the system level, an analysis is performed to evaluate $Y_1, Y_9, Y_{10}, Y_{11}, Y_{12}$ using the values of $X_i, i = 1$ to 6 and $Z_2, Z_3, Z_5, Z_6, Z_7, Z_8$ as inputs to the analysis.

For the CO approach, the system-level optimization problem is as follows:

Minimize: Y_1

Subject to: $J_1 \le 0.0001$

$J_2 \le 0.0001$

$g_1 = Y_9 \le 0.0$

$g_2 = Y_{10} \le 0.0$

$g_3 = Y_{11} \le 0.0$

$g_4 = Y_{12} \le 0.0$

The system-level CO problem has 6 design variables that are coupling parameters:

$Z_2, Z_3, Z_5, Z_6, Z_7, Z_8$

The Electrical subsystem optimization task is as follows:

Minimize: J_1

where $J_1 = \sum_i (Y_i - Z_i)^2; i = 2,3,5,6,7,8$

The electrical task has 4 design variables:

X_1, X_2, X_3, Y_2

The loss subsystem optimization task is as follows:

Minimize: J_2

The loss task has 8 design variables:

$X_4, X_5, X_6, Y_3, Y_5, Y_6, Y_7, Y_8$

At the system level, analysis is performed to evaluate $Y_1, Y_9, Y_{10}, Y_{11}, Y_{12}$ using the subsystem obtained optimal values of $X_i; i = 1,6$ and $Z_2, Z_3, Z_5, Z_6, Z7, Z_8$.

The MDF problem was solved using the method of feasible directions implemented in iSIGHT. The required derivatives were calculated using finite differences with step size of 0.001. The IDF and CO solutions were obtained using the method of feasible directions for the system and subsystem problems. The results are provided in Tables 3.1, 3.2 and 3.3.

Table 3.1: MDF Solutions
(6 design variables, 4 constraints)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation	Work
1	2.03005D+00	+3.38844D-03(2)	1.46687D+00	+3.98444D-03(3)	61 x 5 x 2
2	7.42340D+01	+2.12616D-03(1)	2.69620D+00	+3.96112D-03(3)	90 x 5 x 2
3	1.65931D+00	+2.22721D+00(3)	2.19710D+00	+1.16074D-03(3)	129 x 5 x 2
4	3.50898D+02	-5.20815D-05(4)	4.39826D+00	-6.62049D-05(4)	64 x 5 x 2
5	1.56350D+01	+2.28357D-03(1)	3.17256D+00	+3.57201D-03(3)	96 x 5 x 2
6	7.89477D+01	+1.61542D-02(1)	4.83515D+00	+2.64544D-03(3)	83 x 5 x 2
7	8.20192D+01	+8.05741D-03(1)	2.26158D+00	+3.49565D-03(3)	187 x 5 x 2
8	1.05152D+02	+2.46841D-03(1)	4.58379D+00	+3.99762D-03(3)	114 x 5 x 2
9	4.19526D+01	+3.70250D-03(1)	3.33925D+00	+3.78018D-03(3)	116 x 5 x 2
10	9.53708D+01	+5.87834D-05(1)	3.88211D+00	+3.96192D-03(3)	95 x 5 x 2
11	1.41423D+00	+1.41423D+00(3)	1.30578D+00	+3.99911D-03(3)	127 x 5 x 2
12	3.11182D+02	-2.85777D-05(4)	3.14690D+00	+5.34411D-04(3)	98 x 5 x 2

Note: See page 1, section 2.0 for definition of "**Work**"

Table 3.2: CO Solutions
(6 system variables, 4 ss1 variables, 8 ss2 variables)

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	2.0300	+3.388E-03 (Y10)	1.626	+0.00018 (J1)	97 system iter
2	1.4288	+8.265e-04 (Y10)	1.386	+0.0003 (J1)	33 system iter (770, 1015)=1785
3	311.1	-2.8577e-05 (Y12)	211.38[F]	+0.00046 (J1)	42 system iter (2162, 2419)=4581
4	1.869	+3.622e-03 (Y10)	1.5398	+0.00047 (J1) +0.0025 (Y10)	45 system iter (1109, 1474)=2583

Note: See page 1, section 2.0 for definition of "**Work**"

Note: The superscript "F" added to the value of the final objective indicates failure to converge to a Kuhn-Tucker point for the original problem.

Table 3.3: IDF Solutions
(12 system variables)

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	2.03	+3.388e-03(Y10)	1.323	+0.0004 (J2)	262 x 2
2	1.4272	+8.265e-04 (Y10)	1.14	+0.0004 (J1)	191 x 2
3	311.1	-2.85e-05 (J12)	38.68	+0.00049 (J1)	176 x 2
4	1.869	+3.622e-03 (Y10)	1.4609	+0.00047 (J2)	192 x 2
5	15.635	+2.283e-03 (Y9)	2.803	+0.0004 (J1) +0.0041 (Y11)	195 x 2

Note: See page 1, section 2.0 for definition of "**Work**"

Problem 4: Speed Reducer ([10], [12])

This problem represents the design of a simple gearbox and is posed as an artificial multidisciplinary problem comprising the coupling between gear design and shaft design disciplines.

The design objective is to minimize the speed reducer weight while satisfying a number of constraints posed by gear and shaft disciplines.

For the MDF approach, the optimization problem is defined as:

Minimize:	F (gear box weight)
Subject to:	g_1 (bending stress of gear tooth) ≤ 0.0
	g_2 (contact stress of gear tooth) ≤ 0.0
	g_3, g_4 (transverse deflection of shafts 1,2) ≤ 0.0
	g_5, g_6 (stresses in shafts 1,2) ≤ 0.0
	$g_7 - g_{23}$ (dimensional restrictions)
	g_{24}, g_{25} (dimension requirements for shafts)

Where,

f (objective) =

$$C_1 x_1 x_2^2 \left(C_2 x_3^2 + C_3 x_3 - C_4\right) - C_5\left(x_6^2 + x_7^2\right) x_1 + C_6\left(x_6^3 + x_7^3\right) + C_1\left(x_4 x_6^2 + x_5 x_7^2\right)$$

The MDF problem has 7 design variables:

$2.6 \leq x_1 \leq 3.6$	
$0.7 \leq x_2 \leq 0.8$	$7.3 \leq x_5 \leq 8.3$
$17 \leq x_2 \leq 28$	$2.9 \leq x_6 \leq 3.9$
$7.3 \leq x_4 \leq 8.3$	$5.0 \leq x_7 \leq 5.5$

The MDF analyses involve the calculation of the objective (f) and constraints (g_j) that are all explicit functions of the design variables and some constraints.

For the CO approach, the original problem is reduced into three lower-level subsystems and a system-level coordination problem. The subsystem analyses i/o is shown below.

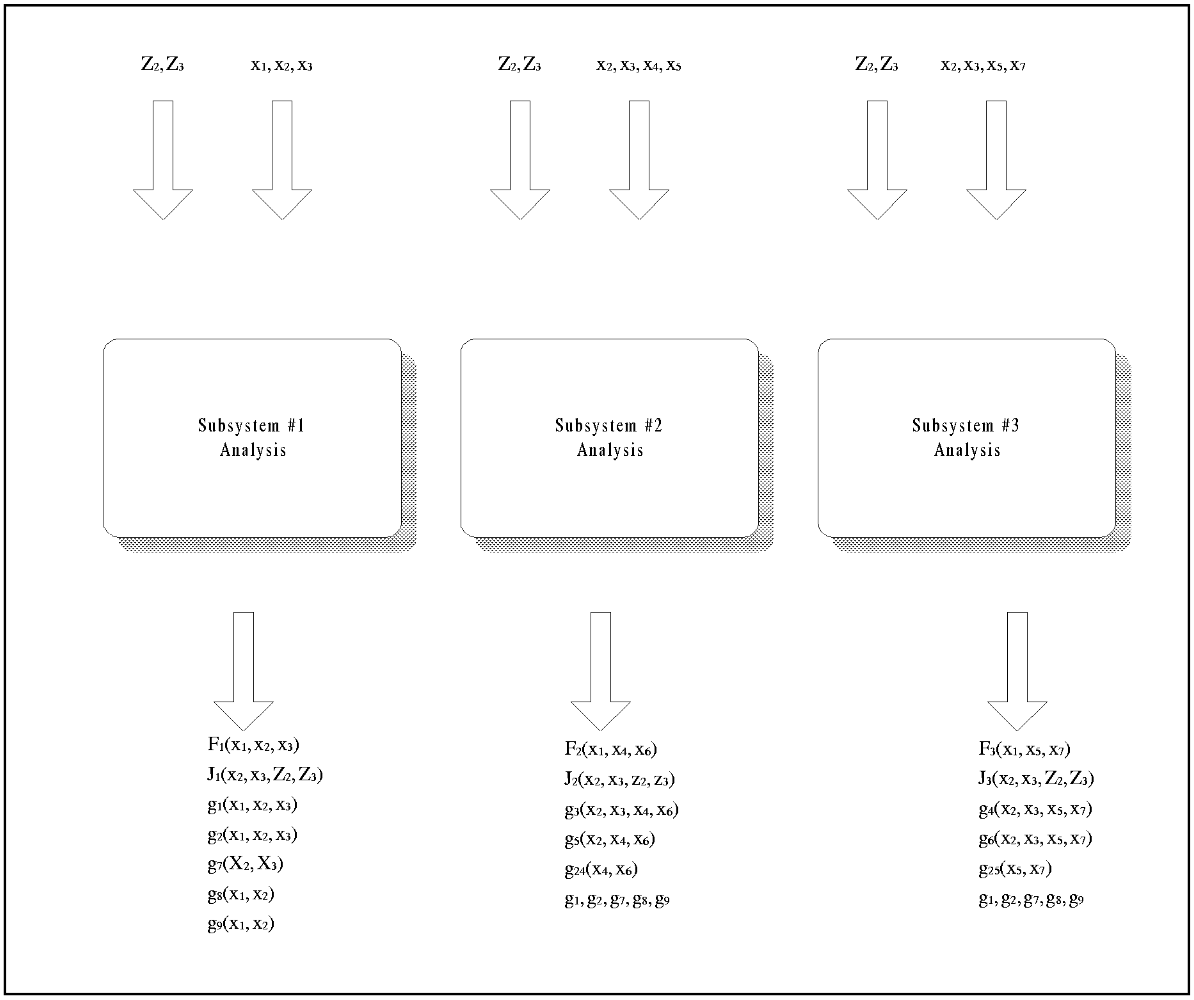

Figure 4.1: Subsystem Analyses Inputs/Outputs

The CO system-level optimization problem is as follows:

Minimize: $F_1 + F_2 + F_3$

Subject to: $J_1 \leq 0.0001; J_2 \leq 0.0001; J_3 \leq 0.0001$

where $J_i = (x_2 - Z_2)^2 + (x_3 - Z_3)^2$

The system-level design variables are Z_2, Z_3

The subsystem 1 optimization task is as follows:

Minimize: $F_1 + J_1$

Subject to: $gj \leq 0.0; \ \ j = 1,2,7,8,9$

The subsystem 1 design variables are x_1, x_2, x_3

The subsystem 2 optimization task is:

Minimize: $F_2 + J_2$

Subject to: $g_j \leq 0.0; \ \ j = 1,2,3,5,7,8,9,24$

The subsystem 2 design variables are x_2, x_3, x_4, x_6

The subsystem 3 optimization task is:

Minimize: $F_3 + J_3$

Subject to: $g_j \leq 0.0; \ \ j = 1,2,4,6,7,8,9,25$

The subsystem 3 design variables are x_2, x_2, x_5, x_7.

The MDF problem is solved using the Method of Feasible Direction (MFD) in iSIGHT. The CO problem is solved using SLP and MFD at the system level while MFD is used to solve the subsystem problems.

The MDF method solutions are provided in Table 4.1. The CO method solutions are provided in Tables 4.2.

An IDF solution is not performed for the speed reducer problem, since any decomposition on this problem is purely on the design variables (inputs) and in the IDF approach all the design variables are considered at the system level (single-level optimization).

Table 4.1: MDF Solutions
(7 design variables, 11 constraints)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation	Work
1	2.99436D+03	+5.96046D-08(11)	2.99436D+03	+5.96046D-08(11)	7 x 1 x 1
2	3.89678D+03	+2.35643D-01(8)	2.99347D+03	+3.11375D-03(5)	97 x 1 x 1
3	3.71309D+03	+1.11526D-01(6)	2.99265D+03	+3.30311D-03(5)	72 x 1 x 1
4	4.02797D+03	+2.67476D-01(8)	2.99320D+03	+2.41172D-03(5)	75 x 1 x 1
5	3.40493D+03	+8.40958D-02(8)	2.99435D+03	+2.47955D-05(5)	81 x 1 x 1
6	4.05869D+03	+1.54719D-01(5)	2.99330D+03	+3.73399D-03(5)	63 x 1 x 1
7	4.17071D+03	+2.35143D-01(8)	2.99288D+03	+3.22282D-03(5)	88 x 1 x 1
8	4.27473D+03	+2.04692D-01(8)	2.99395D+03	+1.54972D-03(5)	101 x 1 x 1
9	5.26058D+03	+3.43947D-01(5)	2.99202D+03	+3.61216D-03(5)	70 x 1 x 1
10	3.66641D+03	+3.08839D-01(5)	2.99330D+03	+3.71677D-03(5)	90 x 1 x 1
11	4.41547D+03	+2.64315D-01(8)	2.99329D+03	+3.72761D-03(5)	95 x 1 x 1
12	5.14732D+03	+2.31500D-01(8)	2.99249D+03	+2.26557D-03(5)	88 x 1 x 1

Note: See page 1, section 2.0 for definition of "**Work**"

Table 4.2: CO Solutions

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	2994.355	0.0	2994.355	0.0	5 system iter
2	3883.807	0.0 0.235 (G8/SS1)	2992.36	0.0	6 system iter
3	3693.27	0.0	2997.40	0.0	5 system iter
4	3980.853	0.0 0.26 (G8/SS1)	2992.16	+0.004	5 system iter
5	3394.65	+0.08 (G9/SS1)	2989.43^F	+0.19 (J1)	5 system iter (445,554,1103)=2102

Note: See page 1, section 2.0 for definition of "**Work**"

Note: The superscript "F" added to the value of the final objective indicates failure to converge to a Kuhn-Tucker point for the original problem.

Problem 5: Combustion of Propane ([2], [12])

This is a chemical equilibrium problem dealing with combustion of propane in air. here are 11 unknowns X_i, i=1,11 which represent the number of moles of each product formed for each mole of propane burned. X_{11} is essentially the sum of the other 10 unknowns. There are 10 products of combustion denoted by equations f_j, j=1,10.

The fixed parameters in the problem are p (pressure in atmospheres) and R (the air to fuel ration). Ideally, we want all the equations f_j's , (j=1,11) to be zero. All the Xi, i=1,11 must be greater than zero.

Equations:

$$f_1(x) = x_1 + x_4 - 3$$

$$f_2(x) = 2x_1 + x_2 + x_4 + x_7 + x_8 + x_9 + 2x_{10} - R$$

$$f_3(x) = 2x_2 + 2x_5 + x_6 + x_7 - 8$$

$$f_4(x) = 2x_3 + x_9 - 4R$$

$$f_5(x) = K_5 x_2 x_4 - x_1 x_5$$

$$f6(x) = K_6 \sqrt{x_2}\sqrt{x_4} - \sqrt{x_1} x_6 \left(\frac{P}{x_{11}}\right)^{1/2}$$

$$f_7(x) = K_7 \sqrt{x_1}\sqrt{x_2} - \sqrt{x_4} x_7 \left(\frac{P}{x_{11}}\right)^{1/2}$$

$$f_8(x) = K_8 x_1 - x_4 x_8 \left(\frac{P}{x_{11}}\right)$$

$$f_9(x) = K_9 K_1 \sqrt{x_3} - x_4 x_9 \left(\frac{P}{x_{11}}\right)^{1/2}$$

$$f10(x) = K_{10}\sqrt{x_1} - \sqrt{x_4} x_{10} \left(\frac{P}{x_{11}}\right)$$

$$f11(x) = x_{11} - \sum_{j-1}^{10} x_j$$

K_5,K_6,K_7,K_9, and K_{10} represent the measured data.

The conventional optimization problem is to solve the set of 11 nonlinear equations (f_j, j=1,11) in 11 unknowns (x_i, i=1,11), given the measured data and fixed parameters.

The NASA MDO web site documents a sample MDO solution for the preceding problem consisting of a system-level problem and three subsystem-level problems. The decomposition is arbitrary and is chosen so that there is coupling between the system and the three subsystems iteratively. The system analyses use a fixed-point iteration with relaxation to find consistent values of the subsystem variables. However, since the relaxation technique implemented is not very robust, the system analysis fails to converge for different starting points. The subsystem analyses involve solving equations algebraically for a term in the system objective. The same decomposition and problem formulation used by NASA is used here for the MDF approach.

The MDF optimization problem is stated as follows:

Find the set of variables, x_i, i=1,3,6,7 that

Minimizes: $f_2(x) + f_6(x) + f_7(x) + f_9(x)$

Such that: $f_j(x) \geq 0.0, \quad j = 2,6,7,9$

Subsystem analyses 1 and 2 involve satisfying the remaining 6 equations f_k=0.0, k=1,3,4,5,8,10 and estimating the remaining variables. For the IDF and CO approaches, a decomposition consisting of 1 system and 2 subsystems is used. Figure 5-1 shows the inputs-outputs of the 2.

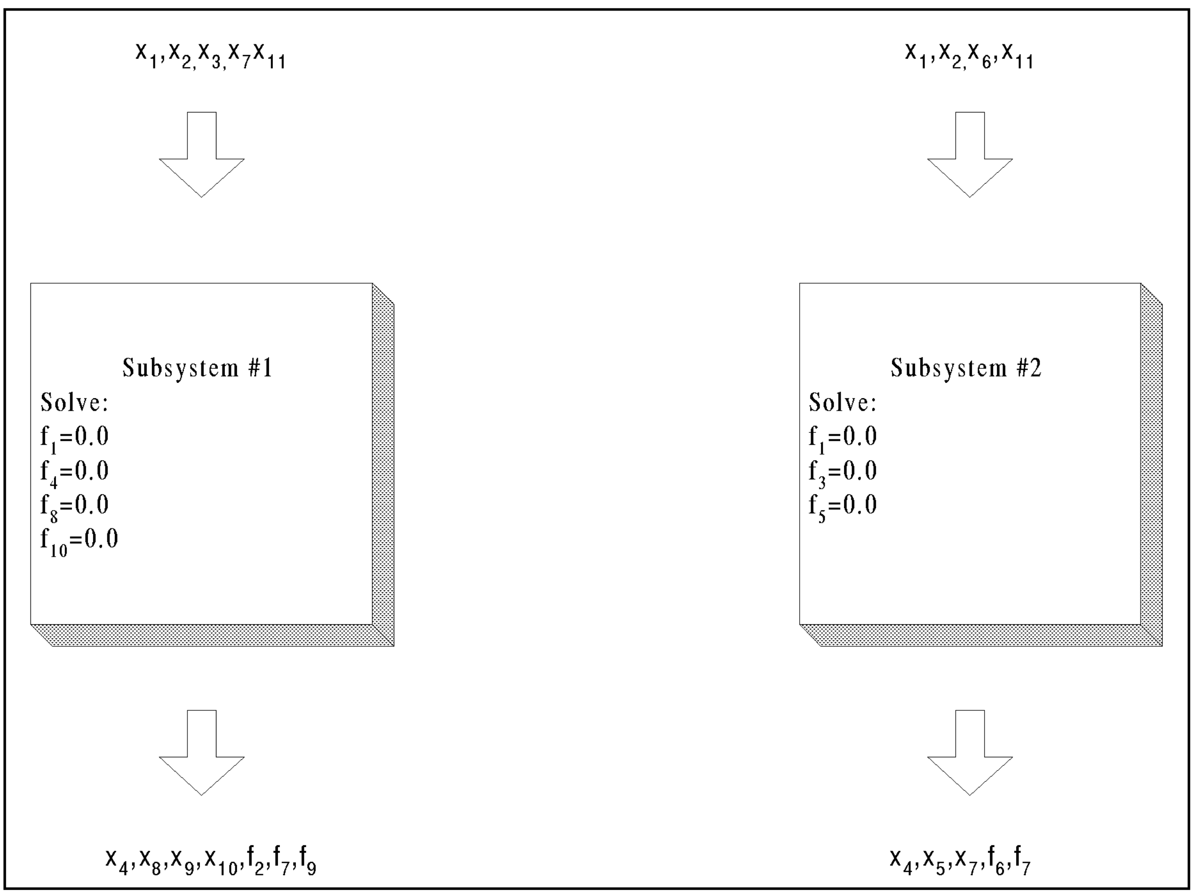

Figure 5.1: Inputs/Outputs of Subsystem Analyses

The IDF approach optimization problem is stated as follows:

Find the set of design variables, $Z^s{}_k, k = 1,2,4,7$ and $x_i = 3,6$

Minimizes: $F(x) = f_2 + f_6 + f_7 + f_9$

Subject to: $f_j \geq 0.0; j = 2,6,7,9$

$J_1 \leq 0.0001$

$J_2 \leq 0.0001$

and bounds on the design variables.

The subsystem evaluations are similar to the CO approach (outline follows).

The CO approach optimization problem is stated as:

Find the set of system design variables, $Z^s{}_k, k = 1,2,4,7$ that:

Minimizes: $F(z) = f_2 + f_6 + f_7 + f_9$

Subject to: $J_1 \leq 0.0001$

$J_2 \leq 0.0001$

$f_j \geq 0.0; j = 2,6,7,9$

and bounds on system variables.

The subsystem 1 optimization task is stated as:

Find the set of design variables, x, that:

Minimizes: $J_1 + f_2 + f_7 + f_9$

Subject to: $f_j \geq 0.0; j = 2,7,9$

and bounds on design variables.

Subsystem 1 has 4 local design variables including the following:

$x_1, x_2, x_{3,} x_7$ and $J_2 = (Z_1 - x_1)^2 + (Z_2 - x_2{}^2) + (Z_4 - x_4)^2 + (Z_7 - x_7)^2$

The subsystem 2 optimization task is stated as:

Find the set of design variables, x, that:

Minimizes: $J_2 + f_6 + f_7$

Subject to: $f_j \geq 0.0, j = 6,7$

and, bounds on design variables.

Subsystem 2 has 3 local design variables including x_1, x_2, x_6 and the following:

$$J_2 = (Z_1 - x_1)^2 + (Z_2 - x_2{}^2) + (Z_4 - x_4)^2 + (Z_7 - x_7)^2.$$

The Method of Feasible directions (MFD) implementation in iSIGHT was used to solve the MDF problem. The required derivatives were calculated using finite differences. For the CO approach, the SLP and MFD techniques were used for solving the system-level problem. The system-level derivatives were calculated analytically. The CO subsystem optimization tasks were solved using MFD. For the IDF approach, all the derivatives were computed using finite differences. The MDF results are tabulated in Table 5.1. The IDF and CO results are given in Tables 5.2 and 5.3.

Table 5.1: MDF Solutions
(4 system variables, 4 constraints)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation	Work
1	11.2374	+1.648 (1)	-0.0045	+0.003 (2)	306 x 14 x 3
2	2.09999	+3.59e-04 (1)	-0.0029	+0.0024 (3)	77 x 14 x 3
3	33.235	+42.739 (1)	-0.00025	+0.0026 (4)	376 x 14 x 3

Note: See page 1, section 2.0 for definition of "**Work**"

Table 5.2: CO Solutions
4 system variables

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	11.237	+1.64 (F2)	0.036	+0.00072 (J1)	112 system iter
2	2.099	+0.0 (J1,J2)	0.0458	+0.00064 (J1)	45 system iter (414,423) = 837
3	33.235	+42.739 (F2)	0.00176	+2.01e-05 (J2)	35 system iter
4	-6.12	+6.75 (F6)	0.0153	+0.00013 (J2)	47 system iter
5	-22.321	+20.20 (F2)	0.00705	+7.447e-05 (J1)	18 system iter

Note: See page 1, section 2.0 for definition of "**Work**"

Table 5.3: IDF Solutions
6 system variables

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	11.237	+1.64 (F2)	4.199	+0.00019	361 x 2
2	2.100	+0.0003 (F2)	0.0029	+0.00009 (J2)	272 x 2
3	33.235	+42.739 (F2)	0.00099	+0.00005 (J2)	254 x 2
4	-6.12	+6.75 (F6)	0.000058	+0.00011 (J2)	307 x 2
5	-22.32	+36.09 (F9)	$+1.34^{F}$	+0.00015 (J2)	541 x 2

Note: See page 1, section 2.0 for definition of "**Work**"

Note: The superscript "F" added to the value of the final objective indicates failure to converge to a Kuhn-Tucker point for the original problem.

Problem 6: Heart Dipole ([6], [12])

The hear dipole problem is formulated from the experimental electrolytic determination of the resultant dipole moment in the heart. The conventional solution procedure is to solve a set of nonlinear equations in 8 unknowns. The conventional heart dipole problem is stated as follows:

Given data $d_{mx}, d_{my}, d_A, d_B, d_C, d_D, d_E, d_F$

Find the values of x_i, such that:

$$
\begin{aligned}
f_1 &= x_1 + x_2 - d_{mx} = 0 \\
f_2 &= x_3 + x_4 - d_{my} = 0 \\
f_3 &= x_5 x_1 = x_6 x_2 - x_7 x_3 - x_8 x_4 - d_A = 0 \\
f_4 &= x_7 x_1 + x_8 x_2 + x_5 x_3 + x_6 x_4 - d_B = 0 \\
f_5 &= x_1\left(x_5^2 - x_7^2\right) - 2x_3 x_5 x_7 + x_2\left(x_6^2 - x_8^2\right) - 2x_4 x_6 x_8 - d_C = 0 \\
f_6 &= x_3\left(x_5^2 - x_7^2\right) + 2x_1 x_5 x_7 + x_4\left(x_6^2 - x_8^2\right) - 2x_2 x_6 x_8 - d_D = 0 \\
f_7 &= x_1 x_5\left(x_5^2 - 3x_7^2\right) + x_3 x_7\left(x_7^2 - 3x_5^2\right) + x_2 x_6\left(x_6^2 - 3x_8^2\right) + x_4 x_8\left(x_8^2 - 3x_6^2\right) d_E = 0 \\
f_8 &= x_3 x_5\left(x_5^2 - 3x_7^2\right) - x_1 x_7\left(x_7^2 - 3x_5^2\right) + x_4 x_6\left(x_6^2 - 3x_8^2\right) - x_2 x_8\left(x_8^2 - 3x_6^2\right) - d_F = 0
\end{aligned}
$$

The NASA MDO web site outlines a sample MDO formulation for the above problem using a system-level problem and two subsystem-level problems. The same NASA problem decomposition and formulation are used here for the MDF solution. The system-level problem is an optimization problem that can be solved by a nonlinear programming algorithm while the 2 subsystem problems are solved iteratively. The system analyses use a fixed-point iteration with relaxation to find consistent values of the subsystem variables. The subsystem analyses involve solving equations algebraically for terms in the system objection function.

The MDF optimization problem is stated as:

Find, $x_i, i = 1,4,6,7$ that

Minimizes: $f_5 + f_6 + f_7 + f_8$

Such that: $f_j \geq 0.0; j = 5,6,7,8$

Subsystem analyses 1 and 2 involve satisfying the remaining 4 equations $f_k = 0.0, k = 1,2,3,4$ and estimating the variables, $x_k, k = 2,3,5,8$.

For IDF and CO approaches, a decomposition consisting of 1 system and 2 subsystems is used. The inputs-outputs of the 2 subsystems are used. The inputs-outputs of the 2 subsystem analyses models are shown in the following figure:

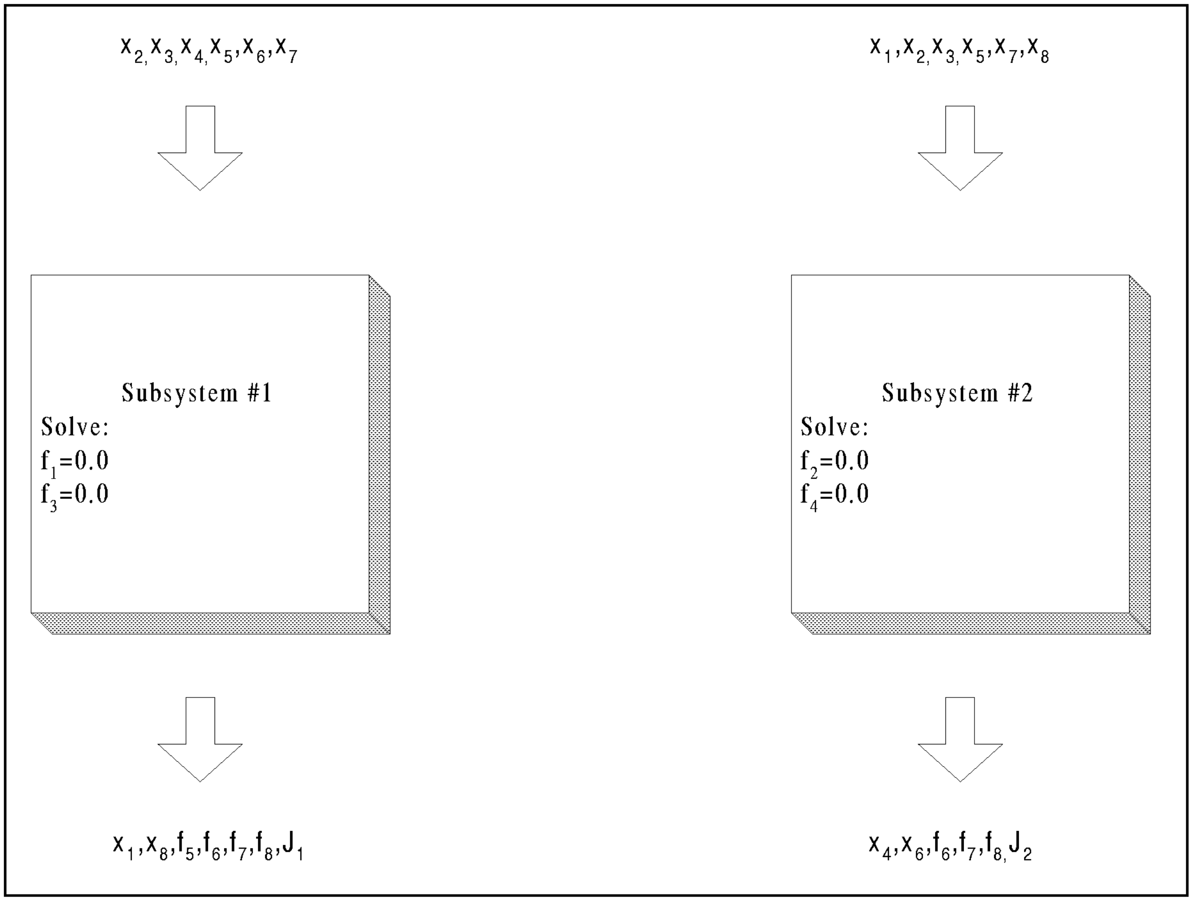

Figure 6.1: Inputs-outputs of Subsystem Analyses

The system analyses for IDF and CO models do not use any fixed-point iteration procedure. Instead f_5, f_6, f_7, f_8 are evaluated directly along with the subsystem analyses described in Figure 6.1.

The CO optimization problem is stated as follows:

Find the set of system-level design variables, $Z^s{}_k, k = 1.....,8$, that:

Minimizes: $F(z) = f_5 + f_6 + f_7 + f_8$

Subject to: $J_1 \le 0.0001$

$J_2 \le 0.0001$

$f_j \ge 0.0; j = 5,6,7,8$

and $J_i = (Z_i - x_i)^2, i = 1,8$

The subsystem 1 optimization task is stated as follows:

Find the set of design variables, $x_j, j = 2,3,4,5,6,7$ that:

Minimizes: J_1

Subject to: $f_j \geq 0.0; j = 5,6,7,8$

and $J_1 = (Z_i - x_i)^2, i = 1,8$

The subsystem 2 optimization task is stated as follows:

Find the set of design variables, $x_j, j = 1,2,3,5,7,8$

Minimizes: J_2

Subject to: $f_j \geq 0.0, j = 5,6,7,8$

and, $J_2 = (Z_i - x_i)^2, i = 1,8$

The Method of Feasible Directions (MDF) implementation in iSIGHT is used for solving the MDF problem. The required derivatives are computed by finite differences. For the CO and IDF approaches, the system-level optimization problem was solved using SLP and MFD techniques. The system-level problem derivatives were computed analytically. For the CO subsystem optimization tasks, MFD and SQP techniques were used.

The MDF results are tabulated in Table 6.1. The CO and IDF results are tabulated in Tables 6.2 and 6.3.

Table 6.1: MDF Solutions
8 design variables, 4 constraints

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation	Work
1	1.01780D+02	+2.80216D+00(2)	5.01214D-07	+6.30955D-06(3)	157 x 16 x 2
2	1.21959D+06	+2.20978D+04(3)	2.47324D-04	+1.43981D-04(3)	105 x 16 x 2
3	2.98644D+07	+3.25137D+07(4)	5.00218D+02	-2.78747D-03(4)	65 x 16 x 2
4	2.66044D+91	+1.23415D+91(4)	2.10243D+07	+1.01051D-04(1)	55 x 16 x 2
5	1.58985D+07	+6.25376D+03(1)	1.57269D+07	+5.97099D+03(1)	44 x 16 x 2
6	5.50505D+06	+5.50505D+06(4)	7.39551D+03	-8.06684D-02(3)	84 x 16 x 2
7	1.75011D+07	+1.62370D+07(4)	9.56141D+00	-3.60878D-06(4)	135 x 16 x 2
8	3.10670D+07	+3.17013D+07(4)	3.53168D+02	-7.37846D-05(2)	82 x 16 x 2
9	1.11673D+05	+1.43861D+02(1)	6.57112D+01	+3.35386D-04(4)	117 x 16 x 2
10	1.19023D+08	+1.16129D+08(4)	2.87504D+06	+1.30661D-05(1)	55 x 16 x 2
11	9.38816D+06	+1.01412D+06(3)	1.85216D+06	-7.81495D-06(1)	44 x 16 x 2
12	4.67166D+06	+1.48592D+05(3)	6.64397D+00	+1.94711D-05(3)	195 x 16 x 2

Note: See page 1, section 2.0 for definition of "**Work**"

Table 6.2: CO Solutions
8 system variables

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	24.5458	+16.301 (F8)	0.02545	+0.00019 (J1)	96 system iter (20034,20091)= 40125
2	0.2583	satisfied	0.0415	+0.00016 (J2)	51 system iter
3	3440302.15	+19207. (J2)	0.019	+0.0046 (J1)	873 system iter
4	1289711.6	+15587. (J1)	12.5^F	+0.44 (J1)	291 system iter
5	-0.00023	+0.0003 (F5)	0.00135	+0.0000	6 system iter

Note: See page 1, section 2.0 for definition of "**Work**"

Note: The superscript "F" added to the value of the final objective indicates failure to converge to a Kuhn-Tucker point for the original problem.

Table 6.3: IDF MDO Solutions
(8 system variables)

Case	Initial Design Objective	Initial Design Max Constraint Violation (System)	Final Design Objective	Final Design Max Constraint Violation (System)	Work
1	24.458	+16.301 (F3)	-4.2e-06	0.0001 (J1)	466 x 2
2	0.2583	satisfied	0.055	0.00019 (J1/J2)	204 x 2
3	3440302.15	+12685 (F5)	1047055.^{F}	0.28 (J1)	1204 x 2
4	1289711.6	+6253. (F5)	339598.^{F}	0.26 (J1)	515 x 2
5	0.2583	satisfied	-2.01e-05	.0001(J1/J2)	289 x 2

Note: See page 1, section 2.0 for definition of "**Work**"

Note: The superscript "F" added to the value of the final objective indicates failure to converge to a Kuhn-Tucker point for the original problem.

Problem 7: Hub Frame ([3])

A 20-member hub frame structure design is considered. The loads for the 2 loading cases, the material properties, and the modal coordinates are fixed, and the design problem is to find the optimum cross-sectional dimensions of the 20 members.

The hub frame analysis consists of the following steps:

1. Determining the area and moment of inertia of each member using the cross-sectional dimensions;

2. Performing a simple frame analysis to calculate the axial forces, shear forces and bending moments applied to each member and, in addition, calculating the system displacements and rotations;

3. Performing a member analysis using as inputs the area, inertia and member forces to calculate the member local stresses and local buckling of the web and flanges of the beam cross-section.

For each member, a total of 19 local stress and bucking constraints are calculated plus 2 system constraints (translational displacement and rotation) are calculated for each loading case. The total number of constraints for a hub frame of 20 members and 2 loading cases is ($(19 * 20 + 2) * 2 = 764$.

For the MDF approach, the optimization problem is stated as follows:

Find the set of design variables, X, that:

Minimizes: Hub frame volume/weight
Subject to: Displacement constraints, local member stress constraints, local buckling constraints, bounds on design variables.

A total of 120 design variables, including 6 cross-sectional dimensions $(b_1, b_2, b_3, h, t_1, t_2)$ for each of the 20 members is considered. The total number of inequality constraints is 764.

For the CO approach, the hub frame design problem is decomposed using a system-level problem and 2 subsystem-level problems. The 20-member frame is decomposed into 2 subsystems of 10 members each and the system level. The system-level variables include the area and moment of inertia for each member in the frame (20 member * 2 variables/member = 40 system variables). The system-level problem formulation consists of finding the system-level design variables that will minimize the hub frame volume while satisfying the subsystems compatibility function (J's) and displacement constraints. As part of the system-level analysis, a frame analysis is performed with the current values of the system design variables to determine the displacements and the internal member forces (axial, shear and bending moment). These member forces and the system-level design variables are used as input to the next step in the system-level analyses which is to perform the 2 subsystem optimizations.

For the subsystem optimization, the design variables include the actual 6 cross-sectional dimensions of the individual members (6/member * 10 members = 60 design variables in each subsystem). The subsystem optimization problem is to find the subsystem design variables that will minimize the compatibility function (J) subject to satisfying the local stress and buckling constraints on each member.

The CO system-level optimization task is stated as follows:

Find the set of design variables $\underline{Z}^s$, that:

Minimizes: Hub frame volume

Subject to: Displacement constraints

$$J1 \leq 0.0001$$

$$J2 \leq 0.0001$$

and bounds on design variables.

$\underline{Z}^s$ = {Area, Inertia of each member = 40 variables}

The CO subsystems optimization task is stated as follows:

Find the set of design variables, $\underline{x}^j$, that:

Minimizes: J_j

Subject to: Local stress constraints, local buckling constraints and bounds on design variables

where, $$J_j = \sum_{i=1}^{10} \left\{ \left(Z_i^s - A_i\right)^2 + \left(Z_i^s - I_i\right)^2 \right\}$$

A total of 60 design variables for each subsystem are considered.

For the MDF approach, the SLP implementation in iSIGHT is used. For the CO system optimization problem, a combination of SLP and Modified Method of Feasible Directions is used. The system-level problem gradients are computed analytically. The subsystem optimization problems are solved using SQP technique in iSIGHT. The results of the MDF and CO approaches are provided in Tables 7.1 and 7.2.

Table 7.1: MDF Solutions
(120 variables, 764 constraints)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation	Work
1	20939.9	+1.1e-03 (375)	11094.1	+1.121e-03 (415)	4365 x 1 x 2
2	23796.1	+2.2521 (375)	12309.3	+9.1e-04 (110)	1578 x 1 x 2
3	24221.9	+1.99 (375)	11293.8	+6.67e-04 (186)	4846 x 1 x 2
4	23688.0	+2.88 (299)	11064.6	+7.00e-04 (319)	4605 x 1 x 2
5	24292.9	+1.79 (375)	11096.2	+1.31e-03 (662)	4850 x 1 x 2
6	25142.2	+1.77 (374)	11622.7	+1.45e-03 (243)	2429 x 1 x 2
7	23060.7	+0.906 (376)	11249.1	+8.4e-04 (338)	3037 x 1 x 2
8	24969.5	+3.82 (375)	11535.7	+.5e-03 (241)	2179 x 1 x 2
9	22641.7	+2.03 (261)	11604.4	+1.30e-03 (434)	4845 x 1 x 2
10	23106.1	+1.313 (167)	12412.6	+2.21e-04 (384)	1700 x 1 x 2

Note: See page 1, section 2.0 for definition of "**Work**"

Table 7.2: CO Solutions
(40 system variables, 60 SS1 variables, 60 SS2 variables)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation	Work
1	20939.9	-0.0005 (J) +0.0011 (SS)	16391.1^F	+0.0047 (J1)	92 (196135,494923) =691058
2	23796.1	+0.00278 (J2) +2.252 (SS)	19322.3^F	+0.00385 (J1)	58
3	24221.9	+0.00039 (J1) +1.99 (SS)	20309.6^F	+0.0026 (J1)	19
4	23688.0	+0.1e-06 (J2) +2.88 (SS)	21527.7^F	+0.0024 (J1)	51

Note: See page 1, section 2.0 for definition of "**Work**"

Note: The superscript "F" added to the value of the final objective indicates failure to converge to a Kuhn-Tucker point for the original problem.

Problem 8: Isomerization Of $\propto$ - Pinene - Collocation Formulation ([2])

This problem involves determination of the reaction coefficients in the thermal isomerization of $\propto$ - pinene [Ref. MINPACK -2 Test Problems]. Collocation is used to approximate the solution of the differential equations that define the kinetics of the problem.

The $\propto$ - pinene problem is formulated as a nonlinear programming (NLP) problem subject to equality constraints, that represent the collocation equations.

The subroutine [Ref. MINPACK - 2]:

diacfj (m, n, x, fvec, fjac, ldfjac, task, nint, sigma)

defines the collocation formulation of the $\propto$ - pinene problem. The parameter "nint" decides the number of design variables and equality constraints in the $\propto$ - pinene - collocation formulation.

m = 25 * nint + 40 (equality constraints)
n = 25 * nint + 5 (design variables)

In this work, a value of 3 is used for nint resulting in 115 equality constraints and 80 design variables. The optimization objective function is calculated as the sum of squares of the first 10 components of the 115 equality constraints in array "fvec".

The NLP problem is now stated as follows:

Find the set of design variables, $\underline{x}_i$, i=1,80, that:

Minimizes: $\sum_{j}^{10}\left(fvec_j\right)^2$

Subject to: h_k=0.0; k=1, 115.

The NLP problem is solved for 6 starting points using the SQP algorithm in iSIGHT. The results are shown in Table 8-1.

Since a meaningful decomposition of the NLP problem is not possible, the decomposition based MDO methodologies (CO, IDF) are not used in solving this problem.

Problem 8: MINPACK 2: Isomerization of ∝-pinene - Collocation formulation:
(80 design variables, 115 equality constraints)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation
1	0.3226E-03	0.6250E+02(G96)	0.3984D-01	-0.7331D+00(G80)
2	0.3581E+16	-0.9439E+08(G34)	0.1020D+02	-0.5741D+01(G88)
3	0.5881E+15	0.1838E+09(G42)	0.3141D-01	0.5043D+01(G95)
4	0.3548E+14	-0.6428E+08(G50)	0.1563D+03	-0.1250D+02(G3)
5	0.1631E+15	-0.6151E+08(G17)	0.7879D+04	-0.8876D+02(G1)
6	0.1128E+17	0.1840E+09(G50)	0.3594D+01	-0.5017D+01(G93)

Problem 9: Propane, Isobutane, n-Butane Nonsharp Separation ([7])

This problem involves a three-component feed mixture that has to be separated into two three-component products. The recoveries of the key components are set to be greater than 0.85 to avoid the distribution of non-key components.

The nonlinear programming problem (NLP) is stated as:

Minimize: $a01 + \propto * x(5) + a02 + \beta * x(13)$

where,

$$\propto = a01 + a21 * x(21) + a31 * x(23) + bA1 * x(31) + bB1 * x(37)$$

$$\beta = a12 + a22 * x(22) + a32 * x(24) + bA2 * x(34) + bB2 * x(40)$$

Subject to:

$$h(1) = x(1) + x(2) + x(3) + x(4) - 300.0 = 0$$
$$h(2) = x(6) - x(7) - x(8) = 0$$
$$h(3) = x(9) - x(10) - x(11) - x(12) = 0$$
$$h(4) = x(14) - x(15) - x(16) - x(17) = 0$$
$$h(5) = x(18) - x(19) - x(20) = 0$$
$$h(6) = x(6) * x(32) - x(21) * x(25) = 0$$
$$h(7) = x(14) * x(41) - x(22) * x(28) = 0$$
$$h(8) = x(9) * x(39) - x(23) * x(27) = 0$$
$$h(9) = x(18) * x(48) - x(24) * x(30) = 0$$
$$h(10) = x(25) - x(5) * x(31) = 0$$
$$h(11) = x(27) - x(5) * x(37) = 0$$
$$h(12) = x(29) - x(5) * x(43) = 0$$
$$h(13) = x(26) - x(13) * x(34) = 0$$
$$h(14) = x(28) - x(13) * x(40) = 0$$
$$h(15) = x(30) - x(13) * x(46) = 0$$
$$h(16) = x(25) - x(6) * x(32) - x(9) * x(33) = 0$$
$$h(17) = x(27) - x(6) * x(38) - x(9) * x39 = 0$$
$$h(18) = x(29) - x(6) * x(44) - x(9) * x(45) = 0$$
$$h(19) = x(26) - x(14) * x(35) - x(18) * x(36) = 0$$
$$h(20) = x(28) - x(14) * x(41) - x(18) * x(42) = 0$$
$$h(21) = x(30) - x(14) * x(47) - x(18) * x(48) = 0$$
$$h(22) = 0.333 * x(1) + x(15) * x(35) - x(25) = 0$$

$$h(23) = 0.333 * x(1) + x(15) * x(41) - x(27) = 0$$
$$h(24) = 0.333 * x(1) + x(15) * x(47) - x(29) = 0$$
$$h(25) = 0.333 * x(2) + x(10) * x(33) - x(26) = 0$$
$$h(26) = 0.333 * x(2) + x(10) * x(39) - x(28) = 0$$
$$h(27) = 0.330 * x(2) + x(10) * x(45) - x(30) = 0$$
$$h(28) = x(44) = 0$$
$$h(39) = x(36) = 0$$
$$h(40) = 0.333 * x(3) + x(7) * x(32) + x(11) * x(33) + x(16) * x(35) + x(19) * x(36) - 30.0 = 0$$
$$h(41) = 0.333 * x(3) + x(7) * x(38) + x(11) * x(39) + x(16) * x(41) + x(19) * x(42) - 50.0 = 0$$
$$h(42) = 0.333 * x(3) + x(7) * x(44) + x(11) * x(45) + x(16) * x(47) + + x(19) * x(48) - 30.0 = 0$$
$$h(43) = x(31) + x(37) + x(43) - 1.0 = 0$$
$$h(44) = x(32) + x(38) + x(44) - 1.0 = 0$$
$$h(45) = x(33) + x(39) + x(45) - 1.0 = 0$$
$$h(46) = x(34) + x(40) + x(46) - 1.0 = 0$$
$$h(47) = x(35) + x(41) + x(47) - 1.0 = 0$$
$$h(48) = x(36) + x(42) + x(48) - 1.0 = 0$$
$$0.85 \le x(21), x(22), x(23), x(24) \le 1.0$$

The NLP problem has a total of 48 design variables, x, and 38 equality constraints. The fixed constants are given by the following:

Coefficient	Column I	Column II
$\alpha 0i$	0.23947	0.75835
$\alpha 1i$	-0.0139904	-0.0661588
$\alpha 2i$	0.0093514	0.0338147
$\alpha 3i$	0.0077308	0.0373349
bAi	-0.0005719	0.0016371
bBi	0.0042656	0.0288996

This NLP was solved for 10 different starting points using the SQP technique in iSIGHT. The results are summarized in Table 9.1.

Since a meaningful decomposition of this NLP problem is not possible, the decomposition - based MDO methodologies (CO, IDF) are not used for solving this problem.

Table 9.1: Propane, Isobutane, n-Butane Separation
(48 design variables, 38 equality constraints)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation	Work
1	1.0401	-45.667(G41)	1.40095	-0.00748(G9)	1574 x 1 x 1
2	3.4047	-279.63(G1)	1.35879	-17.0000(G6)	1552 x 1 x 1
3	2.4236	-276.15(G1)	1.48600	-0.78342(G6)	1623 x 1 x 1
4	3.2534	-282.95(G1)	1.22026	-18.3386(G6)	1550 x 1 x 1
5	2.3927	-273.23(G1)	1.93920	+0.45997(G39)	1544 x 1 x 1
6	1.6256	-279.29(G1)	1.64400	-13.2535(G8)	1592 x 1 x 1
7	2.9781	-287.05(G1)	1.36477	-2.25955(G4)	1565 x 1 x 1
8	2.3719	-280.53(G1)	1.89361	+0.000015(G41)	1549 x 1 x 1
9	2.0935	-289.20(G1)	1.57787	+0.0726(G40)	1568 x 1 x 1
10	1.2871	-281.31(G1)	1.55310	-7.30581(G7)	1362 x 1 x 1

Note: See page 1, section 2.0 for definition of "**Work**"

Problem 10: Three Component Separation – MINLP ([7])

This problem is similar to Problem 9. The composition of the desired products is different. It has additional design variables and 2 inequality constraints apart from the 38 equality constraints. There are also some differences in the objective function.

The nonlinear programming problem (NLP) is stated as:

Minimize: $a01 * x(51) + \propto * x(5) + a02 * x(52) + \beta * x(13)$

where,

$$\propto = a11 + a21 * x(21) + a31 * x(24) + bA1 * x(33) + bB1 * x(39)$$

$$\beta = a12 + a22 * x(49) + a32 * x(50) + bA2 * x(36) + bB2 * x(42)$$

Subject to:

$$h(1) = x(1) + x(2) + x(3) + x(4) - 300.0 = 0$$
$$h(2) = x(6) - x(7) - x(8) = 0$$
$$h(3) = x(9) - x(10) - x(11) - x(12) = 0$$
$$h(4) = x(14) - x(15) - x(16) - x(17) = 0$$
$$h(5) = x(18) - x(19) - x(20) = 0$$
$$h(6) = x(6) * x(32) - x(21) * x(25) = 0$$
$$h(7) = x(14) * x(41) - x(22) * x(28) = 0$$
$$h(8) = x(9) * x(39) - x(23) * x(27) = 0$$
$$h(9) = x(18) * x(48) - x(24) * x(30) = 0$$
$$h(10) = x(25) - x(5) * x(31) = 0$$
$$h(11) = x(27) - x(5) * x(37) = 0$$
$$h(12) = x(29) - x(5) * x(43) = 0$$
$$h(13) = x(26) - x(13) * x(34) = 0$$
$$h(14) = x(28) - x(13) * x(40) = 0$$
$$h(15) = x(30) - x(13) * x(46) = 0$$
$$h(16) = x(25) - x(6) * x(32) - x(9) * x(33) = 0$$
$$h(17) = x(27) - x(6) * x(38) - x(9) * x39 = 0$$
$$h(18) = x(29) - x(6) * x(44) - x(9) * x(45) = 0$$
$$h(19) = x(26) - x(14) * x(35) - x(18) * x(36) = 0$$
$$h(20) = x(28) - x(14) * x(41) - x(18) * x(42) = 0$$
$$h(21) = x(30) - x(14) * x(47) - x(18) * x(48) = 0$$
$$h(22) = 0.333 * x(1) + x(15) * x(35) - x(25) = 0$$

$$h(23) = 0.333 * x(1) + x(15) * x(41) - x(27) = 0$$
$$h(24) = 0.333 * x(1) + x(15) * x(47) - x(29) = 0$$
$$h(25) = 0.333 * x(2) + x(10) * x(33) - x(26) = 0$$
$$h(26) = 0.333 * x(2) + x(10) * x(39) - x(28) = 0$$
$$h(27) = 0.330 * x(2) + x(10) * x(45) - x(30) = 0$$
$$h(28) = x(44) = 0$$
$$h(39) = x(36) = 0$$
$$h(40) = 0.333 * x(3) + x(7) * x(32) + x(11) * x(33) + x(16) * x(35) + x(19) * x(36) - 30.0 = 0$$
$$h(41) = 0.333 * x(3) + x(7) * x(38) + x(11) * x(39) + x(16) * x(41) + x(19) * x(42) - 50.0 = 0$$
$$h(42) = 0.333 * x(3) + x(7) * x(44) + x(11) * x(45) + x(16) * x(47) + + x(19) * x(48) - 30.0 = 0$$
$$h(43) = x(31) + x(37) + x(43) - 1.0 = 0$$
$$h(44) = x(32) + x(38) + x(44) - 1.0 = 0$$
$$h(45) = x(33) + x(39) + x(45) - 1.0 = 0$$
$$h(46) = x(34) + x(40) + x(46) - 1.0 = 0$$
$$h(47) = x(35) + x(41) + x(47) - 1.0 = 0$$
$$h(48) = x(36) + x(42) + x(48) - 1.0 = 0$$
$$0.85 \le x(21), x(22), x(23), x(24) \le 1.0$$

In addition, there are 2 inequality constraints given by:

$$g(1) = x(5) - 300.0 * x(51)$$
$$g(2) = x(13) - 300.0 * x(52)$$

The NLP problem has a total of 52 design variables, x, and 38 equality constraints and 2 inequality constraints. The fixed constants are given by the following:

Coefficient	**Column I**	**Column II**
$\alpha 0i$	0.23947	0.75835
$\alpha 1i$	-0.0139904	-0.0661588
$\alpha 2i$	0.0093514	0.0338147
$\alpha 3i$	0.0077308	0.0373349
bAi	-0.0005719	0.0016371
bBi	0.0042656	0.0288996

This NLP was solved for 6 different starting points using the SQP technique in iSIGHT. The results are summarized in Table 10.1.

Since a meaningful decomposition of this NLP problem is not possible, the decomposition - based MDO methodologies (CO, IDF) are not used for solving this problem.

Table 10.1: Three Component Separation - MINLP:
(52 design variables, 38 equality constraints, 2 inequality)

Case	Initial Design Objective	Initial Design Max Constraint Violation	Final Design Objective	Final Design Max Constraint Violation
1	0.281783	-0.2960e+03(G1)	0.84548D+00	-0.1369D-03(G6)
2	0.39490E+01	-0.2796E+03(G1)	0.55639D+00	-0.5100D+02(G6)
3	0.17138E+01	-0.2741E+03(G1)	0.84136D+00	-0.8480D-05(G5)
4	0.25693E+01	-0.2859E+03(G1)	0.80888D+00	-0.1029D-05(G8)
5	0.36877E+01	-0.2735E+03(G1)	0.86809D+00	-0.2533D-04(G5)
6	0.35661E+01	-0.2711E+03(G1)	0.95908D+00	0.4214D-04(G6)

4.0 Concluding Remarks

The 10 problems, identified by NASA, were solved using an iSIGHT MDO language based implementation of MDF, IDF and CO approaches for several starting points. Not all 10 problems were solved by all methods as some were deemed unsuitable. The problem dimensions are summarized in Table 3.1; convergence to the best known optimal solution from different starting points is summarized in Table 3.2; and representative work done is summarized in Table 3.3.

We realize that the formulation of the problems and their implementation has a direct bearing on performance. Given the limitations of the problems, the testing, and the implementation, we were still able to discern specific trends in the performance of each method that support its theoretical properties. The specifics of the implementation, the analysis of performance and the available conclusions will be presented in detail in [1].

Table 3.1: Problem Dimensionality

Problem #	Ship 1	Epack 2	Power 3	Speed 4	Combustion 5	Heart 6	Hub 7	Iso 8	Prop. Iso. 9	3 Comp Sep. 10
MDF										
# of Variables	6	8	6	7	4	4	120	80	48	52
# of Constraints	7	3	4	11	4	4	764	115 (equality)	38 (equality)	40 (equality)
IDF										
# of Variables	14	12	12	-	6	8	-	-	-	-
# of Constraints	11	5	6	-	6	6	-	-	-	-
CO										
System:										
# of Variables	11	5	6	2	4	8	40	-	-	-
# of Constraints	5	2	6	3	2	6	6	-	-	-
# of Subsystems	5	2	2	3	2	2	2	-	-	-
Total # of Subsystem variables	18	12	12	11	7	12	120	-	-	-

Table 3.2: Convergence Results from the 3 MDO Approaches

Problem #	Ship 1	Epack 2	Power 3	Speed 4	Combustion 5	Heart 6	Hub 7	Iso 8	Prop. Iso. 9	3 Comp Sep. 10
MDF (# Converged/ # attempted)	$^{12}/_{12}$	$^{12}/_{12}$	$^{12}/_{12}$	$^{12}/_{12}$	$^{3}/_{3}$	$^{5}/_{12}$	$^{10}/_{10}$	$^{1}/_{6}$	$^{5}/_{10}$	$^{5}/_{6}$
IDF	$^{4}/_{4}$	$^{4}/_{4}$	$^{4}/_{5}$	-	$^{4}/_{5}$	$^{3}/_{5}$	-	-	-	-
CO	$^{4}/_{4}$	$^{2}/_{4}$	$^{3}/_{4}$	$^{4}/_{5}$	$^{5}/_{5}$	$^{4}/_{5}$	$^{0}/_{5}$	-	-	-

Table 3.3: Average Number of Analyses for Convergence

(No usage of any formal Approximations)

Problem #	Ship 1	Epack 2	Power 3	Speed 4	Combustion 5	Heart 6	Hub 7	Iso 8	Prop. Iso. 9	3 Comp Sep. 10
MDF (including finite diff. calls)	667	275	1025	77	10626	3035	6887	245	1547	1353
IDF (including finite diff. calls)	9640	6019	406	-	694	1071	-	-	-	-
CO System:	152	113	54	5	52	96*	92*	-	-	-
Subsystem: (including finite diff. Calls)	13065	18005	2983	2102*	837*	40125*	691058*	-	-	-

* Not an average number (instead, based on a single data point)

Appendix 1: Implementation Details

All of the 3 methods (MDF, IDF, CO) were implemented in iSIGHT using its MDO Language (MDOL). iSIGHT provides several numerical optimization, genetic search and heuristic search algorithms for solving the optimization problem. These different algorithms can be easily combined together to create an hybrid optimization plan that can be effectively used for solving the optimization problem.

A.1 MDF Implementation

The MDF approach implementation is relatively simple since no decompositions of the optimization problem are involved. The termination criteria for the MDF problem included the satisfaction of Kuhn-Tucker conditions, absolute and relative change in the objective function between successive iterations and maximum number of iterations.

A.2 CO Implementation

The CO method was implemented as an hierarchical optimization model involving a system optimization task and several subsystem optimization tasks in iSIGHT. The CO problem formulation and implementation in iSIGHT are similar to previously published works [4] with the following variations:

(i) At the system level optimization task, the interdisciplinary compatibility constraints (J's) were formulated as inequality constraints (J <= 0.0001) as against strict equality constraints (J = 0.0). J is defined as:

Jj = | Xj - Zj |**2 + | Yj - Zj |**2

(ii) At the system level optimization task, constraints other than compatibility constraints were considered. It is necessary to point out that such consideration of additional constraints, may add significantly to the computational cost if their gradients were to be calculated by finite difference.

(iii) The subsystem optimization objectives for some of the problems included other terms in addition to the compatibility function. Such formulation of the subsystem objectives required appropriate weighting of the different terms of the cumulative objective function.

(iv) The subsystem optimization tasks were not necessarily solved to convergence for each system level evaluation. Instead, the subsystem optimization were run for a minimal number of iterations (1 to 5) and the resulting J's passed back to the system. This did not affect convergence to the MDF solution, however, it has the potential of significantly reducing the computational cost associated with the system level evaluations.

(v) The starting point for the subsystem optimization local variables, for each system level evaluation, was from the previously obtained best design.

A.3 IDF Implementation:

The IDF method was implemented as an hierarchical model involving a system optimization task and several subsystem analysis tasks in iSIGHT. The IDF problem formulation and implementation in iSIGHT are similar to previously published works [5] with the following variation:

(i) At the system optimization task, the compatibility constraints (J's) were formulated as inequality constraints (J <= 0.0001) as against strict equality constraints (J = 0.0). J is defined as

$$Jj = | Xj - Zj |^{**}2 + | Yj - Zj |^{**}2$$

A.4 Sample Description Files:

In order to provide the reader with the implementation details of the 3 MDO methods, the iSIGHT problem description (MDOL-based) files for the Electronic Packaging problem are included below.

MDF Method Description File:

```
###########################################################
#
#    TASK MDF description file for Electronic Packaging
#
#    SYMBOLS
#         System Variable:        YiS
#         Coupling Variable:       Yi
#         Local Variable:          Xi
#
###########################################################
MDOLVersion: 3.0

Task ElectronicPackage

#~~~~~~~~~~~~~~~~~~~~~~~~~~ TaskHeader ~~~~~~~~~~~~~~~~
  TaskHeader ElectronicPackage
      Evaluation: optimize
      ControlMode: expertauto
      Precision: double
      RunCounter: 1
  End TaskHeader ElectronicPackage
```

```
#~~~~~~~~~~~~~~~~~~~~~~~~~~ Inputs ~~~~~~~~~~~~~~~~~~~~
  Inputs ElectronicPackage
      ParameterList DesignGroup
          Type: real
          Parameters
              X1    InitialValue: 0.15
              X2    InitialValue: 0.15
              X3    InitialValue: 0.10
              X4    InitialValue: 0.05
              X5    InitialValue: 1000.0
              X6    InitialValue: 0.009
              X7    InitialValue: 1000.0
              X8    InitialValue: 0.009
      End ParameterList

      P: NStates
          T: integer  I: 12
          D: "Number of initial states"
      P: State
          T: integer  I: 1
          D: "Current initial state"
  End Inputs ElectronicPackage

#~~~~~~~~~~~~~~~~~~~~~~~~~~ Outputs ~~~~~~~~~~~~~~~~~~~~
  Outputs ElectronicPackage
      Parameter: Y1 T: real
      Parameter: H1 T: real
      Parameter: G1 T: real
      Parameter: G2 T: real
  End Outputs ElectronicPackage

#~~~~~~~~~~~~~~~~~~~~~~ Initialization ~~~~~~~~~~~~~~~~~~
  Initialization ElectronicPackage
      Tcl
          global DeltaForInEqualityConstraintViolation
          set DeltaForInEqualityConstraintViolation .004
      End Tcl
  End Initialization ElectronicPackage

#~~~~~~~~~~~~~~~~~~~~~~~~~~ SimCode ~~~~~~~~~~~~~~~~~~~~~
  SimCode EPackageCode
      InputFiles EPackageCode
          FileDescription farFile0
              FileType: standard
              InputFile: "package.in"
              Language: emacs
              Parameters
```

```
            X1 X2 X3 X4 X5 X6 X7 X8
        Instructions
            write $X1
            write $Newline
            write $X2
            write $Newline
            write $X3
            write $Newline
            write $X4
            write $Newline
            write $X5
            write $Newline
            write $X6
            write $Newline
            write $X7
            write $Newline
            write $X8
            write $Newline
        End Instructions
    End FileDescription farFile0
End InputFiles EPackageCode

OutputFiles EPackageCode
    FileDescription farFile2
        FileType: standard
        OutputFile: "test.out"
        Language: emacs
        Parameters
            Y1 H1 G1 G2
        Instructions
            find "Original Objective=  "
            read Y1
            provide $Y1
            find "System level constraints="
            moveto $Line_Start
            moveto line + 1
            moveto word + 1
            read H1
            provide $H1
            moveto $Line_Start
            moveto line + 1
            moveto word + 1
            read G1
            provide $G1
            moveto $Line_Start
            moveto line + 1
            moveto word + 1
```

```
                read G2
                provide $G2
            End Instructions
        End FileDescription farFile2
    End OutputFiles EPackageCode

    SimCodeProcess EPackageCode
        Program: "package.exe"
        ElapseTime: 100s
        Prologue
            WriteInputSpecs: farFile0
        Epilogue
            ReadOutputSpecs: farFile2
    End SimCodeProcess EPackageCode
  End SimCode EPackageCode

#~~~~~~~~~~~~~~~~~~~~~~~~~~ TaskProcess ~~~~~~~~~~~~~~~~~~
  TaskProcess ElectronicPackage
    Control: [ EPackageCode ]
  End TaskProcess ElectronicPackage
#~~~~~~~~~~~~~~~~~~~~~~~~~~ Optimization ~~~~~~~~~~~~~~~~~
  Optimization ElectronicPackage
    PotentialVariables: InputsGroup
    Variables: DesignGroup
    InputConstraints
     Parameter: X1 LB: 0.05  UB: 0.15
     Parameter: X2 LB: 0.05  UB: 0.15
     Parameter: X3 LB: 0.01  UB: 0.10
     Parameter: X4 LB: 0.005 UB: 0.05
     Parameter: X5 LB: 10.0  UB: 1000.0
     Parameter: X6 LB: 0.004 UB: 0.009
     Parameter: X7 LB: 10.0  UB: 1000.0
     Parameter: X8 LB: 0.004 UB: 0.009
    PotentialObjectives: OutputsGroup
    Objectives
        Parameter: Y1
        Direction: minimize
        Weight: 1.0
    OutputConstraints
     Parameter: H1  UB: 0.0
     Parameter: G1  UB: 0.0
     Parameter: G2  UB: 0.0
    OptimizePlan ExploitivePlan
        OptimizeStep mmfd
            Technique: "Modified Method of Feasible
Directions"
                Epilog
```

```
                      Tcl   api_RestoreBestSolution
ElectronicPackage  End Tcl
                  Options
                      NumberOfIterations: 40
                      FiniteDifference: 0.01
                      MinimumFiniteDifference: 0.01
                      PrintLevel: 3552
          Control: [ mmfd ]
  End Optimization ElectronicPackage
#~~~~~~~~~~~~~~~~~~~~~~~~~~ DataStorge ~~~~~~~~~~~~~~~~~~
  DataStorage ElectronicPackage
      Restore: yes
      DataLog: "package.db" Mode: overwrite
      DataLookUp: "package-in.db"
  End DataStorage ElectronicPackage

#~~~~~~~~~~~~~~~~~~~~~~~~~~ Knowledge ~~~~~~~~~~~~~~~~~~~
  Knowledge ElectronicPackage
# This rule is intended to re-initialize the design
variables with
# a new starting point and execute the optimization plan
again:
#
#   If State <= NStates then
#       Initialize design variables,
#       Forget previous best solution,
#       Increment State
        Rule NextState
            Type: knowledgeguided
            Conditions
                Get: i = VariableValue State
                Get: n = VariableValue NStates
                Eval: (test (<= ?i ?n))
            Actions
                Eval: (format nil "CreateStates
[api_GetParameterValue [api_GetTaskName] NStates]")
                Eval: (format nil "ReadState
[api_GetParameterValue [api_GetTaskName] State]")
                Eval: (format nil "api_UnsetBestRunInfo
[api_GetTaskName]")
                Set: VariableValue State (+ ?i 1)
        End Rule NextState
# This rule is here only to override the default
consequence
# rule, since we do not want the latter suspending the
knowledge
# guided rule because the objective is not improving.  That
```

```
# funny action is there just to have an action that does
nothing.
        Rule NextState
            Type: consequence
            Conditions
                Check: KnowledgeGuidedRuleName NextState
            Actions
                Eval: (format nil "")
        End Rule NextState
  End Knowledge ElectronicPackage
#~~~~~~~~~~~~~~~~~~~~~~~~~~~ Procedures ~~~~~~~~~~~~~~~~~~
  Procedures ElectronicPackage
      TclSourceFiles: "msrandom.tcl"
  End Procedures ElectronicPackage
End Task ElectronicPackage
```

IDF Method Description Files:

```
###########################################################
#
#  TASK IDF SYSTEM description file for Electronic Package
#
#    SYMBOLS
#         System Variable:         YiS
#         Coupling Variable:       Yi
#         Local Variable:          Xi
#
###########################################################
MDOLVersion: 3.0

Task EPackage
#~~~~~~~~~~~~~~~~~~~~~~~~ TaskHeader ~~~~~~~~~~~~~~~~~~~~
  TaskHeader EPackage
      Evaluation: optimize
      ControlMode: user
      Precision: double
      RunCounter: 1
  End TaskHeader EPackage
#~~~~~~~~~~~~~~~~~~~~~~~~ Inputs ~~~~~~~~~~~~~~~~~~~~~~~~
  Inputs EPackage
      ParameterList SystemTargets
          Type: real
          Parameters
              X1   I: 1.0
              X2   I: 1.0
              X3   I: 1.0
              X4   I: 1.0
```

```
            X5   I: 1.0
            X6   I: 1.0
            X7   I: 1.0
            X8   I: 1.0
            Y2S  I: 1.0
            Y3S  I: 1.0
            Y11S I: 1.0
            Y12S I: 1.0
        End ParameterList
    End Inputs EPackage
#~~~~~~~~~~~~~~~~~~~~~~~~ Outputs ~~~~~~~~~~~~~~~~~~~~~~~~~
    Outputs EPackage
        ParameterList SystemOutput
            Type: real
            Parameters
                Y1 D1 D2 H1
        End ParameterList
    End Outputs EPackage

#~~~~~~~~~~~~~~~~~~~~~ Initialization ~~~~~~~~~~~~~~~~~~
    Initialization EPackage
        Parameters
            Tcl
                set Y2S(Scale) 0.37149E+03
                set Y3S(Scale) 0.31598E+02
                set Y11S(Scale) 0.36753E+02
                set Y12S(Scale) 0.55072E+02
                set X1(Scale) 0.089646
                set X2(Scale) 0.134049
                set X3(Scale) 0.041800
                set X4(Scale) 0.025096
                set X5(Scale) 325.505845
                set X6(Scale) 0.008432
                set X7(Scale) 25.427021
                set X8(Scale) 0.006920
                global PenaltyMultiplier
                set PenaltyMultiplier 1000000000.0
                global DeltaForInEqualityConstraintViolation
                set DeltaForInEqualityConstraintViolation
0.0001
            End Tcl
    End Initialization EPackage

#~~~~~~~~~~~~~~~~~~~~~~~~ Include ~~~~~~~~~~~~~~~~~~~~~~
    Include From "electIDF.desc"
        Component: ElectricalAnalysis
    End Include
```

```
  Include From "thermalIDF.desc"
      Component: ThermalAnalysis
  End Include

#~~~~~~~~~~~~~~~~~~~~~~ TaskProcess ~~~~~~~~~~~~~~~~~~~~
  TaskProcess EPackage
      Control: [ ElectricalAnalysis ThermalAnalysis ]

      SubTask ElectricalAnalysis
          ParameterMap
              Y2S = Y2S
              Y3S = Y3S
              Y11S= Y11S
              Y12S= Y12S
          InputToSubtask
              Send: X1 X2 X3 X4 X5 X6 X7 X8
              Send: Y2S Y3S Y11S Y12S
          OutputFromSubtask
              Receive: D2
          End SubTask ElectricalAnalysis

      SubTask ThermalAnalysis
          ParameterMap
              Y2S = Y2S
              Y3S = Y3S
              Y11S= Y11S
              Y12S= Y12S
          InputToSubtask
              Send: X1 X2 X3 X4 X5 X6 X7 X8
              Send:  Y2S Y3S Y11S Y12S
          OutputFromSubtask
              Receive: D1 H1 Y1
      End SubTask ThermalAnalysis
  End TaskProcess EPackage

#~~~~~~~~~~~~~~~~~~~~~~ Optimization ~~~~~~~~~~~~~~~~~
  Optimization EPackage
      PotentialVariables: InputsGroup
      Variables: SystemTargets
      InputConstraints
          Parameter: X1   LB: 0.557749    UB: 1.67325
          Parameter: X2   LB: 0.372998    UB: 1.11899
          Parameter: X3   LB: 0.239234    UB: 2.39234
          Parameter: X4   LB: 0.199235    UB: 1.99235
          Parameter: X5   LB: 3.07214E-02 UB: 3.07214
          Parameter: X6   LB: 0.474383    UB: 1.06736
```

```
        Parameter: X7   LB: 0.393282    UB: 39.3282
        Parameter: X8   LB: 0.578035    UB: 1.30058
        Parameter: Y11S LB: 0.01        UB: 2.31
        Parameter: Y12S LB: 0.01        UB: 1.54
        Parameter: Y2S  LB: 0.01        UB: 100.0
        Parameter: Y3S  LB: 0.01        UB: 100.0
    PotentialObjectives: OutputsGroup
    Objectives
        Parameter: Y1
        Direction: minimize
        Weight: 1.0
    OutputConstraints
        Parameter: D1  UB: 0.0001
     Parameter: D2  UB: 0.0001
        Parameter: H1  Eq: 0.00
    OptimizePlan ExploitivePlan
        Prolog
            Tcl
                global DataBaseLog Best
                set Best(ObjectiveandPenalty) 1.E+31
                set Best(Objective) 1.E+31
                set DataBaseLog(Status) append
            End Tcl
        Epilog
            Tcl rerunbest End Tcl
        OptimizeStep dlp1
            Technique: "Sequential Quadratic Programming
- DONLP"
                Options
                    Maxiter: 250
                    Tau0: 1.0
                    Del0: 1.0
                    Epsdif: 0.0001
        OptimizeStep dlp2
            Technique: "Sequential Quadratic Programming
- DONLP"
                Options
                    Maxiter: 250
                    Tau0: 1.0
                    Del0: 1.0
                    Deldif: 0.000001
                    Epsdif: 0.0001
        Control: [ dlp1 dlp2 ]
  End Optimization EPackage

#~~~~~~~~~~~~~~~~~~~~~~ DataStorge ~~~~~~~~~~~~~~~~~~~~~~~~
  DataStorage EPackage
```

```
      Restore: yes
      DataLog: "package.db" Mode: overwrite
      DataLookUp: "package-in.db"
  End DataStorage EPackage

#~~~~~~~~~~~~~~~~~~~~~~ Procedures ~~~~~~~~~~~~~~~~~~~~~~~~
  Procedures EPackage
      TclSourceFiles: "rerunbest.tcl"
  End Procedures EPackage
```

End Task EPackage

Subsystem 1: Thermal Subsystem

```
###########################################################
#
#       TASK IDF Sub-System 2 description file (Thermal)
#
#    SYMBOLS
#         System Variable:          YiS
#         Coupling Variable:        Yi
#         Local Variable:           Xi
#
###########################################################
MDOLVersion: 3.0

Task ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ TaskHeader ~~~~~~~~~~~~~~~~~~~~~
  TaskHeader ThermalAnalysis
      Evaluation: single
      ControlMode: user
      Precision: double
      RunCounter: 1
  End TaskHeader ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Inputs ~~~~~~~~~~~~~~~~~~~~~~~~~
  Inputs ThermalAnalysis
      ParameterList SystemTargetedInput
          Type: real
          Parameters
              X1   I: 1.0
              X2   I: 1.0
```

```
               X3   I: 1.0
               X4   I: 1.0
               X5   I: 1.0
               X6   I: 1.0
               X7   I: 1.0
               X8   I: 1.0
               Y2S  I: 1.0
               Y3S  I: 1.0
               Y11S I: 1.0
               Y12S I: 1.0
       End ParameterList
   End Inputs ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~~ Auxilaries ~~~~~~~~~~~~~~~~~~~
   Auxiliaries ThermalAnalysis
       Parameter: ScaledParmList Type:discrete
   End Auxiliaries ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~~ Outputs ~~~~~~~~~~~~~~~~~~~~~~
   Outputs ThermalAnalysis
       ParameterList Sub1Output
           Type: real
           Parameters
               D1 Y1 Y10 Y11 Y12 Y13 H1
       End ParameterList
   End Outputs ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~~ Initialization ~~~~~~~~~~~~~~~~
   Initialization ThermalAnalysis
       Parameters
           ScaledParmList SystemTargetedInput Sub1Output
       Steps
           Tcl
               set Y1(Scale) 0.68363E+04
               set Y11(Scale) 0.36753E+02
               set Y12(Scale) 0.55072E+02
               set Y2S(Scale) 0.37149E+03
               set Y3S(Scale) 0.31598E+02
               set Y11S(Scale) 0.36753E+02
               set Y12S(Scale) 0.55072E+02
               set X1(Scale) 0.089646
               set X2(Scale) 0.134049
               set X3(Scale) 0.041800
               set X4(Scale) 0.025096
               set X5(Scale) 325.505845
               set X6(Scale) 0.008432
               set X7(Scale) 25.427021
```

```
              set X8(Scale) 0.006920
              global PenaltyMultiplier
              set PenaltyMultiplier 1000000.0
              global DeltaForInEqualityConstraintViolation
              set DeltaForInEqualityConstraintViolation
0.004
              global DeltaForEqualityConstraintViolation
              set DeltaForEqualityConstraintViolation
0.0001
          End Tcl
  End Initialization ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~~ Calcualtions ~~~~~~~~~~~~~~~~~~~~
  Calculations ThermalAnalysis
      Calculation CalcD1
          Parameters
              InputsGroup OutputsGroup
          Tcl
              set t1  [expr ($Y11(V)- $Y11S(V))*($Y11(V)-
$Y11S(V))]
              set t2  [expr ($Y12(V)- $Y12S(V))*($Y12(V)-
$Y12S(V))]
              set D1(V) [expr $t1+$t2]
          End Tcl
      End Calculation CalcD1
  End Calculations ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~~ SimCode ~~~~~~~~~~~~~~~~~~~~~~~~~
  SimCode ThermalAnalysisCode
      InputFiles ThermalAnalysisCode
          FileDescription farFile0
              FileType: standard
              NameValueFile: "thermal-in.nv"
              InputFile: "package.in"
              Language: emacs
              Parameters
                  InputsGroup
              Instructions
                  write $X1
                  write $Newline
                  write $X2
                  write $Newline
                  write $X3
                  write $Newline
                  write $X4
                  write $Newline
                  write $X5
```

```
                write $Newline
                write $X6
                write $Newline
                write $X7
                write $Newline
                write $X8
                write $Newline
                write $Y2S
                write $Newline
                write $Y3S
                write $Newline
            End Instructions
        End FileDescription farFile0
    End InputFiles ThermalAnalysisCode

    OutputFiles ThermalAnalysisCode
        FileDescription farFile2
            FileType: standard
            OutputFile: "package.out"
            NameValueFile: "thermal-out.nv"
            Language: emacs
            Parameters
                OutputsGroup
            Instructions
                read Y1
                provide $Y1
                read Y10
                provide $Y10
                read Y11
                provide $Y11
                read Y12
                provide $Y12
                read Y13
                provide $Y13
                read H1
                provide $H1
            End Instructions
        End FileDescription farFile2
    End OutputFiles ThermalAnalysisCode

    SimCodeProcess ThermalAnalysisCode
        Program: "package.exe"
        ElapseTime: 100s
        Prologue
            Tcl
                UnScaleParameters
            End Tcl
```

```
            WriteInputSpecs: farFile0
        Epilogue
            ReadOutputSpecs: farFile2
            Tcl
                ScaleParameters
            End Tcl
    End SimCodeProcess ThermalAnalysisCode
  End SimCode ThermalAnalysisCode

#~~~~~~~~~~~~~~~~~~~~~~~~ TaskProcess ~~~~~~~~~~~~~~~~~~~
  TaskProcess ThermalAnalysis
    Control: [ ThermalAnalysisCode CalcD1]
  End TaskProcess ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Optimization ~~~~~~~~~~~~~~~~~~
  Optimization ThermalAnalysis
    PotentialVariables: InputsGroup
    Variables: SystemTargetedInput
    PotentialObjectives: OutputsGroup
    Objectives
        Parameter: D1
        Direction: minimize
        Weight: 1.0
    OptimizePlan ExploitivePlan
        Prolog
            Tcl
                global DataBaseLog Best
                set Best(ObjectiveAndPenalty) 1.E+31
                set Best(Objective) 1.E+31
                set DataBaseLog(Status) append
            End Tcl
        Epilog
            Tcl rerunbest End Tcl
        OptimizeStep mmfd
            Technique: "Modified Method of Feasible
Directions"
                Options
                    NumberOfIterations: 40
                    Auto: off
                    FiniteDifference: 0.01
                    MinimumFiniteDifference: 0.01
        OptimizeStep dlp
            Technique: "Sequential Quadratic Programming
- DONLP"
                Options
                    Maxiter: 500
                    Tau0: 1.0
```

```
                      Del0: 1.0
                      Epsdif: 0.0001
          Control: [ dlp mmfd ]
  End Optimization ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ DataStorge ~~~~~~~~~~~~~~~~~~~~
  DataStorage ThermalAnalysis
      Restore: yes
      DataLog: "thermal.db" Mode: overwrite
      DataLookUp: "thermal-in.db"
  End DataStorage ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Procedures ~~~~~~~~~~~~~~~~~~~~
  Procedures ThermalAnalysis
      TclSourceFiles: "rerunbest.tcl" "parmscale.tcl"
  End Procedures ThermalAnalysis
```

End Task ThermalAnalysis

Subsystem 2: Electrical Subsystem

```
###########################################################
#
#    TASK  IDF Sub-System 1 description file (Electrical)
#
#    SYMBOLS
#         System Variable:         YiS
#         Coupling Variable:       Yi
#         Local Variable:          Xi
#
###########################################################
MDOLVersion: 3.0

Task ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ TaskHeader ~~~~~~~~~~~~~~~~~~~~
  TaskHeader ElectricalAnalysis
      Evaluation: single
      ControlMode: user
      Precision: double
      RunCounter: 1
  End TaskHeader ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Inputs ~~~~~~~~~~~~~~~~~~~~~~~~
  Inputs ElectricalAnalysis
```

```
    ParameterList SystemTargets
        Type: real
        Parameters
            X1   I: 1.0
            X2   I: 1.0
            X3   I: 1.0
            X4   I: 1.0
            X5   I: 1.0
            X6   I: 1.0
            X7   I: 1.0
            X8   I: 1.0
            Y2S  I: 1.0
            Y3S  I: 1.0
            Y11S I: 1.0
            Y12S I: 1.0
    End ParameterList
End Inputs ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Auxiliaries ~~~~~~~~~~~~~~~~~~~
Auxiliaries ElectricalAnalysis
    Parameter: ScaledParmList Type:discrete
End Auxiliaries ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Outputs ~~~~~~~~~~~~~~~~~~~~~~~
Outputs ElectricalAnalysis
    ParameterList Sub1Output
        Type: real
        Parameters
            D2 Y2 Y3
    End ParameterList
End Outputs ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~ Initialization ~~~~~~~~~~~~~~~~
Initialization ElectricalAnalysis
    Parameters
        ScaledParmList SystemTargets Sub1Output
    Steps
        Tcl
            set Y2(Scale) 0.37149E+03
            set Y3(Scale) 0.31598E+02
            set X5(Scale) 325.505845
            set X6(Scale) 0.008432
            set X7(Scale) 25.427021
            set X8(Scale) 0.006920
            set Y2S(Scale) 0.37149E+03
            set Y3S(Scale) 0.31598E+02
            set Y11S(Scale) 0.36753E+02
```

```
              set Y12S(Scale) 0.55072E+02
              global PenaltyMultiplier
              set PenaltyMultiplier 1000000.0
              global DeltaForInEqualityConstraintViolation
              set DeltaForInEqualityConstraintViolation
0.004
              global DeltaForEqualityConstraintViolation
              set DeltaForEqualityConstraintViolation
0.0001
          End Tcl
  End Initialization ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Calculations ~~~~~~~~~~~~~~~~~~~
  Calculations ElectricalAnalysis
      Calculation CalcD2
          Parameters
              InputsGroup OutputsGroup
          Tcl
              set t1  [expr ($Y2(V)- $Y2S(V))*($Y2(V)-
$Y2S(V))]
              set t2  [expr ($Y3(V)- $Y3S(V))*($Y3(V)-
$Y3S(V))]
              set D2(V) [expr ($t1 + $t2)]
          End Tcl
      End Calculation CalcD2

      Calculation ElecCalc
          Parameters
              InputsGroup OutputsGroup
          Tcl
              UnScaleParameters
              set Y2(V) [expr ($X5(V)*(1.0 +
$X6(V)*($Y11S(V) - 20.0)))]
              set Y3(V) [expr ($X7(V)*(1.0 +
$X8(V)*($Y12S(V) - 20.0)))]
              ScaleParameters
          End Tcl
      End Calculation ElecCalc
  End Calculations ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ TaskProcess ~~~~~~~~~~~~~~~~~~
  TaskProcess ElectricalAnalysis
      Control: [ ElecCalc CalcD2]
  End TaskProcess ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Optimization ~~~~~~~~~~~~~~~~~
  Optimization ElectricalAnalysis
```

```
      PotentialVariables: InputsGroup
      Variables: SystemTargets
      PotentialObjectives: OutputsGroup
      Objectives
          Parameter: D2
          Direction: minimize
          Weight: 1.0
      OptimizePlan ExploitivePlan
          Prolog
              Tcl
                  global DataBaseLog Best
                  set Best(ObjectiveAndPenalty) 1.E+31
                  set Best(Objective) 1.E+31
                  set DataBaseLog(Status) append
              End Tcl
          Epilog
              Tcl rerunbest End Tcl
          OptimizeStep mmfd
              Technique: "Modified Method of Feasible
Directions"
                  Options
                      NumberOfIterations: 40
                      Auto: off
                      FiniteDifference: 0.01
                      MinimumFiniteDifference: 0.01
          OptimizeStep dlp
              Technique: "Sequential Quadratic Programming
- DONLP"
                  Options
                      Maxiter: 500
                      Tau0: 1.0
                      Del0: 1.0
                      Epsdif: 0.0001
          Control: [ dlp mmfd ]
  End Optimization ElectricalAnalysis
#~~~~~~~~~~~~~~~~~~~~~~~~ DataStorge ~~~~~~~~~~~~~~~~~~~
  DataStorage ElectricalAnalysis
      Restore: yes
      DataLog: "elec.db" Mode: overwrite
      DataLookUp: "elec-in.db"
  End DataStorage ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Procedures ~~~~~~~~~~~~~~~~~~~
  Procedures ElectricalAnalysis
      TclSourceFiles: "rerunbest.tcl" "parmscale.tcl"
  End Procedures ElectricalAnalysis
```

End Task ElectricalAnalysis

CO Method Description Files:

```
###########################################################
#
#       TASK    CO SYSTEM description file for EPackage
#
#    SYMBOLS
#         System Variable:         YiS
#         Coupling Variable:       Yi
#         Local Variable:          Xi
#
###########################################################
MDOLVersion: 3.0

Task EPackage

#~~~~~~~~~~~~~~~~~~~~~~~~ TaskHeader ~~~~~~~~~~~~~~~~~~~
  TaskHeader EPackage
      Evaluation: optimize
      ControlMode: user
      Precision: double
      RunCounter: 1
  End TaskHeader EPackage

#~~~~~~~~~~~~~~~~~~~~~~~~~~ Inputs ~~~~~~~~~~~~~~~~~~~~~~
  Inputs EPackage
      ParameterList SystemTargets
          Type: real
          Parameters
              Y1S  I: -0.1
              Y2S  I: 1.0
              Y3S  I: 1.0
              Y11S I: 1.0
              Y12S I: 1.0
      End ParameterList
  End Inputs EPackage

#~~~~~~~~~~~~~~~~~~~~~~~~~~ Outputs ~~~~~~~~~~~~~~~~~~~~~
  Outputs EPackage
      ParameterList SystemOutput
          Type: real
          Parameters
              Y1S2 D1 D2 H1 Y11 Y21 Y31 Y111 Y121 Y22 Y32
Y112 Y122
      End ParameterList
  End Outputs EPackage
```

```
#~~~~~~~~~~~~~~~~~~~~~~~ Initialization ~~~~~~~~~~~~~~~~~~~
  Initialization EPackage
       Parameters
           Tcl
               set Y1S(Scale) 0.650e+06
               set Y2S(Scale) 0.33976E+03
               set Y3S(Scale) 0.32832E+03
               set Y11S(Scale) 0.36956E+02
               set Y12S(Scale) 0.37032E+02
               set Y1S2(Scale) 0.650e+06
               global PenaltyMultiplier
               set PenaltyMultiplier 1000000000.0
               global DeltaForInEqualityConstraintViolation
               set DeltaForInEqualityConstraintViolation
0.0001
           End Tcl
  End Initialization EPackage

#~~~~~~~~~~~~~~~~~~~~~~~ Calculation ~~~~~~~~~~~~~~~~~~~~
  Calculations EPackage
      Calculation SystemGrad
          Parameters
              SystemTargets SystemOutput
          Steps
              Tcl
         api_SetGradientPartialValue EPackage Y1S  Y1S2 1.0
         api_SetGradientPartialValue EPackage Y2S  Y1S2 0.0
         api_SetGradientPartialValue EPackage Y3S  Y1S2 0.0
         api_SetGradientPartialValue EPackage Y11S Y1S2 0.0
         api_SetGradientPartialValue EPackage Y12S Y1S2 0.0

             api_SetGradientPartialValue EPackage Y1S  D1 \
                      [expr -2.0*($Y11(V)-$Y1S(V))]
             api_SetGradientPartialValue EPackage Y2S  D1 \
                      [expr -2.0*($Y21(V)-$Y2S(V))]
             api_SetGradientPartialValue EPackage Y3S  D1 \
                      [expr -2.0*($Y31(V)-$Y3S(V))]
             api_SetGradientPartialValue EPackage Y11S D1 \
                      [expr -2.0*($Y111(V)-$Y11S(V))]
             api_SetGradientPartialValue EPackage Y12S D1 \
                      [expr -2.0*($Y121(V)-$Y12S(V))]

             api_SetGradientPartialValue EPackage Y1S D2
0.0
             api_SetGradientPartialValue EPackage Y2S  D2 \
                      [expr -2.0*($Y22(V)-$Y2S(V))]
```

```
                api_SetGradientPartialValue EPackage Y3S  D2 \
                         [expr -2.0*($Y32(V)-$Y3S(V))]
                api_SetGradientPartialValue EPackage Y11S D2 \
                         [expr -2.0*($Y112(V)-$Y11S(V))]
                api_SetGradientPartialValue EPackage Y12S D2 \
                         [expr -2.0*($Y122(V)-$Y12S(V))]
                 End Tcl
         End Calculation SystemGrad

         Calculation TaskProcessStatusGrad
             Parameters
                 SystemTargets
             Steps
              Tcl
                     api_SetGradientPartialValue EPackage Y1S
TaskProcessStatus 0.0
                     api_SetGradientPartialValue EPackage Y2S
TaskProcessStatus 0.0
                     api_SetGradientPartialValue EPackage Y3S
TaskProcessStatus 0.0
                     api_SetGradientPartialValue EPackage Y11S
TaskProcessStatus 0.0
                     api_SetGradientPartialValue EPackage Y12S
TaskProcessStatus 0.0
              End Tcl
         End Calculation TaskProcessStatusGrad

         Calculation SysObj
             Parameters
                 Y1S Y1S2
             Steps
                 Tcl
                     set Y1S2(V) $Y1S(V)
                 End Tcl
         End Calculation SysObj
  End Calculations EPackage

#~~~~~~~~~~~~~~~~~~~~~~ Include ~~~~~~~~~~~~~~~~~~~~~~~~~
  Include From "electCO.desc"
      Component: ElectricalAnalysis
  End Include

  Include From "thermalCO.desc"
      Component: ThermalAnalysis
  End Include

#~~~~~~~~~~~~~~~~~~~~~~ TaskProcess ~~~~~~~~~~~~~~~~~~~~~~
```

```
  TaskProcess EPackage
      Control: [ElectricalAnalysis ThermalAnalysis SysObj]
      Gradient
          Control: [ SystemGrad TaskProcessStatusGrad ]
      End Gradient

      SubTask ElectricalAnalysis
          ParameterMap
              Y2S = Y2S
              Y3S = Y3S
                Y11S= Y11S
           Y12S= Y12S
              Y22 = Y2
              Y32 = Y3
              Y112 = Y11
              Y122 = Y12
          InputToSubtask
              Send: Y2S Y3S Y11S Y12S
          OutputFromSubtask
              Receive: D2 Y22 Y32 Y112 Y122
          End SubTask ElectricalAnalysis

      SubTask ThermalAnalysis
          ParameterMap
              Y1S = Y1S
              Y2S = Y2S
              Y3S = Y3S
              Y11S= Y11S
              Y12S= Y12S
              Y11 = Y1
              Y21 = Y2
              Y31 = Y3
              Y111 = Y11
              Y121 = Y12
          InputToSubtask
              Send:  Y2S Y3S Y11S Y12S Y1S
          OutputFromSubtask
              Receive: D1 H1 Y11 Y21 Y31 Y111 Y121
      End SubTask ThermalAnalysis
  End TaskProcess EPackage
#~~~~~~~~~~~~~~~~~~~~~~ Optimization ~~~~~~~~~~~~~~~~~~~~
  Optimization EPackage
      PotentialVariables: InputsGroup
      Variables: SystemTargets
      InputConstraints
          Parameter: Y11S LB: 0.01 UB: 2.30
          Parameter: Y12S LB: 0.01 UB: 2.30
```

```
            Parameter: Y2S  LB: 0.03 UB: 10.0
            Parameter: Y3S  LB: 0.03 UB: 10.0
            Parameter: Y1S  LB: -1.2 UB: -0.001
        PotentialObjectives: OutputsGroup
        Objectives
            Parameter: Y1S2
            Direction: minimize
            Weight: 1.0
        OutputConstraints
         Parameter: D1  UB: 0.0001
         Parameter: D2  UB: 0.0001
        OptimizePlan ExploitivePlan
            Prolog
                Tcl
                    global DataBaseLog Best
                    set Best(ObjectiveandPenalty) 1.E+31
                    set Best(Objective) 1.E+31
                    set DataBaseLog(Status) append
                End Tcl
            Epilog
                Tcl rerunbest End Tcl
            OptimizeStep mmfd
                Technique: "Modified Method of Feasible
Directions"
                    Options
                        NumberOfIterations: 40
                        UserSuppliedGradients: yes
                        ctmin: 0.0001
            OptimizeStep ext
                Technique: "Exterior Penalty"
                    Options
                        NumberOfIterations: 40
                        UserSuppliedGradients: yes
            Control: [ ext mmfd ]
    End Optimization EPackage
#~~~~~~~~~~~~~~~~~~~~~~ DataStorge ~~~~~~~~~~~~~~~~~~~~~
    DataStorage EPackage
        Restore: yes
        DataLog: "package.db" Mode: overwrite
        DataLookUp: "package-in.db"
    End DataStorage EPackage
#~~~~~~~~~~~~~~~~~~~~~~ Procedures ~~~~~~~~~~~~~~~~~~~~~
    Procedures EPackage
        TclSourceFiles: "rerunbest.tcl"
    End Procedures EPackage
```

End Task Epackage

Subsystem 1: Thermal Subsystem

```
###########################################################
#
#       TASK    CO Sub-System 1 description file (Thermal)
#
#    SYMBOLS
#         System Variable:        YiS
#         Coupling Variable:  Yi
#         Local Variable:          Xi
#
###########################################################
MDOLVersion: 3.0

Task ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~~ TaskHeader ~~~~~~~~~~~~~~~~~~~~
  TaskHeader ThermalAnalysis
      Evaluation: optimize
      ControlMode: user
      Precision: double
      RunCounter: 1
  End TaskHeader ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~~ Inputs ~~~~~~~~~~~~~~~~~~~~~~~~~
  Inputs ThermalAnalysis
      ParameterList Sub2Input
          Type: real
          Parameters
              X1  I: 1.0
              X2  I: 1.0
              X3  I: 1.0
              X4  I: 1.0
              X5  I: 1.0
              X6  I: 1.0
              X7  I: 1.0
              X8  I: 1.0
              Y2  I: 1.0
              Y3  I: 1.0
      End ParameterList

      ParameterList SystemTargetedInput
          Type: real
          Parameters
              Y1S  I: -0.1
              Y2S  I: 1.0
              Y3S  I: 1.0
```

```
            Y11S I: 1.0
            Y12S I: 1.0
        End ParameterList
    End Inputs ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~ Auxiliaries ~~~~~~~~~~~~~~~~~~~
    Auxiliaries ThermalAnalysis
        Parameter: ScaledParmList Type:discrete
    End Auxiliaries ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Outputs ~~~~~~~~~~~~~~~~~~~~~
    Outputs ThermalAnalysis
        ParameterList Sub2Output
            Type: real
            Parameters
                D1 D11 Y1 Y10 Y11 Y12 Y13 H1
        End ParameterList
    End Outputs ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~ Initialization ~~~~~~~~~~~~~~~~~~
    Initialization ThermalAnalysis
        Parameters
            Sub2Input ScaledParmList SystemTargetedInput
Sub2Output
        Steps
            Tcl
                set Y2S(Scale) 0.33976E+03
                set Y3S(Scale) 0.32832E+03
                set Y11S(Scale) 0.36956E+02
                set Y12S(Scale) 0.37032E+02
                set X1(Scale) 0.110414
                set X2(Scale) 0.108270
                set X3(Scale) 0.034297
                set X4(Scale) 0.022572
                set X5(Scale) 300.466564
                set X6(Scale) 0.007712
                set X7(Scale) 305.540350
                set X8(Scale) 0.004378
                set Y1(Scale) 0.650E+06
                set Y2(Scale) 0.33976E+03
                set Y3(Scale) 0.32832E+03
                set Y11(Scale) 0.36956E+02
                set Y12(Scale) 0.37032E+02
                global PenaltyMultiplier
                set PenaltyMultiplier 1000000.0
                global DeltaForInEqualityConstraintViolation
```

```
              set DeltaForInEqualityConstraintViolation
0.0001
              global DeltaForEqualityConstraintViolation
              set DeltaForEqualityConstraintViolation
0.0001
          End Tcl
  End Initialization ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~ Calcualtions ~~~~~~~~~~~~~~~~~~~~~~
  Calculations ThermalAnalysis
      Calculation CalcD1
          Parameters
              InputsGroup OutputsGroup
          Tcl
      set t1  [expr ($Y11(V)- $Y11S(V))*($Y11(V)-$Y11S(V))]
      set t2  [expr ($Y12(V)- $Y12S(V))*($Y12(V)-$Y12S(V))]
      set t3  [expr ($Y2(V)- $Y2S(V))*($Y2(V)-$Y2S(V))]
      set t4  [expr ($Y3(V)- $Y3S(V))*($Y3(V)-$Y3S(V))]
      set t5  [expr ($Y1(V)- $Y1S(V))*($Y1(V)-$Y1S(V))]
      set D1(V) [expr $t1+$t2+$t3+$t4+$t5]
      set D11(V) [expr $D1(V) + $Y1(V)]
          End Tcl
      End Calculation CalcD1

      Calculation CheckX4
          Parameters
              Sub2Input
          Tcl
              if { $X4(V) < 0.0 } { set X4(V) 0.653680 }
          End Tcl
      End Calculation CheckX4
  End Calculations ThermalAnalysis

#~~~~~~~~~~~~~~~~~~~~~ SimCode ~~~~~~~~~~~~~~~~~~~~~~
  SimCode ThermalAnalysisCode
      InputFiles ThermalAnalysisCode
          FileDescription farFile0
              FileType: standard
              NameValueFile: "thermal-in.nv"
              InputFile: "package.in"
              Language: emacs
              Parameters
                  Sub2Input
              Instructions
                  write $X1
                  write $Newline
                  write $X2
```

```
            write $Newline
            write $X3
            write $Newline
            write $X4
            write $Newline
            write $X5
            write $Newline
            write $X6
            write $Newline
            write $X7
            write $Newline
            write $X8
            write $Newline
            write $Y2
            write $Newline
            write $Y3
            write $Newline
        End Instructions
    End FileDescription farFile0
End InputFiles ThermalAnalysisCode

OutputFiles ThermalAnalysisCode
    FileDescription farFile2
        FileType: standard
        OutputFile: "package.out"
        NameValueFile: "thermal-out.nv"
        Language: emacs
        Parameters
            Sub2Output
        Instructions
            read Y1
            provide $Y1
            read Y10
            provide $Y10
            read Y11
            provide $Y11
            read Y12
            provide $Y12
            read Y13
            provide $Y13
            read H1
            provide $H1
        End Instructions
    End FileDescription farFile2
End OutputFiles ThermalAnalysisCode

SimCodeProcess ThermalAnalysisCode
```

```
        Program: "package.exe"
        ElapseTime: 100s
        Prologue
            Tcl
                UnScaleParameters
            End Tcl
            WriteInputSpecs: farFile0
        Epilogue
            ReadOutputSpecs: farFile2
            Tcl
                ScaleParameters
            End Tcl
    End SimCodeProcess ThermalAnalysisCode
  End SimCode ThermalAnalysisCode
#~~~~~~~~~~~~~~~~~~~~~ TaskProcess ~~~~~~~~~~~~~~~~~~~~~~~
  TaskProcess ThermalAnalysis
    Control: [ CheckX4 ThermalAnalysisCode CalcD1]
  End TaskProcess ThermalAnalysis
#~~~~~~~~~~~~~~~~~~~~~ Optimization ~~~~~~~~~~~~~~~~~~~~~~
  Optimization ThermalAnalysis
    PotentialVariables: X1 X2 X3 X4 Y2 Y3
    Variables: X1 X2 X3 X4 Y2 Y3
    InputConstraints
        Parameter: X1  LB: 0.452841 UB:  1.35852
        Parameter: X2  LB: 0.461808 UB:  1.38543
        Parameter: X3  LB: 0.291571 UB:  2.91571
        Parameter: X4  LB: 0.221513 UB:  2.21513
        Parameter: Y2  LB: 0.01     UB:  10.0
        Parameter: Y3  LB: 0.01     UB:  10.0
    PotentialObjectives: OutputsGroup
    Objectives
        Parameter: D11
        Direction: minimize
        Weight: 1.0
    OutputConstraints
        Parameter: H1   Eq: 0.00
        Parameter: Y11  UB: 2.38
        Parameter: Y12  UB: 2.22
    OptimizePlan ExploitivePlan
        Prolog
            Tcl
                global DataBaseLog Best
                set Best(ObjectiveAndPenalty) 1.E+31
                set Best(Objective) 1.E+31
                set DataBaseLog(Status) append
            End Tcl
        Epilog
```

```
            Tcl rerunbest End Tcl
          OptimizeStep dlp1
              Technique: "Sequential Quadratic Programming
- DONLP"
                  Options
                      Maxiter: 500
                      Tau0: 1.0
                      Del0: 1.0
                      Epsdif: 0.0001
          OptimizeStep dlp2
              Technique: "Sequential Quadratic Programming
- DONLP"
                  Options
                      Maxiter: 250
                      Tau0: 1.0
                      Del0: 1.0
                      Deldif: 0.000001
                      Epsdif: 0.0001
          Control: [ dlp1 dlp2 ]
  End Optimization ThermalAnalysis
#~~~~~~~~~~~~~~~~~~~~~ DataStorge ~~~~~~~~~~~~~~~~~~~~~~
  DataStorage ThermalAnalysis
      Restore: yes
      DataLog: "thermal.db" Mode: overwrite
      DataLookUp: "thermal-in.db"
  End DataStorage ThermalAnalysis
#~~~~~~~~~~~~~~~~~~~~~ Procedures ~~~~~~~~~~~~~~~~~~~~~~
  Procedures ThermalAnalysis
      TclSourceFiles: "rerunbest.tcl" "parmscale.tcl"
  End Procedures ThermalAnalysis
```

End Task ThermalAnalysis

Subsystem 2: Electrical

```
#############################################################
#
#       TASK CO Sub-System 1 description file (Electrical)
#
#    SYMBOLS
#         System Variable:        YiS
#         Coupling Variable:      Yi
#         Local Variable:         Xi
#
#############################################################
MDOLVersion: 3.0

Task ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ TaskHeader ~~~~~~~~~~~~~~~~~~~~
  TaskHeader ElectricalAnalysis
      Evaluation: optimize
      ControlMode: user
      Precision: double
      RunCounter: 1
  End TaskHeader ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Inputs ~~~~~~~~~~~~~~~~~~~~~~~~
  Inputs ElectricalAnalysis
      ParameterList Sub1Input
          Type: real
          Parameters
              X5  I: 1.0
              X6  I: 1.0
              X7  I: 1.0
              X8  I: 1.0
              Y11 I: 1.0
              Y12 I: 1.0
      End ParameterList
      ParameterList SystemTargets
          Type: real
          Parameters
              Y2S  I: 1.0
              Y3S  I: 1.0
              Y11S I: 1.0
              Y12S I: 1.0
      End ParameterList
  End Inputs ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Auxiliaries ~~~~~~~~~~~~~~~~~~~~
```

```
  Auxiliaries ElectricalAnalysis
      Parameter: ScaledParmList Type:discrete
  End Auxiliaries ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~~ Outputs ~~~~~~~~~~~~~~~~~~~~~~~~~
  Outputs ElectricalAnalysis
      ParameterList Sub1Output
          Type: real
          Parameters
              D2 Y2 Y3
      End ParameterList
  End Outputs ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~ Initialization ~~~~~~~~~~~~~~~~~~~~
  Initialization ElectricalAnalysis
      Parameters
          ScaledParmList SystemTargets Sub1Input Sub1Output
      Steps
          Tcl
              set Y2(Scale) 0.33976E+03
              set Y3(Scale) 0.32832E+03
              set Y11(Scale) 0.36956E+02
              set Y12(Scale) 0.37032E+02
              set Y2S(Scale) 0.33976E+03
              set Y3S(Scale) 0.32832E+03
              set Y11S(Scale) 0.36956E+02
              set Y12S(Scale) 0.37032E+02
              set X5(Scale) 300.466564
              set X6(Scale) 0.007712
              set X7(Scale) 305.540350
              set X8(Scale) 0.004378
              global PenaltyMultiplier
              set PenaltyMultiplier 1000000.0
              global DeltaForInEqualityConstraintViolation
              set DeltaForInEqualityConstraintViolation
0.004
              global DeltaForEqualityConstraintViolation
              set DeltaForEqualityConstraintViolation
0.0001
          End Tcl
  End Initialization ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~ Calculations ~~~~~~~~~~~~~~~~~~~~
  Calculations ElectricalAnalysis
      Calculation CalcD2
          Parameters
              InputsGroup OutputsGroup
```

```
        Tcl
    set t1  [expr ($Y2(V)- $Y2S(V))*($Y2(V)-$Y2S(V))]
    set t2  [expr ($Y3(V)- $Y3S(V))*($Y3(V)-$Y3S(V))]
    set t3  [expr ($Y11(V)- $Y11S(V))*($Y11(V)-$Y11S(V))]
    set t4  [expr ($Y12(V)- $Y12S(V))*($Y12(V)-$Y12S(V))]
    set D2(V) [expr ($t1 + $t2 + $t3 + $t4)]
        End Tcl
    End Calculation CalcD2

    Calculation ElecCalc
        Parameters
            InputsGroup OutputsGroup
        Tcl
            UnScaleParameters
            set Y2(V) [expr ($X5(V)*(1.0 +
$X6(V)*($Y11(V) - 20.0)))]
            set Y3(V) [expr ($X7(V)*(1.0 +
$X8(V)*($Y12(V) - 20.0)))]
            ScaleParameters
        End Tcl
    End Calculation ElecCalc
  End Calculations ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~ TaskProcess ~~~~~~~~~~~~~~~~~~~~
  TaskProcess ElectricalAnalysis
    Control: [ ElecCalc CalcD2]
  End TaskProcess ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~ Optimization ~~~~~~~~~~~~~~~~~~~~
  Optimization ElectricalAnalysis
    PotentialVariables: X5 X6 X7 X8 Y11 Y12
    Variables: X5 X6 X7 X8 Y11 Y12
    InputConstraints
        Parameter: X5  LB:  3.32816E-02 UB:  3.32816
        Parameter: X6  LB:  0.518672    UB:  1.16701
        Parameter: X7  LB:  3.27289E-02 UB:  3.27289
        Parameter: X8  LB:  0.913659    UB:   2.05573
        Parameter: Y11 LB:  0.01        UB: 2.30
        Parameter: Y12 LB:  0.01        UB: 2.30
    PotentialObjectives: OutputsGroup
    Objectives
        Parameter: D2
        Direction: minimize
        Weight: 1.0
    OptimizePlan ExploitivePlan
        Prolog
            Tcl
```

```
                global DataBaseLog Best
                set Best(ObjectiveAndPenalty) 1.E+31
                set Best(Objective) 1.E+31
                set DataBaseLog(Status) append
            End Tcl
        Epilog
            Tcl rerunbest End Tcl
        OptimizeStep dlp
            Technique: "Sequential Quadratic Programming
- DONLP"
                Options
                    Maxiter: 500
                    Tau0: 1.0
                    Del0: 1.0
                    Epsdif: 0.0001
        Control: [ dlp ]
  End Optimization ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~ DataStorage ~~~~~~~~~~~~~~~~~~~~
  DataStorage ElectricalAnalysis
      Restore: yes
      DataLog: "elec.db" Mode: overwrite
      DataLookUp: "elec-in.db"
  End DataStorage ElectricalAnalysis

#~~~~~~~~~~~~~~~~~~~~~~~ Procedures ~~~~~~~~~~~~~~~~~~~~
  Procedures ElectricalAnalysis
      TclSourceFiles: "rerunbest.tcl" "parmscale.tcl"
  End Procedures ElectricalAnalysis
```

End Task ElectricalAnalysis

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The NASA Langley MDO method evaluation study seeks to arrive at a set of guidelines for using promising MDO methods by accumulating and analyzing computational data for such methods. The data are collected by conducting a series of reproducible experiments. This report documents all computational experiments conducted in Phase I of the study. This report is a companion to the paper titled Initial Results of an MDO Method Evaluation Study by N. M. Alexandrov and S. Kodiyalam (AIAA-98-4884).

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