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Improved Modeling in a Matlab-Based Navigation System

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An innovative approach to autonomous navigation is available for low earth orbit satellites. The system is developed in Matlab and utilizes an Extended Kalman Filter (EKF) to estimate the attitude and trajectory based on spacecraft magnetometer and gyro data. Preliminary tests of the system with real spacecraft data from the Rossi X-Ray Timing Explorer Satellite (RXTE) indicate the existence of unmodeled errors in the magnetometer data. Incorporating into the EKF a statistical model that describes the colored component of the effective measurement of the magnetic field vector could improve the accuracy of the trajectory and attitude estimates and also improve the convergence time. This model is identified as a first order Markov process. With the addition of the model, the EKF attempts to identify the non-white components of the noise allowing for more accurate estimation of the original state vector, i.e. the orbital elements and the attitude. Working in Matlab allows for easy incorporation of new models into the EKF and the resulting navigation system is generic and can easily be applied to future missions resulting in an alternative in onboard or ground-based navigation.

INTRODUCTION

The magnetometer has been the focus of numerous studies in low earth navigation in the recent past. The magnetometer, used historically in momentum management, has proven to be a reliable, low cost sensor that has been successful in coarse attitude, orbit, and rate estimation [1,2,3,4,5] and if properly calibrated is capable of accurate fine attitude solutions [6].

The combined attitude and orbit system of Ref. 7, showed promising results with real spacecraft data from three of four Goddard Space Flight Center (GSFC) supported spacecraft. The one satellite that didn't achieve the expected accuracy was the RXTE. Based on the success of Shorshi and Bar-Itzhack in incorporating an additional model to improve the accuracy of the orbit estimation results[6], a statistical model was identified for the RXTE magnetometer data and incorporated into the filter. As in the case of Shorshi and Bar-Itzhack, the identified model is a first-order Markov model.

The EKF presented in Ref. 7 was converted from FORTAN into Matlab. Working in Matlab allowed for easy incorporation of the additional statistical model into the filter. The resulting filter is generic in form and can easily be applied to other missions. With the development of tools to convert Matlab code into C code, there is potential for this type of system to be converted for use in an onboard computer. In

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addition, the filter can be incorporated into the existing MATLAB Attitude Determination System (ADS) used by the GSFC Guidance, Navigation, and Control Center (GNCC) as an additional tool available for coarse ground-based attitude and orbit estimation[8].

This paper will summarize the existing EKF of Ref. 7 in the first section. The second section presents the identified statistical model that describes the non-white noise that exists in the magnetometer data and its augmentation into the EKF. The third section discusses the MATLAB implementation. Finally the fourth section presents the results, following with a discussion of the results in the fifth section.

FILTER MODELS

The EKF is based on the following assumed models:

System Model:

$$\dot{\underline{X}}(t) = \underline{f}(\underline{X}(t), t) + \underline{w}(t) \tag{1}$$

Measurement Model:

$$\underline{y}_k = h_k(\underline{X}(t_k)) + \underline{v}_k \tag{2}$$

where $\underline{w}(t)$ is a zero mean white process noise, $\underline{v}_k$, is a zero mean white sequence measurement error and $\underline{X}(t)$ is the state vector defined as

$\underline{X}^T = [a, e, i, \Omega, \omega, \theta, Cd, \underline{q}^T]$

The first six elements of $\underline{X}(t)$ are the classical Keplerian elements which determine the spacecraft position and velocity, namely; the semi-major axis (a), eccentricity (e), inclination (i), right ascension of the ascending node (Ω), argument of perigee (ω), and the true anomaly (θ). C_d is the drag coefficient and $\underline{q}$ represents the attitude quaternion.

The measurement update is performed using the following

$$\hat{\underline{X}}_{k+1}(+) = \hat{\underline{X}}_{k+1}(-) + K_{k+1}\underline{z}_{k+1} \tag{3}$$

$$P_{k+1}(+) = [I - K_{k+1}H_{k+1}]P_{k+1}(-)[I-K_{k+1}H_{k+1}]^T + K_{k+1}R_{k+1}K^T_{k+1} \tag{4}$$

The Kalman gain, K_{k+1} is computed according to

$$K_{k+1} = P_{k+1}(-)H^T_{k+1}[H_{k+1}P_{k+1}(-)H^T_{k+1} + R_{k+1}]^{-1} \tag{5}$$

where

$$H_{k+1} = \left.\frac{\partial h_{k+1}(\underline{X}_{k+1}(t_{k+1}))}{\partial \underline{X}_{k+1}}\right|_{\underline{X}_{k+1}=\hat{\underline{X}}_{k+1}(-)} \tag{6}$$

P_{k+1} is the estimation error covariance matrix, and R_{k+1} is the covariance matrix of the zero mean white sequence $\underline{v}_{k+1}$.

The effective measurement used by the filter in Eq. (3) is given as

$$\underline{z}_{k+1} = \underline{B}_{m,k+1} - \underline{B}(\hat{\underline{X}}_{k+1}, t_{k+1}) \tag{7}$$

here $\underline{B}_{m,k+1}$ is the magnetic field as measured by the magnetometer in the spacecraft body coordinates. $\underline{B}(\hat{\underline{X}}_{k+1}, t_{k+1})$ is the computed magnetic field vector as a function of the estimated state and time. It is computed using a 10[th] order International Geomagnetic Reference Field model. The derivation of the matrix H_{k+1} resulting in the linear relationship between the effective measurement and the state vector is given in Ref. 7. The update of the attitude follows that of Ref. 7.

The propagation of the state estimate uses the following differential equation based on Eq. (1)

$$\dot{\hat{\underline{X}}}(t) = f(\hat{\underline{X}}(t), t) \tag{8}$$

The updated estimate of the state vector is propagated from time t_k to time t_{k+1} by a numerical solution of the continuous dynamics equation given in Eq. (8). The orbital dynamics are non-linear and describe a central force including J2 effects and drag[5]. The differential equation for the quaternion is linear and requires knowledge of the spacecraft rate, as supplied by the spacecraft gyroscopes[7].

The propagation of the covariance is given as

$$P_{k+1}(-) = A_k(\underline{X}_k(+))P_k(+)A^T_k(\underline{X}_k(+)) + Q_k \tag{9}$$

where Q_k is the covariance matrix of the discretized $\underline{w}(t)$ and A_k is the approximated transition matrix, based on a first order Taylor series expansion of the linearized dynamics. The details of this derivation are given in Ref. 7.

STATISTICAL ERROR MODEL

The filter described above was tested using real spacecraft data from four GSFC sponsored satellites, namely the Compton Gamma Ray Observatory, the Total Ozone Mapping Spectrometer-Explorer Platform, the Earth Radiation Budget Satellite and the RXTE. All but the RXTE satellite gave reasonable estimates for position and attitude within three to four orbits. Following a detailed statistical analysis of a 24-hour sample of RXTE magnetometer data as described in Ref. 9, a statistical model was identified. In an effort to improve the RXTE results this model is augmented with the original EKF. Comparisons of the results obtained with and without this additional model are presented in the following section.

Given that the measurement noise vector, $\underline{v}_b$, given in Eq. (2) above, contains also non-white components it is rewritten as

$$\begin{aligned} \underline{v}_k &= D_b^F \underline{\eta}_F + \underline{\eta}_b \\ \dot{\underline{\eta}}_F &= -T^{-1} \underline{\eta}_F + \underline{\zeta}_F \end{aligned} \tag{10}$$

with $\underline{\eta}_F$ as a first order Markov process in the magnetic spherical coordinates, F, (to eliminate the dependence of the model components on the attitude), transformed to body coordinates through the transformation D_b^F. The matrix T^{-1} contains (diagonally) the inverse of the time constants for each of the three components of $\underline{\eta}_F$. The vectors $\underline{\eta}_b$ and $\underline{\zeta}_F$ are the white noise vectors associated with the measurement and dynamics of the Markov process, respectively. This model is augmented with the original filter equations given above. The resulting model should account for the non-white components of the measurement noise. A detailed derivation of the model given in Eq. (10) is available in Ref. 9.

MATLAB IMPLEMENTATION

The EKF described in Section I was originally developed in Unix-based FORTRAN. Although the main EKF routines were generic, additional routines were necessary to handle the differences of each of the missions, making it necessary to have different executable versions for each mission. In addition, the available memory for processing was limited. Processing multiple orbits of data required skipping data in either the output or the input. Furthermore, the current ground attitude support software in the GNCC is the Matlab ADS. In order to have the EKF available as an additional tool in the ADS, the EKF was converted to Matlab. A major drawback was the reduction in processing time as compared to the FORTRAN version.

Relying on the data format of the Matlab ADS system, the EKF could be made completely generic. The data could be organized into a .mat file and loaded into the program, independent of the mission. Many of the existing routines (both Matlab and ADS routines) could be utilized to perform such tasks as sorting the data, adjusting the time format, processing the IGRF coefficients, evaluating the correlation matrix, propagating the quaternion, etc. And of course, defining the a priori matrices (covariance, process noise, measurement noise, and state) is trivial with Matlab.

The program is divided into 4 basic sections. The first loads and sorts the data and defines the a priori conditions. The processing then enters a main loop and the second section is encountered. This section performs the propagation of the state. The orbital elements are propagated using a 4th order Runge-Kutta routine (this is preferred over the slower Matlab ODE routines). The quaternion is propagated using the kinematic equation described in Ref. 7. The propagation of the covariance follows, performed using Eq. (9) above. The third section performs the measurement update. The measurement update is based on the development in Ref. 7 and Ref. 5. This section of the program required the majority of new routines. The final section saves and plots the results.

The addition of the Markov model involved the addition of the following: 3 Markov components to the state vector; the expansion of the covariance matrix and the dynamic noise covariance; the augmentation of the Markov measurement matrix, defined in Eq. (10) above; and the propagation equation for the Markov components which was performed using the matrix exponential function in MATLAB. Augmenting this model into the EKF in MATLAB was straightforward and required few additional lines of code.

RESULTS

The RXTE data used in the testing consisted of slightly more than 3 orbits of magnetometer and gyro data from September 8, 1996, approximately nine months after the RXTE launch. Reference data is available from the ground based ephemeris, providing the 'truth model' position and velocity, and the onboard computer attitude history generated from star tracker and gyro data providing the 'truth model' quaternion. The initial RSS errors in position, velocity, and attitude are 1451 km, 1.15 km/sec, and 5.5 deg, respectively.

The Markov model was generated from a 24-hour sample of data, which included the data used in the EKF testing. The statistical analysis identified the standard deviation of the covariance matrices R_k and Q_k of Eqs. (5) and (9), as well as the time constants of the matrix T of Eq. (10)[9]. Initially the filter tests were conducted with these identified values. As expected, the filter exhibited divergence which disappeared after some preliminary tuning. The results from this tuning follow.

Figures 1 and 2 show the RSS position errors for the original filter and the filter with the Markov components, respectively, for the first orbit (to see the initial convergence). Figures 3 and 4 show slightly more than 3 orbits, with the vertical axis expanded. The final RSS position errors are approximately 50 km without the Markov model, and 25 km with the Markov model.

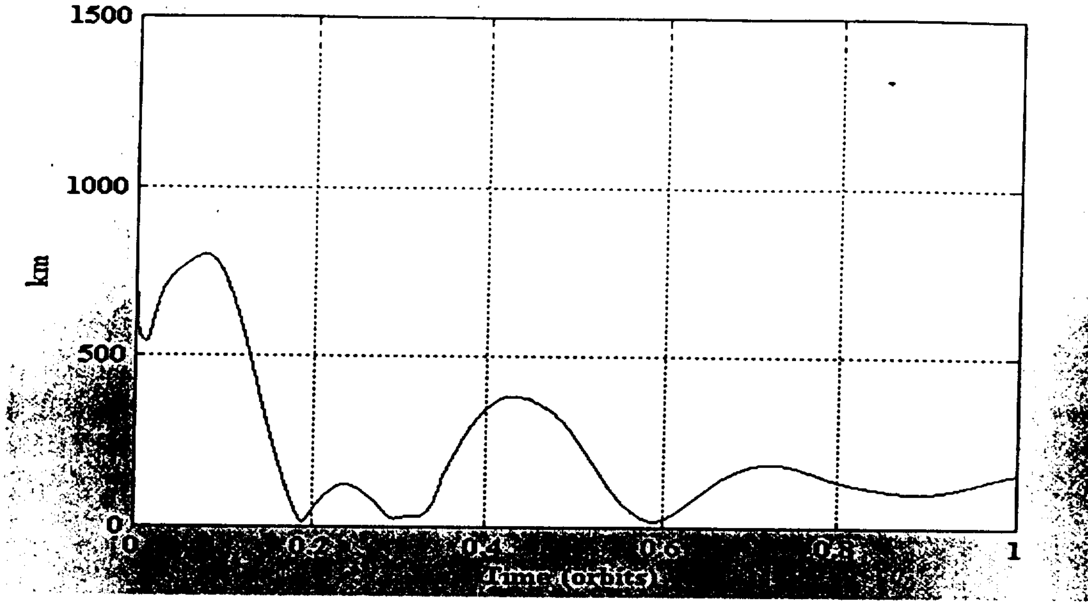

Figure 1. RSS Position Error *Without* Markov model, 1st Orbit

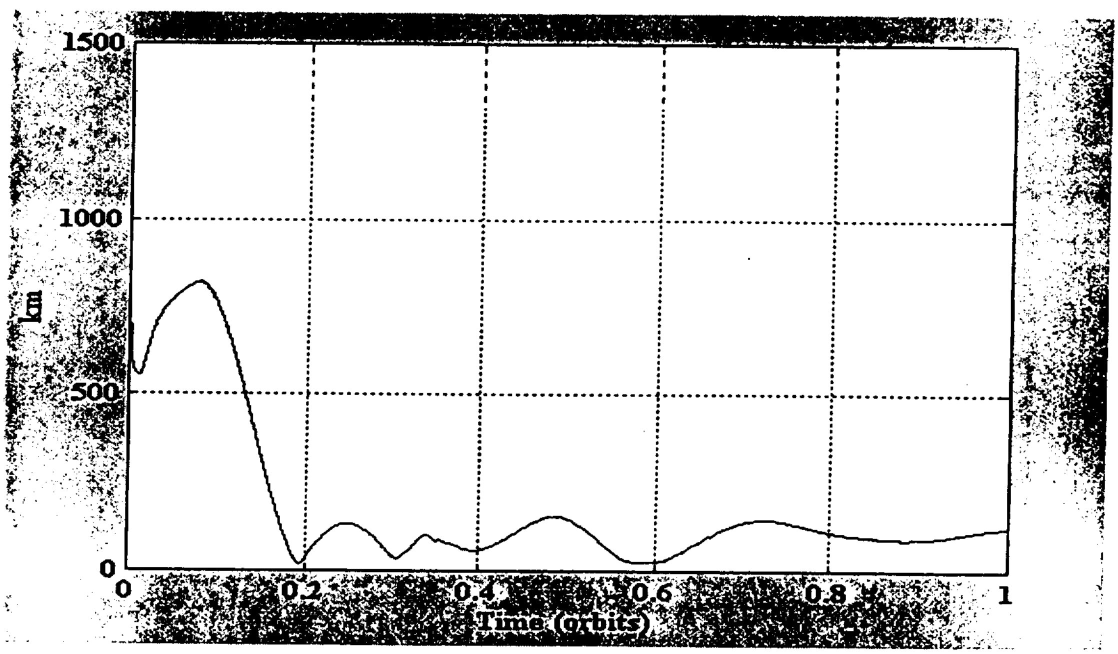

Figure 2. RSS Position Error *With* Markov model, 1st Orbit

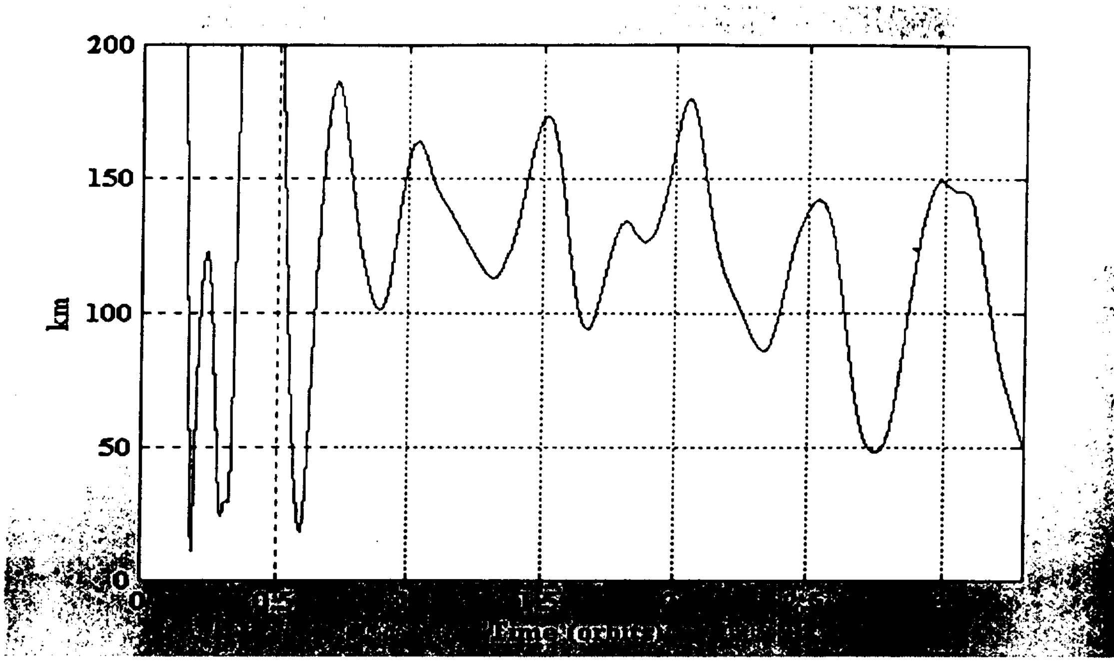

Figure 3. RSS Position Error *Without* Markov model

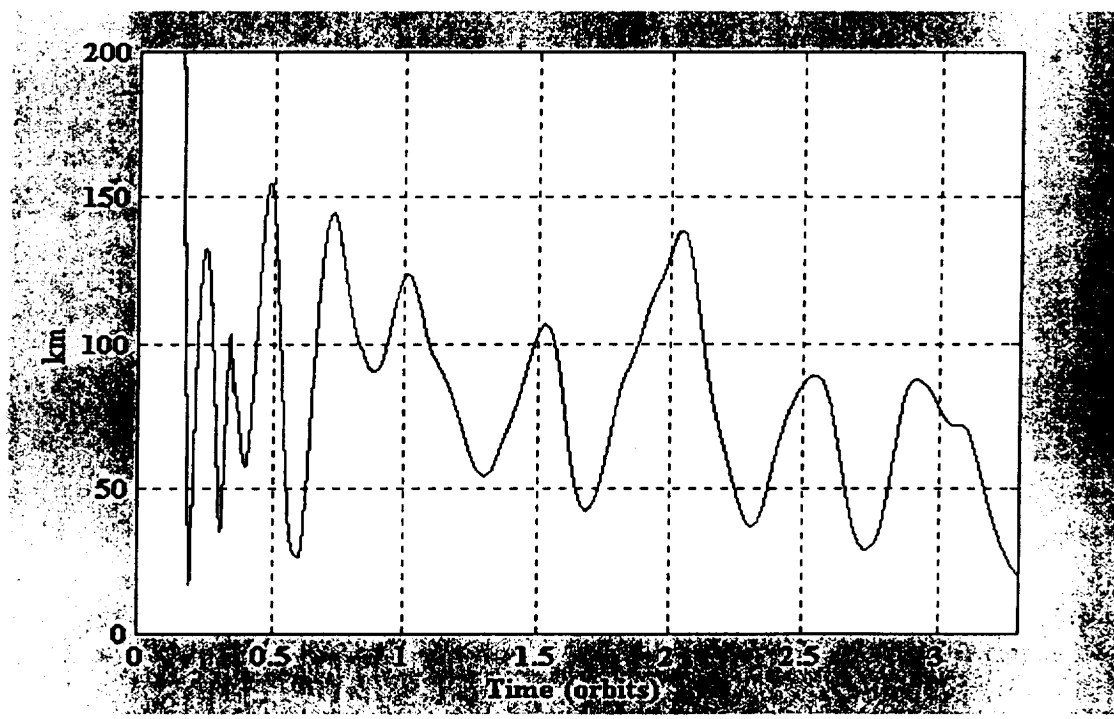

Figure 4. RSS Position Error *With* Markov model

The velocity estimates behave similarly to the position estimates, converging to approximately 0.05 km/sec without the Markov and 0.03 km/sec with the Markov model.

Figures 5 and 6 show the RSS attitude error plots, without and with the Markov model, respectively. The RSS attitude errors are not reduced below 2 deg either with or without the Markov. The oscillations are somewhat reduced with the Markov model.

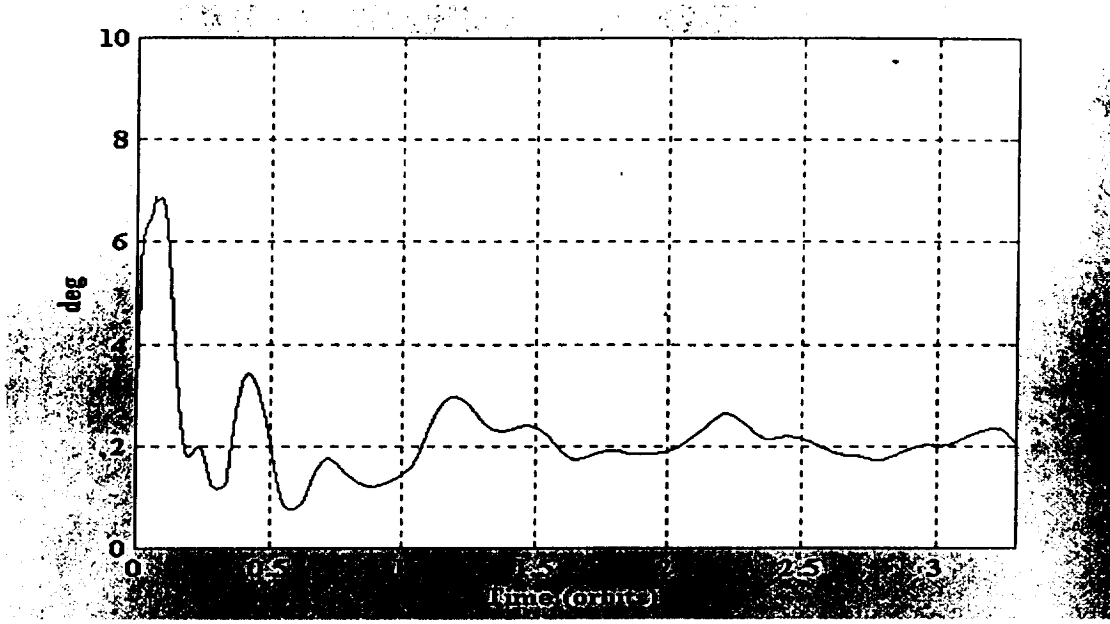

Figure 5. RSS Attitude Error *Without* Markov model

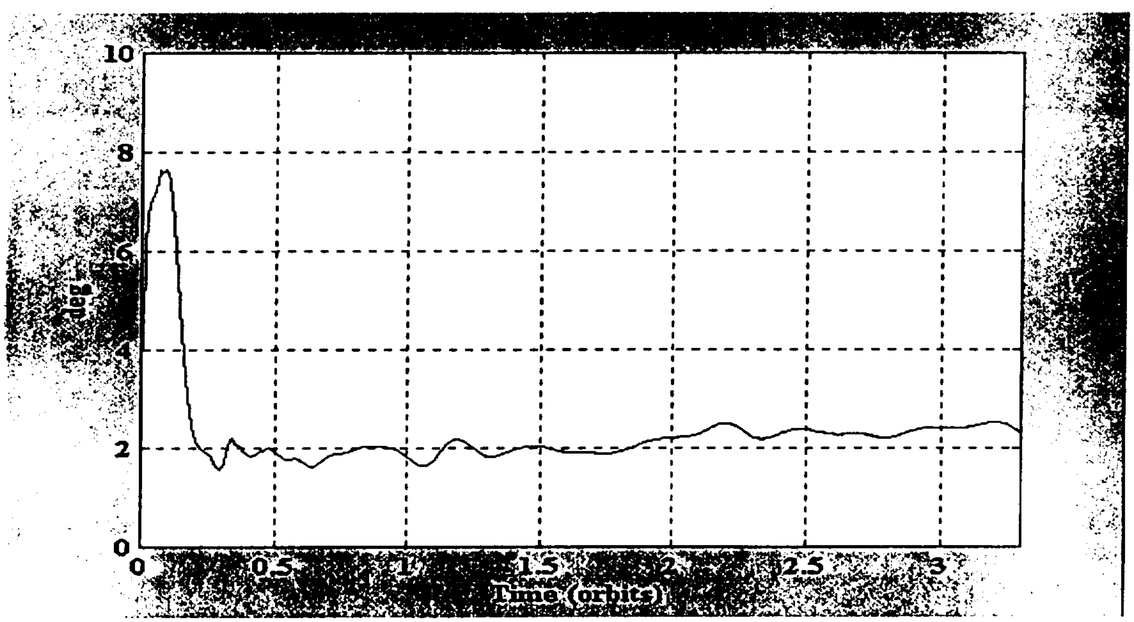

Figure 6. RSS Attitude Error *With* Markov Model

These preliminary tests indicate that the additional Markov model improves the orbit estimates. The oscillations during convergence are reduced, and the RSS error after 3 orbits is reduced by approximately 25 km. The effect on the attitude is seen mainly in the reduction of oscillations. The size of the average RSS attitude error is similar with and without the additional Markov model.

CONCLUSIONS

This paper presents the preliminary results of an enhanced Matlab based EKF which simultaneously estimates spacecraft attitude and orbit, relying on magnetometer and gyro data. The EKF was enhanced by the addition of a first order Markov model. This model was identified by a statistical analysis of the RXTE magnetometer residuals. The results presented indicate that including this model does improve the orbit estimates by approximately 25 km, as well as reduce the oscillations during the initial convergence. The attitude estimates are not dramatically improved by the addition of the model. The average RSS attitude errors still remain at approximately 2 degrees. As with the position estimates, oscillations in the attitude estimates are reduced slightly with addition of the Markov model.

More testing and tuning are necessary to determine if the additional Markov model can improve the results further. Extending the length of the data, including attitude maneuvers, will provide insight into the stability of the EKF and may improve the observability of the state parameters. Based on the results of Shorshi and Bar-Itzhack[5] and the preliminary results presented in this paper, the addition of a statistical model can improve the orbit estimates, and potentially the attitude estimates, when there is evidence of non-white noise components in the sensor noise.

As mentioned, the EFK presented is now available in Matlab. Previous testing of this EKF was done in FORTRAN. With Matlab, many existing routines are incorporated into the code and the overall program is generic. Many of the routines originated in the Matlab ADS, a ground based system developed in the GNCC. In the future, this EKF could be incorporated as an additional tool into the ADS, and can potentially be converted into flight code for use in an onboard computer. Incorporating the additional Markov model in the Matlab EKF was straightforward. Overall the use of Matlab enhanced the testing and development of the attitude and orbit EKF.

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