

# Quasi-Static Probabilistic Structural Analyses Process and Criteria

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## NOMENCLATURE

$K$	= sample size tolerance factor
$L$	= load, kips
$N$	= probabilistic range factor
$n$	= statistical sample number
$R$	= reliability
$S$	= stress, ksi
$SF$	= conventional safety factor
$x$	= specimen value
$y$	= random variable component
$Z$	= safety index
$\eta$	= coefficient of variation, $\sigma/\mu$
$\mu$	= statistical mean
$\sigma$	= standard deviation

### Subscripts

$A$	= applied stress
$k$	= outcome
$R$	= resistive stress
$i$	= $i$ th value
ty	= tensile yield stress





## TECHNICAL PUBLICATION

# QUASI-STATIC PROBABILISTIC STRUCTURAL ANALYSES PROCESS AND CRITERIA

## 1. INTRODUCTION

Increasing emphasis on affordable access to space while maintaining high reliability is driving designers to improved systems analyses and integration methods. These techniques often include designing each substructure to a standard safety factor or specified reliability subjected to its maximum operational environment and then determining its reliability contribution to the total structural systems reliability. Designers advancing these new methods must develop confidence and bridge the natural resistance to change through similar skills, common culture, and verification schemes. The current level of technical maturity and adequacy of reliability analyses techniques varies widely with different materials and failure modes. One of the more mature areas is the quasi-static reliability analyses of polycrystalline materials. This area often comprises the bulk of vehicle inert mass (such as wings, airframes, propellant tanks, skirts, intertanks, etc.) to be optimized. This paper documents an engineering approach and analyses criteria for bridging deterministic structural processes and safety factor assessment with probabilistic input-output data to enable calculation of verifiable quasi-static reliability relationships for metallic substructures.

There was evidence in 1932 that components of successfully designed airplanes did not yield. Since the common structural material was 17ST aluminum alloy having an ultimate-to-yield stress ratio of 1.5, the arbitrary 1.5 safety factor at fracture was universally accepted.<sup>1</sup> Using improved aluminum alloys, and involving historically driven programmatic requirements for optimized performance, the 1.4 safety factor at fracture and 1.0 at yield are now the official NASA standards.<sup>2</sup> These conventional deterministic safety factors are expressed as the ratio of the minimum material resistive stress and maximum allowed applied stress,

$$SF = \frac{S_R}{S_A} \quad (1)$$

Deterministic safety factors may be typically defined at all stages of material degradations (yield, fracture, fatigue, etc.), and at present, safety factors are commonly specified at the substructure level only. This practice implies that if each substructure is designed to a safe margin, then their integrated margins should also produce a safe structural system margin, though the level of system safety is not quantified. But as structural systems become more complex and cost more important, the reliability of each substructure must be defined to compute and optimize the system risk. Nevertheless, the resistance to change remains, as deterministic safety factors are easily incorporated onto applied loads, and structural responses are readily verified through static tests. Over time, structural analysts have developed many quasi-static deterministic techniques and processes and have built great confidence in them. They have served the aerostructural community very well through many years of progressive changes in associated technique.

However, a recent study<sup>3</sup> revealed that current deterministic practices are inherently nonuniformly conservative, while probabilistic structural methods produce leaner structural designs by distributing dispersions in a quantifiable and explicit manner. This discovery, the need to support system reliability with challenging cost mandates, and adaptation of existing deterministic processes and verification methods may provide sufficient inducement to change. Therefore, the emphasis in this study was to develop criteria that implements probabilistic structural data in statistical format throughout the prevailing deterministic process, quantifies substructural risk predictions, and then experimentally verifies the risk predictions with measurable confidence. While many of the techniques previously have been developed and reported, they are repeated here for process continuity, understanding, and convenience.

## 2. DATA CHARACTERIZATION

Data modeling and handling is central to this study. Models are idealized into the simplest mathematical expressions within the physical phenomena of the data and its intended application. Not all data are equally important, as noted by Pareto's principal<sup>4</sup> and as can be distinguished by sensitivity analyses. Data deviations having negligible effect on performance may be reduced to deterministic values. Data deviations having major consequences must be characterized and processed in statistical format throughout the computational process with particular attention to combining their deviations.

Consider the minimum resistive and maximum applied stresses in equation (1), which are derived from observed material and environmental raw data, respectively. The best approach to summarizing a table of data of any distribution is to define the mean about which the data is scattered,

$$\mu = \frac{1}{n} \sum_{i=1}^n x_i . \quad (2)$$

A measure of the actual variation in the set of data is the square root of the variance known as the standard deviation,

$$\sigma = \left[ \frac{1}{n-1} \sum_{i=1}^n (x_i - \mu)^2 \right]^{\frac{1}{2}} . \quad (3)$$

The coefficient of variation is a relative variation or scatter among sets and is defined as the ratio of the standard deviation and the mean,

$$\eta = \frac{\sigma}{\mu} . \quad (4)$$

The coefficient of variation is an effective technique for supporting judgment through comparison with other known events. Coefficients of variation are known to be small for biological phenomena, but often large for natural materials properties. Material property coefficients of variation are generally small for highly controlled manmade materials but are larger for brittle materials. A knowledge of typical coefficients of recurring sources may serve as a guide for judging quality and acceptability of data. Estimates of some common material structural properties characterized by coefficients of variations are 0.05 for metal ultimate strengths and 0.07 for yield stress and for steel fracture toughness.

Another technique used to evaluate raw data is the population probability density distribution, which defines the area shape of the distribution to estimate the probability of a desired value for an assigned range of probability. As shapes become more complex, distribution models become more difficult,

and skills and labor to apply them escalate. Normal distribution is the most widely used because it is the easiest, is the most developed theory, and is representative of most metallic mechanical properties. The normal distribution,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp - \frac{1}{2} \left[ \frac{x_i - \mu}{\sigma} \right]^2 ,$$

for  $x \geq 0$  is noted to be completely defined by only two variables,  $\mu$  and  $\sigma$ . The mean of “ $n$ ” independent observations is believed to approach a normal distribution as “ $n$ ” approaches infinity (central limit theory). Furthermore, for many design considerations only one side of an engineering data distribution is required, and area shapes that are not normally distributed may be developed into a split normal distribution by constructing a mirror image of the useful side about its peak frequency value. The standard deviation is then readily calculated from the constructed symmetrical distribution. Hence, all structural data in this study were assumed to be generalized into normal probability distributions to benefit from existing first order techniques which simplify and expedite design solutions with negligible adverse consequences.

Analytical advantages in using normal probability distributions are the result of having many characteristics well established. The area within a specified number of standard deviations of a probability density plot represents the proportion of the data population captured. One standard deviation (occurring at the inflection point) about the average of a normal distribution is calculated to capture 68.3 percent of the distributed data tabulated and is illustrated in figure 1. Two standard deviations include 95.5 percent of the data, and three standards include 99.7 percent.

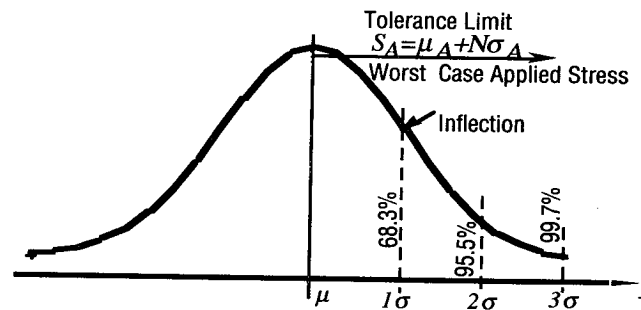


Figure 1. Normal probability density plot.

In structural analysis, normal tolerance limit is a convenient statistical format for specifying and predicting future single observations. The tolerance limit is extended in this study to specify computational input and output data. It defines the normal distribution and specifies a statistical range of deviations about the data's mean. The statistical tolerance limit of the worst case applied stress is noted in figure 1.

### 3. RESISTIVE STRESS

The resistive stress probabilistic distribution is a data characterization of material strength from an appropriate stress test. In many cases, this is a simple uniaxial stress test producing the random sample mean, and standard deviation representing a small proportion of the total population. Given a proportion of the population, the range factor  $k$  of the standard deviation is smaller and the tolerance limits are

$$S = \mu \pm k\sigma .$$

In practice, the population true values of  $\mu$  and  $\sigma$  are not known and so the tolerance limits must be based on the random sample. However, it is possible to determine a consistent  $K$ -factor to assert with a specified confidence that the representation of the population identified in the tolerance limit is statistically valid. In other words, to insure, with a certain percent confidence, that other data from material properties tests conducted on the same number of specimens by different experimenters are contained in the statistically assessed tolerance limit, a  $K$ -factor is specified in the resistive stress tolerance limit,

$$S_R = \mu_R - K\sigma_R , \quad (5)$$

for the lower half (worst case capability) of the distribution and it is plotted in figure 2 to account for the sample size. The  $K$ -factor is shown to decrease as a function of sample size because the uncertainty about the estimates of parameters has narrowed and more reliability can be verified with the same confidence.

This  $K$ -factor is controlled by the designer through the number of test samples specified and often governed by programmatic constraints. Test samples may range from standard uniaxial tensile specimens through more expensive pressure bottles and subscale test articles. The sample size is traded between the initial cost of extensive material sample testing and the recurring cost of lost performance of global structures designed to a larger  $K$ -factor to compensate for small sample property predictions.

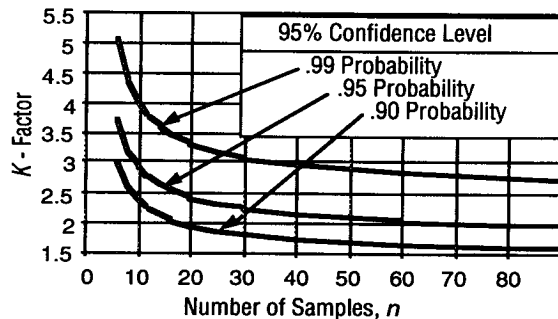


Figure 2.  $K$ -factors for one-sided normal distribution.

Consistent with critical main structures and welds, the stress dispersion is often assumed as  $K=3$ , requiring at least 32 test samples for an A-Basis material. The minimum proportion and degree of confidence must always be designated. An A-Basis property allows that 99 percent of materials produced will exceed the specified value with 95-percent confidence. The B-Basis allows 90 percent with the same 95-percent confidence. Figure 3 illustrates the probability and confidence plot for an A-Basis design.

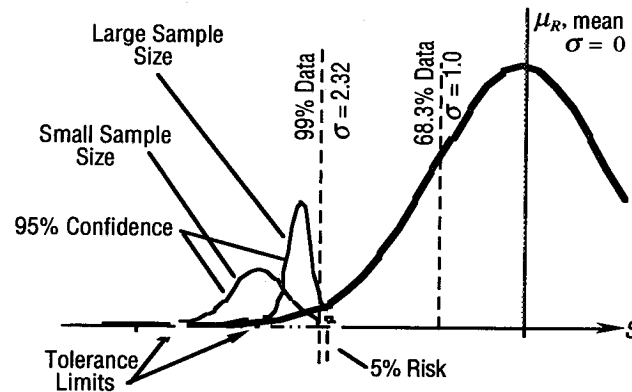


Figure 3. One-sided normal distribution with A-Basis.

Most normally distributed material properties are developed in tolerance limit format as in equation (5). However, they are more often reduced and published as deterministic (single) values that cannot be decomposed again into tolerance limit format as required for reliability analyses. These published deterministic properties are inapplicable for most reliability methods. A preliminary statistical material property may be derived from deterministic data by assuming a typical  $K$ -factor and a common source coefficient of variation.

#### 4. APPLIED STRESS

Aerospace loads modeling uses established computational structural dynamics principles and solution techniques<sup>6</sup> for multi degrees-of-freedom structures. Models assume the structural system to be represented by a network of elements designated along the body possessing mass, damping, and stiffness. Natural and induced environments act as forcing functions at discrete grid points. Launch vehicle forcing functions used to generate ascent generalized forces include: wind speed, shear, gust, and direction; propulsion thrust rise, oscillations, and thrust mismatch; thrust vector control angle and rate; vehicle acceleration and angle of attack; mass distribution; other special trajectory-generated environments. The intensity of these forcing functions vary with events (liftoff, max-q, max-q $\alpha$ , etc.) and time. Static loads include pressure, acoustics, and temperature. The motion of the total quasi-static structure is composed of a system of substructures which are expressed by the linear matrix differential equation

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + [K]\{X(t)\} = \{F(t)\} .$$

The input environments to response analysis are time-dependent and should be statistically characterized. The induced loads output is also time-dependent and of a statistical nature. The resulting internal load,

$$L_g = c_1 F_1 + c_2 F_2 + c_3 F_3 + c_4 F_4 + c_5 F_5 + \dots, \quad (6)$$

is the quasi-static response load at grid point "g" substructure, expressed with forcing functions  $F_i$  in statistical format applied loads

$$F_i = \mu_i + N_i \sigma_i \quad (7)$$

acting along the total structure. The influence coefficients, " $c_i$ ," are time consistent response gains transmitted at grid point  $L_g$ . The applied subjective range factor,  $N_i$ , is autonomously selected by the loads group for each forcing function. The factor may range between 0 and 3, depending on the assumed quality of data, loads sensitivity, and experience. The resulting equation (6) defines a linear combination of the elements of a random vector. An algebraic treatment of the statistical dispersions of equation (7),

$$\sigma_g \neq \frac{1}{\sum_{i=1}^n c_i N_i} \sum_{i=1}^n c_i N_i \sigma_i ,$$

typical within the deterministic method, is clearly a violation of the error propagation law and it poses an inherent source of excessive conservatism.<sup>3</sup> The measure of excess in any region is unique to the types and number of combined dispersions in that region. Conversely, statistically characterized variables that are mutually exclusive may be appropriately defined as a multivariable function by combining their dispersions through the error propagation law.<sup>7</sup>

In applying the error propagation law, let a system performance and its component variables be defined by the formula

$$z = h(y_1, y_2, y_3, y_4, \dots, y_n) .$$

The problem is to obtain an estimate of the system mean, based on the component mean variables. The method consists of expanding the function about each mean by the multivariable Taylor series. The system mean is estimated from

$$\mu_z = h(\mu_1, \mu_2, \mu_3, \mu_4, \dots, \mu_n) , \quad (8)$$

and the variance is approximated to the first order from

$$\sigma_z^2 = \sum_{i=1}^n \left( \frac{\partial h}{\partial y_i} \right)^2 \sigma_i^2 . \quad (9)$$

Hence, when two or more independent variables are added or subtracted, their means are added or subtracted, and their standard deviations are root-sum-squared (RSS). Applying this rule to the sum of a string of tolerance limit loads encountered in equations (6) and (7) gives the combined mean,

$$\sigma_g = \frac{1}{\sum_{i=1}^n c_i N_i} \left[ \sum_{i=1}^n (c_i N_i \sigma_i)^2 \right]^{\frac{1}{2}} . \quad (10)$$

and combined standard deviation

$$\sigma_g = \frac{1}{\sum_{i=1}^n c_i N_i} \left[ \sum_{i=1}^n (c_i N_i \sigma_i)^2 \right]^{\frac{1}{2}} . \quad (11)$$

This computational process is repeated for each substructure grid point and for each unique event time, producing a free-body diagram of the included substructure experiencing maximum probabilistic load response components. The end product of the structural response to environmental excitations is a set of maximum probabilistic design loads, or "limit probabilistic loads," and event times for all the system substructures. The probabilistic applied stress components

$$S_{g,xyz} = \mu_i + N_i \sigma_i , \quad (12)$$

are computed at each grid, and a probabilistic failure criteria is derived.

In order that applied stress components acting at any grid point may be interfaced to the resistive uniaxial stress in equation (1), the applied triaxial stress components must first be reduced into one dimensional (resultant) stress and then indexed to an equivalent uniaxial strength. The complex state of stress at



a point on an oblique surface of a solid may be readily derived<sup>8</sup> by modeling the three normal principal stress components acting along the orthogonal principal axes of a tetrahedron. The sum of forces along each axis provides three linear homogeneous equations to be solved simultaneously. A nontrivial solution of stress on the oblique surface is obtained by setting the resulting determinant of the stress coefficients to zero. The solution to the determinant is reduced to a cubic equation having three combinations of component stresses as coefficients  $I_i$  of the oblique normal stress,

$$S^3 = I_1 S^2 - I_2 S - I_3 = 0 ,$$

known as invariants. The first invariant is the sum of the determinant diagonal which relates to the hydrostatic stress,

$$I_1 = S_1 + S_2 + S_3 ,$$

with a mean stress of  $S_{mean} = I_1/3$ . The second invariant is the sum of the principal minors,

$$I_2 = \frac{I}{2} [(S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2] ,$$

that relates to shear stress. The third invariant is the determinant of the whole matrix. These invariants of the state of stress are defined in statistical format and are noted to be independent of material properties.

## 5. FAILURE CRITERIA

Proceeding with the deterministic process, currently, there is no theory that directly relates multi-axial stresses with uniaxial yield or ultimate stress. However, there are several criteria in which the elastic limit of a multi-axial stress state is empirically related to the uniaxial tensile yielding, and results are reasonably consistent with experimental observations. The Mises yield criterion<sup>9</sup> is based on the minimum strain energy distortion theory which supposes that hydrostatic strain (change in volume) does not cause yielding, but changing shape (shear strain) does cause permanent deformation. Hence, the yield criterion relates the experimental uniaxial tensile elastic limit,  $S_{ty}$ , to the principal shear stresses through the square root of only the second invariant of the stress matrix. In using this second invariant, the Mises initiation of yield criterion is expressed in its familiar form by

$$S_{ty} = \frac{1}{\sqrt{2}} \left[ (S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2 \right]^{\frac{1}{2}}, \quad (13)$$

which depends on a function of all three principal shear stresses. Because of squared terms, it is independent of stress signs and, therefore, it is applicable to compression and tensile combinations of multi-axial stresses. And because of isotropy, the second invariant implies that it is independent of selected axes and may be expressed about any oblique plane

$$S_{ty} = \left[ S_x^2 + S_y^2 + S_z^2 - S_x S_y - S_x S_z - S_y S_z + 3(S_{xy}^2 + S_{xz}^2 + S_{yz}^2) \right]^{\frac{1}{2}}. \quad (14)$$

Using equation (14), the pure shear yield stress reduces to  $S_{sy} = \frac{S_{ty}}{\sqrt{3}}$  and is a good approximation of test results. Having established the yield stress by equations (13) and (14), the criterion also expresses the equivalent applied uniaxial tensile stress over the total elastic and inelastic range about that yield stress,

$$S_{equiv.} = \left[ S_x^2 + S_y^2 + S_z^2 - S_x S_y - S_x S_z - S_y S_z + 3(S_{xy}^2 + S_{xz}^2 + S_{yz}^2) \right]^{\frac{1}{2}}. \quad (15)$$

Recalling that the local multi-axial stresses are in statistical format,

$$S_i = \mu_i + N_i \sigma_i, \quad (16)$$

the probabilistic Mises stress of equation (15) may be appropriately combined through the error propagation laws by expanding the functional relationship in a multivariable Taylor series about a design point (mean) of the system. The mean of the Mises combined applied stresses is determined by substituting equations (16) into (15) and adding the means

$$\mu_A = \left[ \mu_x^2 + \mu_y^2 + \mu_z^2 - \mu_x \mu_y - \mu_y \mu_z - \mu_z \mu_x + 3(\mu_{xy}^2 + \mu_{yz}^2 + \mu_{zx}^2) \right]^{\frac{1}{2}}. \quad (a)$$

The combined standard deviation is calculated from

$$\sigma_A = \left[ \left( \frac{\partial S_A}{\partial S_x} \sigma_x \right)^2 + \left( \frac{\partial S_A}{\partial S_y} \sigma_y \right)^2 + \left( \frac{\partial S_A}{\partial S_z} \sigma_z \right)^2 + 9 \left\{ \left( \frac{\partial S_A}{\partial S_{xy}} \sigma_{xy} \right)^2 + \left( \frac{\partial S_A}{\partial S_{yz}} \sigma_{yz} \right)^2 + \left( \frac{\partial S_A}{\partial S_{zx}} \sigma_{zx} \right)^2 \right\} \right]^{\frac{1}{2}}, \quad (b)$$

and the controlled standard deviation is

$$\begin{aligned} \hat{\sigma}_A = & \left[ \left( \frac{\partial S_A}{\partial S_x} N_x \sigma_x \right)^2 + \left( \frac{\partial S_A}{\partial S_y} N_y \sigma_y \right)^2 + \left( \frac{\partial S_A}{\partial S_z} N_z \sigma_z \right)^2 \right. \\ & \left. + 9 \left\{ \left( \frac{\partial S_A}{\partial S_{xy}} N_{xy} \sigma_{xy} \right)^2 + \left( \frac{\partial S_A}{\partial S_{yz}} N_{yz} \sigma_{yz} \right)^2 + \left( \frac{\partial S_A}{\partial S_{zx}} N_{zx} \sigma_{zx} \right)^2 \right\} \right]^{\frac{1}{2}}. \end{aligned} \quad (c)$$

The probability range factor is calculated from equations (b) and (c)

$$N_A = \frac{\hat{\sigma}_A}{\sigma_A}, \quad (d)$$

and using equations (a) and (b) into (6), the coefficient of variation is

$$\eta_A = \frac{\sigma_A}{\mu_A}. \quad (e)$$

The partials of each term under the radical of equation (15) are given by the chain rule,

$$\frac{d\sqrt{w(S_i)}}{dS_i} = \frac{d\sqrt{w}}{dw} \frac{dw}{dS_i} = \frac{1}{2\sqrt{w}} \frac{dw}{dS_i}. \quad (f)$$

The resulting normal partials are

$$\frac{\partial S_A}{\partial S_x} = \frac{2\mu_x - \mu_y - \mu_z}{2S_A}, \quad \frac{\partial S_A}{\partial S_y} = \frac{2\mu_y - \mu_x - \mu_z}{2S_A}, \quad \frac{\partial S_A}{\partial S_z} = \frac{2\mu_z - \mu_y - \mu_x}{2S_A}, \quad (g)$$

and the shear partials are

$$\frac{\partial S_A}{\partial S_{xy}} = \frac{3\mu_{xy}}{S_A}, \quad \frac{\partial S_A}{\partial S_{yz}} = \frac{3\mu_{yz}}{S_A}, \quad \frac{\partial S_A}{\partial S_{zx}} = \frac{3\mu_{zx}}{S_A}. \quad (h)$$

All partials are evaluated at the system mean. Applying equations (g) and (h) into equation (b) and then into (b), (c), (d), and (e) provides the applied stress parameters of the system in tolerance limit format,

$$S_A = \mu_A(1 + N_A\eta_A), \quad (17)$$

or,

$$S_A = \mu_A + N_A\sigma_A. \quad (18)$$

Having experimentally obtained the probabilistic resistive stress of equation (5), and having calculated the probabilistic applied stress of equation (18), and substituting them into the current deterministic safety factor of equation (1), produces the desired substructure deterministic safety factor in statistical format,

$$SF = \frac{S_R}{S_A} = \frac{\mu_R - K\sigma_R}{\mu_A + N_A\sigma_A}. \quad (19)$$

If the final safety factor of a substructure experiencing maximum operational stress response, equation (19), turns out to be less than the NASA Std. 5001, the substructure must be modified and the probabilistic analysis revised to comply with the specified safety factor. Where near submargins are indicated, increasing the number of material tests to decrease the uncertainty of parameter estimates discussed earlier about the resistive stress  $K$ -factor, may be a preferred option. The option selected would depend on the initial cost of modification, and on recurring costs.

## 6. FAILURE CONCEPT

Having emerged as a critical design parameter, it should be instructive to note how this safety factor plays into the failure concept. Failure occurs when the applied stress on a structure exceeds the resistive stress of the structural material. This simple failure concept integrates the probabilistic interfaces of the material resistive stress with the applied stresses induced from measured environmental data. The probability nature of these interfaces is defined by probabilistic density distributions illustrated in figure 4. Their tail overlap suggests the probability that a weak resistive material will encounter an excessively applied stress to cause failure. The probability of failure is reduced as their tail overlap is reduced through the distributions defined by their normal tolerance limits.

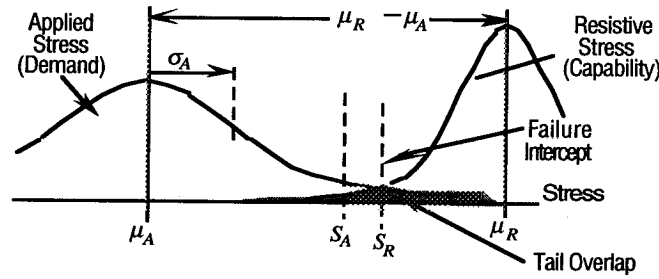


Figure 4. Failure probability concept.

The three-part safety factor expressed in equation (19) is noted to suggest three distinct zones in the failure concept diagram:

- The resistive stress tolerance limit zone:

$$\mu_R - S_R = \mu_R - (\mu_R - K\sigma_R) = K\sigma_R , \quad (20)$$

- The mid or safety factor zone:

$$S_R - S_A = (SF)S_A - S_A = (SF - 1)S_A = (SF - 1)(\mu_A + N\sigma_A) , \quad (21)$$

- The applied stress tolerance limit zone:

$$S_A - \mu_A = (\mu_A + N\sigma_A) - \mu_A = N\sigma_A . \quad (22)$$

The mid zone turns out to be an extension of the applied stress distribution tail produced by the safety factor zone of equation (21),

$$S_R - S_A = S_R - \frac{S_R}{SF} = \left(1 - \frac{1}{SF}\right) S_R .$$

Thus, the primary role of the safety factor in the failure concept is to decrease the applied stress tail overlap by effectively extending the applied stress design tolerance limit in figure 4, and arbitrarily redefining the mean,  $\mu_A$ . Combining equations (21) and (22), the net effective dispersion of the applied stress is

$$N_{eff}\sigma_A = S_R - \mu_A = N\sigma_A + (SF - 1)(\mu_A + N\sigma_A) ,$$

from which the applied stress effective range factor is increased to

$$N_{eff} = \frac{SF - 1}{\eta_A} + N * SF . \quad (23)$$

The effective range factor is noted to be interchangeable with the safety factor. It is also interesting that the probability contributions of the three zones, independently selected by the three disciplines and as defined by equations (20), (21), and (22), are integrated into the difference of the applied and resistive stress distribution means,

$$\mu_R - \mu_A = K\sigma_R + N\sigma_A + (SF - 1)(\mu_A + N\sigma_A) . \quad (24)$$

## 7. FIRST ORDER RELIABILITY

Many advanced techniques are being investigated and are evolving for providing lean, reliable structural designs. In the meantime, assuming split normal probability distributions and defining the safety factor resistive and applied stresses in statistical format as in equation (19) leads to the first order reliability method which is compatible with prevailing practices, codes, and skills.

The concept of failure was introduced in figure 4, in which the distribution tails overlap suggests the probability that a weak resistive material will encounter an excessively applied stress to cause failure. This is to say that the probability of success is reliability and that the reliability is less than 100 percent. Therefore, the probability of interference is the probability of failure and is governed by the difference of their means,  $\mu_R - \mu_A$ . Increasing the difference of the means while holding probability density functions constant decreases the tail interference area and increases the reliability.

Given that the applied and resistive stress probability density functions are independent, they may be combined to form a third random variable density function<sup>10</sup> in  $y = S_R - S_A$ . If  $S_R$  and  $S_A$  are normally distributed random variables, then  $y = S_R - S_A$  are also normally distributed,

$$P_y = \frac{1}{\sigma_y \sqrt{2\pi}} \text{EXP} \left[ -\frac{1}{2} \left[ \frac{y - \mu_y}{\sigma_y} \right]^2 \right], \quad (25)$$

where  $\mu_y = \mu_R - \mu_A$  and  $\sigma_y = \sqrt{\sigma_R^2 + \sigma_A^2}$ . The  $y$ -variable distribution is plotted in figure 5.

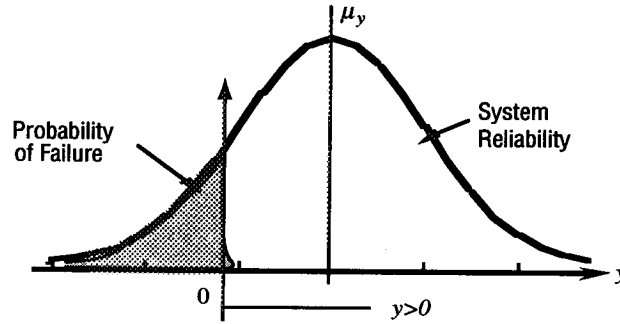


Figure 5. Density function of random variable  $y$ .

The reliability of the third density function expressed in terms of  $y$  is

$$R = P(S_R > S_A) = P(y > 0) = \int_0^{\infty} P_y dy, \quad (26)$$

where  $P_y$  is the  $y$  density function of equation (25). Letting  $Z = \frac{y - \mu_y}{\sigma_y}$ , then  $\sigma_y dz = dy$  and the lower limit of  $Z$  is

$$Z = \frac{K\sigma_R + \sigma_A N_A + (SF - 1)(\mu_A + N_A \sigma_A)}{\sqrt{\sigma_R^2 + \sigma_A^2}}.$$

As  $y$  approaches infinity,  $Z$  approaches infinity, and the reliability of equation (26) is reduced to

$$R = \frac{1}{\sqrt{2\pi}} \int \left[ \exp\left(-\frac{Z^2}{2}\right) \right] dZ. \quad (27)$$

Given the reliability  $R$ , the safety index “ $Z$ ” value is produced, which may then be translated into statistical design parameters through the safety index expression,

$$Z = -Z_l = \frac{\mu_R - \mu_A}{\sqrt{\sigma_R^2 + \sigma_A^2}}. \quad (28)$$

Equation (28) formulates the probability concept. Increasing the safety index and the standard deviations increases the means difference without otherwise affecting the distribution, which decreases tail interference area and the probability of failure. Recognizing that the safety index from equation (28) shares the same difference as of the applied and resistive stress distribution means expressed by equation (24), then substituting it and simplifying reduces the safety index to the desired expression

$$Z = \frac{K\sigma_R + \sigma_A N_A + (SF - 1)(\mu_A + N_A \sigma_A)}{\sqrt{\sigma_R^2 + \sigma_A^2}}. \quad (29)$$

The reliability relationship with the safety index is plotted in figure 6, and the reliability notation  $0.9_n$  represents  $n-9$ 's after the decimal.



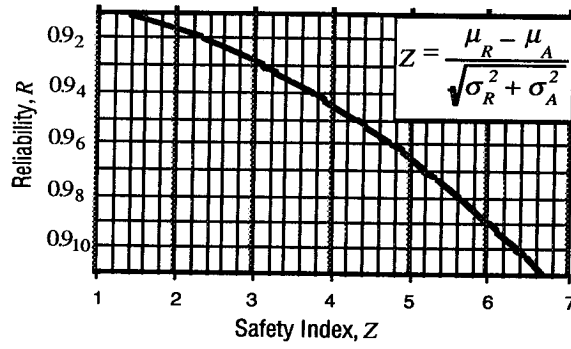


Figure 6. Reliability versus safety index.

This first order safety index provides the structural analysts the faculty of designing substructures to specified reliability (risk) from region to region by adjusting the safety factor to compensate for materials and environmental changes in different regions. It may be a uniform reliability or a series of selected reliabilities associated with maximum levels of elected or systems required risks. It further preserves the current practice to specify the range and safety factors autonomously by loads, materials, and stress specialist, respectively, and then integrates their contributions to the substructure reliability. It retains the current option to design to a standard safety factor and then verifies it.

The reliability may be shown to be an order of magnitude more sensitive to the safety factor than other included parameters,

$$\frac{\partial Z}{Z} = \frac{SF(\mu_a + N\sigma_A)}{K\sigma_R + N\sigma_A + (SF - 1)(\mu_A + N\sigma_A)} \frac{\partial SF}{SF},$$

and its dominance should be a consideration in establishing a reliability criterion.

## 8. RELIABILITY CRITERIA

Developing a reliability criterion is an ongoing concern in the aerostructural community. Arbitrarily selecting a standard reliability, while allowing a lighter-weight vehicle of equivalent system reliability, has no better engineering basis than the current arbitrarily selected standard safety factor. The criterion, ideally, should be based on some compelling physical or economic constraint that precludes debate such as consequence of risk,

$$risk = \sum_k P_k C_k ,$$

where  $P_k$  is the probability of the outcome and  $C_k$  is the consequence of the outcome  $k$ . But risk-consequence assessments have been attempted for decades, requiring great efforts, skills, and data. Results have been too sensitive to simple changes in the course of failure, assumptions, and perceptions.

If current safety factor processes are utilized, the reliability of a structural system for any event and time may still be determined by calculating the probabilistic stresses and safety factor of each substructure in the system for an event and time-consistent environment, and then using equation (29) to determine the reliability contribution of each substructure to an event system reliability. It is expected that designing each substructure to its unique maximum operational environments and to the specified NASA Std. 5001 will result in different structural systems reliabilities at different events, leaving a vehicle with inherent, and unintended, suboptimized weight penalties with respect to structural systems reliability.

Metallic quasi-static structures offer the simplifying attribute of requiring capability only to sustain maximum expected operational overloads environments to satisfy the current NASA Std. 5001 (and, hence, remain traceable to historical practice). There are two potential ways to exploit this attribute and enable leaner vehicle structures. The easiest is to size the structure using a probabilistic set of loads as discussed above. Each substructure must be sized for the maximum operational environments occurring separately on each substructure at different events and times. The reliability analysis then falls out of this sizing technique, and optimization of the structure, consistent with the minimum reliability calculated, may occur, potentially violating the existing NASA Std. 5001 but maintaining a verifiable system level minimum reliability (perhaps consistent with MIL Std. 882C risk assessment considerations).

The other way is to allocate a reliability to the entire structural system, perhaps consistent with a very large sigma no-fail situation. The various components would then have reliabilities allocated through an indentured failure mode, effects, and criticality analysis or fault tree approach to ensure system level considerations. Through this approach, vehicle sizing is a function of the reliability attribution. Other approaches and selected techniques have been promoted and demonstrated for specific systems.<sup>11</sup>

While reliability allocation appears arbitrary, each substructure designed to the NASA Std. 5001 also assumes a reliability. However, the reliability represents an implicit and uncalculated design condition, not a design consideration. The range of reliabilities may be estimated from a parametric analysis. Note that equation (29) consists primarily of applied and resistive stress range factors and coefficients

of variation. Since the resistive stress parameters  $K$  and  $\eta_P$  are usually known from uniaxial test and the applied stress parameters may be estimated from equations (d) and (e), a range of safety indices may be determined from equation (29) for a range of applied and resistive stress parameters ratios. Their reliabilities are obtain from figure 6.

A brief analysis indicated that reliability increases as  $N_A$  increases and decreases as  $\eta_A$  increases. Given a maximum elastic standard safety factor of unity, the safety index expression reduces to

$$Z \geq \frac{K\sigma_R + N\sigma_A}{\sqrt{\sigma_R^2 + \sigma_A^2}} ,$$

and the corresponding reliability range of a structure to yield is around  $0.9_4$ . The reliability of a substructure may be increased for special systems event and time by increasing the range factor  $N$ . Substructure reliability to fracture is in excess of  $0.9_{10}$  but fracture reliability does not represent a design requirement. However, designing to a 1.4 safety factor to fracture is a NASA Std. 5001 requirement.

## 9. RELIABILITY VERIFICATION

Increasing size, complexity, and demand for more reliable and least-cost, high-performance aerospace structures are pressing design analyses, materials, and manufacturing processes to leaner and more innovative methods that often invoke more reliance on experimental verification of their behavior and safety. This compelling shift raises concerns on how thoroughly verification tests are implemented and understood, such as to unknowingly reject a perfectly adequate design or accept a submarginal one, based on a single pass-fail test event.

Fortuitously, the safety factor and reliability relationship of equation (29) allows the structural response and resistive reliabilities to be verified to the same confidence and through the same static test methods and tests as the prevailing deterministic methods. The load versus strain gauge test response verifies that the mechanics model (force-motion) and material properties were properly predicted and that sneak phenomena are screened out. The material response proportion, given by the ratio of equation (20) and (24), and the test response intensity and rate correlation with the mechanics model predictions should help corroborate the activating phenomena. Confirming the yield point verifies the Mises failure criteria application, and assures that the maximum expected environment is operating within the elastic range to satisfy the NASA Std. 5001. The fracture test load should confirm the specified ultimate safety factor of NASA Std. 5001. The fractured surface pattern testifies to the normal, shear, or fatigue type failure mode.

Though safety factors generally are specified at all levels of abrupt material property changes, the safety factor based on polycrystalline yield is difficult to verify. Plastic deformation starts in different locations, numbers, and intensities, and it is hard to detect and determine where and how much deformation has progressed until large enough parts have been affected and detected at specific instrument sites. This phenomenon explains why different gauge lengths in uniaxial tensile tests provide different elastic limits, why yield coefficients of variations are higher than strength variations, and why the elastic limit is more difficult to detect in brittle materials. Exceeding the yield point permanently changes the structural boundary conditions and reduces fatigue life nonlinearly proportional with the exceedence. Therefore, some levels and types of degradation acceptance should be contingent on the consequence of each specific operational case. While the loading instant, location, and nature of yielding may be difficult to experimentally detect, static testing to fracture leaves little doubt.

It should be recalled that the systems of applied static test loads used are the theoretically predicted operational and fracture loads, in which the operational loads can be verified only through flight testing. But real worst-case flight loads may not be realized until late in its flight history. Hence, the 1.4 ultimate safety factor, initially specified to avoid operating in the plastic region of most aerospace structural polycrystalline materials, has subsequently been rationalized to cover such rare operational events in which no statistical design data exists. Its traditional and historical usage now exerts the greatest influence on design and contractually binding acceptance criteria. Furthermore, a test not conducted to fracture provides little information no matter how well the complex structure performs thereafter.

## 10. COST OF RELIABILITY

Reliability of aerostructures is in direct contention with initial and recurring life-cycle cost of all systems engineering design functions, in which the structural designer must optimize the cost of integrating the implied cascading reliability requirements of performance, maintainability, manability, produceability, and availability functions. But quasi-static structures often comprise the bulk of vehicle inert mass, which makes excessive reliability a dominant loss of recurring payload performance, and worthy of a brief observation.

Quasi-static structures are primarily constructed of plates and shell elements in which two of the dimensions that envelop the structural element area are usually optimized for size and shape by the system's operational requirements. The thickness,  $t$ , is controlled by material stress limitations and by the designer-selected safety requirement either from the NASA Std. equation (1) or for a selected reliability specified by equation (29). For example, applying the selected safety factor on the pressure load,  $p$ , a hypothetical tank minimum thickness is approximated from the shape, material physics, and strength of materials theory,

$$t = \frac{p(SF)r}{S_{tu}}$$

The shell weight is then related to the safety factor by

$$W_s = \frac{C_l l r^2 p(SF)}{S_{tu}}$$

The weight performance sensitivity to the safety factor is given by the change in weight to change in the safety factor,

$$\frac{\partial W_s}{W_s} = \frac{C_l l r^2 p \partial(SF) S_{tu}}{C_l l r^2 p(SF) S_{tu}} = \frac{\partial(SF)}{(SF)},$$

resulting in a direct proportionality of 1-percent increase in weight for each percent increase in safety factor. This sensitivity may be a useful rule of thumb for assessing the safety factor penalty to structural element performance subjected to inplane normal stresses.

The ultimate ripple effect of excessive safety factors may be realized from flight performance parameters. Using the well known rocket equation,

$$\Delta V - \Delta V_{loss} = I_{sp} g \ln \frac{W_p}{W_s + W_p + W_{PL}},$$

and assuming the orbital and propulsion parameters are constant, then the mass fraction remains a constant,

$$\text{exp. } \frac{\Delta V - \Delta V_{\text{loss}}}{I_{sp} g} = \frac{W_p}{W_p + W_s + W_{PL}} = C_4 ,$$

and the propellant weight to orbit is

$$W_p = \frac{C_4}{1 - C_4} (W_s + W_{PL}) .$$

The payload and structural weights are interchangeable. The sensitivity of the propellant weight increase to accommodate structural weight increase is

$$\frac{\partial W_p}{W_p} = \frac{W_s}{W_s + W_{PL}} \frac{\partial W_s}{W_s} .$$

Using the weight-to-safety factor relationship developed above, the sensitivity of increased propellant weight consumption to compensate the safety factor increase is

$$\frac{\partial W_p}{W_p} = \frac{W_s}{W_s + W_{PL}} \frac{\partial (SF)}{(SF)} .$$

The ripple effect continues in that increasing the propellant weight further increases the tank size and tank weight, which necessitates more propellant weight, etc. The increased tank size and associated propellant loading facilities represent the initial manufacturing costs. The increased tank and propellant weights to accommodate excess reliability are the recurring costs of lost payload performance. Recognizing the penalties of excessive reliability and potential rippling effects, then it seems not enough for a senior structural analyst to design a reliable structure. His hallmark should be a lean, reliable design such as to create and shift the least excessive conservatism burden downstream onto the vehicle performance and supporting disciplines.

Another source of performance loss is auditing. A static test should prove the article to not be marginally or excessively safe for all the right reasons. A meaningful stress audit should present negative and excessive positive margins and consequences for both cases. While the cause and consequences of negative margins derived from tests are invariably modified, sources and consequences of excessive positive margins are often ignored to avoid design reiterations at the ultimate expense of payload recurring performance loss.

## 11. CONCLUSION

A probabilistic structural design method was developed using an engineering approach to characterize and combine measured structural data into tolerance limit format. Then processing the tolerance limit data through the prevailing deterministic method developed into a probability analysis that lead naturally to a first order reliability technique for quasi-static structures that allows probabilistic optimization and deterministic verification. The probability properties provided the leaner analyses to improve the payload performance and cost. The reliability techniques provided the substructural risk value necessary to calculate its total system reliability. The deterministic methods provided a process that is consistent with current analytical skills, verification practices, and the culture of most structural designers. The complex nature of the structural safety factor was illustrated, and criteria for implementing the probabilistic conversion were developed throughout the process.

## APPENDIX A

The following programs are presented for the structural analysts' information and library. Recognizing that programs are computer and software specific, these are coded in Quick-Basic for their simplicity and application to pocket computers, and because of their easy translation to other languages.

- A.1. Normal probability density distribution program
- A.2. Normality distribution test program
- A.3. Normalizing skewed distribution (Split normal)
  - B. K-factor program
  - C. Safety index programs
  - D. Mises criterion program.

### A.1. Normal Probability Density Distribution Program

```
' NORMAL PROBABILITY DENSITY CURVE
OPEN"CLIP:"FOR OUTPUT AS #1
INPUT "MEAN =",M
INPUT "STD DEVIATION =",SD
INPUT "START =",XS
INPUT "FINISH =",XF
INPUT "INCREMENTS =",NX
DX=(XF-XS)/(NX-1)
FOR I=1 TO NX
  X=XS + (I-1)*DX
  F=EXP(-.5*((X-M)/SD)^2)
  F=F/((2*3.14159*SD)^.5)
  WRITE #3,X,F
  PRINT X,F
NEXT I
CLOSE #1
STOP
```

### A.2. Normality Distribution Test Program

```
' NORMAL DISTRIBUTION TEST
' Kolmogorov-Smirnov (normality test)
' Critical values (n > 30): a=.10, d=.805;
' a=.05, d=.886; a=.01, d=1.03
OPEN"CLIP:"FOR OUTPUT AS #2
```



```

' Input data
CLEAR:INPUT "N=";N
DIM A(N),D(N),Z(N)
FOR I=1 TO N
INPUT A(I)
NEXT I

' sort data
K=N-1
LINE180:FOR X=1 TO K
B=A(X)
IF B<=A(X+1) GOTO line250
A(X)=A(X+1)
A(X+1)=B
Y=1
T=X-1
line250:NEXT X
IF Y=0 GOTO line300
Y=0
K=T
GOTO LINE180
line300:
PRINT "SORT DONE"

'mean and std. deviation
FOR I=1 TO N
C=C+A(I)
D=D+A(I)*A(I)
NEXT I
M=C/N
SD=((D-N*M*M)/(N-1))^.5
PRINT "MEAN=";M
PRINT "STD DEV=";SD

'standardized normal
FOR I=1 TO N
Z(I)=(A(I)-M)/SD
NEXT I

'cumulative normal
FOR I=1 TO N
X=Z(I):T=X
G=EXP(-X*X/2)/SQR(2*3.14159)
A1=.31938:A2=-.35656:A3=1.78147

```

```

A4=-1.82125:A5=1.330427
IF X<0 THEN T=-X
Y=1/(1+.2316419*T)
P=(((A5*Y+A4)*Y+A3)*Y+A2)*Y+A1)*Y
F=1-G*P
IF X<0 THEN F=1-F
DI=I/N      'empirical cumulative
D(I)=ABS (F-DI)
IF DM<D(I) THEN DM=D(I):J=I
NEXT I

'results
FOR I=1 TO N
PRINT D(I)
NEXT I
PRINT "WORST SAMPLE #";J
PRINT "ABS DIFFERENCE, D";DM
CLOSE #2
END

```

Figure 7 is a plot of the "D" critical values for a one-sided distribution. The distribution is not normal if the program test result exceeds the "D" critical value. Most engineering data distributions are one-sided, occurring in the lower or upper sides.

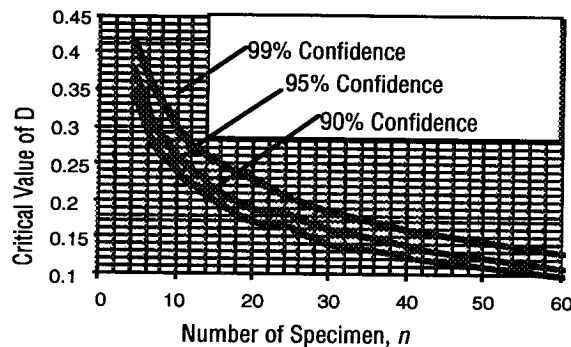


Figure 7. One-sided test critical value of D.

### A.3. Normalizing Skewed Distribution (Split Normal)

Stress data are assumed to be based on a series of observed measurements reduced into a frequency distribution, or probability histograms, shown in figure 8. The base of the histogram is bounded by successive and equal ranges of measured values, and the heights represent the number of observations (frequency) in each range.

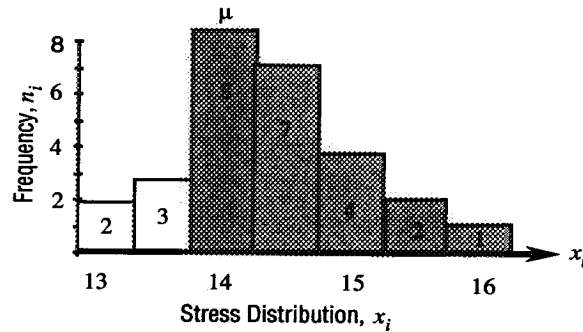


Figure 8. Stress frequency distributions.

To illustrate the direct normalization of a skewed distribution, the stress frequency distribution data of figure 8 is applied to equations (2) through (5). Because the greater stress side defines the worst demand case (applied stress), only data from the shaded right side is used to calculate the normalized distribution variables. The distribution may be normalized by constructing a mirror image of the engaged side about its peak frequency value and calculating the standard deviation from the constructed symmetrical distribution.

The peak frequency from figure 8 distribution is the mean,  $\mu = 14$  ksi.

Sample size is  $\sum n = -n_1 + 2\sum n_i = -8 + 2(8 + 7 + 4 + 2 + 1) = 36$ .

Sum of variations about the mean

$$\begin{aligned}
 v &= 2\sum n_i(x_i - \mu)^2 \\
 2 \times 8(14.0 - 14.0)^2 &= 0 \\
 2 \times 7(14.5 - 14.0)^2 &= 3.5 \\
 2 \times 4(15.0 - 14.0)^2 &= 8.0 \\
 2 \times 2(15.5 - 14.0)^2 &= 9.0 \\
 2 \times 1(16.0 - 14.0)^2 &= 8.0 \\
 v &= 28.5.
 \end{aligned}$$

The variance is  $\sigma^2 = \frac{v}{\sum n-1} = \frac{28.5}{35} = 0.81$ ,

and the standard deviation from equation (3) is  $\sigma = 0.90$  ksi.

The coefficient of variation from equation (40) is  $\eta = \frac{\sigma}{\mu} = \frac{0.90}{14} = 0.065$ .

```

'SPLIT NORMAL DISTRIBUTION
'INPUT DATA
CLEAR: INPUT "PEAK FREQUENCY=",MU
INPUT "NUMBER OF BARS=",N
DIM F(N), X(N)
FOR I=1 TO N
INPUT F(I)
NEXT I
FOR I=1 TO N
INPUT X(I)
NEXT I

'CACULATE
S=0:FOR I=2 TO N
S=S+2 *F(I)
NEXT I
SS=S+F(1)
PRINT"SAMPLE SIZE=",SS
VMU=0:FOR I=1 TO N
VMU=VMU+2*F(I)*(X(I)-MU)^2
NEXT I
PRINT"VARIATIONS FROM MEAN=",VMU
PRINT "MEAN=",MU
SD=(VMU/(SS-1))^0.5
PRINT "STD DEV=",SD
COF=SD/MU
PRINT"COEF OF VAR=",COF

```

## B. K-Factor Program

```
'K- FACTOR
MARIO:
DEFDBL A-Z
INPUT "SAMPLE SIZE=";NS
INPUT "PROPORTION";P
INPUT "CONFIDENCE=";CL
IF NS>90 THEN PRINT "SAMPLE SIZE SHOULD
BE SMALLER THAN 90":WHILE INKEY$="" :WEND
START=TIMER
PI=3.141592654#
'INVERSE NORMAL
Q=1-P:T=SQR(-2*LOG(Q))
A0=2.30753:A1=.27061:B1=.99229:B2=.0481
NU=A0+A1*T:DE=1+B1*T+B2*T*T
X=T-NU/DE
L0: Z=1/SQR(2*PI)*EXP(-X*X/2):IF X>2 GOTO L3
V=25-13*X*X
FOR N=11 TO 0 STEP -1
U=(2*N+1)+(-1)^(N+1)*(N+1)*X*X/V
V=U:NEXT N
F=.5-Z*X/V
W=Q-F:GOTO L2
L3:V=X+30
FOR N=29 TO 1 STEP -1
U=X+N/V
V=U :NEXT N
F=Z/V :W=Q-F :GOTO L2
L2:L=L+1
R=X:X=X-W/Z
E=ABS(R-X)
IF E>.00001 GOTO L0
'END OF INVERSE NORMAL

'CALCULATION OF FACTORIAL
N=NS:NU=N-1
MT=INT(NU/2):UT=NU-2*MT
GT=1
FOR I=1 TO MT-1+UT
KT=I
IF UT=0 GOTO L1
KT=I-.5
L1:GT=GT*KT
```

```

NEXT I
GT=GT*(1+UT*(SQR(PI)-1))
GF=GT*2^(NU/2-1)
'END OF FACTORIAL

'SECANT METHOD
KP=X:J=1:K=KP
K0=K:GOSUB INTEGRATION:SF0=SF
K=K*(1+.0001):K1=K:GOSUB
INTEGRATION:SF1=SF
BEGIN:K=K1-SF1*(K1-K0)/(SF1-SF0)
IF ABS((K1-K)/K1)<.000001 GOTO RESULT
J=J+1:K0=K1:K1=K:SF0=SF1
GOSUB INTEGRATION:SF1=SF:GOTO BEGIN
RESULT:FINISH=TIMER
BEEP:BEEP
PRINT "K =";USING"###.####";K
PRINT "TIME=";FINISH-START;"SECONDS"
'END OF SECANT METHOD
WHILE MOUSE(0)<>(1):WEND
GOTO MARIO

```

```

INTEGRATION:L1=0:L2=10
IF N>40 THEN L2=20
DL=KP*SQR(N):TP=K*SQR(N)
Y=NU/2
M=2:E=0:H=(L2-L1)/2
X=L1:GOSUB FUNCTION
Y0=Y:X=L2:GOSUB FUNCTION
YN=Y:X=L1+H:GOSUB FUNCTION
U=Y:S=(Y0+YN+4*U)*H/3
START:M=2*M
D=S:H=H/2:E=E+U:U=0
FOR I=1 TO M/2
X=L1+H*(2*I-1):GOSUB FUNCTION
U=U+Y
NEXT I
S=(Y0+YN+4*U+2*E)*H/3
IF ABS((S-D)/D)>.00001# GOTO START
SF=S/GF-CL
RETURN
'END OF SIMPSON

```

```

FUNCTION: Z=TP*X/SQR(NU)-DL
T0=Z:G0=1/SQR(2*PI)*EXP(-Z*Z/2)
A1=.3193815:A2=-.3565638:A3=1.781478:
A4=-1.821256:A5=1.330274
IF Z<0 THEN T0=-Z
W=1/(1+.231649*T0)
P1=(((A5*W+A4)*W+A3)*W+A2)*W+A1)*W
PH=1-G0*P1
IF Z<0 THEN PH=1-PH
Y=PH*X^(NU-1)*EXP(-X*X/2)
RETURN

```

### C. Safety Index Programs

‘SAFETY INDEX FROM RELIABILITY

```

‘NORMIN (.5,P,1)
DEFDBL A-Z
LL: INPUT"Probability=";P
PI=3.141593
PI=3.141593
Q=1-P:T=SQR(-2*LOG(Q))
A0=2.30753:a1=.27061
B1=.99229:B2=.0481
NU=A0+a1*T
DE=1+B1*T+B2*T*T
X=T-NU/DE

```

```

‘CUMULATIVE NORMAL
L0: Z=1/SQR(2*PI)*EXP(-X*X/2)
IF X>2 GOTO L1
V=25-13*X*X
FOR N=11 TO 0 STEP-1
U=(2*N+1)+(-1)^(N+1)*(N+1)*X*X/V
V=U:NEXT N
F=.5-Z*X/V
W=Q-F
GOTO L2
L1:V=X+30
FOR N=29 TO 1 STEP-1
U=X+N/V
V=U:NEXT N
F=Z/V:W=Q-F:GOTO L2

```

```

L2:L=L+1
R=X:X=X-W/Z
E=ABS(R-X)
IF E>.001 GOTO L0
PRINT "SAFETY INDEX IS"
PRINT USING "##.####";X
GOTO LL
END

```

'RELIABILITY FROM SAFETY INDEX

```

'NORMIN (0.5,P,1)
DEFDBL A-Z
'INPUT"P=";P:PI=3.141593
PI=3.141593
'Q=1-P:T=SQR(-2*LOG(Q))
'A0=2.30753:A1=.27061
'B1=.99229:B2=.0481
'NU=A0+a1*T
'DE=1+B1*T+B2*T*T
'X=T-NU/DE

```

'CUMULATIVE NORMAL

```

INPUT"X=";X
Z=1/SQR(2*PI)*EXP(-X*X/2)
IF X>2 GOTO L1
V=25-13*X*X
FOR N=11 TO 0 STEP-1
U=(2*N+1)+(-1)^(N+1)*(N+1)*X*X/V
V=U:NEXT N
F=.5-Z*X/V:F=1-F
GOTO L2
L1:V=X+30
FOR N=29 TO 1 STEP-1
U=X+N/V
V=U:NEXT N
F=Z/V:F=1-F
L2:
PRINT F
END

```



## D. Mises Criterion Program

```
'ERROR PROPAGATION METHOD;
' MISES CRITERION
DEFDBL A-Z
INPUT "NUMBER OF NORMAL STRESS=" ,NS
DIM STATIC NSM(3),NSSD(3),NSNF(3),
NSFD(3),LNS(3)

FOR I=1 TO NS
PRINT "NORMAL LOAD MEAN(";I;)"=
INPUT NSM(I)
PRINT "NORMAL LOAD STD. DEVIATION(";I;)"=
INPUT NSSD(I)
PRINT "NORMAL LOAD N-FACTOR(";I;)"=
INPUT NSNF(I)
NEXT I

INPUT "NUMBER OF SHEAR STRESSES=" ,MS
DIM STATIC SSM(3),SSSD(3),SSNF(3),SSFD(3),
LSS(3)
FOR I=1 TO MS
PRINT "SHEAR LOAD MEAN(";I;)"=
INPUT SSM(I)
PRINT "SHEAR LOAD STD. DEVIATION(";I;)"=
INPUT SSSD(I)
PRINT "SHEAR LOAD N-FACTOR(";I;)"=
INPUT SSNF(I)
NEXT I

'CALCULATION OF SYSTEM MEAN
S1=0:FOR I=1 TO NS:S1=S1+NSM(I)^2:NEXT I
S2=0:FOR I=1 TO MS:S2=S2+SSM(I)^2:NEXT I
MZ1= S1-NSM(1)*NSM(2)-NSM(1)*NSM(3)
MZ2=MZ1-NSM(2)*NSM(3)+3*S2
MZ= SQR(MZ2)
'CALCULATION OF DERIVATIVES
NSFD(1)=(2*NSM(1)-NSM(2)-NSM(3))/2/MZ
NSFD(2)=(2*NSM(2)-NSM(1)-NSM(3))/2/MZ
NSFD(3)=(2*NSM(3)-NSM(1)-NSM(2))/2/MZ
FOR I=1 TO MS:SSFD(I)=3*SSM(I)/MZ:NEXT I
```

```

'CALCULATION OF SUM OF SQUARES OF
' NORMAL STRESSES
S3=0:S4=0:FOR I=1 TO NS
S3=S3+(NSFD(I)*NSSD(I))^2
S4=S4+(NSNF(I)*NSFD(I)*NSSD(I))^2
NEXT I

'CALCULATION OF SUM OF SQUARES OF
' SHEAR STRESSES
S5=0:S6=0: FOR I=1 TO MS
S5=S5+(SSFD(I)*SSSD(I))^2
S6=S6+(SSNF(I)*SSFD(I)*SSSD(I))^2
NEXT I

'CALCULATION OF SYSTEM STANDARD
'AND EFFECTIVE DEVIATIONS
SZ=SQR(S3+S5):SN=SQR(S4+S6)
NE=SN/SZ

'CALCULATION OF SYSTEM COEFFICIENT
' OF VARIATION
ETA=SZ/MZ

'CALCULATION OF SYSTEM TOLERANCE LIMIT
TL=MZ+(NE*SZ)

'CALCULATION OF MISES FUNCTION
FOR I=1 TO NS
LNS(I)=(NSM(I)+NSNF(I)*NSSD(I))^2
NEXT I
FOR I=1 TO MS
LSS(I)=(SSM(I)+SSNF(I)*SSSD(I))^2
NEXT I
FM1=0:FOR I=1 TO NS
FM1=FM1+LNS(I):NEXT I
FM2=0:FOR I=1 TO MS
FM2=FM2+LSS(I):NEXT I
FM= SQR(FM1+3*FM2)

PRINT "COMBINED APPLIED STRESSS =" ;FM
PRINT "MEAN =" ;MZ
PRINT "STANDARD DEVIATION =" ;SZ
PRINT "EFFECTIVE N =" ;NE
PRINT "COEFFICIENT OF VARIATION =" ;ETA
PRINT "TOLERANCE LIMIT =" ;TL

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