NAG3-1864: Stability Limits and Dynamics of Nonaxisymmetric Liquid Bridges
Final report

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Objectives:

- Theoretical and experimental investigation of the stability of nonaxisymmetric and nonaxisymmetric bridges contained between equal and unequal radii disks as a function of Bond and Weber number with emphasis on the transition from unstable axisymmetric to stable nonaxisymmetric shapes.
- Numerical analysis of the stability of nonaxisymmetric bridges for various orientations of the gravity vector for equal and unequal disks.
- Experimental and theoretical investigation of large (nonaxisymmetric) oscillations and breaking of liquid bridges.

Task Description:

This project involves both experimental and theoretical work. Static and dynamic experiments are conducted in a Plateau tank which makes a range of static Bond numbers accessible. Theoretical investigation includes both analytical and numerical approaches.

Task Significance

A liquid bridge, or captive drop, is a mass of liquid held by surface tension between two or more solid supports. Liquid bridges occur in a variety of physical and technological situations and a great deal of theoretical and experimental work has been done to determine axisymmetric equilibria for various disk configurations, bridge aspect ratios and rotations. There have also been numerous investigations of the dynamics of axisymmetric liquid bridges subject to different excitations (impulses, vibration, etc.). Such investigations have been motivated both by practical considerations and basic scientific interest. Liquid bridges and drops are important factors when considering propellant management in liquid fuel chambers and in the positioning of liquid masses using surface tension forces. In crystal growth, they are associated with the floating-zone growth technique. Their oscillation and relaxation properties can also be used for viscosity and surface tension measurements of molten materials at high temperatures. Pendular liquid bridges occur widely in the powder technology industry and are a major influence on powder flow process and mechanical properties. In porous media flow, liquid-liquid displacement can lead to evolution of pendant and sessile lobes or lenticular bridges. The formation of liquid bridges from the gel that coats lung micro-airways results in occlusion of the bronchioles and is a precursor to respiratory problems and lung collapse. In addition, liquid bridges have been involved in a number of past microgravity and our research will provide results useful for the quantitative assessment of g-jitter effects on such experiments.

Progress during the period of performance 6/97-2/98

During this period we accomplished the following tasks

- We completed a study of the stability (with respect to arbitrary perturbations) of liquid bridges between unequal disk radii subject to axial gravity on the entire boundary of stability [3]. The stability was examined for all possible values of the liquid volume and disk separation. The parameter defining the disk inequality is K, the ratio between the radii of the smaller and larger disks. Both axisymmetric and nonaxisymmetric perturbations were considered. The parameter space chosen to delimit the stability regions is the Λ-V plane. Here, Λ is the slenderness (ratio of the disk separation to the mean diameter, $2r_0$, of the two support disks), and V is the relative volume (ratio of the actual liquid volume to the volume of a cylinder with a radius equal to r_0). Wide ranges of the Bond number and the ratio K are considered. Emphasis is given to previously unexplored parts of the stability boundaries. In particular, we examined the maximum volume stability limit for bridges of arbitrary Λ and the minimum volume stability limit for small Λ bridges. The maximum volume stability limit was found to have two distinct properties: large values of the critical relative volume at small Λ, and the possibility that stability is lost to axisymmetric perturbations at small values of K. For selected K- values, the maximum Bond number beyond which stability of the bridge is no longer possible for any combination of V and Λ was determined. In addition, the maximum value of the actual liquid volume of a stable bridge that can be held between given disks for all possible disk separations was examined for fixed Bond number. This volume decreases as K decreases and (depending on the sign of the Bond number) tends to the critical volume of a sessile or pendant drop attached to the larger disk. In addition to numerical analysis of this problem, Plateau tank experiments have been performed (for various disk radii) that investigate the theoretically predicted stability limits.

- Further theoretical and experimental investigations on the behavior of the weightless bridge when its axisymmetric shape loses stability were undertaken to investigate our previously obtained theoretical work [4]. The bifurcation of the solutions of the nonlinear equilibrium problem of a weightless liquid bridge with a free surface pinned to the edges of two coaxial equidimensional circular disks is examined. The bifurcation is studied in the neighborhood of the stability boundary for axisymmetric equilibrium states with emphasis on the boundary segment corresponding to nonaxisymmetric critical perturbations. The first approximations for the shapes of the bifurcated equilibrium surfaces are obtained. The stability of the bifurcated states is then determined from the bifurcation structure. Along the maximum volume stability limit, depending on values of the system parameters, loss of stability with respect to nonaxisymmetric perturbations results in either a jump or a continuous transition to stable nonaxisymmetric shapes. The value of the slenderness at which a change in the type of transition occurs is found to be Λ_S = 0.4946. Experimental investigation based on a neutral buoyancy technique agrees with this prediction. It shows that, for $\Lambda < \Lambda_S$, the jump is finite and that a critical bridge undergoes a finite deformation to a stable nonaxisymmetric state.

- Voids and cavities in contained melts and other liquids can occur due to either incomplete filling of the container or due a density change upon freezing or melting. We examined the problem of a melt for which the *wetting angle* α (i.e., the angle formed by the free surface of the liquid at the contact with the *smooth* solid surface) is constant over the entire solid surface (cylinder wall and two circular planar ends). We considered arbitrary values of the wetting angle α, and a wide range of the ratio between the volumes of the liquid and vapor phases. If a connected free surface does not cross the axis of symmetry of the container then it is called a

doubly connected surface. Such a situation might arise when the melt forms a liquid bridge. We considered the shape and stability of a doubly connected surface having contact lines on a lateral cylindrical wall. For this case it was found that only unduloidal free surfaces corresponding to $\alpha > 120°$ can be stable. The stability of such surfaces depends on the value of the dimensionless volume of the vapor cavity $\bar{V}^V$. For a given value of α in the range $120° < \alpha < 180°$, there exists a minimum volume stability limit $\bar{V}^V_{min}$ and a maximum volume stability limit $\bar{V}^V_{max}$. If $\bar{V}^V < \bar{V}^V_{min}$, loss of stability occurs to nonaxisymmetric perturbations, and the vapor cavity shifts to the side of the container. If $\bar{V}^V > \bar{V}^V_{max}$, the stability is lost to axisymmetric perturbations, and the liquid breaks into two disconnected portions. An important feature of the above results is that in the lower unstable region, loss of stability to nonaxisymmetric perturbations will lead to a void that appears to favor one side of the ampoule. In zero or weak gravity the location of this void will be governed by surface tension or surface energy considerations and will not depend on the orientation of the weak gravity vector. In this case any conclusions about the orientation of the quasi-steady low gravity vector that are based on the location of a void will likely be erroneous. This work is still in progress.

- A joint project with J. Meseguer, E.T.S.I. Aeronáuticos, Universidad Politécnica de Madrid, Spain, and F. Zayas, E.T.S.I. Industriales, Universidad de Extremadura, Badajoz, Spain involving an experimental and theoretical study of long liquid bridges subject to nonaxial gravity has shown that there is a self-similar solution to the stability limits for such bridges so that the stability behavior is independent of the slenderness or volume. The expression for the stability limit can be written in a compact form using the reduced axial and lateral Bond numbers defined as follows

$$b_a = (3/2)^2 \lambda^{-3/2} B_a, \tag{1}$$
$$b_l = (\pi/2)\lambda^{-1/2} B_l, \tag{2}$$

where $\lambda = 1 - \Lambda_{max}/\pi + v2$, and $\Lambda_{max} = \pi\{1 - (3/2)^{4/3} B_a{}^{2/3} + v/2 - (\pi B_l)^2/4\}$; and $v = V - 1$ where V is the ratio of the actual bridge volume to the volume of a right circular cylinder. The stability limit is given by the curve

$$b_l = (1 - (b_a)^{2/3})^{1/2}. \tag{3}$$

Transfer of PI to Case Western Reserve University

During this period the PI relocated to Case Western Reserve University (CWRU), Cleveland, Ohio. This involved dismantling the experimental equipment and transporting it to CWRU. The experimental program will be reestablished during the next 3-4 months. During the transition period, Dr. Slobozhanin will remain in Huntsville to avoid interruption to the ongoing theoretical work involving liquid bridge stability in closed containers. Dr. Slobozhanin will keep in close contact with the PI through electronic mail and through visits when necessary.

Publications

Since the start of the grant, five papers have been published in international peer reviewed journals, [1-5], two in reviewed conference proceedings [6,7] and four papers are in preparation [8-11].

[1]   J.I.D. Alexander, S. Delafontaine, A. Resnick, and W.C. Carter, "Stability of nonaxisymmetric liquid bridges, *Microgravity Science and Technology*", **9**, 193-200, 1997.

[2]   L. Slobozhanin, and J.I.D. Alexander, "Stability of an isorotating liquid zone in an axial gravity field", *Physics of Fluids*, **9**, 1880-1892, 1997.

[3]   L. Slobozhanin, J.I.D. Alexander and A. Resnick, "Bifurcation of the equilibrium states of a weightless liquid bridge" , *Physics of Fluids*, **9**, 1893-1905, 1997.

[4]   A. Resnick and J.I.D. Alexander, "An apparatus for the liquid bridge experiments, *Review of Scientific Instruments*", **68**, 1495-1500, 1997.

[5]   J.I.D. Alexander, Y. Zhang, S. Delafontaine and A. Fedoseyev, *"Numerical Simulation of Liquid Bridge Dynamics and Statics"*, in Numerical Methods in Heat and Mass Transfer Problems (Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow, 1997) pp.109-135.

[6]   A. Resnick and J.I.D. Alexander, "A coherent Fourier imaging system for liquid bridge experiments, Optical Pattern Recognition VIII   22-23 April, 1997, *SPIE Proceedings Reprints* Vol. **3073**, 469-477, 1997.

[7]   L. Slobozhanin, J.I.D. Alexander and A. Resnick, "Stability of Liquid bridges under reduced gravity", *Proceedings of the 2nd Symposium on Fluids in Space*, Naples, Italy, April 22-26, 431-439, 1996.

[8]   L. A. Slobozhanin and J. I. D. Alexander, "Combined effect of disk inequality and axial gravity on stability of axisymmetric liquid bridges", accepted, Physics of Fluids, 1998

[9]   L. A. Slobozhanin and J. I. D. Alexander, "Stability of doubly connected axisymmetric free surfaces in a cylindrical container" (in preparation).

[10]   J.I.D. Alexander and Y. Zhang, "A modified VOF method for computing the breaking and merging of liquid bridges" in preparation.

[11]   F. Zayas, J. Meseguer and J.I.D. Alexander, "On the stability limits of non-axisymmetric, cylindrical long liquid bridges" in preparation.

Presentations

J.I.D. Alexander, "Large deformations of liquid bridges", Invited presentation at The Second Symposium on Fluids in Space, Naples, Italy, April 22-26, 1996.

L. Slobozhanin, J.I.D. Alexander and A. Resnick, "Stability of Liquid bridges under reduced gravity", Proceedings of the 2nd Symposium on Fluids in Space, Naples, Italy, April 22-26, 431-439, 1996.

A. Resnick, L. Slobozhanin and J.I.D. Alexander, "Influence of support geometry and gravity on the stability of liquid bridges Division of Fluid Mechanics, American Physical Society, Syracuse, New York, November, 1996.