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MAIN GEOMAGNETIC FIELD MODELS FROM OERSTED AND MAGSAT DATA VIA A RIGOROUS GENERAL INVERSE THEORY WITH ERROR BOUNDS

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This is the final report for NASA grant NAG52967, "Main Geomagnetic Field Models from Oersted and Magsat Data via a Rigorous General Inverse Theory with Error Bounds." The purpose of the grant was to study how prior information about the geomagnetic field can be used to interpret surface and satellite magnetic measurements, to generate quantitative descriptions of prior information that might be so used, and to use this prior information to obtain from satellite data a model of the core field with statistically justifiable error estimates.

The need for prior information in geophysical inversion has long been recognized. Data sets are finite, and faithful descriptions of aspects of the earth almost always require infinite-dimensional model spaces. By themselves, the data can confine the correct earth model only to an infinite-dimensional subset of the model space. Earth properties other than direct functions of the observed data cannot be estimated from those data without prior information about the earth.

Prior information is based on what the observer already knows before the data become available. Such information can be "hard" or "soft". Hard information is a belief that the real earth must lie in some known region of model space. For example, the total ohmic dissipation in the core is probably less that the total observed geothermal heat flow out of the earth's surface. (In principle, ohmic heat in the core can be recaptured to help drive the dynamo, but this effect is probably small.)

"Soft" information is a probability distribution on the model space, a distribution that the observer accepts as a quantitative description of her/his beliefs about the earth. The probability distribution can be a subjective prior in the sense of Bayes or the objective result of a statistical study of previous data or relevant theories. (So far, dynamo theory has not yielded such objective statistical predictions.)

The theoretical framework for the studies carried out under NAG52967 is given in reference 3. That framework includes a quantitative description of the effects of truncating the model space to finitely many dimensions and of propagating the data errors into the model. The framework is sufficiently general to handle both hard and soft prior information, but the work under NAG52967 has focussed on soft priors.

One of our first concerns was whether statistical (soft) priors might be inappropriate for a geomagnetic field **B** that is highly structured at the core-mantle boundary (CMB). In reference 1, Andrew Walker and the Principal Investigator studied core spots, single point sources of flux on the CMB with return flux uniform over the CMB. We found that the 165 spherical harmonic (gauss*) coefficients of B_r at the CMB with degrees between 2 and 12 passed the Kolmogorov-Smirnov (K-S) test for independent, identically normally* distributed random variables with 0 mean. Such gauss coefficients are characteristic of spatial white noise. A small exceptional area of the CMB contained core spots whose gauss coefficients failed the K-S test. The location of this small exceptional region was determined by the coordinate system used to define the spherical harmonics.

The earth's observed gauss coefficients have passed many tests for randomness, with statistical distributions invariant under rotations about the center of the earth. In reference 2, Andrew Walker and the P.I. applied one test (essentially a chi-square test) that suggests a failure of rotational invariance. We found a statistically significant difference between the average values of B_r^2 on the Atlantic and Pacific hemispheres of the CMB, if B_r was calculated from the gauss coefficients with degrees between 2 and 12. In this and other statistical work on the CMB we have ignored the gauss coefficients of degree 1 because the dipole field is well-known to be anomalously strong. We terminated the Atlantic-Pacific comparison at degree 12 to minimize effects of the crustal field.

Based on what was learned in references 1 and 2, we tried to construct a statistical description of the geomagnetic field which could be used as a soft prior for inverting satellite data. That investigation is described in detail in reference 5. Here we give a brief summary. Andrew Walker and the P.I. found a six-parameter gaussian* statistical model of the non-dipole geomagnetic field **B** that fit the observed distribution of 2597 gauss coefficients determined from MAGSAT by Cain, Holter and Sandee (J. Geomag. Geoelect. **42**, 973–987). One parameter in the model is the variance of the satellite errors of measurement, two parameters describe A. Jackson's simplest spatial white-noise model of crustal magnetization (Geophys. J. Int. **103**, 657–674), two parameters describe the strength and apparent depth of a spatial white noise source near the CMB, and one parameter describes the failure of the statistics of the crustal and/or CMB geography to be invariant under rotations about the center of the earth. The data used in the study are insensitive to external fields, and our statistical model gives no information about those fields.

The six parameters of our model normal distribution for the gauss coefficients were fitted to the gauss coefficient data by maximum likelihood. A modified K-S test showed that the statistical model adequately described the data. The K-S test had to be modified by Monte Carlo methods because the classical K-S test assumes that the statistical distribution to be tested is completely known. In the classical test no distribution parameters can be obtained from the data.

The model gives 31 nT^2 as the variance of the errors of measurement. Langel, Ousley & Berbert (Geophys. Res. Letters **9**, 243–245) predicted 34 nT^2 by analysing the known sources of error: magnetometers and satellite position and orientation.

The model gives 2940 nT [2260 nT] for the root-mean-square non-dipole [radial] magnetic field at the CMB. The apparent radius of the white-noise sphere puts that sphere 325 km below the CMB. We show that this discrepancy can be interpreted to mean that the geodynamo produces at the CMB not spatial white noise but a signal with a horizontal correlation length of about 525 km.

The magnetizable part of the crust must be shallower than 40 km in order not to exceed the relevant blocking temperatures. Our model puts the white-noise crust 300 km below the earth's surface. We interpret this depth to crustal white noise as a measure of the horizontal correlation in the statistics of the true crustal magnetization.

In our statistical model the gauss coefficients must have non-zero means. We take this to be an effect of the core and crustal geography. It is remarkable that the magnetic statistics of this geography can be described by a single parameter.

The work described in references 1, 2, 3 and 5 enabled us to carry out an actual inversion of satellite data to produce gauss coefficients for the core field with statistically justifiable error estimates. We had been hoping to use Oersted data for this work, but launch delays forced us to use some of the old MAGSAT data. The inversion is described in reference 6, which contains a table of the gauss coefficients of the core field between degrees 1 and 10 inclusive. Uncertainties produced by the crust, satellite errors and truncation are quoted separately so that the methods of reference 3 will be available to Bayesians and objectivists alike. The uncertainties in the gauss coefficients range from about 0.1 nT at degree 1 to about 6 nT at degree 10. By degree 10, the uncertainties are as large as the coefficients themselves. The dipole moment is determined to about 6 parts in 10^6.

Reference 9 corrects an algebraic and programming error made in reference 6, and shows that the gauss coefficients in that reference fit the observed spatial power spectrum (the Lowes spectrum).

* There is a possibility for confusion in the fact that we use "gauss coefficients" for the spherical harmonic coefficients of the geomagnetic field, while "gaussian" and "normal distribution" are synonymous in statistics. The context should make clear which work of C.F. Gauss is being invoked.

PUBLICATIONS PRODUCED UNDER THIS GRANT

1. Walker, Andrew and Backus, G.E., "Is the non-Dipole Magnetic Field Random?," *Geophys. J. Int.* **124**, 315–319 (1996)

2. Walker, Andrew and Backus, G.E., "On the difference between the average values of B_r^2 in the Atlantic and Pacific Hemispheres," *Geophys. Res. Letters* **23**, 1965–1968 (1996)

3. Backus, G.E., "Trimming and procrastination as geophysical inversion techniques," *Physics of the Earth and Planetary Interiors* **98**, 101–142 (1996)

4. Backus, G.E., Parker, R. and Constable, C., "Foundations of Geomagnetism," Cambridge University Press, 369 pp (1996) (supported mostly by sources outside the grant)

5. Walker, A.D. and Backus, G.E., "A six-parameter statistical model of the Earth's magnetic field," *Geophys. J. Int.* **130**, 693–700 (1997)

6. Walker, A.D., "Statistics of the Earth's Magnetic Field with Applications," PhD thesis, University of California, San Diego (1997)

AGU PRESENTATIONS

(@ denotes a poster)

7. @ Walker, Andrew and Backus, G.E., "On the difference between the average values of B_r^2 in the Atlantic and Pacific Hemispheres," 1995 AGU Fall Meeting, abstract in *EOS* **76**, F166

8. @ Walker, Andrew and Backus, G.E., "A Six-Parameter Model of the Earth's Magnetic Field," 1996 AGU Fall Meeting, abstract in *EOS* **77**, F172

9. Walker, A., Constable, C.G. and Backus, G.E., "Statistical Models of the Earth's Magnetic Field," 1997 AGU Fall Meeting, abstract in *EOS* **78**, F184