

Optimal design of general stiffened composite circular cylinders for global buckling with strength constraints

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Abstract

A design strategy for optimal design of composite grid-stiffened cylinders subjected to global and local buckling constraints and strength constraints was developed using a discrete optimizer based on a genetic algorithm. An improved smeared stiffener theory was used for the global analysis. Local buckling of skin segments were assessed using a Rayleigh–Ritz method that accounts for material anisotropy. The local buckling of stiffener segments were also assessed. Constraints on the axial membrane strain in the skin and stiffener segments were imposed to include strength criteria in the grid-stiffened cylinder design. Design variables used in this study were the axial and transverse stiffener spacings, stiffener height and thickness, skin laminate stacking sequence and stiffening configuration, where stiffening configuration is a design variable that indicates the combination of axial, transverse and diagonal stiffener in the grid-stiffened cylinder. The design optimization process was adapted to identify the best suited stiffening configurations and stiffener spacings for grid-stiffened composite cylinder with the length and radius of the cylinder, the design in-plane loads and material properties as inputs. The effect of having axial membrane strain constraints in the skin and stiffener segments in the optimization process is also studied for selected stiffening configurations. © 1998 Published by Elsevier Science Ltd. All rights reserved.

1. Nomenclature

a	Axial stiffener spacing
b	Transverse stiffener spacing
h	Stiffener height
t	Skin laminate thickness
t_s	Stiffener thickness
N_1	Axial design load
<i>LAMI</i>	Design variable for stacking sequence of skin laminate
<i>ICON</i>	Design variable for stiffening configuration
λ_G	Global buckling load factor
λ_{sk}	Buckling load factor of skin segment
$\lambda_1, \lambda_2, \lambda_3$	Crippling load factor of axial, transverse and diagonal stiffener segment
S_{sk}	Strain level factor for skin segment
S_1, S_2, S_3	Strain level factor for axial, transverse and diagonal stiffener segment

2. Introduction

An aircraft or a launch vehicle in flight is subjected to loads associated with all flight conditions. These

external loads are resisted by the structure and an internal load distribution is established based on the structural layout and external loads. The internal loads, which depend on the location in the aerospace vehicle, may cause either overall buckling of the stiffened fuselage/fuel tank structure, buckling of the skin segment between stiffeners, or crippling of stiffener segment. A stiffened circular cylindrical shell is a widely used structural configuration for an aircraft fuselage or a launch vehicle fuel tank. An efficient and accurate analysis method is needed to develop a buckling-resistant design for the general grid-stiffened composite circular cylindrical shell and to identify the most effective stiffening configurations and stiffener spacings for shells subjected to combined in-plane loading conditions. The identification of structurally efficient stiffening configurations and stiffener spacings also requires integration of optimization techniques with accurate structural analysis methods.

The optimum design of stiffened panels or shells that satisfy buckling constraints either with or without strain constraints has been of considerable interest to researchers since aircraft and launch vehicle structures

consist mostly of stiffened panels or shells. Researchers have used gradient-based optimizer, discrete optimizer, and the simplex method. A gradient-based optimizer were used in Refs [1–8], a discrete optimizer was used in Refs [9–13], and the simplex method was used in Refs [14], [15].

A literature survey of the optimal design of stiffened panels and shells is presented in Ref. [13]. The survey deals with the type of optimizers that have been used, and the type of buckling analysis, and stiffened panel or shell optimized. Type of stiffened panel or shell refers to whether the panel or shell is stiffened axially, orthogonally or in multiple directions (e.g. grid-stiffened panel or shell). According to the review, optimization of panels or shells stiffened in multiple directions were considered in Refs [5], [6], [8], [16]. Grid-stiffened cylinders were considered in Refs [6], [8], [16], and geodesically stiffened flat panels were considered in Ref. [5]. More recently grid-stiffened curved panels were considered in Ref. [13].

The survey also shows that stiffener spacing is treated as a design variable in Refs [1], [2], [7], [8], [14], [15]. A gradient-based optimizer was used in Refs [1], [2], [7], [8], whereas the simplex method [17] was used in Refs [14], [15]. The stiffener panel spacing is a discrete variable since it can be a certain multiple of the length or width of the panel or shell. Both the gradient-based optimizer and the simplex method assume continuous variables and in addition the gradient-based optimizers require derivative information. Therefore as discussed in Jaunky *et al.* [13] rigorous optimization with respect to stiffener spacing as a design variable is best achieved using a discrete optimizer such as the genetic algorithm [11].

Reddy *et al.* [16] studied the buckling response of isogrid and orthogrid cylinders with various stiffener spacing using global and local analyses and concluded that, in general, isogrid cylinders are more efficient than orthogrid cylinders. It is also shown [7] that optimizing axially stiffened panels for different fixed stiffener spacings using a gradient-based optimizer can be a good strategy. For panels or shells stiffened in multiple directions, this approach is tedious since one has to consider many starting points in the optimal design process. The stiffener spacings and stiffening configurations can be treated as design variables in an optimization process provided a discrete optimizer is used. A discrete optimizer such as the genetic algorithm has been used to optimize unstiffened or axially stiffened composite panels [9–13], and for other engineering problems [18]. Haftka *et al.* [9–12] showed the efficiency of the genetic algorithm in dealing with discrete design variables for optimizing composite panels. They treated the skin and/or stiffener laminate as discrete variables since in practical applications, the ply orientations are limited to 0, 90 or $\pm 45^\circ$ and the

laminar thickness can only be integer multiples of commercially available ply thickness. Jaunky *et al.* [13] optimized flat and curved grid-stiffened panels with stiffener height and thickness, skin laminate, stiffener spacings and stiffening configuration as design variables with a global buckling constraint only and showed the efficiency of the genetic algorithm in dealing with all these discrete design variables that affect the buckling response of grid-stiffened panels.

The literature survey [13] also comments on the buckling analyses that have been used. Finite element analysis [6], discrete analysis [5], PASCO [19] type analysis [3], and global and local analyses. The merits of these types of analyses are discussed in Jaunky *et al.* [13]. Most researchers have used global and local analyses, for example Refs [1], [2], [7], [8], [13–16]. Global and local analyses can be made computationally very efficient. Researchers have neglected the anisotropic properties of skin segments in the local buckling analysis and also the curvature of the skin segments. Restrictions on the geometry of the skin segment and loading were imposed in order to obtain a closed-form solution for the buckling response. In some cases the eccentricities of the stiffener were even neglected. The main problem of assessing the local buckling of the skin segment was the non-availability of computationally efficient and accurate buckling analyses for curved skin segments with anisotropic material properties, and different planform geometry (e.g. general triangular and arbitrary quadrilateral geometry). In Jaunky *et al.* [13] accurate buckling analyses [21–23] were used for global and local analyses, and the occurrence of non-simultaneous global and local failures were used as this condition leads to designs that are less sensitive to imperfections as discussed in Simites [15].

The present paper presents the analysis and weight optimization strategy for grid-stiffened composite circular cylindrical shells subjected to axial load and a global buckling design constraint as well as strength constraints using the genetic algorithm. Design variables are the stiffener height and thickness, skin laminate, stiffener spacings and stiffening configuration. It is intended to be a design tool that can be used as a preliminary design stage for grid-stiffened cylinders for aircraft fuselage or launch vehicle application. In the authors' opinion such a tool for cylinders stiffened in multiple directions is lacking. The results are presented and discussed for cylinder designs with and without strength or strain constraints which explains the damage tolerant characteristics of grid-stiffened structures [20].

2.1. Shell buckling analysis

The buckling analysis of grid-stiffened composite shells subjected to combined loads requires several key

steps. In the present study, acceptable designs are those which buckle globally and do not exhibit any local skin buckling or stiffener crippling. The first step in the design process is to assess the global buckling response of a grid-stiffened shell. Once this global buckling response is determined, the second step is to determine the local skin buckling response for the quadrilateral and/or triangular skin segments between the stiffeners. The third step is to determine whether stiffener buckling or stiffener crippling has occurred at this global buckling load level.

The global buckling analysis is based on a Rayleigh–Ritz method using a first-order shear-deformation theory and the improved smeared-stiffener modeling approach discussed in Ref. [21] that accounts for skin-stiffener interactions. It is shown [21] that buckling loads for stiffened panels with different stiffening configurations obtained using the improved smeared stiffener theory are in good agreement with results obtained using detailed finite element analysis. The cylinder is assumed to be simply supported and hence the Rayleigh–Ritz method for the global analysis assumes the following mode shape for the transverse deflection w :

$$w = \sum_{i=1}^N \left(A_{m,n_i} \sin \frac{n_i y}{R} + B_{m,n_i} \cos \frac{n_i y}{R} \right) \sin \frac{m_i \pi x}{L} \quad (1)$$

where L and R are the length and radius of the cylindrical shell, respectively, and N is the number of terms in the Fourier series. The coordinate system for the cylinder is shown in Fig. 1.

The buckling analysis of local skin segments is also based on a Rayleigh–Ritz analysis using a first-order, shear-deformation theory and accounts for material anisotropy. Boundary restraints on the skin segments are provided by the stiffeners and hence, the analysis must be capable of accommodating a variety of boundary conditions and a variety of skin-segment shapes [22], [23]. In most cases, the skin segments for grid-stiffened panels will have either a general parallelogram- or a general triangular-shaped planform. The skin segments are assumed to have simply supported boundary conditions in the present study. It is shown [22], [23] that the buckling analyses presented for arbitrary anisotropic quadrilateral plates and general triangular anisotropic plates can accommodate different boundary conditions, and provide buckling loads that are in very good agreement with finite element analysis and existing solutions. The Rayleigh–Ritz buckling analysis method for global and local analysis makes use of the Sanders–Koiter shell theory [24], [25]. This shell theory provides buckling loads that are in good agreement with finite element analysis compared to other shell theories as shown in Refs [26], [27].

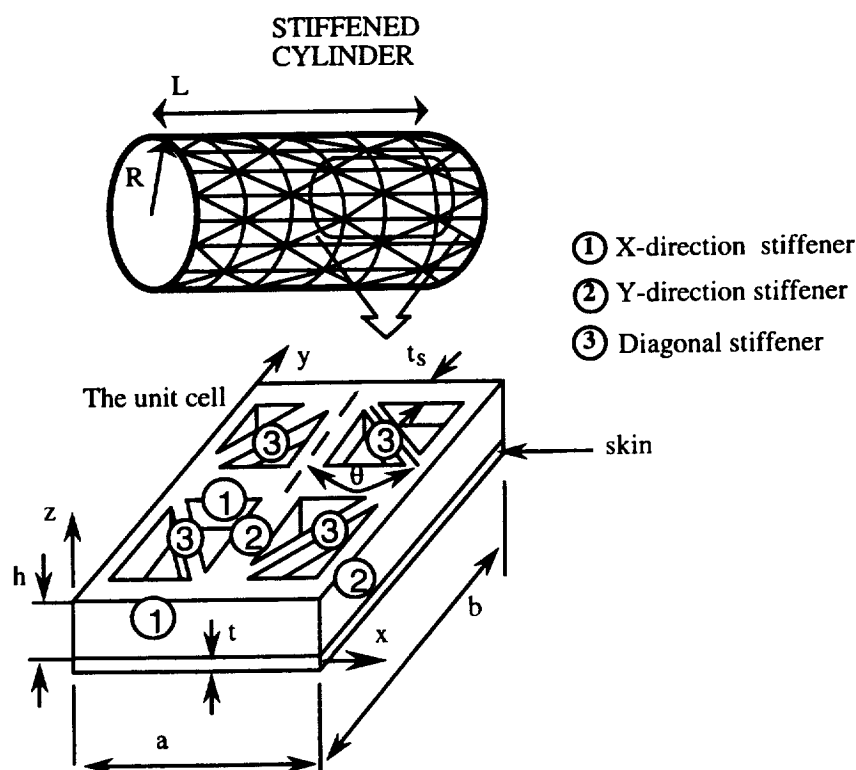


Fig. 1. Unit cell of a grid-stiffened cylinder showing design variables.

In addition to analyzing the local skin segment for buckling, the local stiffener segments must be analyzed to determine whether stiffener crippling will occur. Reference [16] provides a method for determining the crippling load of a stiffener segment. Accordingly, the stiffener segment is assumed to be clamped at the nodes or intersection points of the stiffeners while the edge of the stiffener along the stiffener–skin attachment line is assumed to be simply supported.

The global buckling load is assumed to be a scalar multiple of the design load and has the form

$$N_i = \lambda_G N_1 \quad (2)$$

where N_1 is the applied prebuckling axial load and represents the design load. Once the global buckling load factor (λ_G) has been determined using the improved smeared stiffener theory, the loads acting on the stiffener and skin segments have to be determined by distributing the loads based on the extensional stiffness of the skin and the stiffener. The procedure for distributing the applied loads for a general grid-stiffened panel is discussed in Refs [5], [26] and is the same for a general grid-stiffened circular cylindrical shell. The loads acting on the skin and stiffener segments are computed based on a global load of $N_x = \lambda_G N_1$ and these loads are used to determine the local buckling load factor of the skin (λ_{sk}), local crippling factors of axial stiffener segment (λ_1), transverse stiffener segment (λ_2) and diagonal stiffener segment, (λ_3).

$$\lambda_i = \frac{\text{critical load of local segment}}{\text{load in local segment due to } \lambda_G \times N_1} \quad (3)$$

where $i = 1, 2, 3$ and sk. These local buckling and crippling load factors describe the buckling characteristics of the stiffened cylinder and is as follows

- For $\lambda_{sk}, \lambda_1, \lambda_2, \lambda_3 \geq 1.0$, the cylinder buckles globally at an axial load of $\lambda_G N_1$, i.e. $\lambda_{cr} = \lambda_G$.
- If one of $\lambda_{sk}, \lambda_1, \lambda_2, \lambda_3 < 1.0$, then the stiffened cylinder buckles locally. If $\lambda_{sk} < 1.0$, then skin buckling occurs and if $\lambda_i < 1.0$ then crippling of the axial stiffener occurs. For this case, $\lambda_{cr} = \lambda_i \times \lambda_G$ where $\lambda_i < 1.0$, and i can be any one of sk, 1, 2 or 3.
- If more than one of $\lambda_{sk}, \lambda_1, \lambda_2$ and λ_3 are < 1.0 , then local buckling of the stiffened cylinder occurs and $\lambda_{cr} = \lambda_i \times \lambda_G$ where λ_i is the minimum of any of $\lambda_{sk}, \lambda_1, \lambda_2$ or λ_3 with values < 1.0 .

3. Strain analysis

The critical buckling load of the stiffened cylinder is $\lambda_{cr} N_1$ where λ_{cr} takes on values as discussed previously and based on this load value the loads acting on the skin and stiffener segments are obtained [5], [26]. For

an axial load in the skin segment of N_{sk} , and the loads in axial, transverse and diagonal stiffener segments of N_{x1} , N_{x2} and N_{x3} , respectively, the axial membrane strain the skin and stiffener segments are

$$\epsilon_{xsk}^0 = a_{11}^{(sk)} N_{sk} + a_{12}^{(sk)} N_{x1} + a_{16}^{(sk)} N_{x3} \quad (4)$$

$$\epsilon_{x1}^0 = a_{11}^{(1)} N_{x1}$$

$$\epsilon_{x2}^0 = a_{11}^{(2)} N_{x2}$$

$$\epsilon_{x3}^0 = a_{11}^{(3)} N_{x3}$$

where ϵ_{xsk}^0 , ϵ_{x1}^0 , ϵ_{x2}^0 and ϵ_{x3}^0 are the axial membrane strains in the skin segment, axial, transverse and diagonal stiffener segments respectively. The quantities $a_{ij}^{(sk)}$, $a_{11}^{(1)}$, $a_{11}^{(2)}$ and $a_{11}^{(3)}$ are axial flexibilities of the skin, axial, transverse and diagonal stiffeners respectively.

The strain level factors for the skin, axial, transverse and diagonal stiffener segment are

$$S_{sk} = (\epsilon_{xsk}^0)_{al} / \epsilon_{xsk}^0 \quad (5)$$

$$S_1 = (\epsilon_{x1}^0)_{al} / \epsilon_{x1}^0$$

$$S_2 = (\epsilon_{x2}^0)_{al} / \epsilon_{x2}^0$$

$$S_3 = (\epsilon_{x3}^0)_{al} / \epsilon_{x3}^0$$

where $(\epsilon_{xsk}^0)_{al}$ and $(\epsilon_{xst}^0)_{al}$ are the allowable axial membrane strains in the skin and stiffeners, respectively. The values for $(\epsilon_{xsk}^0)_{al}$ and $(\epsilon_{xst}^0)_{al}$ are taken from D. R. Ambur (pers. commun.).

These global and local buckling analysis methods and the strain analysis have been integrated into a computer code to provide a computationally efficient tool for predicting the buckling load and the strain level factors for grid-stiffened composite circular cylindrical shells subjected to axial compression.

4. Shell design procedure

The design variables for a grid-stiffened composite shell are the axial and transverse stiffener spacings (a , b), the stiffening configuration (*ICON*), which is the combination of axial, transverse and diagonal stiffeners, the skin laminate (*LAMI*), and the height (h) and thickness (t_s) of the stiffener (see Fig. 1). Except for the height of the stiffener, these design variables take on discrete values. Hence gradient-based optimization methods are not suitable for optimal design grid-stiffened shells. Furthermore, owing to manufacturing constraints a ‘family’ of good designs is needed rather than a single point design.

The genetic algorithm is a method for ‘evolving’ a given design problem to a family of near-optimum designs [10], [11], [13], [18]. Stochastic processes are used to generate an initial population of individual

designs and the process then applies principles of natural selection and survival of the fittest to find improved designs. Furthermore, since the discrete design procedure works with a population of designs it can explore a large design space and climb different hills. This is a major advantage as the converged solution may contain many optima of comparable performance. The cost of having a large number of function evaluations is offset by the fact that a large number of optimum solutions are now available. The population or family of good designs produced by using the genetic algorithm may include the global optimum design, rather than a single design. Hence, it is an appropriate tool for designing general grid-stiffened composite shells.

5. Design problem definition

The present design problem is to minimize the weight of a grid-stiffened composite circular cylindrical shell given the design loading condition, the length and radius of the cylinder and the material properties for the skin and stiffeners. The design variables include stiffener spacings (a , b), the stacking sequence of the skin, stiffener layout, stiffener thickness (t_s), and stiffener height (h) as shown in Fig. 1. All stiffeners are assumed to be of the same height and thickness for manufacturing and assembly purposes. The design sought here is a cylinder of minimum weight in a certain design space with buckles globally at the design loads while the axial membrane strain in the skin and the stiffener segments do not exceed the allowable axial membrane strain $(\epsilon_{rsk}^0)_{al}$ and $(\epsilon_{xst}^0)_{al}$ respectively. This design problem can be defined by setting up the optimization procedures in the following way. First, the global buckling load is assumed to be a scalar multiple of design loads and has the form

$$N_x = \lambda_G N_1, \quad (6)$$

where N_1 is the applied in-plane prebuckling load. This value represents the design loads for the grid-stiffened cylinder. Second, the design constraints imposed on panel include:

- The critical buckling load should be greater than or equal to the design loads, that is, $\lambda_G \geq 1$.
- Skin segments should not buckle at the critical buckling load, that is, $\lambda_{sk} \geq 1$.
- Stiffener segments should not cripple at the critical buckling load, that is, $\lambda_1, \lambda_2, \lambda_3 \geq 1$.
- The axial membrane strain in the skin segment should be less than or equal to $(\epsilon_{rsk}^0)_{al}$, that is, $S_{sk} \geq 1$.
- The axial membrane strain in the stiffener segment should be less than or equal to $(\epsilon_{xst}^0)_{al}$, that is, $S_1, S_2, S_3 \geq 1$.

The general form of each constraint equation is written as

$$g_j = \begin{cases} (1/\lambda_j - 1) \leq 0.0 \\ (1/S_j - 1) \leq 0.0 \end{cases} \quad j = 1, \dots, N_c \quad (7)$$

Finally, the 'fitness' expression based on exterior penalty function approach is

$$\begin{aligned} \text{Fitness} &= \left(\frac{Q}{F(\mathbf{X}, r_i)} \right) \\ &= \text{Max} \frac{Q}{W(\mathbf{X}) + r_i \sum_j^{N_c} [|g_j(\mathbf{X})| + g_j(\mathbf{X})]^2} \end{aligned} \quad (8)$$

- where \mathbf{X} = design variable vector
- $F(\mathbf{X}, r_i)$ = modified objective function
- $W(\mathbf{X})$ = weight of panel per unit area
- $r_i \sum_j^{N_c} [|g_j(\mathbf{X})| + g_j(\mathbf{X})]^2$ = penalty function
- Q = normalizing constant
- N_c = number of design constraints
- r_i = penalty parameter
- i = generation or iteration cycle in the optimization procedure.

6. Design process based on genetic algorithm

Implementation of the genetic algorithm [11] is shown schematically in Fig. 2. The design process

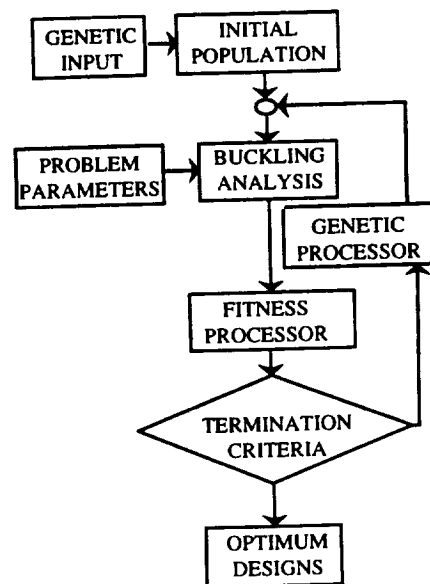


Fig. 2. Flow chart for the optimization procedure using the genetic algorithm.

begins with a random selection of a specified number of designs which comprise the initial population (i.e. first generation) for the genetic algorithm. Material properties, radius and length of the cylinder, boundary conditions of the skin segment and design loadings are input to the analysis processor routine. The buckling analysis is performed which provides the critical eigenvalues for the global buckling response of the grid-stiffened cylinder, the local buckling response of the skin and stiffener segments, and the strain level factors of the skin and stiffener segments. The weight of the grid-stiffened cylinder is also computed. This procedure is repeated for each design configuration in the population. The 'fitness' processor then evaluates the 'fitness' of each design using eqn (8) and assigns a rank based on the fitness expression or objective function. The current population of design configurations is then processed by the genetic operators (crossover, mutation and permutation) to create a new population of design configurations for the next generations which combines the most desirable characteristics of previous generations. Designs from previous generations may be replaced by new ones (i.e. children) except for the 'most fit' designs (i.e. parents) which are always included in the next generation. The process is repeated until design convergence is obtained, which is defined herein by specifying a maximum number of generations (*NSTOP*) that may occur without improvement in the best design. The design procedure will now be demonstrated on grid-stiffened cylinders with and without the axial membrane strain constraint in the skin and stiffener segments to study the influence of constraints on the axial membrane strain on the optimal designs.

6.1. Numerical results for grid-stiffened cylinders

Results are presented for composite grid-stiffened cylinders subjected to axial compression in order to demonstrate the changes in optimal designs due to constraints on the axial membrane strain. These changes will depend on the loading, stiffening configuration (*ICON*), stiffener spacings (*a*, *b*) and the skin laminate (*LAMI*). The cylinders studied in examples 1 and 2 are 7391 mm long, have a radius of 2426 mm and have blade stiffeners made of unidirectional materials [16]. The nominal ply mechanical properties are: $E_{11} = 63218 \text{ N mm}^{-2}$; $E_{22} = 5604$

Table 1
Design space for design variable *ICON* and *LAMI*

Integer value	<i>LAMI</i>	<i>ICON</i>
1	$[\pm 45/0]_{2s}$	Axial stiffeners
2	$[\pm 45/90]_{2s}$	Axial stiffeners
3	$[\pm 45/0/90]_{2s}$	Axial and transverse stiffeners
4	$[\pm 45/0]_{2s}$	Diagonal stiffeners
5	$[\pm 45/90]_{2s}$	Axial and diagonal stiffeners
6	$[\pm 45/0_2/90]_{2s}$	Transverse and diagonal stiffeners
7	$[\pm 45/0/90]_{2s}$	Axial, transverse and diagonal stiffeners
8	$[\pm 45/0_2/90]_{2s}$	No stiffeners

N mm^{-2} ; $G_{12} = G_{13} = G_{23} = 2973 \text{ N mm}^{-2}$ and $\nu_{12} = 0.300$. The mass density of the material ρ , is $1.578 \text{ E-06 kg mm}^{-3}$. The probabilities used for crossover, mutation and permutation are 1.0, 0.10 and 0.95, respectively, and the number of discrete values that each design variable can accommodate is eight, which is a limitation of the FORTRAN code for the genetic algorithm being used. The design variables *ICON* and *LAMI* are described in Table 1

6.2. Example 1

The first example is a cylinder subjected to an axial load of $N_1 = 175.2 \text{ N mm}^{-1}$ which represents a cylinder case studied in Ref. [16]. The design variables are the axial and transverse stiffener spacings, the height and thickness of the stiffeners, the stiffening configuration (*ICON*) and the skin laminate (*LAMI*). The thickness of each ply of the skin laminate for this example is 0.1524 mm. The design space for *a*, *b*, *h* and *t_s* is shown in Table 2. The height and thickness of the stiffener (*h/t*) is chosen such that the aspect ratio of the stiffener is between 4 and 10, and the stiffener thickness is an integer multiple of 0.1524 mm due to manufacturing constraints. The minimum values of the stiffener spacings are also due to manufacturing constraints. A population size of 20 is used, *NSTOP* = 15, and the penalty parameter $r_i = 100000$ at any iteration. The allowable strains $(\epsilon_{\text{ask}}^0)_{\text{al}}$ and $(\epsilon_{\text{rst}}^0)_{\text{al}}$ are 2428 microstrain and 1091 microstrain, respectively (D. R. Ambur, pers. commun.).

The results from design optimization with and without strain constants are shown in Table 3. The minimum weight for this cylinder is 481.53 kg compared to the weights of 479.90 kg for an isogrid

Table 2
Design space for *a*, *b*, *h* and *t_s* for Examples 1 and 2

<i>a</i> , mm	217.4	211.2	205.3	199.7	194.5	189.5	184.8	180.3
<i>b</i> , mm	525.5	508.0	491.6	476.3	461.8	448.2	435.4	423.3
<i>h</i> , mm	10.16	10.46	10.80	11.11	11.43	11.75	12.07	12.38
<i>t_s</i> , mm	1.22	1.37	1.52	1.68	1.83	1.98	2.13	2.29

Table 3
Best designs obtained by genetic algorithm for grid-stiffened cylinders ($N_x = 175.2 \text{ N mm}^{-1}$)

Design variables (mm)	Weight (kg)	λ_i	S_i
$a = 189.5, b = 525.5$ $h = 10.16, t_s = 1.22$ $LAMI = 4, ICON = 6$	481.53	$\lambda_{G1} = 1.008$ $\lambda_{sk} = 1.009$ $\lambda_3 = 55.1$	$S_{sk} = 2.68$ $S_3 = 105.1$
$a = 189.5, b = 491.6$ $h = 10.16, t_s = 1.22$ $LAMI = 4, ICON = 6$	481.72	$\lambda_{G1} = 1.025$ $\lambda_{sk} = 1.011$ $\lambda_3 = 43.4$	$S_{sk} = 2.63$ $S_3 = 81.7$
$a = 189.5, b = 476.3$ $h = 10.16, t_s = 1.22$ $LAMI = 4, ICON = 6$	481.5	$\lambda_{G1} = 1.034$ $\lambda_{sk} = 1.012$ $\lambda_3 = 38.8$	$S_{sk} = 2.61$ $S_3 = 72.6$

cylinder and 477.63 kg for an orthogrid cylinder [16]. Only three optimal designs are identified here and they all buckle globally at their respective λ_{G1} values since λ_{sk} and λ_3 values are greater than 1. For this case, the optimal designs have $ICON = 6$, that is the stiffening configuration has transverse and diagonal stiffeners only, and $LAMI = 4$, which corresponds to a skin laminate which a stacking sequence of $[\pm 45/0_2]_{2s}$, $h = 10.16 \text{ mm}$ and $t_s = 1.22 \text{ mm}$. In this case, the optimal designs without strain constraints are the same as the optimal design with strain constraints. The buckling-resistant design for this load case results in in-plane stiffness values that provide large strain level factors. This analysis suggests that the cylinder design satisfies both buckling and strength requirement. Convergence is obtained after 49 generations for the case without strain constraints and after 77 generations for the case with strain constraints.

6.3. Example 2

The second example is for a grid-stiffened cylinder as in Example 1 but subjected to an axial load of $N_1 = 315.3 \text{ N mm}^{-1}$. For this case, the stiffening configuration with axial and diagonal stiffeners ($ICON$) only has been selected. The design variables are the axial and transverse stiffener spacings, the height and thickness of the stiffeners, and the skin laminate with each ply being 0.2032 mm thick. The design space for the skin laminate is the same as shown in Table 1, and the design space for a, b, h and t_s is shown in Table 2. The material properties, the genetic parameters and the strain allowables for Example 2 are the same as in Example 1. The optimal designs without strain constraints are shown in Table 4, whereas the optimal designs with strain constraints are shown in Table 5. Only three optimal designs are shown here for each case.

All the designs in Tables 4 and 5 buckle globally since λ_{sk}, λ_1 and λ_3 are all greater than one. The

Table 4
Best designs obtained by genetic algorithm for grid-stiffened cylinder with no constraints on strain and $ICON = 5$. ($N_x = 315.3 \text{ N mm}^{-1}$)

Design variables (mm)	Weight (kg)	λ_i
$a = 217.4, b = 423.3$ $h = 11.43, t_s = 1.83$ $LAMI = 4$	634.1	$\lambda_{G1} = 1.0088$ $\lambda_{sk} = 1.078$ $\lambda_1 = 1.11, \lambda_3 = 24.6$
$a = 194.5, b = 435.4$ $h = 10.16, t_s = 1.98$ $LAMI = 3$	634.8	$\lambda_{G1} = 1.0447$ $\lambda_{sk} = 1.051$ $\lambda_1 = 1.04, \lambda_3 = 35.3$
$a = 194.5, b = 423.3$ $h = 10.16, t_s = 1.98$ $LAMI = 3$	635.5	$\lambda_{G1} = 1.0541$ $\lambda_{sk} = 1.077$ $\lambda_1 = 1.04, \lambda_3 = 32.1$

optimal designs obtained with strain constraints are $\approx 4\text{--}6 \text{ kg}$ heavier than the optimal designs obtained without strain constraints for approximately the same global buckling load. The designs in Table 5 have larger axial stiffener spacing (a) than the second and third designs in Table 4. For the designs in Table 5, the strain level factors of the axial stiffeners (S_1), are very close to unity whereas the strain factors for the diagonal stiffeners (S_3) are much larger. The increase in structural weight of the optimal design for this case over the designs in Example 1 is due to the increase in load and the choice of stiffening configuration. Even for this load condition, imposing strain constraints does not result in a significant increase in weight. Convergence is obtained after 18 generations for the case without strain constraints and after 41 generations for the case with strain constraints.

6.4. Example 3

The third example is a grid-stiffened cylinder subjected to an axial load of $N_1 = 350.4 \text{ N mm}^{-1}$. This represents a generic, wide-body transport aircraft

Table 5
Best designs obtained by genetic algorithm for grid-stiffened cylinder with constraints on strain and $ICON = 5$ ($N_x = 315.3 \text{ N mm}^{-1}$)

Design variables (mm)	Weight (kg)	λ_i	S_i
$a = 211.2, b = 423.3$ $h = 11.11, t_s = 1.98$ $LAMI = 4$	638.0	$\lambda_{G1} = 1.0113$ $\lambda_{sk} = 1.101$ $\lambda_1 = 1.36, \lambda_3 = 32.9$	$S_{sk} = 2.07$ $S_1 = 1.08$ $S_3 = 27.2$
$a = 205.3, b = 423.3$ $h = 11.11, t_s = 1.98$ $LAMI = 4$	639.0	$\lambda_{G1} = 1.0100$ $\lambda_{sk} = 1.126$ $\lambda_1 = 1.38, \lambda_3 = 36.2$	$S_{sk} = 2.08$ $S_1 = 1.08$ $S_3 = 29.8$
$a = 211.2, b = 423.3$ $h = 11.75, t_s = 1.98$ $LAMI = 4$	641.5	$\lambda_{G1} = 1.0600$ $\lambda_{sk} = 1.053$ $\lambda_1 = 1.19, \lambda_3 = 28.9$	$S_{sk} = 1.99$ $S_1 = 1.03$ $S_3 = 26.0$

fuselage section between two frames. The radius of the cylinder is 2426 mm and the length is 559 mm. The design variables are the transverse stiffener spacing, the height and thickness of the stiffeners and the skin laminate with each ply being 0.2286 mm thick. The optimization is performed with the stiffening configuration selected to $ICON = 1$ and $ICON = 5$. For the case when $ICON = 5$, the axial stiffener spacing is 159.7 mm. The design space for the skin laminate is the same as shown in Table 1 and the design space for b , h and t_s is shown in Table 6. The material properties and the genetic parameters for Example 3 is the same as in Example 1. Three optimal designs obtained with strain constraints are shown in Table 7 for the axially stiffened cylinders and in Table 8 for the cylinders stiffened with axial and diagonal stiffeners. For axially stiffened cylinders convergence is obtained at 38 gener-

ations, while for the cylinders with axial and diagonal stiffeners, convergence is obtained at 20 generations.

All the optimal designs in Tables 7 and 8 buckle globally and do not violate any strain constraint. However, the optimal designs with $ICON = 5$ are slightly heavier than the optimal designs with $ICON = 1$. The buckling behavior of the first design in Table 7 and Table 8 subjected to an axial load of $N_x = 350.4 \text{ N mm}^{-1}$ and combined transverse compression (N_y), and shear (N_{xy}) is sought to determine their capability to support additional in-plane loads. A load combination for $N_x = 350.4 \text{ N mm}^{-1}$, N_y and N_{xy} is sought in such a way that the cylinder can still support an axial load of $N_x = 350.4 \text{ N mm}^{-1}$, i.e. $\lambda_{cr} \geq 1$ and $\lambda_{cr} \approx 1$. The results are shown in Table 9 for the axially stiffened cylinder and in Table 10 for the grid-stiffened cylinder with $ICON = 5$. It is seen that the grid-

Table 6
Design space for b , h , t_s for Example 3

Axially stiffened cylinder, $ICON = 1$								
b (mm)	217.4	211.2	205.3	199.7	194.5	189.5	184.9	180.3
h (mm)	16.19	16.51	16.83	17.14	17.46	17.78	18.10	18.42
t_s (mm)	2.59	2.74	2.89	3.05	3.20	3.35	3.50	3.66
Grid-stiffened cylinder, $ICON = 5$								
b (mm)	324.3	319.0	311.0	304.8	298.8	293.1	287.6	282.2
h (mm)	13.36	13.65	13.97	14.29	14.67	14.92	15.24	15.56
t_s (mm)	1.68	1.83	1.98	2.13	2.29	2.44	2.59	2.74

Table 7
Best designs obtained by genetic algorithm for axially cylinder with constraints on strain, ($N_x = 350.4 \text{ N mm}^{-1}$)

Design variables (mm)	Weight (kg)	λ_i	S_i
$b = 217.4$, $h = 16.51$ $t_s = 2.59$, $LAMI = 4$	54.4	$\lambda_G = 1.0177$ $\lambda_{sk} = 2.803$, $\lambda_1 = 1.025$	$S_{sk} = 2.310$ $S_1 = 1.200$
$b = 211.2$, $h = 16.19$ $t_s = 2.59$, $LAMI = 4$	54.4	$\lambda_G = 1.0079$ $\lambda_{sk} = 3.004$, $\lambda_1 = 1.073$	$S_{sk} = 2.335$ $S_1 = 1.213$
$b = 211.2$, $h = 16.51$ $t_s = 2.59$, $LAMI = 4$	54.6	$\lambda_G = 1.0285$ $\lambda_{sk} = 2.952$, $\lambda_1 = 1.018$	$S_{sk} = 2.295$ $S_1 = 1.192$

Table 8
Best designs obtained by genetic algorithm for grid-stiffened cylinder with constraints on strain, $ICON = 5$ ($N_x = 350.4 \text{ N mm}$)

Design variables (mm)	Weight (kg)	λ_i	S_i
$b = 319.0$, $h = 13.36$ $t_s = 1.98$, $LAMI = 4$	55.8	$\lambda_G = 1.0368$, $\lambda_{sk} = 2.341$ $\lambda_1 = 1.068$, $\lambda_3 = 24.6$	$S_{sk} = 2.08$ $S_1 = 1.98$ $S_3 = 27.0$
$b = 311.0$, $h = 13.36$ $t_s = 1.98$, $LAMI = 4$	55.9	$\lambda_G = 1.0490$, $\lambda_{sk} = 2.387$ $\lambda_1 = 1.057$, $\lambda_3 = 22.79$	$S_{sk} = 2.064$ $S_1 = 1.072$ $S_3 = 24.67$
$b = 304.8$, $h = 13.36$ $t_s = 1.98$, $LAMI = 4$	56.0	$\lambda_G = 1.0595$, $\lambda_{sk} = 2.424$ $\lambda_1 = 1.049$, $\lambda_3 = 21.44$	$S_{sk} = 2.048$ $S_1 = 1.064$ $S_3 = 22.96$

Table 9
Buckling load factors and strain level factors for axially stiffened cylinder

Loading (N mm^{-1})	$N_x = 350.4$ $N_y = 3.5$ $N_{xy} = 0$	$N_x = 350.4$ $N_y = 2.6$ $N_{xy} = 9.6$	$N_x = 350.4$ $N_y = 0.9$ $N_{xy} = 28.9$	$N_x = 350.4$ $N_y = 0$ $N_{xy} = 38.5$
λ_i	$\lambda_G = 1.0085$ $\lambda_{sk} = 2.710$ $\lambda_1 = 1.034$	$\lambda_G = 1.0108$ $\lambda_{sk} = 2.725$ $\lambda_1 = 1.032$	$\lambda_G = 1.108$ $\lambda_{sk} = 2.757$ $\lambda_1 = 1.032$	$\lambda_G = 1.0087$ $\lambda_{sk} = 2.227$ $\lambda_1 = 1.034$
S_i	$S_{sk} = 2.349$ $S_1 = 1.211$	$S_{sk} = 2.339$ $S_1 = 1.208$	$S_{sk} = 2.330$ $S_1 = 1.208$	$S_{sk} = 2.330$ $S_1 = 1.211$

stiffened cylinder can support up to 70.1 N mm^{-1} of transverse compression and 70.1 N mm^{-1} in shear whereas the axially stiffened cylinder can only support additional loads of 3.5 N mm^{-1} in transverse compression and 38.5 N mm^{-1} in shear without exceeding the strain allowables. It is assumed here that the strains ε_y and γ_{xy} due to N_y and N_{xy} do not produce strains large enough to create failure since N_y and N_{xy} are small compared to N_x . For these loading combinations, both cylinders buckle globally and the constraint on the axial membrane strains in the skin and stiffener are not violated. Hence the grid-stiffened cylinder can sustain larger magnitudes of additional transverse compression and shear loads than the axially stiffened cylinder.

7. Concluding remarks

A minimum-weight design optimization tool with global buckling constraint and constraint on the axial membrane strain in the skin and stiffener segments has been developed for grid-stiffened cylinders using global and local buckling analyses and a genetic algorithm. Design variables used are axial and transverse stiffener spacing, stiffener height and thickness, skin–laminate stacking sequence and stiffening configuration. The present design optimization procedure has been validated by comparing with existing results. Results for grid-stiffened cylinders subjected to axial compression indicate that there is no significant difference in weight between optimal designs obtained with an

without strain constraints for simply supported cylinders that buckle globally. It is also shown that a grid-stiffened cylinder optimized for axial load has an additional load capacity for combined loading larger than for a conventional axially stiffened cylinder. This additional load capability permits load redistribution in the event of structural damage and suggests that a grid-stiffened cylinder is more damage tolerant than an axially stiffened cylinder.

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Table 10
Buckling load factors and strain level factors for grid-stiffened cylinder

Loading (N mm^{-1})	$N_x = 350.4$ $N_y = 70.1$ $N_{xy} = 0$	$N_x = 350.4$ $N_y = 52.6$ $N_{xy} = 17.5$	$N_x = 350.4$ $N_y = 17.5$ $N_{xy} = 52.6$	$N_x = 350.4$ $N_y = 0$ $N_{xy} = 70.1$
λ_i	$\lambda_G = 1.0899$ $\lambda_{sk} = 1.930$ $\lambda_1 = 1.192$ $\lambda_3 = 2.958$	$\lambda_G = 1.0899$ $\lambda_{sk} = 1.991$ $\lambda_1 = 1.191$ $\lambda_3 = 2.929$	$\lambda_G = 1.0898$ $\lambda_{sk} = 2.111$ $\lambda_1 = 1.192$ $\lambda_3 = 2.873$	$\lambda_G = 1.0897$ $\lambda_{sk} = 2.169$ $\lambda_1 = 1.192$ $\lambda_3 = 2.846$
S_i	$S_{sk} = 2.240$ $S_1 = 1.036$ $S_3 = 2.779$	$S_{sk} = 2.173$ $S_1 = 1.036$ $S_3 = 2.752$	$S_{sk} = 2.050$ $S_1 = 1.036$ $S_3 = 2.669$	$S_{sk} = 1.994$ $S_1 = 1.036$ $S_3 = 2.673$

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