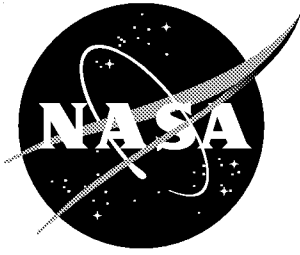# Optimization Strategies for Sensor and Actuator Placement

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April 1999

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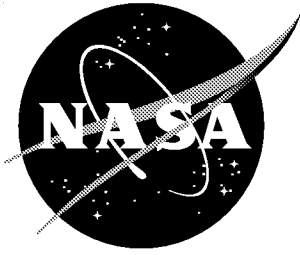
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Optimization Strategies for Sensor and Actuator Placement

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1. Abstract

This paper provides a survey of actuator and sensor placement problems from a wide range of engineering disciplines and a variety of applications. Combinatorial optimization methods are recommended as a means for identifying sets of actuators and sensors that maximize performance. Several sample applications from NASA Langley Research Center, such as active structural acoustic control, are covered in detail. Laboratory and flight tests of these applications indicate that actuator and sensor placement methods are effective and important. Lessons learned in solving these optimization problems can guide future research.

2. Keywords

3. Introduction

Smart structures or smart technologies are the subject of intense research. In the aerospace arena, the term "smart technologies" implies the use of embedded sensors and actuators for active control of airfoil shape, interior noise, or structural vibration. Engineers are considering this approach for vehicles of every size, from large space structures to micro-aircraft, and for products of every size, from automobiles to vacuum cleaners. For example, the NASA Aircraft Morphing program seeks to use smart technologies to increase aircraft system safety, affordability, and environmental compatibility [1]. One aspect of smart technologies is determining the optimum number of active control devices (e.g., piezoelectric actuators) and their placement on the structure. Similarly, the number and placement of sensors (e.g., strain gages) can be critical to the robust functioning of active control systems.

A literature search for optimal actuator or optimal sensor placement methods yields a large number of publications from widely different engineering disciplines. Some of these references describe small optimization problems and employ manual "cut-and-try" optimization techniques or intuitive placement recipes rather than systematic optimization methods. Other references discuss challenging numerical optimization problems and most often use genetic algorithms as the optimization method. This paper will give examples from the literature of actuator and sensor placement problems, will examine common features in these problems, and will describe combinatorial optimization methods used for solving them.

Several sensor or actuator placement problems and their solution methods are described in detail. These problems represent a variety of applications. However, each research effort is part of the Aircraft Morphing program at NASA Langley Research Center and each placement problem is solved with an integer or combinatorial optimization method [2]. The hope is that lessons learned from the cited actuator and sensor placement studies can be applied to future research.

4. A Survey of Documented Actuator and Sensor Placement Problems

This section is a survey of both aerospace and non-aerospace applications in which actuator and sensor placement is a practical concern. Table 1 contains a summary of all the journal articles in this survey. The references are grouped according to the type of application, the size of the optimization problem, and the optimization method used.

Collection 1: Non-aerospace Placement Problems
These five articles demonstrate the breadth of interest in sensor and actuator placement problems. First, Naimimohasses et al. [3] seek the optimal number and placement of sensors to detect the number and size of particles in a solution. Both simulated data sets and experimental data sets are tested. The experimental data models an oil-in-water suspension. The placement procedure employs a neural network. The strategy is to place many sensor devices and to let the neural network eliminate sensors that are not contributing significantly to the characterization of the particles in the solution. Reference 3 indicates that this technique has many uses in the process industry. Second, Oh and No [4] study a sensor placement problem for the safe operation of nuclear reactors. The sensors are needed to estimate the distribution of neutron flux as well as xenon and iodine concentrations in the reactor core over a given time period. Hence, this application is somewhat related to the first. Here, a two-stage procedure is employed. Initially a guess is made, via a sensitivity analysis of the covariance matrix, as to the set of potential sensor locations. Then, the authors seek the minimum number of these sensors required to insure accurate observation of the present state of the nuclear reactor. The trace of the covariance matrix is the main performance measure in this regard. One test case involved 52 potential sensor locations from which 8 to 14 locations were selected. The third application is reported by Reis et al. [5] and concerns the location of control valves to reduce

water leakage in a water supply network. Both the locations of the valves (which serve as actuators) and the valve settings are sought. The authors use a genetic algorithm (GA) approach and the performance measure requires solution of a linear programming problem. Their demonstration problem consists of 37 pipes and three valves. The fourth application is a hyperthermia cancer treatment examined by Mattingly et al. [6]. Sensors measure temperature in the tumor, but no optimization problem is formulated with respect to the placement or number of sensors. However, an actuator placement problem is addressed. The actuators damp modes that are generated by ultrasound heating. The main goal of this research is to validate a reduced-order model. An idealized cylindrical tumor with 450 states is modeled with 26 states, thus allowing fast reevaluation of the damping performance of selected sets of actuators. Four actuators are selected by an exhaustive search over all possible sets of four actuator locations. The fifth example, another thermal system, is similar to the tumor example. However, the focus in reference 7 is to determine sensor locations and sampling times, as well as the duration of the experiment, to minimize uncertainty in measured temperatures. A variant of the Fisher information matrix is used to estimate the uncertainties.

Collections 2–5: Aerospace Placement Problems
The articles in this subsection consider sensor and actuator placement on flexible structures for vibration control or noise attenuation. For a recent survey of active noise control with an emphasis on non-aerospace applications, the interested reader is referred to reference 8 by Hansen.

Collection 2: manual optimization techniques and intuitive placement recipes. The studies presented employ manual methods rather than systematic optimization methods. Several of these studies [9–13] are posed on a beam with actuators to damp out vibrations. Reference 9 presents a study of the effect of the placement of a pair of actuators, one above and one below the beam, on control efficiency. The experimental structure is the same in reference 10, but the effect of actuator thickness and length (as well as location) is examined. A complete enumeration of all potential locations is performed; the objective is to find the location that excites the maximum structural response. That is, no particular external disturbance is assumed. Reference 11 also locates two actuators on a beam, but here the actuators are not paired. Five potential locations for the two actuators are tested on the top of the beam. As in reference 10, an actuator power factor determines the effectiveness of one location over another. The placement of a single sensor-actuator pair on a laminated beam is considered in reference 12. The performance measure is a weighted sum of the modal damping over several frequencies and is referred to as the structural damping index (SDI). Reference 13 seeks good locations for sensor-actuator pairs without reference to any control law. Examples include a beam and a plate. As in the other references, only two actuators are placed. The performance measure maximizes the norm of the controllability gramian. A review of the literature is provided, and many details are given concerning the effects of both transient and persistent structural disturbances on actuator location.

The remaining references in collection 2 [14–17] describe small, experimental models of plates rather than beams. The performance of a variety of types of actuators is reported in references 14 and 15. The purpose of the actuators is to suppress noise transmitted through the plate. One to three actuators are placed and no optimization is done with regard to placement. In reference 16, a single collocated sensor-actuator is placed for vibration control of a laminated plate. A gradient descent procedure minimizes an SDI to determine the best location. In reference 17, a set of six location patterns on a plate are tested. The performance of each pattern is cataloged for degree of controllability (three measures) and for degree of observability (three measures). Sensors and actuators are collocated and no optimization is performed.

Collection 3: combinatorial optimization techniques for larger problems. The studies in these references [18–25] include a larger number of potential locations, consider complex structural models, and often use GA as the optimization method. For example, a cantilevered truss structure with 15 nodes and 40 truss elements and a finite element model of a beam are examined in reference 18. Twenty-one different methods of sensor placement are tested and five measures of performance are provided. In reference 19, two variations of GA compete in placing four actuators (out of 24 in one experiment and out of 128 in another experiment) on a cylinder to control noise. A different approach is taken in reference 20 where a 28 node steel truss structure is examined. Two sensors (actuators) out of 36 are selected by maximizing the correlation of the Hankel singular values of each sensor with the Hankel singular values of the disturbance. First, a correlation performance index ranks potential locations, and then correlations between locations are considered. Another ranking scheme called effective independence (EFI) is developed in reference 21 for sensor placement on flexible space structures. EFI is a heuristic procedure that attempts to maximize the determinant of the Fisher information matrix. The procedure was tested on a space station photovoltaic

array with 321 potential sensor locations from which 15 were selected. An enclosed, three-dimensional region is the subject of reference 22. A single frequency acoustic source is inserted through an opening in one end of the enclosure and generates the primary noise field. A GA is the optimization procedure that selects 14 sensors (out of 22) and 14 actuators (out of 90) to control the noise inside the enclosed region. Another active noise control problem is presented in reference 23. Here the enclosure simulates the interior of a small aircraft. A total of 48 sensors (out of 80) and 26 actuators (out of 52) are placed via a GA. It is not clear whether a single or multiple frequency noise source is considered. A GA is also used in reference 25 to place 26 actuators (out of 80) to control the noise in the interior of some (undisclosed) structure. It is assumed that a control law has been specified and that the sensors are fixed. The performance of the GA is compared against a greedy heuristic. A fourth active noise control model in reference 24 focuses on a multiple frequency noise source and the placement of one to three actuators. Optimization is again performed by a GA. Sensor placement is considered as well.

Collection 4: continuous optimization techniques. An actuator placement problem for a flexible space structure is given in reference 26. The actuators are placed to suppress structural vibrations. A standard nonlinear programming package selects three sensor-actuator sites out of 98 potential sites. The performance measure for the selection of sensor-actuator placements is defined in terms of the finite transmission zeros of the system. In reference 27, a flat, double-walled panel representative of an aircraft fuselage provides the test base for the placement of four sensors and two actuators. A single loudspeaker generates a multiple frequency, primary noise source. The performance of five nonlinear programming methods and a GA are compared for this active noise control problem.

Collection 5: design optimization plus placement techniques. The last four references [28-31] include the sensor and actuator placement problems within a traditional design optimization framework. The procedure in reference 28 seeks collocated sensor-actuator locations, truss member cross-sectional areas, and controller feedback gains while optimizing five distinct objective functions. The procedure is tested on a 12-member tetrahedral truss with five to seven sensor-actuator pairs to be placed. Solution methodologies include gradient-based searches and GA's. Optimal placement and sizing of piezoelectric actuators is the subject of reference 29. A general nonlinear programming procedure is used. The experimental structure is a plate. Actuators (one to three) are placed to attenuate noise from a simulated noise source measured at nine fixed sensor locations. A 90-bar truss topology optimization plus the placement of four actuators is considered by Liu in reference 30. A variety of optimization methods are tested, including sequential linear programming and simulated annealing (SA). A hybrid method is used by Chattopadhyay et al. [31] to design a composite plate to simulate an aircraft wing for active control tests. The design variables include stacking sequence for the composite plate, actuator locations, and control gains.

5. Combinatorial Optimization Methods

This section proposes several combinatorial optimization methods that are well suited for solving the variety of actuator and sensor placement problems surveyed in the preceding section. In general, the problems surveyed above can be stated as "Given a set of N possible locations, find the subset of M locations where $M << N$ which provides the best possible performance." The performance measure to be optimized can be quite complicated. For example, aerospace vehicles require active damping over a range of vibration modes that may vary with operating conditions. The optimization method must identify a set of locations that balances the expected performance over all conditions. Additionally, the method may impose constraints on geometry, for example, to prohibit adjacent locations or to encourage an even distribution of mass. Moreover, the method may include operational constraints related to power consumption or to physical limitations of the sensor and actuator devices. An optimization method is especially valuable when the performance tradeoffs are not intuitive, when the constraints are critical, and when the number of possible combinations of M locations exceeds that which can be examined manually.

For very small combinatorial problems, complete enumeration is a viable option and finding the global optima can be guaranteed. For larger problems, a variety of heuristic search methods, including simulated annealing, tabu search (TS), or genetic algorithms are available. These methods can not guarantee convergence to the global optima, but can uncover useful local optima after examining a tiny percentage of all possible combinations of N locations taken M at a time. Each of the heuristic methods has advantages. This paper emphasizes the importance of judicious choice of design variables, optimization formulation, and solution method to fit each problem.

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Each of the heuristic search methods requires a cost function to be minimized (or performance function to be maximized) and a mapping between design variable values and physical locations. For simplicity, only the minimization problem will be discussed, recognizing that it is easy to transform one problem into the other. Furthermore, the unconstrained minimization problem is discussed, because the constraints can be implemented by adding a penalty term to the cost function. There are at least two choices for the design variable mapping. One choice is a set of N design variables, x, such that $x_i = 1$ if location i is selected and $x_i = 0$ otherwise. The other choice is a set of M design variables, x, such that $1 \leq x_i \leq N$ and $x_i = j$ if location j is selected. Both methods provide a way to randomly generate subsets and to identify subsets that are nearest neighbors. Subsets are nearest neighbors if they select $M - 1$ identical locations.

The simplest optimization method is a random search where a number of subsets are generated at random and the subset that has lowest cost is reported. The random search algorithm is inefficient because the same subset can be generated and evaluated more than once and because the information from previous subsets is not used to guide the search.

The SA method is one attractive alternative to random search. The SA method is easy to implement, and robust values for the parameters that define the method can be found [32]. The algorithm starts by selecting a single random subset and seeks to improve the cost function by moving to one of the nearest neighbors of the selected subset. If the neighbor has a lower cost, then the move is permanent and the neighbor becomes the new selected subset. If the neighbor has a higher cost, then the permanence of the move is a probabilistic event. The SA algorithm is particularly effective if the best solutions tend to cluster in one part of the design space. Tabu search [33] is like SA in that TS requires a neighborhood structure and a performance measure to evaluate neighbors. However, whereas SA relies on probabilistic events to search for good solutions, TS is deterministic and incorporates search history as well as structural features of the model to drive it towards high-quality solutions. References 34–38 give examples of placement problems solved by either TS or SA.

Unlike SA and TS, GA's are initialized by generating a large number of random subsets (e.g., ref. 39). These initial subsets are combined and mutated to search for improved subsets. The GA methods are easy to adapt for use on parallel processing computer architectures. Compared to other heuristic search methods, a parallel GA can reduce the elapsed time required for solution of a large combinatorial problem. The GA is particularly effective at finding optimal solutions that are widely scattered throughout the design space (i.e., when the best solutions are not neighbors). References 40 and 41 give examples of placement problems solved by GA's.

6. Aerospace Applications

Three applications of sensor or actuator placement optimization are described in this section. The first involves active structural acoustic control (ASAC), where actuators attached to the aircraft fuselage are controlled in order to reduce noise in the passenger compartment. The second involves optimum sensor placement for system identification and control of aeroelastic structures. The last involves smart technologies that can produce a quasi shape change in an aircraft wing to provide three-axis control. In the first two applications, the placement of actuators and sensors in consideration of operational constraints is accomplished by using TS. In the third application, minimization of the number of actuators to provide an acceptable level of control authority is accomplished by using GA.

Of the three applications, ASAC has received the most study at NASA Langley [42–48], with both laboratory and flight experiments providing evidence that the placement of actuators and sensors is critical to success. This application will be highlighted. However, the optimization formulations and the lessons learned in the study of ASAC are applicable to many other sensor and actuator placement problems. This point is illustrated by considering the placement of sensors for system identification and the placement of actuators for three-axis flight control. These are new areas of application for combinatorial search methods and few results are available. However, the formulation of the problems will be compared with those used in the ASAC research.

Active Structural Acoustic Control

ASAC is a promising technique for reducing noise and vibration in aircraft interiors by active control of actuators bonded to the fuselage skin. The goal of the optimum placement method is to select actuator and sensor locations that reduce noise at several discrete frequencies produced by a typical propeller aircraft.

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A simplified model of the ASAC problem is used to explain the process. Assume that an aircraft fuselage is represented as a cylinder with rigid end caps (fig. 1) and that a propeller is represented as an acoustic point source with a frequency related to the propeller blade passage frequency. Piezoelectric actuators bonded to the fuselage skin are represented as line force distributions in the axial and azimuthal directions. With this simplified model, the point source produces predictable pressure waves that are exterior to the cylinder. These periodic pressure changes cause predictable structural vibrations in the cylinder wall and predictable noise levels in the interior space. The noise level at any interior microphone location depends on the control forces applied at each piezoelectric actuator location. For a given set of sensor and actuator locations, the control forces that minimize the average acoustic response are easy to calculate [42]. However, methods for choosing good locations for the sensors and the actuators are needed.

In accordance with the notation used in reference 43, the ASAC optimization problem is to minimize the sum of squared pressures at a set of N_p interior microphones:

$$E = \sum_{m=1}^{Np} \Lambda_m \Lambda_m^*$$

(1)

where * indicates the complex conjugate. The response at microphone m is given as

$$\Lambda_m = \sum_{k=1}^{Nc} H_{mk} c_k + p_m$$

(2)

where p_m is the response with no active control and H_{mk} is a complex-valued transfer matrix that represents the response at microphone m that results from one unit control force ($|c_k| = 1$) at actuator k. The values in the transfer matrix can be collected experimentally [43], or they can be simulated [42].

The cost function can be written either as in equation (1) or on a decibel scale to compare the interior pressure norms with and without ASAC:

$$\text{Level} = 10 \log \left(\frac{\sum_{m=1}^{Np} \Lambda_m \Lambda_m^*}{\sum_{m=1}^{Np} p_m p_m^*} \right)$$

(3)

A negative level represents a decrease in sound pressure level at N_p sensors, caused by the action of the N_c piezoelectric actuators.

For a fixed set of actuators and sensors, the forces c_k that minimize either equation (1) or equation (3) can be determined by solving a complex least-squares problem [42].

The composite cylinder model shown in figure 2 was used to test the importance of optimized actuator and sensor locations. This laboratory model has eight piezoelectric actuators bonded to the cylinder and 462 potential locations for microphone sensors. In one experiment, the four best and four worst actuators were tested [44]. For each actuator set, the eight best sensors were determined by TS with the steps given below. These test results were compared with previous test results in which the actuators and sensors were selected by using modal methods.

In the case of the composite cylinder experiment, the goal of the optimization is to pick the four best actuators and the eight best sensors for use in active noise control. Tabu search may be used to select four out of eight actuators; however, enumeration is a better option because only 70 possible combinations exist. In contrast, approximately 5×10^{16} combinations of 8 sensors can be selected from 462 candidates. Therefore, after the four actuators have been identified, TS is used to select the best microphones to use with those actuators. Whether choosing actuators or sensors, equations (1)–(3) are used to compare the candidates.

Given 462 possible sensor locations and a transfer matrix $\mathbf{H}^a$ that includes the response due to each of the four selected actuators, the optimal placement procedure uses tabu search to converge to the best subset of eight locations. As each proposed subset is considered, the matrix $\mathbf{H}$ is assembled by extracting the appropriate eight rows of $\mathbf{H}^a$. Then, the vector of control forces that minimizes E (see eq. (1)) is calculated, and the corresponding noise level (see eq. (3)) is used to determine the cost of the proposed move [43].

Note that the optimal control forces c_k are calculated with the 8 × 4 matrix **H**, and the noise reduction is calculated with the 462 × 4 matrix $\mathbf{H}^a$. In other words, the goal is to reduce noise at all 462 microphones, but only eight microphones are connected to the controller. The optimization procedure is summarized as follows:

1. Select 4 actuators; evaluate all combinations
2. Form 462 × 4 matrix $\mathbf{H}^a$
3. Choose 8 sensors at random
4. Form **H** matrix by collecting 8 rows from $\mathbf{H}^a$
5. Predict control forces c_k that minimize eq. (1)
6. Predict noise reduction with c_k and $\mathbf{H}^a$
7. Select 8 new sensors with TS
8. Repeat from step 4

Test results are reported in reference 42 and summarized here (see table 2). As expected, the control forces predicted in step 5 of the optimization procedure and the noise reduction predicted in step 6 fail to match the observed control forces or noise reduction. Possible explanations for the differences include premature convergence of the optimal control algorithm and errors in the measured transfer functions. However, the trends are well predicted.

Table 2 is representative of the results presented in reference 42. Active noise control was tested for a series of single frequency noise sources. Table 2 summarizes the results for the 275 Hz source. Notice that the observed noise reduction is less than predicted. However, the four best actuators provide 3.5 dB more noise reduction than the four worst and, in addition, perform better than those selected by modal methods.

The method summarized in steps 1–8 has been extended and used to plan ASAC flight tests on a Beech 1900 aircraft (fig. 3). Three harmonics of the blade passage frequency are targeted for reduction. Preliminary flight test results [46] indicate that a 14 dB reduction in overall sound pressure levels (as in eq. (3)) can be achieved at the first target frequency. More modest noise reduction was observed in two other frequencies.

Two types of extensions to the basic TS procedure were required to plan the Beech flight tests. These extensions allow multiple objective functions and constraints. For example, the goal of the flight tests is to reduce interior noise at three frequencies simultaneously, whereas the goal of the laboratory tests is to reduce noise at one frequency at a time. Moreover, there are constraints on the placement of sensors because some locations are overly sensitive to measurement noise or some subsets contain insufficient sensors to characterize the response field. Also, there are constraints on the placement of actuators, because the c_k calculated in step 5 above is often too large to be produced by the available actuators. These extensions to the TS method are discussed in references 43 and 45.

The lessons learned from applying TS to the ASAC problem are summarized here and discussed in reference 43. The most important lesson is that the cost function must be inexpensive to evaluate but does not have to be accurate. In the ASAC problem, the cost function is not the expected measured value of the overall sound pressure level, but is rather a lower bound on the expected value. Similarly, the constraints on c_k are crude estimates of the required control effort. Tabu search uses the penalized cost function to determine which of two neighbors has the better performance. Thus, the accuracy of the penalized cost function is not important as long as the trends are properly represented. However, the use of constraints is very important. Without constraints, the TS procedure favors sets of locations that provide marginally better performance while using orders of magnitude more power. Similarly, when optimizing sensor placement without constraints, TS favors sets of locations that appear to be marginally better, but that require unrealistically sensitive measurement accuracy.

Related Actuator and Sensor Placement Problems
The second aerospace application of combinatorial optimization involves the placement of sensors for system identification. Kammer studied this problem as part of the early planning for the international space station [49]. However, the same system identification techniques may be useful in the Aircraft Morphing program, where large numbers of sensors and actuators will be used for aeroservoelastic control. The basic

problem is to correctly observe, with a limited number of sensors, all important vibration modes in a flexible structure.

The optimization problem is easily stated in mathematical terms. Use a structural finite element model to produce a matrix $\mathbf{X}^a$ of eigenvectors for the n most important vibration modes at the m potential sensor locations. Alternately, produce the matrix $\mathbf{X}^a$ by using measured data. Choose a subset of k sensor locations and form the reduced matrix $\mathbf{X}$ by selecting rows from $\mathbf{X}^a$. The best subset is the one that maximizes the determinant of the Fisher information matrix $\mathbf{X}^T\mathbf{X}$.

Kincaid has successfully applied TS to the system identification problem [38]. He adapted a TS code originally developed for the ASAC problem. The cost function is evaluated with an existing discrete D-optimal design code called DOPT, developed by Miller and Nguyen [50]. This code implements an efficient neighborhood search for the maximum determinant of $\mathbf{X}^T\mathbf{X}$.

A lesson learned in applying TS to the system identification problem is that it is hard to improve on the solutions found by the older techniques for the problems cited by Kammer. However, the new TS algorithm that uses DOPT is much more efficient than the GA-based techniques previously used [21]. Therefore, TS can be used when the design space is very large. The efficiency is gained by substituting an approximate cost function for the determinant of $\mathbf{X}^T\mathbf{X}$ whenever possible.

The third combinatorial optimization problem involves placement of flow control actuators for three-axis control of an aircraft. Scott explains that conventional control devices like flaps and ailerons have gaps between the wing and the control surface that contribute to leakage and protuberance drag [51]; these gaps can cause increased aerodynamic noise and reduced stealth. Flow control actuators potentially allow a seamless aircraft with no moving external control surfaces, but rather hundreds of small ports capable of aerodynamically morphing the shape of the wing as needed. Obviously, the number and placement of these ports affect the cost and complexity of such a vehicle.

A GA-based method is being developed [52] to minimize the number of actuators needed for active flow control. A 3-D panel code (PMARC) [53] predicts the aerodynamic force and moment coefficients for the various combinations of actuator locations. The number of possible combinations is large, but the GA approach finds good combinations without trying all of them. A good combination is one that satisfies a three-condition flight control design. The first condition requires a pitching moment of at least −0.05 while the roll and yaw moments are constrained to very small absolute values. Similarly, the second condition requires a significant roll moment uncoupled from pitch and yaw, and the third condition requires yaw without pitch or roll. The GA is required to find the minimum number of installed actuators that can be turned on and off to produce these three conditions on demand.

This research is in its early stages, but one important lesson can be explained. This lesson is that the design variables and the actuator location numbers are not necessarily the same. Initially, the GA-based method used a set of design variables with values of either 0 or 1 to indicate selected locations. This method ignores the fact that, to produce pitch without yaw or roll, an even number of actuators with a certain side-to-side symmetry is required. Similarly, the other two conditions imply different kinds of placement symmetry. Due to the poor choice of design variables, the initial method was inefficient. The neighborhood structure is such that good subsets tend to have inferior neighbors because, by changing one design variable value, the symmetry can be lost. Similarly, if two excellent parents are mated, the offspring are usually inferior because each offspring violates symmetry requirements. The initial method was improved by defining design variables in such a way that a symmetric set of locations was guaranteed. For example, if the first design variable has a value of 1, then several actuator locations with the proper symmetry are activated. For the three-condition flight control design, three sets of design variables are used to generate three sets of actuator locations with symmetry appropriate for each of the flight control conditions. The total number of actuators is then determined by counting those actuators required by one or more conditions.

7. Concluding Remarks
Actuator and sensor placement problems occur in engineering systems from nuclear power plants to the International Space Station. A survey of these applications highlights both the differences and the similarities in these systems. Differences include actuator number and type (e.g., valves, loudspeakers, and

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piezoelectric devices) and sensor number and type (e.g., microphones, accelerometers, and strain gages). Other differences include the performance measures although in general the performance measures are calculated by using a transfer matrix of responses at each sensor due to a set of actuator inputs.

The algorithms for optimal placement of actuators and sensors appear to be very similar regardless of the application. All can be posed as selecting a subset of locations from a large set of candidate locations. This paper describes several combinatorial optimization methods like tabu search and genetic algorithms that are effective in solving these problems. These optimization methods are particularly important when the performance tradeoffs are not intuitive and when the number of combinations is large enough to preclude enumeration.

Researchers at NASA Langley have solved a number of actuator and sensor placement problems and believe that the lessons learned will be helpful in future applications. One lesson is to formulate the problem carefully so that nonsensical configurations can be discarded without evaluation. Another lesson is to approximate the performance measure and use this faster technique to evaluate as many configurations as possible. The last lesson is to include all important constraints, such as limits on the force available to actuators and the sensitivity of sensors.

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9. Tables and Figures

Table 1. Summary of optimal placement journal articles

First Author	Journal	Volume (Issue)	Year	Optimization Method	Potential Locations	Number Selected
Naimimohasses	*Measurement Science Tech.*	**6**(9)	1995	other	6	2
Oh	*Nuclear Eng. and Design*	**152**(1)	1994	other	52	14
Reis	*J. Water Resources Plan. & Manag.*	**123**(6)	1997	GA	25	3
Mattingly	*Int. J. Hyperthermia*	**14**(4)	1998	none	NA	4
Emery	*J. Heat Transfer*	**119**(4)	1997	none	NA	NA
Sunar	*AIAA Journal*	**34**(10)	1996	none	NA	2
Liang	*J. Intelligent Mat. Sys. & Structures*	**6**(4)	1995	none	8	3
Bhargava	*J. Intelligent Mat. Sys. & Structures*	**6**(3)	1995	none	5	2
Kang	*AIAA Journal*	**34**(9)	1996	none	NA	2
Hac	*J. Sound and Vibration*	**167**(2)	1993	none	10	2
Wang	*J. Acoustical Society of America*	**90**(5)	1991	none	NA	3
Kang	*AIAA Journal*	**36**(9)	1998	NLP	NA	1
Papadopoulos	*AIAA Journal*	**36**(2)	1998	other	NA	NA
Simpson	*Noise Control Engineering J.*	**44**(4)	1996	GA	40	15
Gawronski	*J. Sound and Vibration*	**208**(1)	1997	other	36	2
Kammer	*J. Sound and Vibration*	**171**(1)	1994	other	321	15
Pottie	*Internoise 96*		1996	GA	90	14
Manolas	*Internoise 96*		1996	GA	80	48
Wang	*J. Acoustical Society of America*	**99**(5)	1996	GA	NA	3
Katsikas	*Mechanical Sys. & Signal Processing*	**9**(6)	1995	GA	80	26
Maghami	*IEEE T. on Aero. & Elect. Sys.*	**29**(2)	1993	NLP	98	3
De Fonseca	*SPIE*	**3041**	1997	NLP	NA	4
Dhingra	*Int. J. for Num. Methods in Eng.*	**38**(20)	1995	GA	12	5
Wang	*J. Intelligent Mat. Sys. and Structures*	**5**(1)	1994	NLP	NA	3
Liu	*J. Aerospace Eng.*	**10**(3)	1997	SA	90	4
Chattopadhyay	*SPIE*	**3329**	1998	GA	NA	NA
Kincaid	*Location Science*	**1**(2)	1993	SA	1608	32
Kincaid	*J. Combinatorial Optimization*	**1**(3)	1997	TS	102	32
Furuya	*J. of Spacecraft and Rockets*	**33**(3)	1996	GA	1608	8

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Table 2. Optimization results at 275 Hz

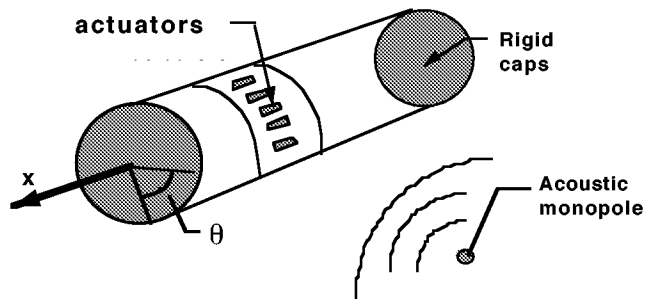| Selection criteria	Predicted reduction	Measured reduction
Best 4 actuators and best sensors	−5.7 dB	−3.9 dB
Worst 4 actuators and best sensors	−0.5 dB	−0.4 dB
Modal method	N/A	−2.7 dB

Figure 1. Schematic of simplified cylinder, point source, and actuator model.
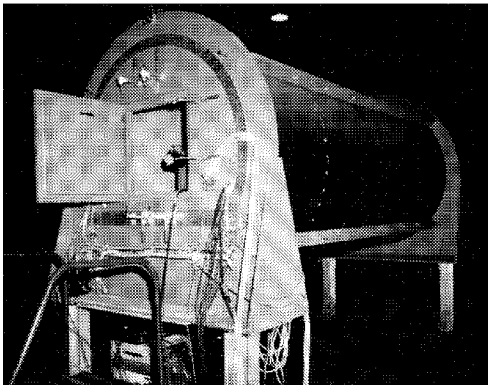

Figure 2. Composite cylinder at NASA Langley.

Figure 3. Beech 1900 aircraft used for flight testing.

REPORT DOCUMENTATION PAGE		Form Approved OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE April 1999	3. REPORT TYPE AND DATES COVERED Technical Memorandum	
4. TITLE AND SUBTITLE Optimization Strategies for Sensor and Actuator Placement			5. FUNDING NUMBERS WU 522-32-31-03
6. AUTHOR(S) Sharon L. Padula and Rex K. Kincaid			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) NASA Langley Research Center Hampton, VA 23681-2199			8. PERFORMING ORGANIZATION REPORT NUMBER L-17839
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, DC 20546-0001			10. SPONSORING/MONITORING AGENCY REPORT NUMBER NASA/TM-1999-209126
11. SUPPLEMENTARY NOTES Kincaid: College of William and Mary, Williamsburg, VA			
12a. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified-Unlimited Subject Category 59 Distribution: Standard Availability: NASA CASI (301) 621-0390			12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

This paper provides a survey of actuator and sensor placement problems from a wide range of engineering disciplines and a variety of applications. Combinatorial optimization methods are recommended as a means for identifying sets of actuators and sensors that maximize performance. Several sample applications from NASA Langley Research Center such as active structural acoustic control are covered in detail. Laboratory and flight tests of these applications indicate that actuator and sensor placement methods are effective and important. Lessons learned in solving these optimization problems can guide future research.

14. SUBJECT TERMS Optimization; Optimal Placement; Sensors; Actuators; Smart technologies			15. NUMBER OF PAGES 17
			16. PRICE CODE A03
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL