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A NEW MERIT FUNCTION FOR EVALUATING THE FLAW TOLERANCE OF COMPOSITE LAMINATES - PART II, ARBITRARY SIZE HOLES AND CENTER CRACKS

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ABSTRACT

In a previous paper [1], a new merit function for determining the strength performance of flawed composite laminates was presented. This previous analysis was restricted to circular hole flaws that were large enough that failure could be predicted using the laminate stress concentration factor. In this paper, the merit function is expanded to include the flaw cases of an arbitrary size circular hole or a center crack. Failure prediction for these cases is determined using the point stress criterion. An example application of the merit function is included for a wide range of graphite/epoxy laminates.

INTRODUCTION

In a previous paper [1], a new merit function was determined to examine the flaw tolerance of composite laminates. In brief summary, it was determined that the strength performance of a uniaxially loaded infinite width orthotropic sheet could be optimized by maximizing the new merit function, ψE_x. In this function, E_x is the longitudinal extension modulus of the laminate and ψ is the flaw tolerance factor used to predict the reduction in strength of a flawed laminate. The values of ψ are ≤ 1 and can be used to account for various flaw effects such as holes, cracks, or impact damage. For circular hole flaws that were large enough that failure could be predicted using the laminate stress concentration factor, it was shown in [1] that ψ was equal to $1/k$, where k is the orthotropic stress concentration factor. Therefore, the merit function for the case of a large circular hole is E_x/k. For a small circular hole or a center crack, since failure cannot be predicted through the use of a simple stress concentration factor, the same merit function cannot be used.

In this paper, the merit function is expanded to include the flaw cases of an arbitrary size hole or a center crack. In order to predict the failure strength of an infinite width plate in the presence of these flaws, the point stress criterion [3] is used. After development of the merit function for each of the flaw cases,

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examples are presented of the application of the merit function to a wide range of graphite/epoxy laminates.

DESIGN OF A LAMINATE FOR IMPROVED FLAW TOLERANCE IN THE PRESENCE OF A CIRCULAR HOLE

For an infinite width orthotropic sheet containing a hole of radius R, if a uniaxial stress σ is applied parallel to the y-axis (Figure 1), then the normal stress, σy, along the x-axis is approximated [5] by

$$\sigma_y(x,0) = \frac{\sigma}{2}\left\{2 + \left(\frac{R}{x}\right)^2 + 3\left(\frac{R}{x}\right)^4 - (k-3)\left[5\left(\frac{R}{x}\right)^6 - 7\left(\frac{R}{x}\right)^8\right]\right\} \quad (1)$$

where k is the orthotropic stress concentration factor for an infinite width orthotropic plate [2]

$$k = 1 + \sqrt{2\left(\sqrt{\frac{E_{11}}{E_{22}}} - v_{12}\right) + \frac{E_{11}}{G_{12}}}. \quad (2)$$

The subscripts 1 and 2 denote the material principle axis.

The stress distribution given by (1) is actually a modification of the isotropic stress distribution solution. For isotropic materials, the stress concentration factor, k, equals 3 and expression (1) is exact. Since (1) is an approximation of the stress distribution in front of a hole for anisotropic materials, the accuracy of this expression was tested before use. The approximate stress distribution was compared with the exact stress distribution solution from Savin [4]. The details of this comparison are contained in Appendix A.

In order to predict failure in the presence of a circular hole, the point stress failure criterion [3] has been used. Using the point stress criterion, failure of a flawed material is assumed to occur when the normal stress, σy, at a certain distance, d_0, ahead of the flaw reaches the ultimate strength of the unflawed laminate, σo, or when

$$\sigma_y(x,o)\big|_{x=R+d_0} = \sigma_o. \quad (3)$$

d_0 is a material parameter that needs to be experimentally determined for each laminate under consideration.

Using the point stress criterion along with the stress distribution given by (1) results in the flawed to unflawed strength ratio for an infinite width plate containing a circular hole as

$$\frac{\sigma_N^\infty}{\sigma_o} = \frac{2}{\left\{2 + \xi_1^2 + 3\xi_1^4 - (k-3)\left(5\xi_1^6 - 7\xi_1^8\right)\right\}} \tag{4}$$

where

$$\xi_1 = \frac{R}{(R + d_0)} \tag{5}$$

In (4), σ_N^∞ is the ultimate strength of the flawed laminate.

Using the methodology presented in [1], expression (4) is used to define the flaw tolerance factor for the flaw case of a circular hole, ψ_{CH}, as

$$\psi_{CH} = \frac{2}{\left\{2 + \xi_1^2 + 3\xi_1^4 - (k-3)\left(5\xi_1^6 - 7\xi_1^8\right)\right\}} \tag{6}$$

As R —> 0, then ψ_{CH} —> 1 which, as expected, implies that there will be no reduction in strength. As R becomes large (> 2.5 in), then ψ_{CH} —> 1/k. This is shown in Figure 2 where the value ψ_{CH}*k is plotted against hole radius for three values of the stress concentration factor. Since ψ_{CH} —> 1/k, then ψ_{CH}*k → 1. For a hole diameter larger than 5 in, each of the curves shown in Figure 2 approach this limit with the error only reaching a maximum of 11% for the k=5 curve. Since expression (6) returns the expected limits of the strength reduction, it is used herein for a circular hole of any size.

For the flaw case of a circular hole, the merit function is

$$\psi_{CH} E_x = \frac{2E_x}{\left\{2 + \xi_1^2 + 3\xi_1^4 - (k-3)\left(5\xi_1^6 - 7\xi_1^8\right)\right\}} \tag{7}$$

DESIGN OF A LAMINATE FOR IMPROVED FLAW TOLERANCE IN THE PRESENCE OF A CENTER CRACK

For a center crack of length 2c in an infinite width orthotropic sheet under an axial load σ (Figure 1), the exact anisotropic solution for the normal stress, σ_y, in front of the crack [2] is

$$\sigma_y(x,0) = \frac{\sigma x}{\sqrt{x^2 - c^2}} \tag{8}$$

Using the point stress criterion along with (8) results in the flawed to unflawed strength ratio for the flaw case of a center crack [3] as

$$\frac{\sigma_N^\infty}{\sigma_o} = \sqrt{1 - \xi_3^2} \tag{9}$$

where

$$\xi_3 = \frac{c}{(c + d_0)}. \tag{10}$$

Using (9), the flaw tolerance factor for the flaw case of a center crack, ψ_{CC}, can be defined as

$$\psi_{CC} = \sqrt{1 - \xi_3^2}. \tag{11}$$

It should be noted from (11) that ψ_{CC} is independent of laminate properties and is only a function of the crack size and d_0. This implies that for a given crack size and d_0, ψ_{CC} is a constant. Therefore, in the case of a center crack, the merit function will be an optimum by maximizing the laminate property E_x.

The merit function for the flaw case of a center crack is

$$\psi_{CC} E_x = \left(\sqrt{1 - \xi_3^2}\right) E_x. \tag{12}$$

Example Application of Merit Function

For the example application of the merit function, the same baseline laminate identified in Table 1 of reference [1] was used. To show the application of the merit function, the baseline laminate was perturbed from the initial configuration of $[\pm 45/0_2/\pm 45/0_2/\pm 45/0/90]_{2s}$ to a different configuration. The resulting laminate properties and flaw tolerance factor were then determined for each of the resulting laminates.

To do this, the baseline laminate was considered to consist of only three layers. These three layers each have thickness t_l and consist of all the 0° plies, all the

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|±45°| plies, and all the 90° plies respectively. Using $\bar{t}_l = t_l / h$, h = laminate thickness, the relative values of the $\bar{t}_l$'s were changed, and the angle of the ±45° ply was changed to some angle $\pm\theta$. The laminate stiffness properties were then computed using equations (3.1) through (3.5) from [1] and the stiffness constants defined in Appendix A of [1] also. The resulting flaw tolerance and merit function for circular hole flaws was then determined using equations (2),(5), (6) and (7). For center crack flaws, equations (10) and (12) were used.

For each of the flaw types, the dimensionless thickness $\bar{t}_{90}$ was left constant at $\bar{t}_{90}$ = .083, its value in the baseline laminate. The values of $\bar{t}_0$ and $\bar{t}_\theta$ were then varied from 0 to .917 while holding the sum $\bar{t}_0 + \bar{t}_\theta$ = .917 a constant, and θ was varied from 20° to 60°. The result these changes have on the laminate stiffness E_X is shown in Figure 3. As expected, E_X increases with increasing $\bar{t}_0$ and decreasing θ. The resulting changes in the individual merit functions are discussed in the following sections. A value of d_0 = .04 inches was used in all calculations since this value has been shown to give good results for graphite/epoxy laminates [3,6].

Circular Holes

Three hole sizes were used: R= .125 in., R=.5 in. and R= 2.5 in.. The R=2.5 in. hole size is the same as that presented in [1]. The flaw tolerance factors for $\theta = \pm45$ and variable $\bar{t}_0$ are shown in Figure 4, while the effect of varying θ is shown in Figures 5, 6 and 7. A major difference between the small and large hole cases is the fact that the flaw tolerance factor increases with increasing $\bar{t}_0$ for the small hole case (R=.125), while it decreases with increasing $\bar{t}_0$ for the larger hole cases. In addition, decreasing θ improves the flaw tolerance factor for small holes, while worsening the flaw tolerance factor for larger holes. The resulting values of the merit function are shown in Figures 8, 9 and 10.

For the R=.125 case (Figure 8), it can be seen that the optimum values of the merit function will occur when $\bar{t}_0$ is increased to its maximum value of .917 and when θ is decreased to 20°. Significant increases in the merit function compared to the baseline laminate are possible.

For the R=.5 case (Figure 9), optimum values for the merit function also occur at high values of $\bar{t}_0$ and low values of θ, but not at the extremes of these parameters. Significant increases in the merit function over the baseline laminate are still possible.

The results for the R=2.5 case (Figure 10) are identical to those presented in [1]. Optimum values of the merit function now occur at various combinations of $\bar{t}_0$ and θ. Increases in the merit function are still possible, but are not as significant as in the smaller hole cases.

Center Cracks

Three different crack sizes were examined: cracks with half lengths of c=.125 in., c=.5 in and c= 2.5 in.. Since the values of ψ are a constant for any given crack size, the optimum values of the merit function will occur when E_X is maximized. The values of ψ for each case are shown in Figure 4. The resulting merit function values are shown in Figures 11, 12 and 13. As intuitively expected, larger cracks reduce the strength more than smaller cracks. The merit function for each crack size is optimum at high values of $\bar{t}_0$ and lower values of θ since these values maximize E_X. In each case, significant increases in the merit function compared to the baseline laminate are possible.

Discussion

The example application of the merit function uses the same graphite-epoxy laminate as presented in reference [1]. In addition to large holes, small holes and center cracks were studied in this example. From this example it was seen that:

1) For the flaw case of a small hole (R≤ .125 in), significant increases in the merit function over the baseline laminate are possible. The large increases are driven by the fact that increasing the longitudinal stiffness E_X also increases the flaw tolerance factor for the small hole case. For the larger hole cases, ψ_{CH} decreases with increasing E_X and the increases in the merit function are not as significant.

2) For the case of a center crack, the merit function is optimized by increasing the longitudinal stiffness E_X of the laminate since the flaw tolerance factor is a constant not dependent on laminate properties. Large cracks reduce the laminate strength more than small cracks.

Increases in the merit function are again accompanied by significant decreases in the shear stiffness G_{xy} and fluctuations in the major Poisson's ratio as detailed in [1]. These changes may not be acceptable depending upon the application.

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As detailed in Appendix A, values of the merit function at high values of $\bar{i}_0$ may not be accurate for the circular hole flaws. This is due to the fact that the accuracy of the approximate stress distribution (equation 1) is not very good for highly fiber-dominated laminates. Since the merit function derived for center cracks uses the exact stress distribution (equation 8), the values of the merit function at high $\bar{i}_0$ values should still be accurate in the center crack examples.

It should also be noted that the results presented herein are for infinite width plates only. The effects of finite width are beyond the scope of this paper, and should be examined further.

CONCLUDING REMARKS

In this paper, the new merit function ψE_x developed in [1] was expanded to include the flaw cases of arbitrary size circular holes and center cracks in an infinite width orthotropic sheet. In reference [1], the merit function was limited to holes large enough that failure could be predicted using the laminate stress concentration factor. Since failure of a plate containing a small hole or a center crack cannot be predicted through the use of a simple stress concentration factor, a different merit function must be used.

In the present paper, the merit function for the flaw cases of an arbitrary size circular hole or a center crack were derived using the point stress failure criterion. An example of the use of the merit function was presented for each of the flaw cases using a wide range of graphite/epoxy laminates.

From this example, it was shown that significant increases in the merit function compared to a baseline laminate are possible. The increases are tempered by the flaw size, with larger flaw sizes having smaller increases in the merit function. For center crack flaws, increases in the merit function are governed by increases in the laminate longitudinal stiffness E_x only and not the flaw tolerance factor. This is due to the fact that the flaw tolerance factor for center cracks is a constant not dependent on laminate properties. Accompanying the increases in the merit function are corresponding decreases in the laminate shear stiffness and changes in the major Poisson's ratio. These effects are fully detailed in reference [1]. The effect of finite width on the results contained herein needs to be further examined.

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4 Savin, G.N., "Stress Distribution Around Holes," NASA TT F-607, Translation of "Raspredeleniye Napryazheniy Okolo Otverstiy," "Naukova Dumka" Press, Kiev, 1968.

5 Konish, H.J. and J.M. Whitney, "Approximate Stresses in an Orthotropic Plate Containing a Circular Hole," Journal of Composite Materials, Vol. 9, April 1975, pp. 157-166.

6 Rhodes, M.D., Mikulas, M.M. Jr., and P.E. McGowan, "Effect of Orthotropic Properties and Panel Width on the Compression Strength of Graphite-Epoxy Laminates with Holes," AIAA Journal, Vol. 22, No. 9, September 1984, pp.1283-1292.

7 Poe, C.C. Jr., "Fracture Toughness of Fibrous Composite Materials," NASA Technical Paper 2370, November 1984.
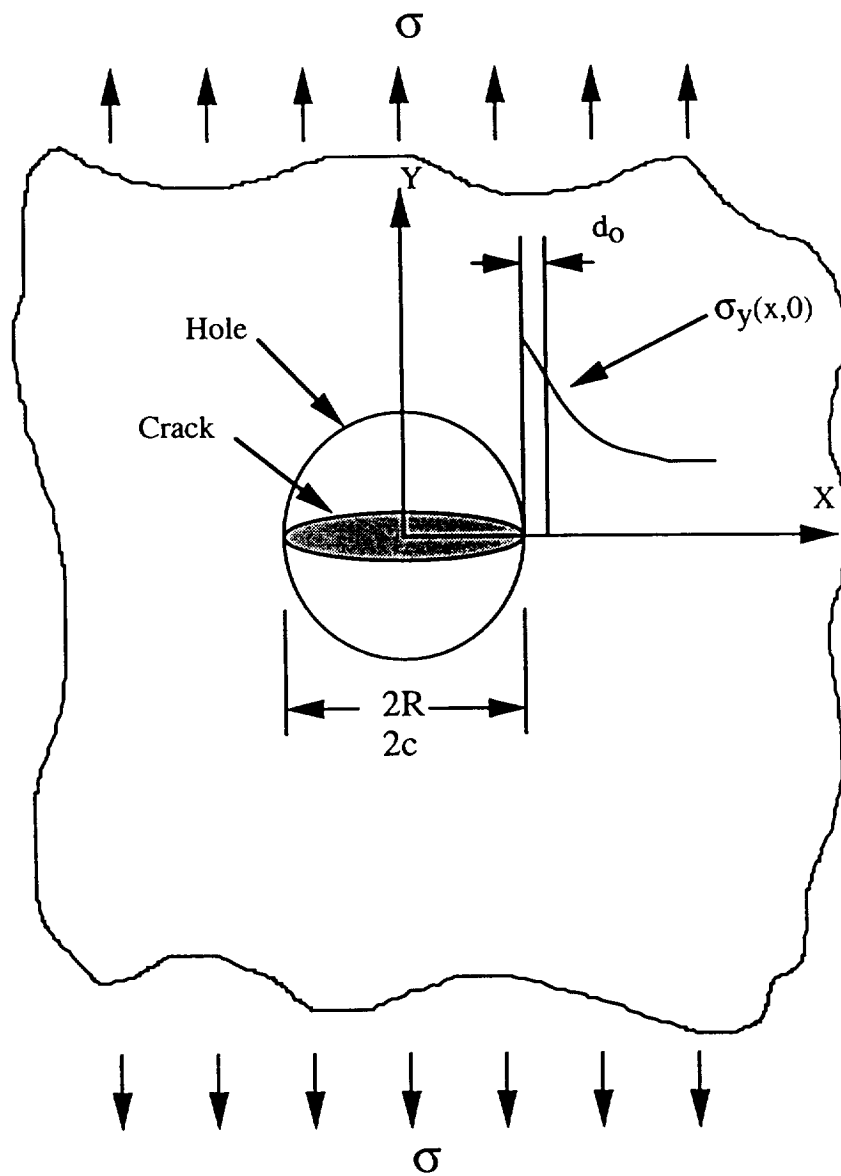
Figure 1. Infinite width plate containing a hole or crack under a uniaxial load.
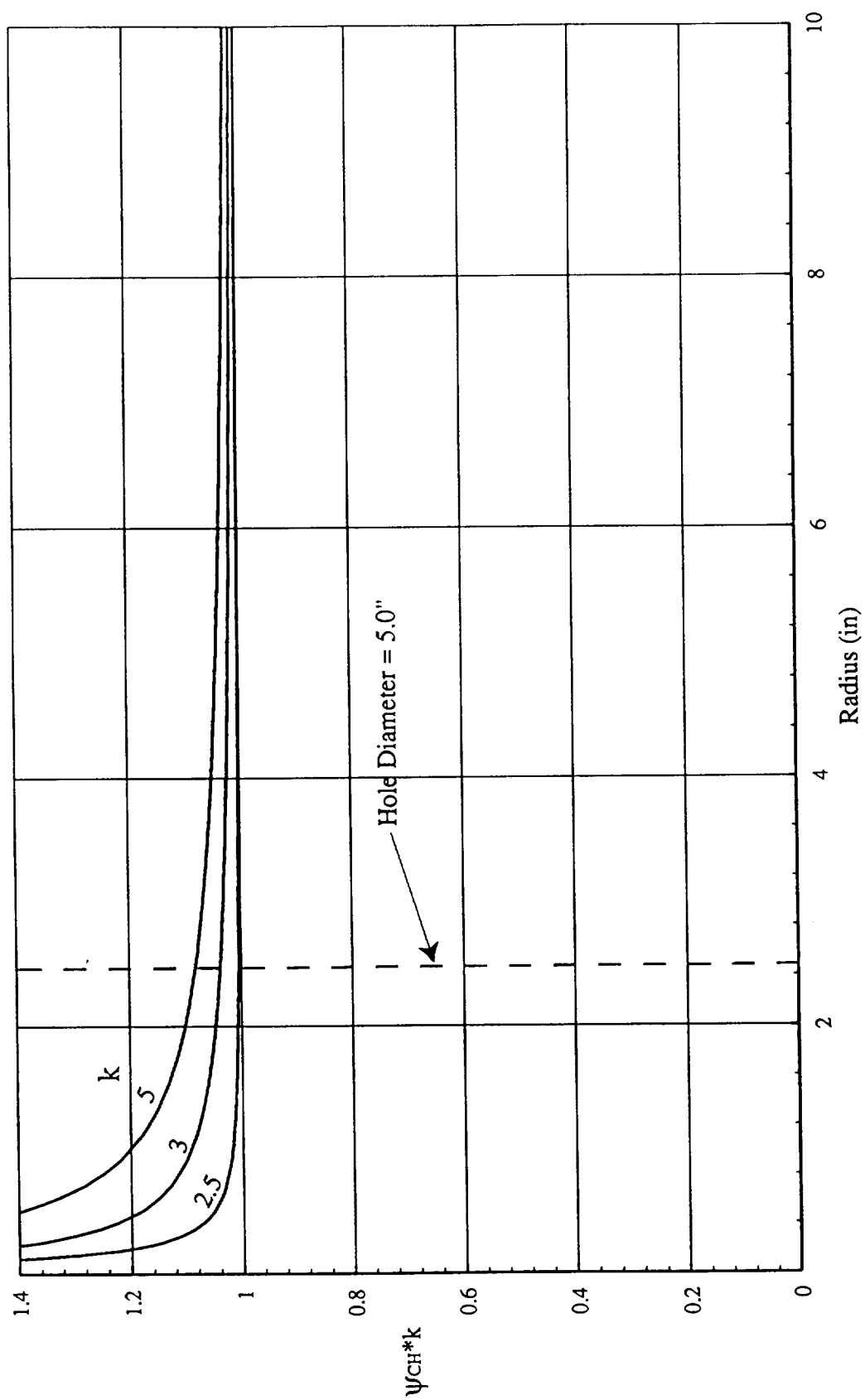
Figure 2 - Flaw tolerance factor approaches 1/k with increasing hole radius.
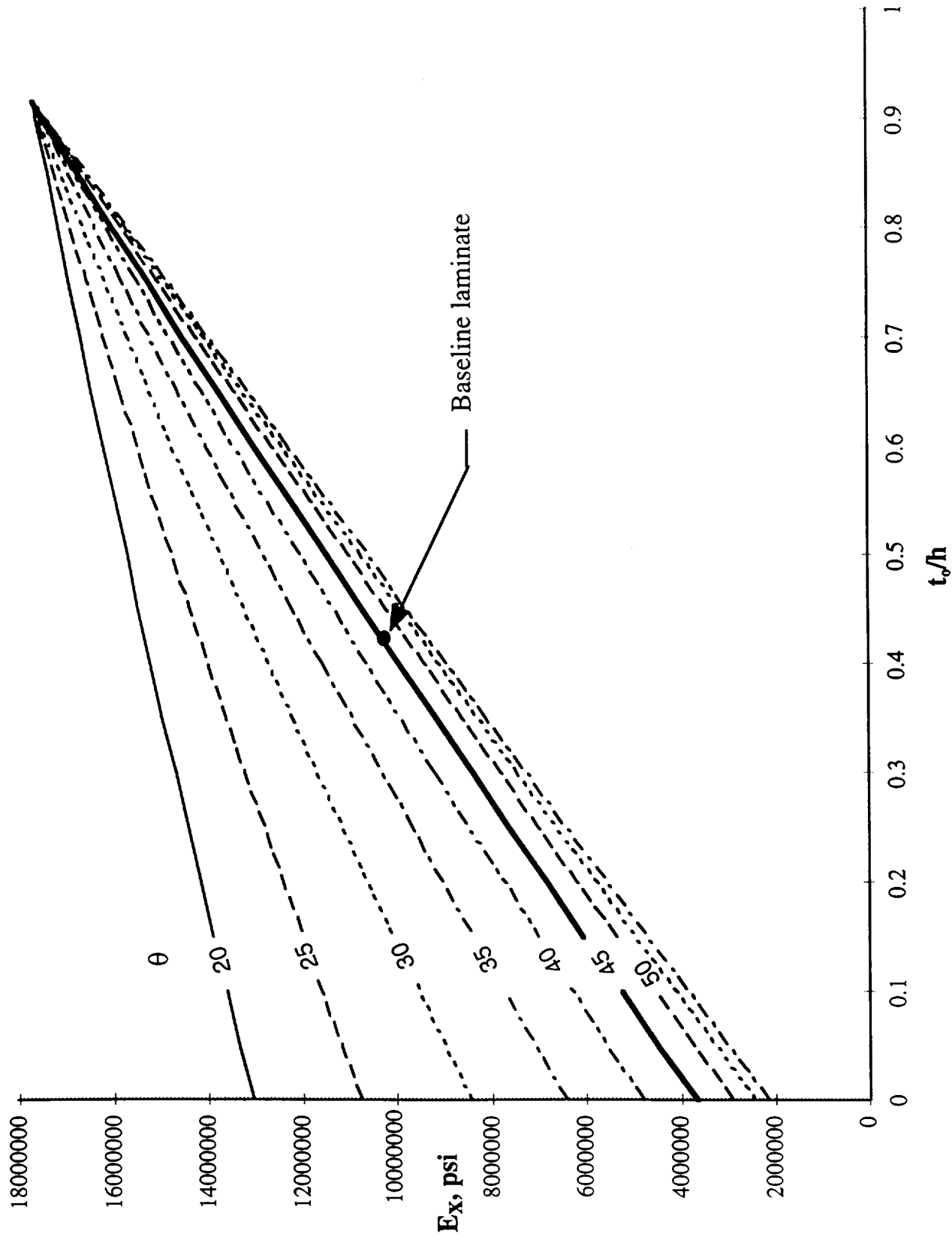
Figure 3 - Extensional Modulus for example laminate.
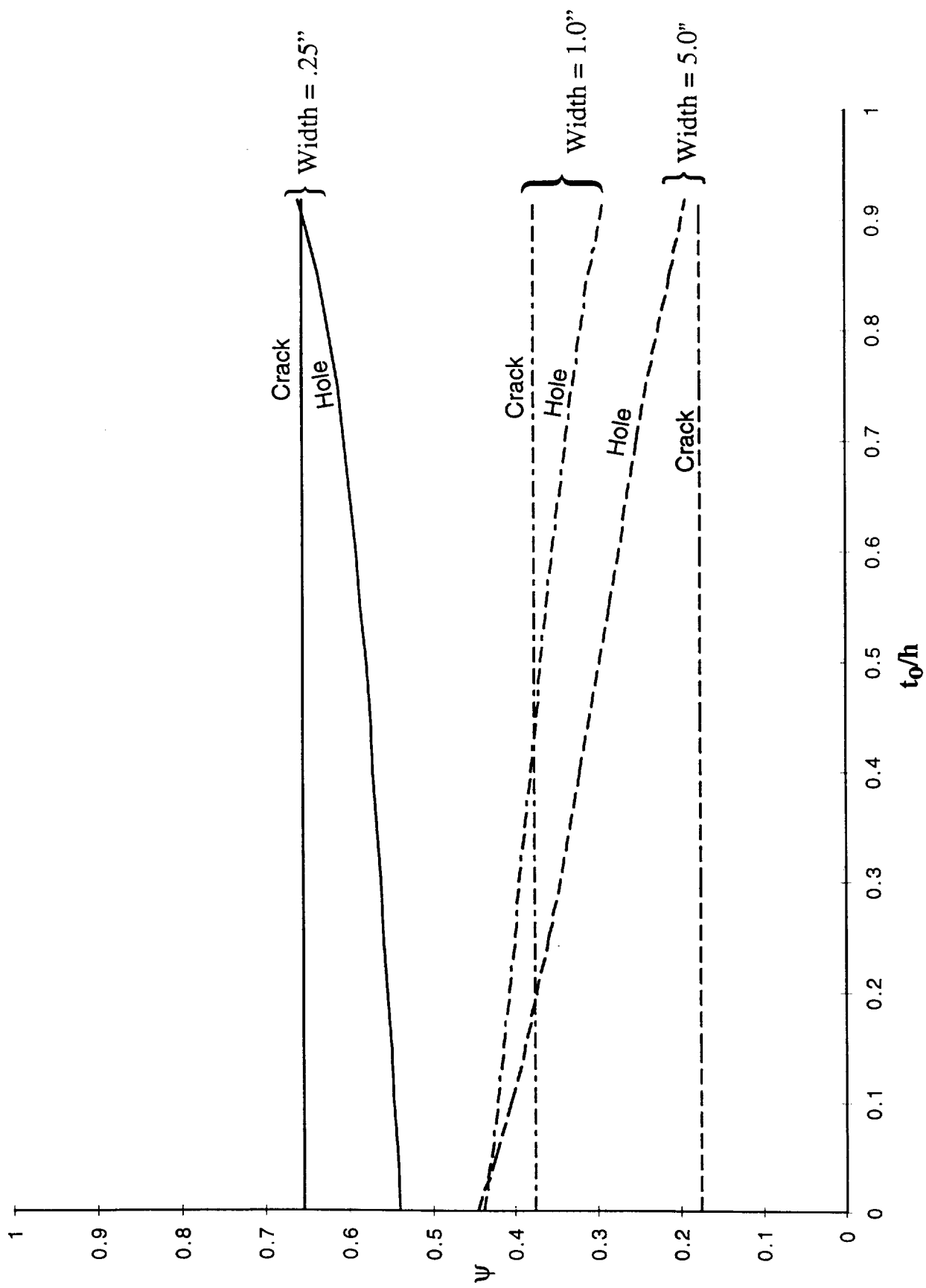
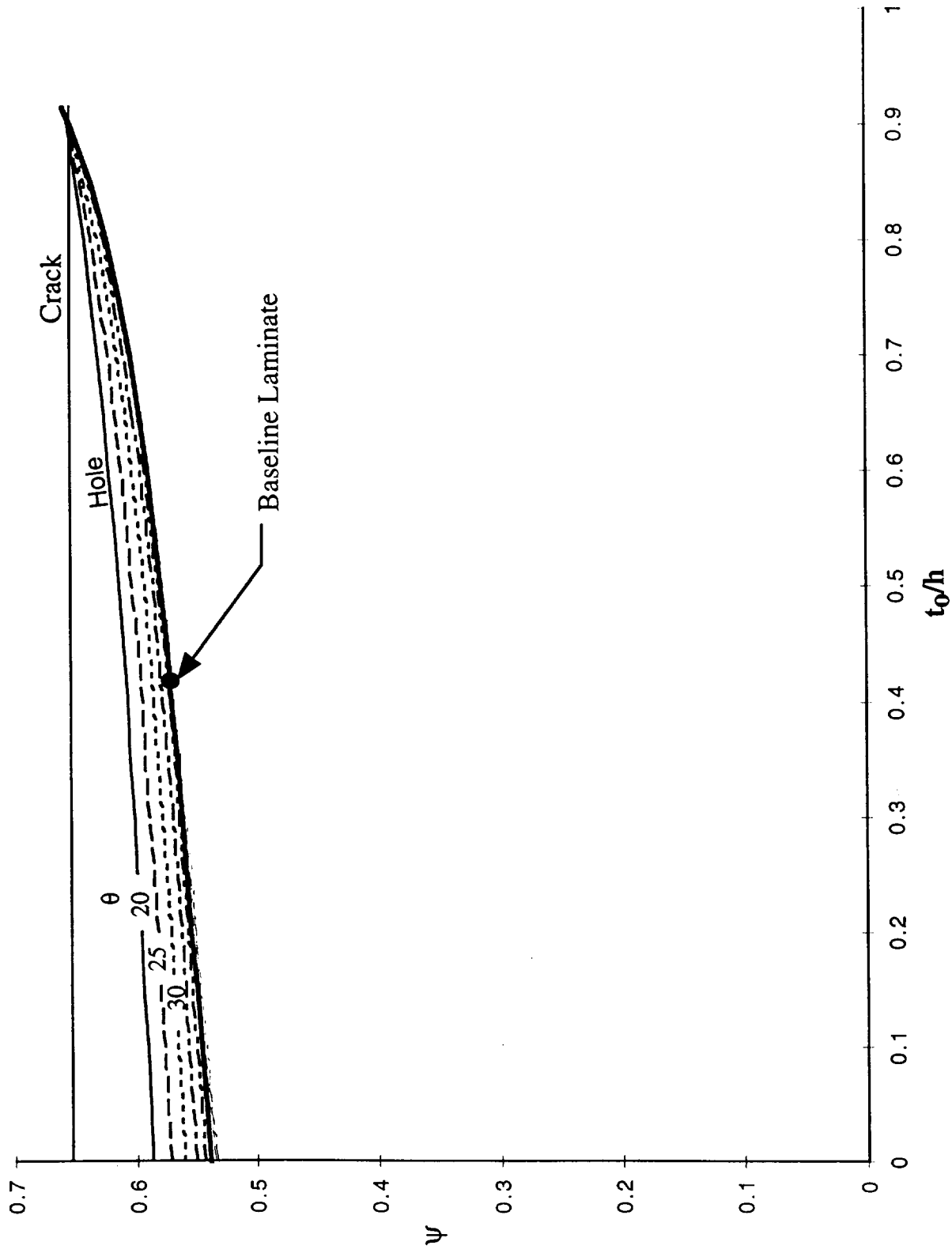Figure 4 - Behaviour of the flaw tolerance factor for circular hole flaws and center cracks.

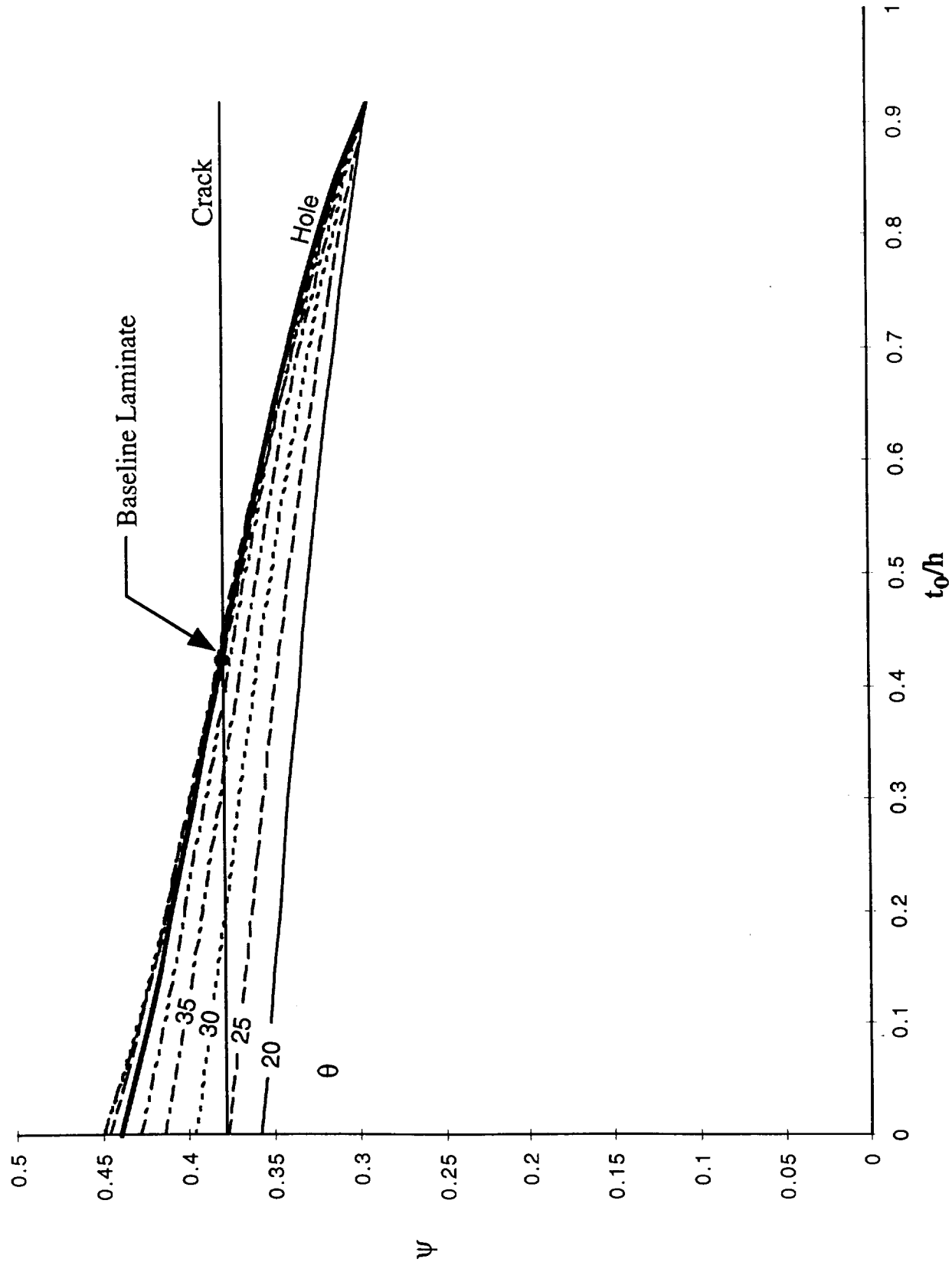Figure 5 - Flaw tolerance factor for a flaw width of .25".

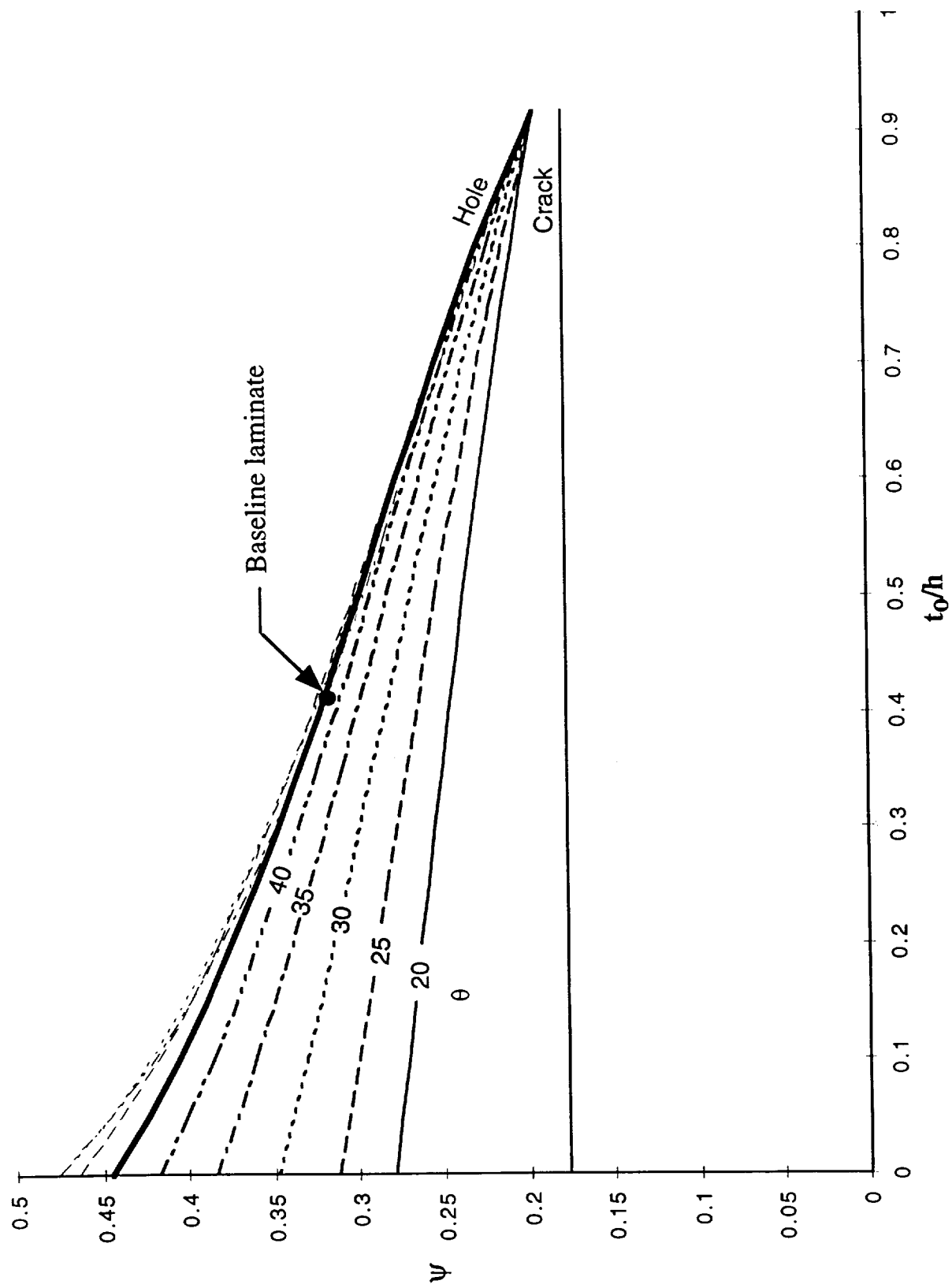Figure 6 - Flaw tolerance factor for a flaw width of 1.0".
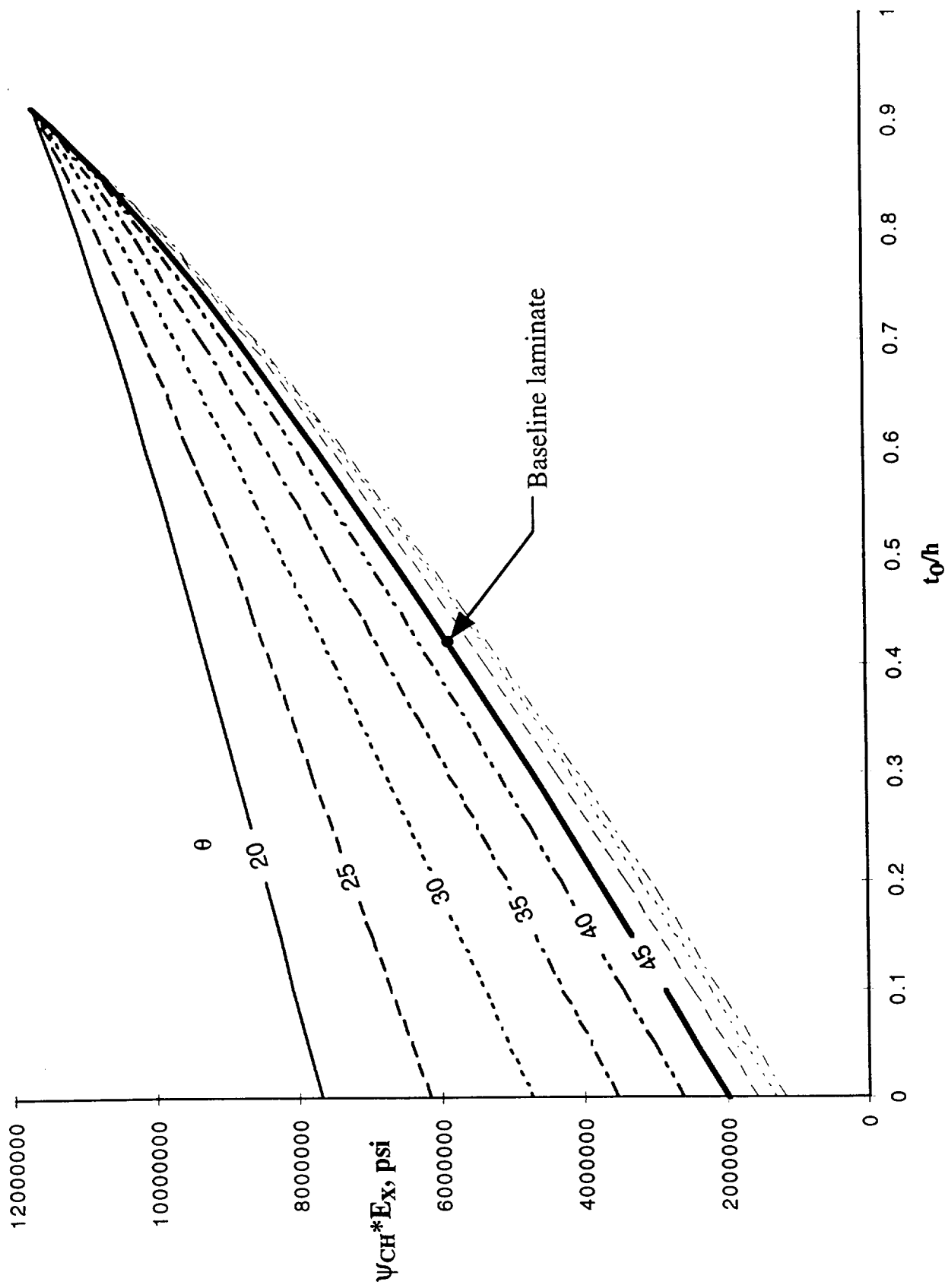
Figure 7 - Flaw tolerance factor for a flaw width of 5.0".

Figure 8 - Merit function for example laminate, hole radius =.125".
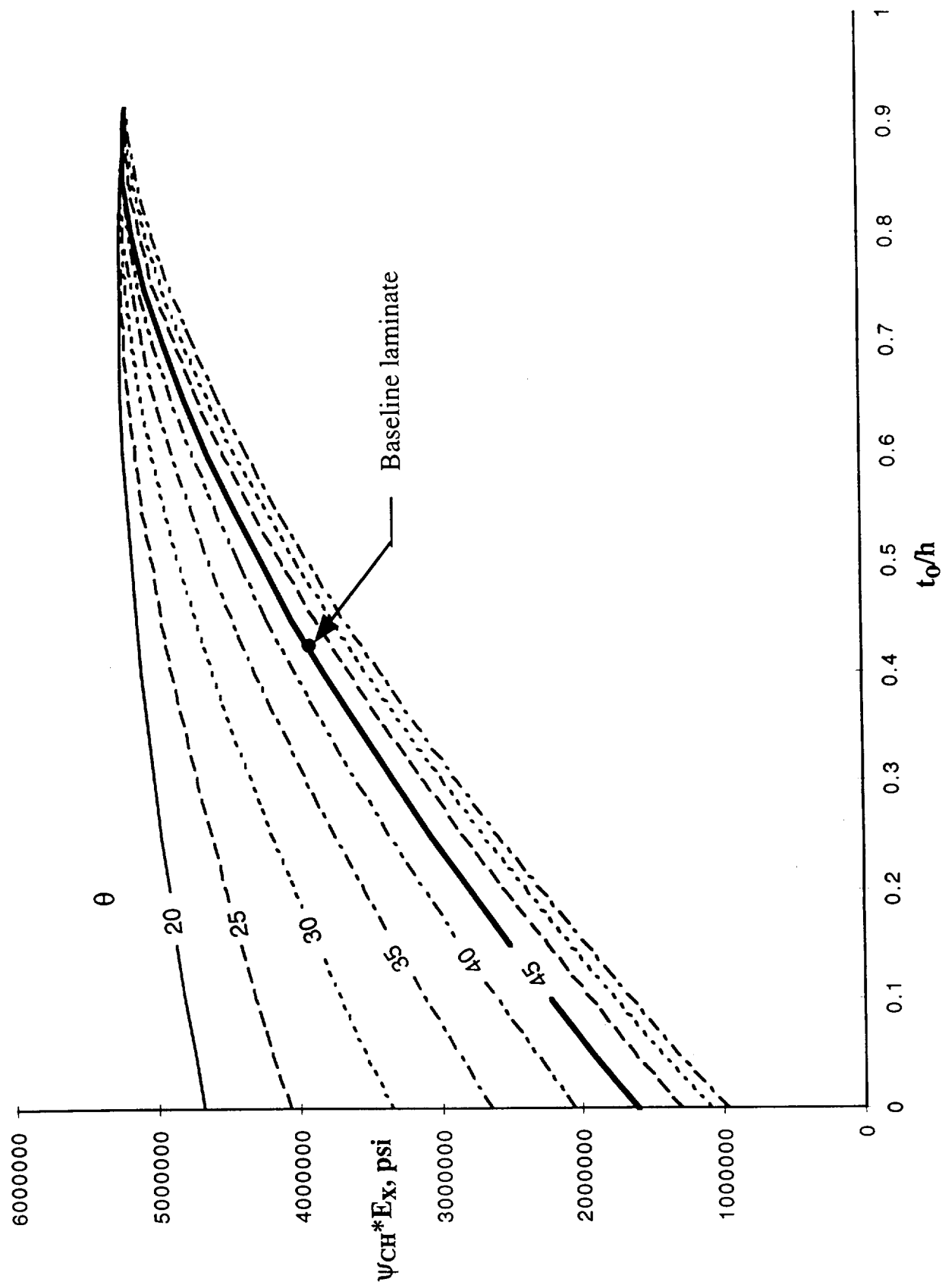
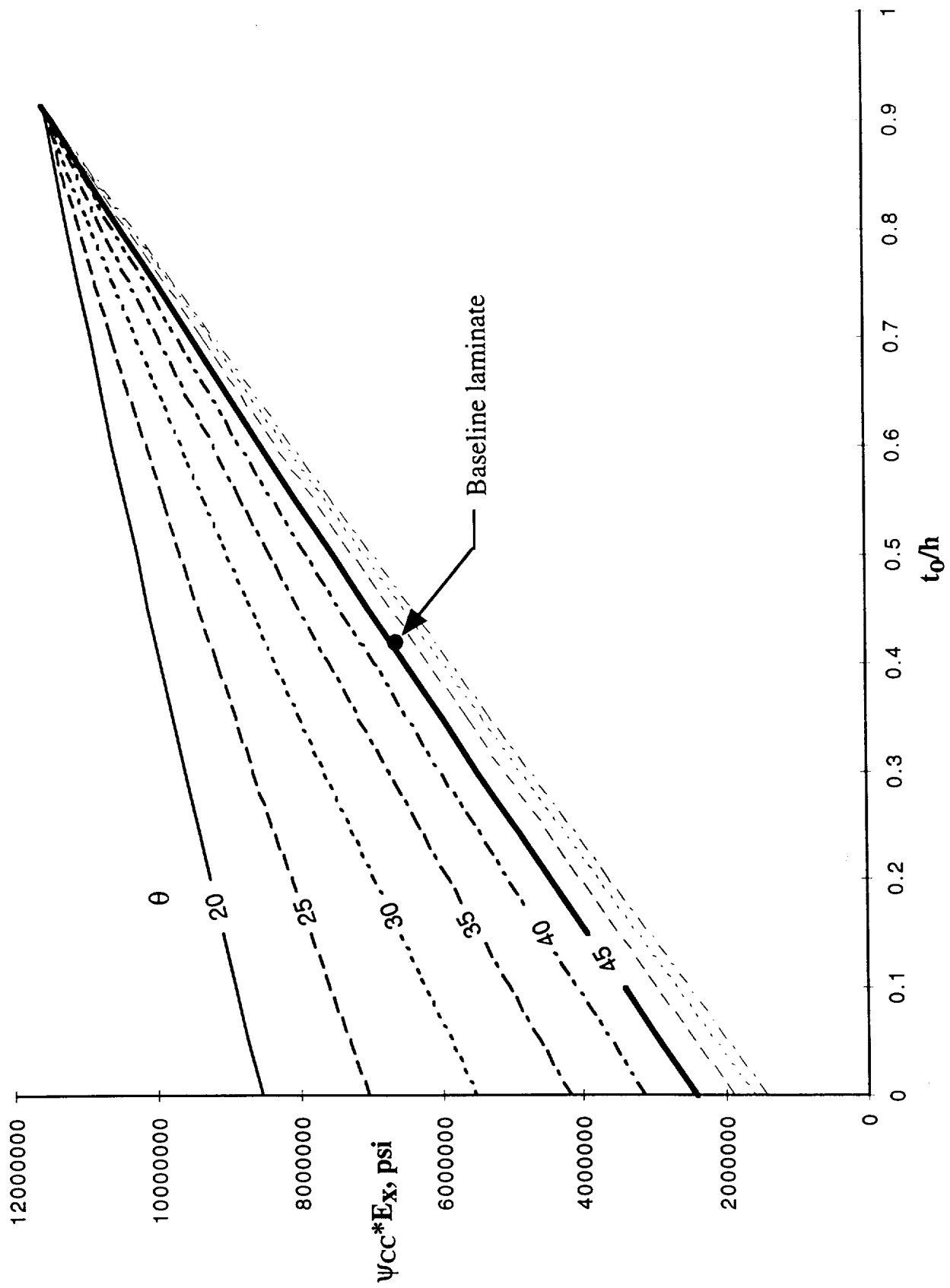Figure 9 - Merit function for example laminate, hole radius = .5".
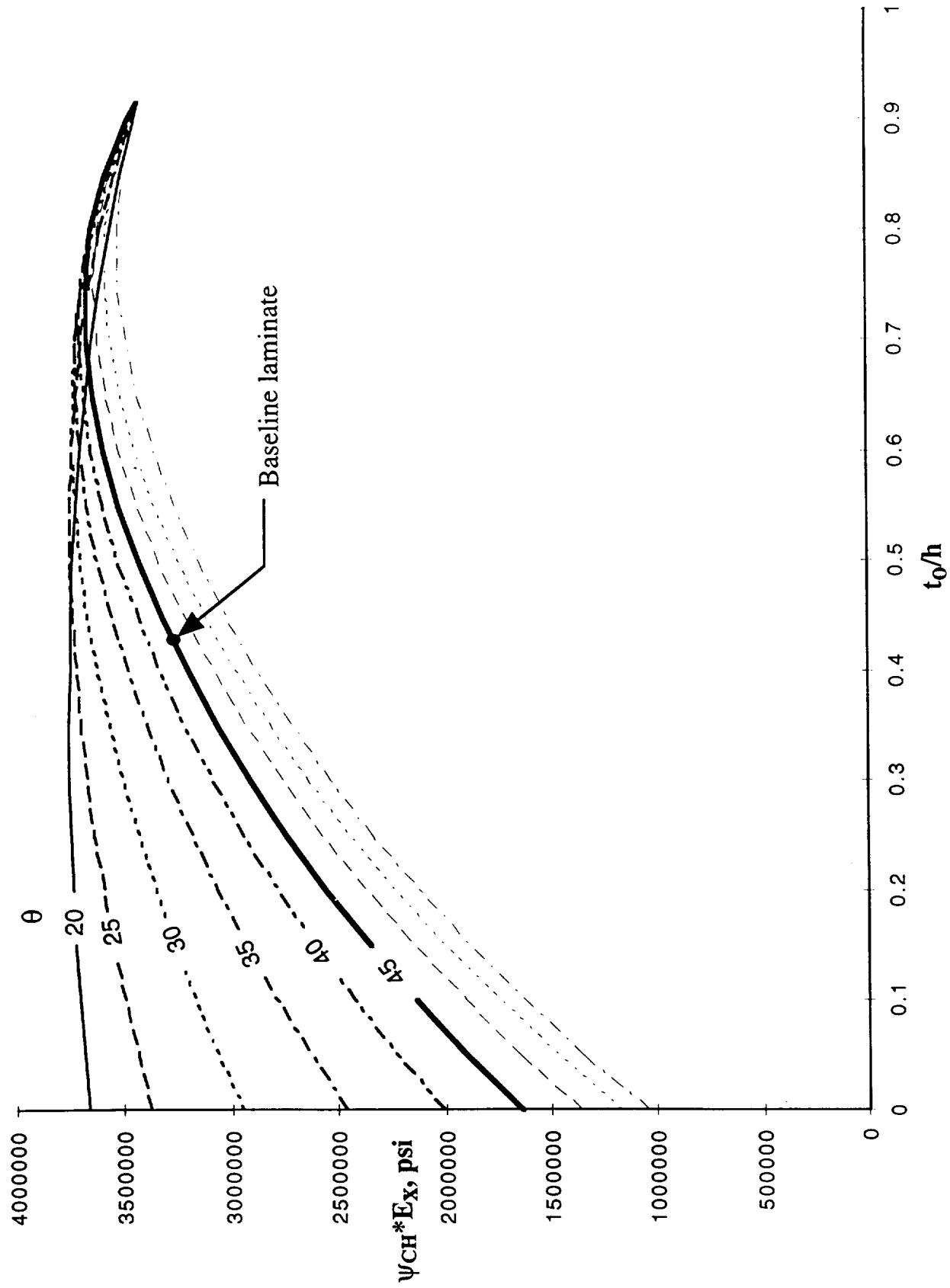
Figure 10 - Merit function for example laminate, hole radius = 2.5".

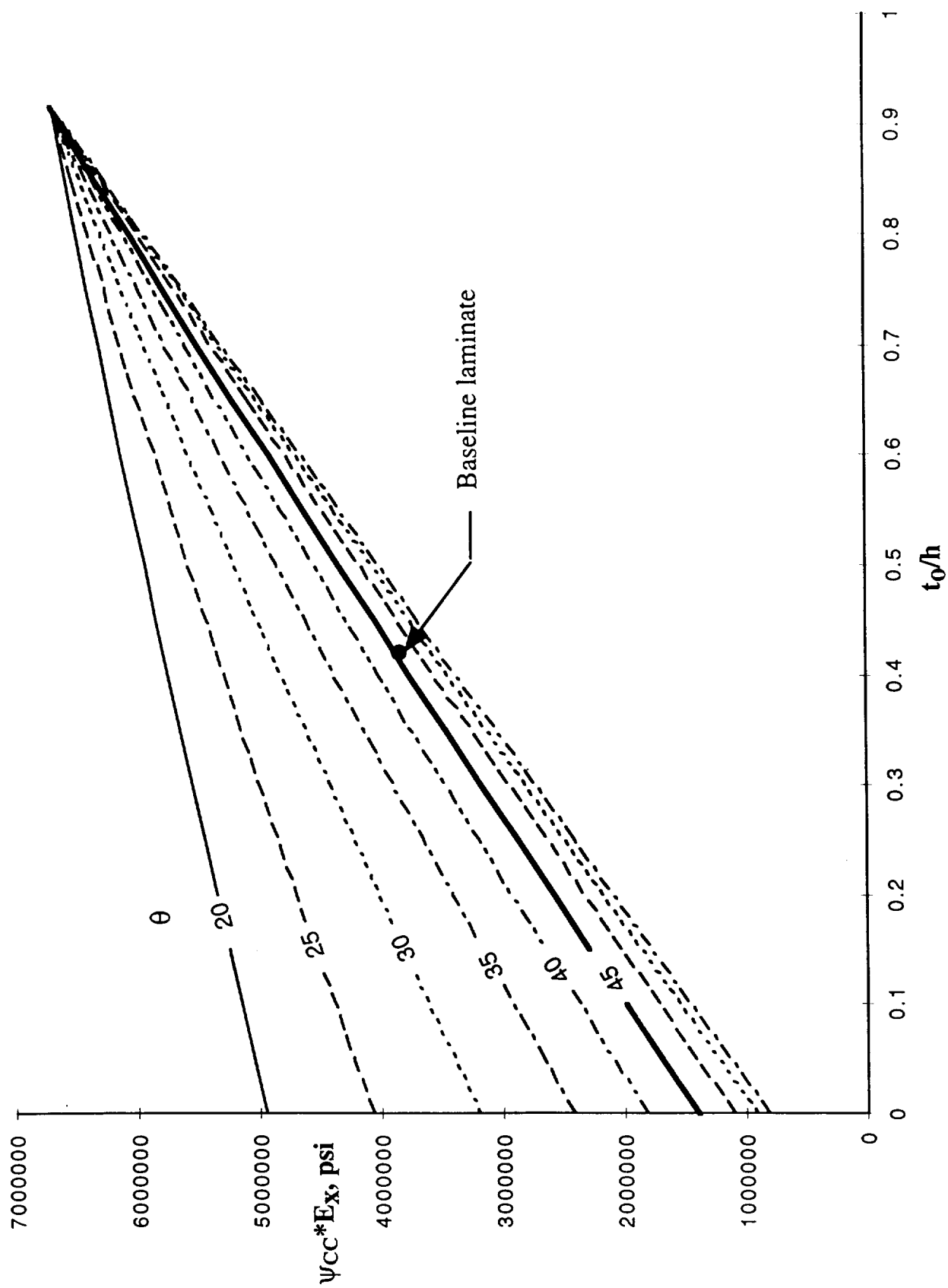Figure 11 - Merit function for example laminate, crack half-width = .125".

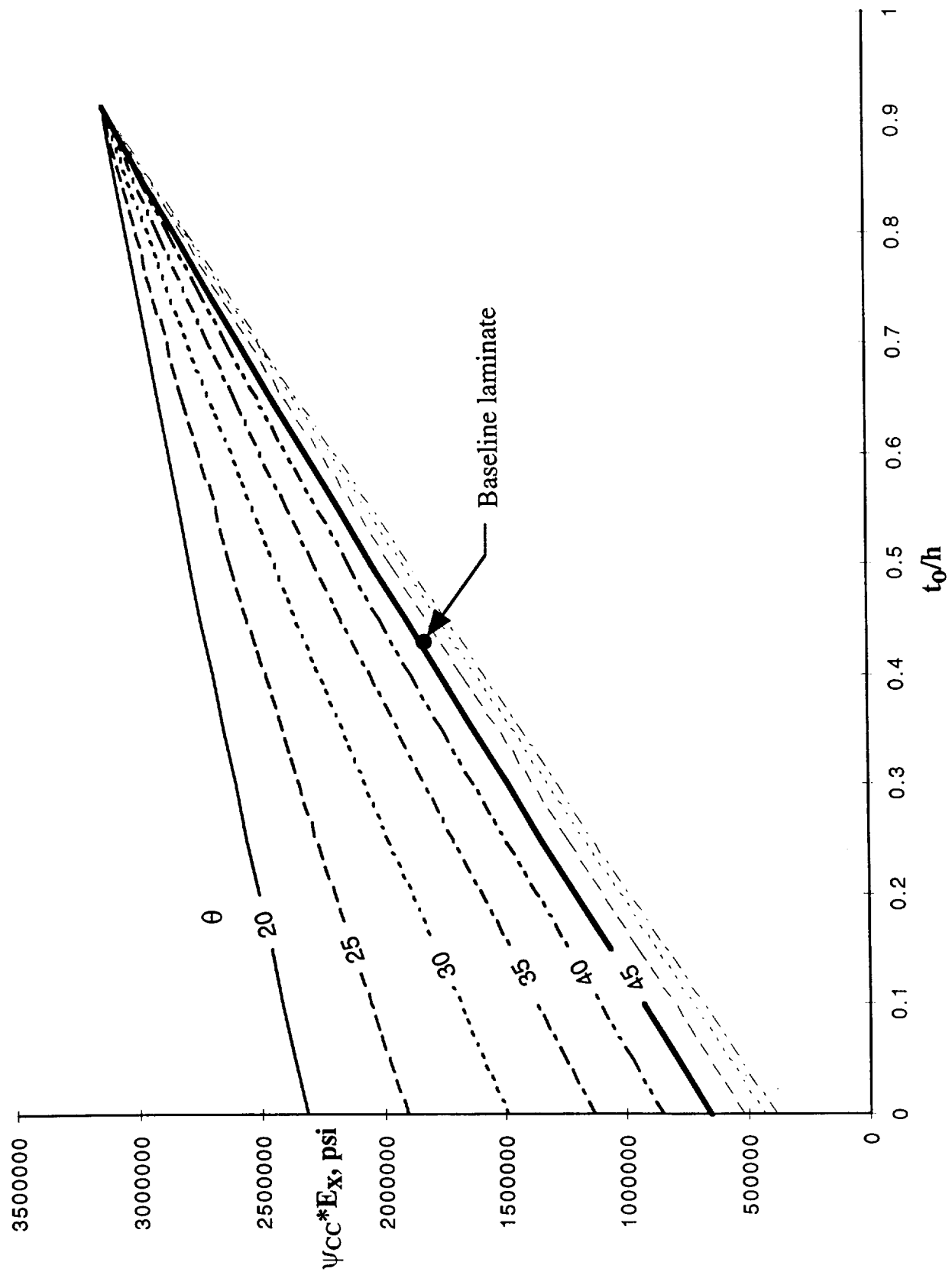Figure 12 - Merit function for example laminate, crack half-width = .5".

Figure 13 - Merit function for example laminate, crack half-width = 2.5".

Appendix A: Comparison of the Approximate and Exact Stress Distribution Solutions for an Orthotropic Plate.

In order to test the accuracy of the approximate stress distribution solution shown in equation (1) of this paper, the solution obtained using the approximate expression was compared with the exact stress distribution obtained using the complex variable mapping approach by G. Savin [4]. The normal stress distribution in front of a circular hole in an infinite orthotropic plate under an axial load σ was computed for a broad range of laminates using both methods, and the error between the solutions was determined.

The exact normal stress distribution, σ_y, along the x-axis in front of a circular cutout of radius R in an infinite width anisotropic plate under an axial load σ (Figure 1) is given by

$$\frac{\sigma_y(\xi,0)}{\sigma} = 1 + Re\left[\frac{1}{S_1 S_2}\left\{\frac{-S_2(1-iS_2)}{\sqrt{\xi^2-1-S_1^2}\left(\xi+\sqrt{\xi^2-1-S_1^2}\right)} + \frac{S_1(1-iS_2)}{\sqrt{\xi^2-1-S_2^2}\left(\xi+\sqrt{\xi^2-1-S_2^2}\right)}\right\}\right] \quad (1A)$$

where

$$\xi = \frac{x}{R}. \quad (2A)$$

For an infinite width orthotropic plate, S_1 and S_2 are the roots of

$$S^4 + \left(\frac{E_1}{G} - 2\upsilon_1\right)S^2 + \frac{E_1}{E_2} = 0 \quad (3A)$$

where the subscripts 1 and 2 denote the material principle axes which are aligned along x and y respectively. The roots of (3A) can fall into the following three cases:

1) $S_1 = i\beta_1$, $S_2 = i\beta_2$, $\beta_1, \beta_2 > 0$.
2) $S_1 = S_2 = i\beta$, $\beta > 0$.
3) $S_1 = \alpha + i\beta$, $S_2 = -\alpha + i\beta$, $\alpha, \beta > 0$.

These roots are for the case of the principle axis parallel to the x-axis and the loading normal to the x-axis. To rotate the principle axis by 90°, the original roots S_1 and S_2 should be replaced by $1/S_1$ and $1/S_2$. For the three root cases the following substitutions need to be made:

Case 1) replace β_1 and β_2 by $1/\beta_1$ and $1/\beta_2$.
Case 2) replace β by $1/\beta$.
Case 3) replace α by $\alpha/(\alpha^2+\beta^2)$, β by $\beta/(\alpha^2+\beta^2)$.

In order to compute the exact stress distribution, the software package Mathematica was used. After first finding the roots of (3A) for the 16 different laminates shown in Table 1A, the exact and approximate stress distributions were computed and compared for each of the laminates. The material properties for the laminates were obtained from references [6] and [7]. The stress distributions calculated using both methods and the error between the two solutions are shown in Figures 1A through 16A. In all of the figures, the dashed line represents the approximate stress distribution and the solid line is the exact stress distribution.

The results of this investigation show that the approximate expression does provide very accurate solutions except in a few laminate cases. These cases include highly fiber-dominated laminates (mostly $0°$ plies) or when the laminate consists mostly of $\pm45°$ plies. Yet even in ±45 laminates (i.e. Poe 11, 15) the error between the exact and approximate solutions is quite low ($\sim6\%$). Since these laminates are the extremes of laminate design, their use is fairly limited. For the broader general class of laminates, the approximate stress distribution is accurate enough and has been used in this paper.

The desire behind using the approximate expression given by equation (1) is that it is a simple closed form solution for the stress distribution that can be easily used without having to resort to specialized mathematical software such as Mathematica. While obtaining the exact solutions has become much easier with the aid of such packages, the exact solution technique is still very cumbersome and the use of simpler formulas greatly simplifies the analysis.

Laminate	E_X(GPa)	E_Y(GPa)	G_{XY} (GPa)	ν_{XY}
Rhodes Orthotropic (Ref. 6) [±45/0$_2$/±45/0$_2$/±45/0/90]$_{2s}$	70.2	34.8	20.2	.48
Rhodes Quasi-Isotropic (Ref. 6)	53.3	53.3	20.2	.32
Poe 1 (Ref. 7) [0]$_{8T}$	129	10.9	5.65	.312
Poe 2 [0$_2$/90/0]$_S$	100	40.7	5.65	.0836
Poe 3 [0$_2$/45/0$_2$/-45/0$_2$]$_S$	103	17.9	12.7	.551
Poe 4 [90/0]$_{2s}$	70.5	70.5	5.65	.0482
Poe 5 [90/0/90/0/45/0/-45/0]$_S$	76.4	47.4	12.7	.214
Poe 6 [45/0/-45/0]$_S$	75.3	23.3	19.7	.649
Poe 8 [±45/0/±45/0]$_S$	50.0	25.6	26.0	.698
Poe 9 [45/0/-45/90]$_S$	51.4	51.4	19.7	.307
Poe 10 [90/45/90/-45]$_S$	23.3	75.3	19.7	.201
Poe 11 [±45]$_{2s}$	19.6	19.6	33.7	.735
Poe 12 [0]$_{8T}$	109	8.32	4.82	.314
Poe 13 [45/0/-45/0]$_{2s}$	63.4	19.2	16.5	.654
Poe 14 [45/0/-45/90]$_{2s}$	43.1	43.1	16.5	.303
Poe 15 [±45]$_{2s}$	16.7	16.7	28.2	.730

Table 1A: Laminates used to investigate the accuracy of the approximate solution.
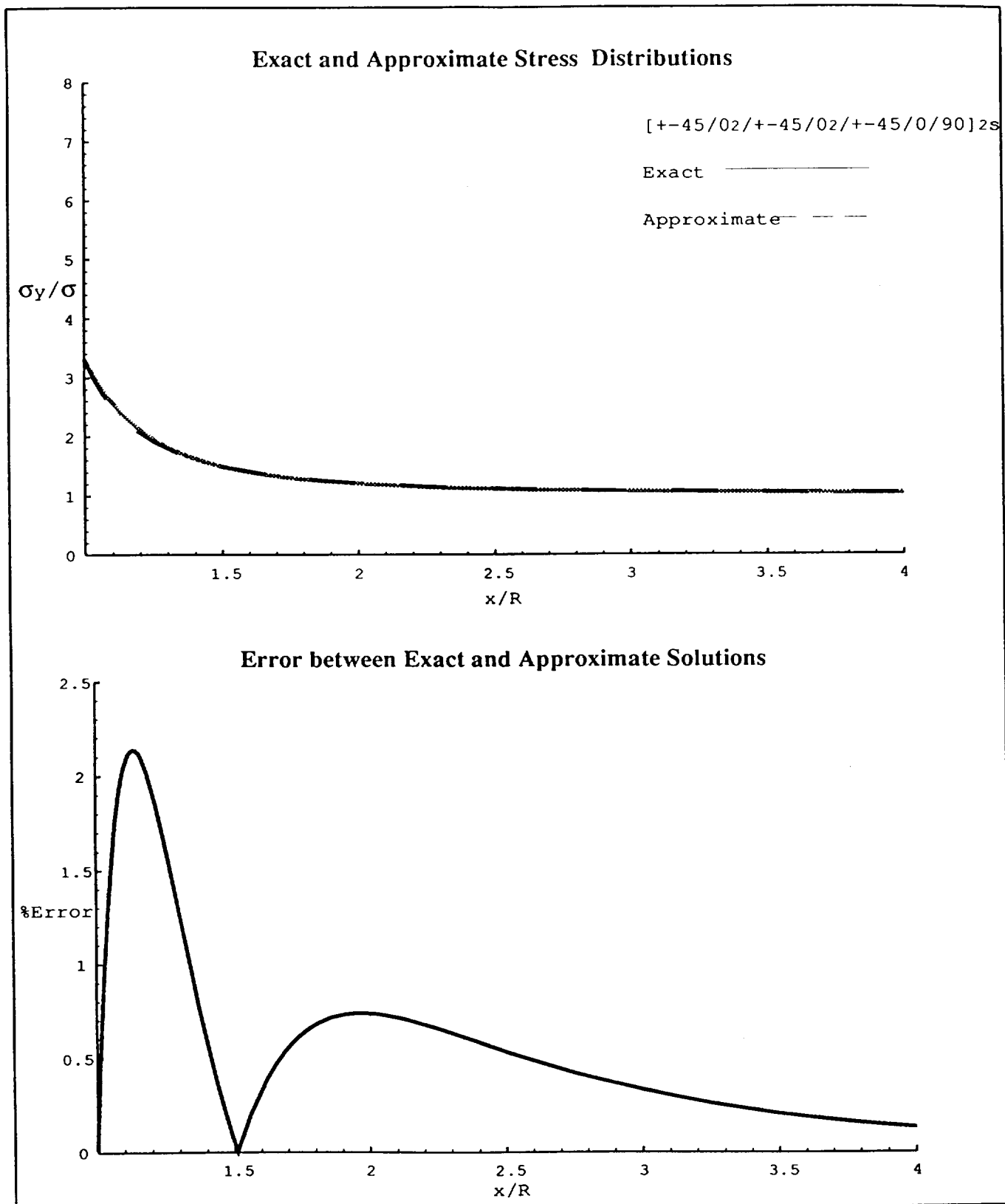
Figure 1A: Comparison of Exact and Approximate Solutions for Rhodes Orthotropic Laminate.
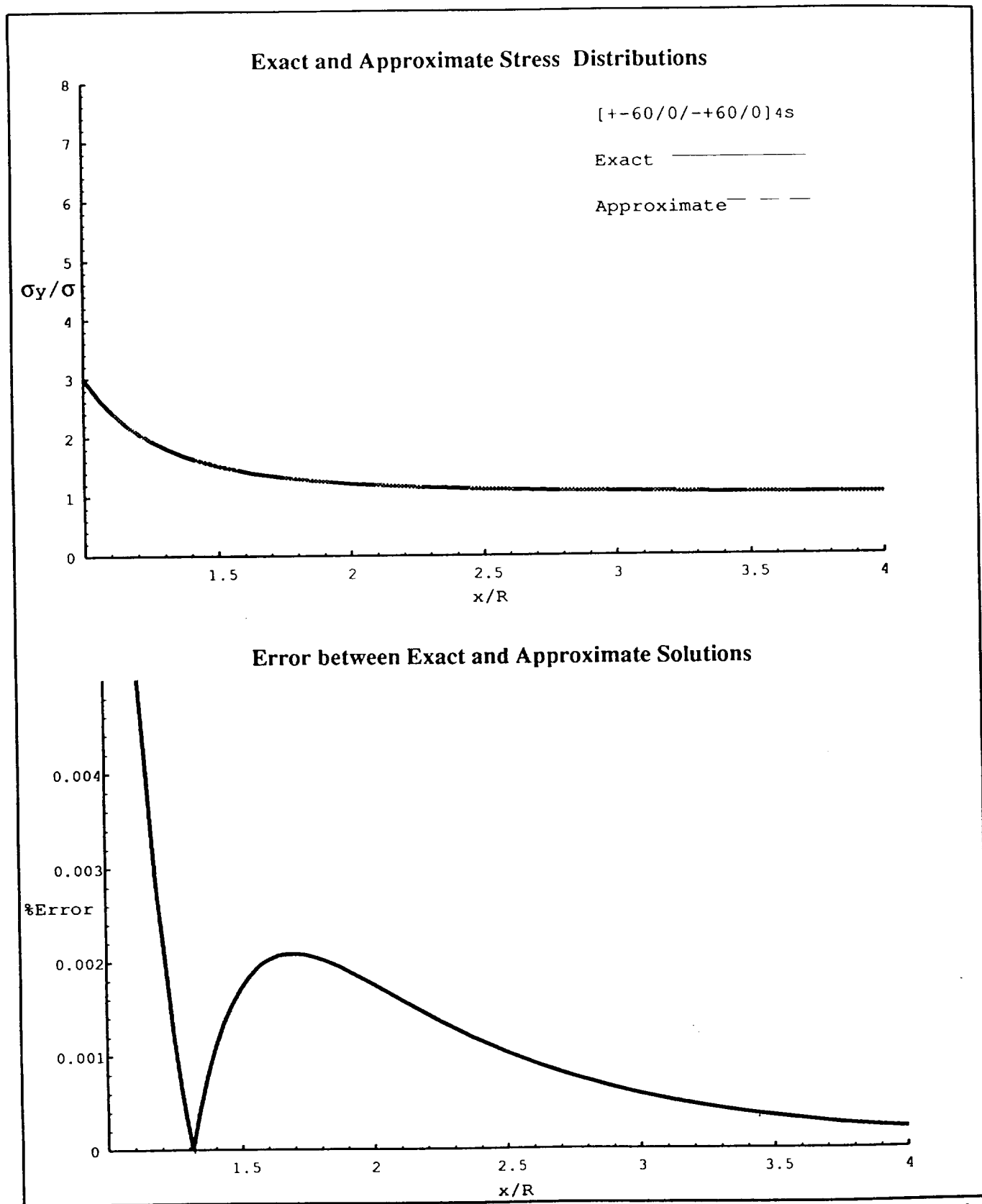
Figure 2A: Comparison of Exact and Approximate Solutions for Rhodes Quasi-Isotropic Laminate.
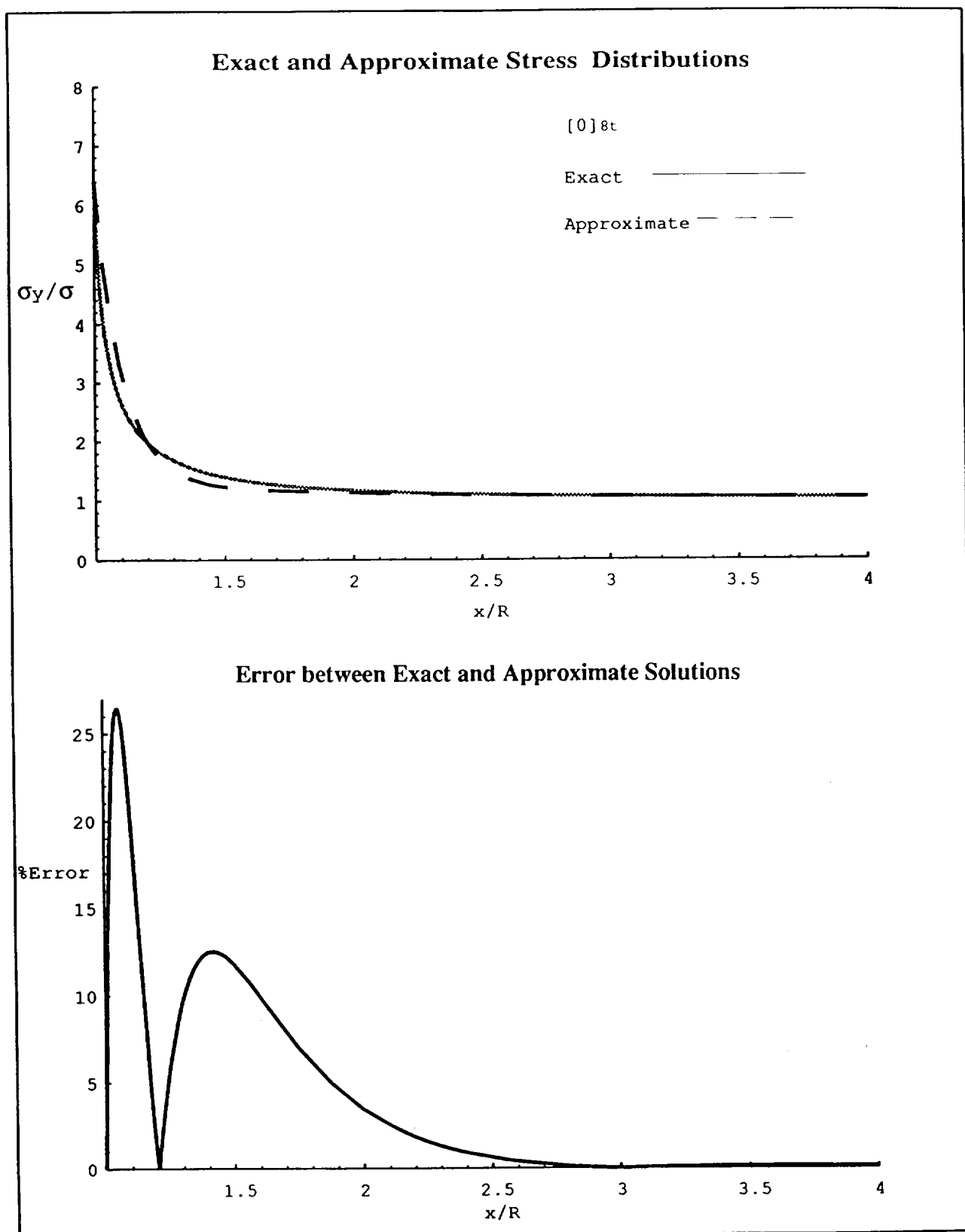
Figure 3A: Comparison of Exact and Approximate Solutions for Poe Laminate 1.
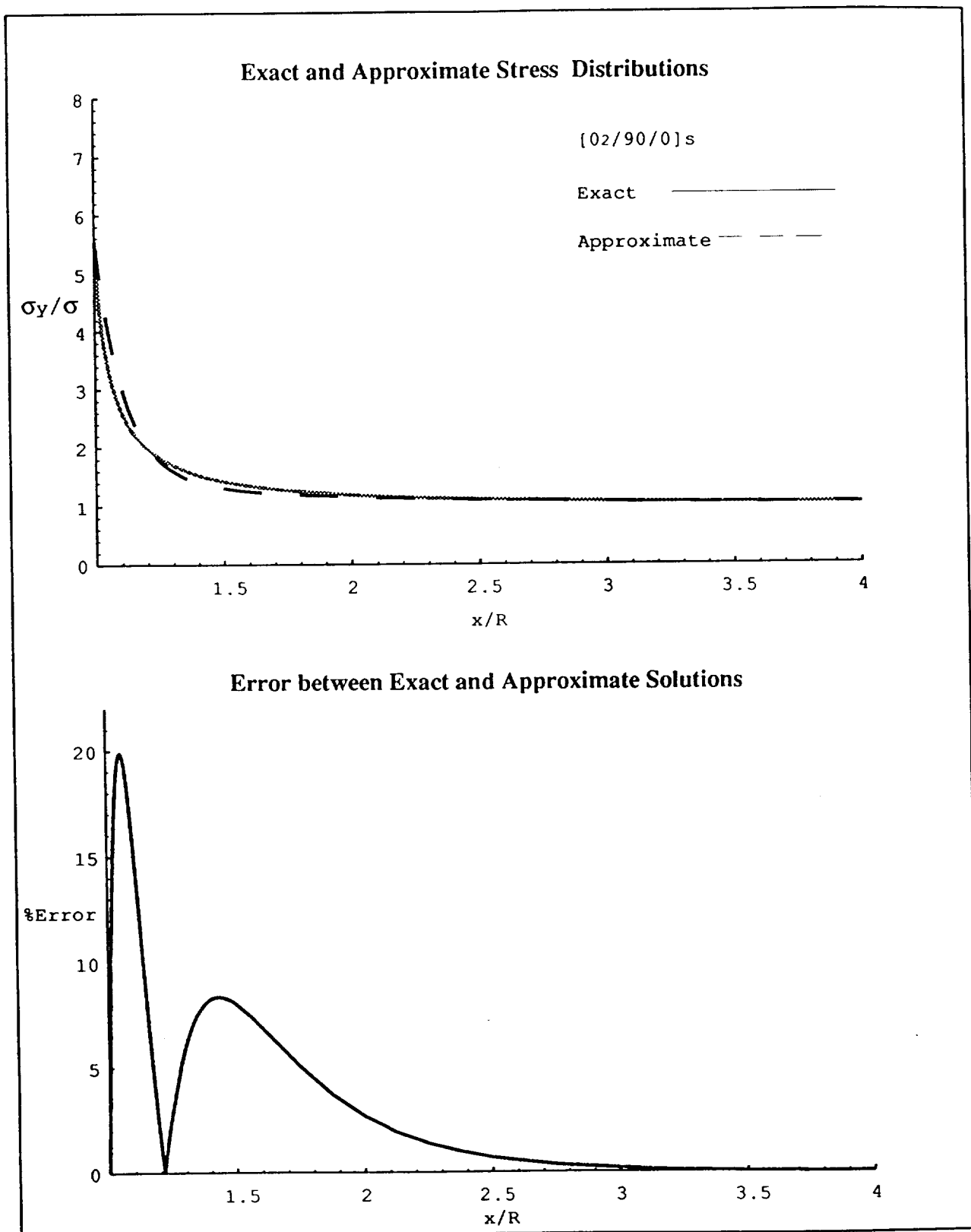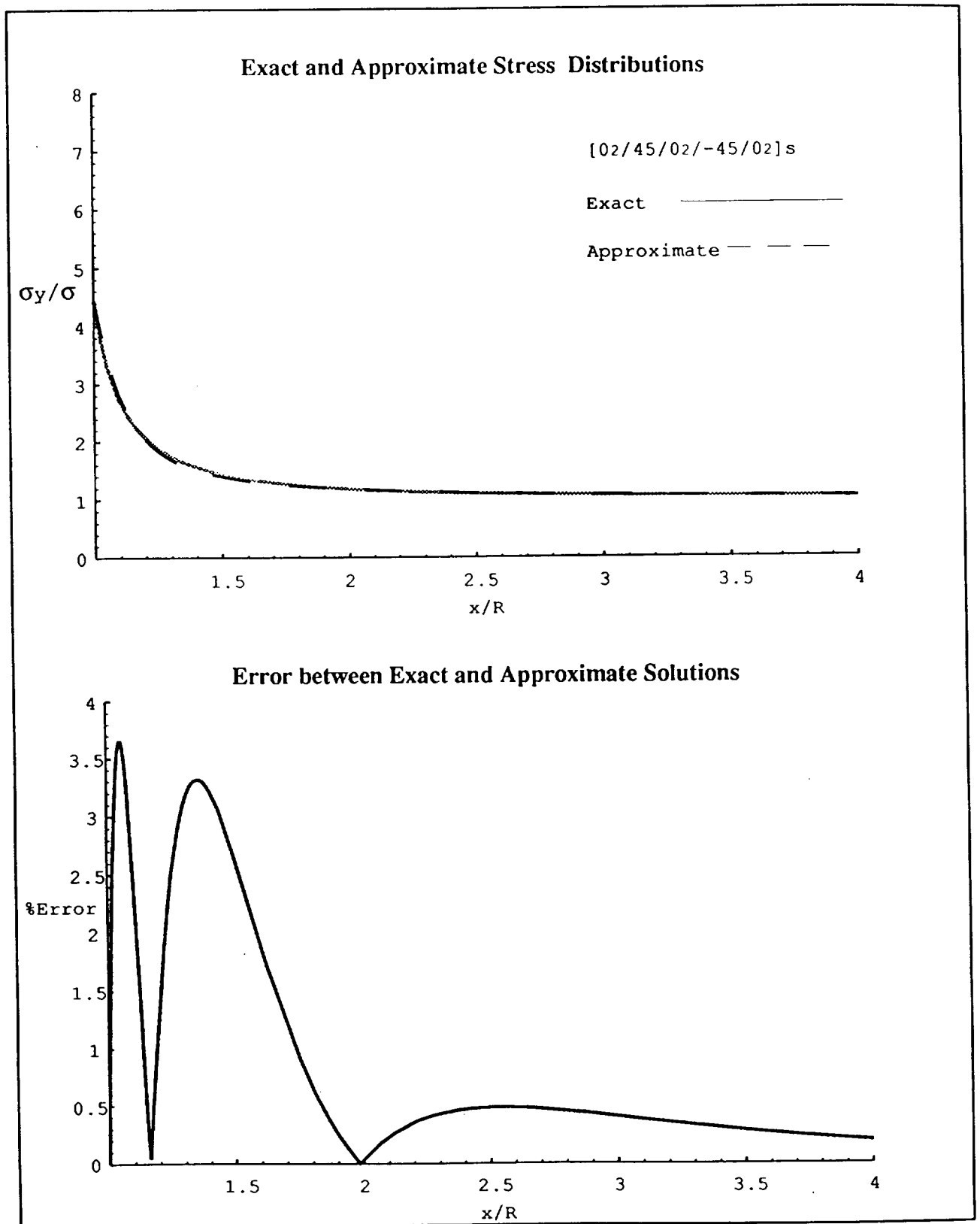
Exact and Approximate Stress Distributions

[02/90/0]s

Exact ————————

Approximate — — —

σ_y/σ

x/R

Error between Exact and Approximate Solutions

%Error

x/R

Figure 4A: Comparison of Exact and Approximate Solutions for Poe Laminate 2.

Exact and Approximate Stress Distributions

$[0_2/45/0_2/-45/0_2]s$

Exact ————————

Approximate — — —

σy/σ

x/R

Error between Exact and Approximate Solutions

%Error

x/R

Figure 5A: Comparison of Exact and Approximate Solutions for Poe Laminate 3.
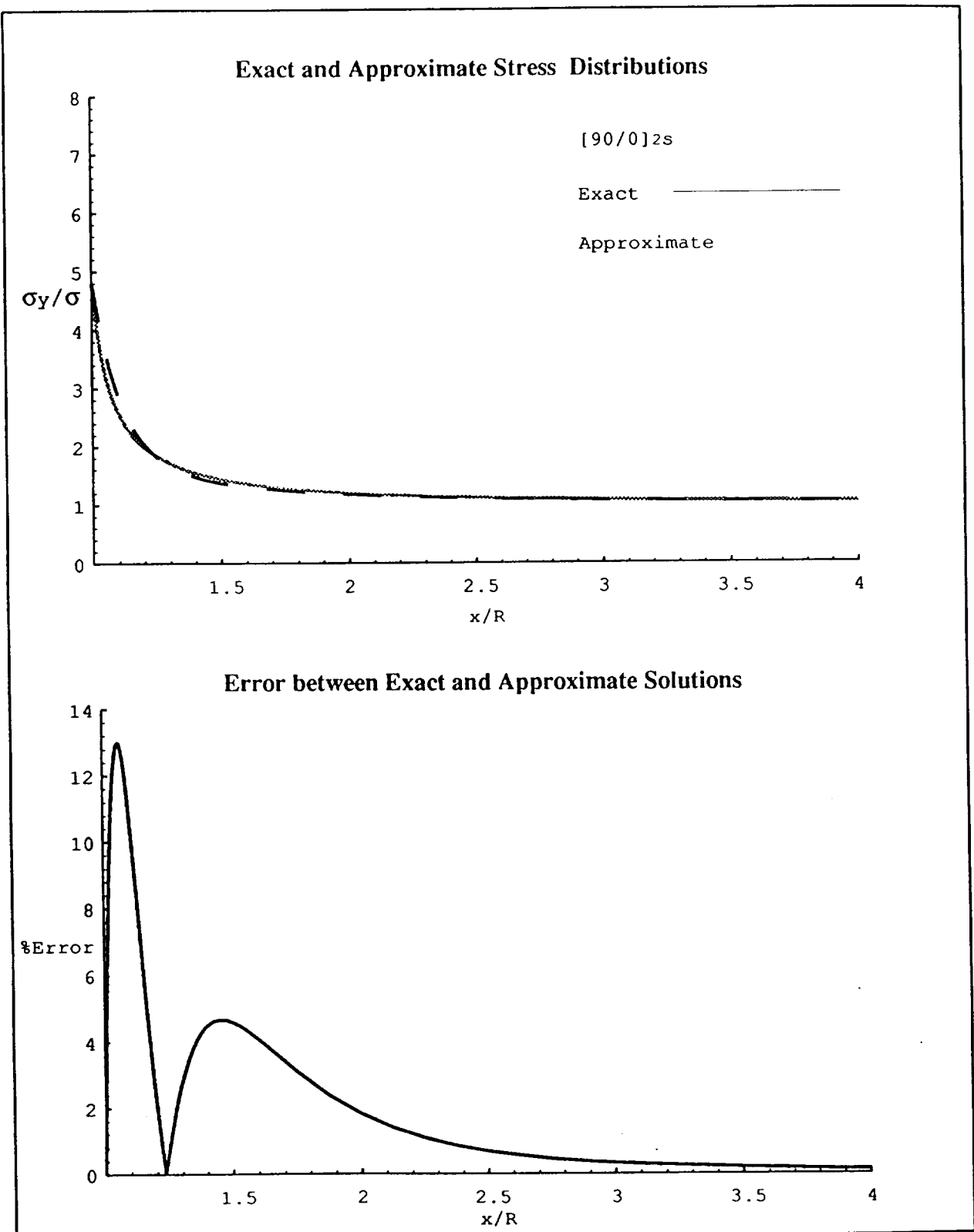
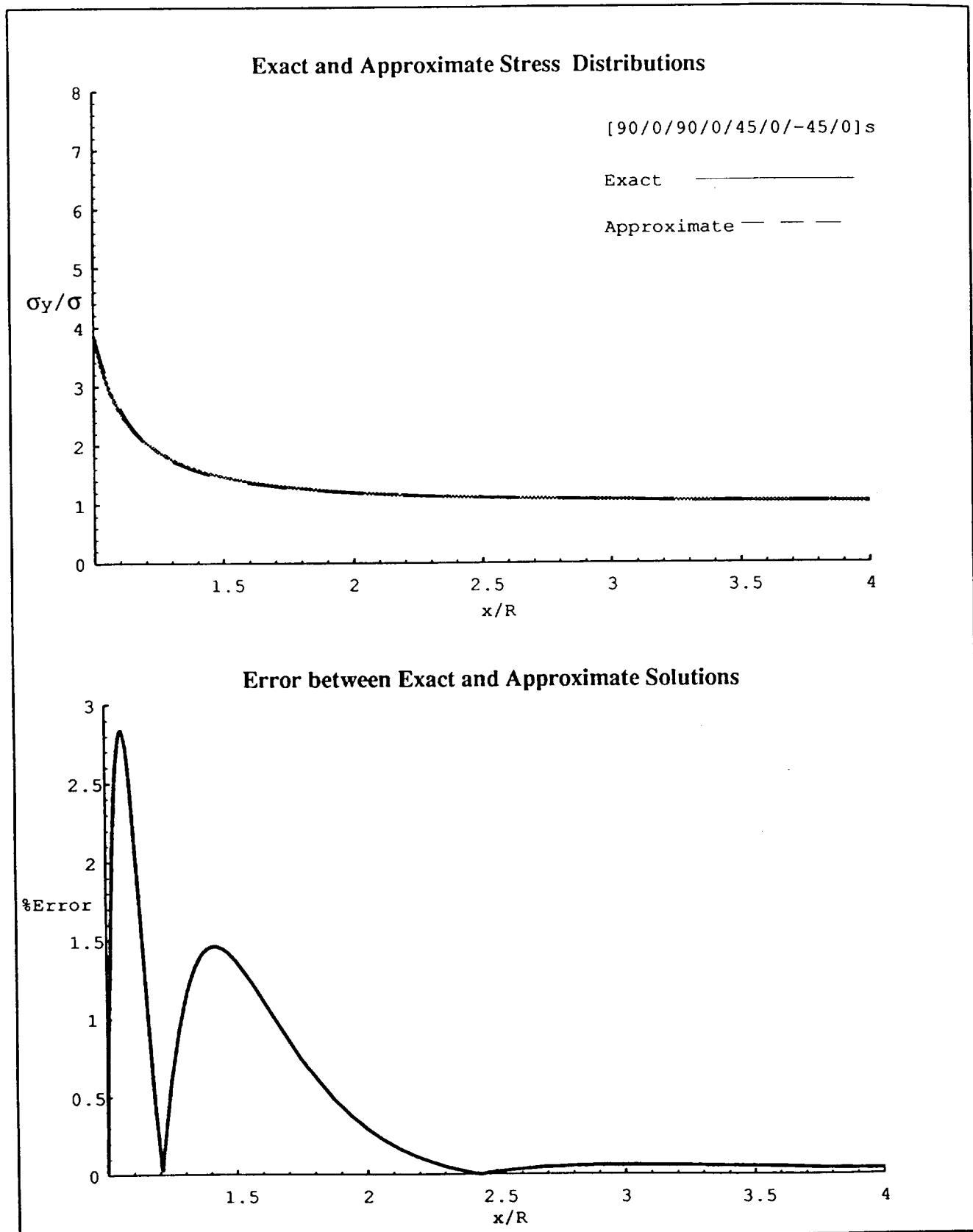Figure 6A: Comparison of Exact and Approximate Solutions for Poe Laminate 4.

Figure 7A: Comparison of Exact and Approximate Solutions for Poe Laminate 5.
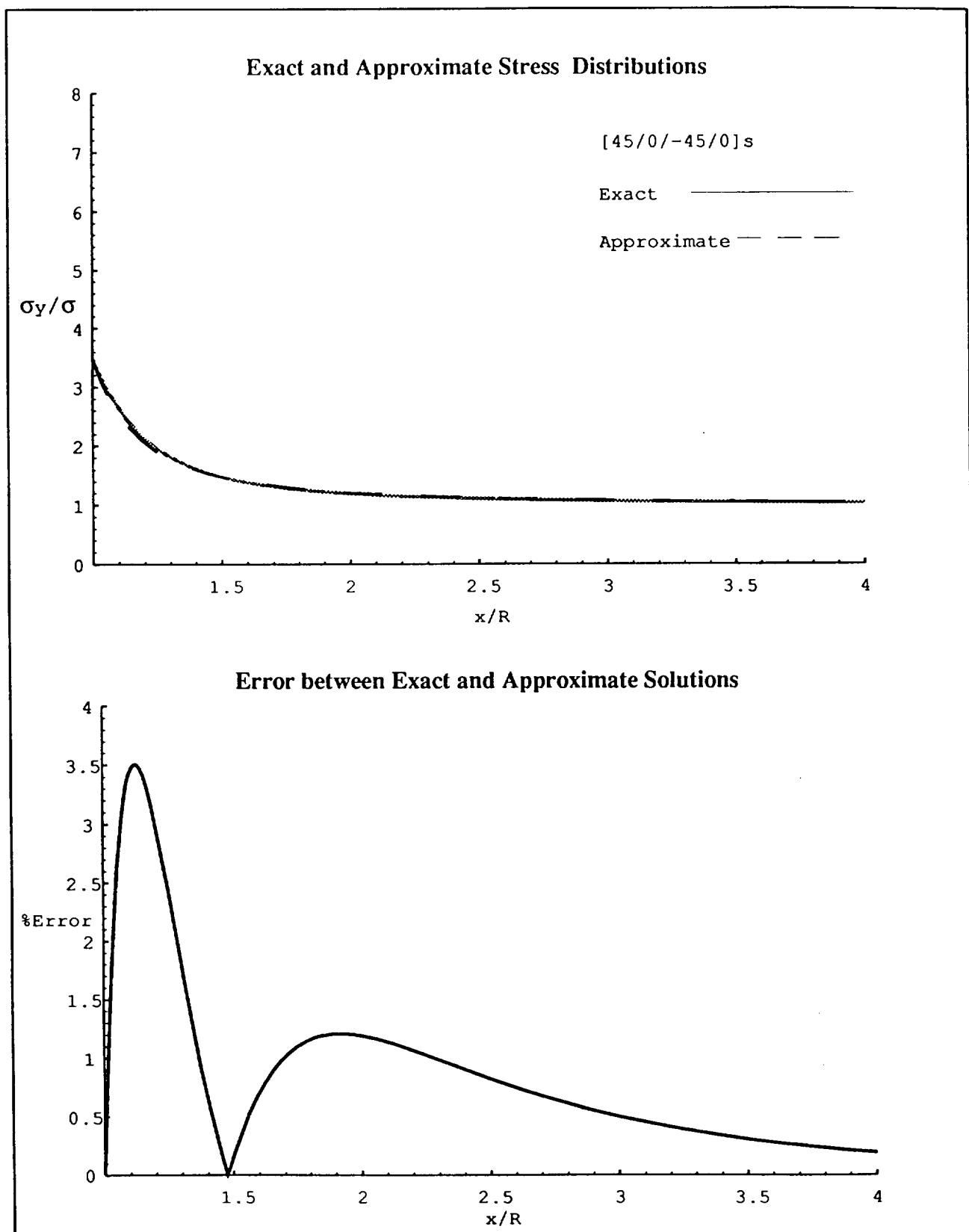
Figure 8A: Comparison of Exact and Approximate Solutions for Poe Laminate 6.
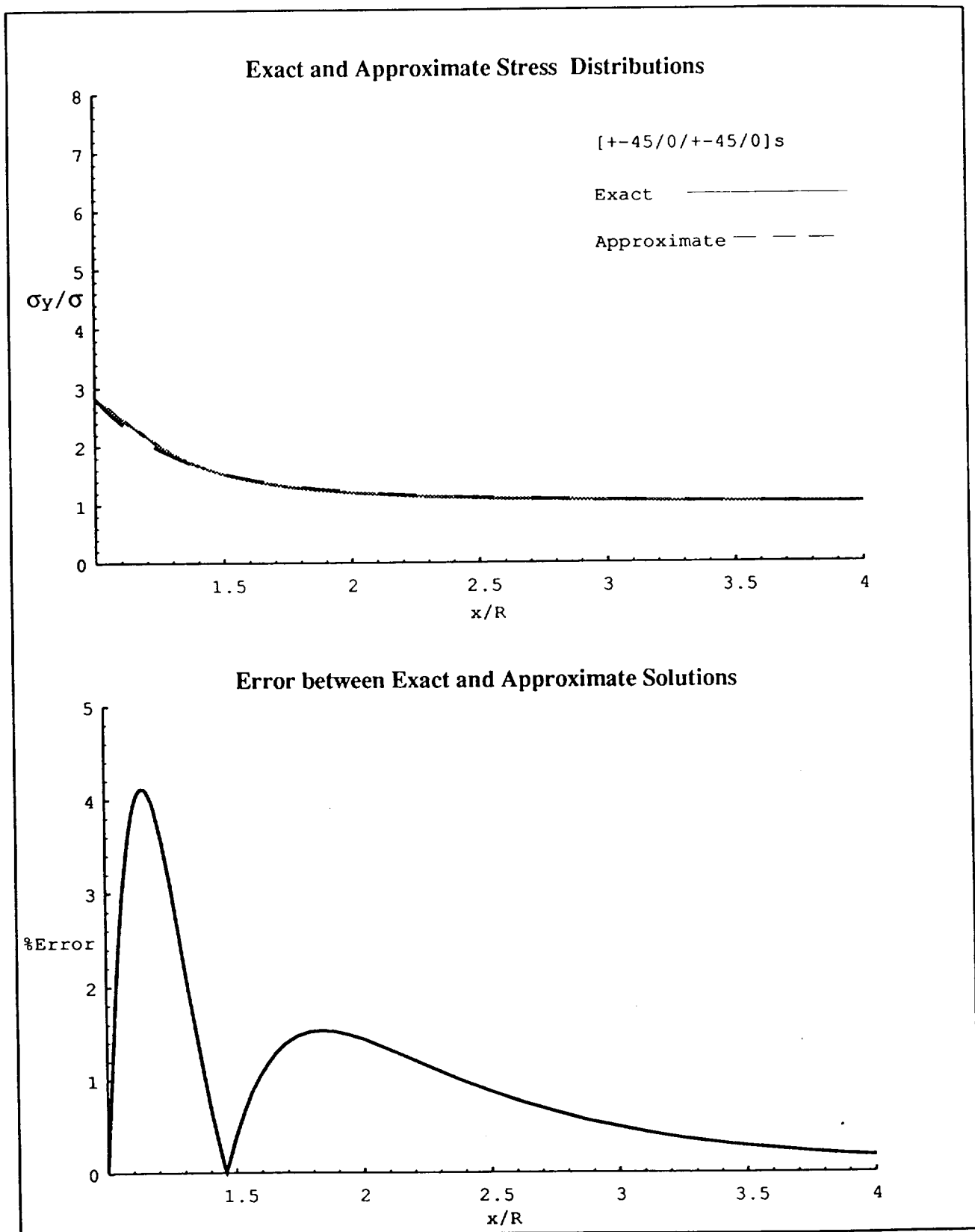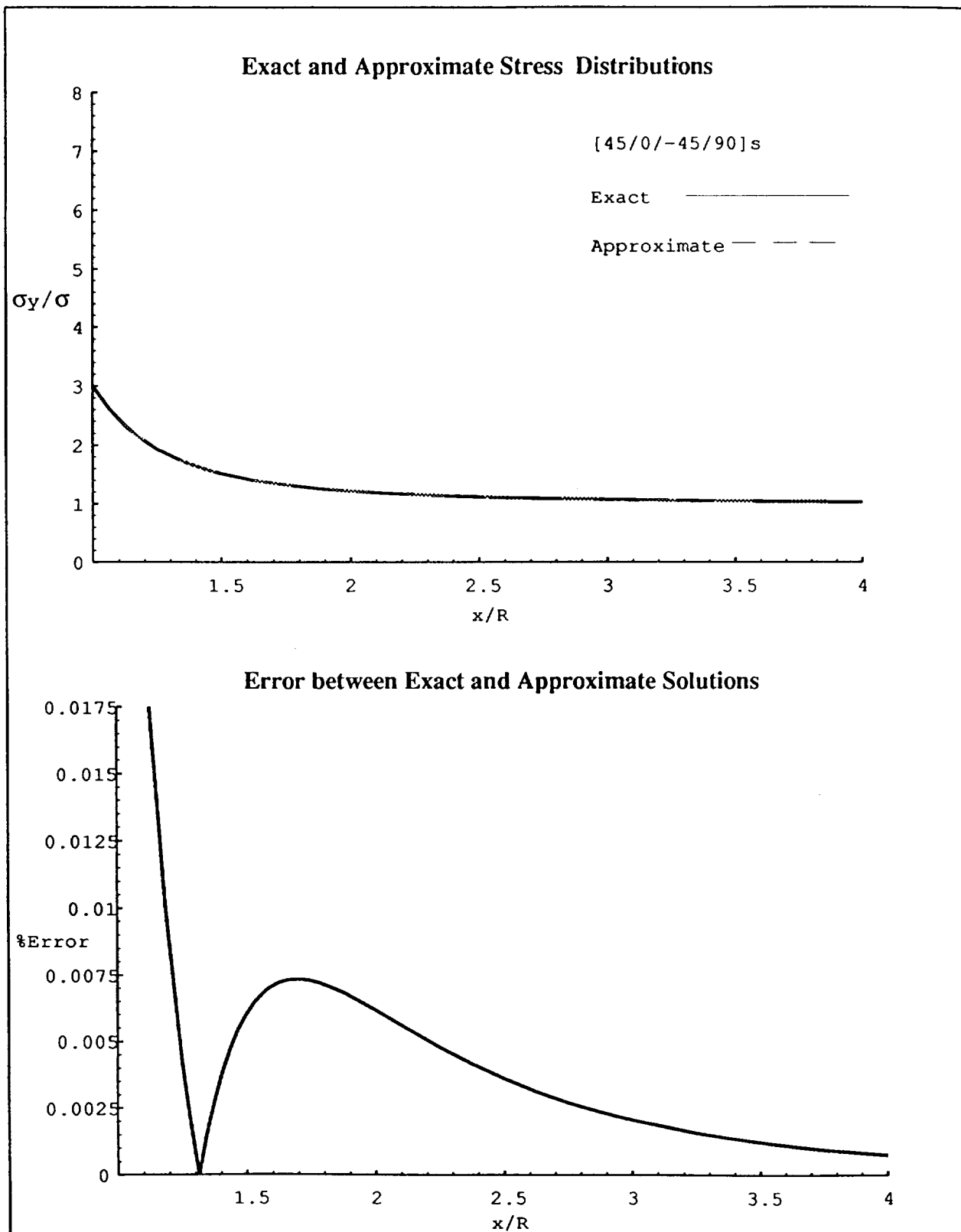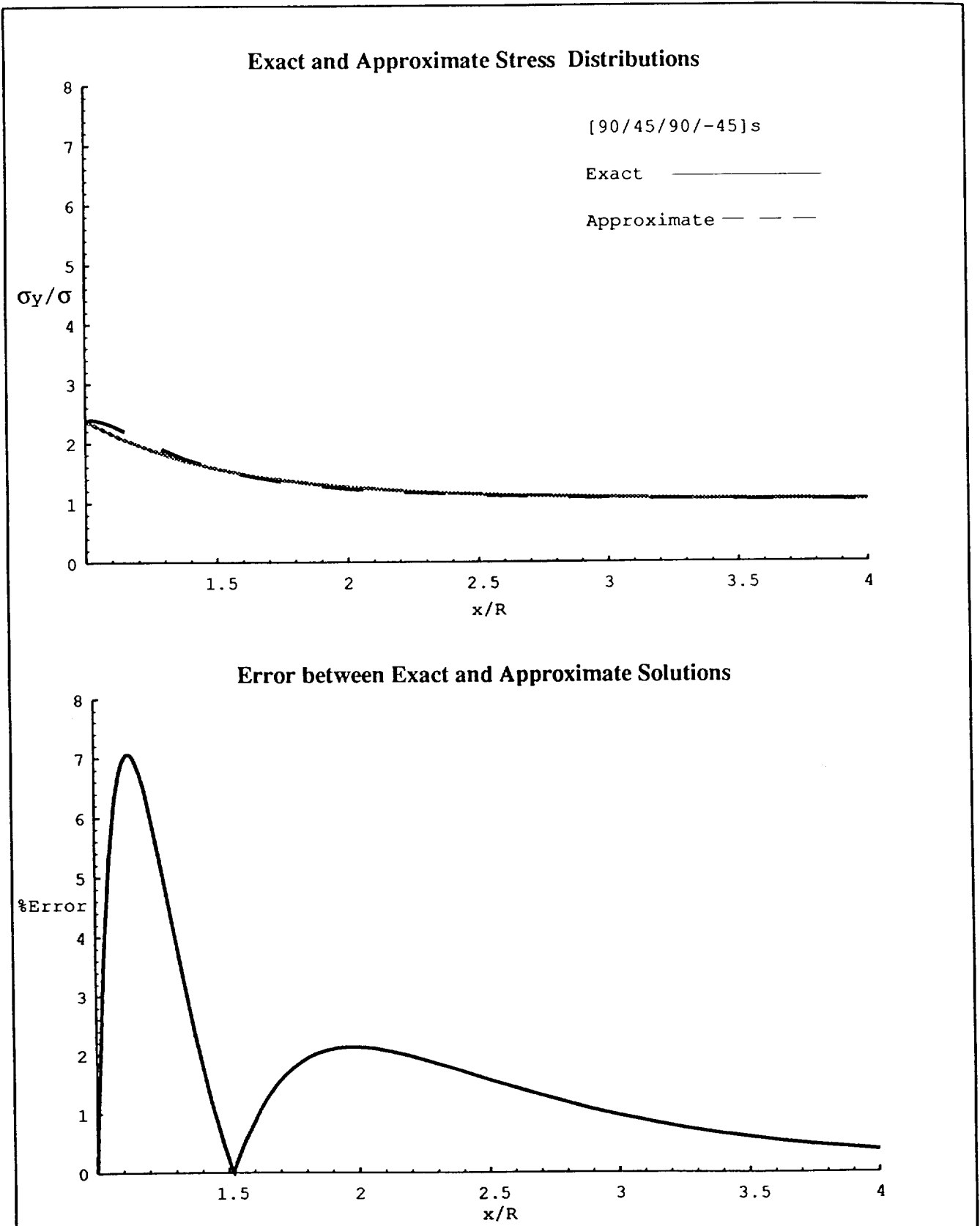
Exact and Approximate Stress Distributions

[+-45/0/+-45/0]s

Exact ————————

Approximate — — —

Error between Exact and Approximate Solutions

Figure 9A: Comparison of Exact and Approximate Solutions for Poe Laminate 8.

Exact and Approximate Stress Distributions

[45/0/-45/90]s

Exact —————————

Approximate — — — —

σy/σ

x/R

Error between Exact and Approximate Solutions

%Error

x/R

Figure 10A: Comparison of Exact and Approximate Solutions for Poe Laminate 9.

Exact and Approximate Stress Distributions

[90/45/90/-45]s

Exact ————————

Approximate — — —

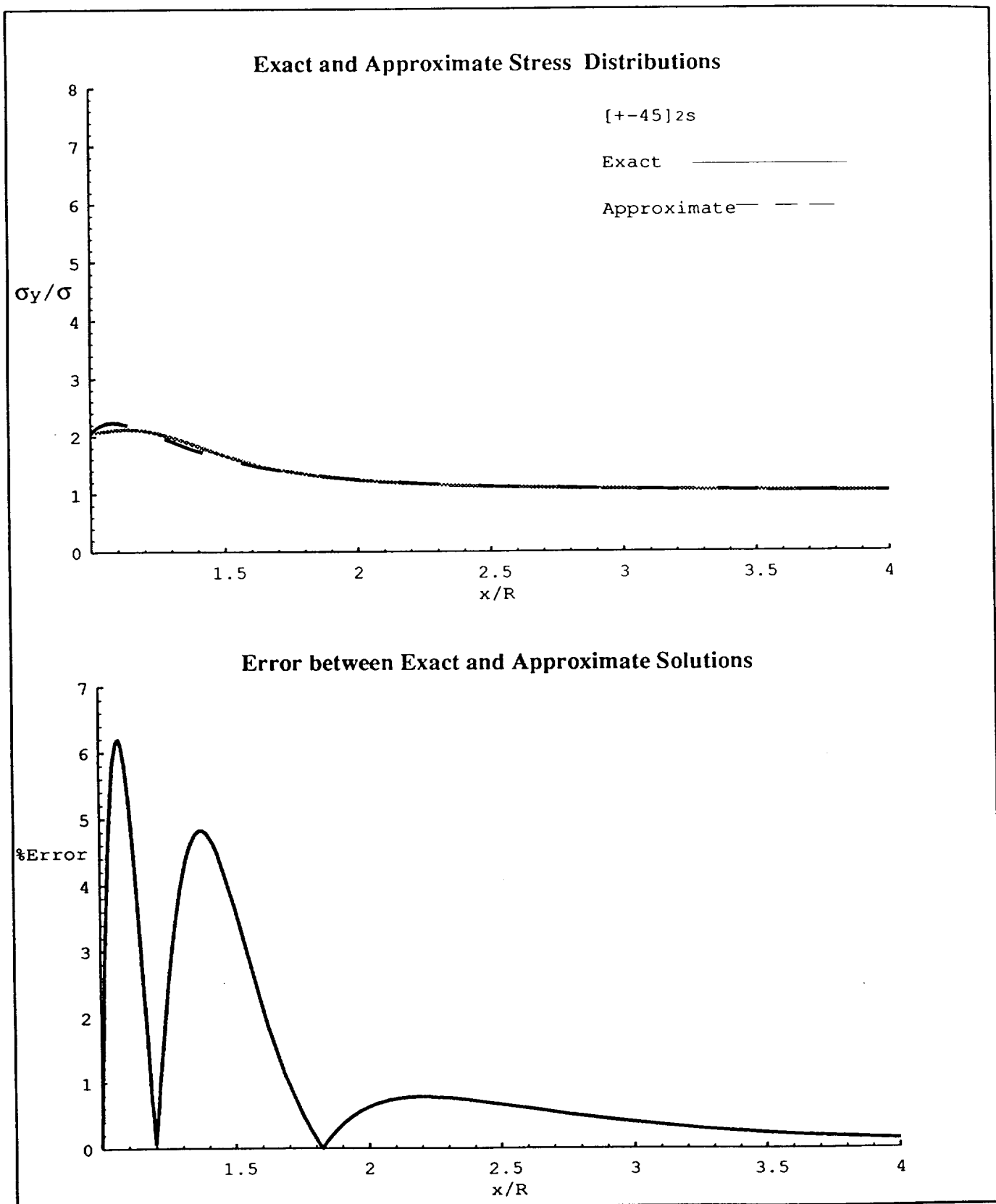**Error between Exact and Approximate Solutions**

Figure 11A: Comparison of Exact and Approximate Solutions for Poe Laminate 10.

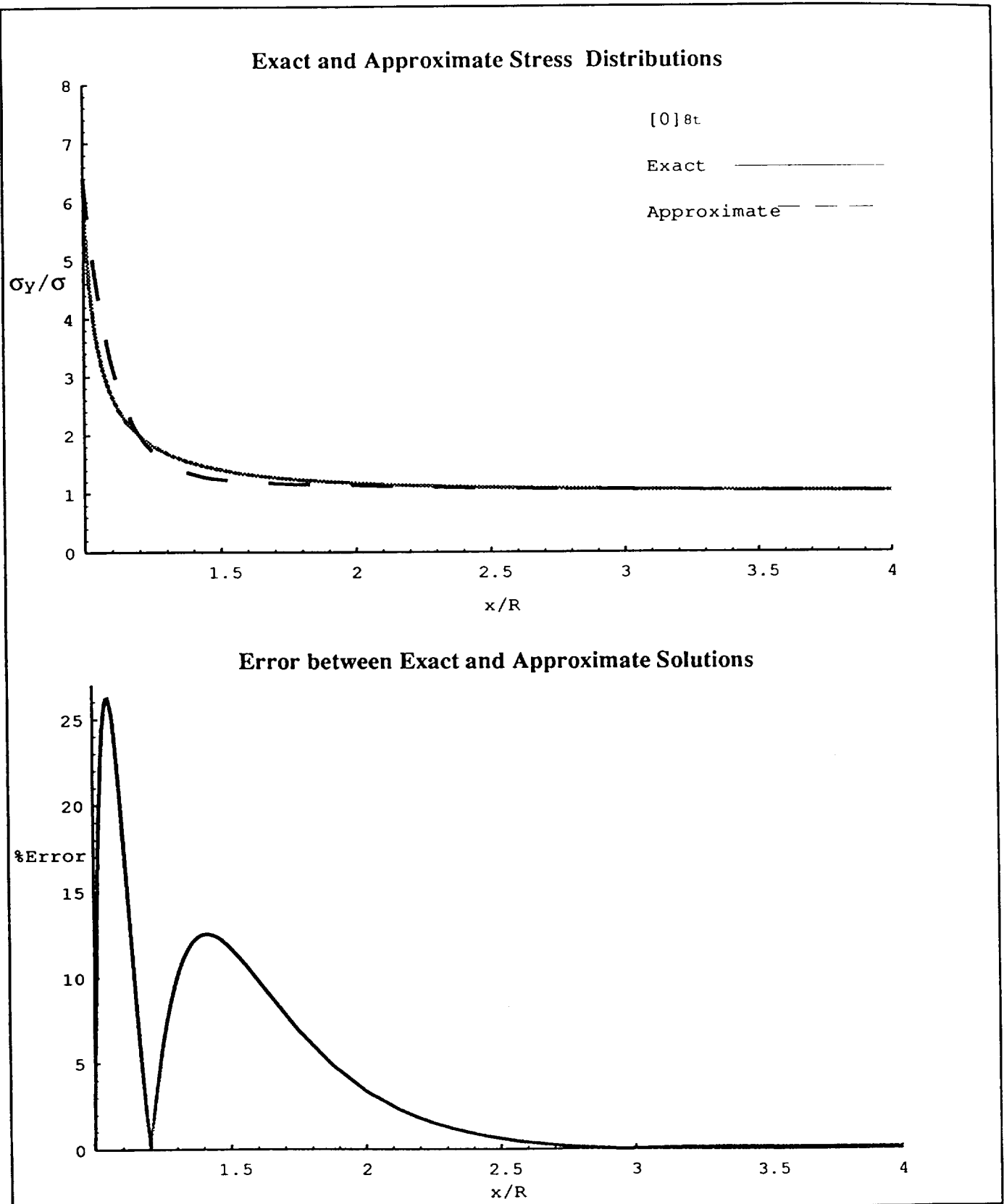**Figure 12A: Comparison of Exact and Approximate Solutions for Poe Laminate 11.**

Exact and Approximate Stress Distributions

[0]8t

Exact —————————

Approximate — — —

σy/σ

x/R

Error between Exact and Approximate Solutions

%Error

x/R

Figure 13A: Comparison of Exact and Approximate Solutions for Poe Laminate 12.

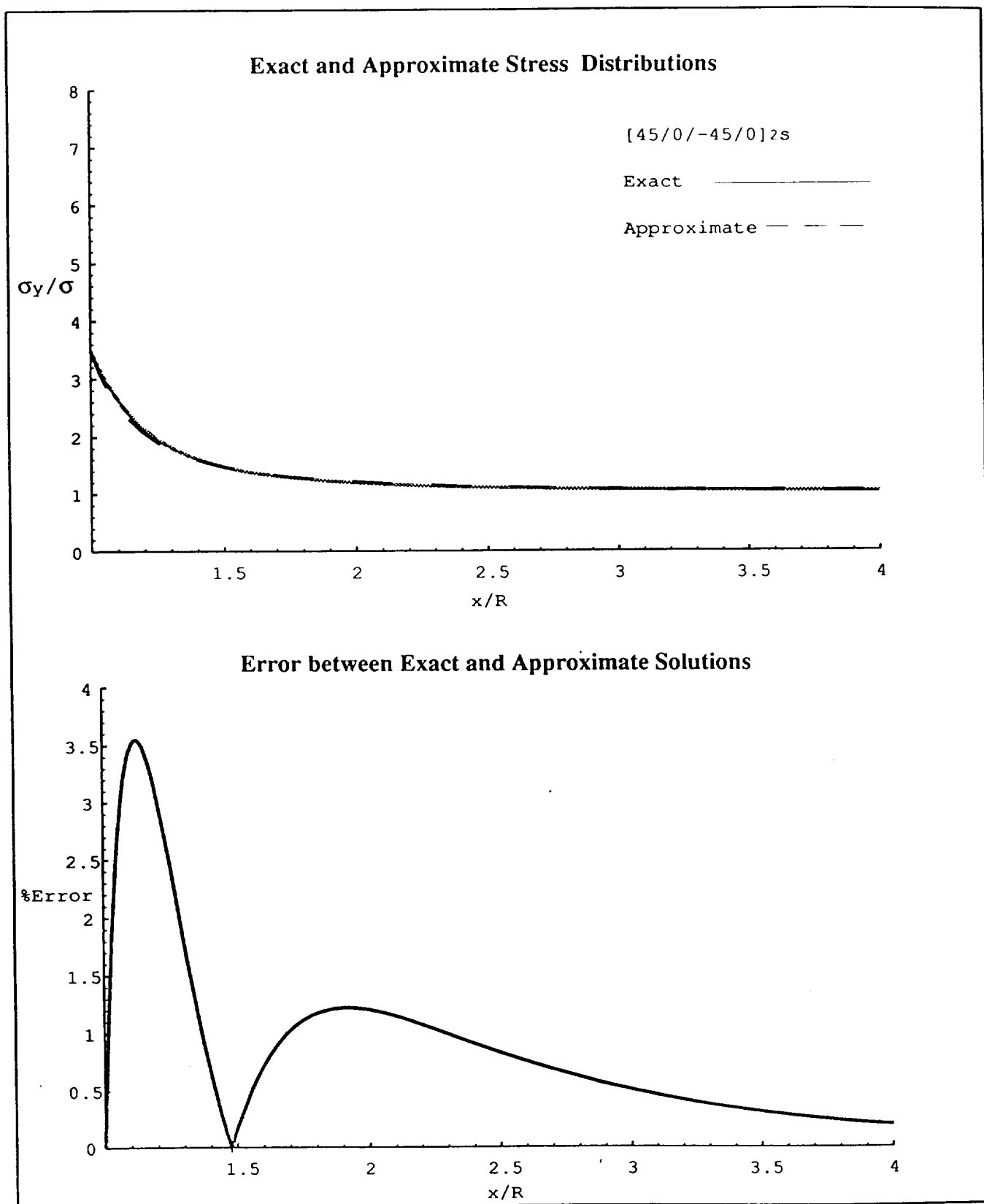**Figure 14A: Comparison of Exact and Approximate Solutions for Poe Laminate 13.**
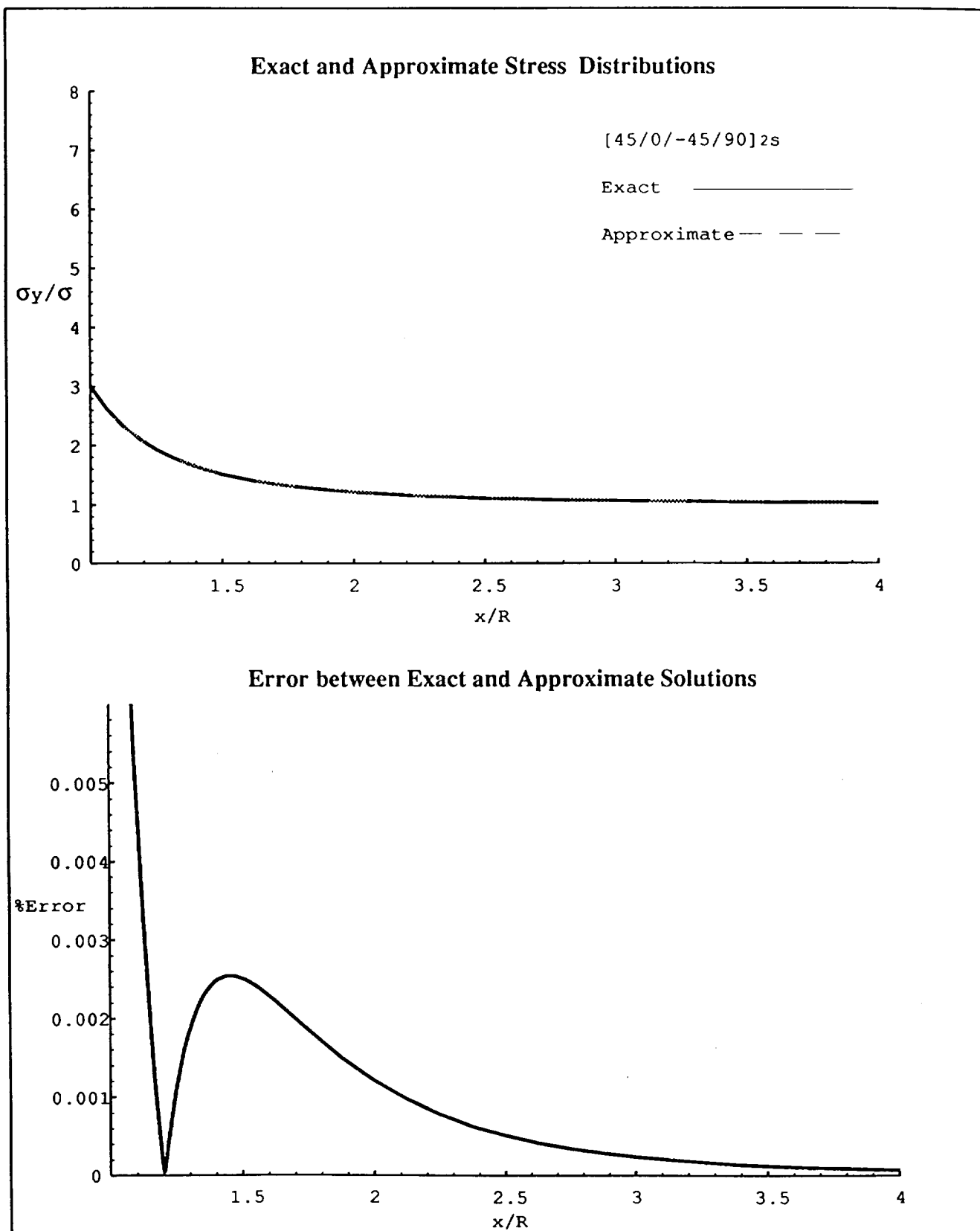
Exact and Approximate Stress Distributions

$[45/0/-45/90]2s$

Exact —————————

Approximate — — — —

$\sigma y/\sigma$

x/R

Error between Exact and Approximate Solutions

%Error

x/R

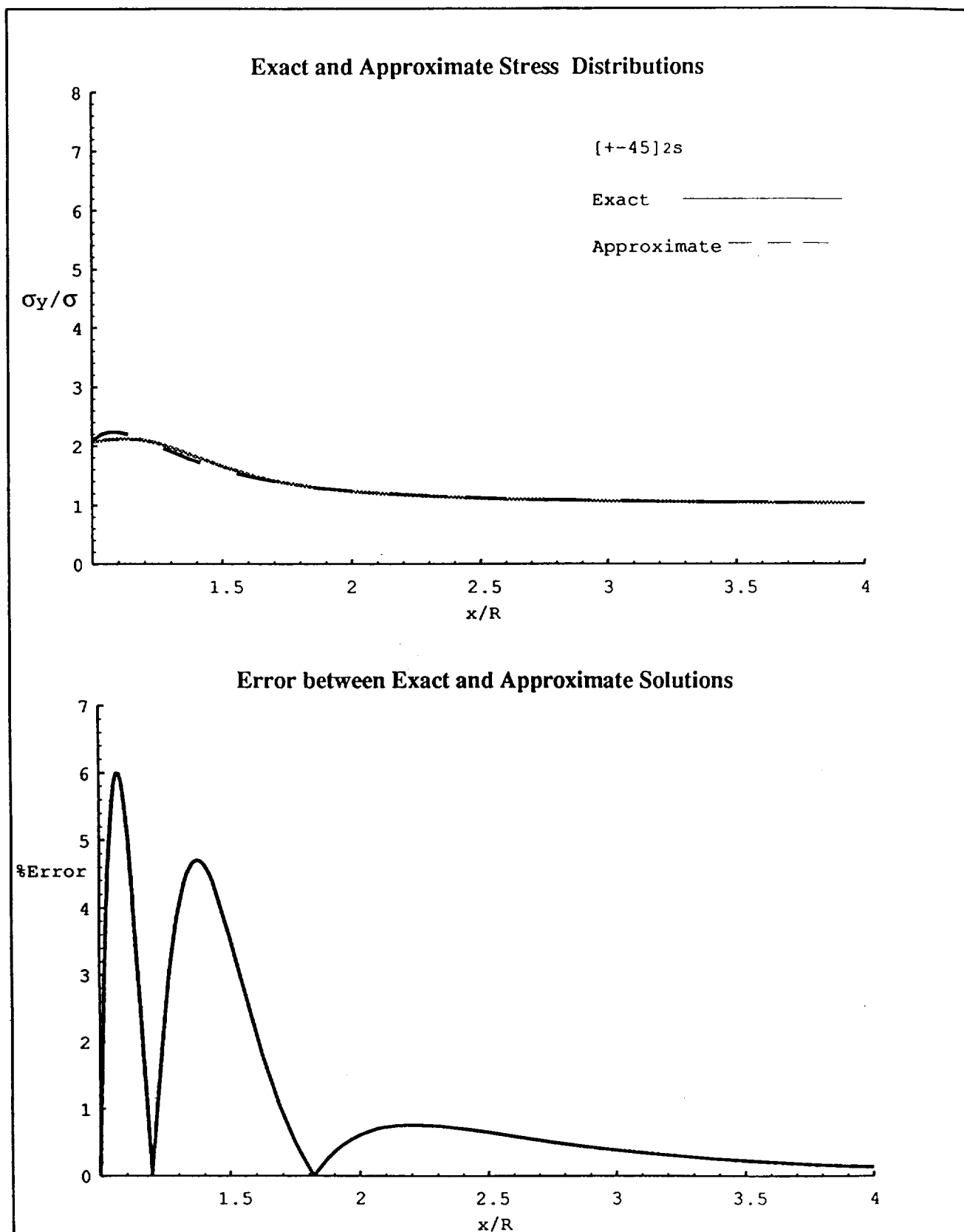**Figure 15A:** Comparison of Exact and Approximate Solutions for Poe Laminate 14.

Figure 16A: Comparison of Exact and Approximate Solutions for Poe Laminate 15.