Analysis of Unsteady Flow Phenomena: Shock - Vortex and Shock - Boundary Layer Interactions

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I. INTRODUCTION

The interaction of shock waves with vortices has received much attention in the past, mainly because shock-vortex interaction closely models the interaction of a shock wave with the coherent structures of a turbulent flow-field, and is a key feature in the broad-band shock noise for supersonic jets in off-project conditions.

Chu and Kovasznay[1] have shown that a weak disturbance in a viscous heat conducting fluid can be decomposed as the sum of three basic modes, namely acoustic, vortical and entropy mode; the interaction of any of these modes with a shock wave gives rise to all three disturbance modes downstream of the shock. The vortical mode is important since it constitues the basis of the coherent structures that have been observed to dominate turbulence for low- to moderate-flow speed. Hollingsworth et al.[2] have experimentally investigated the interaction of a cylindrical shock-induced starting vortex with a plane normal shock, and have shown that the interaction generates a cylindrical acoustic pulse that exhibits a quadrupolar structure consisting of four alternate compression and expansion regions centered around the transmitted vortex. The investigations of Hollingsworth and Richards have been extended by Dosanjh and Weeks[3] that have analyzed the interaction of a columnar spiral vortex with a normal shock wave, thus obtaining quantitative measurements and confirming the generation of a progressive cylindrical wavefront of alternate compression-expansion nature. However, the measured acoustic pressure field exhibited deviations from the purely quadrupolar directivity pattern predicted by the linear theory, and the authors arrived at a semiempirical relation for the wavefront pressure amplitude distribution (by considering a combination of monopole, dipole and quadrupole sources) to fit the measured data. Naumann and Hermanns[4] have experimentally addressed the non-linear aspects of shock-vortex interaction, and have shown that the interaction causes both a diffraction and a reflection of the shock with a pattern consisting of either a regular- or a Mach-reflection depending on the shock and the vortex strengths.

An attempt to theoretically explain the production of sound from the shock-vortex inter-

action was carried out by Ribner[5], who decomposed the vortex as the sum of sinusoidal radially symmetric shear waves, thus obtaining the acoustic pressure field generated through the shock-vortex interaction as the superposition of the pressure waves produced through the interaction of the plane shear waves with the shock. Ribner's analysis shows the occurrence of a precursor wave that obeys causality, and its duration depends upon the value of the circulation, being finite in the case of a zero-circulation, and infinite in the case of a finite-circulation vortex. In addition, Ribner showed that the predicted acoustic field is a growing cylindrical sound wave centered at the transmitted vortex that is partly cut off by the shock.

Pao and Salas[6] have numerically studied two-dimensional shock-vortex interactions by means of a two-step MacCormack scheme for the solution of the Euler equations, and have confirmed the findings of Ribner[5], Dosanjh and Weeks[3] and Naumann and Hermanns[4] regarding the quadrupole structure of the acoustic field generated through a shock-vortex interaction. In addition, they have also shown that the pressure field behind the shock can be separated into three contributions: *i)* one corresponding to the shock itself; *ii)* a contribution due to the vortex; and *iii)* the acoustic pressure field induced by the shock-vortex interaction. A computational study of the shock-vortex interaction has been reported by Meadows et al.[7]. These authors have shown the importance of solving the nonlinear governing equations in their conservation form for strong interactions that produce a complex pattern of reflected and diffracted shocks. Ellzey et al.[8] have numerically analyzed the interaction of a shock with a compressible, isothermal, zero-circulation vortex for different shock and vortex strengths. The simulations confirm the quadrupolar nature of the acoustic field that is generated in the early stages of the interaction. In addition those authors have shown that, depending on the strength of the interaction, the distortion of the shock may produce reflected shocks that eventually merge with the quadrupolar component to give an acoustic wave consisting of a stronger compression region in the immediate vicinity of the shock front. Ellzey and Henneke[9] have addressed the issue

of the origins of acoustic noise in shock-vortex interactions, and have shown that shock distortion and vortex deformation are the mechanisms responsible for the development of the acoustic field. When shock distortion is not significant, vortex compression is the controlling mechanism for the formation of the quadrupole pattern. In addition, they observe that, as the shock strength increases, an initially circular vortex becomes elliptical, with a major- to minor- axis ratio equal to the density ratio across the shock. Erlebacher et al.[10] have carried out a study on the non-linear effects which result from the interaction of planar shock waves with vortices, for shock Mach numbers ranging from 2 to 8 and for vortex Mach numbers up to $M_v = 0.1346$. In particular, these authors found that the onset of nonlinearities occurs when $M_v\,M_s^- \approx 0.12$ (where M_s^- is the shock Mach number). More recently, Inoue and Hattori[11] have reported a study on the sound generated through the interaction of a shock wave with either a single vortex or a pair of counter-rotating vortices. In particular, they solve the Navier-Stokes equations by means of compact finite difference schemes based on sixth-order accurate Padé formulas[12], and present results for weak shock strengths (the highest shock Mach number being $M_s^- = 1.2$) and rather low Reynolds numbers based on the size of the vortex core ($Re = 400$ and 800). For the case of a single vortex (relevant to the present study) they observe the formation of a first (the so-called precursor) and a second sound both having a quadrupole nature. They speculate that a third sound may arise, even though not predicted in their simulations, most likely because of the limitations in the extent of the computational domain and simulation time. The comparison of the circumferential pressure distribution with the measured (Ref.[3]) and inviscid numerical (Ref.[8]) results shows very small effect of the Reynolds number, thus suggesting that the generation of sound is essentially due to an inviscid mechanism.

The objective of the present paper is to analyze shock-vortex interactions with an emphasys on the characterization of shock and vortex deformations and sound production. The approach relies on the solution of the Euler equations in their integral formulation by means of a fourth-order-accurate weighted essentially non oscillatory scheme.

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II. NUMERICAL METHODOLOGY

The simulation of the sound field in the presence of shock waves is a challenging task for a numerical algorithm since it requires the capability to compute disturbances of small amplitude and short wavelength, and at the same time to handle flow discontinuities.[13] In the present work we solve the inviscid compressible flow equations for mass, momentum and energy conservation. A finite volume formulation has been used that relies on a fourth-order weighted essentially non oscillatory (WENO) scheme for the discretization of the inviscid fluxes.[14,15] This approach is employed in order to enforce conservation properties for the discretized equations, and to obtain high resolution properties far from the shocks. Time integration is performed by means of an explicit four stage Runge-Kutta algorithm optimized to achieve maximum time accuracy. The 2D Euler equations of compressible gas dynamics are written as

$$\mathbf{w}_t + \mathbf{f}(\mathbf{w})_x + \mathbf{g}(\mathbf{w})_y = 0, \tag{1}$$

where

$$\mathbf{w} = (\rho, \rho\, u, \rho\, v, \rho\, E)$$

$$\mathbf{f} = (\rho\, u, \rho\, u^2 + p, \rho\, u\, v, \rho u(E + p/\rho))$$

$$\mathbf{g} = (\rho\, v, \rho\, u\, v, \rho\, v^2 + p, \rho v(E + p/\rho)),$$

and $p = \rho\,(\gamma - 1)\,(E - (u^2 + v^2)/2)$. The system of Eqns. (1) is integrated on a cartesian grid of rectangular cells

$$S_{i,j} \equiv [x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}] \times [y_{j-\frac{1}{2}}, y_{j+\frac{1}{2}}], \qquad 1 \leq i \leq N_x, 1 \leq j \leq N_y,$$

yielding

$$
\begin{aligned}
\frac{d\overline{\overline{\mathbf{w}}}_{i,j}}{dt} = & -\frac{1}{\Delta x_i}\left[\frac{1}{\Delta y_j}\left(\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} \mathbf{f}\left(\mathbf{w}\left(x_{i+\frac{1}{2}}, y, t\right)\right) dy - \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} \mathbf{f}\left(\mathbf{w}\left(x_{i-\frac{1}{2}}, y, t\right)\right) dy\right)\right] \\
& -\frac{1}{\Delta y_j}\left[\frac{1}{\Delta x_i}\left(\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathbf{g}\left(\mathbf{w}\left(x, y_{j+\frac{1}{2}}, t\right)\right) dx - \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathbf{g}\left(\mathbf{w}\left(x, y_{j-\frac{1}{2}}, t\right)\right) dx\right)\right], \tag{2}
\end{aligned}
$$

where $\overline{(\cdot)}$ is the cell average, *i.e.*

$$\overline{\overline{\mathbf{w}}}_{i,j} \equiv \frac{1}{\Delta x_i \Delta y_j} \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathbf{w}(\xi, \eta, t) \, \mathrm{d}\xi \, \mathrm{d}\eta.$$

Equation (2) is then approximated by the following conservative scheme

$$\frac{\mathrm{d}\overline{\overline{\mathbf{w}}}_{i,j}}{\mathrm{d}t} = -\frac{1}{\Delta x_i} \left(\hat{\mathbf{f}}_{i+\frac{1}{2},j} - \hat{\mathbf{f}}_{i-\frac{1}{2},j} \right) - \frac{1}{\Delta y_j} \left(\hat{\mathbf{g}}_{i,j+\frac{1}{2}} - \hat{\mathbf{g}}_{i,j-\frac{1}{2}} \right),$$

where the numerical fluxes $\hat{\mathbf{f}}_{i+\frac{1}{2},j}$ $\hat{\mathbf{g}}_{i,j+\frac{1}{2}}$ are defined at two gaussian points (per cell face) to achieve fourth order accuracy; for example $\hat{\mathbf{f}}_{i+\frac{1}{2},j}$ is

$$\hat{\mathbf{f}}_{i+\frac{1}{2},j} = \sum_{\alpha=1,2} \gamma_\alpha \, \mathbf{h} \left(\mathbf{w}^- \left(x_{i+\frac{1}{2}}, y_j \pm \beta_\alpha \, \Delta y_j \right), \mathbf{w}^+ \left(x_{i+\frac{1}{2}}, y_j \pm \beta_\alpha \, \Delta y_j \right) \right), \tag{3}$$

where $\gamma_1 = \gamma_2 = 1/2$ and $\beta_1 = -\beta_2 = -\sqrt{3}/6$. The numerical flux function $\mathbf{h}$ has been obtained by performing a local characteristic projection in the direction normal to the cell face, and using for each characteristic field Roe's approximate Riemann solver. For example, the left and right state for the vector unknown $\mathbf{w}^\pm(x_{i+\frac{1}{2}}, y_j + \beta_\alpha \Delta y_j)$ are determined by performing two successive one-dimensional WENO reconstructions of the cell averages $\overline{\overline{\mathbf{w}}}_{i,j}$ first in the y- and then in the x- direction.

Referring for example to a one-dimensional situation, given the x-direction averages $\overline{v}_i$ of any variable v, for each cell (i), the upwind biased 5-th order reconstruction at the cell boundaries (denoted by $v^-_{i+\frac{1}{2}}$ and $v^+_{i-\frac{1}{2}}$) is obtained as a convex combination of three second-order polynomials over the three-point stencils centered around $i-1$, i and $i+1$, respectively

$$v^-_{i+\frac{1}{2}} = \sum_{r=0,2} \omega_r \, v^{(r)}_{i+\frac{1}{2}}, \qquad v^+_{i-\frac{1}{2}} = \sum_{r=0,2} \omega_r \, v^{(r)}_{i-\frac{1}{2}}, \tag{4}$$

where

$$v^{(r)}_{i+\frac{1}{2}} = \sum_{j=0,2} c_{rj} \overline{v}_{i-r+j},$$

and the i-th cell weights ω_r are chosen in the form

$$\omega_r = c_r / \sum_{m=0,2} c_m, \qquad c_r = \tilde{\omega}_r / \left(\epsilon_0 + S_r \right)^2,$$

where S_r is a measure of the smoothness of the solution on the r-th stencil, ϵ_0 is a small positive constant, and the $\tilde{\omega}_r$ are weights that yield 5-th order accuracy for a smooth solution (for more details see Refs.[15,16]).

III. RESULTS AND DISCUSSION

The aim of the study is to contribute to the understanding of the mechanism of the sound generation through shock-vortex interactions. Extensive simulations have been carried out to analyze the influence of the shock strength and the vortex intensity in terms of shock and vortex Mach numbers. The objective of the simulations is three-fold: *i)* to evaluate the non-linear effects of shock-vortex interactions; and *ii)* to characterize the formation of second and third sounds.

IV. TEST CONDITIONS

In the present work we refer to the problem of a moving planar shock interacting with a cylindrical vortex whose axis is aligned with the shock. However, for computational purposes all simulations are carried out considering a vortex crossing a steady planar shock (positioned at $x = 0$). The computational domain extends, respectively, 10 and 40 vortex radii upstream and downstream of the shock in the x-direction and ± 20 vortex radii in the y-direction (where the $+$ and $-$ are measured with respect to the direction of vortex propagation). At the far field and at the outflow, in order to avoid spurious reflections of waves into the computational domain we have employed non-reflecting boundary conditions (Refs.[17,18]). All computations have been carried out on a 500×500 grid that was selected through a grid sensitivity analysis at a given shock and vortex Mach number. All simulations are initialized assuming a homentropic Taylor vortex placed at five vortex radii upstream of the shock. Let $(x_{v,0}, 0)$ be the initial vortex location, the initial flow

field at a point (x, y) is given by

$$u_0(x, y) = -v_{max} \frac{y}{r_v} \exp \left(\frac{1 - \xi^2}{2} \right)$$

$$v_0(x, y) = v_{max} \frac{x - x_{v,0}}{r_v} \exp \left(\frac{1 - \xi^2}{2} \right)$$

$$\rho_0(x, y) = \left[1 - \frac{\gamma - 1}{2} M_v^2 \exp \left(1 - \xi^2 \right) \right]^{\frac{1}{\gamma - 1}} \tag{5}$$

$$p_0(x, y) = \left[1 - \frac{\gamma - 1}{2} M_v^2 \exp \left(1 - \xi^2 \right) \right]^{\frac{\gamma}{\gamma - 1}}$$

where $\xi = \sqrt{(x - x_{v,0})^2 + y^2}/r_v$.

Several simulations have been carried out in the range of shock Mach number $1.01 \leq M_s^- \leq 2.0$ and vortex Mach number $0.05 \leq M_v \leq 1.0$.

V. CONCLUSIONS

Shock wave - vortex interactions have been analyzed through extensive numerical simulations. The approach relies on the numerical solution of the Euler equations by means of a weighted-ENO scheme. A parametric study has been carried out to determine the effects of both shock strength and vortex intensity, with the aim to characterize the shock and vortex deformations and the sound generation mechanism. The results show that for weak vortices (*i.e.* vortex Mach number $M_v \simeq 0.1 \div 0.2$) weak interactions occur regardless of the shock strength. Strong interactions will always occur whenever a shock wave interacts with a "strong" vortex (*i.e.* for $M_v \geq 0.1 \div 0.2$) giving rise to either a regular reflection pattern or a Mach reflection pattern depending upon the value of the shock Mach number.

The vortex deformation is found to depend upon the shock strength; in particular, an initially cylindrical vortex is deformed into an elliptical one and nutates with a nearly constant angular velocity. The ratio of major- to minor-axis of the (elliptical) vortex is found to scale with the square root of the density ratio across the shock. The sound generated through shock-vortex interactions is found to exhibit a three-stage evolution:

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in the first stage a (precursor) sound wave having a dipolar directivity is generated due to shock deformation; then, in the second stage the shock undergoes a restoring mechanism and the sound directivity changes into a quadrupolar one. However, the first two stages occur on a (rather short) time scale that corresponds to the duration of the interaction. In the third stage two secondary sounds are generated; the characteristic times of formation of the second and the third sound are found to depend on the speed of sound behind the shock, and they are increasing functions of the vortex Mach number, while they decrease with the shock strength. The simulations also show that the onset of nonlinearities occurs when $M_v M_s^- \approx 0.1$.

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