A Preliminary Model for Spacecraft Propulsion Performance Analysis based on Nuclear Gain and Subsystem Mass-Power Balances

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Acknowledgments

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Space Flight Requirements

- Omniplanetary space flight requires new high-performance propulsion systems based on nuclear energy.

- Over the last several decades, many propulsion concepts have discussed one-month missions to Mars and one-year missions to the outer planets.

- Such missions entail large mission velocities and vehicle accelerations, which in turn require both high exhaust velocities (and therefore, specific impulses) and extremely low mass-power ratios, e.g.:

$$I_{sp} \geq 10^4 \text{ to } 10^5 \text{ sec} \qquad \alpha \leq 10^{-2} \text{ kg/kW}$$

Spacecraft Energy "Gain"

- High performance electric propulsion appears capable of enabling multi-month transits to Mars and the near-earth asteroids; however, the mass-power ratio (α) of these "power-limited" systems appears too high to achieve large accelerations for outer planet missions.

- Higher accelerations demand energy "gains" from nuclear reactions in the propellant.

  - Such energy "gains" must account for power required to "drive" nuclear reactions — this type of system is "gain-limited" in that driver power can be a significant fraction of total power produced.

"Control Theory" for Performance

- The concept of energy "gain" for propulsion systems implies that an approach analogous to control theory may be useful in evaluating the performance of such systems — in effect, treating propulsion systems as "power circuits".
  - First, derive expressions for mission trip time and distance as functions of parameters including (*required*) I_{sp} and α.
  - Next, derive expressions for α for both power- and gain-limited systems.
  - Last, connect the mission relations to the power systems relations.

Mission Assumptions

- Treat I_{sp} and α as independent parameters that characterize a propulsion system
  - Treat vehicle acceleration as a parameter dependent upon I_{sp} and α
- Assume round-trip mission between points A and B
  - Goal is to minimize trip time τ_{RT} for the distance D_{AB}
  - Assume accelerations far greater than local acceleration of the sun
  - Assume constant thrust accelerations
  - Assume zero velocity at points A and B
    - These points permit assumption of a straight-line trajectory where $D_{AB} = D_{BA}$

Round Trips: Time & Distance

- If we begin with $\Delta\tau = \frac{m_{\text{propellant expended}}}{\dot{m}}$, we can obtain trip times in each direction:

$$\tau_{AB} = \frac{gI_{sp}}{T/m_{A2}} \frac{m_{A2}}{m_B}\left(\frac{m_B}{m_{A1}} - 1\right) \qquad \tau_{BA} = \frac{gI_{sp}}{T/m_{A2}}\left(\frac{m_{A2}}{m_B} - 1\right)$$

- Using $D_{if} = \frac{1}{\dot{m}}\int_{m_i}^{m_f} V dm$, we can obtain the distance in each direction:

$$D_{AB} = \frac{\left(gI_{sp}\right)^2}{T/m_{A2}} \frac{m_{A2}}{m_B}\left(\sqrt{\frac{m_B}{m_{A1}}} - 1\right)^2 \qquad D_{BA} = \frac{\left(gI_{sp}\right)^2}{T/m_{A2}}\left(\sqrt{\frac{m_{A2}}{m_B}} - 1\right)^2$$

Trip Times = $f(D_{AB}, I_{sp}, T/m_{A2})$

- With straight-line trajectories, $D_{AB}=D_{BA}$, and the mass ratios can be eliminated to yield both round-trip and one-way trip times as functions of I_{sp} and D_{AB}:

$$\tau = \frac{D_{AB}}{gI_{sp}} \cdot (h + kU)$$

$$\text{where } (h,k) = \begin{Bmatrix} (4,4) & \text{for Round Trip} \\ (3,2) & \text{for One Way} \end{Bmatrix} \text{ and } U = \frac{gI_{sp}}{\sqrt{(T/m_{A2})D_{AB}}}$$

Vehicle acceleration, T/m_{A2}

- The acceleration T/m_{A2} is related to the system mass-power ratio α. We can use the following expression for final (burnout) mass m_{A2}, together with relations for power output and propellant mass:

$$m_{A2} = m_{pay} + \alpha \cdot P_{out} + \beta \cdot m_{prop}$$

$$m_{pay} = m_{A2}\lambda_{pay} \;\; ; \;\; P_{out} = TV_e/2 \;\; ; \;\; m_{prop} = T\tau/V_e$$

- Substitution enables solving for the acceleration in terms of γ:

$$\frac{1}{T/m_{A2}} = \frac{1}{1-\lambda_{pay}}\left(\alpha\frac{gI_{sp}}{2} + \beta\frac{\tau}{gI_{sp}}\right)$$

Trip Times = $f(I_{sp}, D_{AB}, \alpha)$

- This leads to a generalized expression relating trip time and I_{sp}:

$$\tau = \frac{1}{I_{sp}} \cdot \left[X \pm \sqrt{Y + Z \cdot I_{sp}^3} \right]$$

$$X = \left(\frac{D_{AB}}{2g} \right) \left(2h + k^2 \frac{\beta}{1 - \lambda_{pay}} \right)$$

$$Y = \left(\frac{kD_{AB}}{2g} \right)^2 \left[4h + k^2 \frac{\beta}{1 - \lambda_{pay}} \right] \frac{\beta}{1 - \lambda_{pay}}$$

$$Z = \frac{k^2}{2} \frac{gD_{AB}}{1 - \lambda_{pay}} \alpha$$

Optimized I_{sp} yields Optimal τ

- An optimal τ is obtained by taking the derivative with respect to I_{sp} and solving:

$$\left[\left(I_{sp}\right)_{OPT}\right]^3 = \frac{1}{Z}\iota_{OPT}^3$$

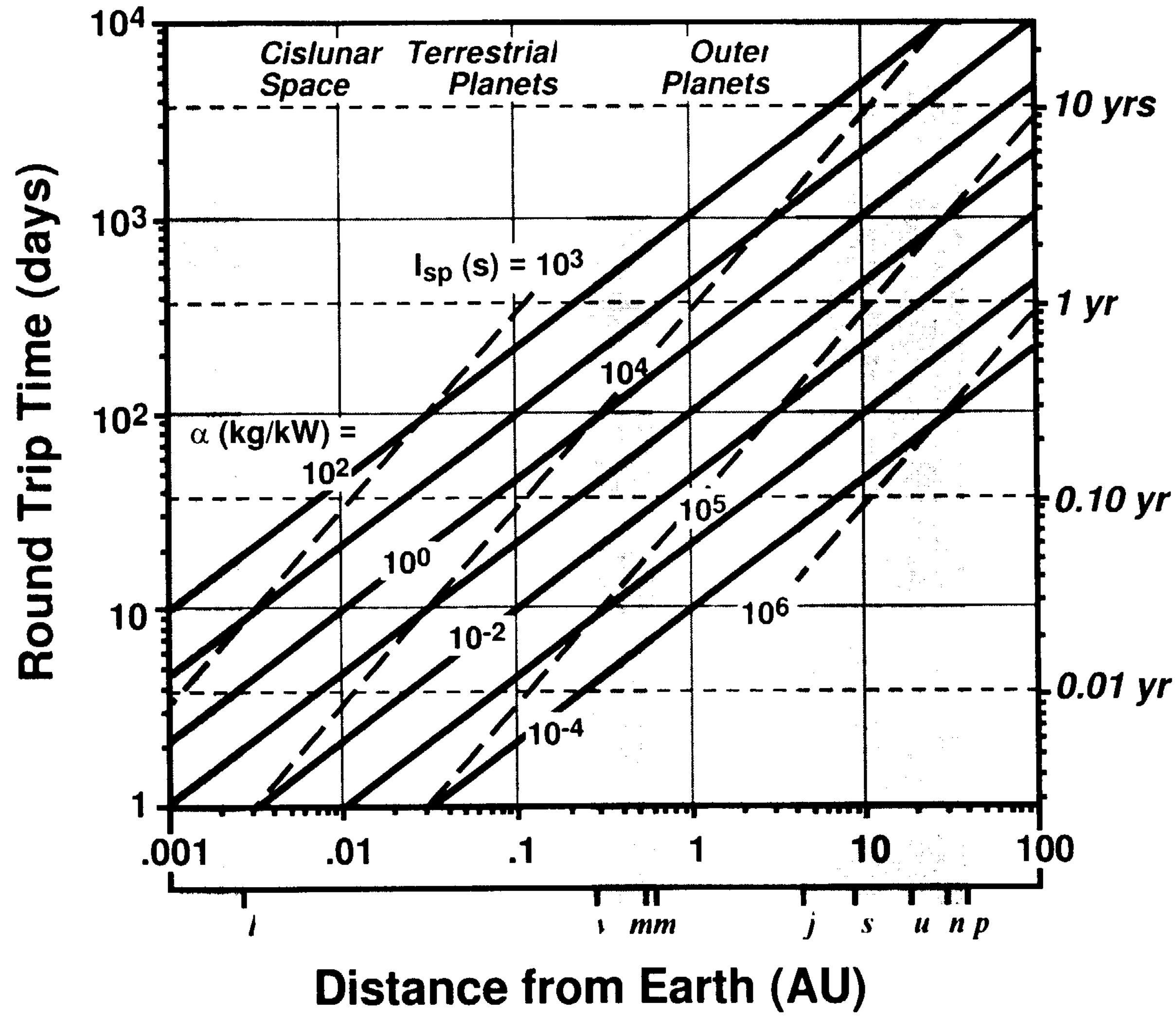
$$\iota_{OPT}^3(X,Y) = \left(\frac{2X}{9}\right)\cdot\left[\left(X - \frac{3Y}{X}\right) + \sqrt{X^2 + 3Y}\right]$$

- Substitution of this optimized I_{sp} yields the optimal trip time:

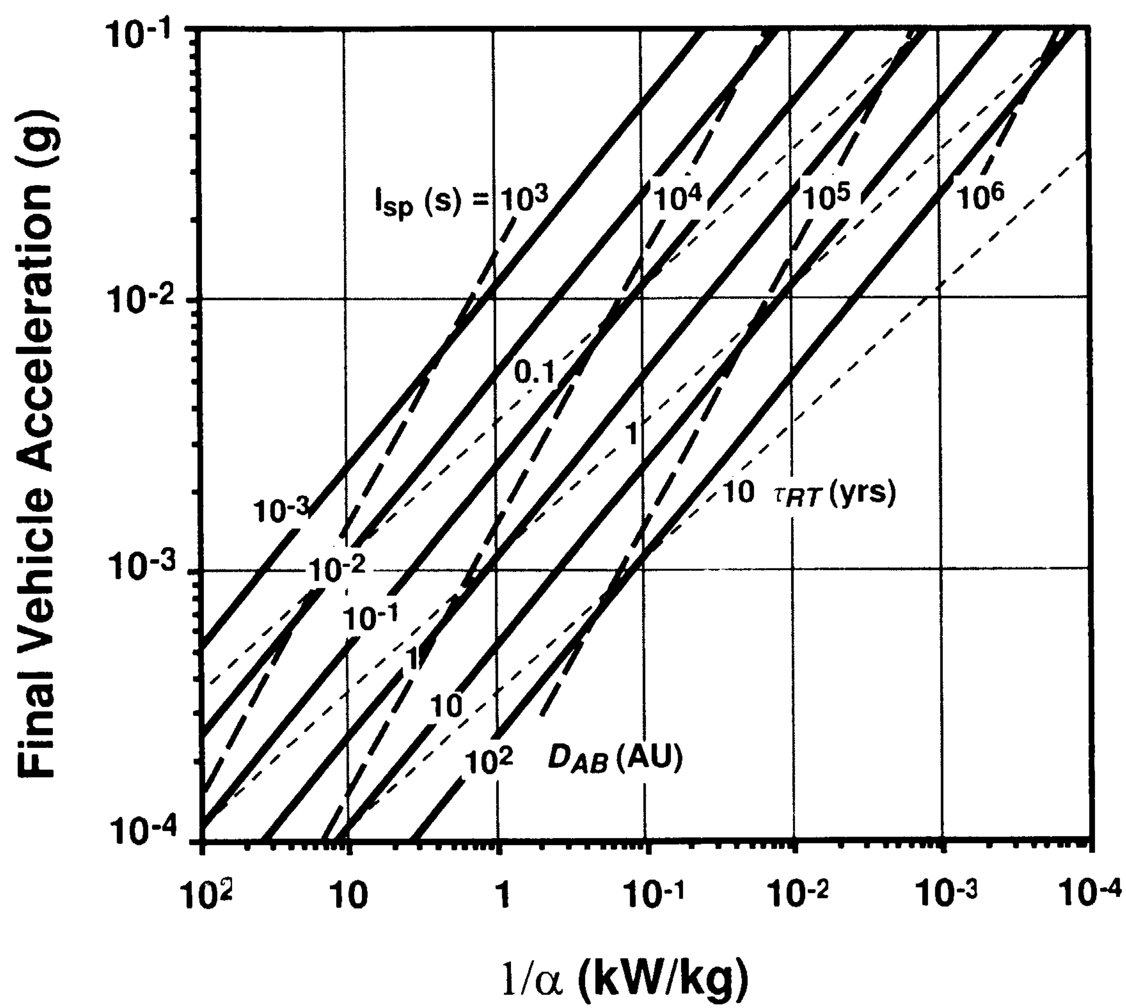
$$\tau_{OPT} = \frac{Z^{1/3}}{\iota_{OPT}}\left[X + \sqrt{Y + \iota_{OPT}^3}\right]$$

- Note that Z is proportional to α: this means that τ_{OPT} varies as $\alpha^{1/3}$.

Minimizing Trip Times



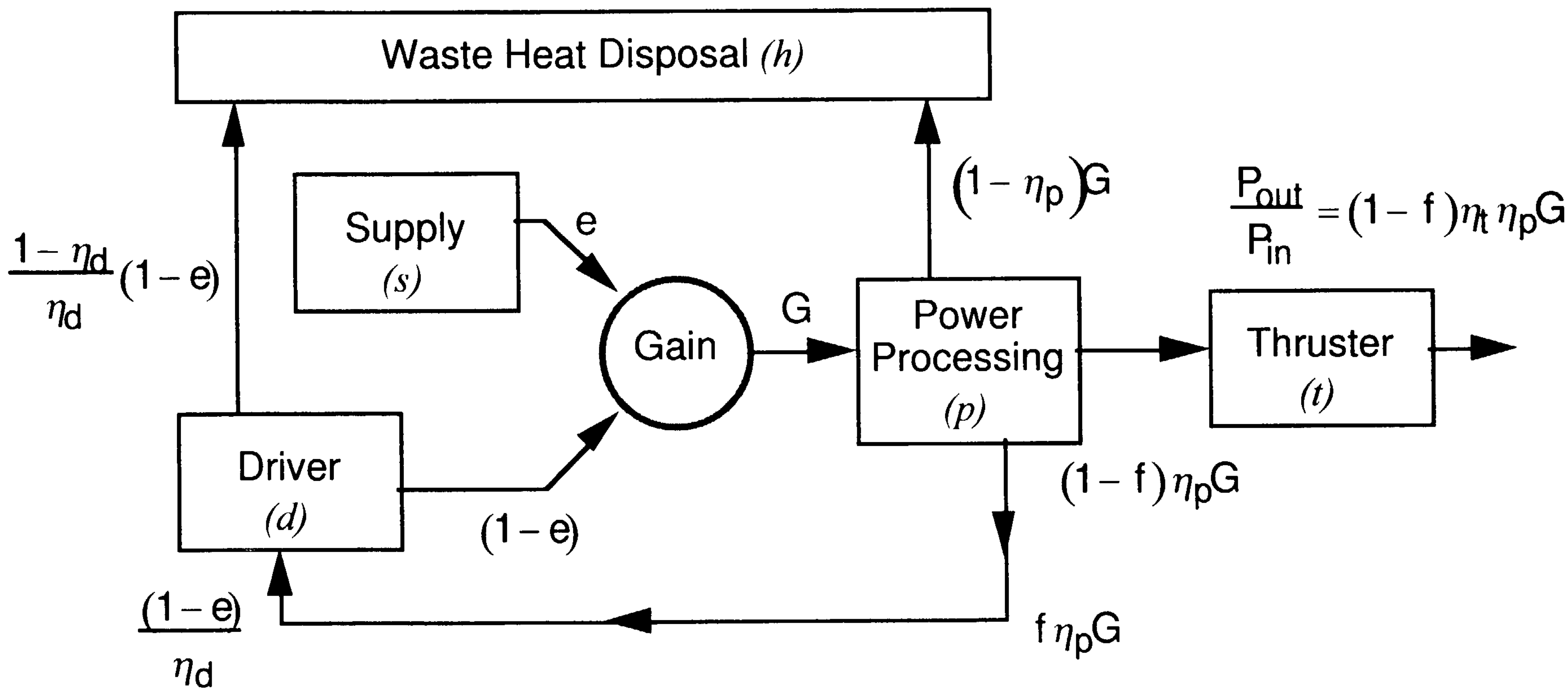
Required Final Accelerations



Fast missions require low α, high I_{sp}

- The previous two figures emphasize that fast missions to the outer planets necessitate very low α and very high I_{sp} values:
  - One year to Jupiter: $\alpha \sim 10^{-1}$ kg/kW; $I_{sp} \sim 70000$ sec
  - One year to Pluto: $\alpha \sim 10^{-3}$ kg/kW; $I_{sp} \sim 300000$ sec
- These values are beyond the limits of power-limited systems — including even high-performance electric propulsion
- Instead, consider the influence of gain-limited systems, with spacecraft gain illustrated by the following power system schematic (or “power circuit”) …

Power System Schematic



Power flows as fractions of P_{in}

- The input power to the nuclear process — P_{in} — is obtained from two sources:
  - Fractional power from an onboard source: e
  - Fractional power from a driver powered from system: $1 - e$
    - If $e = 1$ — solely power-limited
    - If $e = 0$ — solely gain-limited
- The power flows in the schematic are represented as fractions of this input power:
  - Fraction of power needed to power driver: f
  - Subsystem mass-power ratios: $\hat{\alpha}_D, \hat{\alpha}_P, \hat{\alpha}_T, \hat{\alpha}_S, \hat{\alpha}_H$
  - Subsystem component efficiencies (always < 1) : $\eta_D,\ \eta_P,\ \eta_T$

Conservation of Mass/Power

- Using a similar equation for conservation of mass as used previously, we can substitute the mass-power ratio α multiplied by the fractional power to yield the mass for each subsystem.
  - Example: [mass of power supply subsystem]: $m_S = (\hat{\alpha}_S)(e)P_{in}$
- Summing all subsystem masses and dividing through by output power yields an equation for the overall system mass-power ratio, α:

$$\alpha = \frac{\left\{ \begin{array}{l} \hat{\alpha}_S \eta_D e + \hat{\alpha}_D (1-e) + \hat{\alpha}_P \eta_D G + \hat{\alpha}_T \left[G \eta_P \eta_D - (1-e) \right] + \\ \hat{\alpha}_H \left[(1-\eta_D)(1-e) + \eta_D (1-\eta_P) G \right] \end{array} \right\}}{\eta_T \left[G \eta_P \eta_D - (1-e) \right]}$$

Special Cases of Power Systems

- This equation is capable of modeling the α of either power-limited or gain-limited systems.

- For solely power-limited systems ($e = 1$, $G = 1$):

$$\alpha_{P-L} = \alpha_{POWER-LIMITED} = \frac{\hat{\alpha}_S + \hat{\alpha}_P \eta_P + \hat{\alpha}_H (1 - \eta_P)}{\eta_T \eta_P} + \hat{\alpha}_T$$

- For solely gain-limited systems ($e = 0$):

$$\alpha_{G-L} = \alpha_{GAIN-LIMITED} = \frac{\hat{\alpha}_D \eta_D + \hat{\alpha}_P \eta_D \eta_P G + \hat{\alpha}_H \left[(1 - \eta_D) + \eta_D (1 - \eta_P) G \right]}{\eta_T (\eta_P \eta_D G - 1)} + \hat{\alpha}_T$$

Limits on Values of Gain

- In order to have a net positive input power for thrust production and driver operation, the denominator of this equation must have a positive value. This condition results in:

$$G > G_{MIN} \quad ; \quad \text{where} \quad G_{MIN} = \frac{1}{\eta_P \eta_D}$$

- **Progressively higher values of *G* above this minimum result in successively lower mass-power ratios, α.**

- In the limit where gain goes to infinity, there is a minimum of α:

$$\alpha_{G\infty} = \hat{\alpha}_T + \frac{\hat{\alpha}_P \eta_P + \hat{\alpha}_H (1 - \eta_P)}{\eta_T \eta_P}$$

A Simplified form of $\alpha_{G\text{-}L}$

- In the limit where gain goes to zero, the value of α has no physical significance:

$$\alpha_{G0} = \hat{\alpha}_T - \frac{\hat{\alpha}_D \eta_D + \hat{\alpha}_H (1 - \eta_D)}{\eta_T}$$

- However, substitution of α_{G0} and $\alpha_{G\infty}$ simplifies the $\alpha_{G\text{-}L}$ power balance into a more compact form emphasizing gain-driven and gain-independent parameters:

$$\alpha_{G-L} = \frac{\frac{G}{G_{MIN}} \alpha_{G\infty} - \alpha_{G0}}{\frac{G}{G_{MIN}} - 1}$$

Given α, calculate needed Gain

- Inverting the compact $\alpha_{G\text{-}L}$ equation for G yields an equation stating the G required for a given $\alpha_{G\text{-}L}$ (or α):

$$G = G_{MIN} \frac{\alpha - \alpha_{G0}}{\alpha - \alpha_{G\infty}}$$

- The lower the value of $\alpha_{G\infty}$ — implying lower subsystem α's and higher η_P — the smaller the value of gain G required to meet a mission.

Summary so far ...

- For very fast missions with straight-line trajectories, it has been shown that mission trip time is proportional to the cube root of α.
- Analysis of spacecraft power systems via a power balance and examination of gain vs. mass-power ratio has shown:
  - A minimum gain is needed to have enough power for thruster and driver operation
  - Increases in gain result in decreases in overall mass-power ratio, which in turn leads to greater achievable accelerations.
    - However, subsystem mass-power ratios and efficiencies are crucial: less efficient values for these can partially offset the effect of nuclear gain.
    - Therefore, it is of interest to monitor the progress of gain-limited subsystem technologies.
    - It is also possible that power-limited systems with sufficiently low α may be competitive for such ambitious missions.