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ASYMPTOTIC BEHAVIOR OF ENSEMBLE-AVERAGED LINEAR DISTURBANCES IN HOMOGENEOUS SHEAR FLOW

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ABSTRACT

In order to expand the predictive capability of single-point turbulence closure models to account for the early-stage transition regime, a methodology for the formulation and calibration of model equations for the ensemble-averaged disturbance kinetic energy and energy dissipation rate is presented. The calibration is based on homogeneous shear flow where disturbances can be described by rapid distortion theory (RDT). The relationship between RDT and linear stability theory is exploited in order to obtain a closed set of modeled equations. The linear disturbance equations are solved directly so that the numerical simulation yields a database from which the closure coefficients in the ensemble-averaged disturbance equations can be determined.

INTRODUCTION

Demands on the range of applicability of turbulence modeling are increasing, and with these increasing demands has come the need to develop models which are better sensitized to the transition process within the context of the traditional Reynolds-averaged turbulence modeling. Thus, a common mathematical framework linking the laminar regime, with its linear disturbances, and the fully turbulent regime, with its stochastic fluctuations, needs to be provided.

As a first step toward the development of such a mathematical framework, Thacker, Gatski and Grosch (1999) studied the behavior of homogeneous, isotropic decaying disturbances. In turbulence modeling, this flow is used to calibrate the destruction-of-dissipation term in the modeled transport equation for the turbulent dissipation rate. In the linear disturbance case, the same functional form of the turbulence model was found but a new value of the model coefficient for the destruction-of-dissipation term was required.

For consistency, disturbances are defined as deviations from the ensemble-mean in all flow regimes. The laminar regime is defined as the region of the flow in which the ensemble-mean velocity is a stationary solution of the Navier-Stokes equation. The disturbances in this regime are small enough in amplitude that their nonlinear interactions can be neglected, and their evolution is completely predictable from their initial state. Traditionally, in laminar stability theory, disturbance fields are studied through the linear Orr-Sommerfeld equation, which describes the evolution of individual infinitesimal disturbance modes. Even when a quantity, such as the disturbance energy, is studied, it is the evolution of the instantaneous quantity rather than an ensemble average that is investigated. In contrast, the turbulent regime is defined as the region where the flow is subject to stochastic fluctuations, arising from nonlinear interactions, which render the behavior of the disturbances unpredictable. In this case, the disturbance field is traditionally studied through (modeled) transport equations which describe the evolution of mean turbulent correlations.

In this work, a set of (ensemble) mean disturbance transport equations, capable of describing the behavior of the linear disturbance fields, is developed. The approach is founded on the observation that, even in the laminar regime, every flow is subject to an inevitable uncertainty in initial conditions. Therefore, although each individual disturbance evolves deterministically, a probability distribution describing the initial ($t = 0$) energy distribution of the modes must be introduced for the calculation of ensemble-mean properties. This approach is similar to rapid distortion theory (RDT) in that it is based on linearized disturbance equations; however, the realm of application is different. RDT traditionally considers flows at higher Reynolds numbers in which the turbulence is fully developed and the effects of the molecular viscosity can be neglected, using linearized equations to study the behavior of the disturbances under rapid (strong) distortion. The approach taken here, on the other hand, considers linear disturbances in the early stages of transition where viscous effects must be taken into account. In addition, RDT is usually applied to short time evolution since in the turbulence case the nonlinear interactions cause sufficient growth of the fluctuations to render the linear approximation invalid after a few eddy turnover times. In this linearized disturbance case, no such limitation on the time duration is encountered because viscous effects result in the decay of the disturbance field at large times after a relatively small (initial) energetic growth.

More recent extensions of RDT by Salhi, Cambon and Speziale (1997) have also exploited the connection with linear stability theory. They studied quadratic flows in a rotating frame to gain better insight into the dynamics so that a generalized stability criterion applicable to turbulent flows could be developed. They also considered the effect on single-point closure modeling – specifically the deficiencies in predicting elliptic flows. While the mathematical framework is similar in this study, the region of interest here is the early-stage transition regime. Nevertheless, this commonality further substantiates the basic assumption that a mathematical framework can be developed which will provide a set of transport equations capable of describing the flow (in a mean statistical sense) in the early-stage transition regime.

In this study, the earlier analysis of Thacker et al. (1999) is applied to the case of mean homogeneous shear. Homogeneous shear flow is commonly used as a calibration flow for turbulence models because both turbulent transport and viscous terms can be removed from the transport equations for the turbulent correlations. The purpose of this study is to use the solution of the disturbance evolution equations for mean homogeneous shear flow as a database in the calibration of the evolution equations for the ensemble-averaged disturbance correlations. As a first step, the focus is on a simple disturbance kinetic energy and disturbance dissipation rate (two-equation) description. In such a two-equation description of homogeneous shear, the terms in the kinetic energy equation are exact and require no modeling; whereas, in the dissipation rate equation both the production-of-dissipation and destruction-of-dissipation terms require modeling. The two closure coefficients associated with these terms are determined from the analysis presented here. Utilization of the disturbance evolution results as a reliable database is supported by the DNS results of Lee, Kim, and Moin (1990) who studied homogeneous shear flow at a high-shear rate. They showed that RDT results compared very well with the simulation results over the time period examined. The results herein also show good agreement with the DNS results of Lee et al. (1990), and are found to apply at much later times due to the energetic decay of the disturbance field in the parameter range studied. Thus, this database will be used to provide insight into the asymptotic behavior of important dynamic variables, as well as to provide the necessary information for the closure model calibration. The resulting closed disturbance dissipation rate equation can then be used in the formulation of a transition-sensitized turbulence model.

LINEAR THEORY AND ENSEMBLE-AVERAGED CORRELATIONS

In this section, transport equations for the ensemble-averaged linear disturbance second-moments in homogeneous shear flow are constructed. The theoretical development presented here parallels that of Townsend (1970) who described the structure of turbulence in a free shear flow as a product of the finite distortion of parcels of turbulent fluid. In this study, the relationship of RDT with linear stability theory (Speziale et al. 1996, Salhi et al. 1997) is expanded to include an analysis of the transport equations for the ensemble-averaged disturbance kinetic energy and the disturbance energy dissipation rate. As in the turbulence case, such model equations require closure through the specification of closure constants. In the two-equation $K-\varepsilon$ formulation, only the disturbance dissipation rate equation contains modeled terms which have unknown closure constants. The homogeneous shear flow is used as a calibration flow for the production-of-dissipation and destruction-of-dissipation terms.

In terms of dimensional coordinates $(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ in a fixed frame, the mean velocity is given by $\tilde{U}_1 = S\tilde{x}_2$, $\tilde{U}_2 = \tilde{U}_3 = 0$, where the mean shear S = constant, and the disturbance velocity and pressure are denoted by $\hat{u}_j$ and $\hat{p}$, respectively. For the problem of homogeneous shear, it is convenient to work (Rogallo 1984) in a moving frame $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$, in which the local mean velocity is zero and the coordinates are given by

$$\begin{aligned} \hat{x}_1 &= \tilde{x}_1 - (\, S\hat{t}\,)\tilde{x}_2 \\ \hat{x}_2 &= \tilde{x}_2 \\ \hat{x}_3 &= \tilde{x}_3\,, \end{aligned} \tag{1}$$

where $\hat{t}$ is time. The mean shear S and the viscosity ν determine the time scale $T = S^{-1}$, length scale $L = \sqrt{\nu S^{-1}}$, velocity scale $U = \sqrt{\nu S}$, and pressure scale $P = U^2 = \nu S$ for the flow. This leads to the introduction of dimensionless variables $x_j = \tilde{x}_j/L$ (fixed frame), $x'_j = \hat{x}_j/L$ (moving frame), $t = \hat{t}/T$, $u_j = \hat{u}_j/U$, and $p = \hat{p}/P$. The linear approximation is based on the assumption that the nonlinear terms $u_j \frac{\partial u_i}{\partial x_j}$, in the disturbance momentum equations, can be neglected with respect to both the viscous term $\nu\nabla^2 u_i$ and the mean shear term $u_j \partial U_i/\partial x_j$. This approximation remains valid as long as the amplitude of the disturbance velocity remains small compared with the velocity scale $\sqrt{\nu S}$.

An arbitrary solution of the incompressibility condition and linearized Navier-Stokes equations in the moving frame can be written as a linear superposition

$$\begin{aligned} u_j(\mathbf{x}', t) &= \int d^3\mathbf{k}\; f_j(k, t) e^{i\mathbf{k}\cdot\mathbf{x}'} \\ p(\mathbf{x}', t) &= \int d^3\mathbf{k}\; p(\mathbf{k}, t) e^{i\mathbf{k}\cdot\mathbf{x}'}, \end{aligned} \tag{2}$$

where the mode amplitudes $f_j(k,t)$ and $p(\mathbf{k},t)$ satisfy

$$k_1 f_1 + k_2' f_2 + k_3 f_3 = 0 \quad (3)$$

$$\dot{f}_1 + f_2 + ik_1 p = -\mathcal{K}^2 f_1 \quad (4)$$

$$\dot{f}_2 + ik_2' p = -\mathcal{K}^2 f_2 \quad (5)$$

$$\dot{f}_3 + ik_3 p = -\mathcal{K}^2 f_3 , \quad (6)$$

with $k_2' = k_2 - tk_1$, $\mathcal{K}^2 = k_1^2 + k_2'^2 + k_3^2$, and the wave numbers (k_1, k_2, k_3) are constant in time. The solution for the velocity disturbance modes can be written in the form

$$f_i(\mathbf{k},t) = M_{ij}(\mathbf{k},t) f_j(\mathbf{k},0) e^{-q(\mathbf{k},t)} , \quad (7)$$

where the exponent

$$q(\mathbf{k},t) = \int \mathcal{K}^2 \, dt = \frac{1}{3} k_1^2 \, t^3 \; - \; k_1 \, k_2 \, t^2 \; + \; (k_i k_i) \, t \quad (8)$$

arises from the viscous damping of all modes. The transfer matrix M has non-vanishing components

$$\begin{aligned} M_{11} &= M_{33} = 1 \\ M_{22} &= \frac{k^2}{\mathcal{K}^2} \\ M_{12} &= \frac{k_3^2 k^2 \,(\Theta - \Theta_0)}{k_1 k_H^3} + \frac{k_1^2 t (k_1^2 - k_2 k_2' + k_3^2)}{k_H^2 \mathcal{K}^2} \\ M_{32} &= \frac{k_3 k^2 (\Theta_0 - \Theta)}{k_H^3} + \frac{k_1 k_3 t (k_1^2 - k_2 k_2' + k_3^2)}{k_H^2 \mathcal{K}^2}, \end{aligned} \quad (9)$$

with

$$\begin{aligned} k_H &= \left(k_1^2 + k_3^2\right)^{\frac{1}{2}} \\ \Theta_0 &= \tan^{-1}\left(\frac{k_2}{k_H}\right) \\ \Theta &= \tan^{-1}\left(\frac{k_2'}{k_H}\right) \end{aligned} \quad (10)$$

The initial mode amplitudes $f_i(\mathbf{k},0)$ are elements of an ensemble with mean zero, and covariance $\langle f_i(\mathbf{k},0) f_j(\mathbf{l},0)\rangle$. Assuming that the initial disturbance field is homogeneous, isotropic, and has finite energy K_0, the initial covariance is given by the same expression as that used by Thacker et al. (1999) for the case of zero mean shear with no boundary

$$\langle f_i(\mathbf{k},0) f_j(\mathbf{l},0)\rangle = \delta^3(\mathbf{k} + \mathbf{l}) \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \times K_0 \mathcal{P}(\mathbf{k}), \quad (11)$$

where

$$\mathcal{P}(\mathbf{k}) = \frac{2}{3}\left(\frac{a^5}{\pi^3}\right)^{1/2} k^2 e^{-ak^2} \quad (12)$$

is the probability distribution for disturbance second-moments, and the parameter a, related to the variance of the distribution, will be given a physical interpretation in what follows. This probability distribution is consistent with a k^4 low wave number behavior for the energy spectrum $E(k)$ and, in the case of linear disturbances in zero-mean-shear flow, leads to a $t^{-\frac{5}{2}}$ decay law for the mean disturbance kinetic energy.

The disturbance mode covariance is related to the energy spectrum tensor by

$$\langle f_i(\mathbf{k},t) f_j(\mathbf{l},t)\rangle = E_{ij}(\mathbf{k},t)\delta^3(\mathbf{k}+\mathbf{l}), \quad (13)$$

so it follows from Eq.(7) that

$$E_{ij}(\mathbf{k},t) = M_{il}(\mathbf{k},t) \; M_{jm}(\mathbf{k},t) \times E_{0lm}(\mathbf{k}) e^{-2q(\mathbf{k},t)}, \quad (14)$$

where, as a result of (11),

$$E_{0ij}(\mathbf{k}) = E_{ij}(\mathbf{k},0) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) K_0 \mathcal{P}(\mathbf{k}) \quad (15)$$

is the initial energy spectrum tensor.

The energy spectrum tensor is the fundamental quantity from which all disturbance second-moments are derived by integration over wave number space. Of particular interest are the disturbance stress tensor

$$\tau_{ij}(t) = \int d^3\mathbf{k} \; E_{ij}(\mathbf{k},t) , \quad (16)$$

the disturbance kinetic energy $K(t) = \tau_{ii}/2$, and the disturbance dissipation rate

$$\varepsilon(t) = \int d^3\mathbf{k} \; \mathcal{K}^2 \; E_{ii}(\mathbf{k},t). \quad (17)$$

A physical interpretation can be given to the parameter a, which appears in the probability distribution (12), by performing the integral in (17) at time zero to obtain $\varepsilon_0 = 5K_0/a$. In terms of the dimensional disturbance kinetic energy $\hat{K} = \nu S K$, and the dimensional dissipation rate $\hat{\varepsilon} = \nu S^2 \varepsilon$ the parameter a is given by

$$a = 5\left(\frac{S\hat{K}_0}{\hat{\varepsilon}_0}\right) = 5\eta_0 \quad (18)$$

It will be shown that this quantity, a ratio of the disturbance time scale to the mean flow time scale, regulates the subsequent growth or decay of linear disturbances in homogeneous shear flow. In the turbulence case, the variable $\eta = S\hat{K}/\hat{\varepsilon}$ also plays a critical role. Jongen and Gatski (1998) have shown that for turbulent homogeneous shear flow, η reaches an equilibrium value which can be analytically connected to the variation of the ratio of kinetic energy production to dissipation rate.

MODELED DISTURBANCE TRANSPORT EQUATIONS

The ensemble-mean kinetic energy and dissipation rate for linear disturbances satisfy evolution equations which can be derived from the linearized Navier-Stokes equations (in the fixed frame). In this homogeneous flow, gradients of all ensemble-mean quantities vanish, and the (dimensionless) kinetic energy and dissipation rate equations reduce to

$$\dot{K} = -\tau_{12} - \varepsilon = \mathcal{P}_K - \varepsilon, \tag{19}$$

$$\dot{\varepsilon} = \mathcal{P}_\varepsilon - \mathcal{D}_\varepsilon, \tag{20}$$

where $\mathcal{P}_K(= -\tau_{12})$ is the kinetic energy production,

$$\begin{aligned} \mathcal{P}_\varepsilon &= -2\left(\left\langle \frac{\partial u_i}{\partial x_1}\frac{\partial u_i}{\partial x_2}\right\rangle + \left\langle \frac{\partial u_1}{\partial x_i}\frac{\partial u_2}{\partial x_i}\right\rangle\right) \\ &= -2\int d^3\mathbf{k}\left[k_1 k_2' \; E_{ii} + \mathcal{K}^2 \; E_{12}\right] \end{aligned} \tag{21}$$

is the production-of-dissipation, and

$$\begin{aligned} \mathcal{D}_\varepsilon &= 2\left\langle \frac{\partial^2 u_i}{\partial x_j \partial x_k}\frac{\partial^2 u_i}{\partial x_j \partial x_k}\right\rangle \\ &= 2\int d^3\mathbf{k}\mathcal{K}^4 \; E_{ii} \end{aligned} \tag{22}$$

is the destruction-of-dissipation. From Eqs. (19) and (20), it is seen that only the disturbance dissipation rate equation requires closure models. In the standard $K-\varepsilon$ formulation, the production-of-dissipation is modeled by

$$\mathcal{P}_\varepsilon = -C_{\varepsilon 1}\frac{\varepsilon}{K}\tau_{12}, \tag{23}$$

which gives

$$C_{\varepsilon 1} = -\mathcal{P}_\varepsilon \frac{K}{\varepsilon\tau_{12}} = \frac{K}{\varepsilon}\frac{\mathcal{P}_\varepsilon}{\mathcal{P}_K}, \tag{24}$$

for the production-of-dissipation coefficient. The destruction-of-dissipation is modeled by

$$\mathcal{D}_\varepsilon = C_{\varepsilon 2}\frac{\varepsilon^2}{K}, \tag{25}$$

which yields

$$C_{\varepsilon 2} = \frac{K}{\varepsilon^2}\mathcal{D}_\varepsilon \tag{26}$$

for the destruction-of-dissipation coefficient.

Recall that the components E_{ij} of the energy spectrum tensor are completely defined by Eqs. (7) through (15). With the solution of these equations, all the quantities of interest, including $\mathcal{P}_\varepsilon$ and $\mathcal{D}_\varepsilon$, can be obtained by carrying out the integration in Eqs. (16), (17), (21), and (22). These three-dimensional integrations were initially performed numerically using cubic and spherical grids. It was found that although the convergence was good at small times, at times greater than about $10S^{-1}$, the results became strongly dependent on the grid resolution used (even for grids with as many as 250 million points). The numerical evaluation of the integrals in Eqs. (16), (17), (21), and (22) is therefore greatly simplified, and the accuracy significantly enhanced, by performing the radial integration analytically.

For the chosen probability distribution (12) the radial integrals in Eqs. (16), (17), (21), and (22) are Gaussian and can easily be carried out analytically. Convergence tests show that the resulting angullar integrals had to be evaluated on grids with a spacing of 1/16 of a degree.

In the next section the results of these computations are used to analyze the ensemble-averaged transport equations for the disturbance kinetic energy and dissipation rate. Of particular interest is the dissipation rate equation Eq. (20) which requires closure. It will be shown that closure models for $\mathcal{P}_\varepsilon$ and $\mathcal{D}_\varepsilon$, with constant closure coefficients, can be determined through an analysis of the long-time behavior of the disturbance correlations.

RESULTS

Figure 1 shows the evolution of the disturbance kinetic energy for different initial values of η_0. For a small value of η_0 (= 0.6), the energy decays monotonically in time. A transition between this monotonic decay and substantial growth of the disturbance energy occurs for values of η_0 between 1.2, where the energy reaches a minimum and grows briefly, and 2.4, where the energy grows to a magnitude just equal to its initial value before decreasing. The temporal evolution shown in Fig. 1 for the moderate shear case $\eta_0 = 6$, which corresponds to the equilibrium value for η in turbulent homogeneous shear flow, contrasts with the exponential energetic growth (Tavoularis 1985) characteristic of the turbulent case. Here the disturbance kinetic energy grows to a little beyond three times its initial value. Therefore, if the initial disturbance kinetic energy is considerably less than the natural energy scale νS, the linear approximation should remain valid for all times. Calculations have also been carried out for $\eta_0 = 12$ and 18. The evolution of K with time for these other η_0 values (not shown) is very similar to that for $\eta_0 = 6$. The only difference is that, as expected, the maximum value of K reached before the final decay is larger than for $\eta_0 = 6$; for $\eta_0 = 12$, $K_{max} = 6.95K_0$, and for $\eta_0 = 18$, $K_{max} = 10.82K_0$. The initial value of $\eta_0 = 18$ is a high-shear case which is very close to the value used in the DNS study of Lee et al. (1990) ($\eta_0 \approx 17$). Using the numerical simulation results, Lee et al. also compared with RDT and found very good agreement over the time interval examined ($0 \leq t \leq 12$). The high-shear limiting value $\eta_0 \to \infty$ corresponds to the case traditionally considered in RDT calculations (Rogers 1991), where viscosity is neglected. The absence of viscosity allows the energy to grow monotonically, rendering at some finite time the linear approximation invalid.

Figure 2 shows qualitatively similar trends for the disturbance dissipation rate $\varepsilon(t)$ at the corresponding values of $\eta_0 = 0.6$ and 6.0. For the small initial value $\eta_0 = 0.6$, the dissipation rate also decays monotonically. At the moderate shear rate value $\eta_0 = 6$, $\varepsilon(t)$ grows after a brief initial period of decay, reaches a peak and then decays at a more rapid rate than the disturbance kinetic energy. Similar behavior was found for the higher shear cases of $\eta_0 = 12$ and 18.

Since the main focus here is on disturbances which can eventually grow (and lead to turbulence), the results for the initial value of $\eta_0 = 0.6$ are not of interest. In addition, while this weak shear case yields decaying disturbances, the validity of neglecting the nonlinear terms does come into question. As Cambon and Scott (1999) have pointed out, the contribution of the nonlinear term, while small, may have a cumulative effect on the dynamics over a period of time. In addition, the linearized, weak shear case may be inconsistent because the product term containing the weak mean shear is retained while the nonlinear terms, possibly of the same magnitude, are omitted. These considerations dictate that subsequent results will focus on the moderate- and high-shear cases.

Fig. 3 shows the mapping of the anisotropy invariants $II(= -b_{ij}b_{ij}/2)$ and $III(= b_{ik}b_{kl}b_{lj}/3)$, where $b_{ij}(t) = \tau_{ij}(t)/(2K(t)) - \delta_{ij}/3$ are the stress anisotropies. Recall that initially, an isotropic distribution of the disturbance energy is assumed. The figure shows that after $t \approx 2$, the (realizable) disturbance field migrates toward the two-component (2C) state. After $t \approx 10$ the disturbance field is 2C and monotonically evolves towards a one-component (1C) state. Thus, the structure of the disturbance field is significantly altered by the imposition of the mean shear. Also shown in Fig. 3 is the invariant map trajectory of the DNS results of Lee et al. (1990). As can be seen, at early times ($t \lesssim 2$) both the DNS trajectory and that from the present calculations, with $\eta_0 = 6$ and 18, are in phase, and at later times, even though the DNS evolution lags behind the the current results, all of these evolve toward the same 1C state. While the results shown in Fig. 3 are for the $\eta_0 = 6$ and 18 cases, the same trend toward a 1C limit was found to hold for the case of $\eta_0 = 0.6$. These disturbance field results are in sharp contrast to the equilibrium state reached for the turbulent case where the fixed point invariant values are $III_\infty \approx 0.0043$ and $II_\infty \approx 0.064$.

Of particular importance for turbulence modeling are the closure coefficients $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$. Figure 4 shows the evolution of these disturbance dissipation rate coefficients. Results for three initial values of η_0 are shown which suggest that a limiting range of values at large times (and large η_0) can be reached. For $C_{\varepsilon 1}$ there is a modest change in the value at $t = 50$ with changes in η_0: for $\eta_0 = 6$, $C_{\varepsilon 1} = 2.23$, for $\eta_0 = 12$, $C_{\varepsilon 1} = 2.08$, and for $\eta_0 = 18$, $C_{\varepsilon 1} = 2.01$. For $C_{\varepsilon 2}$, the values range about $\pm 3\%$ from a mean of 2.46. Although it was not possible to determine analytically the limiting values for $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$, numerical simulations for large values of η_0 at large times showed that the limiting values were $C_{\varepsilon 1} \approx 2.0$ and $C_{\varepsilon 2} \approx 2.5$. These values are in contrast to the usual values obtained for turbulent closure models where $C_{\varepsilon 1} \approx 1.45$ and $C_{\varepsilon 2}$ lies in the range of 1.83 to 1.92.

In the turbulent case, the destruction-of-dissipation coefficient was deduced from an analysis of the decay of isotropic turbulence (e.g. Comte-Bellot and Corrsin 1971). In the analysis of the decay of homogeneous, isotropic linear disturbances, Thacker et al. (1999) determined a value for $C_{\varepsilon 2}$ of 1.4 in a boundary-free case. This is in sharp contrast to the value of 2.5 found in this homogeneous shear flow. An explanation for this difference may lie in the structure of the disturbance field. In the previous decay studies, the disturbance field was isotropic, and remained isotropic in the absence of mean shear. However, as the results shown here indicate, the imposition of mean shear quickly produces an anisotropic field (see Fig. 3) which eventually drives the flow close to a 1C state (Fig. 3). Thus, a more relevant flow with which to form a comparison may be the decay of anisotropic turbulence. Dakos and Gibson (1990) studied such a flow and deduced from the decay of the turbulence the destruction-of-dissipation rate coefficient $C_{\varepsilon 2}$. Their anisotropic decay data yielded a value for $C_{\varepsilon 2}$ of 2.18 which is relatively close to the value of 2.5 found here for $C_{\varepsilon 2}$. Of course, in spite of fundamental differences in the dynamics associated with each study, it is interesting to note that the introduction of anisotropy in either disturbance field (linear or turbulent) significantly increases the value of the coefficient $C_{\varepsilon 2}$.

CONCLUSIONS

Closure models for the production-of-dissipation rate and destruction-of-dissipation rate in a linearized disturbance transport equation for the dissipation rate have been developed. The study has extended the approach of Thacker et al. (1999) to mean homogeneous shear flow in which deterministic solutions of the linearized Navier-Stokes equation are combined with a probability distribution that accounts for the uncertainty in initial conditions, to obtain mean transport equations for disturbance correlations in the laminar regime.

The temporal evolution of the disturbance kinetic energy and the disturbance dissipation rate were shown to depend on the magnitude of the initial value of the time scale ratio η. The temporal evolution for both the kinetic energy and dissipation rate was monotonically decaying for values of $\eta_0 < 1.2$. For larger values of η_0, both quantities initially decayed, then grew with time, and then subsequently decayed at large times. This is in contrast to the turbulent, homogeneous shear case where the quantities displayed exponential growth. Nevertheless, it was possible to find approximate values for the dissipation rate coefficients $C_{\varepsilon 1} \approx 2.0$ and $C_{\varepsilon 2} \approx 2.5$, which were considerably larger than the corresponding turbulent values.

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References

Cambon, C., and Scott, J. F., 1999, "Linear and Nonlinear Models of Anisotropic Turbulence," *Annu. Rev. Fluid Mech.*, Vol. 31, pp. 1–53.

Comte-Bellot, G., and Corrsin, S., 1971, "Simple Eulerian Time Correlation of Full- and Narrow-Band Velocity Signals in Grid-Generated, 'Isotropic' Turbulence," *J. Fluid Mech.*, Vol. 48, pp. 273–337.

Dakos, T., and Gibson, M. M., 1990, "The Decay of Anisotropic Homogeneous Turbulence," In *Engineering Turbulence: Modelling and Experiments*, (W. Rodi and E. N. Ganic eds.), Elsevier, Amsterdam, pp. 73–81.

Jongen, T., and Gatski, T. B., 1998, "A New Approach to Characterizing the Equilibrium States of the Reynolds Stress Anisotropy in Homogeneous Turbulence," *Theoret. Comput. Fluid Dynamics*, Vol. 11, pp. 31–47. Erratum: *Theoret. Comput. Fluid Dynamics*, Vol. 12, pp. 71–72.

Lee, M. J., Kim, J., and Moin, P., 1990, "Structure of Turbulence at High Shear Rate," *J. Fluid Mech.*, Vol. 216, pp. 561–583.

Rogallo, R. S., 1984, "Numerical Experiments in Homogeneous Turbulence," *NASA Technical Memorandum 81315*.

Rogers, M. M., 1991, "The Structure of a Passive Scalar Field with a Uniform Mean Gradient in Rapidly Sheared Homogeneous Turbulent Flow" *Phys. Fluids A*, Vol. 3, pp. 144–154.

Salhi, A., Cambon, C., and Speziale, C. G., 1997, "Linear Stability Analysis of Plane Quadratic Flows in a Rotating Frame with Applications to Modeling," *Phys. Fluids*, Vol. 9, pp. 2300–2309.

Speziale, C. G., Abid, R., and Blaisdell, G. A., 1996, "On the Consistency of Reynolds Stress Turbulence Closures with Hydrodynamic Stability Theory," *Phys. Fluids*, Vol. 8, pp. 781–788.

Tavoularis, S., 1985, "Asymptotic Laws for Transversely Homogeneous Turbulent Shear Flows," *Phys. Fluids*, Vol. 28, pp. 999–1001.

Thacker, W. D., Gatski, T. B, and Grosch, C. E., 1999, "Analyzing Mean Transport Equations of Turbulence and Linear Disturbances in Decaying Flows," *Phys. Fluids*, Vol. 11, pp. 2626–2631.

Townsend, A. A., 1970, "Entrainment and the Structure of Turbulent Flow," *J. Fluid Mech.*, Vol. 41, pp. 13–46.

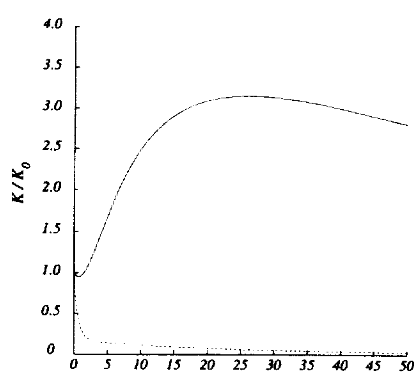

Figure 1. Temporal evolution of disturbance kinetic energy K for initial conditions: - - - -, $\eta_0 = 0.6$; ———, $\eta_0 = 6$.

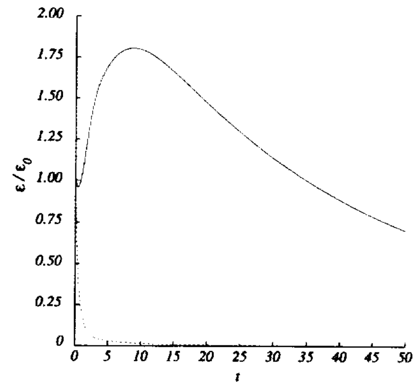

Figure 2. Temporal evolution of disturbance dissipation rate ε for initial conditions: - - - -, $\eta_0 = 0.6$; ———, $\eta_0 = 6$.

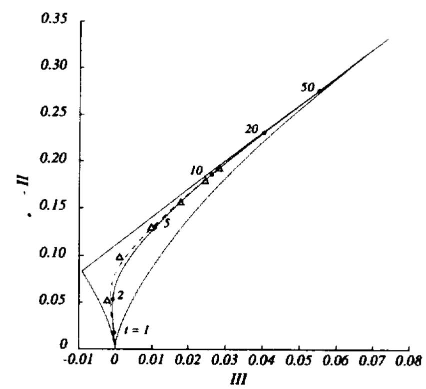

Figure 3. Invariant mapping of disturbance anisotropy tensor at various times for initial conditions: —•—, $\eta_0 = 6$ (times as labeled); - - - -, $\eta_0 = 18$; $\triangle$, $\eta_0 \approx 17$ from DNS of Lee et al. (1990) at $t = 2, 4, 6, 8, 10, 12$.

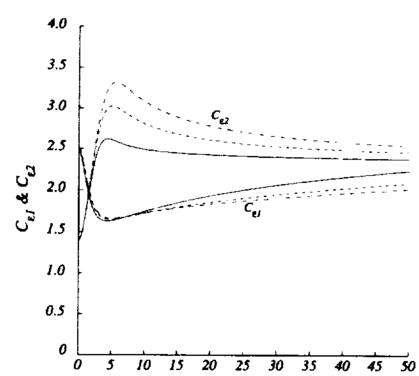

Figure 4. Temporal evolution of closure coefficients $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ for initial conditions: ———, $\eta_0 = 6$; - - - - -, $\eta_0 = 12$; – – – –, $\eta_0 = 18$.