Precision Interval Estimation of the Response Surface by means of An Integrated Algorithm of Neural Network and Linear Regression

Final Report

May--September, 1999

Ching F. Lo, Professor
and
J. L. Zhao, Graduate Research Assistant
Mechanical, Aerospace and Engineering Science
University of Tennessee Space Institute
Tullahoma, TN 37388

Under
Grant NAG-1-2185
NASA/Langley Research Center
Hampton, VA 23681

September, 1999

Preface

The work presented herein was conducted by the University of Tennessee Space Institute, Tullahoma, TN for the NASA/Langley Research Center Grant NAG-1-2185. Mr. Richard DeLoach is the technical Officer. The TST Alpha Jet Model data was provided by Mr. William L. Sickles, Sverdrup Technology Inc./AEDC Group. The results of the research were obtained under the direction of the principal investigator Dr. Ching F Lo, Professor of Mechanical, Aerospace and Engineering Science. The other contributor Mr. Jun-Long Zhao, Graduate Research Assistant, was supported by UTSI Sponsored Research Program for Graduate Student. To Mr. Deloach, we acknowledge his technical discussion and support. To Mr. Sickles, we appreciate his effort to provide the TST Alpha Jet Model data. The research was performed from May to September 1999 and the manuscript of the final report was submitted on September 30, 1999.

TABLE OF CONTENTS

PAGE NO.

Precision Interval Estimation of the Response Surface by means of An Integrated Algorithm of Neural Network and Linear Regression[&]

Ching F. Lo[%] and J. L. Zhao[#]
University of Tennessee Space Institute
Tullahoma, TN 37388

Abstract

The integration of Radial Basis Function Networks and Back Propagation Neural Networks with the Multiple Linear Regression has been accomplished to map nonlinear response surfaces over a wide range of independent variables in the process of the Modern Design of Experiments. The integrated method is capable to estimate the precision intervals including confidence and predicted intervals. The power of the innovative method has been demonstrated by applying to a set of wind tunnel test data in construction of response surface and estimation of precision interval.

& This work was supported by NASA/Langley Research Center Grant: NAG-1-2185 with Technical Officer, Mr. Richard DeLoach

% Professor of Mechanical, Aerospace and Engineering Science

Graduate Research Assistant, Department of MAES

1. Introduction

The integration of Modern Design of Experiments (MDOE) and Neural Networks methodology should improve the capability the current form of MDOE. The application of the enhanced MDOE to aerospace ground testing could increase the productivity by reducing the test time and increasing data precision. Furthermore, the wind tunnel flow characteristics may be comprehended by analyzing and modeling the interpolation data using the newly innovative approach. The effort to increase productivity of ground test clearly should be benefit to NASA as well as the whole aerospace technical community.

The application of Modern Design of Experiments methodology to aerospace wind tunnel testing has been initiated at NASA/Langley Research Center and other organizations in the past years. The methodology comparing with the conventional "One Factor at A Time" test has the potential to save wind tunnel test time and to increase precision by eliminating "block effects" from the unknown variance in the test data. The method could also determine factor interaction effects that provide the fundamental model of the test article flow phenomena.

There are many ways to select test data points based on available experimental designs. Most of Response Surfaces are constructed by utilizing a polynomial model of a given order function. This type of model are well suited for a finite range of the region of the independent variables, and in complex situations that can occur in certain aerodynamic testing, this may not cover the entire range of interest. The construction of piecewise-continuous response surfaces is necessary over contiguous truncated inference subspaces. The neural networks may overcome this limitation to cover several variables in the broad ranges.

In the present report, we have replaced the polynomial model by a Neural Network in constructing the Response Surface. The range of test design points could be enlarged to cover most test variables required range since the neural networks are capable to map a highly nonlinear hyper-surface. The identification of underlying model could also be studied from the response surface. In addition to data interpolation performed by the neural networks, the analysis of derivative functions, variable sensitivity and interaction effects could be investigated even though we have not included in the present effort.

The widely used algorithms of neural networks in mapping function or constructions of response surfaces are the "Back Propagation" and "Radial Basis Function Networks". Since the model of neural networks is **nonlinear** in nature, the precision confidence interval analysis is not feasible by adapting the linear regression approach based on the statistical theorem. But the importance of the precision interval of a constructed response surface is recognized for the MDOE application.

Without the capability to estimate confidence or prediction intervals, the neural network is unable to provide the fitting goodness characteristics or the imperfection in the model. The confidence interval is the way to identify the systematical errors in the model or the model adequacy. Furthermore, the prediction confidence interval is able to estimate or forecast the uncertainty of response surfaces of the future observations. This is particular important to know the expected uncertainty since the data are not even available in a region of interest. Therefore, the major effort of this grant has been devoted to construct a special type of neural networks to be able to compute the precision intervals of response surfaces.

Two neural network algorithms, which are capable to modeling response surfaces, have been selected to integrate with the multiple linear regression to compute the precision intervals. In Sections 2-4, the "radial basis function networks" is introduced to compute the precision intervals. An example of application to modeling force data of the Alpha Jet Model in various angles of attack is given. In Sections 5-6, the integration of back propagation neural networks and multiple linear regression has been constructed to compute the precision confidence interval. The integrated method has been applied to tunnel force data of the Alpha Jet Model with three typical variables, Mach number, Reynolds Number and Angle of attack, which consists of 25 test configurations conducted in the National Transonic Facility at NASA/Langley Research Center. Each test configurations ranges 10 to 22 points of angles of attack. The total number of test points is over four hundred sets of data. The integrated method is able to mapping all force data into a single neural network. The precision confidence intervals, which are associated this neural network, are computed for the Alpha Jet Model data.

The resulting response surfaces and precision intervals obtained by these two algorithms are quite satisfactory. These methods are ready to be utilized to other design of test data. Specially, the integrated algorithm is more powerful to construct response surfaces of a lager sets of data.

2. The Radial Basis Function Networks

The Radial Basis Function Network (RBFN) with N inputs and a scalar output, which is depicted in Figure 1, can be expressed for a function approximation as

$$F(x_j, w_i) = \sum_{i=1}^{k} w_i \, \phi_i \left(\sum_{j=1}^{N} (x_j - c_{ij})^2 \right) + w_o \qquad (1)$$

Where x_j 's are the inputs, ϕ_i 's are the given basis function and w_i 's are the weights. The Gaussian function is chosen as the Basis Function as shown in Figure 1. The Gaussian Function with a radial-basis function argument that is used to form a network is called Radial Basis Function Networks (RBFN). The Gaussian function of the input variable x_j 's is of the form

$$\phi_i(r_i^2) = e^{-\frac{r_i^2}{2\sigma_i^2}} \qquad (2)$$

Where

$$r_i^2 = \sum_{j=1}^{N} (x_j - c_{ij})^2 \qquad (3)$$

c_{ij} 's are the centers or mean values and σ_i the standard deviation of a normal distribution function of statistics.

By specifying a set of inputs, x_j 's and the corresponding desired output F, the values of the weights w_i 's can be determined using the linear **Least Squares Method** (LSM).

The above-described RBFN is a special case of Multiple Linear Regression models. The F is the desired output and is called as the Response. The ϕ_i is known as **regressors** which are a specified function of inputs x_j 's.

The pattern unit (or regressors) in a RBF Network consists of center, c_{ij} and deviation, σ_i for each Gaussian function. A clustering algorithm is applied by Moody & Darken (Ref. 1) to determine the value of centers and a nearest neighbor heuristic to determine the deviation, σ_i. The **Linear Regression** or a gradient descent algorithm evaluates the weights of the output function. The linear regression will be used in the present application. Therefore the confidence interval and predictive confidence interval can be determined by the available statistical method for this radial basis function network.

The description given herein has only a single (scalar) output for notational simplicity. There is no limitation of number of outputs. To extend the multiple outputs, another sets of weights should be introduced for additional desired outputs.

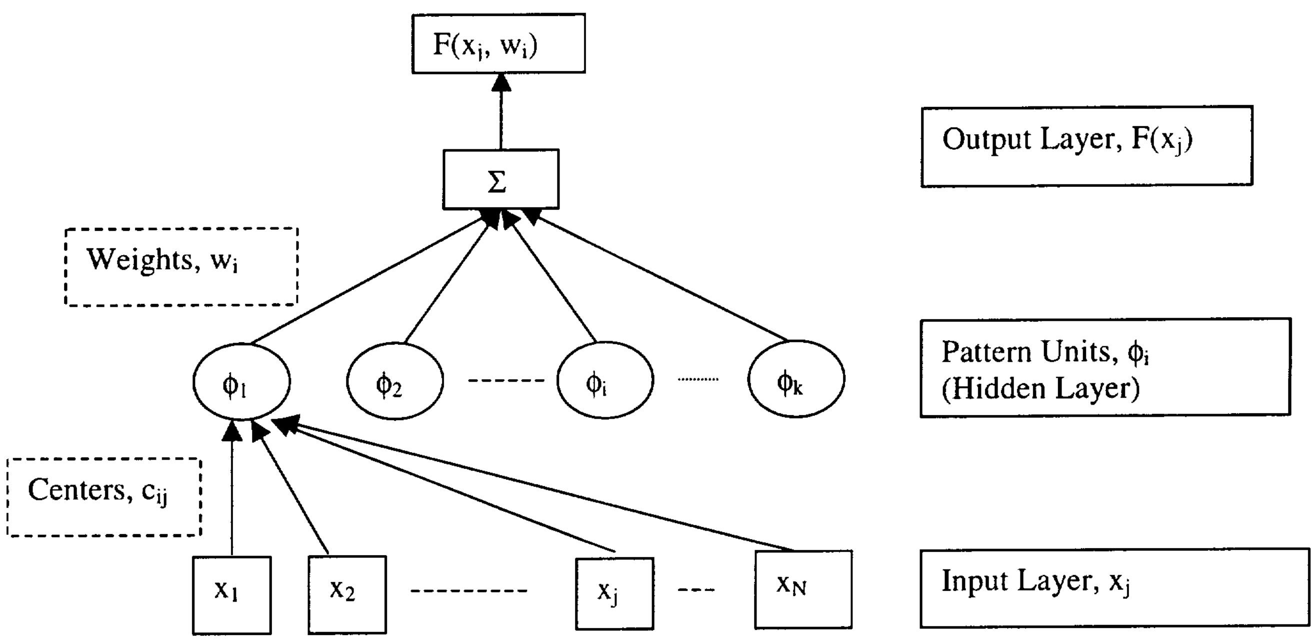


Figure 1. Structure of a Radial Basis Function Network

3. Implementation of RBFN and Computation of Precision Intervals

The RBFN is constructed in a commercial Neural Network software package--NeuralWork Professional II published by Neural Ware, Inc. (Ref. 2). The algorithm is based on Moody and Darken paper (Ref. 1). As the trained RBNF is accepted, the values of the regressors are determined for specified inputs and the desired output. Then the trained RBFN are converted to a C-language code. With these sets of inputs and corresponding output (observations), the Precision Intervals for mean response can be obtained by the linear regression analysis. The Confidence Interval and Prediction Interval are able to compute that are based on the standard formulae given in the linear regression references (e.g. Ref. 3). The formulae are listed as follows:

A 100(1-α) percent half-width of **confidence interval (CIHW)** on the mean response at a particular point $\phi_{11}, \phi_{12}, \phi_{13}, \ldots\ldots\ldots \phi_{1k}$ is expressed as

$$t_{\alpha/2,n-p}\sigma \sqrt{x_1'(X'X)^{-1}x_1} \tag{4}$$

Where $t_{\alpha/2,n-p}$ is t-distribution quantiles,
σ^2 is sum of squares residual/degree of freedom
and

$$x_1' = [1, \phi_{11}, \phi_{12}, \ldots\ldots \phi_{1k}]$$

$$X' = [x_1, x_2, \ldots\ldots\ldots x_n]$$

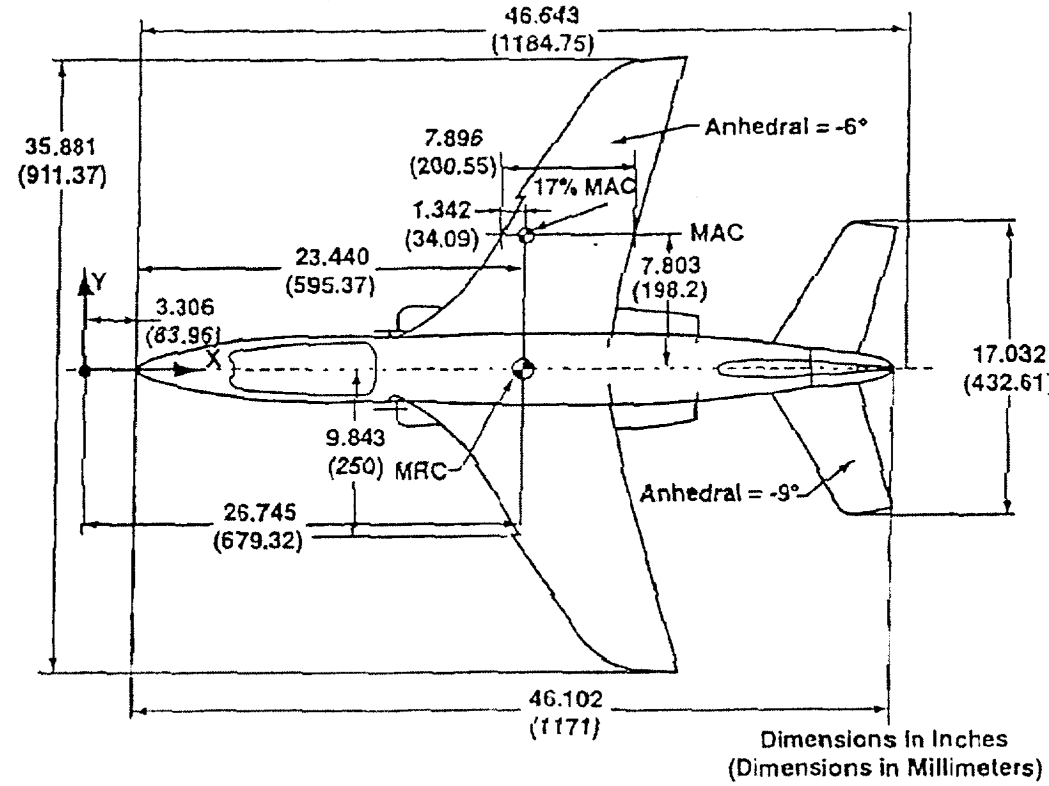
A 100(1-α) percent half-width of **prediction interval (PIHW)** for the future observation is of the form

$$t_{\alpha/2,n-p}\sigma \sqrt{1 + x_1'(X'X)^{-1}x_1} \tag{5}$$

These formulae are coded C-language and combined with the C code trained RBFN produced by NeuralWork.

4. RBFN Application to Tunnel Data of The Alpha-Jet model

An illustrated example is utilized a set of force coefficients obtained on the TST Alpha-Jet model from the AEDC Tunnel 16T as shown in Figure 2 published in AIAA paper No. 98-2878 (Ref. 4). The force coefficients are taken at Mach Number 0.8 and Chord Reynolds Number 1.5 millions under transition-free configuration in the present application. The angle of attack ranges from -4 to 10 degree.



TST model dimensions, top view.

Figure 2. Transonic Technology Wing (TST Alpha Jet) Model (from Ref 4.: Laster, M.L., Stanewsky, E, Sinclair, D.W. and Sickles, W. L. "Reynolds Number Scaling at Transonic Speeds," AIAA paper 98-2878, 1998)

These force data including lift, drag and pitching moment coefficients are modeled by the RBFN. The results obtained from the RBFN and the original data are plotted in Figure 3. The comparison of RBFN results and tunnel data is within the accuracy of tunnel measurement. The results of 95% Confidence Interval Half Width on the response surface from Eq. (4) are shown in Figure 4 as the error band by the Coef-HI (or -UPPER) and Coef-LO (-LOWER). The results are satisfactory as expected. The 95% CIHW and Coef-Residual, which is defined as

$$\text{Coef-Residual} = \text{Coef-RFBN} - \text{Coef-DATA}$$

are plotted in Figure 5 as well as tabulated in Table 1 in Appendix B. The prediction interval was not computed in this example since we do not have sufficient data to reserve as the test file. The prediction interval formula will be applied in the later example.

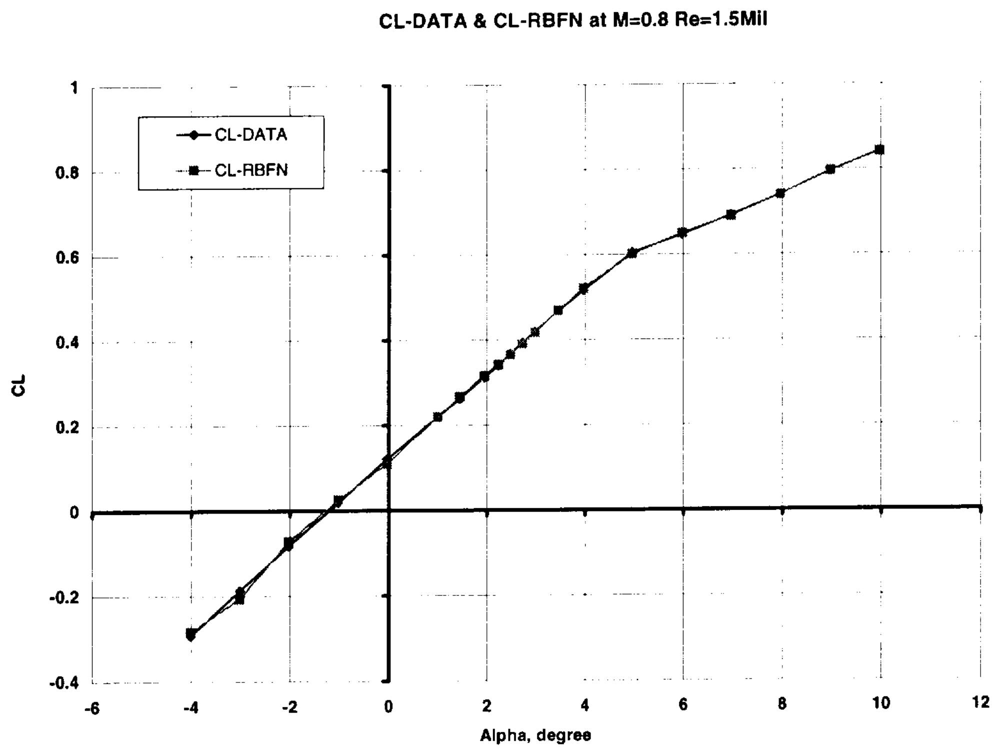


Figure3(a) Lift Coefficient, CL

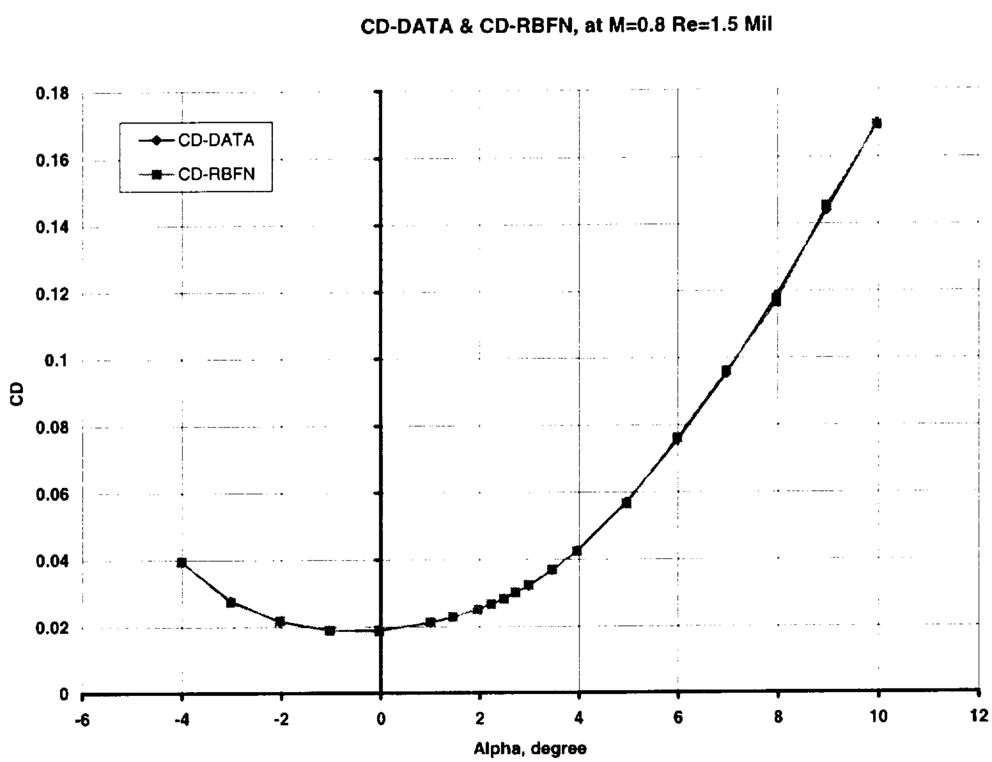


Figure3(b). Drag Coefficient, CD

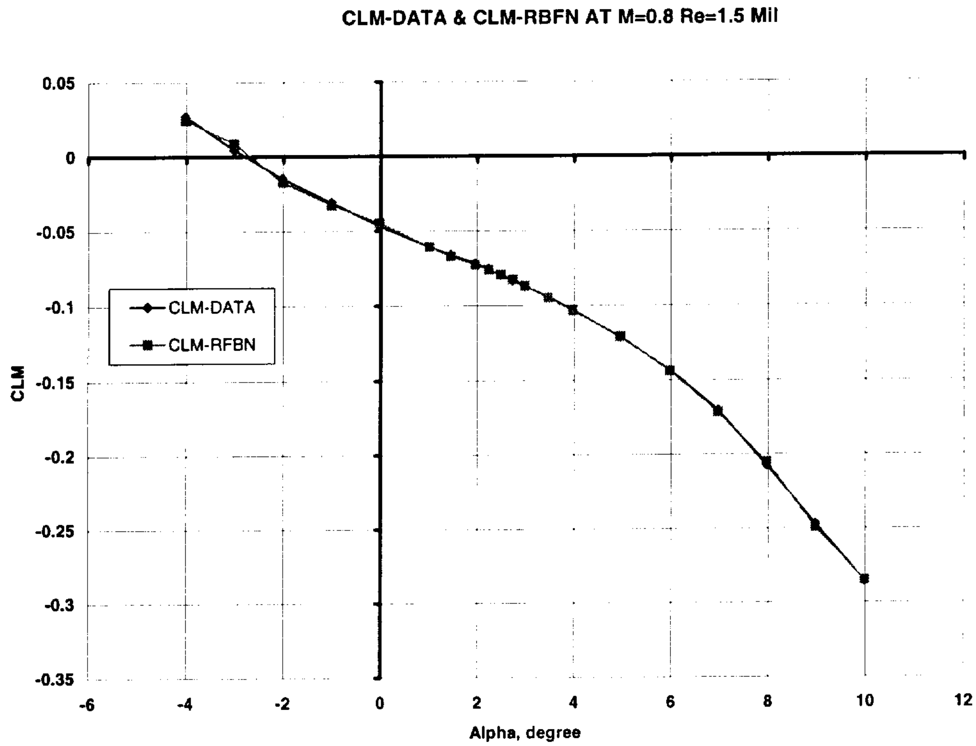
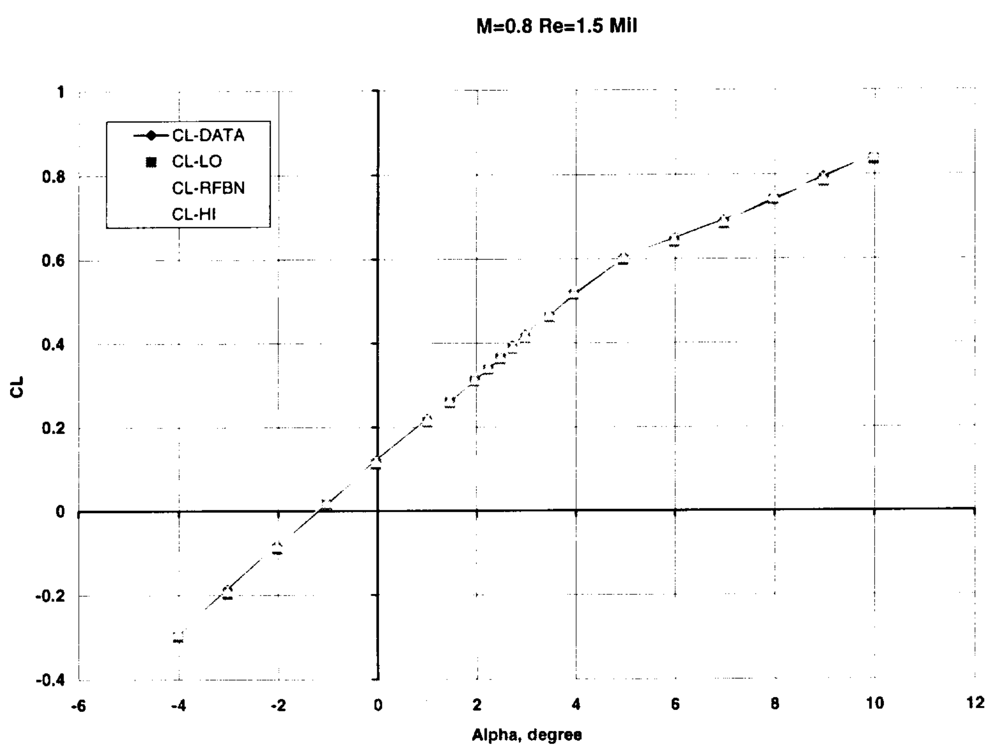


Figure3(c). Pitching Moment Coefficient, CLM
Figure. 3. Comparison of RBFN Results of Force Coefficients with Tunnel Data at M=0.8 and Re=1.5 Million
Figure4(a) Lift Coefficient CL



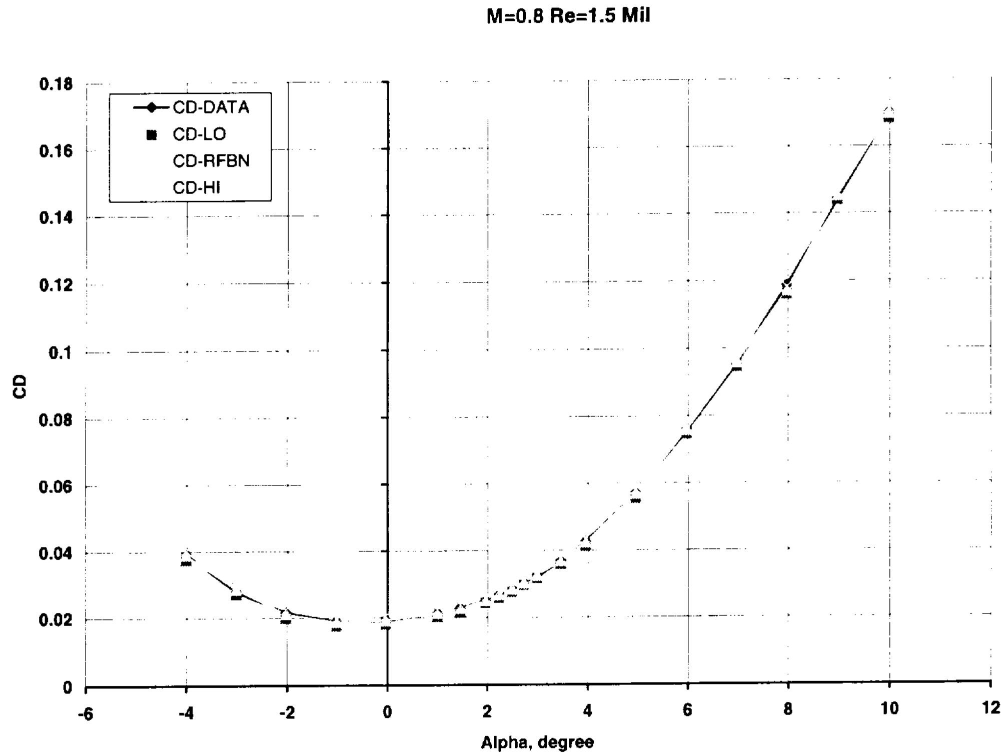


Figure4(b). Drag Coefficient, CD

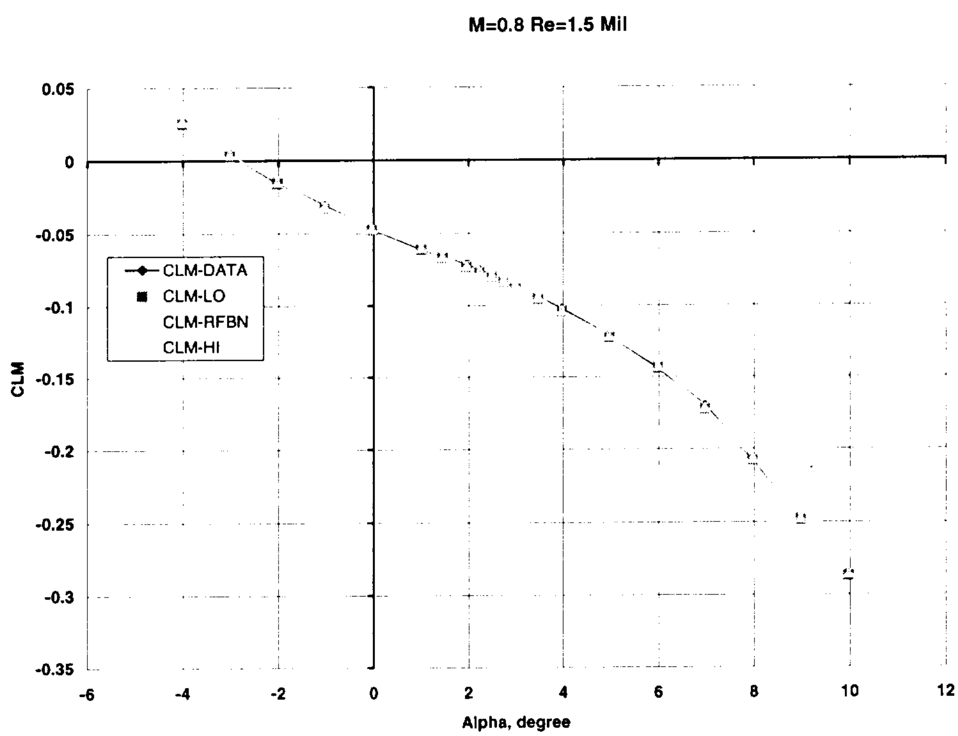


Figure4(c). Pitching Moment Coefficient, CLM

Figure 4. Confidence Intervals with RBFN and DATA at M=0.8 and Re=1.5 Million.

Note: The Coef-HI and Coef-LO are indicated the range of the 95% Confidence intervals.

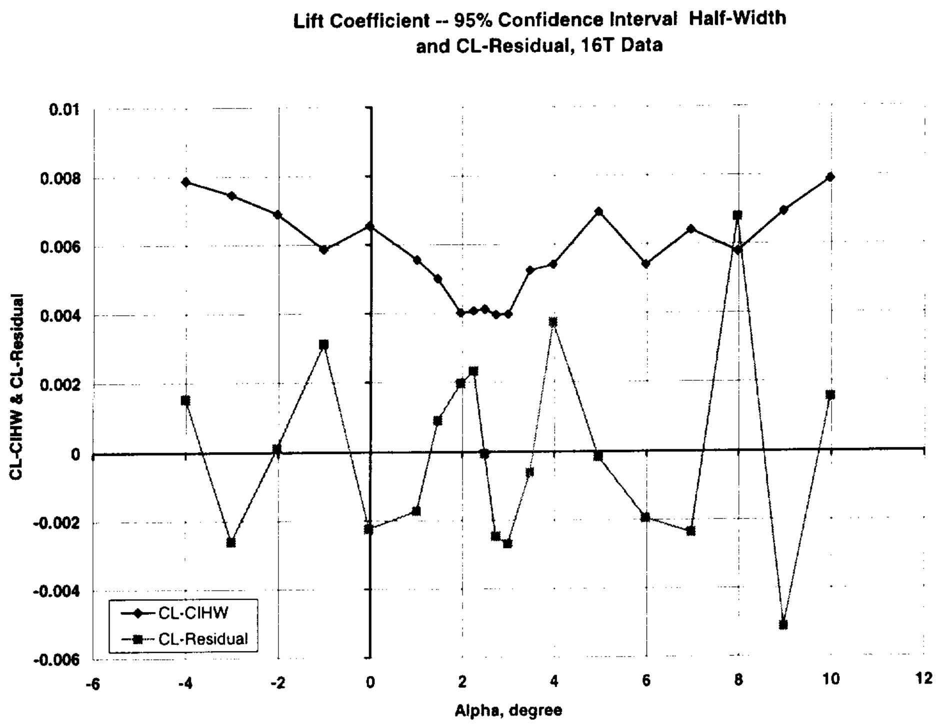


Figure 5(a). Lift Coefficient, RMS(CL-Residual)=0.0027

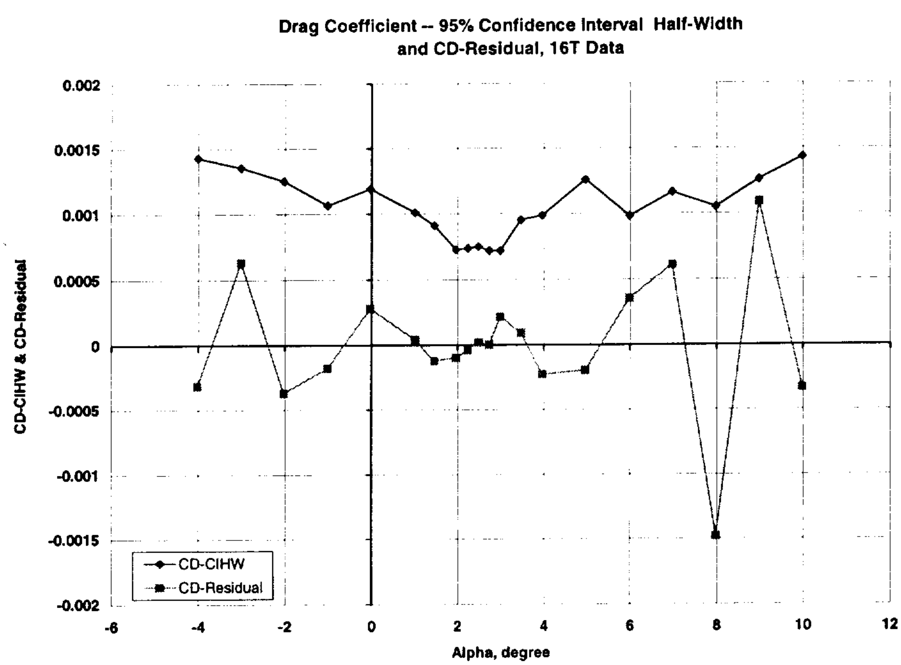


Figure 5(b). Drag Coefficient, RMS(CD-Residual)=0.0005

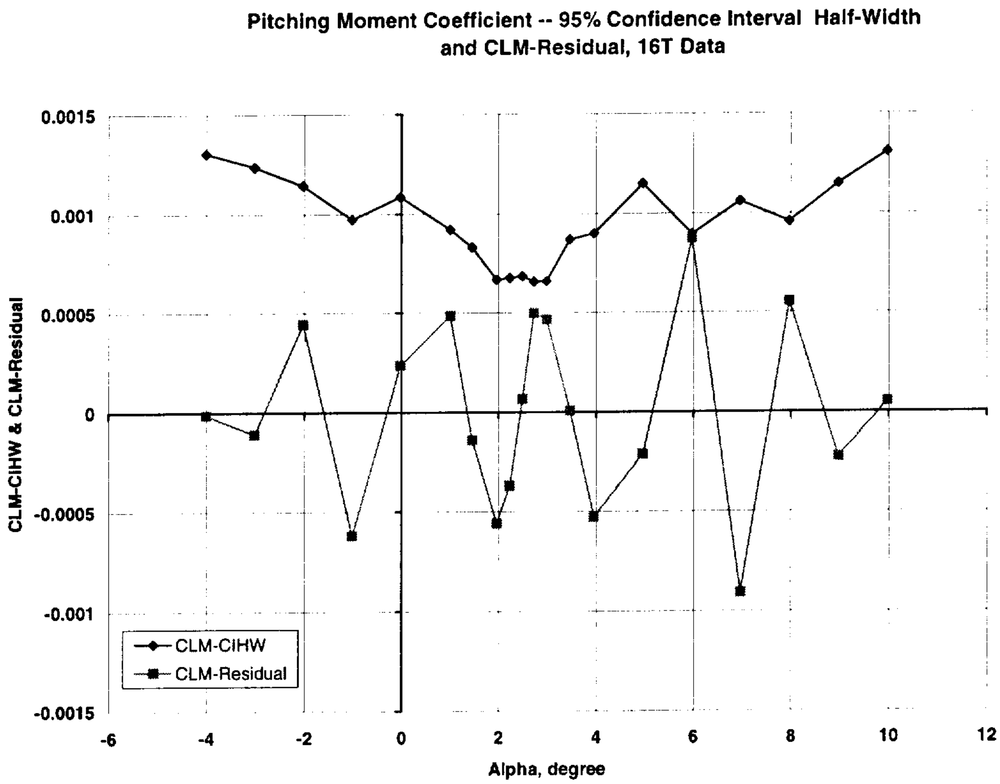


Figure 5(c). Pitching Moment Coefficient, RMS (CLM-Residual)=0.0005

Figure 5 The 95% Confidence Interval Half-Width and RBFN Coef-Residuals.

Data Uncertainty for Tunnel 16T ΔCL=0.0048, ΔCD=0.0009 & ΔCLM=0.0025.
See TABLE 1 in Appendix B.

5. Integration of Back Propagation Neural Network and Multiple Linear Regression

Although the RBFN is capable to map multiple variable function with the rapid training process, the majority of function mapping has been carried out by Back Propagation (BP) Neural Networks. It is well known that the BP Neural Network has the powerful capability of function mapping to model the nonlinear response surface in a large numbers of parameters in a wide range. Thus the determination of the precision intervals for the BP nets is also necessary in the function mapping of neural networks application.

The typical back propagation network has an input layer, an output layer and one or more hidden layers. The network relationship is a non-linear function. The analysis of confidence and prediction intervals based on the statistical method is not available. The concept of integration of the linear regression method into the **last hidden layer** of the back propagation network (i.e. the hidden layer just before the output layer) is enable to map the response surface but also to evaluate the confidence and prediction intervals of the response surface. This is a special case of linear regression model as named in Ref. 5. The integration of the process is described as follows.

The first step is to train a selected design of a back-propagation neural network for the desired response surface with its inputs. After the network is satisfactorily trained, the linear regression will be incorporated in the trained BP nets. The processing elements of the last hidden in the nets will be treated as **regressor variables** in a multiple linear regression model. With the trained weights of the BP nets, each observation will provide the value of regressor variables, ϕ_i, and the response, F, known as the desired output for each input data set as

$$F = \sum_{i=1}^{K} w_i \phi_i + w_0 + \varepsilon \qquad (6)$$

Where the ϕ_i 's are **regressor** variables functions of the input layer data and all weights in the trained Back Propagation (BP) Neural Network. The w_i 's are known as the **regression coefficients**. The error (or residual) of the regression is ε. The input variables in the cases of wind tunnel data include Mach number, Reynolds number, Angle of Attack and so on. The outputs, F, are force coefficients, e.g. lift, drag and pitching moment, for model force data.

In addition, a modified Back Propagation Net can be enhanced by introducing functions in terms of input variables linearly independent. The modified net, which is called as Functional-link nets (Ref 6), is to enhance the original representation of input. The additional dimensions produced by these functions may be learned more readily in the hyperplanes. These functions typically consist of outer-product and functional enhanced modes of input variables. Some of the superior qualities of Functional-link nets have been demonstrated in the supervised train net by many examples in the literature. This technique of a simple representation of the net is illustrated in Figure 6. The Functional-link Back Propagation nets have been applied in the present work to map all force coefficients of a force model in the next section.

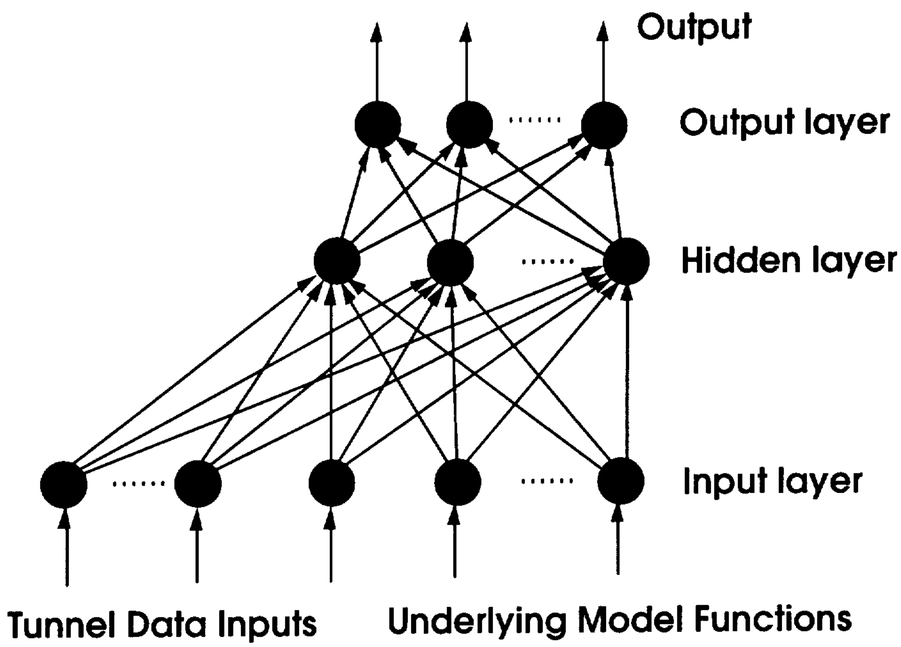


Figure 6. A Functional-Link Back Propagation Neural Network with Regular Tunnel Variable Data Inputs and Underlying Model Functions in terms of Regular Data as Additional Inputs.

6. Implementation of the Integrated Algorithm and Precision Intervals

The design of Back Propagation (B-P) Nets has been constructed in a commercial Neural Network software package--NeuralWork Professional II published by Neural Ware, Inc. The Functional-link B-P net was trained by NeuralWork package. The resulting B-P net was converted into a C-language code. The computation process of the linear regression algorithm is coded in C-language code, which is integrated with the input values computed from the last hidden layer of the trained B-P. The integrated B-P and linear regression algorithm is ready to accept the input to map the response surface.

Confidence Interval on the Mean Response. The confidence interval in multiple regression on the mean response is given in Equation (4) of Section 3. For each particular point on the surface, the formula of a confidence interval on the mean response has been programmed.

Prediction of New Response. A regression model can also predict future observations on the response surface at the specified point. The formula of a 100(1-α)% predication interval for the future observation given in Equation (5) of Section 3 is also coded to compute a given observation point.

7. Application of BP Neural Network to Tunnel Data of the Alpha Jet Model

The force data sets of the Alpha Jet Model from the NASA/Langley NTF Tunnel have been selected to apply the integration of B-P Neural Networks and Linear Regression. The Alpha Jet model is shown in Figure 2 (Ref. 4). The data range of Mach Number from 0.6 to 0.9 and Chord Reynolds Number from 2.7 to 10 millions for transition-free configurations are available for the present investigation. The angle of attack ranges from -4 to 10 degree for most test conditions.

The uniform distribution design was chosen as shown in Figure 7 with half of all database for the training cases of the neural network. The remaining half set of data are reserved for the testing cases. The force and moment coefficients plots of some typical **training** cases for Mach Numbers 0.6, 0.8 and 0.9 are shown in Figures 8-10(in Appendix A). It can be seen that the comparison of Neural Network-LSM prediction and tunnel data is good for those typical training flow conditions. The Coef-HI (-Upper) and Coef-LO (-Lower) are also plotted in Figures 8-10 for the range of the 95% confidence intervals. In the **testing** case, the prediction interval of 95% confidence is plotted as Coef-HI-P and Coef-LO-P in Figures 11-13(in Appendix A) along with Neural Network-LSM prediction and tunnel data for Mach Numbers 0.6, 0.8 and 0.9 for various Reynolds numbers. The results are satisfactory as expected. The details of the precision intervals are plotted as follows. The 95% **confidence interval** half-width and Coef-Residual at tunnel conditions M=0.6, 0.8 and 0.9 are plotted in Figures 14-16 (in Appendix A) and only M=0.8 case, as a representative example, is tabulated in TABLE 2 (in Appendix B). The 95% **prediction interval** half-width and Coef-Residual at tunnel conditions M=0.6, 0.8 and 0.9 are plotted in Figures 17-19(in Appendix A) and only M=0.8 case, as a representative example, is tabulated in TABLE 3 (in Appendix B).

Reynolds No.	Mach Number				
(million)	0.6	0.8	0.835	0.86	0.9
2.7					
3.3		Training set			
3.9		Testing set			
4.5					
10					

Figure 7. Alpha Jet Model Test Matrix in NTF Tunnel ranges for Mach number from 0.6 to 0.9 and Reynolds Number from 2.7 to 10 million. The shaded (blue) block is the selected training data sets and the white blocks are the testing data sets.

8. Concluding Remarks

The multiple linear regression has been integrated in the neural network algorithms--Radial Basis Function Network and Back Propagation Network. Both neural networks, which have nonlinear characteristics, are capable to construct the nonlinear response surfaces of data sets in the wide range of variables obtained from the Modern Design of Experiments. The confidence precision interval including confidence interval and prediction interval of the response surface is determined by the linear regression analysis. Applications of the Radial Basis Function Network and Back Propagation integrated method to the force data sets of an Alpha Jet Model have shown the satisfactory data mapping results. The innovative algorithms are ready to be applied to construct response surfaces and estimate precision intervals as part of the procedure in the Modern Design of Experiments

References

1. Moody, J. and Darken, C. J. "Fast Learning in Networks of Locally-tuned Processing Units," Neural computation 1, 281-294, 1989

2."NeuralWork Profession II/Plus," NeuralWare, Inc. Pittsburgh, PA., (http://neuralware.com/index.htm)

3. Montgomery, D.C. and Peck, E. A. "Introduction to Linear Regression Analysis," John Wiley & Sons, Inc., second edition 1992

4. Laster, M.L., Stanewsky, E, Sinclair, D.W. and Sickles, W. L. "Reynolds Number Scaling at Transonic Speeds," AIAA paper 98-2878, 1998

5. Chen, S, Cowan, C. F. N. and Grant, P. M. "Orthogonal Least Square Algorithm for Radial Basis function Networks," IEEE on Neural Networks, Vol. 2. No. 2, March 1991

6. Pao, Y. H. "Adaptive Pattern Recognition and Neural Networks," Addison -Wesley, 1989

APPENDIX A

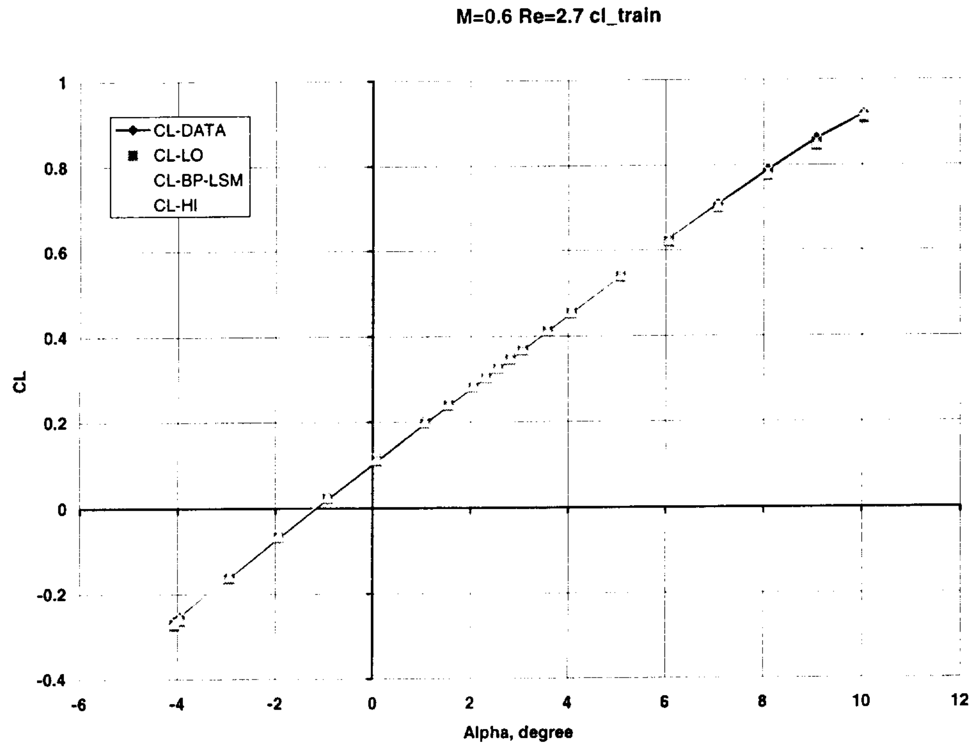


Figure 8(a). Lift Coefficient

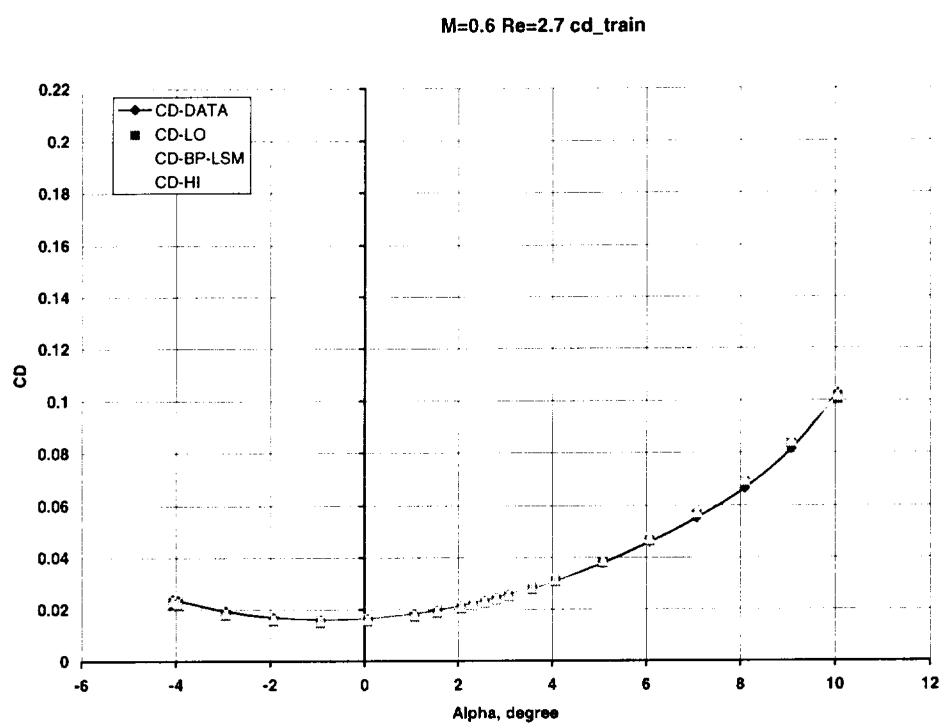


Figure 8 (b). Drag Coefficient

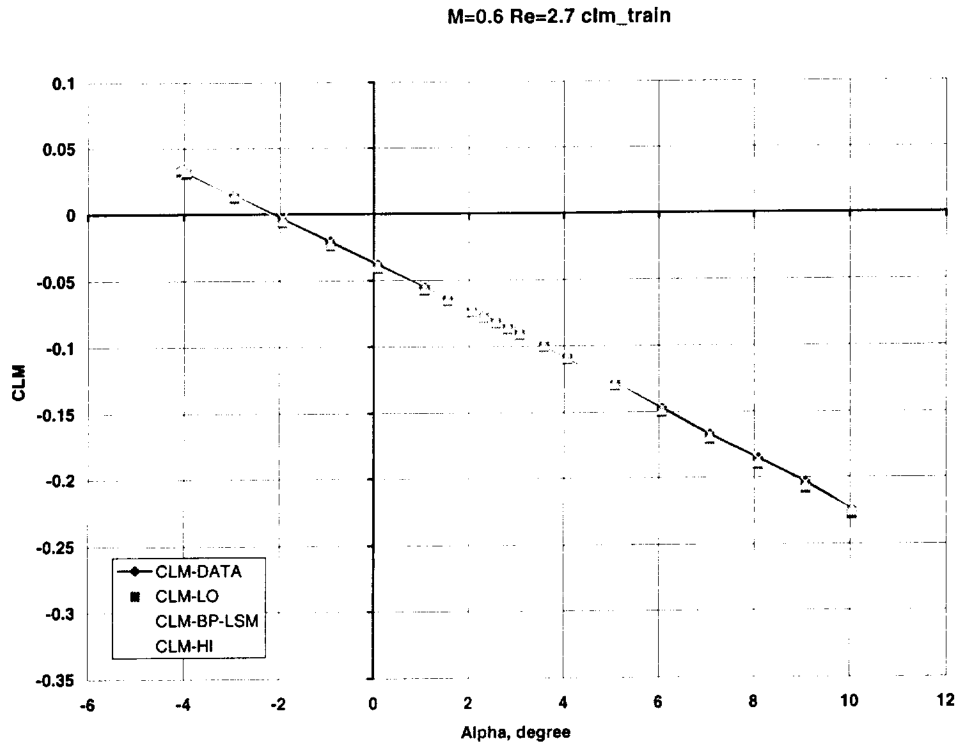


Figure 8(c). Pitching Moment Coefficient
Figure 8. Training Results: Comparison of Neural Networks-LSM Prediction and Tunnel Data at Mach 0.6 Reynolds Number 2.7 million for Force Coefficients.
Note: Coef-LO and Coef-HI are 95% Confidence Intervals for Neural Networks-LSM 's Response Surface.

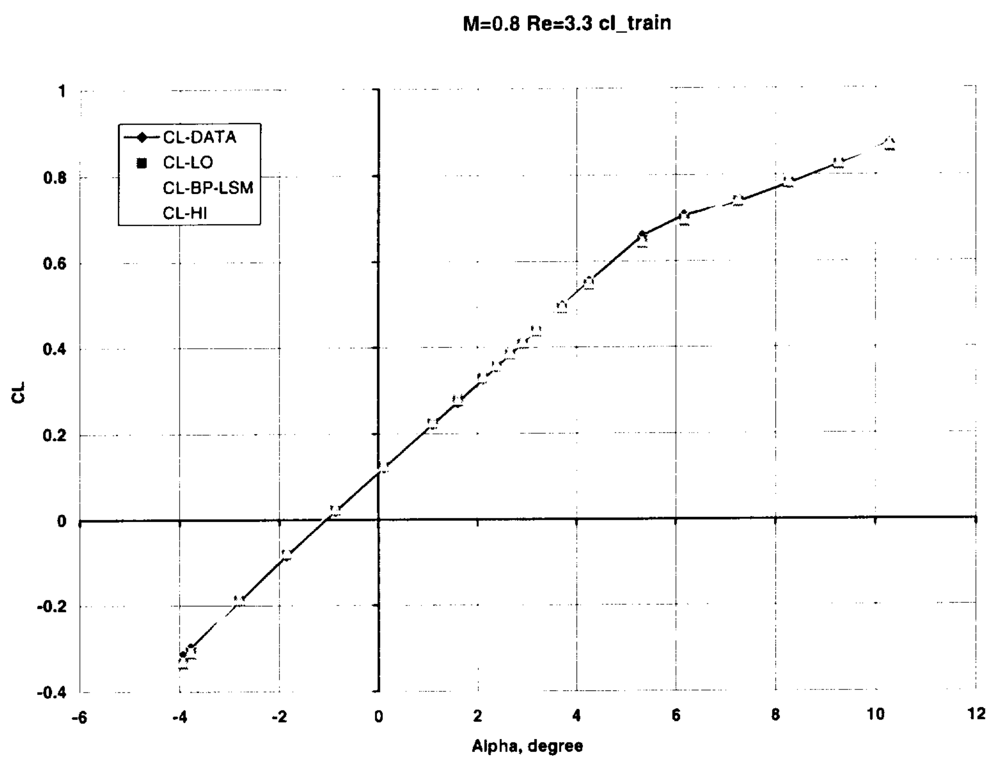


Figure 9(a). Lift Coefficient

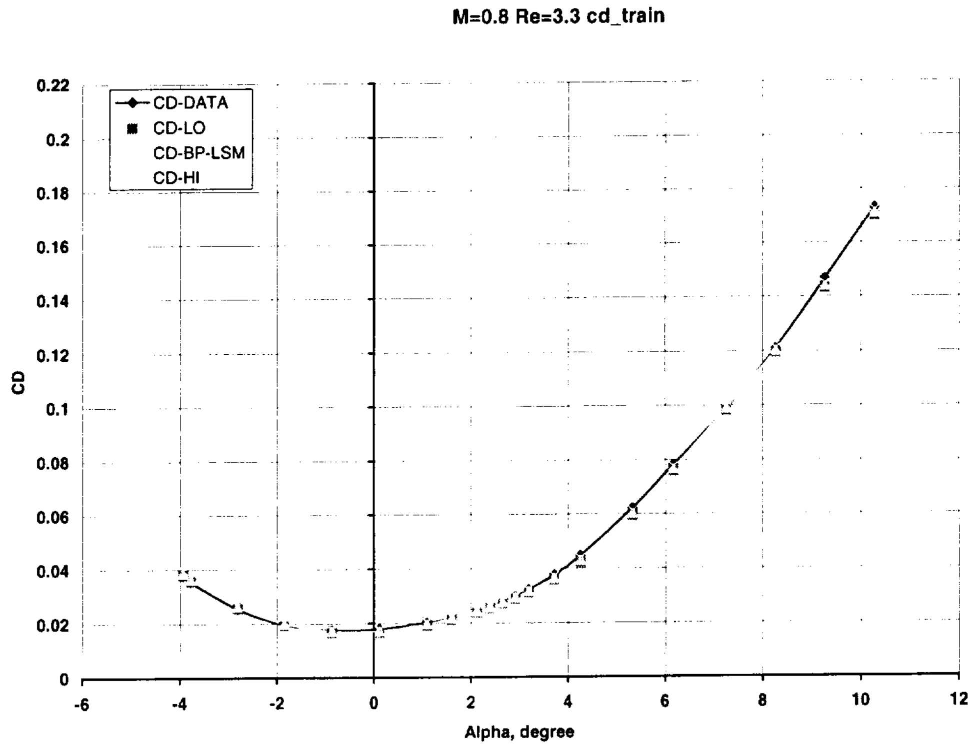


Figure 9 (b). Drag Coefficient

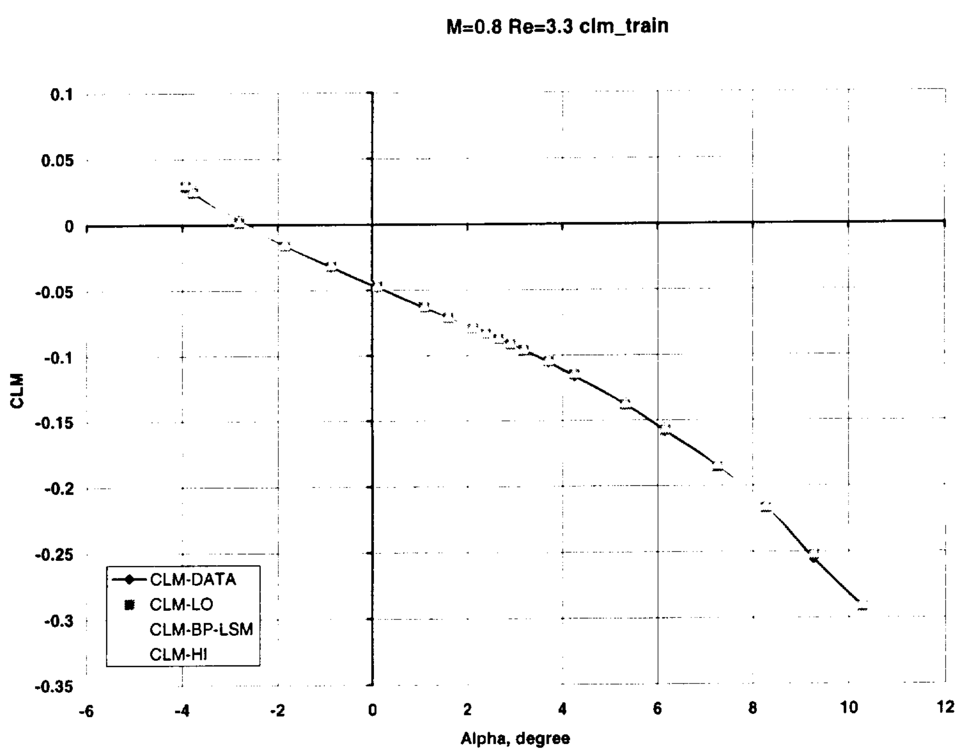


Figure 9 (c). Pitching Moment Coefficient

Figure 9. Training Results: Comparison of Neural Networks-LSM Prediction and Tunnel Data at Mach 0.8 Reynolds Number 3.3 mil for Force Coefficients.

Note: Coef-LO and Coef-HI are 95% Confidence Intervals for Neural Networks-LSM 's Response Surface.

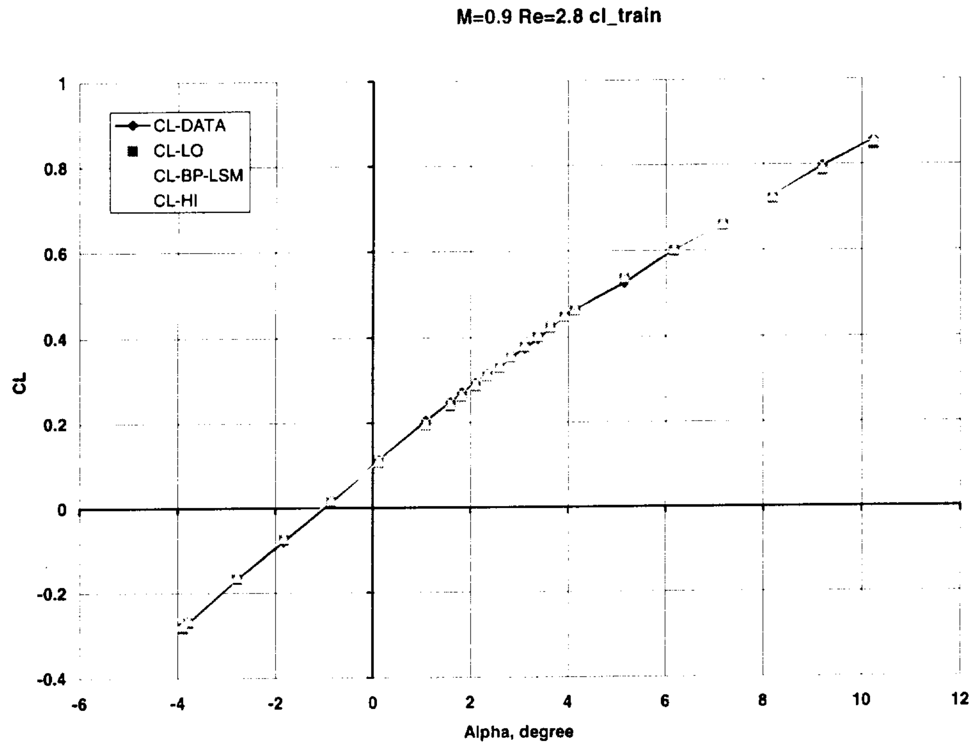


Figure 10(a). Lift Coefficient

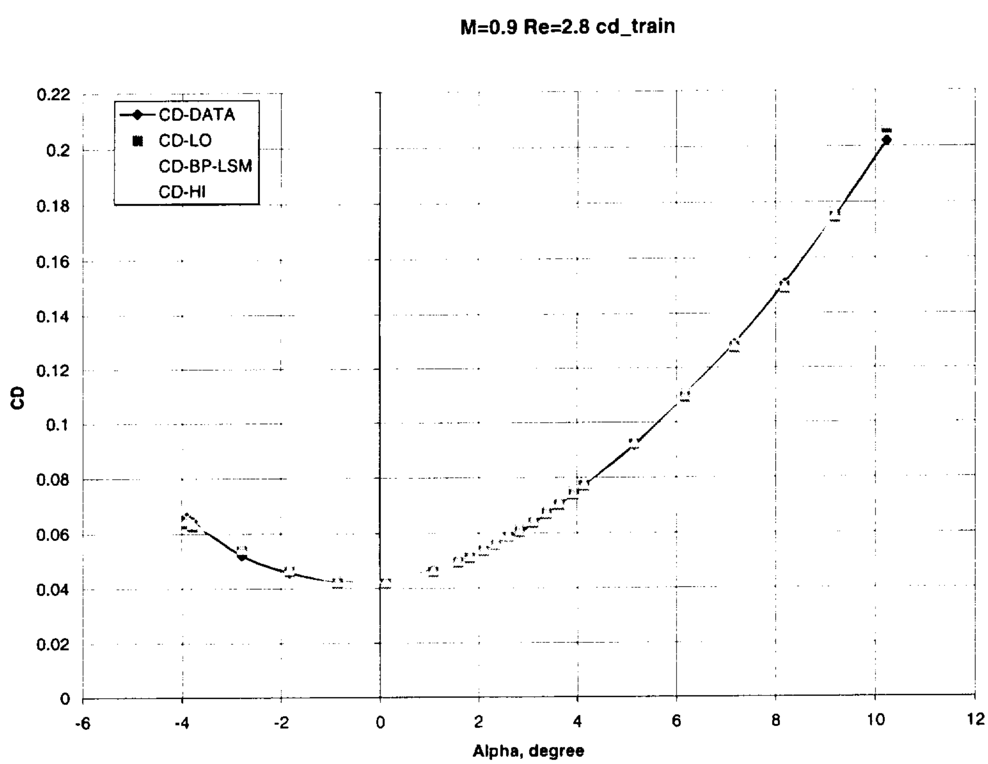


Figure 10 (b). Drag Coefficient

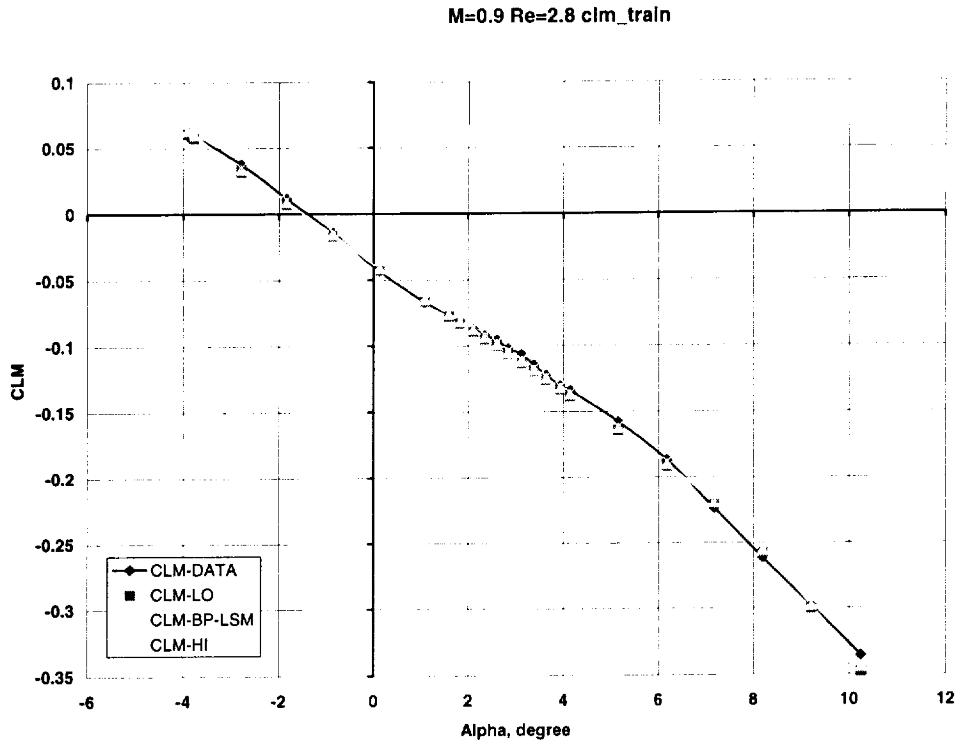


Figure 10(c). Pitching Moment Coefficient
Figure 10. Training Results: Comparison of Neural Networks-LSM Prediction and Tunnel Data at Mach 0.9 Reynolds Number 2.8 mil for Force Coefficients
Note: Coef-LO and Coef-HI are 95% Confidence Intervals for Neural Networks-LSM 's Response Surface.

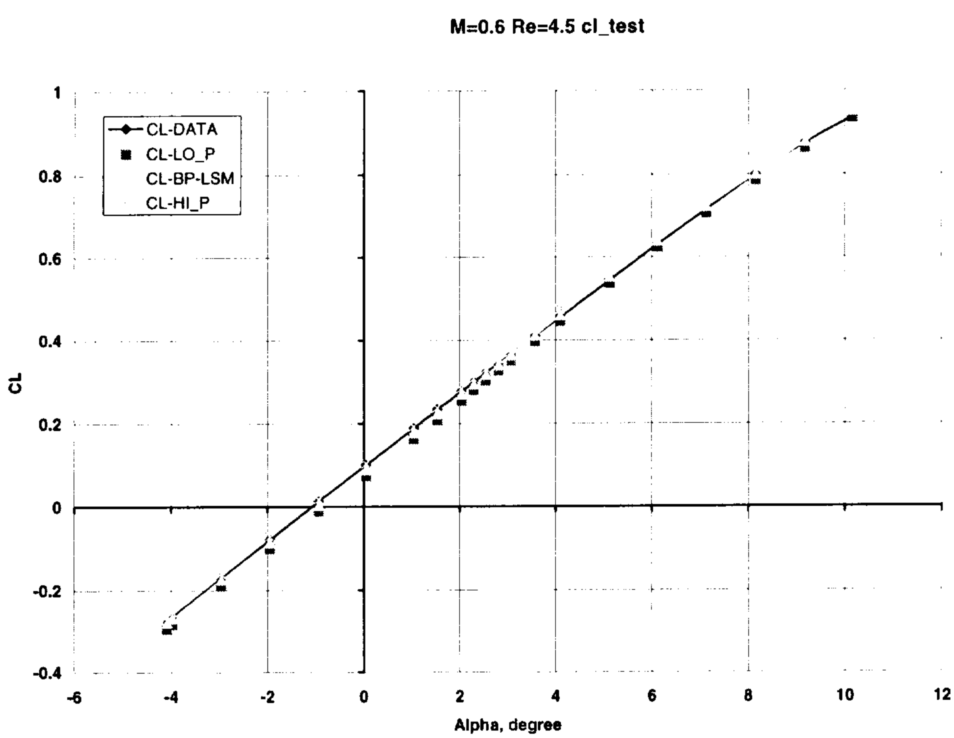


Figure 11(a). Lift Coefficient

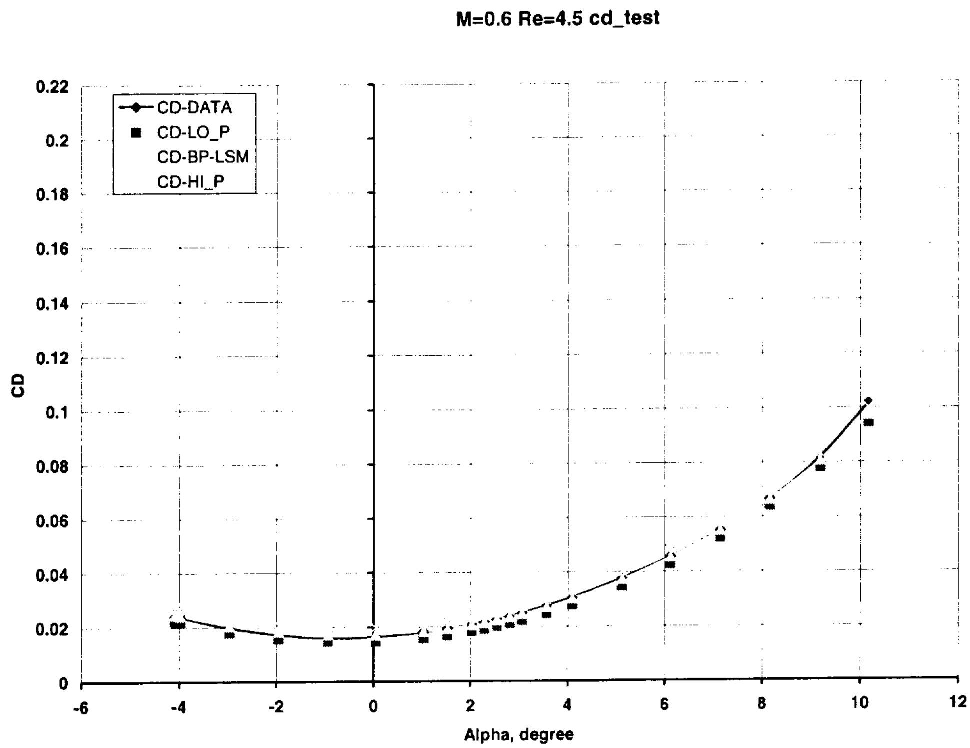


Figure 11(b). Drag Coefficient

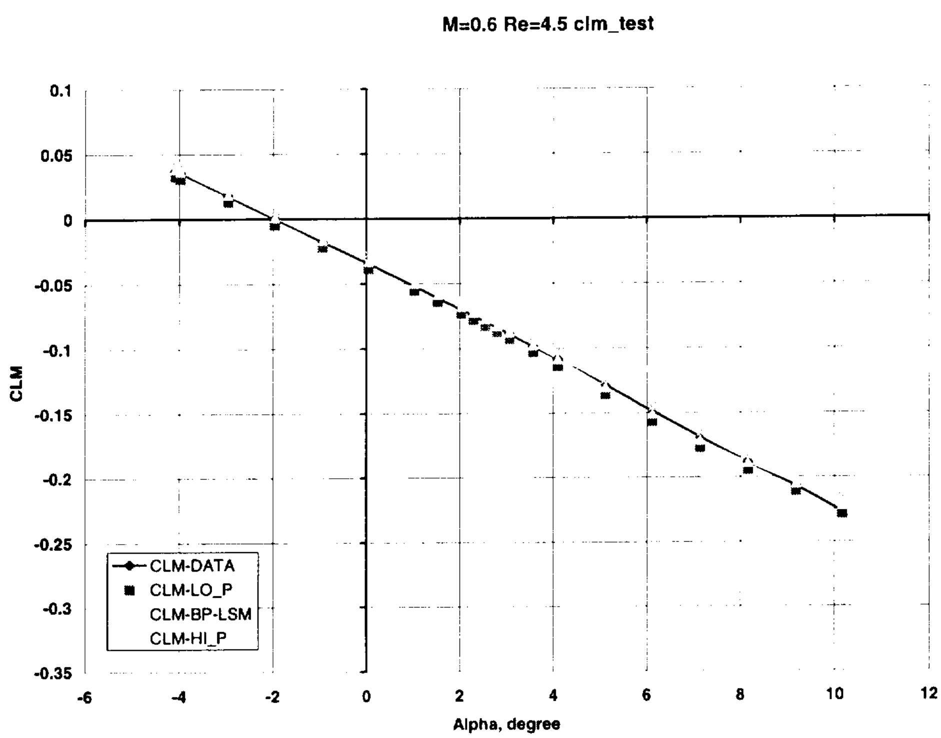


Figure 11 (c). Pitching Moment Coefficient

Figure 11. Test Results: Comparison of Neural Networks-LSM Prediction and Tunnel Data at Mach 0.6 Reynolds Number 4.5 mil for Force Coefficients Note: Coef -LO-P and Coef -HI-P are 95% Prediction Intervals for Neural Networks-LSM 's Future Observation Response Surface

.

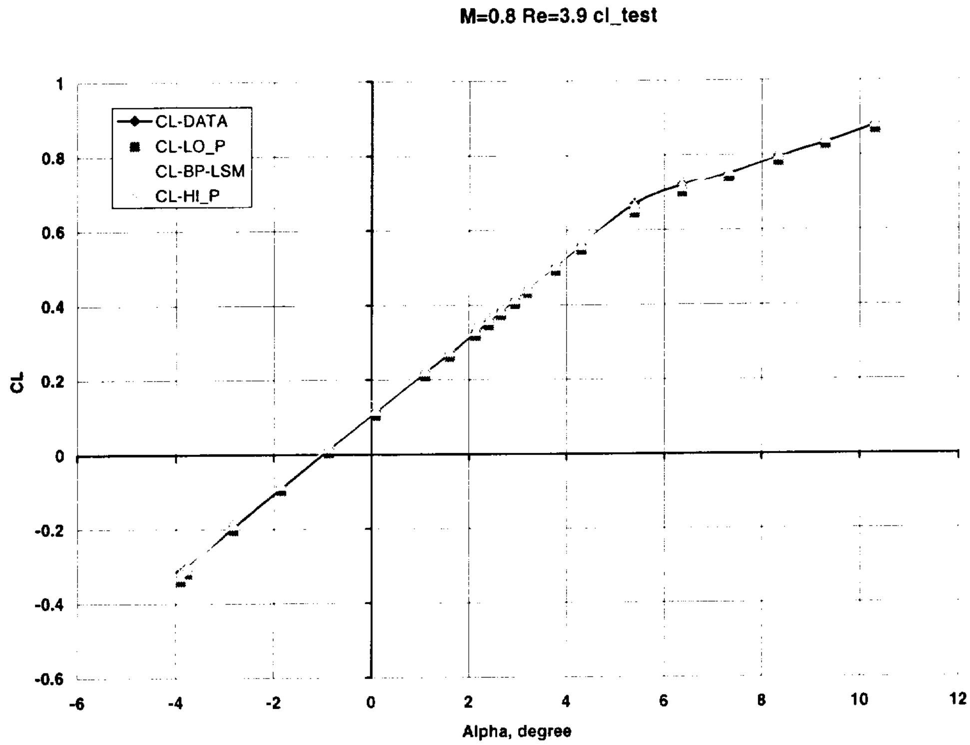


Figure 12(a). Lift Coefficient

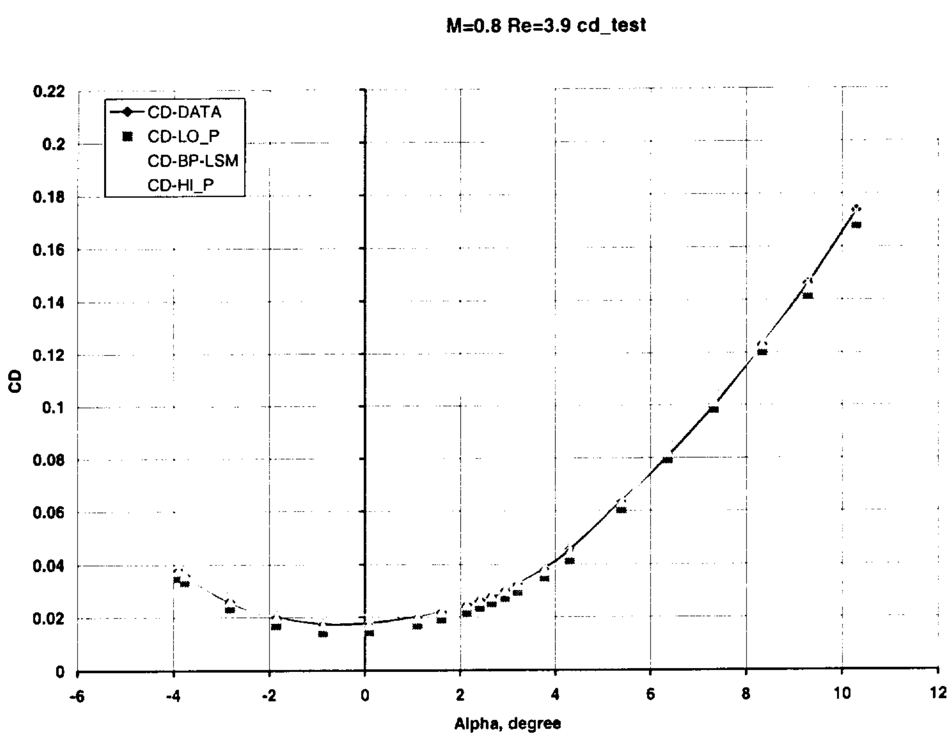


Figure 12(b). Drag Coefficient

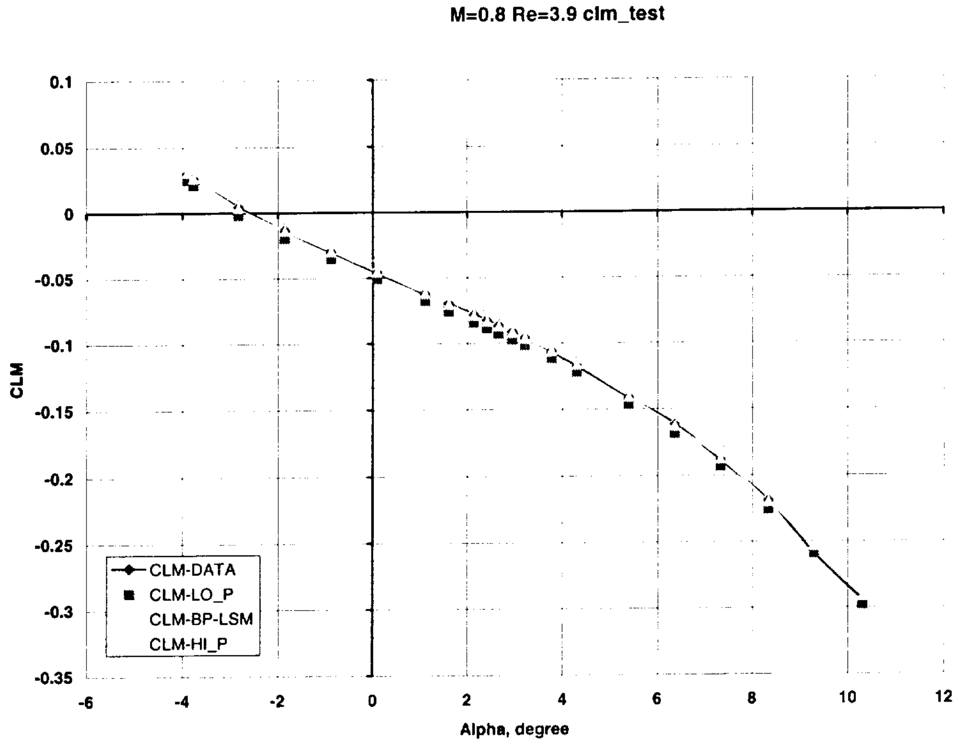


Figure12.(c). Pitching Moment Coefficient
Figure 12. Test Results: Comparison of Neural Networks-LSM Prediction and Tunnel Data at Mach 0.8 Reynolds Number 3.9 mil for Force Coefficients
Note: Coef -LO-P and Coef -HI-P are Prediction Intervals for Neural Networks-LSM 's Future Observation Response Surface.

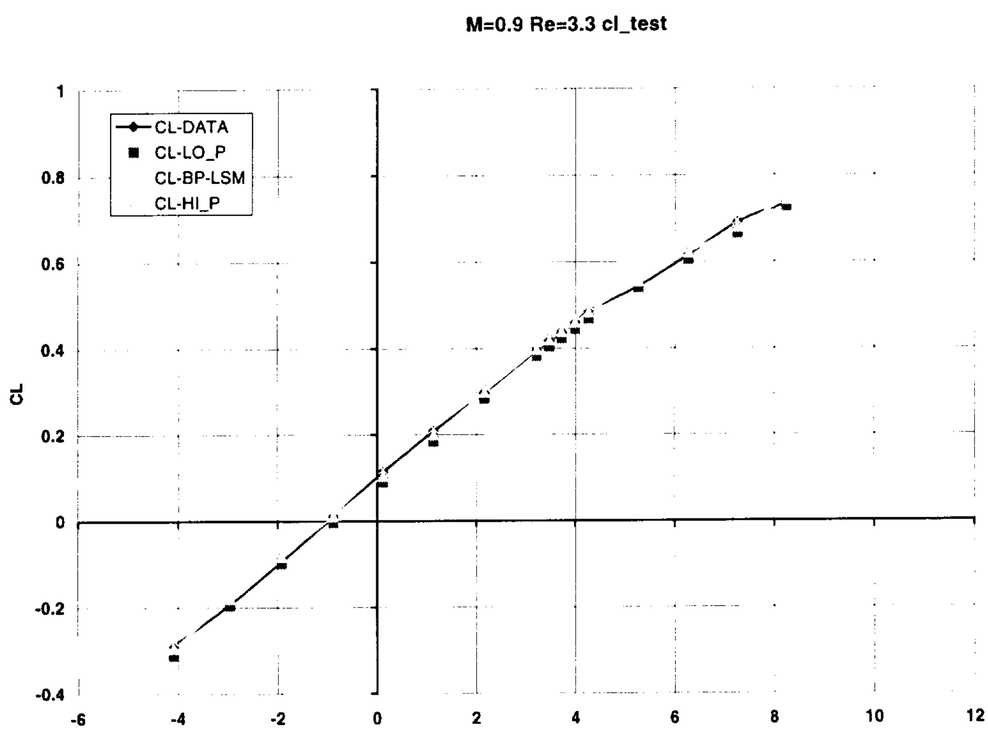


Figure 13(a). Lift Coefficient

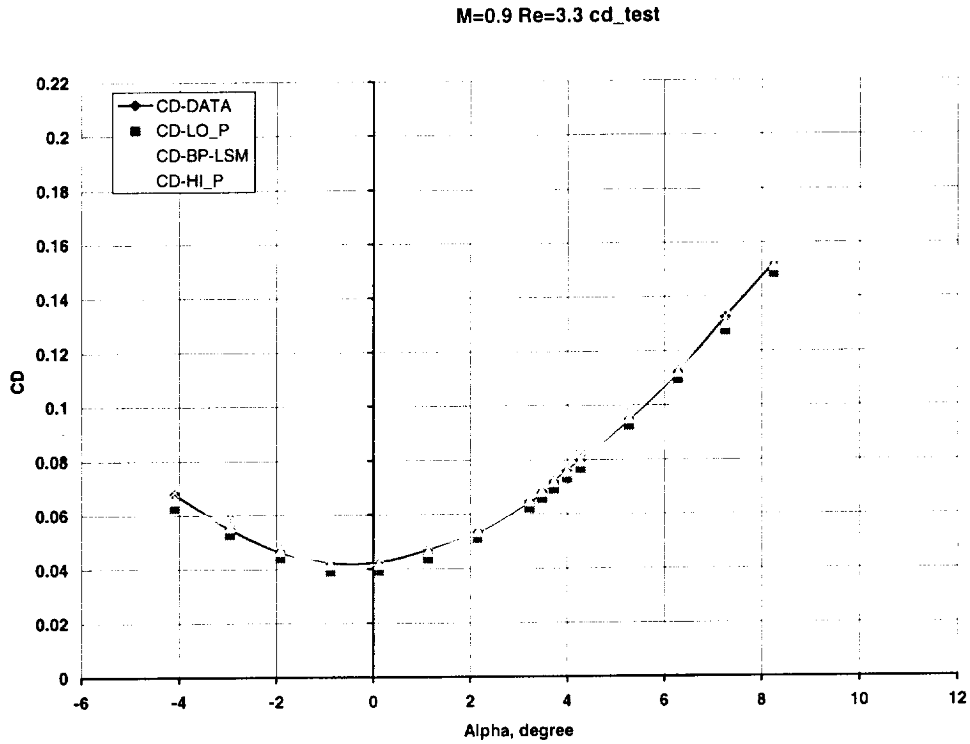


Figure 13(b). Drag Coefficient

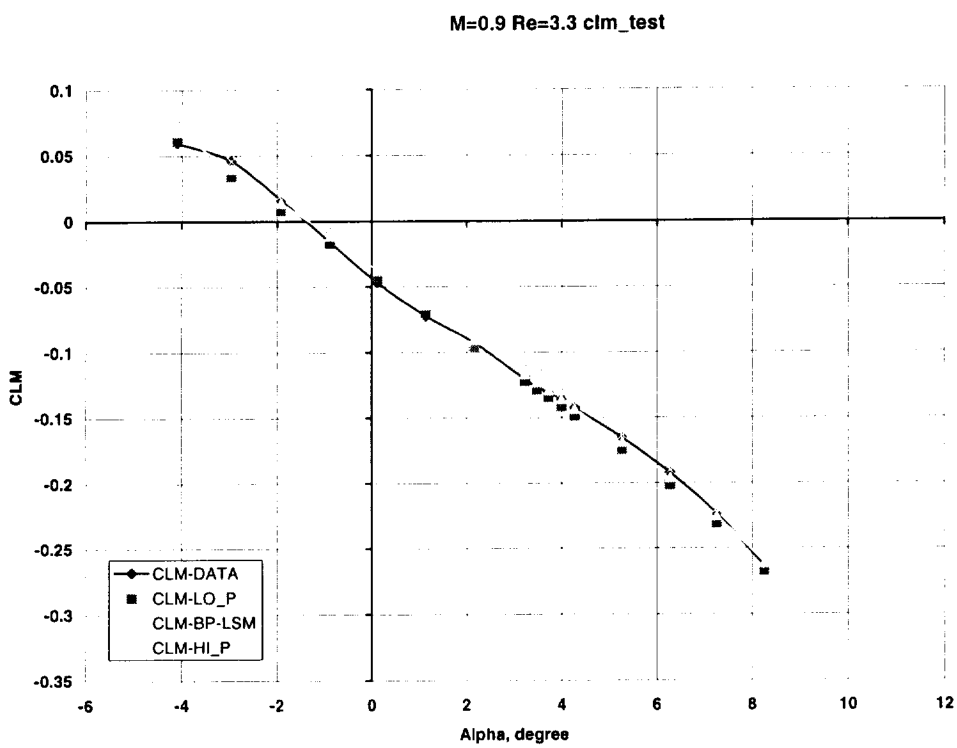
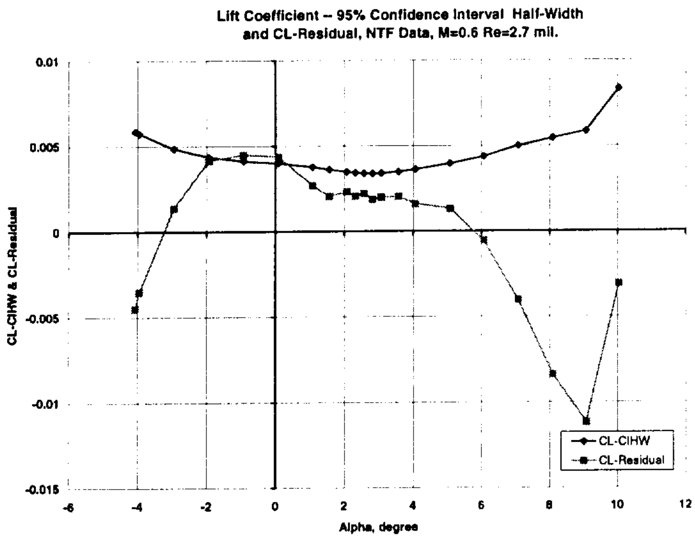


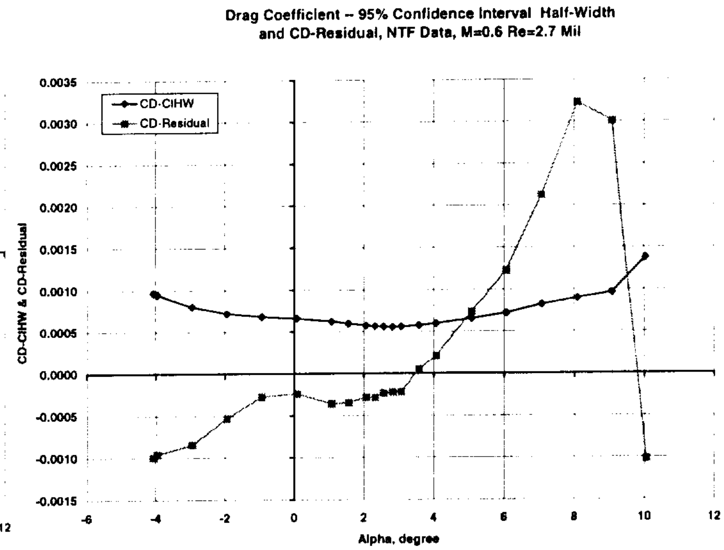
Figure13 (c). Pitching Moment Coefficient

Figure 13. Test Results: Comparison of Neural Networks-LSM Prediction and Tunnel Data at Mach 0.9 Reynolds Number 3.3 mil for Force Coefficients

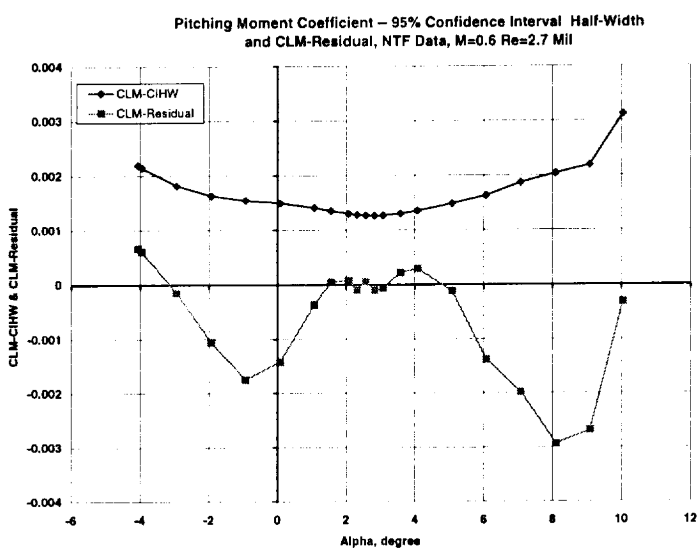
Note: Coef -LO-P and Coef -HI-P are 95% Prediction Intervals for Neural Networks-LSM 's Future Observation Response Surface.



(a) Lift Coefficient

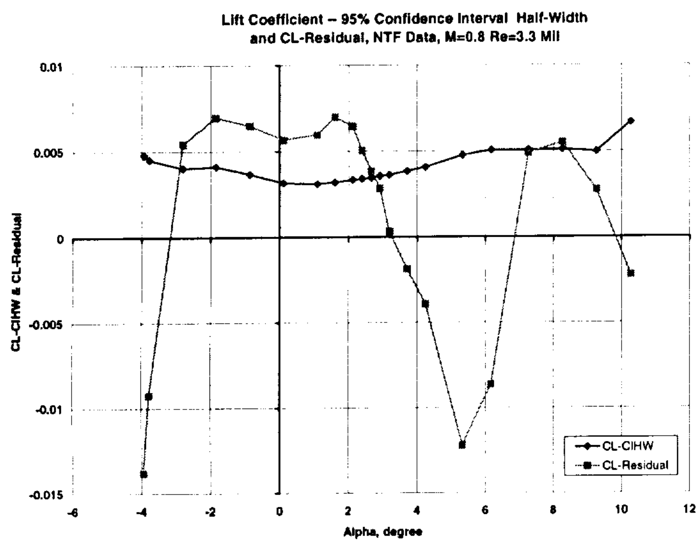


(b) Drag Coefficient

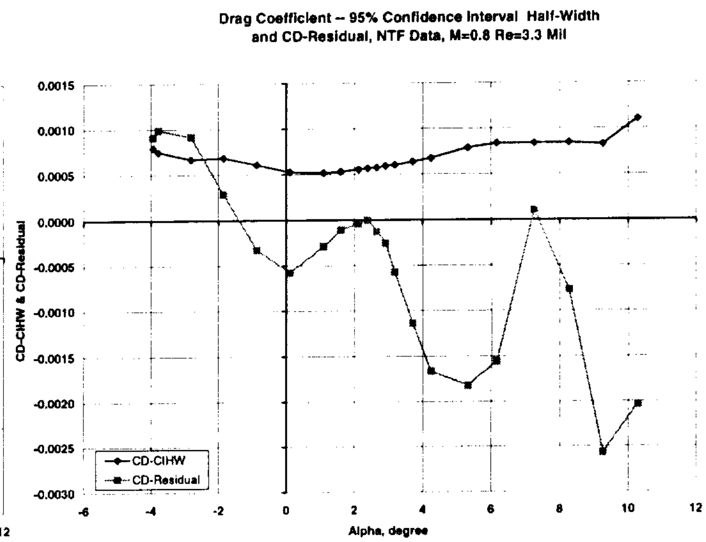


(c) Pitching Moment Coefficient

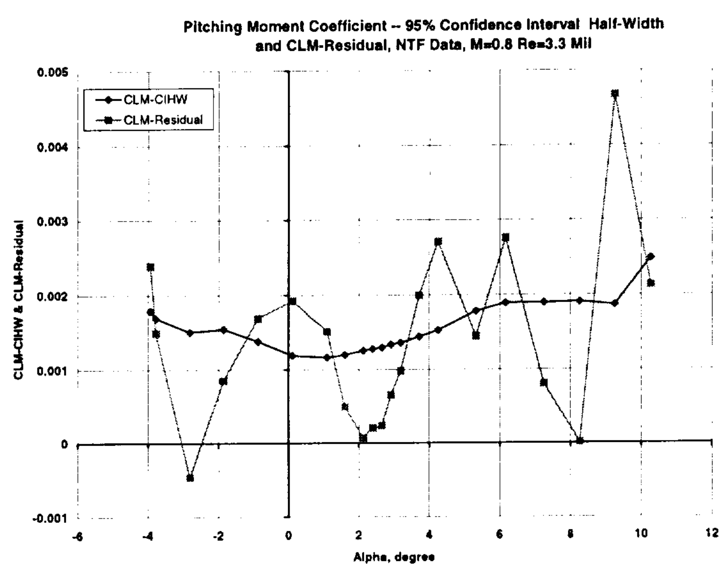
Figure 14. The 95% Confidence Interval Half-Width and BP-LSM Coef-Residuals at M=0.6 Re=2.7 Mil



(a) Lift Coefficient

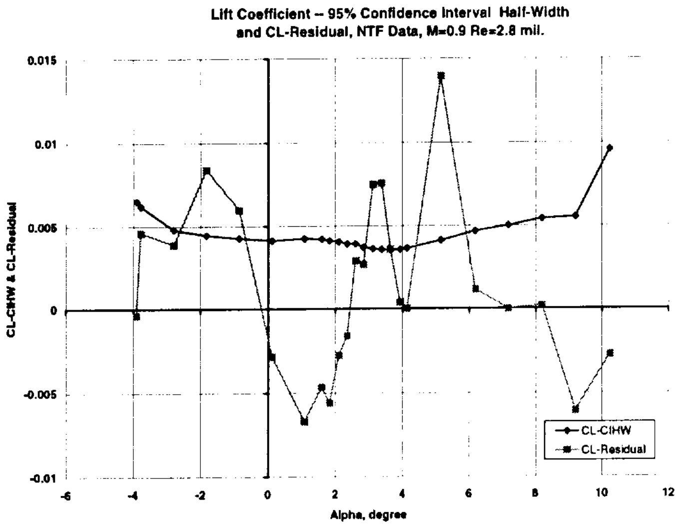


(b) Drag Coefficient

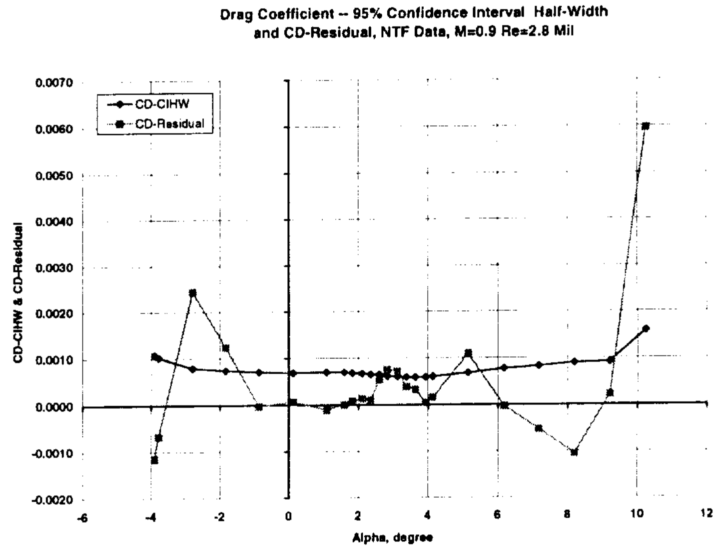


(c) Pitching Moment Coefficient

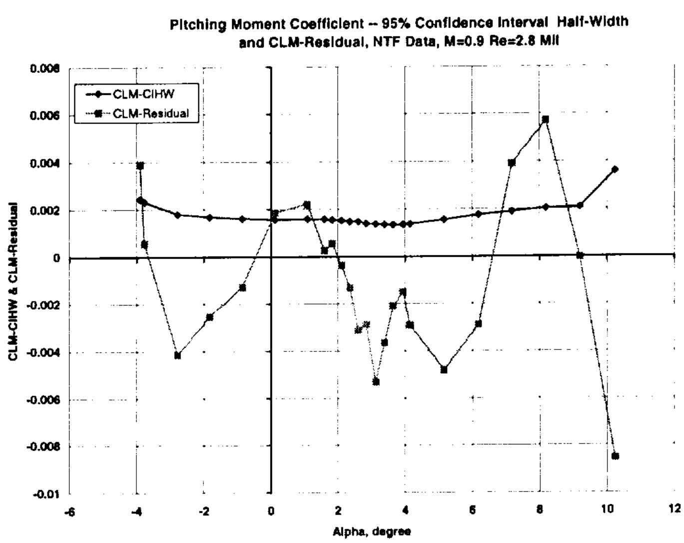
Figure 15. The 95% Confidence Interval Half-Width and BP-LSM Coef-Residuals at M=0.8 Re=3.3 Mil



(a) Lift Coefficient

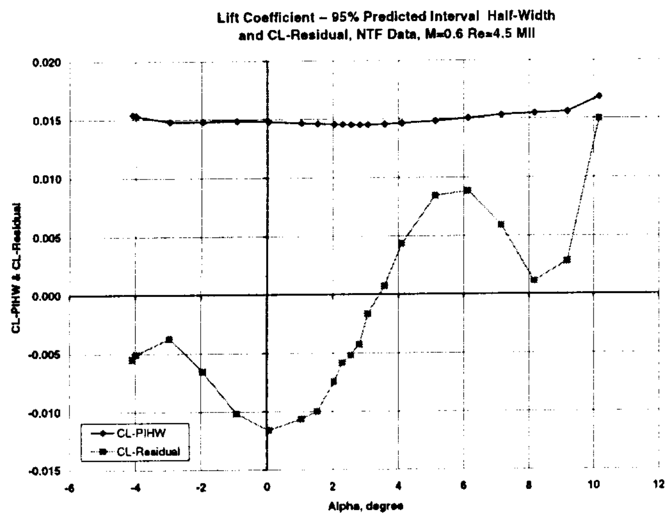


(b) Drag Coefficient

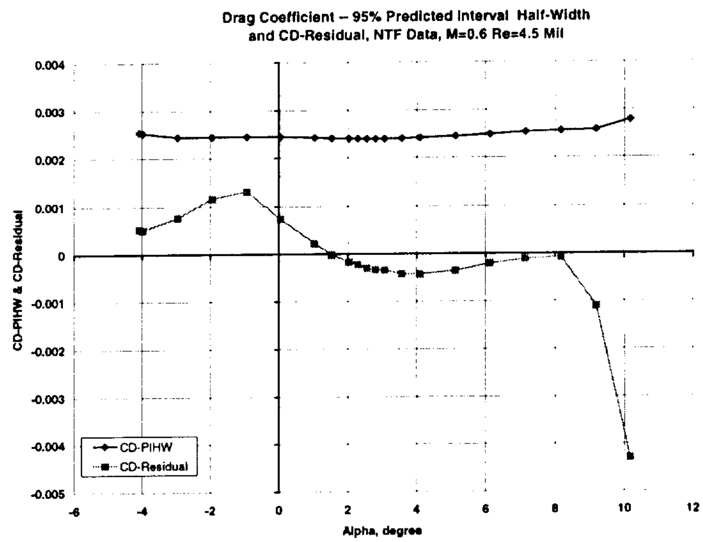


(c) Pitching Moment Coefficient

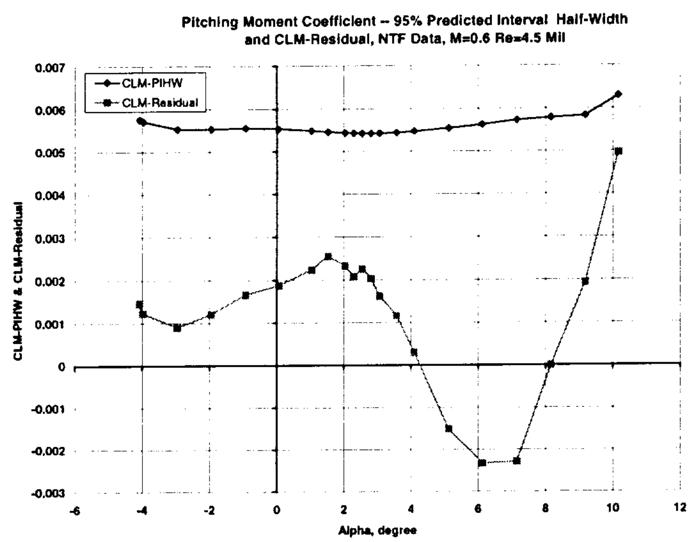
Figure 16. The 95% Confidence Interval Half-Width and BP-LSM Coef-Residuals at M=0.9 Re=2.8 Mil



(a) Lift Coefficient

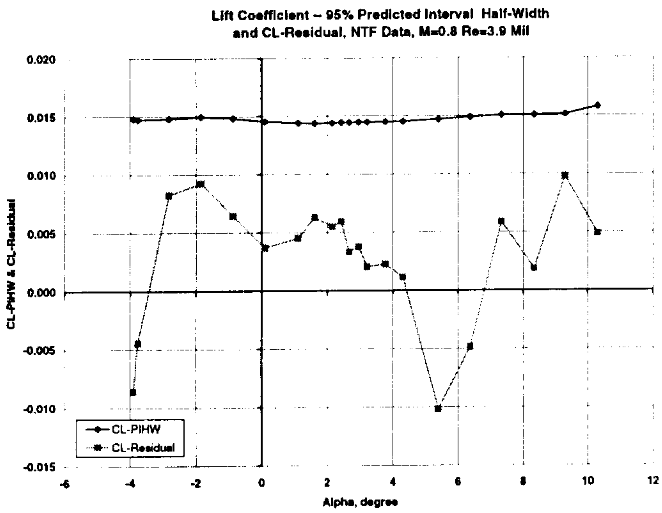


(b) Drag Coefficient

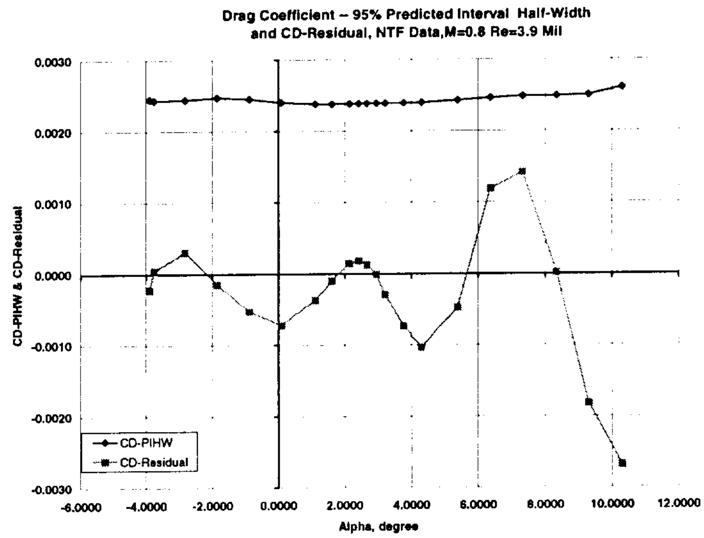


(c) Pitching Moment Coefficient

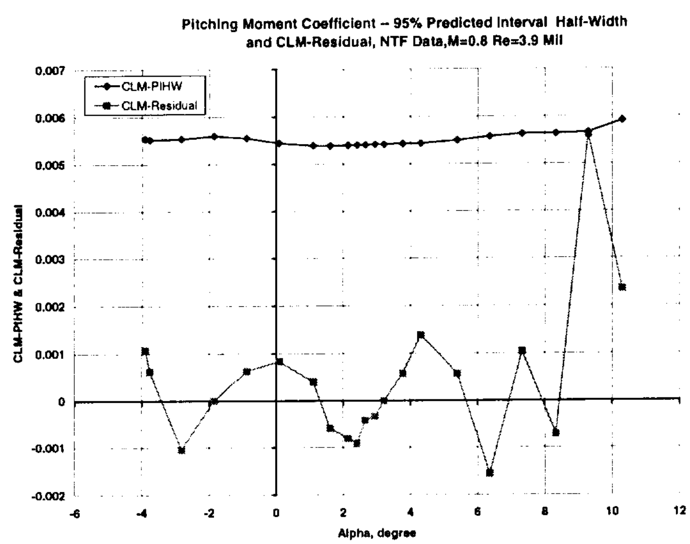
Figure 17. The 95% Prediction Interval Half-Width and BP-LSM Coef-Residuals at M=0.6 Re=4.5 Mil



(a) Lift Coefficient

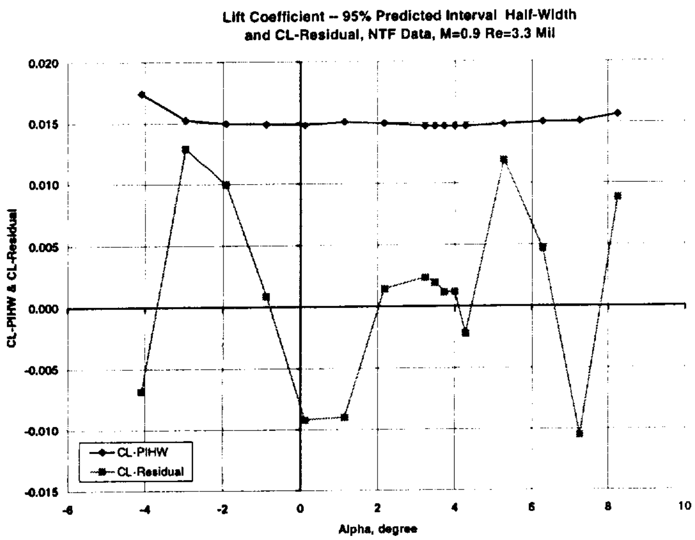


(b) Drag Coefficient

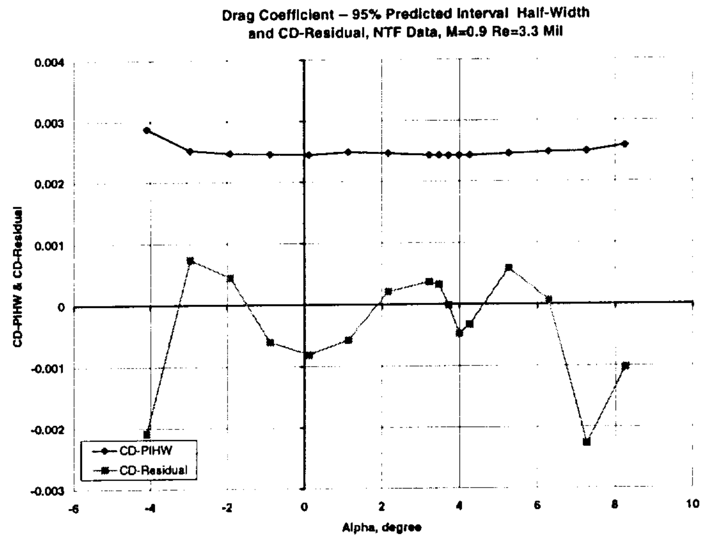


(c) Pitching Moment Coefficient

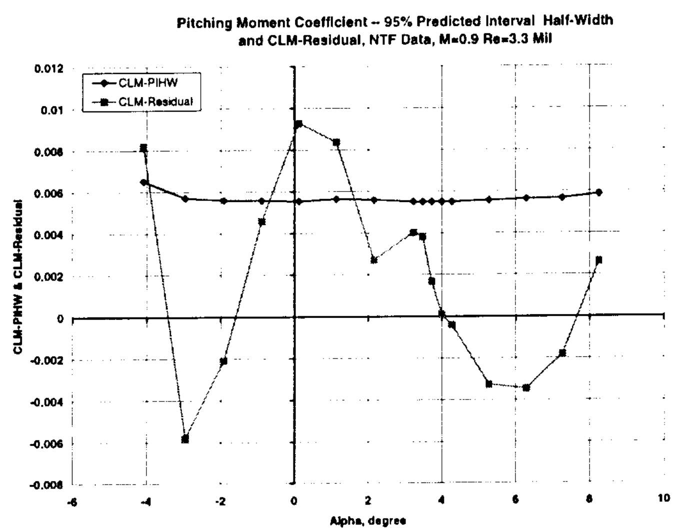
Figure 18. The 95% Prediction Interval Half-Width and BP-LSM Coef-Residuals at M=0.8 Re=3.9 Mil



(a) Lift Coefficient



(b) Drag Coefficient



(c) Pitching Moment Coefficient

Figure 19. The 95% Prediction Interval Half-Width and BP-LSM Coef-Residuals at M=0.9 Re=3.3 Mil

APPENDIX B
TABLES

TABLE 1 RFBN Results and 95% Confidence Intervals

LIFT COEFFICIENT-16T Data

ALPHA	CL-DATA	CL-LOWER	CL-RFBN	CL-UPPER	CL-CIHW	CL-Residual
-4.0130	-0.2932	-0.2995	-0.2917	-0.2838	0.0079	0.0015
-3.0185	-0.1870	-0.1970	-0.1896	-0.1821	0.0075	-0.0026
-2.0290	-0.0832	-0.0900	-0.0831	-0.0762	0.0069	0.0001
-1.0236	0.0178	0.0150	0.0209	0.0267	0.0059	0.0031
-0.0328	0.1197	0.1109	0.1174	0.1240	0.0065	-0.0023
1.0050	0.2197	0.2124	0.2180	0.2235	0.0056	-0.0017
1.4569	0.2623	0.2581	0.2632	0.2682	0.0050	0.0009
1.9608	0.3119	0.3098	0.3139	0.3179	0.0040	0.0020
2.2347	0.3388	0.3371	0.3411	0.3452	0.0041	0.0023
2.4881	0.3663	0.3621	0.3662	0.3704	0.0041	-0.0001
2.7303	0.3928	0.3863	0.3903	0.3942	0.0040	-0.0025
2.9874	0.4190	0.4123	0.4163	0.4203	0.0040	-0.0027
3.4625	0.4671	0.4613	0.4665	0.4717	0.0052	-0.0006
3.9636	0.5165	0.5148	0.5202	0.5256	0.0054	0.0037
4.9560	0.6044	0.5973	0.6042	0.6111	0.0069	-0.0002
5.9787	0.6472	0.6399	0.6453	0.6507	0.0054	-0.0020
6.9699	0.6915	0.6827	0.6891	0.6955	0.0064	-0.0024
7.9735	0.7391	0.7401	0.7459	0.7517	0.0058	0.0068
8.9746	0.7938	0.7817	0.7886	0.7956	0.0069	-0.0051
9.9786	0.8411	0.8348	0.8427	0.8506	0.0079	0.0016

DRAG COEFFICIENT-16T Data

ALPHA	CD-DATA	CD-LOWER	CD-RFBN	CD-UPPER	CD-CIHW	CD-Residual
-4.0130	0.0393	0.0376	0.0390	0.0404	0.0014	-0.0003
-3.0185	0.0278	0.0270	0.0284	0.0298	0.0014	0.0006
-2.0290	0.0216	0.0199	0.0212	0.0224	0.0013	-0.0004
-1.0236	0.0188	0.0176	0.0187	0.0197	0.0011	-0.0002
-0.0328	0.0189	0.0180	0.0192	0.0204	0.0012	0.0003
1.0050	0.0212	0.0202	0.0213	0.0223	0.0010	0.0000
1.4569	0.0228	0.0218	0.0227	0.0236	0.0009	-0.0001
1.9608	0.0251	0.0243	0.0250	0.0257	0.0007	-0.0001
2.2347	0.0267	0.0259	0.0266	0.0274	0.0007	0.0000
2.4881	0.0284	0.0276	0.0284	0.0291	0.0007	0.0000
2.7303	0.0303	0.0295	0.0303	0.0310	0.0007	0.0000
2.9874	0.0322	0.0317	0.0324	0.0331	0.0007	0.0002
3.4625	0.0368	0.0359	0.0369	0.0378	0.0010	0.0001
3.9636	0.0426	0.0414	0.0424	0.0434	0.0010	-0.0002
4.9560	0.0570	0.0555	0.0568	0.0581	0.0013	-0.0002
5.9787	0.0755	0.0748	0.0758	0.0768	0.0010	0.0004
6.9699	0.0953	0.0947	0.0959	0.0970	0.0012	0.0006
7.9735	0.1185	0.1160	0.1170	0.1180	0.0010	-0.0015
8.9746	0.1439	0.1437	0.1450	0.1462	0.0013	0.0011
9.9786	0.1700	0.1682	0.1697	0.1711	0.0014	-0.0003

TABLE 1 (continued)

PITCHING MOMENT COEFFICIENT-16T Data

ALPHA	CLM-DATA	CLM-LOWER	CLM-RFBN	CLM-UPPER	CLM-CIHW	CLM-Residual
-4.0130	0.0268	0.0255	0.0268	0.0281	0.0013	0.0000
-3.0185	0.0044	0.0031	0.0043	0.0055	0.0012	-0.0001
-2.0290	-0.0150	-0.0157	-0.0146	-0.0134	0.0011	0.0004
-1.0236	-0.0311	-0.0327	-0.0318	-0.0308	0.0010	-0.0006
-0.0328	-0.0468	-0.0477	-0.0466	-0.0455	0.0011	0.0002
1.0050	-0.0607	-0.0612	-0.0603	-0.0593	0.0009	0.0005
1.4569	-0.0659	-0.0669	-0.0660	-0.0652	0.0008	-0.0001
1.9608	-0.0720	-0.0732	-0.0725	-0.0718	0.0007	-0.0006
2.2347	-0.0757	-0.0768	-0.0761	-0.0754	0.0007	-0.0004
2.4881	-0.0796	-0.0802	-0.0795	-0.0788	0.0007	0.0001
2.7303	-0.0834	-0.0836	-0.0829	-0.0823	0.0007	0.0005
2.9874	-0.0872	-0.0874	-0.0867	-0.0861	0.0007	0.0005
3.4625	-0.0946	-0.0954	-0.0945	-0.0937	0.0009	0.0000
3.9636	-0.1029	-0.1043	-0.1034	-0.1025	0.0009	-0.0005
4.9560	-0.1212	-0.1225	-0.1214	-0.1202	0.0011	-0.0002
5.9787	-0.1438	-0.1439	-0.1430	-0.1421	0.0009	0.0009
6.9699	-0.1705	-0.1724	-0.1714	-0.1703	0.0011	-0.0009
7.9735	-0.2074	-0.2078	-0.2069	-0.2059	0.0010	0.0006
8.9746	-0.2473	-0.2487	-0.2476	-0.2464	0.0011	-0.0002
9.9786	-0.2857	-0.2870	-0.2857	-0.2844	0.0013	0.0001

Where
ALPHA= angle of attack, CL= lift coef, CD= drag coef, CLM= pitch moment coef.
Coef-DATA= force coef obtained from Tunnel 16T
Coef-LOWER= force coef for Lower bound of 95% confidence interval
Coef-UPPER= force coef for Upper bound of 95% confidence interval
Coef-RBFN= force coef modeled by RBFN
Coef-CIHW= force coef of 95% confidence interval half-width
Coef-Residual = (Coef-RBFN) - (Coef-DATA)
Data Uncertainty for Tunnel 16T: $\Delta CL=0.0048$, $\Delta CD=0.0009$ & $\Delta CLM=0.0025$.

TABLE 2 BackPropagation-Least Squares Method Results and 95% Confidence Intervals

LIFT COEFFICIENT-NTF

Mach	Rec/Mil	ALPHA	CL-DATA	CL-LOWER	CL-BP-LSM	CL-UPPER	CL-CIHW	CL-Residual
0.8	3.3	-3.9364	-0.3166	-0.3352	-0.3304	-0.3256	0.0048	-0.0138
0.799	3.3	-3.7808	-0.2990	-0.3127	-0.3082	-0.3037	0.0045	-0.0092
0.8	3.3	-2.8116	-0.1903	-0.1889	-0.1848	-0.1808	0.0040	0.0054
0.799	3.3	-1.8513	-0.0839	-0.0810	-0.0769	-0.0728	0.0041	0.0070
0.8	3.3	-0.8728	0.0191	0.0219	0.0256	0.0293	0.0037	0.0065
0.8	3.3	0.1058	0.1204	0.1229	0.1261	0.1293	0.0032	0.0057
0.8	3.3	1.0935	0.2209	0.2237	0.2268	0.2300	0.0031	0.0060
0.8	3.3	1.6027	0.2717	0.2755	0.2787	0.2819	0.0032	0.0070
0.799	3.3	2.1217	0.3249	0.3280	0.3314	0.3347	0.0033	0.0064
0.799	3.3	2.3894	0.3536	0.3552	0.3586	0.3620	0.0034	0.0050
0.8	3.3	2.6561	0.3820	0.3825	0.3859	0.3894	0.0035	0.0039
0.799	3.3	2.9117	0.4088	0.4081	0.4117	0.4152	0.0036	0.0029
0.8	3.3	3.1873	0.4399	0.4366	0.4403	0.4439	0.0036	0.0003
0.8	3.3	3.7086	0.4957	0.4900	0.4938	0.4976	0.0038	-0.0019
0.8	3.3	4.2492	0.5535	0.5454	0.5495	0.5536	0.0041	-0.0039
0.8	3.3	5.3241	0.6582	0.6413	0.6460	0.6508	0.0047	-0.0122
0.8	3.3	6.1646	0.7058	0.6921	0.6972	0.7022	0.0050	-0.0086
0.8	3.3	7.2550	0.7373	0.7372	0.7422	0.7473	0.0051	0.0049
0.8	3.3	8.2638	0.7791	0.7795	0.7846	0.7897	0.0051	0.0055
0.8	3.3	9.2636	0.8256	0.8234	0.8284	0.8333	0.0050	0.0027
0.799	3.3	10.2772	0.8726	0.8638	0.8704	0.8771	0.0067	-0.0022

DRAG COEFFICIENT-NTF

Mach	Rec/Mil	ALPHA	CD-DATA	CD-LOWER	CD-BP-LSM	CD-UPPER	CD-CIHW	CD-Residual
0.8	3.3	-3.9364	0.0377	0.0378	0.0386	0.0394	0.0008	0.0009
0.799	3.3	-3.7808	0.0354	0.0356	0.0364	0.0371	0.0007	0.0010
0.8	3.3	-2.8116	0.0254	0.0256	0.0263	0.0269	0.0007	0.0009
0.799	3.3	-1.8513	0.0196	0.0193	0.0199	0.0206	0.0007	0.0003
0.8	3.3	-0.8728	0.0175	0.0166	0.0172	0.0178	0.0006	-0.0003
0.8	3.3	0.1058	0.0179	0.0168	0.0173	0.0178	0.0005	-0.0006
0.8	3.3	1.0935	0.0200	0.0192	0.0197	0.0202	0.0005	-0.0003
0.8	3.3	1.6027	0.0219	0.0212	0.0218	0.0223	0.0005	-0.0001
0.799	3.3	2.1217	0.0244	0.0238	0.0244	0.0249	0.0006	0.0000
0.799	3.3	2.3894	0.0259	0.0254	0.0259	0.0265	0.0006	0.0000
0.8	3.3	2.6561	0.0278	0.0271	0.0277	0.0282	0.0006	-0.0001
0.799	3.3	2.9117	0.0297	0.0289	0.0294	0.0300	0.0006	-0.0003
0.8	3.3	3.1873	0.0323	0.0311	0.0317	0.0323	0.0006	-0.0006
0.8	3.3	3.7086	0.0377	0.0359	0.0365	0.0372	0.0006	-0.0011
0.8	3.3	4.2492	0.0447	0.0423	0.0430	0.0437	0.0007	-0.0017
0.8	3.3	5.3241	0.0621	0.0595	0.0603	0.0611	0.0008	-0.0018
0.8	3.3	6.1646	0.0782	0.0758	0.0766	0.0774	0.0008	-0.0016
0.8	3.3	7.2550	0.0988	0.0981	0.0989	0.0998	0.0008	0.0001
0.8	3.3	8.2638	0.1210	0.1194	0.1203	0.1211	0.0008	-0.0008
0.8	3.3	9.2636	0.1465	0.1431	0.1440	0.1448	0.0008	-0.0026
0.799	3.3	10.2772	0.1730	0.1699	0.1710	0.1721	0.0011	-0.0020

TABLE 2 (Continued)
PITCHING MOMENT DOEFFICIENT-NTF

Mach	Rec/Mil	ALPHA	CLM-DATA	CLM-LOWER	CLM-BP-LSM	CLM-UPPER	CLM-CIHW	CLM-Residual
0.8	3.3	-3.9364	0.0286	0.0292	0.0310	0.0328	0.0018	0.0024
0.799	3.3	-3.7808	0.0246	0.0244	0.0261	0.0278	0.0017	0.0015
0.8	3.3	-2.8116	0.0032	0.0012	0.0027	0.0042	0.0015	-0.0005
0.799	3.3	-1.8513	-0.0156	-0.0162	-0.0147	-0.0132	0.0015	0.0009
0.8	3.3	-0.8728	-0.0323	-0.0319	-0.0306	-0.0292	0.0014	0.0017
0.8	3.3	0.1058	-0.0481	-0.0473	-0.0462	-0.0450	0.0012	0.0019
0.8	3.3	1.0935	-0.0634	-0.0630	-0.0619	-0.0607	0.0012	0.0015
0.8	3.3	1.6027	-0.0704	-0.0711	-0.0699	-0.0687	0.0012	0.0005
0.799	3.3	2.1217	-0.0780	-0.0792	-0.0780	-0.0767	0.0013	0.0001
0.799	3.3	2.3894	-0.0823	-0.0834	-0.0821	-0.0808	0.0013	0.0002
0.8	3.3	2.6561	-0.0866	-0.0876	-0.0863	-0.0850	0.0013	0.0002
0.799	3.3	2.9117	-0.0909	-0.0916	-0.0902	-0.0889	0.0013	0.0007
0.8	3.3	3.1873	-0.0957	-0.0961	-0.0947	-0.0933	0.0014	0.0010
0.8	3.3	3.7086	-0.1053	-0.1048	-0.1033	-0.1019	0.0014	0.0020
0.8	3.3	4.2492	-0.1161	-0.1149	-0.1134	-0.1119	0.0015	0.0027
0.8	3.3	5.3241	-0.1379	-0.1383	-0.1365	-0.1347	0.0018	0.0014
0.8	3.3	6.1646	-0.1587	-0.1578	-0.1559	-0.1540	0.0019	0.0028
0.8	3.3	7.2550	-0.1846	-0.1857	-0.1838	-0.1819	0.0019	0.0008
0.8	3.3	8.2638	-0.2150	-0.2169	-0.2150	-0.2131	0.0019	0.0000
0.8	3.3	9.2636	-0.2555	-0.2526	-0.2508	-0.2489	0.0019	0.0047
0.799	3.3	10.2772	-0.2915	-0.2919	-0.2894	-0.2869	0.0025	0.0021

Where
Mach = Mach Number, Rec/Mil =Chord Reynolds Number per Million
ALPHA= angle of attack, CL= lift coef, CD= drag coef, CLM= pitch moment coef.
Coef-DATA= force coef obtained from NTF/NASA
Coef-LOWER= force coef for Lower bound of 95% confidence interval
Coef-UPPER= force coef for Upper bound of 95% confidence interval
Coef-BP-LSM= force coef modeled by BP-LSM
Coef-CIHW= force coef of 95% confidence interval half-width
Coef-Residual = Coef-BP-LSM - Coef-DATA

TABLE 3 BackPropagation-Least Squares Method Results and 95% Prediction Intervals

LIFT COEFFICIENT-NTF

Mach	Rec/Mil	ALPHA	CL-DATA	CL-LOWER_P	CL-BP-LSM	CL-UPPER_P	CL-PIHW	CL-Residual
0.8	3.9	-3.9013	-0.3168	-0.3402	-0.3254	-0.3106	0.0148	-0.0086
0.8	3.9	-3.7644	-0.3024	-0.3215	-0.3068	-0.2921	0.0147	-0.0045
0.8	3.9	-2.8233	-0.1965	-0.2030	-0.1883	-0.1735	0.0148	0.0082
0.8	3.9	-1.8545	-0.0910	-0.0967	-0.0817	-0.0668	0.0149	0.0093
0.8	3.9	-0.8802	0.0127	0.0043	0.0191	0.0340	0.0148	0.0064
0.8	3.9	0.0862	0.1143	0.1035	0.1180	0.1326	0.0145	0.0037
0.8	3.9	1.1056	0.2189	0.2090	0.2234	0.2378	0.0144	0.0045
0.801	3.9	1.6079	0.2696	0.2615	0.2758	0.2902	0.0144	0.0063
0.8	3.9	2.1415	0.3263	0.3174	0.3318	0.3462	0.0144	0.0055
0.8	3.9	2.4150	0.3547	0.3462	0.3607	0.3751	0.0144	0.0059
0.8	3.9	2.6602	0.3832	0.3721	0.3865	0.4010	0.0144	0.0033
0.8	3.9	2.9476	0.4131	0.4024	0.4169	0.4313	0.0145	0.0038
0.801	3.9	3.2125	0.4438	0.4313	0.4458	0.4603	0.0145	0.0021
0.8	3.9	3.7706	0.5030	0.4908	0.5053	0.5198	0.0145	0.0023
0.8	3.9	4.3036	0.5601	0.5467	0.5612	0.5757	0.0145	0.0012
0.801	3.9	5.3909	0.6694	0.6445	0.6592	0.6739	0.0147	-0.0102
0.8	3.9	6.3637	0.7197	0.6999	0.7148	0.7297	0.0149	-0.0049
0.8	3.9	7.3268	0.7500	0.7408	0.7558	0.7709	0.0150	0.0059
0.802	3.9	8.3356	0.7968	0.7836	0.7986	0.8137	0.0150	0.0019
0.8	3.9	9.2931	0.8335	0.8281	0.8432	0.8583	0.0151	0.0098
0.8	3.9	10.3017	0.8806	0.8697	0.8855	0.9012	0.0158	0.0049

DRAG COEFFICIENT-NTF

Mach	Rec/Mil	ALPHA	CD-DATA	CD-LOWER_P	CD-BP-LSM	CD-UPPER_P	CD-PIHW	CD-Residual
0.8	3.9	-3.9013	0.0380	0.0353	0.0378	0.0402	0.0024	-0.0002
0.8	3.9	-3.7644	0.0360	0.0336	0.0361	0.0385	0.0024	0.0000
0.8	3.9	-2.8233	0.0258	0.0237	0.0261	0.0285	0.0024	0.0003
0.8	3.9	-1.8545	0.0199	0.0173	0.0198	0.0223	0.0025	-0.0001
0.8	3.9	-0.8802	0.0175	0.0145	0.0170	0.0194	0.0024	-0.0005
0.8	3.9	0.0862	0.0178	0.0147	0.0171	0.0195	0.0024	-0.0007
0.8	3.9	1.1056	0.0201	0.0173	0.0197	0.0221	0.0024	-0.0004
0.801	3.9	1.6079	0.0219	0.0194	0.0218	0.0242	0.0024	-0.0001
0.8	3.9	2.1415	0.0244	0.0222	0.0246	0.0269	0.0024	0.0001
0.8	3.9	2.4150	0.0261	0.0239	0.0262	0.0286	0.0024	0.0002
0.8	3.9	2.6602	0.0277	0.0255	0.0279	0.0303	0.0024	0.0001
0.8	3.9	2.9476	0.0300	0.0276	0.0300	0.0324	0.0024	0.0000
0.801	3.9	3.2125	0.0326	0.0299	0.0323	0.0347	0.0024	-0.0003
0.8	3.9	3.7706	0.0385	0.0354	0.0377	0.0401	0.0024	-0.0007
0.8	3.9	4.3036	0.0455	0.0420	0.0444	0.0468	0.0024	-0.0010
0.801	3.9	5.3909	0.0636	0.0607	0.0632	0.0656	0.0024	-0.0005
0.8	3.9	6.3637	0.0808	0.0795	0.0820	0.0845	0.0025	0.0012
0.8	3.9	7.3268	0.0998	0.0988	0.1013	0.1037	0.0025	0.0014
0.802	3.9	8.3356	0.1230	0.1205	0.1230	0.1255	0.0025	0.0000
0.8	3.9	9.2931	0.1462	0.1419	0.1444	0.1469	0.0025	-0.0018
0.8	3.9	10.3017	0.1736	0.1683	0.1709	0.1735	0.0026	-0.0027

TABLE 3 (Continued)

PITCHING MOMENT DOEFFICIENT-NTF

Mach	Rec/Mil	ALPHA	CLM-DATA	CLM-LOWER_P	CLM-BP-LSM	CLM-UPPER_P	CLM-PIHW	CLM-Residual
0.8	3.9	-3.9013	0.0295	0.0250	0.0306	0.0361	0.0055	0.0011
0.8	3.9	-3.7644	0.0259	0.0210	0.0265	0.0321	0.0055	0.0006
0.8	3.9	-2.8233	0.0047	-0.0019	0.0036	0.0092	0.0055	-0.0010
0.8	3.9	-1.8545	-0.0140	-0.0196	-0.0140	-0.0084	0.0056	0.0000
0.8	3.9	-0.8802	-0.0304	-0.0353	-0.0298	-0.0242	0.0055	0.0006
0.8	3.9	0.0862	-0.0462	-0.0508	-0.0454	-0.0399	0.0054	0.0008
0.8	3.9	1.1056	-0.0624	-0.0674	-0.0620	-0.0566	0.0054	0.0004
0.801	3.9	1.6079	-0.0697	-0.0756	-0.0702	-0.0649	0.0054	-0.0006
0.8	3.9	2.1415	-0.0780	-0.0842	-0.0788	-0.0734	0.0054	-0.0008
0.8	3.9	2.4150	-0.0823	-0.0886	-0.0832	-0.0778	0.0054	-0.0009
0.8	3.9	2.6602	-0.0868	-0.0926	-0.0872	-0.0818	0.0054	-0.0004
0.8	3.9	2.9476	-0.0915	-0.0972	-0.0918	-0.0864	0.0054	-0.0003
0.801	3.9	3.2125	-0.0964	-0.1018	-0.0964	-0.0910	0.0054	0.0000
0.8	3.9	3.7706	-0.1066	-0.1115	-0.1060	-0.1006	0.0054	0.0006
0.8	3.9	4.3036	-0.1177	-0.1217	-0.1163	-0.1109	0.0054	0.0014
0.801	3.9	5.3909	-0.1414	-0.1463	-0.1408	-0.1353	0.0055	0.0006
0.8	3.9	6.3637	-0.1613	-0.1684	-0.1628	-0.1573	0.0056	-0.0016
0.8	3.9	7.3268	-0.1886	-0.1932	-0.1875	-0.1819	0.0056	0.0010
0.802	3.9	8.3356	-0.2191	-0.2254	-0.2198	-0.2142	0.0056	-0.0007
0.8	3.9	9.2931	-0.2586	-0.2587	-0.2530	-0.2474	0.0057	0.0056
0.8	3.9	10.3017	-0.2940	-0.2975	-0.2916	-0.2857	0.0059	0.0024

Where

Mach = Mach Number, Rec/Mil = Chord Reynolds Number per Million

ALPHA= angle of attack, CL= lift coef, CD= drag coef, CLM= pitch moment coef.

Coef-DATA= force coef obtained from NTF/NASA

Coef-LOWER-P= force coef for Lower bound of 95% prediction interval

Coef-UPPER-P= force coef for Upper bound of 95% prediction interval

Coef-BP-LSM= force coef modeled by BP-LSM

Coef-PIHW= force coef of 95% prediction interval half-width

Coef-Residual = Coef-BP-LSM - Coef-DATA