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NASA

Guidelines for Design and Analysis of Large, Brittle Spacecraft Components

1 September 1993

Prepared by

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Technology Operations

Prepared for

NASA/JOHNSON SPACE CENTER
Houston, TX 77058

Engineering and Technology Group

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THE AEROSPACE CORPORATION
El Segundo, California

1993

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OF LARGE, BRITTLE SPACECRAFT COMPONENTS

Prepared

E. Y. Robinson

Approved

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Center

FOREWORD

This document was prepared for NASA/Johnson Space Center (JSC), Houston, Texas, under MIPR Number T-9315R, through SMC/FMBR.

The program monitor for NASA/JSC was Ms. Karen Edelstein.

There were two related parts to this work. The first, conducted at The Aerospace Corporation, was to develop and define methods for integrating the statistical theory of brittle strength with conventional finite element stress analysis, and to carry out a limited laboratory test program to illustrate the methods. The second part, separately funded at Aerojet Electronic Systems Division, was to create the finite element postprocessing program for integrating the statistical strength analysis with the structural analysis. The second part was monitored by Capt. Jeff McCann of USAF/SMC, as Special Study No. 11 of Contract F04701-86-C-0029, which authorized Aerojet to support Aerospace on this work requested by NASA. This second part is documented in Appendix A.

The activity at Aerojet was guided by the Aerospace methods developed in the first part of this work. This joint work of Aerospace and Aerojet stemmed from prior related work for the Defense Support Program (DSP) Program Office, to qualify the DSP sensor main mirror and corrector lens for flight as part of a shuttle payload. These large brittle components of the DSP sensor are provided by Aerojet.

This document defines rational methods for addressing the structural integrity and safety of large, brittle, payload components, which have low and variable tensile strength and can suddenly break or shatter. The methods are applicable to the evaluation and validation of such components, which, because of size and configuration restrictions, cannot be validated by direct proof test.

Comments and suggestions are welcomed and should be sent to:

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ACKNOWLEDGMENTS

The author acknowledges the early sustaining level of support provided by the Defense Support Program (DSP) office during the DSP shuttle payload safety reviews, enabling the investigation of more general aspects of the analysis of large shatterable space systems components.

The author also acknowledges the collaborative efforts of Wayne Ely and Troan C. Nguyen of Aerojet Electronic Systems Division, who performed the complementary part of this task and authored Appendix A of this document. Extra efforts were required by all parties to bring the work to this level of completion.

In addition, the author thanks Ms. Dana Speece, who carried out the glass strength test program used to illustrate the typical test and analysis methods. This program is reported in detail in Appendix C.

Finally, the author also thanks the management of the Structural Materials Department, who provided supplementary funding support to allow completion of this report, and the reviewing editors, Mike Meyer and Jackie Naiditch, whose suggestions were valuable in improving the form and readability of this document.

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1. INTRODUCTION

The requirements and procedures in this document shall apply to all customer hardware components designated for launch by the Space Shuttle Program (SSP). Specifically, these procedures and requirements shall apply to certain hardware items that cannot meet the requirements set forth in NSTS 14046, Payload Verification Requirements. This document is applicable to all SSP users.

1.1 SCOPE

The fracture toughness of brittle ceramic and glass materials is so low that critical crack sizes are not readily inspectable. Such materials exhibit a pronounced sensitivity of strength to size and stress distribution. They are also prone to high data scatter. In the case of very large components for which a proof test is not feasible or practical, this document provides a rational, consistent method for estimating the Factor of Safety, based on testing of small convenient test articles. The method is based on the Weibull theory of brittle strength which considers brittle strength to be controlled by the weakest flaw distributed throughout that portion of the material that is subject to tensile stresses. The method is based on the volume distribution of flaws, which is known to give conservative estimates of strength of large structures.

1.2 PURPOSE

This document specifies the method for estimating the Factor of Safety of structural components manufactured from brittle materials such as glass or ceramics. The method shall be used only according to the flow chart shown in Figure 1. The guideline summary for determining the Factor of Safety is given in Figure 2.

1.3 INTENDED USE

This document is intended for use by SSP customers planning to use large components made of brittle materials. This document is invoked by NSTS 14046 as per Figure 1.

1.4 NATIONAL AERONAUTICS AND SPACE ADMINISTRATION APPROVALS

Program approvals that are required by this document shall be obtained through the Space Shuttle/Payloads Working Group.

1.5 PRECEDENCE

NSTS 14046, Payload Verification Requirements, defines the structural certification requirements for payloads and other hardware flown on the SSP. If there is any conflict between NSTS 14046 and this document, NSTS 14046 shall take precedence. If the customer cannot meet the NSTS 14046 requirements for brittle components, this document is invoked.

2. DOCUMENTS

None specified.

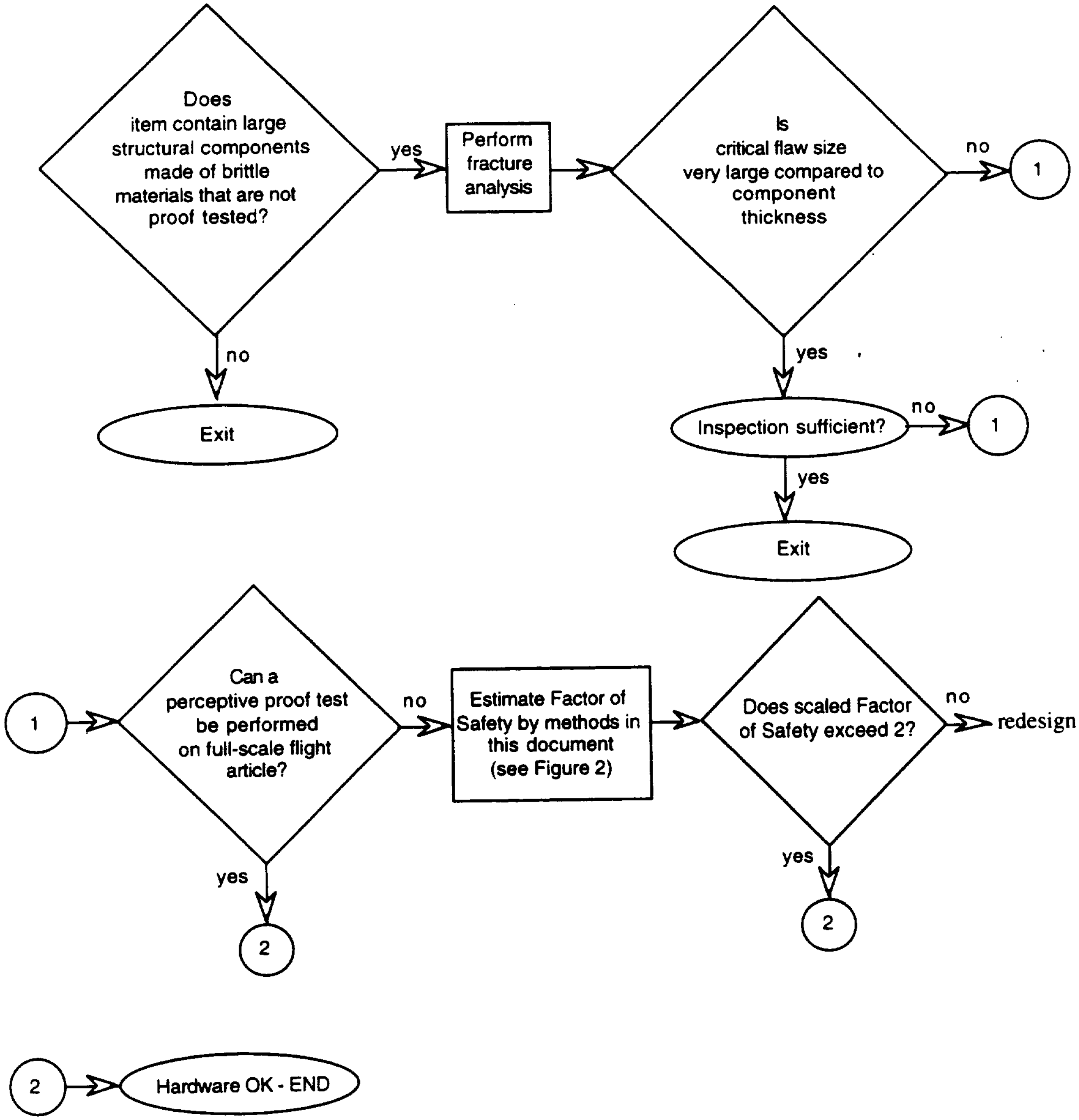

Figure 1. Decision Flow Chart

1. Select Small Test Specimens

- Material - representative of component
- Surface Preparation - representative of component
- Number of Specimens - 15 minimum, preferably 30 per test type
- Configuration - beams of circular or rectangular cross section
- Size - one beam size is acceptable, two sizes are preferred

2. Conduct Specimen Tests

- Flexure - 3 point and 4 point
- Size Effects - two types minimum, varying flexure and/or span

3. Specimen Data Analysis

- Data Pooling - combine median normalized groups
- Weibull Modulus - pooled data for estimate, confirm by size and stress distribution tests
- Specimen A-allowable Design based on m-value and pooled sample size (subsection 3.2.1.4)
- Specimen R-integral - compute for selected test types

4. Component Numerical Analysis

- FE Analysis Postprocessing - max operating stress reference, Ki's, element volumes (subsection 3.2.2)
- Component R-integral - use maximum element principal tensile stresses over element volume
- Optional Adjustments - equivalent principal tensions, actual stress gradient in elements (subsection 3.2.2.1)

5. Component Factor of Safety

- Expected Component Strength (at peak stress) - equate specimen and component Risks of Rupture
- Allowable Component Peak Stress - apply A-level knockdown factor for sample size and m
- Estimated Component Factor of Safety - ratio of A-allowable to operating stress at peak stress location

Figure 2. Factor of Safety Estimation Procedure

3. REQUIREMENTS AND PROCEDURES

3.1 STRUCTURAL AND STATISTICAL ANALYSIS REQUIREMENTS

NASA safety procedures require the implementation of a fracture control plan. Dye penetrant is used to inspect for critical crack sizes, and proof tests are used to screen out certain crack sizes. Metallic structures exhibit detectable cracks, but glassy, brittle materials have very small critical cracks, on the order of a few thousandths of an inch, which are not readily detectable by conventional inspection techniques. These more brittle materials have low strength and high data scatter, which are not usually correlated to an observable crack. Proof testing of an operational brittle component is the preferred approach, but if such testing is not practical (e.g., because of size and configuration of the component), then analytical, conservative verification of adequate structural margins is required in accordance with this document.

Verification of brittle component structural integrity, by analysis and suitable specimen testing, must account for the principal brittle behavior characteristics: the size effect, the effect of stress distribution, and the effect of high data scatter. A statistical approach is necessary, requiring the testing of small representative specimens to provide statistical parameters. In order to predict the allowable stress and the corresponding Factor of Safety of a full-scale component, its strength must be estimated from specimen test data. These data must be properly scaled to account for actual component size and operating stress distributions.

The method defined herein to verify brittle structure integrity is based on the concept of Risk of Rupture, derived from the Weibull theory of brittle failure. It combines finite element structural analysis with the theory of brittle material failure to predict the Factor of Safety of a large brittle component. The theoretical foundations and procedures for verifying the strength margins of full-scale components are addressed in the following subsections.

3.1.1 WEIBULL THEORY OF BRITTLE STRENGTH

The prediction of strength for large, brittle space system structures must account for strength reduction from effects of size, stress distribution, surface fabrication condition, and inherent scatter of brittle strength. The theory of strength of brittle materials used in this document was developed in its present form by W. Weibull, who published his well-known seminal paper in 1939.

The Weibull theory addressed the discrepancy between observed material strength and the theoretical strength that far exceeds observed values. Practical strength was limited, in his view, by intense flaws distributed throughout the material, causing high stress concentration and failure when the tensile stress exceeded the local strength at a severe flaw. Unlike metals, brittle materials have shear strengths exceeding tensile strength and will not shear to relieve local high stress concentrations. Rather, tensile fracture initiates at the weakest defect, and the brittle material behaves like a chain that fails at its weakest link. The Weibull theory of brittle strength is a "weakest link" or series model, derived from the statistics of extreme values in a random sample. Consequences of this theory are high strength scatter, a strength decrease with increasing size, and a dependence of strength on the distribution of applied tensile stress.

The classical Weibull distribution is utilized here as the rational procedure for predicting the strength of very large, brittle structures. The procedure is based on tests of small convenient and representative test specimens, and a conservative statistical analysis of data scatter. The Weibull theory results in a power law, exponential statistical distribution. The exponential term of the Weibull distribution is termed the Risk of Rupture. This term allows direct prediction of strengths at equal probability of failure for various sized components subject to various stress distributions. A guideline procedure is developed in this document to combine the Weibull theory with the usual finite element structural analysis, in order to predict the strength and Factor of Safety of large, brittle components.

The finite element structural analysis that is commonly conducted on space system structures is modified to perform a numerical integration of the Weibull Risk of Rupture, which we call the R–integral, for the operational component. The Risk of Rupture is also computed for the representative test articles, and the respective Risks of Rupture are equated to determine the expected reduction in strength of the large operational component and thereby its Factor of Safety at design operating conditions.

3.1.2 WEIBULL ANALYSIS OF BRITTLE STRENGTH

The classical Weibull distribution for strength of brittle materials is expressed as

$$S = \exp - \left[\int_V \left(\frac{\sigma}{\sigma_o}\right)^m dV \right]$$

where S is the cumulative probability of surviving the applied tensile stress acting over the volume V; the subscript o denotes a scaling factor; and the exponent m is termed the Weibull modulus. The Weibull Modulus is a statistical shape factor of the distribution and is inversely proportional to the coefficient of variation, cv. An accurate approximation is given by $m \approx 1.2/\text{cv}$. The equation above gives the strength distribution of a brittle material.

The exponential term, the Risk of Rupture, R, is expressed as the R-integral:

$$R = \left[\int_V \left(\frac{\sigma}{\sigma_o}\right)^m dV \right]$$

This R-integral determines the probability of failure for any stress distribution and volume.

3.1.2.1 SIZE EFFECT FOR UNIFORM TENSION

Consider two specimens of similar material but of different size that are tested for strength by uniform tensile stresses throughout their volumes, V_1 and V_2. The respective R-integrals are given by

$$R_1 = \left(\frac{\sigma_1}{\sigma_o}\right)^m V_1 \qquad\qquad R_2 = \left(\frac{\sigma_2}{\sigma_o}\right)^m V_2$$

Since the Risks of Rupture are equal at equal probabilities of failure, the effect of size on strength is determined by equating R_1 and R_2 and eliminating the scale factor, σ_o, in the resulting ratio:

$$\left(\frac{\sigma_1}{\sigma_2}\right) = \left(\frac{V_2}{V_1}\right)^{1/m}$$

If V_2 is larger than V_1, its relative strength will be diminished. The effect will be more pronounced with greater strength scatter (lower m-value). The same relationship will hold for any stress distribution that is scaled linearly between the two specimen sizes.

3.1.2.2 STRESS DISTRIBUTION EFFECT FOR TENSION AND BENDING

The Risk of Rupture, R_{UB}, for uniform bending in a rectangular beam specimen having volume V_{UB} in tension, and the Risk of Rupture, R_T, for the uniform tension specimen, are given by

$$R_{UB} = \left(\frac{\sigma_1}{\sigma_o}\right)^m \frac{V_{UB}}{(m+1)} \qquad R_T = \left(\frac{\sigma_2}{\sigma_o}\right)^m V_T$$

Equating the Risks of Rupture gives the relative strengths of the two cases as

$$\frac{R_T}{R_{UB}} = \left[\left(\frac{V_{UB}}{V_T}\right)\frac{1}{(m+1)}\right]^{(1/m)}$$

If both specimens have equal total volume in tension, the effect of the change in stress distribution from bending to uniform tension is a function only of the Weibull m-value. The effect is intensified by higher scatter, which corresponds with a lower m-value.

3.1.2.3 RISK OF RUPTURE FOR LINEAR STRESS GRADIENT

A common case is that of a finite element, subject to a linear tensile stress distribution like a beam, from maximum stress at one surface to minimum stress at the opposite surface. The R-integral for this element (the j-th element) with linear gradient is given in subsection 3.2.2.1, with the following result:

$$R_j = V_j\left(\frac{\sigma_{max}}{\sigma_o}\right)^m\left[\frac{1 - \left(\frac{\sigma_{min}}{\sigma_{max}}\right)^{(m+1)}}{(m+1)\left(1 - \frac{\sigma_{min}}{\sigma_{max}}\right)}\right]$$

This particular relation is a guideline for determining the finite element mesh fineness when computing the Risk of Rupture of an operational structure by the numerical method. This relation may be used to correct excessive conservatism associated with the assumption of uniform tension throughout the volume.

3.1.3 INTEGRATING STRUCTURAL AND STATISTICAL ANALYSES

The finite element (FE) structural analysis is used to compute the Risk of Rupture by numerical approximation of the R-integral. A representative tensile stress is selected for each finite element and assumed to act uniformly throughout that element volume (see the uniform tension case discussed in subsection 3.1.2.1). The Risk of Rupture of the total structure, R_c, is computed by the summation:

$$R_c = \sum_{V_T}\left(\frac{\sigma_i}{\sigma_o}\right)^m \Delta V_i$$

For initial computation, the maximum principal tensile stress in the element should be used for σ_i, giving an inherently conservative result. Other options for σ_i are the average tensile stress, a selected characteristic stress between the maximum and minimum, or a substructured detailed numerical integration of the R-integral within the element. For biaxial principal tension, the summation is carried out separately for each principal stress.

This equation for R_c lends itself to simple postprocessing analysis using the finite element stress output files and the element geometry files for individual volume computation. If the analysis prediction, based on the maximum element tensile stress, leads to an acceptable Factor of Safety, then there is no need to proceed further, since a more accurate analysis will produce an even lower Risk of Rupture and higher margins. This approximation of component Risk of Rupture is more conservative for higher stress gradients.

The i-th element's tensile stress (its maximum or another representative element stress) should be normalized to the maximum (critical) operating tensile stress in the operational component as

$$\sigma_i = \sigma_{C_{max}} K_i$$

Substituting in the prior equation gives the component Risk of Rupture at design operating conditions:

$$R_c = \left[\frac{\sigma_{c\,(max)}}{\sigma_o}\right]^m \sum_{V_T} (K_i^m)\, \Delta V_i$$

This computation is now combined with data from test samples of the brittle material, which provide reference strength values with information on the Weibull modulus, the scatter, and the effects of size and stress distribution. Test specimens and specimen data analysis are addressed in a subsection 3.2. The specimens are most likely to be beams in bending, whose R-value is given directly by equations such as those in subsection 3.1.2.2. We note that, in the general case, the Risk of Rupture of the test specimens will be known from direct analysis and evaluation of the R-integral:

$$R_{test} = \int_V \left(\frac{\sigma}{\sigma_o}\right)^m dV$$

The stress anywhere in the test specimen can be expressed in terms of the maximum stress in the test specimen (e.g., the maximum outer fiber stress in a beam) and a dimensionless geometric function, $F(x,y,z)$, describing the applied stress distribution. The specimen volume subject to tension, V_T, is also used for normalization, leading to a nondimensional geometric function to be integrated:

$$R_{test} = \left[\frac{\sigma_{test(max)}}{\sigma_o}\right]^m V_T \int_{V_T} [F(x,y,z)]^m \frac{dV}{V_T}$$

For most common types of tests and specimen shapes, this function is integrable in closed form.

3.1.4 ESTIMATING COMPONENT STRENGTH

The Risks of Rupture of the test specimens and of the large component are equated in order to compare the strengths at the same probability of failure. The test specimen strength, $\sigma_{test(max)}$, and the maximum operating stress, $\sigma_{c(max)}$, are known. Thus, we have

$$\frac{R_{test(max)}}{R_{c(max)}} = \frac{\left[\frac{\sigma_{test(max)}}{\sigma_o}\right]^m V_T \int_{V_T} [F(x,y,z)]^m \frac{dV}{V_T}}{\left[\frac{\sigma_{c\,(max)}}{\sigma_o}\right]^m \sum (K_i^m)\, \Delta V_i} = 1$$

The scaling factor, σ_o, is never explicitly evaluated, but is eliminated by the ratio of the two Risks of Rupture given by the previous equation. The relative strength, shown below, gives the strength of the large brittle structural component in terms of the test specimen strength. This strength prediction accounts for the effects of size, stress distribution, and statistical scatter (implicit in the Weibull modulus, *m):*

$$\frac{\sigma_{c\,(max)}}{\sigma_{test(max)}} = \left(\frac{V_T \int_{V_T} [F\,(x,y,z)]^m \frac{dV}{V_T}}{\sum\,(K_i^m)\,\Delta V_i} \right)^{(1/m)}$$

The total component volume that is subject to tension, V_{cT}, is the sum of the element volumes, ΔV_i, and these volumes may be normalized also, using $Q_i = \Delta V_i / V_{cT}$, giving the normalized expression:

$$\frac{\sigma_{c\,(max)}}{\sigma_{test(max)}} = \left(\left(\frac{V_{test}}{V_{cT}} \right) \frac{\int_{V_T} [F\,(x,y,z)]^m \frac{dV}{V_T}}{\sum\,(K_i^m) Q_i} \right)^{(1/m)}$$

3.1.4.1 EXAMPLE OF ANALYSIS PROCEDURE - INITIAL DATA

This example utilizes methods and computations that are discussed in more detail in subsection 3.2. Consider a large spacecraft mirror made of ultralow expansion (ULE) glass. Reference strength and statistics are taken from laboratory tests of mirror quality uniform bend specimens, 0.125 in. high by 0.25 in. wide, with a 2 in. center span, giving a total midspan volume, V_{ub}, of 0.0625 in^3. Surface preparation is representative of the flight mirror. Groups of at least 15, preferably 25 to 30, specimens are tested. The statistical analysis is based on pooling observations from various types of tests, as discussed in subsection 3.2.1.3, and fitting the median-normalized distribution with an appropriate Weibull distribution and *m*-value. The data for ULE glass show a scatter of 20% to 25% coefficient of variation, which corresponds to $m = 5$. For this example, the ULE glass test specimen median strength, σ_{UB}, is 10 ksi. For a sample size of 25 specimens and $m = 5$, the A-allowable knockdown factor is about 0.26, as shown in subsection 3.2.1.4.

3.1.4.2 EXAMPLE - DETAILED COMPUTATIONS FOR FACTOR OF SAFETY

The Risk of Rupture for the test specimens is given by:

$$R_{UB} = \left(\frac{\sigma_{UB}}{\sigma_o} \right)^m \left(\frac{V_{UB}}{2(m+1)} \right)$$

and the Risk of Rupture for the component is given by:

$$R_c = \left(\frac{\sigma_{c\,(max)}}{\sigma_o} \right)^m \sum (K_i^m) \Delta V_i$$

These Risks of Rupture are equated to give the strength relation

$$\frac{\sigma_{c(max)}}{\sigma_{test(max)}} = \left[\left(\frac{V_{UB}}{2}\right)\frac{1}{(m+1)\ \Sigma\left(K_i^{\,m}\right)\Delta V_i}\right]^{\frac{1}{m}}$$

The output files from the finite element structural analysis of the mirror were postprocessed (see Appendix A) to give the value $\Sigma\ (Ki)^m\ Vi = 0.113$ which, in turn, leads to the predicted component strength relative to the test specimens

$$\frac{\sigma_{c(max)}}{\sigma_{test(max)}} = \left[\left(\frac{0.0625}{2}\right)\frac{1}{(6)(0.113)}\right]^{\left(\frac{1}{5}\right)} = 0.54$$

The A-level design stress for the component is 10,000 × 0.54 × 0.26 = 1404 psi. In this particular instance, an operating peak stress of about 1000 psi in the component would correspond to a 1.4 Factor of Safety. This Factor of Safety is inherently conservative, being based on the Weibull volume flaw distribution. Note that the projected component Factor of Safety of 1.4 corresponds to a Factor of Safety of 7 from the small test specimens' strength.

3.2 PROCEDURES

Tests conducted on suitable small specimens are used to determine the Weibull modulus for the material. Data processing methods allow the pooling of tests from several different types of specimen and sizes. Pooling also provides a more confident basis for selecting the Weibull modulus, permitting a lower knockdown factor for the A-level design allowable. An A-level design allowable factor is determined for the test specimens (subsection 3.2.1.4) on the basis of the corresponding probability for the normal distribution. The Risk of Rupture integral (R-integral) is determined from the geometry of the test specimen and the applied test stresses. The R-integral is estimated for the large operational component, and the relative strength of the large structure is computed by the equations of subsection 3.1.4. This computation yields the A-allowable strength value of the large structural component, and is compared to the operational stresses to determine the Factor of Safety. The procedure contains several aspects that make it inherently conservative, as discussed in the following subsections. Appendix C illustrates some of the statistical strength effects and the data processing.

3.2.1 DEVELOPING MATERIAL DATA

The following subsections address test specimen configuration, recommended number of test specimens, data normalization, and A-level allowable.

3.2.1.1 TEST SPECIMEN CONFIGURATION

The typical type of specimen used to generate design data for brittle glassy materials is the flex bar. Tests may be conducted in simple bending (3 point) or in uniform bending (4 point). Specimens may be rectangular or circular cross sections. Both length and cross-section size may be changed to explore the effects of size and stress distribution (ASTM flex testing guidelines are available). The specimen material should be representative of the large structure. Surface preparation, texture, and environment should simulate that of the operational component. It is well known that surface etching can substantially increase the strength of glass, and that surface coatings can preserve this improvement for substantial periods of time in ambient environments. If such strength is critical for the flight component, and extended storage is a possibility, then witness specimens should be prepared and kept in the same environments as the flight article. These specimens may be tested to verify preflight strength and margins of safety.

3.2.1.2 GUIDELINE FOR NUMBER OF TEST SPECIMENS

The Weibull modulus of brittle materials such as glass and ceramics varies typically between $m = 4$ and $m = 10$, with polished glass giving values of around 5. Controlled surface grinding and sawing decreases the strength and also the scatter, so that the Weibull modulus may increase substantially from the values of highly polished defect-free surfaces. The test articles should be processed to represent the critical surfaces of flight hardware.

In order to develop a guideline for sample size, a Monte Carlo study was conducted (see Appendix B) in which random samples were drawn from an ideal Weibull distribution with $m = 5$. Replicate sample sizes of 10, 15, and 25 were generated and analyzed to estimate Weibull modulus.

These semiempirical Monte Carlo trials show that the parent distribution is reasonably recovered from samples of 15, although, of course, consistency is improved with the larger sample size. Therefore, the sample size should exceed 15 for each particular test type. Samples of 30 are preferred, especially if A-level design criteria are to be rigidly imposed. Normalization and pooling techniques may be used to develop a larger sample size for improving the Weibull modulus estimate and also to decrease the A-level knockdown factor.

To verify the consistency between the statistical variations and the effects of size and stress distribution, tests should be conducted with different sizes and stress distributions. These different test groups can be pooled by normalization to create a larger population base for statistical estimates.

3.2.1.3 DATA NORMALIZATION AND POOLING

The classical form of the Weibull distribution is

$$S = exp - \left[\int_V \left(\frac{\sigma}{\sigma_o}\right)^m dV \right]$$

As noted in subsection 3.1.3, the exponential part of the distribution can be rewritten in terms of a reference maximum stress and a dimensionless geometric function that accounts in general for specimen shape and applied stress distribution:

$$R = \left\{ \left(\frac{\sigma}{\sigma_o}\right)^m V \int_{V_T} \left[F(x,y,z)\right]^m \frac{dV}{V} \right\} = Ln(1/S)$$

The median strength value, σ_{med}, corresponding to S = 0.5, is given by

$$Ln(2) = \left\{ \left(\frac{\sigma_{med}}{\sigma_o}\right)^m V \int_{V_T} \left[F(x,y,z)\right]^m \frac{dV}{V} \right\}$$

Eliminating σ_o gives the general median normalized Weibull distribution:

$$S = exp - \left[Ln(2) \left(\frac{\sigma}{\sigma_{med}}\right)^m \right]$$

This normalized distribution allows pooling different sets of data on the same material in order to estimate the Weibull modulus, m. Each group of data is normalized to the respective median value, and then combined into a larger pooled population to investigate the statistical strength distribution. This larger sample size benefits the knockdown factor for A-level allowables (see subsection 3.2.1.4).

3.2.1.4 A-LEVEL DESIGN ALLOWABLES FACTORS

The conventional engineering design requirement for primary structure is the A-level design basis, defined to have a reliability of 99% with 95% confidence. A less stringent alternative is the B-level design basis, defined to have a reliability of 90% with 95% confidence. The design-allowable knockdown levels are dependent on the reference data base. The B-level is used for secondary structure, for multiple load path designs, and for cases where the design data base is known to be excessively scattered. A guideline knockdown factor table is provided in MIL-HBK5, based on the normal distribution. The knockdown factor is very sensitive to the number of test specimens. For example, the traditional 3-sigma knockdown, used by most designers, is equivalent to an A-level only if there are over 30 test specimens in the design data generation program.

The MIL-HBK5 guideline is based on the standardized normal distribution, and the A-level knockdown is given in terms of a number of standard deviations, KA, from the mean or median value. For metallic primary structures the variability is typically very small, on the order of a few percent at most, and the A-level allowable is around 90% of the median strength value. For brittle materials like glass and ceramics, the variability is much greater, and the normal distribution is physically inadmissible as a rational description of material behavior. Table 1 gives the A-level normal probability of survival, S, for various combinations of sample size and scatter. These A-level probabilities are used to establish the corresponding knockdown factors for the Weibull distribution having coefficients of variation CV and equivalent Weibull Moduli, *m*. The factors are tabulated and plotted below in Figure 3.

Table 1. A-level Equivalent Knockdown Factors for Weibull Distributions

	Sample size =	10	15	25	50	100	inf.
	No. of std, devs. KA =	3.98	3.52	3.16	2.86	2.68	2.33
	Normal Dist A-level S =	0.99997	0.99978	0.99921	0.99788	0.99632	0.9901
CV	m	Weibull A-Allowable as fraction of median, Ka					
0.24	5	0.134	0.2	0.258	0.314	0.351	0.428
0.15	8	0.285	0.365	0.429	0.485	0.52	0.588
0.12	10	0.366	0.447	0.508	0.561	0.592	0.654
0.086	14	0.488	0.562	0.616	0.661	0.688	0.739
0.043	28	0.698	0.75	0.785	0.813	0.829	0.859
0.033	36	0.756	0.8	0.828	0.851	0.865	0.889

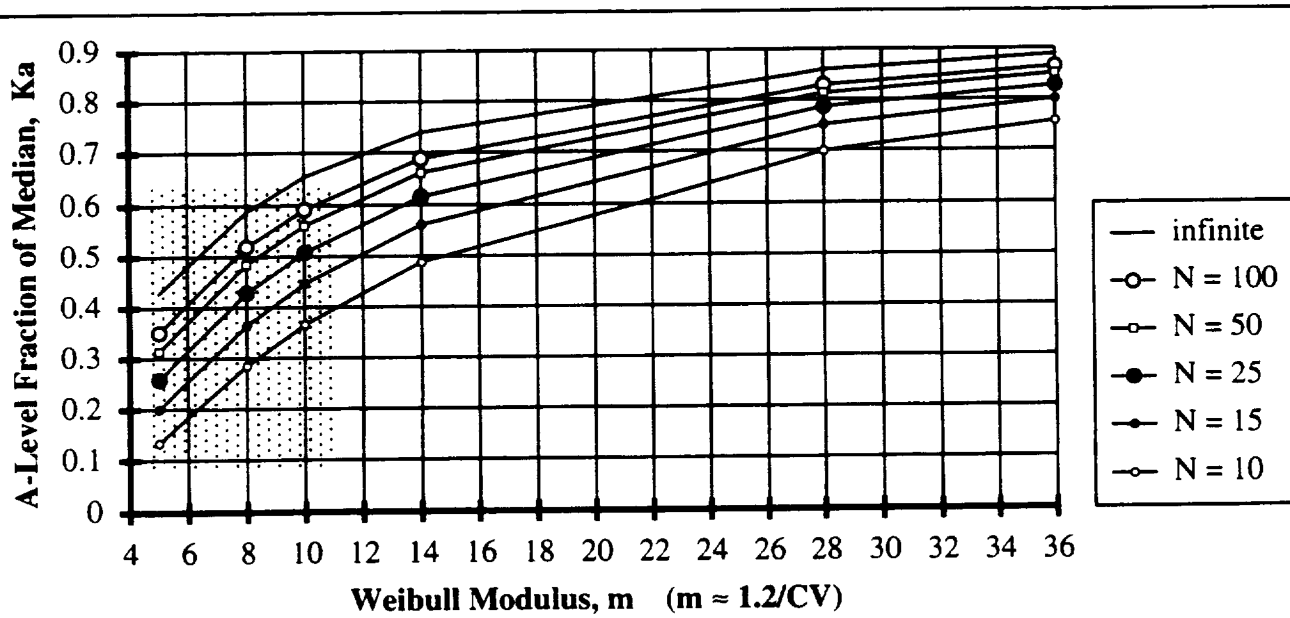

Figure 3. A-Level Allowables for Weibull Distribution

3.2.2 COMPUTING THE COMPONENT FACTOR OF SAFETY

This subsection presents several basic steps to arrive at the component Factor of Safety: (1) limiting the stress gradient in the finite elements; (2) determining the appropriate mesh fineness; (3) using the FE analysis to get the component Risk of Rupture ; (4) postprocessing analysis files; (5) estimating the component's expected strength, and (6) determining the component A-level allowable strength and corresponding Factor of Safety by comparison with test specimen data.

3.2.2.1 GUIDELINE FOR LIMITING ELEMENT STRESS GRADIENT

The conservative approach for numerical computation of the Risk of Rupture is to use the maximum principal tension in the finite element as effectively constant throughout the element. This approach may be excessively conservative when there is a significant stress gradient, and the resulting predicted strength reduction may be very large. A more accurate approach is necessary to avoid excessively pessimistic predictions. The computations in this subsection provide a more accurate analysis of the Risk of Rupture for a linear stress gradient within the element.

Consider the schematic finite element shown in Figure 4.

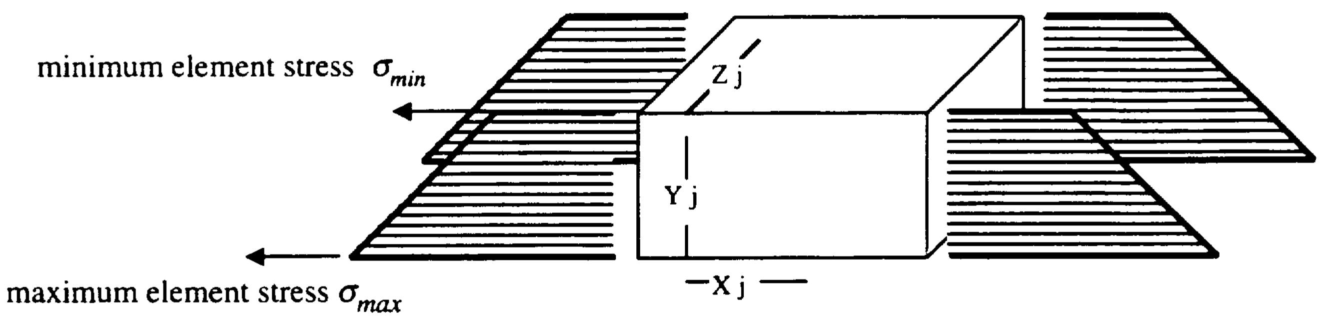

Figure 4. Finite Element with Linear Gradient

The Risk of Rupture is defined as

$$R = \int_V \left(\frac{\sigma}{\sigma_o}\right)^m dV$$

where $dV_j = Zj\, Xj\, dY$, and the stress varies only in the Y-direction

$$R = \frac{1}{(\sigma_o)^m} \int_{Y_j} \left[\sigma_{min} + (\sigma_{max} - \sigma_{min}) \frac{Y}{Y_j}\right]^m Z_j X_j \, dy$$

Rewriting R in volume-normalized form (see subsection 3.1.3) gives

$$R = \frac{V_j}{(\sigma_o)^m} \int_{y=0}^{y=y_j} \left[\sigma_{min} + (\sigma_{max} - \sigma_{min})\frac{Y}{Y_j}\right]^m \frac{dY}{Y_j}$$

Letting $W = Y/Y_j$, $A = \sigma_{min}$, and $B = \sigma_{max} - \sigma_{min}$, we have

$$R = \frac{V_j}{(\sigma_o)^m} \int_{W=0}^{W=1} (A + BW)^m \, dW$$

Integration and rearrangement lead to the following expression, which is equated to the Risk of Rupture for equivalent uniform tension throughout the element volume

$$R = V_j \left(\frac{\sigma_{max}}{\sigma_o}\right)^m \left[\frac{1 - \left(\frac{\sigma_{min}}{\sigma_{max}}\right)^{(m+1)}}{(m+1)\left(1 - \frac{\sigma_{min}}{\sigma_{max}}\right)}\right] = V_j \left(\frac{\sigma_{eq}}{\sigma_o}\right)^m$$

This expression defines an equivalent uniform stress, σ_{eq}, which produces the same Risk of Rupture as the linear stress gradient. The graph of this relation, shown in Figure 5, provides a basis for more refined R-integral computation, if needed. Using the uniform equivalent stress computation in regions of linear stress gradients will give more accurate and less pessimistic estimates:

$$\frac{\sigma_{eq}}{\sigma_{max}} = \left[\frac{1 - \left(\frac{\sigma_{min}}{\sigma_{max}}\right)^{(m+1)}}{(m+1)\left(1 - \frac{\sigma_{min}}{\sigma_{max}}\right)}\right]^{\frac{1}{m}}$$

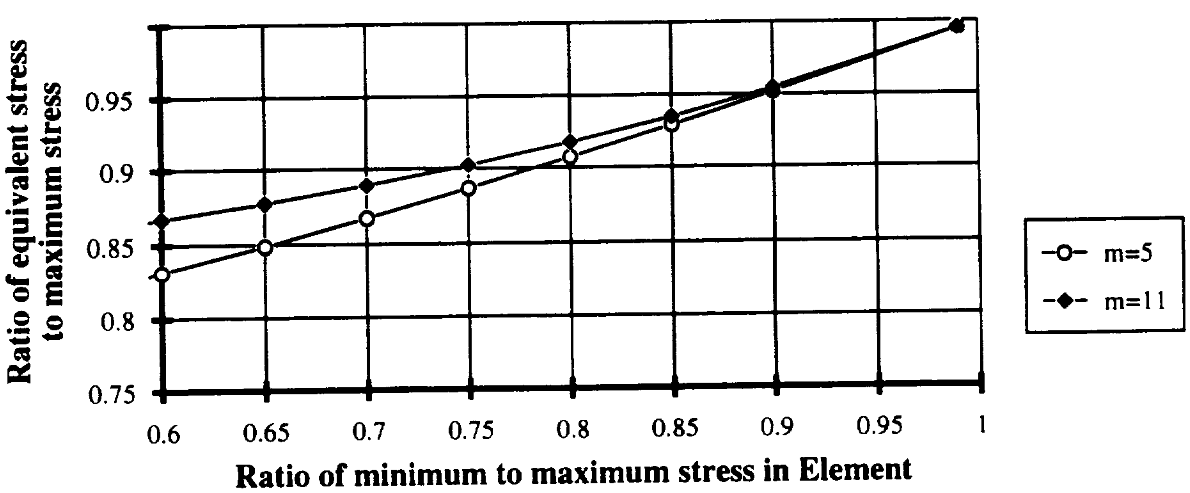

Figure 5. Equivalent uniform stress for linear gradient

3.2.2.2 GUIDELINE FOR DETERMINING MESH FINENESS

For most cases of finite-element structural analysis of large, brittle components, the finite element mesh will be sufficiently fine for adequate estimates of the Risk of Rupture by the maximum principal tensile stress acting uniformly in the element volume. Where the stress varies significantly through the element, the previous subsection provides a guideline to improve the accuracy of the Risk of Rupture computation. If gradients through the finite elements are sharp and possibly nonlinear, or highly skewed with respect to the element boundaries, it may be appropriate to remesh the FE grid more finely in the local tensile stress regions or to introduce conservative analysis in this region.

If the ratio of minimum-to-maximum tensile stress in an element is less than 0.80, or the stress gradient is not congruent with the element boundaries, or the correction of subsection 3.2.2.1 is not deemed adequate, the finite element mesh should be refined. It is recommended that minimum-to-maximum stress ratio exceed 0.80.

3.2.2.3 FE ANALYSIS OF COMPONENT AND R-INTEGRAL

Appendix A provides a computerized procedure to estimate the Risk of Rupture from the FE stress analysis. The procedure uses the maximum principal tension uniformly distributed within the element volume. The Weibull modulus, *m*, must be derived from specimen testing.

The FE analysis files provide the volume of each element, the principal tensile stresses, and the average tensile stress in the element, the *K* value (ratio of element peak tension to the overall peak tensile stress in the component). The individual element Risk of Rupture for the i-th element is given by

$$R_i (\sigma_o)^m = (K_i)^m (\Delta V_i)$$

The ultimate objective is the component Risk of Rupture, Rc, given by the equation noted in subsection 3.1.3, with V_T denoting the whole volume in tension:

$$R_c = \left[\frac{\sigma_{c\,(max)}}{\sigma_o}\right]^m \sum_{V_T} (K_i^m)\, \Delta V_i$$

3.2.2.4 FE ANALYSIS POSTPROCESSING

A postprocess procedure may be set up, using Appendix A as a guide, to compute and tabulate the parameters discussed in the previous paragraph: the volume, maximum and minimum principal tensions in each element, the equivalent element stress (subsection 3.2.2.1), the *K* value, and the individual scaled Risk of Rupture value. The computations should be presented in a table of file columns, as shown below.

The columns of individual finite element volume, and total scaled Risk of Rupture results should be added to give the total component volume and scaled Risk of Rupture.

An example tabulation for each element is given below (numbers are arbitrary, m = 5, see Appendix A):

elem. No.	elem. vol.	max. tensile	min. tensile	equiv. tensile	K-ratio	K^m	$K^m \Delta V_i$
112	0.002	3000	2400	2715	0.93	0.696	0.00139
	...						...
	total vol.						total

This table applies only to elements in tension, and there will be two columns for K if both principal stresses are tensile, as noted in subsection 3.1.3.

3.2.2.5 DETERMINING THE COMPONENT FACTOR OF SAFETY

This procedure is identical with that shown in subsection 3.1.4.1, where equating the specimen and component Risks of Rupture gives the ratio of expected strengths at the same probability of failure. The test specimen Risk of Rupture is usually determined in closed form, in terms of a geometric function reflecting the tensile stress distribution throughout the specimen, which is usually simple enough to allow the integration, as discussed in subsections 3.1.2 and 3.1.4. The relative reference strength of the component to the specimen is given by

$$\frac{\sigma_{c\,(max)}}{\sigma_{test(max)}} = \left(\frac{V_T \displaystyle\int_{V_T} \left[F\,(x,y,z)\right]^m \frac{dV}{V_T}}{\displaystyle\sum_{V_T} (K_i^m)\,\Delta V_i} \right)^{(1/m)}$$

The integrand, which varies with test type, is denoted by $F*$, and the component strength is given by:

$$\sigma_{c\,(max)} = \sigma_{test\,(max)} \left[\frac{V_T\,F*}{\displaystyle\sum_{V_T} (K_i)^m \Delta V_i} \right]^{\frac{1}{m}}$$

In order to estimate the Factor of Safety, the reference component strength, $\sigma_{c(max)}$, in the above equation must be reduced to the A-level allowable strength in the component. The A-level allowable component strength, $\sigma_{c\ allow}$, is determined from the knockdown factors (denoted here by K_a) defined in subsection 3.2.1.4 and Figure 3.

The Factor of Safety is given by the ratio of A-level allowable strength to the maximum operational stress, $\sigma_{op(max)}$, in the component:

$$\mathrm{FOS} = \frac{\sigma_{c\ allow}}{\sigma_{op(max)}} = \frac{K_a\left[\sigma_{test(max)}\right]}{\sigma_{op(max)}} \left[\frac{V_T\,F*}{\displaystyle\sum_{V_T} (K_i)^m \Delta V_i} \right]^{\frac{1}{m}}$$

A particular form of this computation was used in the example of subsection 3.1.4.1.

This equation addresses the scatter and allowable knockdown factors for the Weibull distribution, through K_a. It also addresses the effect of stress distribution and size difference between test specimens and operational component, through the bracketed factor and the Weibull modulus, m.

APPENDICES

A. POST-PROCESSING ANALYSIS FOR ESTIMATING THE R-INTEGRAL

B. GUIDELINE FOR SAMPLE SIZE BY MONTE CARLO SIMULATION

C. ILLUSTRATIVE GLASS TESTING

APPENDIX A

POST-PROCESSING ANALYSIS FOR ESTIMATING THE R-INTEGRAL

A.1 User Manual for Risk of Rupture Code

A.2 Finite Element Analysis Post-processing

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APPENDIX A.1

USER MANUAL FOR
RISK OF RUPTURE CODES

TABLE OF CONTENTS

1. Introduction

Previous studies (References 1 and 2) indicate that the strength of a brittle structure can be estimated by determining the term $\Sigma K^m{}_i V_i$, using results of a finite element analysis of the structure and Weibull parameter. The process of calculation of this summation is very time-consuming and therefore it needs to be computerized. A computer program, called WEIBULL, was developed for this purpose. This program will determine the subject summation if the volume and corresponding tensile stress of each element in the analysis model are known. Another computer program, called READFILE, was developed to transfer the values of volumes and stresses of all elements in the model, from the structural analysis output file, to two data files which will be used later as input files for the WEIBULL program.

This manual will familiarize you with the basics of the two programs READFILE and WEIBULL. It will help you get started in two steps. First, it will provide a quick overview of the two programs. Second, it will guide you through a hands-on example. Listings of the two programs are also included in the appendix.

2. Overview Of The READFILE Program

READFILE is a computer program written in VAX C language. Its function is to scan through the output file of the NASTRAN analysis run and transfer all information concerning the volumes and corresponding stresses of all elements in the model to two separate data files.

The program was prepared as generally as possible so that it can be used to obtain any data for any group of elements, and can be used for other structural analysis output files with little modifications. To serve that purpose, the program was developed such that it can read in any file (INPUT FILE) and write it, or a portion of it, in the form of two columns (in our application, one column is for the element ID numbers, the other is for either the corresponding volumes or stresses), to another file (OUTPUT FILE). To allow the user the option to name his/her output file, the names of the two input and output files are to supplied in the command line that invokes the program, However,to pass the file names to the program as arguments, the program must be installed as a DCL foreign command. As explained in page 5-16 of "Guide to VAX C" (for VAX C Version 3.0), the program name can be assigned to a symbol that is later used to invoke the program. For example, to assign the program to the symbol ECHO, the following command can be used:

```
$ ECHO == "$ DISK$AES204:[D4300.NGUYENT]READFILE.EXE"
```

Now, the program can be run by typing at the command line the following:

```
$ ECHO InputFileName OutputFileName
```

Example: `$ ECHO NASTRAN.F06 VOLUME.DAT`

If the user specifies an existing name for the output file, data will be added to the end of the given file. Next, the program will prompt the user for additional data that locate the portion in the Input File to be read. These data include:

1. The string (FIRST STRING) locating the start of the file portion of interest, e.g. PAGE.
2. The farthest-to-the-left string (SECOND STRING) from which the position of the two columns to be read are referenced.
3. The starting value of a group of incrementing numbers attributing to the first string, e.g. StartingPageNumber.
4. The ending value of the group of incrementing numbers attributing to the first string, e.g. EndingPageNumber.
5. The number of lines from the line containing the first string to the top line of the two subject columns (vertical reference for both columns).
6. The number of character spaces from the beginning of the second string to the beginning of the first column (horizontal reference for the first column).
7. The number of character spaces from the beginning of the second string to the beginning of the second column (horizontal reference for the second column).

Figure 1 shows partial listing of a typical data file with the required data input for the program READFILE.

There might be other unnecessary data that coincidentally fit the given locating data input. These data will also be displayed in the two columns and therefore, they must be removed by editing the output files before the files can be used as input files for the program WEIBULL.

WEIBULL PARAMETER STUDY
MIRROR

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1 2 3 4 5 6 7

12345678901234567890

ELEMENT TYPE - QUAD4

ELEMENT ID	AREA	VOLUME	ELEMENT ID	AREA	VOLUME	ELEMENT ID	AREA	VOLUME	ELEMENT ID	AREA	VOLUME
161	.046032	.011508	162	.046032	.016111	163	.154977	.023246	164	.154977	.023246
165	.155767	.023365	166	.155739	.023361	167	.155796	.023369	168	.155725	.023359
178	.131601	.0329	180	.131601	.04606	187	.155223	.023283	188	.155222	.023283
189	.110597	.027649	192	.110597	.038709	195	.831455	.124718	202	.155805	.023371
203	.155939	.023391	204	.155805	.023371	205	.155939	.023391	206	.155939	.023391
207	.155939	.023391	208	.155805	.023371	209	.155939	.023391	210	.155939	.023391
211	.155805	.023371	212	.155939	.023391	213	.155939	.023391	214	.155222	.023283
215	.155222	.023283	216	.155222	.023283	217	.155222	.023283	218	.055965	.013991
219	.055965	.019588	229	.131437	.032859	231	.131437	.046003	244	.154977	.023246
251	.154977	.023246	261	.110477	.027619	266	.110477	.038667	269	.831431	.124715
277	.155743	.023361	278	.155743	.023361	279	.155743	.023361	280	.155743	.023361
281	.155743	.023361	282	.155743	.023361	283	.155743	.023361	284	.155743	.023361
285	.155743	.023361	290	.155743	.023361	291	.155743	.023361	292	.155743	.023361
293	.154977	.023246	294	.154977	.023246	295	.154977	.023246	296	.154977	.023246
297	.055978	.013994	298	.055978	.019592	309	.155767	.023365	326	.155739	.023361
327	.155796	.023369	328	.155725	.023359	329	.155805	.023371	330	.155805	.023371
331	.155805	.023371	332	.155805	.023371	339	.155805	.023371	340	.155805	.023371
353	.155805	.023371	354	.155805	.023371	385	.045937	.016078	386	.110611	.038714
390	.045937	.011484	391	.110611	.027653	393	.055887	.019561	396	.055887	.013972
399	.045927	.016074	400	.045927	.011482	401	.155767	.023365	402	.155796	.023369
403	.862044	.215511	408	3.85966	.578949	412	.155805	.023371	413	.155805	.023371
414	.155805	.023371	415	.155805	.023371	418	.07973	.019932	421	.07973	.027905
433	.622392	.093359	436	.62228	.093342	438	.155939	.023391	439	.155939	.023391
440	.155939	.023391	441	.155939	.023391	442	.13185	.032963	443	.13185	.046148
444	.862199	.21555	449	3.85961	.578942	453	.155743	.023361	454	.155743	.023361
455	.155743	.023361	456	.155743	.023361	459	.079857	.019964	468	.079857	.02795
497	.622337	.093351	501	.622283	.093342	503	.154977	.023246	504	.154977	.023246
505	.154977	.023246	506	.154977	.023246	507	.131437	.032859	508	.131437	.046003
509	.622212	.093332	516	.622444	.093367	528	.131677	.046087	533	.131677	.032919
557	.110692	.038742	558	.155767	.023365	561	.155796	.023369	562	.110692	.027673
565	.831911	.124787	584	.055919	.019572	585	.155739	.023361	586	.155805	.023371
587	.155591	.023339	608	.155771	.023366	609	.155739	.023361	610	.155805	.023371
611	.155591	.023339	612	.155725	.023359	613	.155805	.023371	614	.155581	.023337
615	.155771	.023366	616	.155796	.023369	617	.155725	.023359	618	.155805	.023371
619	.155581	.023337	620	.155796	.023369	621	.055919	.01398	625	.674477	.101172
626	.879145	.219786	627	4.03624	.605436	628	.155222	.023283	629	.155222	.023283
630	.155222	.023283	631	.155222	.023283	632	.079533	.019883	633	.079533	.027837
636	.622145	.093322	637	.842298	.210574	638	3.86534	.5798	639	.155743	.023361
640	.155743	.023361	641	.155743	.023361	642	.155743	.023361	643	.079857	.019964
644	.079857	.02795	660	.292011	.102204	668	.291567	.102048	669	.861808	.215452
671	.622438	.093366	673	3.85952	.578929	689	.291509	.102028	696	.291509	.102028
698	.862392	.215598	700	.622503	.093375	702	3.86001	.579002	711	3.85983	.578974
715	.861799	.21545	720	.079714	.0279	721	.155805	.023371	722	.155805	.023371
723	.155805	.023371	724	.155805	.023371	727	.079714	.019929	739	.622412	.093362
742	.622279	.093342	744	.131628	.04607	745	.155591	.023339	746	.155591	.023339

Data Input for the program READFILE:

PAGE	ELEMENT	66	75	8	3	20
1	2	3	4	5	6	7

Figure 1 - A typical data file with required data input for the program READFILE

3. Overview Of The WEIBULL Program

WEIBULL is written in VAX FORTRAN language. Input files to this program consist of the 'VOLUME.DAT' and the 'STRESS.DAT' data files which are generated by the program READFILE. After reading these two input files, knowing the volumes and the corresponding stresses of all elements in the model, the program will determine the maximum tensile stress of the entire structure. Finally, the program will calculate the summation $\Sigma K^m_i V_i$ where V_i is the volume of the ith element and K_i is the ratio of the tensile stress of the ith element to the maximum tensile stress of the entire structure. The first two DIMENSION statements in the program can be modified for larger analysis problem. As shown, the program can take 2000 elements with 2000 values for volumes and 4000 values for stresses (There are a maximum and a minimum stress value for each element); also, the element ID numbers for these 2000 elements cannot exceed the number 3000.

To execute this program, the following command can be typed:

```
$    RUN WEIBULL
```

The program will prompt the user for the value of Weibull parameter:

```
ENTER WEIBULL PARAMETER (#.#)
```

After the user enters the Weibull parameter (using format #.#, e.g. 5.2), the program will perform the calculations and write out the results in a file called WEIBULL.OUT. The calculated value of $\Sigma K^m_i V_i$ in this file can be used in estimating the strength of the structure as described previously in References (1) and (2).

4. Example

To process the portion of NASTRAN output file as shown in Table 1, the following steps can be performed:

- To assign the program READFILE to a symbol called ECHO:

  ```
  $ ECHO == "$ DISK$AES204:[D4300.NGUYENT]READFILE.EXE"
  ```

- To obtain a file containing volumes of all elements in the model:

  ```
  $ ECHO NASTRAN.F06 VOLUME.DAT
  Input control parameters:  PAGE ELEMENT 65 65 51 3 20
  ```

  This computer run will create a data file called VOLUME.DAT that contains volumes for the first column of elements as shown in Table 2. Note that this file might have to be edited to remove any undesired data. Additional runs have to be performed for the remaining elements. Data inputs for these runs are:

  ```
  Input control parameters:  PAGE ELEMENT 65 65 51 35 52
  Input control parameters:  PAGE ELEMENT 65 65 51 67 84
  Input control parameters:  PAGE ELEMENT 65 65 51 99 116
  Input control parameters:  PAGE ELEMENT 66 75 8 3 20
  Input control parameters:  PAGE ELEMENT 66 75 8 35 52
  Input control parameters:  PAGE ELEMENT 66 75 8 67 84
  Input control parameters:  PAGE ELEMENT 66 75 8 99 116
  ```

  Table 3 shows the complete VOLUME.DAT file.

- To obtain a file containing principal stresses of all elements in the model:

  ```
  $ ECHO NASTRAN.F06 STRESS.DAT
   Input control parameters:  PAGE ELEMENT 305 409 9 3 86
  ```

  Table 4 shows a partial listing of the STRESS.DAT file.

- To run program WEIBULL:

  ```
  $ RUN WEIBULL
   ENTER WEIBULL PARAMETER (#.#)
   5.0
  ```

  Table 5 shows the results of this run (file WEIBULL.OUT).

TABLE 1 - An Excerpt From NASTRAN Output File

This excerpt shows how volumes and resulted stresses of all elements in the model are presented in the NASTRAN output file.

SEQUENCE PROCESSOR OUTPUT

THERE ARE 1002 POINTS DIVIDED INTO 1 GROUP(S).

CONNECTION DATA

ELEMENT TYPE	NUMBER	ASSEMBLY TIME(SEC)
QUAD4	744	704.09
TRIA3	918	434.38

TOTAL MATRIX ASSEMBLY TIME FOR 1662 ELEMENTS IS 1138.47 SECONDS.

ORIGINAL PERFORMANCE DATA

SUPER(GROUP) ID	NO. GRIDS	AV. CONNECTIVITY	C-AVERAGE	C-RMS	C-MAXIMUM	P-GROUPS	P-AVERAGE	DECOMP TIME(SECS) (6.0 DOF/GRID)
0	1002	9.16	45.62	49.63	81	0	0.00	3519.774

RESEQUENCED PERFORMANCE DATA

SUPER(GROUP) ID	NO. GRIDS	AV. CONNECTIVITY	C-AVERAGE	C-RMS	C-MAXIMUM	P-GROUPS	P-AVERAGE	DECOMP TIME(SECS) (6.0 DOF/GRID)	METHOD
0	1002	9.16	54.06	57.62	84	0	0.00	4715.691	ACTIVE

0 -- AS THE ORIGINAL SEQUENCE FOR THE ABOVE GROUP IS BETTER, IT WILL BE RETAINED AND USED. -- 3519.774 ORIGINAL

ELEMENT TYPE = QUAD4

ELEMENT ID	AREA	VOLUME	ELEMENT ID	AREA	VOLUME	ELEMENT ID	AREA	VOLUME	ELEMENT ID	AREA	VOLUME
21	.155222	.023283	22	.155939	.023391	23	.155222	.023283	24	.155939	.023391
25	.155743	.023361	26	.154977	.023246	27	.155743	.023361	28	.154977	.023246
29	.155805	.023371	30	.155805	.023371	31	.155805	.023371	32	.155223	.023283
33	.155939	.023391	34	.155805	.023371	37	.155222	.023283	38	.155939	.023391
47	.155805	.023371	48	.155805	.023371	49	.155805	.023371	50	.155805	.023371
51	.155743	.023361	52	.154977	.023246	53	.155743	.023361	54	.155743	.023361
55	.155743	.023361	56	.155743	.023361	57	.154977	.023246	58	.155743	.023361
69	.155743	.023361	70	.155743	.023361	71	.155743	.023361	72	.155743	.023361
75	.04583	.011458	79	.110804	.027701	100	.04583	.016041	101	.110804	.038781
103	.055965	.013991	106	.055965	.019588	109	.045988	.011497	110	.045988	.016096
111	.155222	.023283	112	.155222	.023283	115	.046032	.011508	116	.110477	.027619
120	.046032	.016111	121	.110477	.038667	123	.055978	.013994	158	.055978	.019592

ELEMENT TYPE = QUAD4

ELEMENT ID	AREA	VOLUME	ELEMENT ID	AREA	VOLUME	ELEMENT ID	AREA	VOLUME	ELEMENT ID	AREA	VOLUME
161	.046032	.011508	162	.046032	.016111	163	.154977	.023246	164	.154977	.023246
165	.155767	.023365	166	.155739	.023361	167	.155796	.023369	168	.155725	.023359
178	.131601	.0329	180	.131601	.04606	187	.155223	.023283	188	.155222	.023283
189	.110597	.027649	192	.110597	.038709	195	.831455	.124718	202	.155805	.023371
203	.155939	.023391	204	.155805	.023371	205	.155939	.023391	206	.155939	.023391
207	.155939	.023391	208	.155805	.023371	209	.155939	.023391	210	.155939	.023391
211	.155805	.023371	212	.155939	.023391	213	.155939	.023391	214	.155222	.023283
215	.155222	.023283	216	.155222	.023283	217	.155222	.023283	218	.055965	.013991
219	.055965	.019588	229	.131437	.032859	231	.131437	.046003	244	.154977	.023246
251	.154977	.023246	261	.110477	.027619	266	.110477	.038667	269	.831431	.124715
277	.155743	.023361	278	.155743	.023361	279	.155743	.023361	280	.155743	.023361
281	.155743	.023361	282	.155743	.023361	283	.155743	.023361	284	.155743	.023361
285	.155743	.023361	290	.155743	.023361	291	.155743	.023361	292	.155743	.023361
293	.154977	.023246	294	.154977	.023246	295	.154977	.023246	296	.154977	.023246
297	.055978	.013994	298	.055978	.019592	309	.155767	.023365	326	.155739	.023361
327	.155796	.023369	328	.155725	.023359	329	.155805	.023371	330	.155805	.023371
331	.155805	.023371	332	.155805	.023371	339	.155805	.023371	340	.155805	.023371
353	.155805	.023371	354	.155805	.023371	385	.045937	.016078	386	.110611	.038714
390	.045937	.011484	391	.110611	.027653	393	.055887	.019561	396	.055887	.013972
399	.045927	.016074	400	.045927	.011482	401	.155767	.023365	402	.155796	.023369
403	.862044	.215511	408	3.85966	.578949	412	.155805	.023371	413	.155805	.023371
414	.155805	.023371	415	.155805	.023371	418	.07973	.019932	421	.07973	.027905
433	.622392	.093359	436	.62228	.093342	438	.155939	.023391	439	.155939	.023391
440	.155939	.023391	441	.155939	.023391	442	.13185	.032963	443	.13185	.046148
444	.862199	.21555	449	3.85961	.578942	453	.155743	.023361	454	.155743	.023361
455	.155743	.023361	456	.155743	.023361	459	.079857	.019964	468	.079857	.02795
497	.622337	.093351	501	.622283	.093342	503	.154977	.023246	504	.154977	.023246
505	.154977	.023246	506	.154977	.023246	507	.131437	.032859	508	.131437	.046003
509	.622212	.093332	516	.622444	.093367	528	.131677	.046087	533	.131677	.032919
557	.110692	.038742	558	.155767	.023365	561	.155796	.023369	562	.110692	.027673
565	.831911	.124787	584	.055919	.019572	585	.155739	.023361	586	.155805	.023371
587	.155591	.023339	608	.155771	.023366	609	.155739	.023361	610	.155805	.023371
611	.155591	.023339	612	.155725	.023359	613	.155805	.023371	614	.155581	.023337
615	.155771	.023366	616	.155796	.023369	617	.155725	.023359	618	.155805	.023371
619	.155581	.023337	620	.155796	.023369	621	.055919	.01398	625	.674477	.101172
626	.879145	.219786	627	4.03624	.605436	628	.155222	.023283	629	.155222	.023283
630	.155222	.023283	631	.155222	.023283	632	.079533	.019883	633	.079533	.027837
636	.622145	.093322	637	.842298	.210574	638	3.86534	.5798	639	.155743	.023361
640	.155743	.023361	641	.155743	.023361	642	.155743	.023361	643	.079857	.019964
644	.079857	.02795	660	.292011	.102204	668	.291567	.102048	669	.861808	.215452
671	.622438	.093366	673	3.85952	.578929	689	.291509	.102028	696	.291509	.102028
698	.862392	.215598	700	.622503	.093375	702	3.86001	.579002	711	3.85983	.578974
715	.861799	.21545	720	.079714	.0279	721	.155805	.023371	722	.155805	.023371
723	.155805	.023371	724	.155805	.023371	727	.079714	.019929	739	.622412	.093362
742	.622279	.093342	744	.131628	.04607	745	.155591	.023339	746	.155591	.023339

etc...

SUBCASE 2

S T R E S S E S   I N   T R I A N G U L A R   E L E M E N T S   (T R I A 3)

ELEMENT ID.	FIBRE DISTANCE	STRESSES IN ELEMENT COORD SYSTEM NORMAL-X	NORMAL-Y	SHEAR-XY	PRINCIPAL STRESSES (ZERO SHEAR) ANGLE	MAJOR	MINOR	VON MISES
2125	-1.750000E-01	4.110807E+01	-6.212570E+01	5.315520E+01	22.9206	6.358422E+01	-8.460185E+01	1.287625E+02
	1.750000E-01	1.510899E+02	7.836072E+01	4.434182E+01	25.3225	1.720714E+02	5.737916E+01	1.517486E+02
2126	-1.250000E-01	-7.366570E+01	-4.730748E+01	1.382998E+01	66.8098	-4.138274E+01	-7.959045E+01	6.894563E+01
	1.250000E-01	7.549992E+01	2.759513E+01	-1.055081E+01	-11.8865	7.772074E+01	2.537432E+01	6.864589E+01
2127	-1.750000E-01	-7.198855E+01	-9.406401E+01	-2.453019E+01	-32.8870	-5.612718E+01	-1.099254E+02	9.520529E+01
	1.750000E-01	8.318729E+01	7.079819E+01	-3.290694E+01	-39.6696	1.104777E+02	4.350783E+01	9.639294E+01
2128	-1.250000E-01	-4.371668E+01	-6.528427E+01	-1.316716E+01	-25.3414	-3.748095E+01	-7.152000E+01	6.196204E+01
	1.250000E-01	1.944046E+01	7.012230E+01	9.425488E+00	79.7987	7.181843E+01	1.774433E+01	6.479489E+01
2130	-1.250000E-01	-4.568937E+01	-2.442888E+01	-2.851418E+01	-55.2229	-4.627880E+00	-6.549036E+01	6.330343E+01
	1.250000E-01	1.283609E+01	3.354234E+01	2.522351E+01	56.1580	5.045481E+01	-4.076384E+00	5.261158E+01
2132	-1.750000E-01	-5.769400E+01	-5.315635E+01	-4.693439E+01	-46.3838	-8.435975E+00	-1.024144E+02	9.846778E+01
	1.750000E-01	4.215576E+01	5.197731E+01	1.025728E+01	57.7916	5.843876E+01	3.569431E+01	5.102197E+01

etc...

TABLE 2 - A Partial Listing of
File VOLUME.DAT

This listing shows the result of the first computer run of the program READFILE in reading the volumes of those elements listed in the first column of the NASTRAN output file as shown in Table 1.

21	.023283
25	.023361
29	.023371
33	.023391
47	.023371
51	.023361
55	.023361
69	.023361
75	.011458
103	.013991
111	.023283
120	.016111

TABLE 3 - A Complete Listing of File VOLUME.DAT

This listing shows the result of all the computer runs of the program READFILE in reading volumes of all elements listed in the NASTRAN output file as shown in Table 1.

21	.023283
25	.023361
29	.023371
33	.023391
47	.023371
51	.023361
55	.023361
69	.023361
75	.011458
103	.013991
111	.023283
120	.016111
22	.023391
26	.023246
30	.023371
34	.023371
48	.023371
52	.023246
56	.023361
70	.023361
79	.027701
106	.019588
112	.023283
121	.038667
23	.023283
27	.023361
31	.023371
37	.023283
49	.023371
53	.023361
57	.023246
71	.023361
100	.016041
109	.011497
115	.011508
123	.013994
24	.023391
28	.023246
32	.023283
38	.023391
50	.023371
54	.023361
58	.023361
72	.023361
101	.038781
110	.016096
116	.027619
158	.019592
161	.011508
165	.023365
178	.0329
189	.027649
203	.023391
207	.023391
211	.023371
215	.023283
219	.019588
251	.023246
277	.023361
281	.023361
285	.023361
293	.023246
297	.013994
327	.023369
331	.023371
353	.023371

TABLE 4 - A Partial Listing of File STRESS.DAT

This listing shows the result of the execution of the progam READFILE in reading the principal stresses of all elements in the model.

21	4.426900E-01
	7.970184E+01
22	5.133324E+01
	8.377534E+01
23	7.405219E+01
	7.253616E+01
24	7.167787E+01
	5.694920E+01
25	2.026727E+00
	1.028487E+00
26	-1.588351E+00
	1.703049E+00
27	-1.486169E-01
	1.591354E+01
28	5.494180E+00
	1.188584E+00
29	5.886485E+01
	3.975111E+01
30	7.369182E+01
	7.505077E+01
31	1.432576E+02
	6.465224E+01
32	6.482987E+01
	1.318105E+02
33	3.140154E+01
	6.927631E+01
34	6.429088E-01
	9.962685E+01
37	1.261949E+02
	-1.018380E+01
38	1.180051E+02
	5.399296E+01
47	6.312255E+01
	8.120956E+01
48	5.844344E+01
	2.862135E+01
49	7.362578E+00
	3.135316E+01
50	8.913837E+01
	9.577346E+01
51	7.964378E+00
	1.192653E+01
52	1.233275E+01
	1.989032E+01
53	1.030616E+00
	2.020785E+00
54	1.590810E+01
	-1.356768E-01
55	1.188783E+01
	7.946831E+00
56	-6.499645E+00
	-4.051637E+00
57	7.290546E+00
	-2.351501E+00
58	-4.044339E+00
	-6.470842E+00
69	1.848735E+00
	1.339524E+00
70	-1.491037E+00
	-2.181815E+00
71	1.184860E+00
	1.320040E+01
72	-4.207570E+00
	5.070053E-01
75	2.972058E+01
	5.701298E+01

TABLE 5 - A Partial Listing of File WEIBULL.OUT

This listing shows the result of the execution of the program WEIBULL using the VOLUME.DAT and STRESS.DAT files as input.

MAXIMUM TENSILE STRESS, PSI = [illegible]

ELEM	VOLUME	TENSILE STR	VI*(KI**M)
21	0.0232830	79.7018	0.94625E-06
22	0.0233910	83.7753	0.12197E-05
23	0.0232830	74.0522	0.65517E-06
24	0.0233910	71.6779	0.55924E-06
25	0.0233610	2.0267	0.10095E-13
26	0.0232460	1.7030	0.42083E-14
27	0.0233610	15.9135	0.30127E-09
28	0.0232460	5.4942	0.14706E-11
29	0.0233710	58.8648	0.20873E-06
30	0.0233710	75.0508	0.70320E-06
31	0.0233710	143.2576	0.17819E-04
32	0.0232830	131.8105	0.11706E-04
33	0.0233910	69.2763	0.47163E-06
34	0.0233710	99.6268	0.28986E-05
37	0.0232830	126.1949	0.94161E-05
38	0.0233910	118.0051	0.67636E-05
47	0.0233710	81.2096	0.10431E-05
48	0.0233710	58.4434	0.20136E-06
49	0.0233710	31.3532	0.89476E-08
50	0.0233710	95.7735	0.23797E-05
51	0.0233610	11.9265	0.71234E-10
-	-	-	-
-	-	-	-
-	-	-	-
2113	0.0435390	39.3876	0.52156E-07
2114	0.0402030	28.2241	0.90987E-08
2115	0.0804050	10.7030	0.14271E-09
2116	0.0402030	10.6147	0.68458E-10
2117	0.0402030	67.3811	0.70563E-06
2118	0.0804050	72.5601	0.20436E-05
2119	0.0402030	249.9942	0.49606E-03
2120	0.0599980	79.4675	0.24027E-05
2121	0.0184430	37.6463	0.17622E-07
2122	0.0184430	125.9023	0.73727E-05
2123	0.0137610	23.3820	0.12153E-08
2124	0.0137610	29.8033	0.40888E-08
2125	0.0376600	164.3263	0.57022E-04
2126	0.0269000	80.6225	0.11579E-05
2127	0.0295490	114.8054	0.74470E-05
2128	0.0211060	75.1257	0.63823E-06
2130	0.0142050	53.2308	0.76715E-07
2132	0.0198870	70.0470	0.42378E-06
0	0.0000000	0.0000	0.00000E+00

SIGMA(VI*(KI**M)) = 0.11311E+00

5. References

1. IOM #19/92, "Strength Estimte For Brittle (Glasslike) Structures Using Weibull Statistics And Finite Element Analysis", T. C. Nguyen to J. W. Provins, dated 8 June 92.

2. IOM #37/92, "Strength Estimate for DSP Primary Mirror Using Weibull Statistics And Finite Element Analysis - Sensor Element", T. C. Nguyen to J. W. Provins, dated 5 August 92.

3. IOM #42/92, "Brittle Structures Study - Post Processing Codes For Estimating Strength Of Brittle Structures", T. C. Nguyen to J. W. Provins, dated 24, September, 92.

6. Appendix

A.1.1 Listing of C program READFILE.C

A.1.2 Listing of FORTRAN program WEIBULL.FOR

APPENDIX A.1.1 - Listing of C Program
READFILE.C

```
/* PROGRAM READFILE.C

   This program is written in VAX C computer language. It reads in a file
   (INPUT FILE) and write it, or a portion of it, in the form of two
   columns, to another file (OUTPUT FILE).

   The names of the two INPUT & OUTPUT files are to be supplied in the
   command line that invokes the program. In addition, to pass the file
   names to the program as arguments, the program must be installed as a
   DCL foreign command. As explained in page 5-16 of "Guide to VAX C" (for
   VAX C Version 3.0), the program name can be assigned to a symbol that
   is later used to invoke the program. For example, if the symbol is ECHO,
   the program can be run by typing at the command line the following:

         $ ECHO InputFileName OutputFileName

   Data supplied to the program include:

     1. The string (FIRST STRING) locating the start of the file portion of
        interest, e.g. PAGE;
     2. The farthest-to-the-left string (SECOND STRING) from which the
        position of the two columns to be read are referenced;
     3. The starting value of the group of incrementing numbers attributing
        to the first string, e.g. StartingPageNumber;
     4. The ending value of the group of incrementing numbers attributing
        to the first string, e.g. EndingPageNumber;
     5. The number of lines from the line containing the first string to the
        top line of the two subject columns (vertical reference for both
        columns);
     6. The number of character spaces from the beginning of the second
        string to the beginning of the first column (horizontal reference
        for the first column);
     7. The number of character spaces from the beginning of the second
        string to the beginning of the second column (horizontal reference
        for the second column).

          Example: PAGE ELEMENT 64 72 7 3 20

*/
#include stdlib
#include stdio
#include string

#define TRUE 1
#define FALSE 0

main(int argc, char **argv)
{
   FILE *infile, *outfile;
   char *tmp, *ptr1, *ptr2, *ptr3;
   char line[140], str1[20], str2[20], outstr1[8]="", outstr2[14]="";
   int pgstrt, pgend, lnstrt, offset1, offset2;
   int first=TRUE;
   int pg=0;
   int ln=0;

   if(argc != 3)
   {
      printf("\n%s%s%s\n","Usage: ", argv[0], " infile outfile");
      exit(1);
   }
   infile=fopen(argv[1], "r");
   outfile=fopen(argv[2], "a");
   printf("\nInput control parameters: ");
   scanf("%s %s %d %d %d %d %d", str1, str2, &pgstrt, &pgend, &lnstrt, &offset1,
 &offset2);
```

```
   while(fgets(line, 140, infile) != NULL)
   {
      ++ln;
      if(line[0]=='\n')
         continue;
      if((tmp=strstr(line, str2)) != NULL)
      {
         if(first)
         {
            ptr1=tmp;
            first=FALSE;
         }
         else if(tmp<ptr1)
            ptr1=tmp;
      }
      if(strstr(line, str1))
      {
         ++pg;
         ln=1;
         continue;
      }
      if((pg>=pgstrt) && (pg<=pgend) && (ln>=lnstrt))
      {
         ptr2=ptr1+offset1;
         ptr3=ptr1+offset2;
         strncpy(outstr1, "\0", 8);
         strncpy(outstr1, ptr2, 7);
/*
   The two numbers 14 and 13 in the following two 'strncpy' functions can be
   changed to 11 and 10, respectively, to read the values of elementary volumes
   in the second column. Numbers 14 and 13 are for stress values.
*/
         strncpy(outstr2, "\0", 14);
         strncpy(outstr2, ptr3, 13);
         strncat(outstr1, outstr2, 14);
         fputs(outstr1, outfile);
         fputc('\n', outfile);
      }
   }
   free(outstr1);
   free(outstr2);
   fclose(infile);
   fclose(outfile);
}
```

APPENDIX A.1.2 - Listing of FORTRAN Program
WEIBULL.FOR

```
C----------------------------------------------------------------------
C  This program calculates the summation of Vi*(Ki**m) where Vi is
C  the volume of the ith element and Ki is the ratio of the tensile
C  stress of the ith element to the maximum tensile stress of the
C  entire structure.
C  Input files to this program consist of the 'VOLUME.DAT' and the
C  'STRESS.DAT' data files. These files can be generated from the
C  NASTRAN output file with the use of the program 'READFILE.C'.
C  After reading these two input files, knowing the volume and the
C  corresponding tensile stress of each individual element, the
C  program will determine the maximum tensile stress of the entire
C  structure.
C  Finally, the program will calculate the ratio Ki and the subject
C  summation in a do-loop.
C----------------------------------------------------------------------
      DIMENSION IELV(2000),IELS(4000),V(3000),S(3000)
      DIMENSION SEL(2),IS(4000),STRESS(4000)
      REAL M
      OPEN(UNIT=1,FILE='VOLUME.DAT',STATUS='OLD')
      OPEN(UNIT=2,FILE='STRESS.DAT',STATUS='OLD')
      OPEN(UNIT=3,FILE='WEIBULL.OUT',STATUS='NEW')
C----------------------------------------------------------------------
C  Receive value of Weibull parameter m from the terminal

      PRINT 5
  5   FORMAT(1X,'ENTER WEIBULL PARAMETER (#.#)')
      ACCEPT 10, M
 10   FORMAT(F3.0)
C----------------------------------------------------------------------
C  Read in volumes of all elements

      READ(1,20,END=25) (IELV(I),V(IELV(I)),I=1,2000)
 20   FORMAT(I5,2X,F10.6)
C----------------------------------------------------------------------
C  Read in the two stress values of all elements and also determine
C  the larger value between these two values. If the larger value is
C  negative, use zero (0) as the tensile stress for this particular
C  element.
C  Each element has only 1 value for volume but 2 values (minimum &
C  maximum) for stress. Therefore, if IMAX is the total number of
C  elements (determined after reading in VOLUME.DAT file), then the
C  total number of stress values to be read in is 2xIMAX.

 25   IMAX=I
      I=1
      N=0
      DO 60 J=1,(2*IMAX)
        READ(2,30,END=60) IS(J),STRESS(J)
 30     FORMAT(I5,2X,E13.6)
        N=N+1
        SEL(N)=STRESS(J)
        IF (N .EQ. 2) THEN
          IF (SEL(1) .GT. SEL(2)) THEN
            S(IELS(I))=SEL(1)
          ELSE
            S(IELS(I))=SEL(2)
          END IF
          IF (S(IELS(I)) .LT. 0.) S(IELS(I))=0.
          N=0
          I=I+1
        ELSE
          IELS(I)=IS(J)
        END IF
 60   CONTINUE
C----------------------------------------------------------------------
```

```
C  Determine the maximum tensile stress for the entire structure

      SMAX=0.
      DO 74 I=1,IMAX
        IF (S(IELS(I))-SMAX)74,74,72
 72     SMAX=S(IELS(I))
 74   CONTINUE
C------------------------------------------------------------------
C  Determine the summation of Vi*(Ki**m) and print out the results

      TOTSIG=0.0
      WRITE(3,75)SMAX
 75   FORMAT(/,31H MAXIMUM TENSILE STRESS, PSI = ,F10.3,/)
      WRITE(3,80)
 80   FORMAT(/,41H  ELEM   VOLUME   TENSILE STR  VI*(KI**M),/)
      DO 100 I=1,IMAX
        SIGMA=V(IELS(I))*((S(IELS(I))/SMAX)**M)
        TOTSIG=TOTSIG+SIGMA
        WRITE(3,90)IELS(I),V(IELS(I)),S(IELS(I)),SIGMA
 90     FORMAT(2X,I4,2X,F9.7,2X,F9.4,2X,E12.5)
 100  CONTINUE
      WRITE(3,110)TOTSIG
 110  FORMAT(/,22H  SIGMA(VI*(KI**M)) = ,E12.5)
C------------------------------------------------------------------
      CLOSE(UNIT=1)
      CLOSE(UNIT=2)
      CLOSE(UNIT=3)
      STOP
      END
```

APPENDIX A.2

FINITE ELEMENT ANALYSIS
POST-PROCESSING RESULTS

TABLE OF CONTENTS

Section 1

SUMMARY

This report presents the results of the finite element analyses and NASTRAN post-processing codes performed by Aerojet Electronic Systems Division in support of the Brittle Structures Study conducted by The Aerospace Corporation.

Various finite element analysis runs have been performed for a simple 4-point, flex specimen and the sensor primary mirror. Results obtained in the 4-point flex specimen analysis indicate that, for the same aspect ratio, the calculated risk of rupture decreases as the number of elements across the beam thickness increases, i.e., for a constant aspect ratio, mesh fineness does improve the calculated value of the risk of rupture. Also, for thin elements (aspect ratio of 4 or above), changing the element aspect ratio will not affect the risk of rupture significantly.

In the primary mirror analysis, the strength of the mirror was estimated with the use of the NASTRAN output file and the post-processing codes. An estimated strength of 5063 psi was determined for the mirror for a Weibull parameter of 5.0 and a material modulus of rupture of 10815 psi.

The two post-processing codes were developed for use on the VAX computer system. Listings of the codes and user's manual are also included in this report.

Section 2

ANALYSIS OF A SIMPLE 4-POINT FLEX SPECIMEN

The theoretical risk of rupture R for a 4-point flex specimen was derived in Reference 1 as:

$$R = (m+3)V\sigma^{m}_{MOR}/6(m+1)^2\sigma^{m}_{o}$$

where m is Weibull parameter, σ_{MOR} is modulus of rupture, σ_o is a material constant, and V is the specimen volume.

The risk of rupture based on a finite element analysis of the specimen is

$$R_{FE} = \sum^{n} \sigma_i^{m} V_i / \sigma^{m}_{o}$$

where n is the number of elements.

A ratio of R_{FE}/R can be calculated to eliminate σ_o as:

$$R_{FE}/R = 6(m+1)^2 \sum^{n} \sigma_i^{m} V_i / (m+3)V\sigma^{m}_{MOR}$$

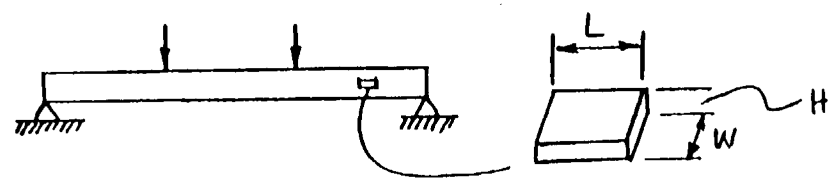
For a given Weibull parameter m, this ratio can be determined for a particular finite element analysis run. This ratio should be greater than 1.0 because the calculated risk of rupture was obtained by applying the peak stress in a finite element to the entire volume of that element while the theoretical value was calculated by integrating the stress linearly over the entire element volume.

Five different analysis runs have been performed with the use of the NASTRAN computer program. Details of the analysis assumptions and boundary conditions were described in Reference 1. A summary of the results obtained from these runs was shown in Table 1.

Results of the two analysis runs #1 and #2 indicated that, for the same aspect ratio (=1), the R_{FE}/R ratio was reduced significantly (48.3%) as the number of elements across the beam thickness increased. The two analysis runs #2 and #3 indicated that, for the same proportional increase in the number of elements across the beam thickness, i.e. 2 times, the ratio R_{FE}/R was reduced only 38.4%, instead of 48.3%, if the aspect ratio was increased at the same time to 1.6. There can be only two reasons for this behavior: either the aspect ratio is inversely proportional to the reduction of the ratio R_{FE}/R, or the rate of reduction of the ratio R_{RE}/R is slower for a higher number of elements across the beam thickness. The results of the two analysis runs #4 and #5 eliminate the first reason because, for the same number of elements across the beam thickness, increasing the aspect ratio (from 4 to 8 for outer elements) does not increase the ratio R_{RE}/R. In fact, it does reduce this ratio, although not significantly, from 2.006 to 2.000.

Table 1 - Summary of Calculation Results

Analysis Number	Total Number of Elements	No. of Elements Across Beam Thickness	Element Size W x L x H	Aspect Ratio L/H	$\Sigma S_i^5 V_i$ ($x10^{15}$)	$\frac{R_{FE}}{R_{THEORETICAL}}$
1	120	2	.25X.25X.25	1	.41730	6.010
2	960	4	.125X.125X.125	1	.21595	3.110
3	2400	8	.125X.1X.0625	1.6	.13304	1.916
4	240 (Outer) 240 (Inner) 240 (Center)	6	.125x.25x.03125 .125x.25x.09375 .125x.25x.125	8 2.67 2	.13892	2.00
5	240 (Outer) 240 (Inner) 240 (Center)	6	.25x.125x.03125 .25x.125x.09375 .25x.125x.125	4 1.33 1	.13937	2.006



Section 3

ANALYSIS OF SENSOR PRIMARY MIRROR

The primary mirror was analyzed by the finite element method, with the use of the NASTRAN computer program. Details of the analysis assumptions and boundary conditions were described in Reference 2.

For a Weibull parameter of 5, a modulus of rupture of 10815 psi and the same risk of rupture as that of a 3-point bend specimen, the estimated strength was determined as 5063 psi. A margin of safety of 5.0 was obtained when the maximum principal tensile stress of 602 psi found in the mirror was compared with the calculated MOR of 5063 psi, using a conservative factor of safety of 1.40.

This analysis also illustrated the application of the post-processing codes in determining the risk of rupture of a brittle structure.

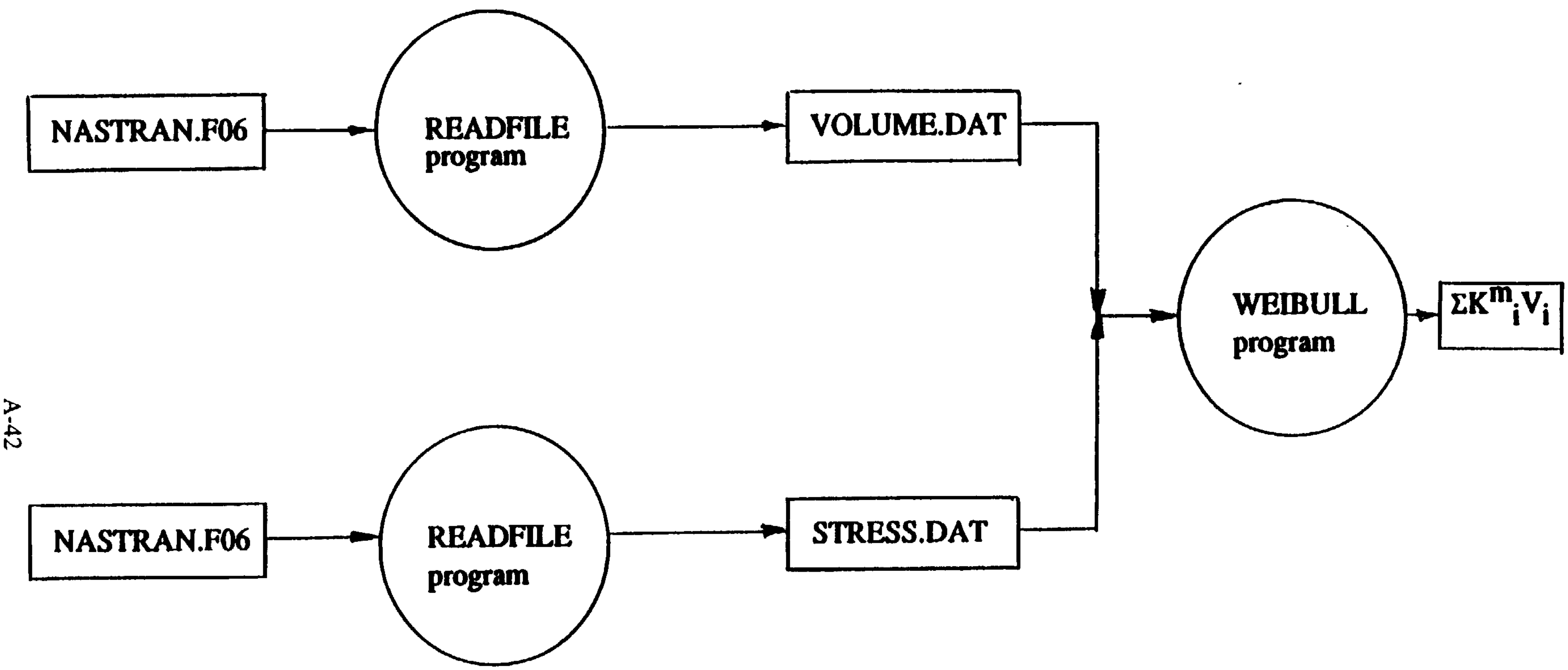
Section 4

POST-PROCESSING CODES FOR ESTIMATING STRENGTH OF BRITTLE STRUCTURES

Figure 1 outlines the procedure for estimating the strength of brittle structures using NASTRAN output file. Basically, there are two post-processing codes for this task. A computer program called READFILE, written in Vax C computer language, is used to scan through the NASTRAN output file to extract only information concerning the volumes and pertinent stresses of all elements in the analyzed model. Another computer program called WEIBULL, written in Vax FORTRAN computer language, is used to calculate the term $\Sigma K_i^m V_i$ required in the strength estimation, using the output files of the READFILE program as data input.

Listings of the two computer programs READFILE and WEIBULL as well as the user's manual are shown in the Appendix.

FIGURE 1 - PROCEDURE FOR ESTIMATING STRENGTH OF BRITTLE STRUCTURES



Section 5

REFERENCES

1. IOM #19/92, "Strength Estimte For Brittle (Glasslike) Structures Using Weibull Statistics And Finite Element Analysis", T. C. Nguyen to J. W. Provins, dated 8 June 92.

2. IOM #37/92, "Strength Estimate for DSP Primary Mirror Using Weibull Statistics And Finite Element Analysis - Sensor Element", T. C. Nguyen to J. W. Provins, dated 5 August 92.

3. IOM #42/92, "Brittle Structures Study - Post Processing Codes For Estimating Strength Of Brittle Structures", T. C. Nguyen to J. W. Provins, dated 24, September, 92.

4. IOM #43/92, "Brittle Structures Study - User's Manual For Risk of Rupture Codes", T. C. Nguyen to J. W. Provins, dated 29 September 92.

Section 6

APPENDIX

MAXIMUM TENSILE STRESS, PSI = 602.109

ELEM	VOLUME	TENSILE STR	VI*(KI**M)
21	0.0232830	79.7018	0.94625E-06
22	0.0233910	83.7753	0.12197E-05
23	0.0232830	74.0522	0.65517E-06
24	0.0233910	71.6779	0.55924E-06
25	0.0233610	2.0267	0.10095E-13
26	0.0232460	1.7030	0.42083E-14
27	0.0233610	15.9135	0.30127E-09
28	0.0232460	5.4942	0.14706E-11
29	0.0233710	58.8648	0.20873E-06
30	0.0233710	75.0508	0.70320E-06
31	0.0233710	143.2576	0.17819E-04
32	0.0232830	131.8105	0.11706E-04
33	0.0233910	69.2763	0.47163E-06
34	0.0233710	99.6268	0.28986E-05
37	0.0232830	126.1949	0.94161E-05
38	0.0233910	118.0051	0.67636E-05
47	0.0233710	81.2096	0.10431E-05
48	0.0233710	58.4434	0.20136E-06
49	0.0233710	31.3532	0.89476E-08
50	0.0233710	95.7735	0.23797E-05
51	0.0233610	11.9265	0.71234E-10
52	0.0232460	19.8903	0.91450E-09
53	0.0233610	2.0208	0.99476E-14
54	0.0233610	15.9081	0.30075E-09
55	0.0233610	11.8878	0.70086E-10
56	0.0233610	0.0000	0.00000E+00
57	0.0232460	7.2905	0.60502E-11
58	0.0233610	0.0000	0.00000E+00
69	0.0233610	1.8487	0.63751E-14
70	0.0233610	0.0000	0.00000E+00
71	0.0233610	13.2004	0.11832E-09
72	0.0233610	0.5070	0.98896E-17
75	0.0114580	57.0130	0.87217E-07
79	0.0277010	100.8231	0.36469E-05
100	0.0160410	360.5690	0.12354E-02
101	0.0387810	247.8088	0.45796E-03
103	0.0139910	160.1197	0.18608E-04
106	0.0195880	322.9987	0.87020E-03
109	0.0114970	58.2293	0.97256E-07
110	0.0160960	335.6024	0.86590E-03
111	0.0232830	52.0905	0.11284E-06
112	0.0232830	70.6760	0.51883E-06
115	0.0115080	11.0466	0.23920E-10
116	0.0276190	29.4989	0.77958E-08
120	0.0161110	16.0924	0.21971E-09
121	0.0386670	23.2394	0.33120E-08
123	0.0139940	43.5327	0.27647E-07
158	0.0195920	14.4523	0.15609E-09
161	0.0115080	11.0143	0.23573E-10
162	0.0161110	16.1635	0.22461E-09
163	0.0232460	1.7037	0.42159E-14
164	0.0232460	5.4923	0.14681E-11
165	0.0233650	80.3536	0.98905E-06
166	0.0233610	90.8078	0.18228E-05
167	0.0233690	77.7085	0.83677E-06
168	0.0233590	74.4274	0.67413E-06
178	0.0329000	125.3183	0.12850E-04
180	0.0460600	324.2745	0.20870E-02
187	0.0232830	90.3605	0.17724E-05
188	0.0232830	74.4452	0.67274E-06

189	0.0276490	105.9809	0.46713E-05
192	0.0387090	199.8043	0.15576E-03
195	0.1247180	52.4846	0.62765E-06
202	0.0233710	84.1983	0.12497E-05
203	0.0233910	146.8229	0.20167E-04
204	0.0233710	84.4952	0.12719E-05
205	0.0233910	83.2654	0.11830E-05
206	0.0233910	68.8708	0.45798E-06
207	0.0233910	57.5045	0.18586E-06
208	0.0233710	44.3496	0.50670E-07
209	0.0233910	24.0117	0.23593E-08
210	0.0233910	40.2911	0.31385E-07
211	0.0233710	87.1247	0.14825E-05
212	0.0233910	111.2161	0.50293E-05
213	0.0233910	77.8139	0.84326E-06
214	0.0232830	179.6447	0.55047E-04
215	0.0232830	108.0944	0.43419E-05
216	0.0232830	86.3850	0.14153E-05
217	0.0232830	174.5607	0.47687E-04
218	0.0139910	194.7728	0.49558E-04
219	0.0195880	275.3161	0.39154E-03
229	0.0328590	30.1655	0.10371E-07
231	0.0460030	20.1548	0.19333E-08
244	0.0232460	19.8511	0.90552E-09
251	0.0232460	7.2940	0.60645E-11
261	0.0276190	29.4117	0.76812E-08
266	0.0386670	23.2728	0.33359E-08
269	0.1247150	3.7619	0.11873E-11
277	0.0233610	1.8560	0.65020E-14
278	0.0233610	0.0000	0.00000E+00
279	0.0233610	1.9601	0.85400E-14
280	0.0233610	1.9686	0.87279E-14
281	0.0233610	2.4241	0.24710E-13
282	0.0233610	2.4211	0.24556E-13
283	0.0233610	13.1866	0.11770E-09
284	0.0233610	5.5419	0.15432E-11
285	0.0233610	5.5538	0.15598E-11
290	0.0233610	0.5279	0.12103E-16
291	0.0233610	0.0000	0.00000E+00
292	0.0233610	0.0000	0.00000E+00
293	0.0232460	0.7740	0.81615E-16
294	0.0232460	2.8541	0.55636E-13
295	0.0232460	22.6784	0.17621E-08
296	0.0232460	9.2210	0.19583E-10
297	0.0139940	43.5106	0.27577E-07
298	0.0195920	14.4652	0.15679E-09
309	0.0233650	131.1854	0.11471E-04
326	0.0233610	117.4653	0.66018E-05
327	0.0233690	135.1503	0.13315E-04
328	0.0233590	73.8566	0.64867E-06
329	0.0233710	102.8267	0.33949E-05
330	0.0233710	57.5587	0.18658E-06
331	0.0233710	79.3327	0.92803E-06
332	0.0233710	146.5386	0.19955E-04
339	0.0233710	102.1202	0.32799E-05
340	0.0233710	87.4326	0.15089E-05
353	0.0233710	64.5958	0.33214E-06
354	0.0233710	37.5089	0.21927E-07
385	0.0160780	372.0858	0.14490E-02
386	0.0387140	260.3514	0.58518E-03
390	0.0114840	60.9093	0.12166E-06
391	0.0276530	105.3071	0.45254E-05
393	0.0195610	311.9973	0.73075E-03
396	0.0139720	159.1526	0.18028E-04
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400	0.0114820	62.3111	0.13629E-06

181	0.0435710	43.0515	0.81426E-07
182	0.0138070	28.7559	0.34305E-08
183	0.0435480	40.9795	0.63595E-07
184	0.0600770	190.0004	0.18798E-03
185	0.0600060	180.6355	0.14583E-03
186	0.0184840	260.1563	0.27835E-03
190	0.0137570	37.4426	0.12793E-07
191	0.0142150	79.5501	0.57223E-06
193	0.0184180	102.4760	0.26301E-05
194	0.0199010	167.8349	0.33490E-04
196	0.0489560	38.3733	0.51472E-07
197	0.0489650	53.6033	0.27382E-06
198	0.0530580	12.5997	0.21290E-09
199	0.0529400	12.0736	0.17163E-09
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201	0.0684870	85.6999	0.40007E-05
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234	0.0435520	6.7026	0.74445E-11
235	0.0600170	2.7212	0.11317E-12
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243	0.0600170	2.6685	0.10262E-12
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265	0.0142050	19.4562	0.50045E-09
267	0.0184550	7.4086	0.52051E-11
268	0.0198870	13.2644	0.10319E-09
270	0.0489790	6.7515	0.86825E-11
272	0.0489790	2.5823	0.71067E-13
273	0.0530440	4.0089	0.69408E-12
274	0.0530440	4.0203	0.70396E-12
275	0.0685510	3.8777	0.75948E-12
276	0.0685510	3.9172	0.79893E-12
299	0.0518700	24.9427	0.63279E-08
300	0.1154510	21.6855	0.69963E-08
301	0.0684680	118.8325	0.20502E-04
305	0.1153950	3.6759	0.97869E-12
307	0.0519110	7.7968	0.18901E-10
308	0.0685370	7.4720	0.20171E-10
333	0.0401880	24.7717	0.47370E-08
334	0.0401880	91.0507	0.31779E-05
335	0.0401880	324.7816	0.18352E-02
336	0.0803790	94.0793	0.74858E-05
337	0.0401880	38.9626	0.45599E-07
338	0.0803760	19.7499	0.30519E-08
355	0.0269220	102.8971	0.39242E-05
356	0.0213100	107.4125	0.38502E-05
357	0.0376900	235.1353	0.34233E-03
358	0.0298340	161.4145	0.41309E-04
359	0.0269520	21.3741	0.15193E-08
360	0.0213520	23.3281	0.18640E-08
361	0.0377330	20.3909	0.16809E-08
382	0.0298930	17.4171	0.60545E-09
383	0.0184110	127.3038	0.77787E-05
384	0.0184480	283.2312	0.42489E-03
387	0.0198380	205.6019	0.92100E-04
388	0.0137270	32.5217	0.63105E-08

389	0.0137550	24.5017	0.15349E-08
392	0.0141700	71.5160	0.33497E-06
394	0.0299130	209.9194	0.15408E-03
395	0.0376570	168.7013	0.65022E-04
397	0.0213670	92.4149	0.18200E-05
398	0.0268980	92.4586	0.22966E-05
404	0.1116420	48.8500	0.39244E-06
405	0.1114970	51.2995	0.50056E-06
406	0.0090950	47.0503	0.26500E-07
407	0.1515250	97.9981	0.17306E-04
409	0.2892490	41.4138	0.44527E-06
410	0.1514340	154.4556	0.16822E-03
411	0.0122410	145.4318	0.10063E-04
416	0.0217410	68.6482	0.41884E-06
417	0.0141800	77.1360	0.48931E-06
419	0.0304370	276.0533	0.61658E-03
420	0.0198510	222.8562	0.13789E-03
422	0.1152300	22.7230	0.88210E-08
423	0.0518590	31.4349	0.20115E-07
424	0.0181160	24.0205	0.18306E-08
425	0.0686090	151.5704	0.69355E-04
426	0.0227580	67.9678	0.41713E-06
427	0.0183490	63.0316	0.23069E-06
428	0.0274800	75.8565	0.87218E-06
429	0.0091460	95.3805	0.91233E-06
430	0.0246840	136.1614	0.14599E-04
431	0.0368640	98.6581	0.43540E-05
432	0.0122670	165.1936	0.19069E-04
434	0.1516430	34.3735	0.91953E-07
435	0.1516760	57.1923	0.11728E-05
437	0.2891140	55.3242	0.18935E-05
445	0.1114230	9.1382	0.89724E-10
446	0.1115840	12.0453	0.35754E-09
447	0.0091530	20.2235	0.39126E-09
448	0.2891830	3.2800	0.13873E-11
450	0.1515780	6.5038	0.22289E-10
451	0.1513560	6.7298	0.26402E-10
452	0.0123010	6.7399	0.21619E-11
457	0.0217620	23.1023	0.18097E-08
458	0.0142050	20.9005	0.71590E-09
460	0.0304660	8.8796	0.21252E-10
461	0.0198870	6.1396	0.21922E-11
469	0.0180880	6.4482	0.25480E-11
470	0.0519170	7.7187	0.17974E-10
477	0.1154140	3.6614	0.95966E-12
487	0.0227620	7.1380	0.53299E-11
490	0.0685480	7.4063	0.19303E-10
491	0.0274310	16.5986	0.43674E-09
492	0.0183040	12.6909	0.76144E-10
493	0.0091530	20.8482	0.45554E-09
494	0.0123010	15.7013	0.14834E-09
495	0.0369360	9.8427	0.43117E-10
496	0.0246000	12.5340	0.96165E-10
498	0.1516410	3.0124	0.47535E-12
500	0.1516280	3.1156	0.56252E-12
502	0.2892070	3.1814	0.11911E-11
510	0.0091830	75.9812	0.29386E-06
511	0.0123160	120.5791	0.39669E-05
517	0.0091530	14.5969	0.76645E-10
518	0.0123010	9.5964	0.12650E-10
519	0.0245720	177.9416	0.55393E-04
520	0.0227380	103.5984	0.34288E-05
521	0.0123300	193.9610	0.42772E-04
522	0.0368490	157.2102	0.44715E-04
523	0.0182840	56.1489	0.12894E-06
524	0.0180860	22.0346	0.11871E-08

401	0.0233650	59.7035	0.22397E-06
402	0.0233690	73.1964	0.62046E-06
403	0.2155110	22.3981	0.15352E-07
408	0.5789490	51.6339	0.26850E-05
412	0.0233710	138.0500	0.14808E-04
413	0.0233710	133.2142	0.12390E-04
414	0.0233710	214.4527	0.13396E-03
415	0.0233710	186.1878	0.66078E-04
418	0.0199320	79.1515	0.78248E-06
421	0.0279050	376.7610	0.26769E-02
433	0.0933590	42.1739	0.15740E-06
436	0.0933420	50.0601	0.37082E-06
438	0.0233910	55.2038	0.15154E-06
439	0.0233910	101.0155	0.31089E-05
440	0.0233910	162.1275	0.33110E-04
441	0.0233910	144.1108	0.18372E-04
442	0.0329630	148.5507	0.30132E-04
443	0.0461480	412.3028	0.69480E-02
444	0.2155500	4.6265	0.57737E-11
449	0.5789420	1.1474	0.14547E-13
453	0.0233610	2.4985	0.28742E-13
454	0.0233610	0.0000	0.00000E+00
455	0.0233610	39.0357	0.26756E-07
456	0.0233610	0.0000	0.00000E+00
459	0.0199640	30.3037	0.64469E-08
468	0.0279500	12.4090	0.10392E-09
497	0.0933510	6.4218	0.12884E-10
501	0.0933420	6.5180	0.13877E-10
503	0.0232460	0.7729	0.81006E-16
504	0.0232460	2.8531	0.55537E-13
505	0.0232460	22.6412	0.17477E-08
506	0.0232460	9.2206	0.19578E-10
507	0.0328590	30.1066	0.10270E-07
508	0.0460030	20.1417	0.19270E-08
509	0.0933320	47.0776	0.27273E-06
516	0.0933670	6.1734	0.10579E-10
528	0.0460870	375.9257	0.43723E-02
533	0.0329190	125.3597	0.12878E-04
557	0.0387420	199.8009	0.15588E-03
558	0.0233650	73.0882	0.61578E-06
561	0.0233690	95.4369	0.23380E-05
562	0.0276730	110.3095	0.57114E-05
565	0.1247870	53.7088	0.70473E-06
584	0.0195720	252.9732	0.25623E-03
585	0.0233610	94.1685	0.21860E-05
586	0.0233710	97.5941	0.26147E-05
587	0.0233390	117.9468	0.67319E-05
608	0.0233660	224.7500	0.16932E-03
609	0.0233610	75.6282	0.73036E-06
610	0.0233710	104.0440	0.36007E-05
611	0.0233390	178.2462	0.53065E-04
612	0.0233590	63.7545	0.31091E-06
613	0.0233710	83.4034	0.11918E-05
614	0.0233370	83.1755	0.11739E-05
615	0.0233660	177.8895	0.52597E-04
616	0.0233690	114.6390	0.58469E-05
617	0.0233590	46.2726	0.62618E-07
618	0.0233710	34.8518	0.15185E-07
619	0.0233370	22.0063	0.15220E-08
620	0.0233690	81.8037	0.10818E-05
621	0.0139800	195.2444	0.50122E-04
625	0.1011720	46.0492	0.26472E-06
626	0.2197860	18.0667	0.53459E-08
627	0.6054360	26.8578	0.10692E-06
628	0.0232830	78.0720	0.85338E-06
629	0.0232830	123.3643	0.84064E-05

630	0.0232830	256.3172	0.32550E-03
631	0.0232830	199.8312	0.93752E-04
632	0.0198830	83.9030	0.10447E-05
633	0.0278370	434.2596	0.54324E-02
636	0.0933220	7.3950	0.26079E-10
637	0.2105740	10.9691	0.42255E-09
638	0.5798000	0.0000	0.00000E+00
639	0.0233610	2.5072	0.29248E-13
640	0.0233610	0.0000	0.00000E+00
641	0.0233610	39.0066	0.26657E-07
642	0.0233610	0.0000	0.00000E+00
643	0.0199640	30.3577	0.65046E-08
644	0.0279500	12.4173	0.10427E-09
660	0.1022040	159.0363	0.13139E-03
668	0.1020480	185.4772	0.28306E-03
669	0.2154520	30.2729	0.69222E-07
671	0.0933660	53.8971	0.53659E-06
673	0.5789290	88.3714	0.39428E-04
689	0.1020280	14.5643	0.84487E-09
696	0.1020280	14.5658	0.84530E-09
698	0.2155980	4.6421	0.58730E-11
700	0.0933750	5.9973	0.91548E-11
702	0.5790020	1.0643	0.99924E-14
711	0.5789740	52.3787	0.28844E-05
715	0.2154500	22.9475	0.17324E-07
720	0.0279000	305.4926	0.93806E-03
721	0.0233710	139.0924	0.15375E-04
722	0.0233710	121.1846	0.77186E-05
723	0.0233710	128.6367	0.10402E-04
724	0.0233710	229.7188	0.18892E-03
727	0.0199290	78.9530	0.77260E-06
739	0.0933620	43.2397	0.17832E-06
742	0.0933420	51.3148	0.41967E-06
744	0.0460700	496.0957	0.17493E-01
745	0.0233390	199.7697	0.93833E-04
746	0.0233390	37.6936	0.22441E-07
747	0.0233370	107.3771	0.42095E-05
766	0.0233370	165.0343	0.36103E-04
769	0.0329070	150.5441	0.32154E-04
770	0.0933350	48.0003	0.30053E-06
773	0.2570260	24.8640	0.30864E-07
774	0.6109550	19.0333	0.19284E-07
775	0.1858560	12.6591	0.76350E-09
776	1.7044400	7.3048	0.44798E-09
778	0.3893420	25.0199	0.48238E-07
779	0.5821870	8.9241	0.41639E-09
785	0.1831580	6.5898	0.28761E-10
786	0.2570790	8.6376	0.15619E-09
799	0.6024580	8.4639	0.33067E-09
800	1.6785400	2.5495	0.22846E-11
802	0.5818820	4.4588	0.12959E-10
804	0.3893500	3.2115	0.16807E-11
813	0.0933300	7.2180	0.23107E-10
814	0.2106340	10.8970	0.40897E-09
815	0.5798540	0.0000	0.00000E+00
816	0.1011600	45.8890	0.26012E-06
817	0.6054130	27.2224	0.11437E-06
818	0.2198140	18.0948	0.53883E-08
819	0.0278990	269.6993	0.50305E-03
820	0.0233660	140.4136	0.16116E-04
821	0.0233660	57.1828	0.18052E-06
822	0.0233690	116.3444	0.62949E-05
823	0.0233690	272.8081	0.44622E-03
824	0.0199280	84.9126	0.11116E-05
831	0.1833610	4.1196	0.27491E-11
835	1.2411799	8.8975	0.87458E-09

525	0.0091550	85.5898	0.53136E-06
526	0.0274150	64.0304	0.37286E-06
527	0.0123090	184.0253	0.32827E-04
529	0.0091410	64.2062	0.12604E-06
535	0.0600020	195.4610	0.21632E-03
552	0.0599360	189.1378	0.18332E-03
553	0.0184800	268.5122	0.32595E-03
554	0.0435080	42.4859	0.76106E-07
555	0.0137930	28.8578	0.34882E-08
556	0.0434860	40.3303	0.58631E-07
559	0.0184130	107.7350	0.33770E-05
560	0.0198700	177.5673	0.44324E-04
563	0.0137430	38.9064	0.15481E-07
564	0.0141930	81.5692	0.64763E-06
566	0.0489880	38.3216	0.51160E-07
579	0.0489960	54.9314	0.30966E-06
580	0.0685770	74.3001	0.19622E-05
581	0.0684910	91.2473	0.54747E-05
582	0.0530430	12.6686	0.21873E-09
583	0.0529890	12.0994	0.17363E-09
622	0.0685120	119.5060	0.21103E-04
623	0.0518770	26.0396	0.78482E-08
624	0.1154350	21.7302	0.70678E-08
634	0.0217250	76.8046	0.73370E-06
635	0.0304160	437.0663	0.61300E-02
645	0.0217620	23.1316	0.18212E-08
646	0.0304660	8.9065	0.21576E-10
647	0.0376640	272.8000	0.71907E-03
648	0.0298970	175.5947	0.63068E-04
649	0.0269030	98.1456	0.30958E-05
650	0.0213550	107.2188	0.38237E-05
651	0.2257350	15.1856	0.23035E-08
652	0.2465830	30.0651	0.76542E-07
653	0.1144010	49.4772	0.42863E-06
654	0.0160660	125.6718	0.63639E-05
655	0.1321710	3.0841	0.46598E-12
656	0.3147640	46.8553	0.89826E-06
657	0.3148850	68.6184	0.60531E-05
658	0.0509670	121.6266	0.17142E-04
659	0.0160880	111.5325	0.35086E-05
661	0.0121970	210.7339	0.64054E-04
662	0.0121970	50.0217	0.48269E-07
663	0.0121970	214.9320	0.70694E-04
664	0.0243930	117.2967	0.68441E-05
665	0.0365900	44.3592	0.79416E-07
666	0.0160640	106.9236	0.28369E-05
667	0.1066140	277.5808	0.22202E-02
670	0.1115590	57.9149	0.91850E-06
672	0.1515030	109.1145	0.29611E-04
674	0.0091300	54.7983	0.57007E-07
675	0.1113360	59.8515	0.10805E-05
676	0.0122450	298.9284	0.36933E-03
677	0.1516200	117.9841	0.43802E-04
678	0.0141590	86.9649	0.88997E-06
679	0.0198230	152.6815	0.20784E-04
680	0.2464410	12.6700	0.10168E-08
681	0.2257050	10.2152	0.31725E-09
682	0.0160480	41.0317	0.23585E-07
683	0.1144130	16.7628	0.19135E-08
684	0.3145180	1.8127	0.77778E-13
685	0.1320200	0.0000	0.00000E+00
686	0.0510140	15.0790	0.50255E-09
687	0.3149230	9.7929	0.35842E-09
688	0.0160480	34.3052	0.96349E-08
690	0.0121830	8.9805	0.89925E-11
691	0.0121830	0.0000	0.00000E+00

692	0.0121830	9.1516	0.98822E-11
693	0.0243650	21.8121	0.15201E-08
694	0.0365480	0.3517	0.24858E-17
695	0.0160480	34.2871	0.96094E-08
697	0.1065780	18.2001	0.26894E-08
699	0.1114280	9.0494	0.85450E-10
701	0.1516090	6.5533	0.23155E-10
703	0.1115890	12.0238	0.35436E-09
704	0.0091530	20.1804	0.38711E-09
705	0.1513880	6.7468	0.26743E-10
706	0.0123010	6.7997	0.22595E-11
707	0.0142050	20.9404	0.72276E-09
708	0.0198870	6.1146	0.21480E-11
709	0.2892640	42.5429	0.50939E-06
710	0.1514510	103.0647	0.22256E-04
712	0.0122740	152.4344	0.12765E-04
713	0.1515740	159.8592	0.19996E-03
714	0.1115890	50.0700	0.44374E-06
716	0.0091150	47.1970	0.26975E-07
717	0.1115890	51.6670	0.51917E-06
718	0.0304260	278.0693	0.63920E-03
719	0.0198760	247.3778	0.23268E-03
725	0.0217330	70.9958	0.49535E-06
726	0.0141970	78.8533	0.54691E-06
728	0.0686030	156.6051	0.81658E-04
729	0.0227500	75.8979	0.72403E-06
730	0.1152950	22.7703	0.89182E-08
731	0.0518810	32.9057	0.25292E-07
732	0.0181070	25.8616	0.26469E-08
733	0.0246440	146.8762	0.21286E-04
734	0.0368370	106.1794	0.62822E-05
735	0.0122830	185.9680	0.34524E-04
736	0.0183270	65.0589	0.26993E-06
737	0.0274670	77.0634	0.94336E-06
738	0.0091640	96.2401	0.95607E-06
740	0.1516460	34.1683	0.89242E-07
741	0.1516810	58.8514	0.13531E-05
743	0.2891280	57.0912	0.22160E-05
771	0.0122970	134.0865	0.67352E-05
772	0.0091760	76.5468	0.30473E-06
777	0.4745200	18.5047	0.13011E-07
780	0.1865360	17.3818	0.37399E-08
781	0.2784940	13.2729	0.14497E-08
782	0.0492160	0.6486	0.71370E-16
783	0.0159900	135.0671	0.90828E-05
784	0.0509670	319.4559	0.21427E-02
805	0.1864700	9.7517	0.20780E-09
806	0.4606140	10.2309	0.65243E-09
807	0.0492300	1.5744	0.60184E-14
808	0.2704380	3.5609	0.19564E-11
809	0.0160480	40.9950	0.23480E-07
811	0.0510140	15.0913	0.50459E-09
825	0.0304200	602.1090	0.30420E-01
826	0.0217280	80.8517	0.94862E-06
832	0.4842870	19.4998	0.17253E-07
833	0.9081000	17.5827	0.19284E-07
834	0.3183360	55.3980	0.20988E-05
836	0.1144280	60.4571	0.11679E-05
837	0.3183040	54.3501	0.19075E-05
838	0.1646570	82.5667	0.79841E-05
840	0.3155370	78.8579	0.12159E-04
841	0.0511890	63.8621	0.68709E-06
842	0.0511110	48.0784	0.16592E-06
846	0.1070400	145.7774	0.89047E-04
848	0.3842850	88.2949	0.26059E-04
865	0.3152100	167.7688	0.52940E-03

866	0.3151920	142.5500	0.23444E-03
867	0.2464050	35.4003	0.17310E-06
869	0.2258440	18.2957	0.58504E-08
872	0.3145020	94.2814	0.29606E-04
873	0.1320780	96.0558	0.13648E-04
875	0.3183150	13.4330	0.17593E-08
876	0.8823590	14.8288	0.79946E-08
877	0.4777750	9.4853	0.46356E-09
878	0.1143960	16.6570	0.18536E-08
879	0.3182560	13.4135	0.17463E-08
900	0.1646400	7.0833	0.37097E-10
901	0.3149230	6.2821	0.38936E-10
903	0.0510140	4.3659	0.10226E-11
904	0.0510140	4.3632	0.10194E-11
905	0.1065780	12.7120	0.44705E-09
906	0.3851600	6.0263	0.38682E-10
907	0.3149530	9.8223	0.36387E-09
908	0.3149530	6.2834	0.38981E-10
909	0.2257400	10.1637	0.30937E-09
910	0.2464130	12.5801	0.98107E-09
911	0.1320520	0.0093	0.11881E-24
912	0.3145540	1.8422	0.84343E-13
913	0.3146910	47.8484	0.99734E-06
914	0.1321430	2.6026	0.19940E-12
915	0.0509580	124.1499	0.18992E-04
916	0.3148390	61.8980	0.36149E-05
917	0.2466060	30.7968	0.86329E-07
918	0.2258470	15.4459	0.25091E-08
919	0.0160700	130.5747	0.77079E-05
920	0.1144000	51.0481	0.50113E-06
923	0.0122010	206.2144	0.57493E-04
924	0.1065880	259.9428	0.15985E-02
925	0.0121970	202.5979	0.52608E-04
926	0.0121970	47.1247	0.35820E-07
927	0.0365900	41.9036	0.59737E-07
928	0.0243930	117.9803	0.70459E-05
929	0.0160860	115.0354	0.40948E-05
930	0.0160780	109.9725	0.32680E-05
932	0.1514880	117.9686	0.43736E-04
934	0.1115550	59.5596	0.10565E-05
936	0.1516040	134.4833	0.84271E-04
937	0.0122270	394.4528	0.14754E-02
938	0.1113710	60.8411	0.11732E-05
939	0.0091240	55.2661	0.59443E-07
940	0.0198320	146.3056	0.16799E-04
941	0.0141660	90.0847	0.10620E-05
943	1.1767300	12.9492	0.54139E-08
944	1.6165700	18.4388	0.43540E-07
946	1.2244800	19.4595	0.43175E-07
947	1.9297600	3.3785	0.10734E-10
948	2.1986101	1.2219	0.75679E-13
949	1.9984699	5.4409	0.12041E-09
956	0.4606990	10.2179	0.64841E-09
959	0.1864670	9.6444	0.19661E-09
960	0.2703550	3.5646	0.19661E-11
961	0.0492490	1.6230	0.70088E-14
967	0.0492040	0.9762	0.55126E-15
969	0.2784260	12.9118	0.12626E-08
970	0.1864480	17.3575	0.37121E-08
971	0.4743960	18.4333	0.12758E-07
972	0.0509180	308.7125	0.18041E-02
973	0.0160160	141.8360	0.11617E-04
987	0.0269070	16.7728	0.45136E-09
988	0.0213580	17.1313	0.39823E-09
991	0.0376700	3.5983	0.28714E-12
992	0.0299020	3.6825	0.25589E-12

995	1.5643800	29.7713	0.46234E-06
998	1.5647300	28.8662	0.39629E-06
999	0.9082240	20.8388	0.45100E-07
1001	1.5621001	27.6798	0.32074E-06
1003	1.5597800	27.5758	0.31429E-06
1005	2.1912100	18.8569	0.66018E-07
1006	2.1916900	33.4433	0.11586E-05
1009	0.1864080	21.1940	0.10073E-07
1010	0.4843500	23.7746	0.46489E-07
1011	2.1883299	2.6290	0.34730E-11
1012	2.1848099	4.4287	0.47033E-10
1013	0.0491930	103.1303	0.72520E-05
1018	0.4741310	27.0716	0.87115E-07
1020	0.2785800	30.3969	0.91353E-07
1022	0.8822390	14.7827	0.78702E-08
1024	1.5185200	21.2517	0.83179E-07
1027	1.5186501	21.2392	0.82942E-07
1030	1.5145900	25.2079	0.19481E-06
1032	2.1275499	4.0258	0.28431E-10
1033	2.1277101	4.0488	0.29252E-10
1052	0.4777200	9.3905	0.44080E-09
1055	2.1215501	2.1665	0.12796E-11
1057	0.3155010	77.5320	0.11169E-04
1059	0.0511760	60.1485	0.50911E-06
1060	0.3182440	56.2126	0.22571E-05
1061	0.9080540	17.7597	0.20273E-07
1062	0.4842480	19.6380	0.17872E-07
1063	0.1144600	63.1756	0.14555E-05
1064	0.3181960	55.1421	0.20499E-05
1065	0.1646100	79.8596	0.67564E-05
1066	0.0511450	44.3579	0.11099E-06
1067	0.1070180	140.0803	0.72940E-04
1068	0.3149990	176.3819	0.67952E-03
1069	0.3152810	137.9280	0.19888E-03
1070	0.3841930	85.4212	0.22080E-04
1072	0.1320500	99.6916	0.16431E-04
1085	0.3144710	95.2641	0.31178E-04
1090	0.2259490	19.4525	0.79527E-08
1091	0.2465410	36.7213	0.20802E-06
1092	1.2245899	19.6339	0.45148E-07
1095	1.6166700	18.3939	0.43015E-07
1097	1.1767800	13.0036	0.55289E-08
1098	1.9983701	5.1794	0.94124E-10
1099	2.1986499	1.2080	0.71468E-13
1101	1.9293801	3.3438	0.10192E-10
1104	0.0183040	16.9348	0.32216E-09
1105	0.0180980	16.7148	0.29837E-09
1106	0.0137600	15.3169	0.14659E-09
1107	0.0142060	15.4943	0.16031E-09
1108	0.0274640	18.1167	0.67730E-09
1109	0.0091470	19.2724	0.30731E-09
1110	0.0137600	13.6588	0.82661E-10
1118	0.0091450	19.3566	0.31402E-09
1124	0.0402000	6.8811	0.78366E-11
1125	0.0402000	3.3410	0.21148E-12
1126	0.0402000	8.8923	0.28244E-10
1127	0.0804010	3.9273	0.94923E-12
1128	0.0804040	5.8988	0.72566E-11
1132	0.0402000	5.4360	0.24114E-11
1134	0.0245990	3.0419	0.80962E-13
1151	0.0184390	4.8920	0.65284E-12
1152	0.0227340	4.8127	0.74170E-12
1153	0.0198890	3.1072	0.72788E-13
1154	0.0123040	3.7927	0.12201E-12
1155	0.0368990	3.7201	0.33220E-12
1156	0.0184400	6.1204	0.20011E-11

839	2.1402500	8.1418	0.96759E-09
868	0.2200730	21.9315	0.14110E-07
870	0.1011650	59.3007	0.93746E-06
871	0.6054270	78.2281	0.22413E-04
874	1.2431300	1.2646	0.50806E-13
902	2.0850699	2.5126	0.26386E-11
921	0.1021730	153.2708	0.10921E-03
922	0.1021170	172.1784	0.19526E-03
931	0.5789520	91.4252	0.46730E-04
933	0.2154670	31.4672	0.84004E-07
935	0.0933660	54.4171	0.56297E-06
942	1.9001600	4.7553	0.58386E-10
945	1.9007500	6.0586	0.19608E-09
950	1.5182101	14.6255	0.12838E-07
951	1.4899600	3.6554	0.12288E-10
952	0.2570420	8.8920	0.18056E-09
953	0.1832020	6.6679	0.30514E-10
954	0.6024890	8.4721	0.33230E-09
955	1.6782600	2.7434	0.32954E-11
957	0.3893540	3.2113	0.16802E-11
958	0.5820160	4.5873	0.14939E-10
962	0.1858970	12.7206	0.78242E-09
963	0.2570440	25.0880	0.32282E-07
964	1.7045300	7.7308	0.59478E-09
965	0.6110870	18.9590	0.18915E-07
966	0.5821840	8.9579	0.42433E-09
968	0.3891940	25.0929	0.48927E-07
974	0.1833750	4.3649	0.36716E-11
975	0.0233620	1.2843	0.10314E-14
976	0.0233580	4.6290	0.62730E-12
977	0.0233660	1.5365	0.25288E-14
978	0.0233580	0.1590	0.29979E-19
979	0.0233580	1.1318	0.54814E-15
980	0.0233560	0.9702	0.25366E-15
981	0.0233560	8.7732	0.15340E-10
982	0.0233660	3.3315	0.12117E-12
983	0.0233580	0.3936	0.27888E-17
984	0.0233620	7.8284	0.86795E-11
985	0.0139650	21.0745	0.73358E-09
986	0.0276580	13.8946	0.18100E-09
989	0.0195520	5.2798	0.10137E-11
990	0.0387210	3.1661	0.15567E-12
993	1.8977400	22.3946	0.13508E-06
994	1.2412000	8.4434	0.67304E-09
996	1.8657900	39.1229	0.21609E-05
997	1.8979501	38.7784	0.21031E-05
1000	1.8690200	4.8467	0.63162E-10
1002	1.8634900	10.2354	0.26453E-08
1004	1.8378600	0.9210	0.15389E-13
1007	2.1400900	40.1590	0.28247E-05
1008	0.5821110	16.6712	0.94725E-08
1014	0.1859780	14.5312	0.15226E-08
1015	0.2570410	54.4492	0.15545E-05
1016	0.6113970	33.1231	0.30804E-06
1017	0.3891060	43.6489	0.77904E-06
1019	1.7045400	50.3317	0.69573E-05
1021	1.8451101	0.0000	0.00000E+00
1023	1.2430900	1.3309	0.65592E-13
1025	1.8685499	4.7642	0.57951E-10
1026	1.8452200	0.0000	0.00000E+00
1028	1.8100899	1.4280	0.13583E-12
1029	1.8639300	0.7278	0.48112E-14
1031	2.0852301	2.5258	0.27089E-11
1056	1.2411799	8.7949	0.82528E-09
1058	2.1401401	7.4255	0.61050E-09
1071	0.6054460	81.5005	0.27511E-04

1086	0.1011650	62.0678	0.11776E-05
1088	0.2201240	22.5538	0.16233E-07
1093	1.9007500	5.9742	0.18280E-09
1094	1.9001100	4.8973	0.67641E-10
1102	1.4899600	3.6513	0.12220E-10
1103	1.5181500	14.7788	0.13525E-07
1111	0.0114790	14.9988	0.11011E-09
1112	0.0233580	5.0592	0.97832E-12
1117	0.0233580	11.9090	0.70702E-10
1119	0.0329040	17.4811	0.67876E-09
1120	0.0233620	7.3882	0.64989E-11
1121	0.0233660	3.2570	0.10822E-12
1122	0.0233660	5.6540	0.17061E-11
1123	0.0233620	6.2185	0.27450E-11
1157	0.0160710	4.7423	0.48708E-12
1159	0.0460660	6.8111	0.85325E-11
1160	0.0233580	0.6146	0.25882E-16
1161	0.0233580	3.6228	0.18419E-12
1162	0.0233580	2.0776	0.11426E-13
1163	0.0233360	4.8416	0.78448E-12
1164	0.0233580	0.0000	0.00000E+00
1165	0.0233340	3.2205	0.10216E-12
1186	0.0233580	9.9809	0.29235E-10
1187	0.0233580	5.4304	0.13938E-11
1188	0.0233340	6.3712	0.30954E-11
1189	0.0233580	0.2290	0.18599E-18
1190	0.0233580	8.4371	0.12619E-10
1191	0.0233360	3.3164	0.11830E-12
1192	0.0233350	0.0788	0.89818E-21
1193	0.0233340	1.3440	0.12931E-14
1194	0.0233340	2.4907	0.28265E-13
1195	0.0233350	6.3512	0.30474E-11
1196	0.0139580	23.0139	0.11387E-08
1197	0.0195410	0.8770	0.12812E-15
1198	1.5181700	30.0000	0.46617E-06
1201	0.0114840	15.3350	0.12307E-09
1202	0.0160770	2.6495	0.26527E-13
1203	0.0233580	4.4152	0.49522E-12
1204	0.0233560	4.9182	0.84928E-12
1206	1.8882999	9.2958	0.16563E-08
1207	1.8586800	17.7920	0.41875E-07
1208	1.8382699	3.7630	0.17526E-10
1211	1.8884000	10.6849	0.33234E-08
1215	1.9002700	4.4787	0.43270E-10
1216	1.8636500	23.0562	0.15344E-06
1217	1.8690200	23.4406	0.16714E-06
1222	1.8341900	2.3147	0.15400E-11
1224	1.8017800	6.1720	0.20392E-09
1226	1.8295701	0.7364	0.50083E-14
1229	0.1834780	13.5218	0.10480E-08
1230	0.2570310	36.4434	0.20879E-06
1231	0.6025190	27.1091	0.11147E-06
1232	1.6782100	27.8543	0.35557E-06
1234	1.8997500	18.0047	0.45421E-07
1238	1.8099800	1.4005	0.12325E-12
1240	1.8341700	2.2196	0.12486E-11
1242	1.8579100	0.7248	0.46954E-14
1245	1.8691800	4.7803	0.58962E-10
1247	1.8639300	0.7351	0.50564E-14
1249	1.2410200	7.8977	0.48186E-09
1250	1.8977500	21.7618	0.11704E-06
1252	1.8979501	37.4059	0.17564E-05
1253	1.8659500	38.1531	0.19062E-05
1257	1.8378500	0.9248	0.15707E-13
1259	1.8635600	10.0328	0.23937E-08
1260	2.1402199	39.7397	0.26804E-05

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1199	1.9981800	30.0455	0.61824E-06
1200	1.2244000	26.7059	0.21017E-06
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1209	2.1853399	4.0780	0.31145E-10
1210	2.1799400	3.1031	0.79258E-11
1212	1.5567000	33.2870	0.80391E-06
1213	1.5601600	30.5231	0.52232E-06
1214	1.5566500	31.9446	0.65434E-06
1218	1.6160200	25.2758	0.21067E-06
1219	1.5624100	33.3349	0.81267E-06
1220	2.1980801	9.8407	0.25633E-08
1221	2.1887801	9.1559	0.17796E-08
1223	1.5108700	30.2742	0.48553E-06
1225	1.5087700	30.9132	0.53823E-06
1227	2.1158400	0.8602	0.12591E-13
1228	2.1126201	0.0000	0.00000E+00
1233	0.4605790	24.6374	0.52833E-07
1235	1.1765000	26.9427	0.21107E-06
1236	0.2701030	11.2634	0.61874E-09
1237	1.9292099	5.6385	0.13894E-09
1239	1.5145100	25.2331	0.19577E-06
1241	1.5108401	30.2540	0.48390E-06
1243	2.1214600	2.1504	0.12327E-11
1244	2.1158199	0.8094	0.92896E-14
1246	1.5622300	27.6403	0.31848E-06
1248	2.1885099	2.4820	0.26046E-11
1251	0.9079890	21.4194	0.51730E-07
1254	1.5647900	28.9964	0.40532E-06
1255	1.5644701	29.8672	0.46986E-06
1256	2.1913300	17.6830	0.47876E-07
1258	2.1847799	4.2226	0.37060E-10
1261	2.1917901	31.6206	0.87554E-06
1262	1.5597800	27.6245	0.31708E-06
1264	0.1864370	21.7977	0.11593E-07
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1266	0.0491730	103.8257	0.74968E-05
1271	0.2784090	30.6362	0.94948E-07
1273	0.4742010	27.7543	0.98682E-07
1274	0.0518560	14.4833	0.41760E-09
1275	0.0435350	11.6453	0.11782E-09
1276	0.1153600	10.4289	0.17984E-09
1277	0.0530490	12.2126	0.18211E-09
1279	0.1115900	20.2284	0.47760E-08
1280	0.1114960	20.2075	0.47473E-08
1281	0.0091450	19.3492	0.31341E-09
1282	0.0435370	12.7943	0.18861E-09
1283	0.0137630	13.5177	0.78497E-10
1284	0.0599890	5.4677	0.37043E-11
1285	0.0599960	4.9595	0.22748E-11
1286	0.0184460	2.2434	0.13245E-13
1291	0.0217260	18.2317	0.55301E-09
1292	0.0142040	20.7198	0.68543E-09
1294	0.0123010	7.0335	0.26755E-11
1295	0.0304170	5.5345	0.19959E-11
1296	0.0198850	6.2207	0.23407E-11
1301	0.0137580	15.2774	0.14469E-09
1302	0.0142040	15.6549	0.16877E-09
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1305	0.0198850	0.9457	0.19010E-15
1307	0.0489900	4.4647	0.10983E-11
1308	0.0489920	5.2837	0.25495E-11
1309	0.0530470	12.9146	0.24082E-09
1310	0.0685520	6.2978	0.85822E-11
1311	0.0685520	2.6106	0.10504E-12
1312	0.0685540	6.5650	0.10564E-10

1314	0.1514970	3.5016	0.10078E-11
1315	0.2892280	6.5538	0.44192E-10
1316	0.1514950	6.3761	0.20174E-10
1322	0.0268960	17.5617	0.56773E-09
1323	0.0213560	17.4068	0.43126E-09
1325	0.0376540	2.1856	0.23728E-13
1326	0.0298990	1.3464	0.16718E-14
1327	1.9984100	30.5917	0.67659E-06
1329	1.2245100	27.2820	0.23387E-06
1344	0.1516590	8.6073	0.90538E-10
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1347	0.0304150	4.5305	0.73360E-12
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1353	2.1165199	0.0000	0.00000E+00
1354	2.1219001	0.0000	0.00000E+00
1356	1.5086800	30.4900	0.50236E-06
1357	1.5113600	31.6157	0.60326E-06
1358	1.5148500	30.2934	0.48835E-06
1367	0.4776080	19.9204	0.18931E-07
1369	0.8822050	25.1032	0.11113E-06
1370	1.5183800	31.8804	0.63186E-06
1373	2.1273301	2.2166	0.14386E-11
1375	2.1127400	0.0000	0.00000E+00
1379	1.5088400	30.9657	0.54284E-06
1387	0.1864150	20.3614	0.82442E-08
1388	0.2257330	21.7882	0.14006E-07
1389	0.0491890	9.7107	0.53671E-10
1390	0.1320870	3.8441	0.14011E-11
1392	2.1797400	0.0000	0.00000E+00
1394	1.5565200	33.2706	0.80183E-06
1396	2.1851699	3.7651	0.20893E-10
1399	2.1796300	2.7063	0.39981E-11
1400	1.5600600	30.7234	0.53965E-06
1401	1.5563900	31.9534	0.65513E-06
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1408	2.1885200	8.9161	0.15583E-08
1409	0.2699240	11.3478	0.64183E-09
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1415	0.1153560	9.6787	0.12381E-09
1416	0.2258380	21.9076	0.14401E-07
1417	0.2465250	20.2734	0.10669E-07
1418	0.1144520	21.1624	0.61386E-08
1419	0.0160750	21.0660	0.84272E-09
1420	0.0518420	13.6283	0.30797E-09
1421	0.0181010	15.6310	0.21343E-09
1422	0.0685520	0.5436	0.41104E-16
1423	0.0227420	1.5283	0.23958E-14
1424	0.0160740	20.9806	0.82573E-09
1425	0.3145630	5.5490	0.20912E-10
1426	0.3149420	5.1731	0.14744E-10
1427	0.0510210	5.0272	0.20701E-11
1429	0.0122020	1.2444	0.46016E-15
1430	0.0122020	2.5178	0.15603E-13
1431	0.0122060	2.6461	0.20007E-13
1432	0.0244030	1.2148	0.81567E-15
1433	0.0366050	3.3672	0.20021E-12
1434	0.0160710	22.3482	0.11321E-08
1435	0.1065930	3.0779	0.37207E-12
1437	0.0183040	16.0032	0.24278E-09
1438	0.0274600	16.8401	0.46994E-09
1439	0.0091530	18.2620	0.23493E-09
1440	0.0246030	0.9409	0.22928E-15

1263	0.5821540	15.3403	0.62493E-08
1267	0.1861130	15.7895	0.23080E-08
1268	0.2570410	55.6088	0.17272E-05
1269	1.7046000	51.0028	0.74339E-05
1270	0.3890730	44.7244	0.87979E-06
1272	0.6117200	33.9794	0.35015E-06
1278	0.2154820	17.3325	0.42594E-08
1287	0.0233620	1.4120	0.16567E-14
1288	0.0233660	0.0000	0.00000E+00
1289	0.0233660	6.8017	0.42983E-11
1290	0.0233620	0.6496	0.34160E-16
1293	0.0199170	21.7796	0.12334E-08
1297	0.0278840	5.0609	0.11699E-11
1298	0.0233560	3.6319	0.18650E-12
1299	0.0233580	0.5289	0.12214E-16
1300	0.0276520	14.5029	0.22419E-09
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1318	0.0233660	3.7405	0.21620E-12
1319	0.0233660	2.3671	0.21942E-13
1320	0.0233620	1.7300	0.45753E-14
1321	0.0329190	18.2820	0.84956E-09
1324	0.0460870	3.8773	0.51034E-12
1328	1.5182700	30.6631	0.52006E-06
1330	0.2570310	36.6956	0.21611E-06
1331	0.1835370	13.7458	0.11382E-08
1332	1.6782700	28.0026	0.36515E-06
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1334	0.0933520	17.2085	0.17802E-08
1335	0.0933620	9.4132	0.87194E-10
1336	0.0933410	22.9646	0.75334E-08
1337	0.2106930	21.5716	0.12436E-07
1338	0.0233580	0.8940	0.16858E-15
1339	0.0233580	1.9993	0.94297E-14
1340	0.0233580	5.4788	0.14570E-11
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1342	0.0199110	23.2202	0.16984E-08
1343	0.0278750	6.0356	0.28212E-11
1345	0.5799110	11.2208	0.13035E-08
1348	0.1832460	8.4894	0.10210E-09
1349	1.8293999	4.5162	0.43433E-10
1351	1.8019600	14.0874	0.12633E-07
1352	1.8643500	1.1905	0.56336E-13
1355	1.8343199	8.4270	0.98507E-09
1359	2.0029199	0.0000	0.00000E+00
1360	1.8543600	5.2949	0.97521E-10
1361	1.3322500	7.0852	0.30060E-09
1362	2.0026100	0.0000	0.00000E+00
1363	1.8541900	3.2026	0.78941E-11
1364	1.4305600	32.4165	0.64709E-06
1365	1.4303300	25.3919	0.19078E-06
1366	1.4897500	0.9161	0.12149E-13
1368	1.8100300	14.0229	0.12402E-07
1371	1.8450700	11.4170	0.45226E-08
1372	2.0847700	2.9716	0.61041E-11
1374	1.8017600	5.9708	0.17278E-09
1376	1.8545200	5.3096	0.98896E-10
1377	2.0031400	0.0000	0.00000E+00
1378	1.3325800	0.0000	0.00000E+00
1380	1.4307100	32.4087	0.64637E-06
1381	0.0933410	22.8515	0.73497E-08
1382	0.1831080	8.4670	0.10069E-09
1383	0.2106480	21.4288	0.12027E-07
1384	0.5798920	11.1662	0.12720E-08

1385	0.3891730	0.5266	0.19924E-15
1386	0.5820120	15.1317	0.58344E-08
1391	1.8882200	9.1598	0.15386E-08
1393	1.8295799	0.6888	0.35855E-14
1395	1.8382200	3.9442	0.22172E-10
1397	1.8884000	10.3280	0.28042E-08
1398	1.8584200	17.4484	0.37980E-07
1402	1.9004400	4.2419	0.32983E-10
1404	1.8690200	23.2124	0.15916E-06
1405	1.8635000	23.0806	0.15424E-06
1411	1.8999300	18.2257	0.48282E-07
1414	0.0933610	6.3178	0.11875E-10
1428	0.1020520	4.5123	0.24124E-11
1436	0.1020380	4.3513	0.20114E-11
1452	0.3891350	0.5293	0.20424E-15
1454	0.5820550	15.1960	0.59599E-08
1459	1.4898700	0.8377	0.77674E-14
1460	1.8585200	3.2907	0.90624E-11
1462	1.8344400	8.2825	0.90353E-09
1475	1.8099600	3.4323	0.10896E-10
1478	1.8682600	1.2549	0.73467E-13
1482	2.0028701	0.0000	0.00000E+00
1483	1.8542000	8.0220	0.77837E-09
1484	1.3324400	4.3878	0.27383E-10
1486	1.8017900	2.3450	0.16144E-11
1487	1.4305201	26.9972	0.25924E-06
1489	0.8670410	8.1191	0.38655E-09
1490	0.8668910	6.1931	0.99796E-10
1491	0.8670410	8.3004	0.43169E-09
1504	0.8668780	8.2050	0.40735E-09
1506	1.2431500	8.6993	0.78264E-09
1510	1.8449200	5.6003	0.12843E-09
1511	1.2429500	1.8696	0.35875E-12
1516	2.0848200	2.1116	0.11059E-11
1523	1.3324100	6.9627	0.27553E-09
1524	0.8668910	6.2009	0.10043E-09
1525	2.0029299	0.0000	0.00000E+00
1526	1.8292400	4.5775	0.46454E-10
1528	1.8541900	3.0684	0.63733E-11
1530	1.4305600	25.3615	0.18967E-06
1531	0.2154670	17.2450	0.41526E-08
1532	0.0933520	17.1196	0.17347E-08
1535	0.5789110	9.0922	0.45455E-09
1538	1.8641800	1.2619	0.75370E-13
1540	1.8343199	8.2532	0.88759E-09
1541	1.8017900	13.9997	0.12244E-07
1546	1.8450700	11.3390	0.43703E-08
1548	1.8099600	14.0667	0.12597E-07
1549	2.0849099	2.9194	0.55869E-11
1551	0.0933500	12.3427	0.33791E-09
1552	0.2154630	15.7565	0.26442E-08
1553	0.5789310	1.6030	0.77432E-13
1590	1.2429800	8.7569	0.80881E-09
1629	1.8293101	5.3459	0.10093E-09
1630	1.8882300	5.8308	0.16082E-09
1631	1.8636100	4.1877	0.30330E-10
1634	1.8642600	3.2602	0.86766E-11
1639	1.8691100	6.1338	0.20507E-09
1640	1.8999200	7.0060	0.40525E-09
1642	1.4898100	0.9532	0.14818E-13
1646	0.8668700	8.0077	0.36067E-09
1647	2.0030799	0.0000	0.00000E+00
1648	1.3325200	3.6372	0.10718E-10
1649	1.8543600	7.9406	0.73974E-09
1651	1.4306700	27.0094	0.25986E-06
1652	1.8293200	5.3904	0.10520E-09

1441	0.0369040	1.1342	0.87510E-15
1442	0.0123020	0.6576	0.19120E-16
1443	0.1516520	5.3646	0.85144E-11
1444	0.2892280	2.6017	0.43566E-12
1445	0.1320590	3.8256	0.13674E-11
1446	0.0091510	18.1804	0.22967E-09
1447	0.0091510	18.0135	0.21932E-09
1448	0.0142060	21.7057	0.86489E-09
1449	0.0122990	2.7286	0.23508E-13
1450	0.0122990	3.6893	0.10623E-12
1451	0.0198880	3.9222	0.23328E-12
1453	0.0491690	9.7927	0.55953E-10
1455	0.1864440	20.5173	0.85659E-08
1456	0.0160770	21.1501	0.85980E-09
1457	0.0510300	5.1513	0.23389E-11
1458	0.4776650	20.1122	0.19863E-07
1461	2.1165099	0.0000	0.00000E+00
1463	2.1219001	1.6355	0.31374E-12
1477	2.1271901	0.6280	0.26252E-14
1479	1.5113100	28.6626	0.36945E-06
1480	1.5148200	29.9277	0.45957E-06
1481	1.5182400	28.7079	0.37408E-06
1485	2.1123400	0.0000	0.00000E+00
1488	1.5085300	25.9037	0.22232E-06
1505	0.2463890	20.1656	0.10383E-07
1507	0.1144210	21.0353	0.59549E-08
1508	0.3182980	22.6048	0.23739E-07
1509	0.8819470	27.0055	0.16008E-06
1512	0.3852550	3.1884	0.16041E-11
1513	0.3182820	20.8731	0.15936E-07
1514	0.1646430	0.0776	0.58449E-20
1515	0.3148600	2.8107	0.69796E-12
1517	0.3149870	1.0056	0.40923E-14
1518	0.0510300	1.4997	0.48917E-14
1519	0.0509900	1.3083	0.24696E-14
1520	0.1066160	3.5424	0.75151E-12
1521	0.3145950	5.5530	0.20991E-10
1522	0.3151540	5.1795	0.14846E-10
1527	2.1122899	0.0000	0.00000E+00
1529	1.5084701	30.5652	0.50851E-06
1533	0.1114610	20.0995	0.46204E-08
1534	0.1115950	20.1175	0.46467E-08
1536	0.1515110	6.3856	0.20326E-10
1537	0.1515120	3.5098	0.10197E-11
1539	2.1217401	0.0000	0.00000E+00
1542	2.1162100	0.0000	0.00000E+00
1543	1.5147099	30.4072	0.49755E-06
1544	1.5111099	31.6493	0.60637E-06
1545	0.8819880	25.2211	0.11374E-06
1547	1.5184200	32.0184	0.64567E-06
1550	2.1274099	2.2007	0.13877E-11
1554	0.3181900	22.7590	0.24551E-07
1591	0.1144500	20.1379	0.47897E-08
1592	0.3181900	20.9604	0.16267E-07
1593	0.1645970	0.0498	0.63703E-21
1615	0.1115450	18.5657	0.31091E-08
1616	0.1514900	0.2640	0.24559E-17
1617	0.3149490	2.7935	0.67708E-12
1618	0.0510240	1.2877	0.22829E-14
1619	0.1065950	3.5369	0.74550E-12
1620	0.0510170	1.4922	0.47693E-14
1621	0.3851630	3.1727	0.15646E-11
1622	0.3149540	3.9399	0.37785E-11
1623	0.3149500	0.9972	0.39251E-14
1624	0.1115460	17.9531	0.26289E-08
1625	0.1515110	2.9841	0.45303E-12

1626	0.2465100	18.1734	0.61750E-08
1627	0.3145540	3.0062	0.97585E-12
1628	1.5562700	25.1273	0.19699E-06
1632	1.5600500	25.6115	0.21724E-06
1633	1.5624200	28.1378	0.34824E-06
1635	2.1794200	0.0000	0.00000E+00
1636	2.1851299	3.4072	0.12679E-10
1637	2.1887300	0.3921	0.25626E-15
1638	2.1981101	3.4827	0.14231E-10
1641	1.9296900	0.6929	0.38950E-14
1643	1.6161799	25.1674	0.20621E-06
1644	1.1765600	23.7879	0.11324E-06
1645	0.4776630	19.9907	0.19270E-07
1650	1.5086100	25.8554	0.22027E-06
1653	1.5562700	25.1159	0.19654E-06
1655	2.1124899	0.0000	0.00000E+00
1656	2.1794701	0.0000	0.00000E+00
1657	0.0091520	19.2530	0.30594E-09
1658	0.0160480	20.9445	0.81732E-09
1659	0.0141970	20.6040	0.66617E-09
1660	0.0244030	1.2160	0.81981E-15
1661	0.0366050	3.3766	0.20303E-12
1662	0.0160590	20.8523	0.80004E-09
1663	0.0160740	22.2213	0.11005E-08
1664	0.1144510	20.0409	0.46755E-08
1665	0.0160740	21.0448	0.83844E-09
1666	0.1066190	3.0902	0.37967E-12
1667	0.0122020	2.6423	0.19859E-13
1668	0.0122020	1.2440	0.45928E-15
1669	0.0122020	2.5239	0.15790E-13
1670	0.1865250	20.6394	0.88277E-08
1672	0.2257190	20.0071	0.91435E-08
1673	0.2464820	18.0821	0.60207E-08
1674	0.0491860	2.5202	0.63191E-13
1675	0.3150020	3.9478	0.38171E-11
1676	0.1320910	0.0000	0.00000E+00
1677	0.3146270	3.0051	0.97428E-12
1678	0.0510690	5.0263	0.20703E-11
1681	0.0510400	5.1562	0.23507E-11
1682	0.2700370	2.2484	0.19607E-12
1684	0.0123190	7.0404	0.26928E-11
1685	0.0198760	6.2316	0.23603E-11
1688	2.1162100	0.0000	0.00000E+00
1690	1.5111099	28.6859	0.37090E-06
1691	0.0274770	18.0346	0.66241E-09
1692	0.0183260	16.8518	0.31472E-09
1693	0.0518340	14.4085	0.40675E-09
1694	0.0181070	16.6311	0.29112E-09
1695	0.1152960	10.3795	0.17551E-09
1697	0.1516540	8.5755	0.88876E-10
1698	0.2892140	6.5463	0.43937E-10
1699	0.0091520	19.2573	0.30628E-09
1700	0.0091300	19.1720	0.29884E-09
1701	0.0369260	3.7296	0.33674E-12
1702	0.0246390	3.0349	0.80165E-13
1703	0.0227410	4.8085	0.73873E-12
1704	0.0685600	6.5562	0.10494E-10
1705	0.0123200	5.5710	0.83544E-12
1706	0.0122890	3.8038	0.12365E-12
1708	2.1273799	0.6230	0.25226E-14
1710	2.1217301	1.6578	0.33577E-12
1712	1.5184000	28.7649	0.37785E-06
1713	1.5147200	29.9589	0.46194E-06
1716	0.8819690	27.0919	0.16266E-06
1719	0.2258320	20.0828	0.93224E-08
1722	0.1320600	0.0000	0.00000E+00

1724	0.1864450	20.7047	0.89643E-08
1725	0.4776540	20.0545	0.19579E-07
1726	0.0491700	2.5250	0.63769E-13
1727	1.5599300	25.5872	0.21620E-06
1728	1.5645700	25.6116	0.21787E-06
1732	1.5648000	25.6111	0.21788E-06
1734	2.1849899	3.4140	0.12804E-10
1735	2.1914799	4.6161	0.58039E-10
1736	2.1917901	4.6049	0.57347E-10
1738	0.9080850	18.8379	0.27221E-07
1739	0.4840060	20.5574	0.22455E-07
1742	1.2243700	22.7502	0.94290E-07
1744	1.9980299	1.9933	0.79450E-12
1745	0.4608080	22.8143	0.35990E-07
1747	0.4742590	22.1188	0.31728E-07
1751	0.2784180	5.9569	0.26389E-10
1754	1.5624599	28.1428	0.34856E-06
1756	2.1887901	0.4199	0.36124E-15
1758	0.0217240	18.1403	0.53924E-09
1760	0.1114530	17.8749	0.25700E-08
1761	0.0091300	17.9214	0.21329E-09
1762	0.0141880	21.5983	0.84264E-09
1764	0.0217340	17.5669	0.45945E-09
1768	0.1115980	18.4857	0.30441E-08
1772	0.1513690	2.9938	0.46006E-12
1774	0.1515640	0.2732	0.29162E-17
1775	0.0122650	3.7000	0.10747E-12
1776	0.0198640	3.9315	0.23578E-12
1777	0.0304130	5.5403	0.20061E-11
1778	0.0304270	4.5406	0.74207E-12
1780	0.0137740	15.2369	0.14295E-09
1781	0.0142280	15.4151	0.15649E-09
1782	0.0137740	13.6023	0.81047E-10
1783	0.0530000	12.1451	0.17697E-09
1784	0.0435970	11.5758	0.11451E-09
1786	0.0213130	17.0371	0.38659E-09
1788	0.1516500	5.3292	0.82371E-11
1789	0.0489590	4.4612	0.10933E-11
1791	0.1153700	9.6329	0.12092E-09
1792	0.0530630	12.8480	0.23474E-09
1793	0.2892130	2.5970	0.43175E-12
1794	0.0685080	0.5418	0.40425E-16
1795	0.0685990	2.6072	0.10443E-12
1796	0.0685480	6.2915	0.85384E-11
1797	0.0489590	5.2792	0.25369E-11
1798	0.0269260	16.6793	0.43922E-09
1799	0.0184440	4.8854	0.64860E-12
1800	0.0184440	6.1138	0.19909E-11
1801	0.0600590	5.4665	0.37046E-11
1802	0.0199190	3.0882	0.70695E-13
1804	0.0298390	3.6974	0.26054E-12
1805	0.0376960	3.6144	0.29384E-12
1808	1.9294500	0.6930	0.38974E-14
1809	2.1984999	3.4874	0.14330E-10
1811	1.1766200	23.8367	0.11442E-06
1812	1.6164600	25.1897	0.20716E-06
1814	0.2699330	2.2493	0.19637E-12
1818	0.4606560	22.8746	0.36456E-07
1821	0.1863250	17.6970	0.40869E-08
1822	0.0491520	27.7730	0.10263E-07
1824	0.9079300	18.8290	0.27153E-07
1825	0.3182060	42.2149	0.53909E-06
1828	0.3842100	65.5409	0.58716E-05
1829	0.1646080	62.5947	0.19988E-05
1830	0.3182710	42.2128	0.53907E-06
1831	1.2244400	22.7558	0.94411E-07

1832	0.4843440	20.5494	0.22427E-07
1835	1.9982200	1.9997	0.80746E-12
1837	0.3155700	16.4416	0.47911E-08
1838	0.1070340	94.6909	0.10296E-04
1839	0.3155390	16.3964	0.47253E-08
1840	0.0511170	16.0770	0.69376E-09
1841	0.0511170	16.0537	0.68876E-09
1842	0.1144750	37.5743	0.10834E-06
1843	0.2465310	24.0969	0.25311E-07
1844	0.2257850	14.5936	0.18886E-08
1846	0.3145270	30.4388	0.10385E-06
1847	0.3149070	36.6389	0.26274E-06
1848	0.1320310	30.4680	0.43805E-07
1869	0.0091760	18.1022	0.22539E-09
1870	0.0274600	16.7685	0.46004E-09
1871	0.0091520	18.1809	0.22973E-09
1881	0.0183290	15.9325	0.23778E-09
1882	0.0518340	13.5628	0.30059E-09
1883	0.0180870	15.5613	0.20856E-09
1885	0.0123250	2.7381	0.23969E-13
1886	0.0368850	1.1361	0.88225E-15
1887	0.0122950	0.6726	0.21379E-16
1888	0.0246200	0.9398	0.22804E-15
1889	0.0227320	1.5278	0.23908E-14
1893	0.0436010	12.7316	0.18430E-09
1894	0.0804970	3.9230	0.94517E-12
1895	0.0402490	8.8827	0.28126E-10
1896	0.0137070	13.4352	0.75821E-10
1904	0.0804970	5.8940	0.72355E-11
1905	0.0402490	6.8664	0.77627E-11
1906	0.0402490	3.3290	0.20794E-12
1907	0.0137500	15.2044	0.14118E-09
1908	0.0600710	4.9561	0.22699E-11
1909	0.0402490	5.4291	0.23989E-11
1910	0.0184380	2.1696	0.11201E-13
1911	0.0183790	2.2416	0.13144E-13
1922	0.2784090	5.9517	0.26273E-10
1924	0.4742030	22.1330	0.31827E-07
1927	0.1864310	17.6988	0.40913E-08
1928	0.0491720	27.8096	0.10335E-07
1929	0.0091530	39.4838	0.11099E-07
1930	0.1115320	42.8848	0.20443E-06
1931	0.0160750	83.1267	0.80627E-06
1932	0.0142050	61.1754	0.15380E-06
1933	0.0123010	78.3909	0.46014E-06
1934	0.1514660	77.4018	0.53173E-05
1935	0.0510220	100.4040	0.65787E-05
1936	0.0198870	148.8433	0.18359E-04
1937	0.0366160	37.8432	0.35912E-07
1938	0.0122050	170.0750	0.21946E-04
1939	0.0122080	175.4319	0.25634E-04
1940	0.0122080	59.0357	0.11062E-06
1941	0.1065990	176.7520	0.23238E-03
1942	0.1144570	37.5503	0.10798E-06
1943	0.0160750	83.1104	0.80547E-06
1944	0.0160760	66.7461	0.26911E-06
1945	0.0244090	81.1157	0.10832E-05
1946	0.0160760	66.7365	0.26892E-06
1947	0.2258220	14.5846	0.18831E-08
1949	0.2464990	24.1003	0.25325E-07
1950	0.1320670	30.5086	0.44109E-07
1951	0.3145650	30.5178	0.10522E-06
1952	0.3149380	36.6822	0.26432E-06
1954	0.0510220	100.6116	0.66469E-05
1957	0.1115360	37.8156	0.10899E-06
1958	0.1514670	30.3043	0.48918E-07

1654	1.8582600	2.2077	0.12314E-11
1671	0.5820530	6.6109	0.92873E-10
1679	0.1019830	4.5053	0.23919E-11
1680	0.1020690	4.3517	0.20129E-11
1683	0.3892840	2.6765	0.67565E-12
1686	1.3326000	4.4439	0.29185E-10
1687	1.8582600	3.3427	0.97999E-11
1689	1.8017900	2.2683	0.13673E-11
1696	0.0933620	9.3696	0.85191E-10
1707	1.8684200	1.2808	0.81370E-13
1709	1.8099600	3.3645	0.98601E-11
1711	1.8343199	8.2680	0.89560E-09
1714	1.2429500	1.8544	0.34445E-12
1715	1.8450400	5.6299	0.13186E-09
1717	2.0848501	2.1093	0.11000E-11
1718	0.2106910	19.4804	0.74690E-08
1720	0.0933430	17.1779	0.17643E-08
1721	0.5799120	1.8802	0.17218E-12
1723	0.5820500	6.6298	0.94208E-10
1729	1.8882500	5.8334	0.16117E-09
1730	1.8380300	4.5378	0.44688E-10
1731	1.8659900	13.1247	0.91829E-08
1733	1.8382300	4.5311	0.44366E-10
1737	1.8979200	18.1561	0.47317E-07
1740	1.5181299	11.8913	0.45612E-08
1741	1.9001700	3.4626	0.11951E-10
1743	2.1398399	10.0218	0.27335E-08
1746	0.6119780	24.9952	0.75447E-07
1748	0.1836770	8.5284	0.10471E-09
1749	0.6030620	24.1702	0.62863E-07
1750	1.7047600	21.8764	0.10794E-06
1752	1.6784000	10.1924	0.23329E-08
1753	1.8641800	3.2840	0.89977E-11
1755	1.8636100	4.1858	0.30261E-10
1757	0.0329600	17.3886	0.66212E-09
1759	0.0198730	21.6456	0.11933E-08
1763	0.0199150	23.0845	0.16497E-08
1765	0.0278800	6.0486	0.28523E-11
1766	0.0278220	5.0670	0.11743E-11
1767	0.2106730	19.3972	0.73102E-08
1769	0.2155240	15.6827	0.25836E-08
1770	0.1832200	5.6203	0.12983E-10
1771	0.5798860	1.8605	0.16336E-12
1773	0.5789050	1.5820	0.72477E-13
1779	0.0461440	6.8191	0.85977E-11
1785	0.0276340	13.8233	0.17625E-09
1787	0.0933580	6.2816	0.11538E-10
1790	0.1246630	1.8118	0.30752E-13
1803	0.0386870	3.1561	0.15309E-12
1806	1.4898700	0.9621	0.15521E-13
1807	1.8999200	7.0105	0.40654E-09
1810	1.8692501	6.1656	0.21046E-09
1813	0.3891440	2.6852	0.68649E-12
1815	0.1832330	5.6311	0.13110E-10
1816	0.2570310	24.0653	0.26216E-07
1817	0.6028450	24.2375	0.63720E-07
1819	1.6782600	10.2234	0.23684E-08
1820	0.3891250	23.0296	0.31852E-07
1823	1.8977200	18.1738	0.47543E-07
1826	1.2411300	9.1602	0.10115E-08
1827	1.2411799	9.1472	0.10044E-08
1833	1.9005400	3.4830	0.12310E-10
1834	1.5181201	11.8942	0.45667E-08
1836	2.1399500	10.0324	0.27482E-08
1845	0.5819080	15.8324	0.73149E-08
1849	0.2570140	24.0047	0.25886E-07

1850	0.0933380	17.1046	0.17268E-08
1851	0.2570750	34.7446	0.16448E-06
1852	0.1011690	35.5879	0.72977E-07
1853	0.2200660	16.3710	0.32701E-08
1854	0.1861210	7.0726	0.41622E-10
1855	0.6054720	32.5112	0.27790E-06
1856	0.0232800	6.7786	0.42101E-11
1857	0.0233880	2.4716	0.27256E-13
1858	0.0139770	20.9549	0.71361E-09
1859	0.0233580	5.4053	0.13619E-11
1860	0.0232800	0.0000	0.00000E+00
1861	0.0233880	1.3327	0.12425E-14
1862	0.0233580	0.0000	0.00000E+00
1863	0.0233580	5.4501	0.14193E-11
1864	0.0233580	1.9877	0.91592E-14
1865	0.0232800	1.4145	0.16655E-14
1866	0.0233580	0.8918	0.16648E-15
1867	0.0233880	0.0770	0.80156E-21
1868	0.0233580	2.0726	0.11287E-13
1872	0.0329010	18.1854	0.82689E-09
1873	0.0232800	2.3648	0.21756E-13
1874	0.0232800	3.7389	0.21495E-12
1875	0.0232800	3.4189	0.13742E-12
1876	0.0232800	0.6513	0.34477E-16
1877	0.0233580	0.0000	0.00000E+00
1878	0.0233880	6.3373	0.30209E-11
1879	0.0460610	3.8850	0.51512E-12
1880	0.0232800	1.7459	0.47718E-14
1884	0.0933470	12.2824	0.32972E-09
1890	0.0195680	5.2497	0.98592E-12
1891	0.0233580	8.4124	0.12436E-10
1892	0.0114950	14.9266	0.10763E-09
1897	0.0114580	15.2438	0.11918E-09
1898	0.0232800	3.3215	0.11892E-12
1899	0.0233580	5.0324	0.95270E-12
1900	0.0233880	8.7437	0.15104E-10
1901	0.0232800	1.5270	0.24426E-14
1902	0.0233580	0.1615	0.32446E-19
1903	0.0233880	0.9616	0.24302E-15
1912	0.0160920	4.7414	0.48728E-12
1913	0.0160410	2.6486	0.26418E-13
1914	0.0232800	7.8300	0.86579E-11
1915	0.0233580	11.8777	0.69779E-10
1916	0.0233880	0.4010	0.30654E-17
1917	0.0232800	1.2766	0.99750E-15
1918	0.0233580	4.6153	0.61813E-12
1919	0.0233880	1.1304	0.54537E-15
1920	0.1835530	8.5395	0.10533E-09
1921	1.7045900	21.9057	0.10865E-06
1923	0.6117670	25.0181	0.75767E-07
1925	0.2570360	34.7739	0.16515E-06
1926	0.3891210	23.0333	0.31878E-07
1948	0.5820530	15.8575	0.73750E-08
1953	0.1020480	43.1852	0.19369E-06
1955	0.1020480	43.2324	0.19475E-06
1956	0.2154290	20.0618	0.88465E-08
1959	0.5788840	46.5437	0.15978E-05
1960	0.0933420	43.9082	0.19250E-06
1961	0.0139770	22.8853	0.11087E-08
1962	0.0233880	6.3061	0.29474E-11
1963	0.0233580	9.9538	0.28841E-10
1964	0.0233880	3.2239	0.10293E-12
1965	0.0233580	3.6135	0.18184E-12
1966	0.0233880	4.8302	0.77704E-12
1967	0.0233580	0.6146	0.25886E-16
1968	0.0233880	3.2813	0.11243E-12

1969	0.0233580	0.2303	0.19130E-18
1972	0.0277010	14.4425	0.21995E-09
1976	0.0195680	0.8852	0.13438E-15
1977	0.0387820	1.1985	0.12117E-14
1979	0.0232800	5.6326	0.16678E-11
1980	0.0232800	3.2510	0.10683E-12
1981	0.0232800	7.3769	0.64264E-11
1982	0.0233880	3.6017	0.17913E-12
1983	0.0233880	4.9129	0.84583E-12
1984	0.0232800	6.2109	0.27189E-11
1985	0.0233880	0.5391	0.13462E-16
1986	0.0233880	4.4153	0.49593E-12
1987	0.1011620	35.5865	0.72957E-07
1988	0.1861300	7.0599	0.41250E-10
1989	0.6054780	32.5304	0.27872E-06
1990	0.2200870	16.3713	0.32707E-08
1994	0.0331610	106.0262	0.56146E-05
1996	0.0199120	58.7083	0.17548E-06
2000	0.0464250	146.7927	0.39985E-04
2002	0.0278770	235.4761	0.25504E-03
2004	0.0933530	36.7275	0.78833E-07
2008	0.0233580	88.7951	0.16293E-05
2009	0.0233580	123.9459	0.86342E-05
2010	0.0233580	78.2117	0.86381E-06
2011	0.0233580	91.9572	0.19408E-05
2012	0.0233580	74.3120	0.66889E-06
2013	0.0233580	136.5578	0.14017E-04
2016	0.0233580	92.4742	0.19960E-05
2017	0.0233580	141.6054	0.16806E-04
2018	0.0278770	235.4312	0.25479E-03
2022	0.0199120	58.7051	0.17544E-06
2024	0.2154930	20.0464	0.88155E-08
2026	0.5789420	46.5808	0.16043E-05
2039	0.0933530	43.9269	0.19293E-06
2040	0.0276530	67.6953	0.49678E-06
2042	0.0139690	133.6837	0.75367E-05
2043	0.0233340	70.5600	0.51571E-06
2044	0.0233340	61.2995	0.25521E-06
2045	0.0233580	123.8701	0.86078E-05
2046	0.0233580	92.0342	0.19490E-05
2047	0.0233580	136.6712	0.14075E-04
2048	0.0233340	48.7362	0.81072E-07
2049	0.0387140	175.5809	0.81635E-04
2051	0.0195570	215.6952	0.11538E-03
2052	0.0233340	72.9330	0.60846E-06
2053	0.0233580	141.7087	0.16867E-04
2054	0.0933600	36.7292	0.78858E-07
2055	0.1247400	37.9712	0.12442E-06
2066	0.0233520	53.3205	0.12718E-06
2067	0.0233610	0.1175	0.65996E-20
2068	0.0233520	44.5207	0.51613E-07
2069	0.0233610	33.2610	0.12017E-07
2070	0.0233520	49.0783	0.84023E-07
2071	0.0233610	55.3771	0.15373E-06
2072	0.0233520	72.5040	0.59123E-06
2073	0.0233610	96.0196	0.24094E-05
2077	0.0464250	146.7523	0.39930E-04
2078	0.0233580	92.7590	0.20269E-05
2079	0.0233580	74.6397	0.68377E-06
2080	0.0233580	78.2534	0.86611E-06
2081	0.0233580	88.6965	0.16203E-05
2082	0.0331610	105.9967	0.56068E-05
2097	0.0114810	37.7357	0.11101E-07
2098	0.0233580	116.6511	0.63754E-05
2099	0.0233580	60.3882	0.23704E-06
2100	0.0139690	133.5779	0.75069E-05

2101	0.0233340	70.5324	0.51470E-06
2102	0.0233610	0.1036	0.35271E-20
2103	0.0233340	61.3103	0.25544E-06
2104	0.0233610	33.2769	0.12046E-07
2105	0.0233340	48.7640	0.81303E-07
2106	0.0233610	55.5227	0.15576E-06
2107	0.0233340	73.3447	0.62583E-06
2108	0.0233610	96.2013	0.24323E-05
2109	0.0160740	192.2858	0.53393E-04
2110	0.0233580	50.6124	0.98027E-07
2111	0.0233580	31.6076	0.93114E-08
2112	0.0195570	215.6618	0.11529E-03
2129	0.0276530	67.6572	0.49538E-06
2131	0.0387140	176.0955	0.82839E-04
2133	0.0114810	37.7159	0.11072E-07
2134	0.0160740	192.5351	0.53740E-04
2135	0.0233580	116.6034	0.63623E-05
2136	0.0233580	60.4268	0.23780E-06
2137	0.0233580	31.8292	0.96424E-08
2138	0.0233520	53.2088	0.12585E-06
2139	0.0233520	44.5728	0.51916E-07
2140	0.0233520	49.4113	0.86912E-07
2141	0.0233520	72.9366	0.60908E-06
2142	0.0233580	50.8660	0.10051E-06
41	0.0402360	24.0036	0.40515E-08
42	0.0402360	89.1952	0.28704E-05
43	0.0402360	38.3270	0.42050E-07
44	0.0804730	19.3963	0.27917E-08
45	0.0402360	319.1187	0.16827E-02
46	0.0804730	92.3530	0.68317E-05
59	0.0402150	0.7222	0.99832E-16
60	0.0402150	5.0965	0.17472E-11
61	0.0402150	2.0059	0.16501E-13
66	0.0804300	0.9344	0.72385E-15
67	0.0804300	5.3624	0.45065E-11
68	0.0402150	17.0458	0.73130E-09
73	0.0137190	31.2577	0.51728E-08
74	0.0136990	26.3360	0.21931E-08
97	0.0141940	69.7949	0.29706E-06
98	0.0184090	121.7058	0.62117E-05
99	0.0183810	275.0823	0.36585E-03
102	0.0198720	194.2922	0.69525E-04
104	0.0213410	92.5359	0.18297E-05
105	0.0269100	95.3198	0.26758E-05
107	0.0298780	187.2963	0.87021E-04
108	0.0376750	144.6097	0.30107E-04
113	0.0137550	6.9694	0.28580E-11
114	0.0137550	12.0606	0.44354E-10
117	0.0142050	19.5369	0.51091E-09
118	0.0184550	8.4460	0.10023E-10
119	0.0184550	7.3310	0.49381E-11
122	0.0198870	13.2009	0.10074E-09
136	0.0213520	23.4010	0.18934E-08
157	0.0269520	21.4128	0.15331E-08
159	0.0298930	17.3914	0.60099E-09
160	0.0377330	20.3804	0.16765E-08
169	0.0183090	54.4544	0.11078E-06
170	0.0180730	20.3166	0.79052E-09
171	0.0274160	63.3734	0.35413E-06
172	0.0091540	84.7080	0.50449E-06
173	0.0245900	167.1497	0.40543E-04
174	0.0227280	96.1825	0.23641E-05
175	0.0123240	166.9662	0.20208E-04
176	0.0368310	149.9947	0.35336E-04
177	0.0091650	64.2383	0.12669E-06
179	0.0123360	181.5475	0.30743E-04

1970	0.0213310	17.3228	0.42046E-09
1971	0.0269090	17.4744	0.55404E-09
1973	0.0142280	15.5804	0.16507E-09
1974	0.0298630	1.3547	0.17219E-14
1975	0.0376720	2.2041	0.24760E-13
1978	0.0199190	0.9456	0.19027E-15
1991	0.0274580	50.9500	0.11913E-06
1992	0.0090470	65.6081	0.13897E-06
1993	0.0092580	53.6744	0.52117E-07
1995	0.0217290	53.2646	0.11772E-06
1997	0.0368940	45.9921	0.95939E-07
1998	0.0121590	86.3551	0.73784E-06
1999	0.0124430	92.1100	0.10425E-05
2001	0.0304200	142.1117	0.22281E-04
2003	0.1516440	37.3255	0.13883E-06
2005	0.2892090	34.2603	0.17250E-06
2006	0.0211060	75.1624	0.63978E-06
2007	0.0269000	80.6192	0.11576E-05
2014	0.0295490	114.4376	0.73284E-05
2015	0.0376600	162.5900	0.54072E-04
2019	0.1115380	42.8695	0.20408E-06
2020	0.0091530	39.4624	0.11069E-07
2021	0.0142050	61.1612	0.15362E-06
2023	0.0217290	53.2477	0.11754E-06
2025	0.1115410	37.7932	0.10867E-06
2027	0.1514980	30.4110	0.49795E-07
2028	0.1514980	77.5103	0.53559E-05
2029	0.0123010	78.5578	0.46506E-06
2030	0.0198870	149.1036	0.18520E-04
2031	0.0304200	142.1956	0.22347E-04
2032	0.0518390	20.3172	0.22678E-08
2033	0.0181030	13.9942	0.12277E-09
2034	0.0183040	41.7296	0.29268E-07
2035	0.1153240	19.6971	0.43207E-08
2036	0.0685390	86.2216	0.41271E-05
2037	0.0227370	39.6563	0.28179E-07
2038	0.0245940	53.6270	0.13784E-06
2041	0.0142050	53.2651	0.76963E-07
2050	0.0198870	69.8566	0.41805E-06
2056	0.1516580	37.3066	0.13849E-06
2057	0.0490010	33.9737	0.28025E-07
2058	0.2892330	34.3011	0.17355E-06
2059	0.0685490	86.2399	0.41321E-05
2060	0.0490040	50.0068	0.19364E-06
2061	0.0685520	40.7200	0.96980E-07
2062	0.0685530	40.7478	0.97313E-07
2063	0.1153500	19.6934	0.43176E-08
2064	0.0530440	12.3815	0.19504E-09
2065	0.0530440	12.3807	0.19498E-09
2074	0.0518510	20.3069	0.22625E-08
2075	0.0137610	23.4055	0.12214E-08
2076	0.0184430	37.4916	0.17263E-07
2083	0.0274580	50.9216	0.11880E-06
2084	0.0090470	65.5830	0.13870E-06
2085	0.0092580	53.6534	0.52015E-07
2086	0.0183040	41.7042	0.29179E-07
2087	0.0181030	13.9855	0.12239E-09
2088	0.0121590	86.7573	0.75519E-06
2089	0.0369010	46.1182	0.97280E-07
2090	0.0124430	92.3576	0.10566E-05
2091	0.0246000	53.7570	0.13955E-06
2092	0.0227430	39.6713	0.28239E-07
2093	0.0435390	39.3802	0.52107E-07
2094	0.0137610	29.8137	0.40959E-08
2095	0.0599980	79.4063	0.23935E-05
2096	0.0184430	125.7512	0.73285E-05

2113	[illegible]	39.3876	0.52156E-07
2114	0.0402030	28.2241	0.90987E-08
2115	0.0804050	10.7030	0.14271E-09
2116	0.0402030	10.6147	0.68458E-10
2117	0.0402030	67.3811	0.70563E-06
2118	0.0804050	72.5601	0.20436E-05
2119	0.0402030	249.9942	0.49606E-03
2120	0.0599980	79.4675	0.24027E-05
2121	0.0184430	37.6463	0.17622E-07
2122	0.0184430	125.9023	0.73727E-05
2123	0.0137610	23.3820	0.12153E-08
2124	0.0137610	29.8033	0.40888E-08
2125	0.0376600	164.3263	0.57022E-04
2126	0.0269000	80.6225	0.11579E-05
2127	0.0295490	114.8054	0.74470E-05
2128	0.0211060	75.1257	0.63823E-06
2130	0.0142050	53.2308	0.76715E-07
2132	0.0198870	70.0470	0.42378E-06
0	0.0000000	0.0000	0.00000E+00

SIGMA(VOLUME(I)) = 0.67303E+03

SIGMA(VI*(KI**M)) = 0.11311E+00

APPENDIX B

GUIDELINE FOR SAMPLE SIZE BY MONTE CARLO SIMULATION

MONTE CARLO GENERATION OF WEIBULL SAMPLES

The Weibull Distribution can be simply expressed in the median normalized form as

$$F = 1 - \mathrm{Exp} - (\mathrm{Ln}2)\, X^{m}$$

where X is the median normalized variable and F is the cumulative probability.

The probability F may be viewed as a uniformly distributed random number, and a random observation from a Weibull distribution may be computed by

$$X = \left[\frac{\mathrm{Ln}\,\frac{1}{(1-F)}}{\mathrm{Ln}\,(2)} \right]^{(1/m)}$$

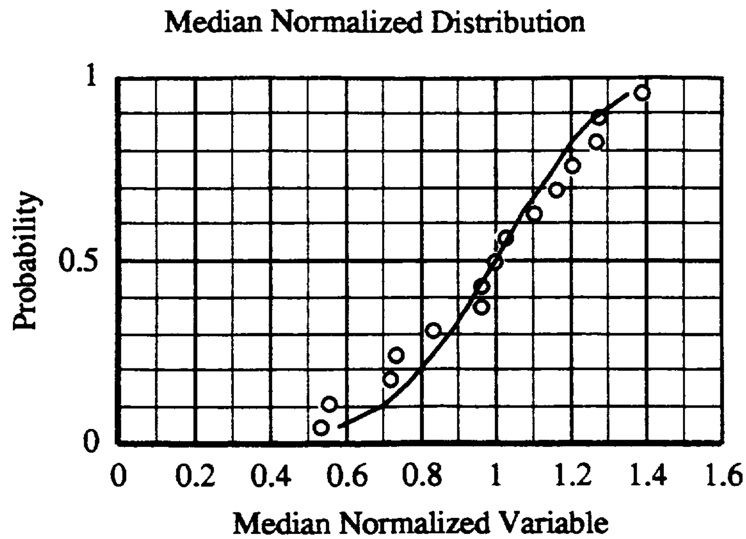
The value of the Weibull Modulus, *m*, the shape factor, may be freely selected, and the uniformly distributed random number, F, of range 0 to 1, will generate a random observation from that parent population.

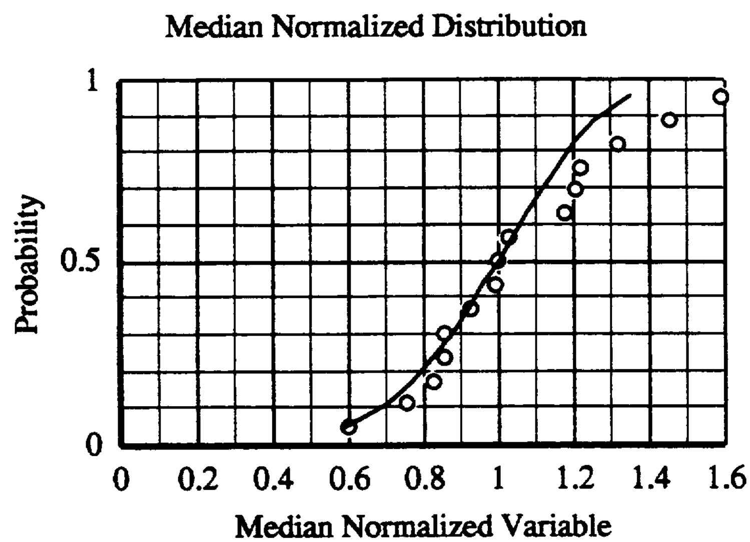
This procedure was used to generate samples of 15, 20, and 30 observations each, from a parent population with $m = 5$. The resulting "observed" data were fitted by the curve representing the parent population. The data lead back to the parent population fairly consistently with larger sample sizes, but do show occasional significant gyration with a sample at lower sample sizes. A size of 15 seems to be the minimum for estimating the population. There is a significant benefit of pooling two or three groups of 15 samples each, as recommended in the test program.

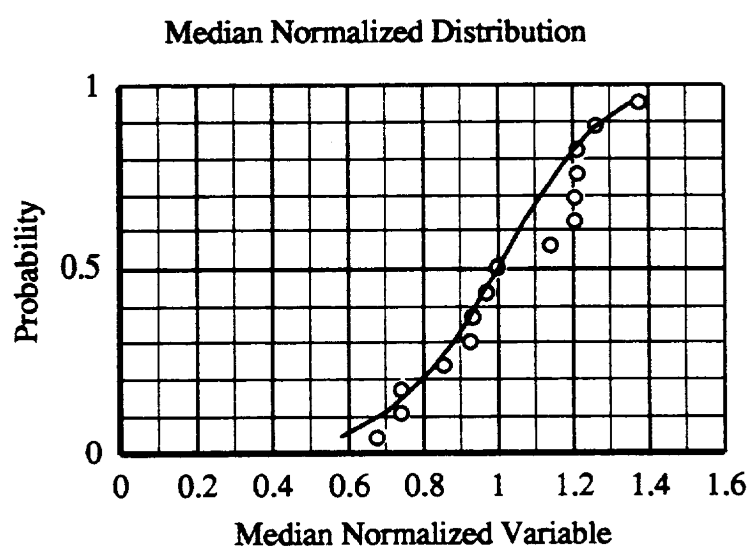
It should be noted that this Monte Carlo simulation is idealized. In the real world cases, there will be other experimental and material factors which effectively confuse the "parent" population parameters. Even with large samples of real experimental data, the inclination to assume complex (e.g., bimodal) parent distributions may be in error, since observed values may very well be random samples of simple unimodal parent populations, further clouded by some experimental variations.

The summary results of these Monte Carlo runs are plotted in this appendix, and may be readily repeated by the user, to compare the kind of experimental results one can expect with different Weibull parent populations.

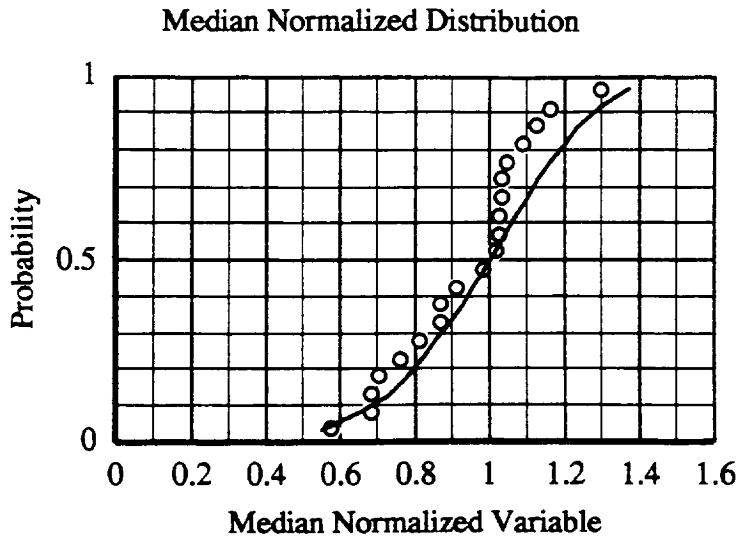
Monte Carlo Trials
m = 5 (parent)
N = 15

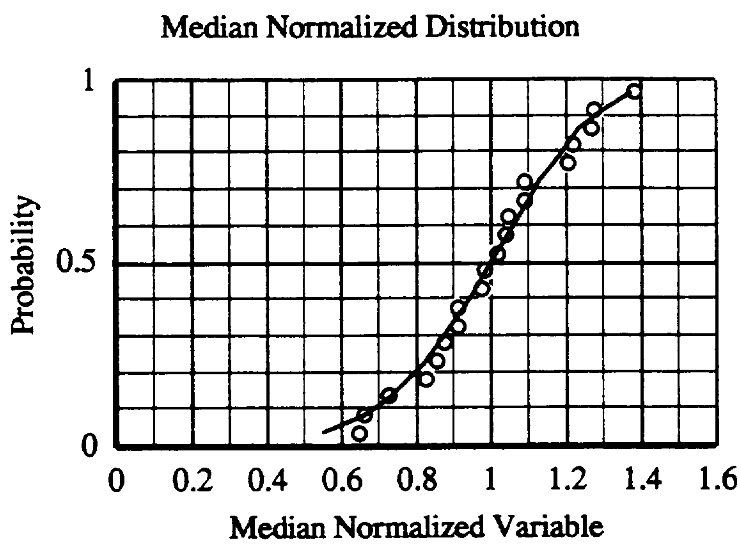


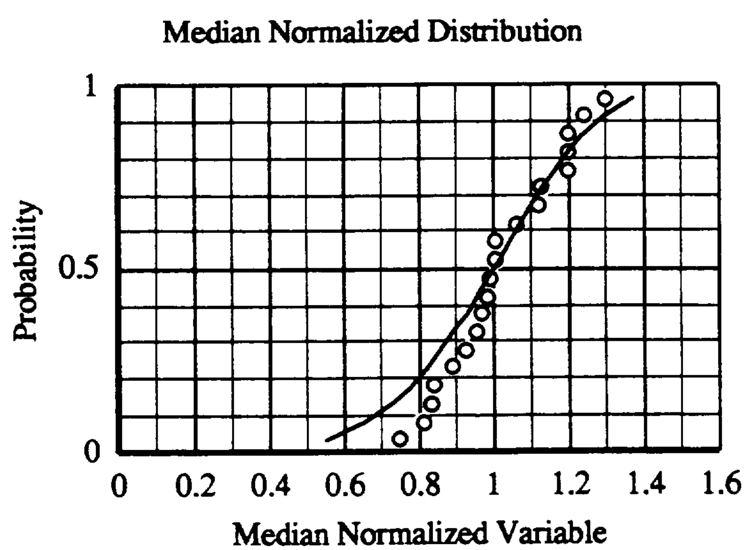




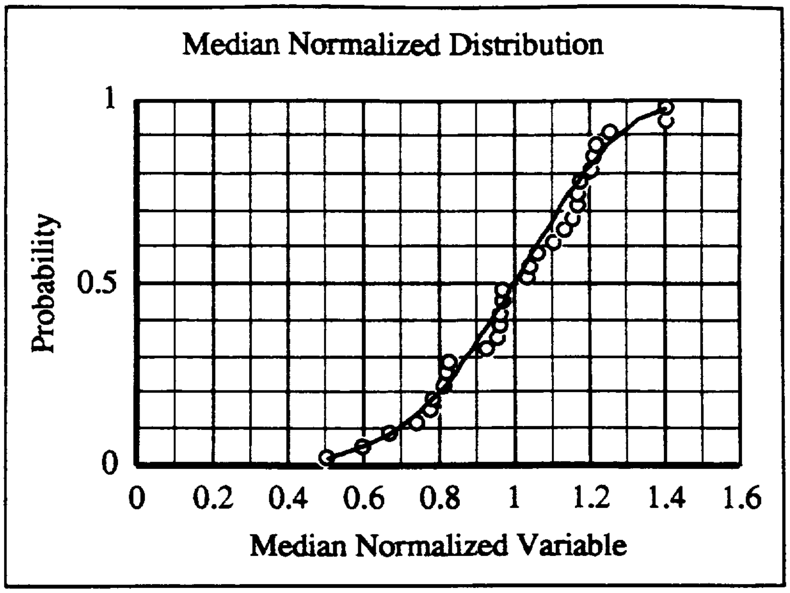
Monte Carlo Trials
m = 5 (parent)
N = 20

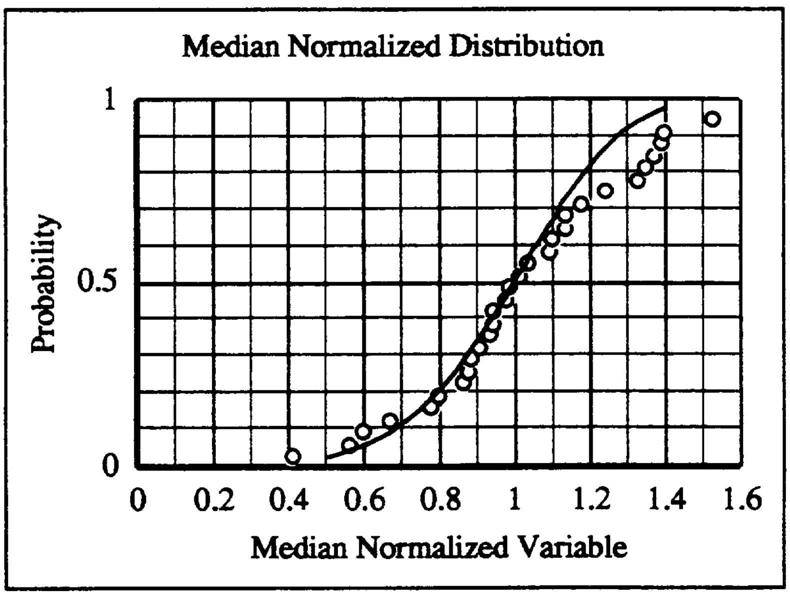


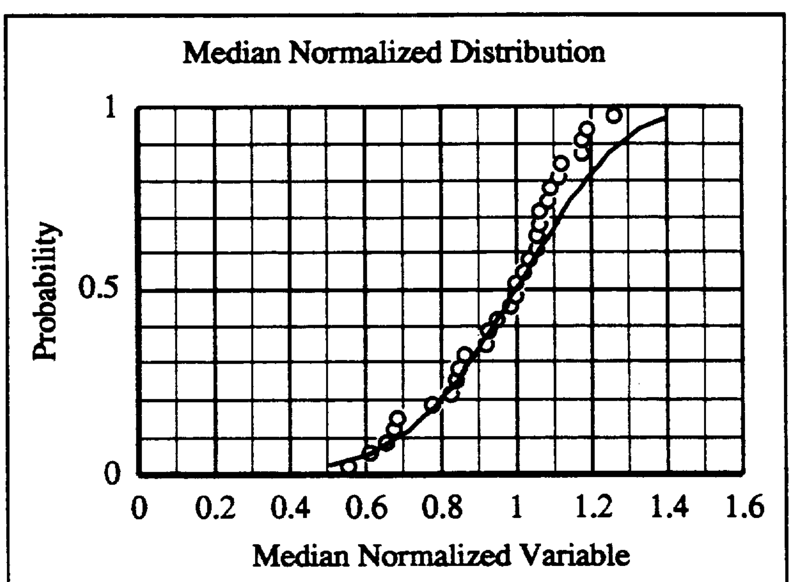




Monte Carlo Trials
m = 5 (parent)
N = 30







APPENDIX C

ILLUSTRATIVE GLASS TESTING

GLASS TEST SPECIMENS AND TESTING

Round rods of commercial Pyrex glass of nominally 4 mm diameter were readily available for the illustrative tests. These off-the-shelf specimens exhibited some variations in diameter and roundness which would not be present in test articles for a program addressing the Factor of Safety for a critical large brittle space system component. Individual test specimen diameters were measured and used to calculate the Modulus of Rupture (MOR) at failure for each test. The as received condition reflected surface flaws and damage typical of normal handling of glass in our glass shop. The effect of surface flaws was briefly investigated by etching the Pyrex rods in HF.

The etching procedure was as follows: rods were immersed in an agitated 10% HF solution, rinsed with DI water, rinsed with isopropanol, sprayed with dry nitrogen, and dipped in grease (Apiezon N diluted in hexane). Each specimen was placed in an individual plastic bag and stored for periods up to several weeks before testing. Testing was conducted with specimens taped on the compression face and tested within a plastic bag, to preserve the fragments for fracture initiation site location if possible. Specimens etched for 6 hours exhibited a frosty, irregular, and pitted appearance. The beneficial effect of etch polishing to remove surface flaws was eventually counteracted by these other irregularities and there was an optimum HF etching condition for the Pyrex rods.

All tests were conducted in bending, either 3-point or 4-point bending at a rate of 0.02 in. per minute.

A summary of all the illustrative glass tests is given in Table C-1 below.

Table C-1
Summary of Glass Tests

Description Glass (Pyrex) rods Diam.= 0.154 in. typ.	3 pt Flexure	4 pt Flexure	Etching time min, hrs	Storage period	Number Tested	Median Strength ksi	CV	Weibull Modulus m
Group No. 1& 2 AR		√	As Rec'd	0	21	11.50	0.21	no fit
Group No. 1 Etched		√	10 min	NA	12	19.80	0.22	5
Group No. 2 Etched		√	1 hour	5 days	22	36.00	0.39	5
Group No. 3 AR	√		As Rec'd	10 days	15	13.30	0.18	5
Group No. 3 Etched	√		2 hours	10 days	15	93.00	0.17	5
Group No. 3 Etched		√	2 hours	10 days	20	68.00	0.22	5
Group No. 4 Etched	√		6 hours	4 weeks	15	82.90	0.26	5
Group No. 4 Etched		√	6 hours	4 weeks	15	71.30	0.34	5
3 pt load span, L = 1.9 in.								
4 pt load outer span, Lo = 1.9 in. inner span, Li = 1 in.								
As received rod diameter = 0.154 in. ± 0.02 in. typical								

GLASS ROD BENDING TESTS

A number of experiments were conducted to illustrate, in an abbreviated fashion, some of the behavior characteristics of brittle materials and the effects addressed by the Weibull Theory of brittle failure: the size and stress distribution effect, the importance of proper surface preparation, and the data scatter. The methods of data analysis are also illustrated by these experiments.

To predict the relative strength between 3- and 4-point bending we equate the respective Risks of Rupture in accordance with the considerations in subsection 3.1.2.2. The R-integral for 3-point center-span bending is evaluated and shown below.

$$R_4 = \left(\frac{\sigma_4}{\sigma_o}\right)^m \frac{V_{T4}}{(m+1)} \qquad R_3 = \left(\frac{\sigma_3}{\sigma_o}\right)^m \frac{V_{T3}}{(m+1)^2}$$

The ratio of strengths is then given by

$$\frac{\sigma_4}{\sigma_3} = \left[\frac{V_{T3}}{V_{T4}(m+1)}\right]^{\frac{1}{m}}$$

As noted in the main document, the prediction of strength reduction by the Weibull volume flaw distribution should be inherently conservative. This conclusion is verified by the illustrative test data which are summarized in Table C-2 below.

THE EFFECT OF SIZE AND STRESS DISTRIBUTION

The outer span of the bending configuration was the same in all bending tests. The effective volume of material subject to tension was larger in the 4-point tests than in the 3-point tests despite the ratio of outer span to inner span. The Risk of Rupture associated with failure outside the center span was ignored, and in fact, no such failures were observed. The strength comparison for these conditions is shown below. The predicted size effect on strength accounts for the larger volume (1.9 in. span) of the 3-point tests, but with linear stress distribution, compared with the smaller central span (1.0 in.) of the 4-point tests at uniform bending.

Table C-2 Stress Distribution Effects

	3-point strength ksi	4-point strength ksi	Strength Ratio 4 pt/3 pt	Remarks
As received	13.30	11.50	0.86	Median strength ratio, 4pt to 3pt: V_T (3 point)/V_T (4 point) = 1.90 Weibull volume factor: $[1.90/(m+1)]^{(1/m)}$ For m= 5, ratio is 0.80
Etched 2 hours	93.00	82.90	0.89	
Etched 6 hours	83.00	71.30	0.86	
	1.9" span	1.9" outer span 1.0" inner span		

The data are perfectly consistent in a higher strength for the lower Risk of Rupture. The strength effects predicted by the Weibull Theory, based on volume flaw distribution and a fit of the data with m = 5, are consistently conservative.

EFFECT OF SURFACE PREPARATION AND SURFACE FLAW REMOVAL

Etched specimens were treated in 10% HF for various times, rinsed with DI water and isopropanol, then sprayed with dry nitrogen and dipped into Apiezon N grease dissolved in hexane, sprayed again with dry nitrogen, and individually bagged. As shown in Table C-1, specimens were stored from several days up to 4 weeks before testing. Table C-1 and Figure C-1 show the effect of the HF etching. Etching for 2 hours initially removed strength-controlling flaws, leading to increased strength by nearly an order of magnitude, possibly to the point where sub-surface volume flaws might be the controlling defects. Further etching, to 6 hours, degraded the surface and produced a decrease of strength accompanied by some increased scatter, although the statistical distribution was not much altered, as discussed later.

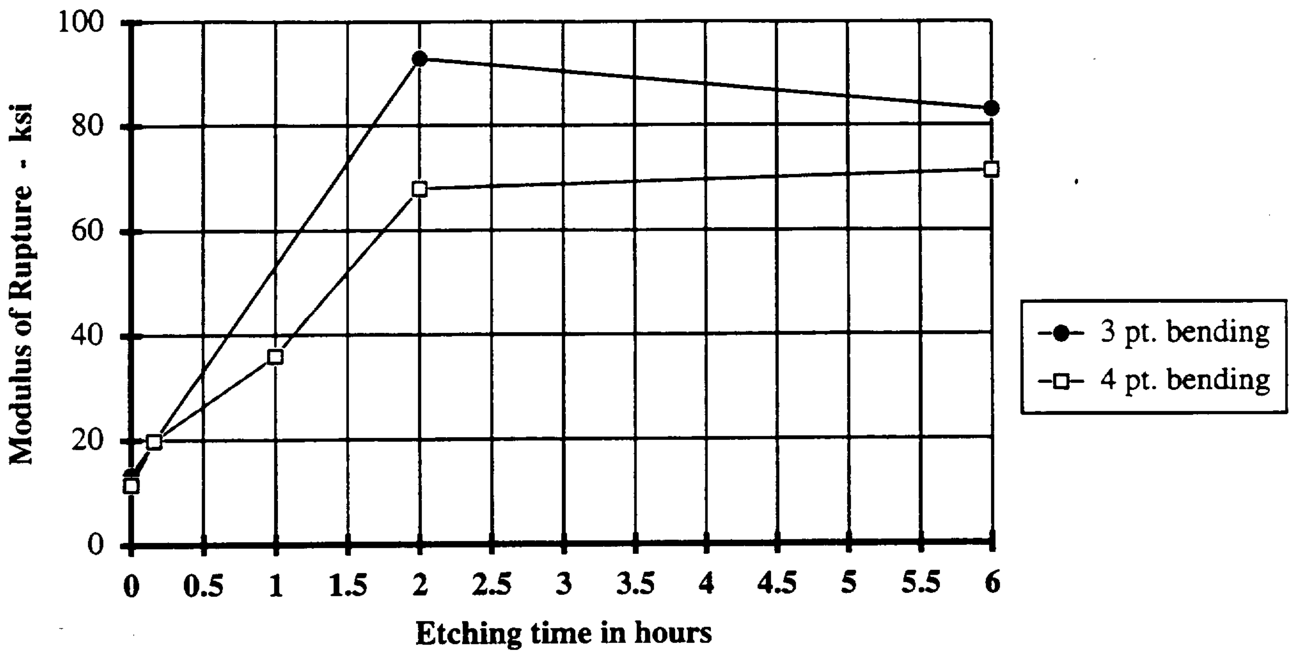


Figure C1. Etching Effect on Strength of Pyrex Rods

STATISTICAL STRENGTH DISTRIBUTION

All of the test data from the tests summarized in Table C-1 are tabulated in Tables C-3a and C-3b. The data have been sorted, normalized, and individually fitted with Weibull distributions as shown in Figure C-2, to seek out the most appropriate distribution fit. All the data seem to be reasonably well represented by a Weibull modulus, $m = 5$, even after etching and increasing strength dramatically. The individual samples are small, and the individual fits are not highly accurate. Table C-3 shows the pooling of <u>all</u> the glass data to create a pseudopopulation of 120 observations. Figure C-3 shows that $m = 5$ is a very good fit for these integrated data, and therefore a design-allowable knockdown factor for this glass can follow the top curve of Figure 3 in subsection 3.2.1.4. The reference strength data for design will depend on surface preparation. The observed strength distribution of each test group is plotted in Figure C-2.

Table C-3a. Glass Data Ranked for Statistical Analysis

As received		j	MOR	x/xmed	F
3 pt bend		1	11221	0.842	0.045
		2	11488	0.862	0.110
N=	15	3	11690	0.878	0.175
		4	11924	0.895	0.240
Med=	13320	5	12105	0.909	0.305
		6	12487	0.937	0.370
Avg=	14323	7	13213	0.992	0.435
		8	13320	1.000	0.500
CV=	0.18	9	13892	1.043	0.565
		10	16115	1.210	0.630
		11	16315	1.225	0.695
		12	16874	1.267	0.760
		13	17536	1.317	0.825
		14	17901	1.344	0.890
		15	18763	1.409	0.955

10 min. etch		j	MOR	x/xmed	F
4 pt bend		1	10970	0.554	0.056
		2	13120	0.663	0.137
N=	12	3	14060	0.710	0.218
		4	18280	0.924	0.298
Med=	19790	5	18470	0.933	0.379
		6	19190	0.970	0.460
Avg=	18793	7	20390	1.030	0.540
		8	20520	1.037	0.621
CV=	0.22	9	20690	1.045	0.702
		10	21800	1.102	0.782
		11	22120	1.118	0.863
		12	25900	1.309	0.944

As received		j	MOR	x/xmed	F
4 pt bend		1	8654	0.750	0.033
		2	9505	0.824	0.079
N=	21	3	9590	0.831	0.126
		4	10269	0.890	0.173
Med=	11541	5	10278	0.891	0.220
		6	10378	0.899	0.266
Avg=	12566	7	10544	0.914	0.313
		8	10792	0.935	0.360
CV=	0.21	9	11393	0.987	0.407
		10	11470	0.994	0.453
		11	11541	1.000	0.500
		12	12317	1.067	0.547
		13	12977	1.124	0.593
		14	13364	1.158	0.640
		15	14324	1.241	0.687
		16	15238	1.320	0.734
		17	15443	1.338	0.780
		18	16139	1.398	0.827
		19	16165	1.401	0.874
		20	16510	1.431	0.921
		21	17005	1.473	0.967

1 hr etch		j	MOR	x/xmed	F
4 pt bend		1	20313	0.564	0.031
		2	21763	0.604	0.076
N=	22	3	22819	0.633	0.121
		4	23832	0.661	0.165
Med=	36046	5	25113	0.697	0.210
		6	27424	0.761	0.254
Avg=	37919	7	28626	0.794	0.299
		8	31073	0.862	0.344
CV=	0.39	9	32126	0.891	0.388
		10	32167	0.892	0.433
		11	36017	0.999	0.478
		12	36075	1.001	0.522
		13	36954	1.025	0.567
		14	37132	1.030	0.612
		15	40863	1.134	0.656
		16	40908	1.135	0.701
		17	42242	1.172	0.746
		18	43646	1.211	0.790
		19	51687	1.434	0.835
		20	61547	1.707	0.879
		21	67394	1.870	0.924
		22	74497	2.067	0.969

Table C-3b. Glass Data Ranked for Statistical Analysis

2 hr etch	j	MOR	x/xmed	F
4 pt bend	1	46726	0.686	0.034
	2	48213	0.708	0.083
N= 20	3	49618	0.729	0.132
	4	51922	0.762	0.181
Med= 68105	5	51987	0.763	0.230
	6	61115	0.897	0.279
Avg= 68734	7	62473	0.917	0.328
	8	62773	0.922	0.377
CV= 0.22	9	65189	0.957	0.426
	10	65652	0.964	0.475
	11	70559	1.036	0.525
	12	72699	1.067	0.574
	13	74135	1.089	0.623
	14	74313	1.091	0.672
	15	74396	1.092	0.721
	16	76381	1.122	0.770
	17	85298	1.252	0.819
	18	88476	1.299	0.868
	19	94931	1.394	0.917
	20	97826	1.436	0.966

2 hr etch	j	MOR	x/xmed	F
3 pt bend	1	60784	0.650	0.045
	2	73151	0.782	0.110
N= 15	3	77126	0.825	0.175
	4	78715	0.842	0.240
Med= 93505	5	84564	0.904	0.305
	6	89134	0.953	0.370
Avg= 92270	7	90720	0.970	0.435
	8	93505	1.000	0.500
CV= 0.17	9	97340	1.041	0.565
	10	99020	1.059	0.630
	11	100465	1.074	0.695
	12	101614	1.087	0.760
	13	103331	1.105	0.825
	14	113300	1.212	0.890
	15	121281	1.297	0.955

6 hr etch	j	MOR	x/xmed	F
4 pt bend	1	35590	0.499	0.045
	2	38874	0.545	0.110
N= 15	3	40064	0.562	0.175
	4	43527	0.610	0.240
Med= 71332	5	51408	0.721	0.305
	6	57146	0.801	0.370
Avg= 65954	7	60907	0.854	0.435
	8	71332	1.000	0.500
CV= 0.34	9	71786	1.006	0.565
	10	71840	1.007	0.630
	11	71937	1.008	0.695
	12	78740	1.104	0.760
	13	84560	1.185	0.825
	14	105173	1.474	0.890
	15	106426	1.492	0.955

6 hr etch	j	MOR	x/xmed	F
3 pt bend	1	36789	0.444	0.045
	2	37568	0.453	0.110
N= 15	3	63624	0.767	0.175
	4	68406	0.825	0.240
Med= 82911	5	70966	0.856	0.305
	6	72252	0.871	0.370
Avg= 76385	7	73716	0.889	0.435
	8	82911	1.000	0.500
CV= 0.26	9	83624	1.009	0.565
	10	84247	1.016	0.630
	11	87018	1.050	0.695
	12	87830	1.059	0.760
	13	93754	1.131	0.825
	14	96703	1.166	0.890
	15	106360	1.283	0.955

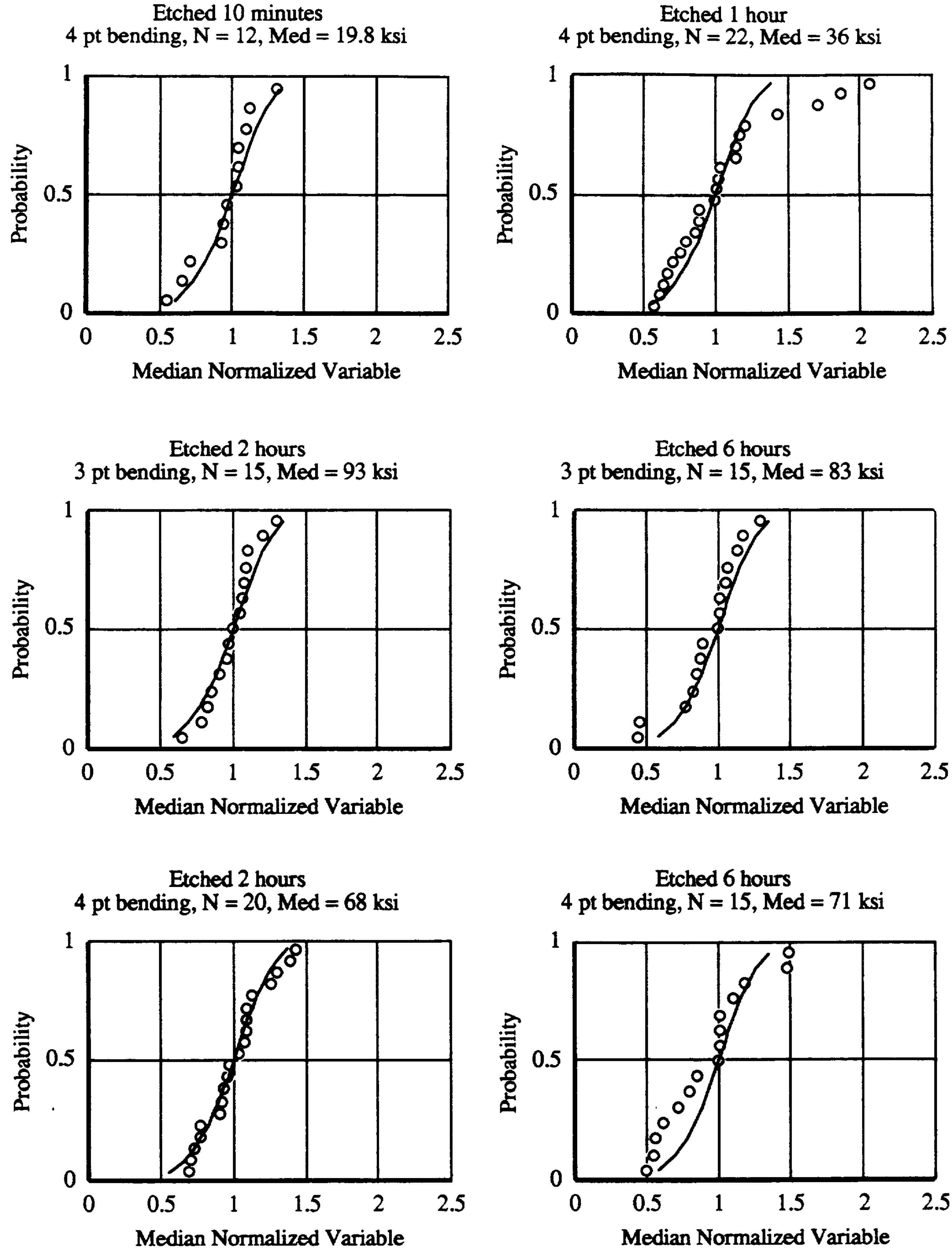


Figure C-2. Weibull Distribution with m = 5 for Glass Data

Table C-3c. Pooled Glass Data Ranked for Statistical Analysis

Pooled Data		x/xmed	F	m est	fit-norm	
Median normalized	1	0.444	0.006	5.88	0.385	
	2	0.453	0.014	4.91	0.460	
N= 120	3	0.499	0.022	4.92	0.505	
	4	0.545	0.031	5.11	0.538	
Med= 1	5	0.554	0.039	4.84	0.565	
	6	0.562	0.047	4.61	0.587	
Avg= 1	7	0.564	0.056	4.35	0.607	
	8	0.604	0.064	4.66	0.625	
CV= 0.27	9	0.610	0.072	4.50	0.641	
	10	0.633	0.081	4.62	0.656	
m = 5	11	0.650	0.089	4.66	0.669	
	12	0.661	0.097	4.63	0.682	
	13	0.663	0.105	4.45	0.694	
	14	0.686	0.114	4.64	0.705	
	15	0.697	0.122	4.63	0.716	
	16	0.708	0.130	4.64	0.726	
	17	0.710	0.139	4.49	0.736	
	18	0.721	0.147	4.49	0.745	
	19	0.729	0.155	4.46	0.754	
	20	0.750	0.164	4.71	0.763	
	21	0.761	0.172	4.76	0.771	
	22	0.762	0.180	4.60	0.779	
	23	0.763	0.189	4.44	0.787	
	24	0.767	0.197	4.35	0.794	
	25	0.782	0.205	4.50	0.802	
	26	0.794	0.213	4.60	0.809	
	27	0.801	0.222	4.59	0.816	
	28	0.824	0.230	5.02	0.823	
	29	0.825	0.238	4.85	0.830	
	30	0.825	0.247	4.65	0.836	
	31	0.831	0.255	4.62	0.843	
	32	0.842	0.263	4.76	0.849	
	33	0.854	0.272	4.95	0.855	
	34	0.856	0.280	4.80	0.861	
	35	0.862	0.288	4.80	0.867	
	36	0.871	0.297	4.93	0.873	
	37	0.889	0.305	5.49	0.879	
	38	0.890	0.313	5.25	0.885	
	39	0.891	0.321	5.01	0.890	
	40	0.891	0.330	4.77	0.896	
	41	0.892	0.338	4.56	0.901	
	42	0.897	0.346	4.51	0.907	
	43	0.899	0.355	4.32	0.912	
	44	0.904	0.363	4.28	0.918	
	45	0.914	0.371	4.44	0.923	
	46	0.917	0.380	4.32	0.928	
	47	0.922	0.388	4.23	0.933	
	48	0.924	0.396	4.00	0.938	
	49	0.933	0.404	4.21	0.944	Etc to row 121
	50	0.935	0.413	3.93	0.949	

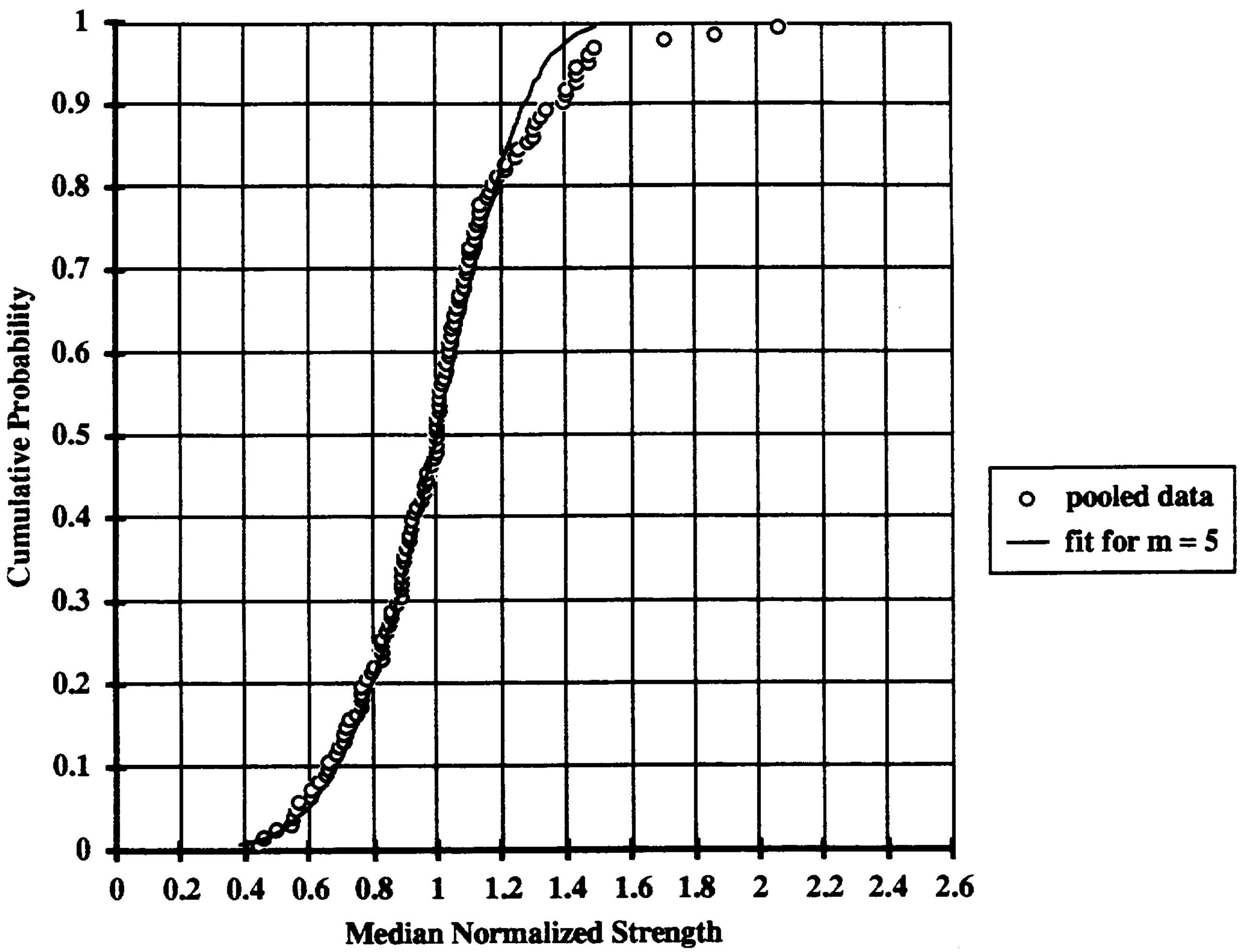


Figure C-3. Weibull Distribution for Pooled Glass Rod Data

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