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# DETERMINING USABILITY VERSUS COST AND YIELDS OF A REGIONAL TRANSPORT

Slobodan Gvozdenovic  
University of Belgrade

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## NOMENCLATURE

$\alpha, \beta, g$  and  $d$  - represent statistical coefficients for one particular aircraft type determined by using Performance Manual

$a, b, c$  and  $d$  - coefficients depend upon the type of cruise (HSC or LSC) and flight altitude

DOC - direct operating costs

$DOC_{AT}$  - direct operating cost per aircraft trip

$e, f, e_1$  and  $f_1$  - coefficients which are determined for each aircraft type and configuration, reserve fuel and flight regime

$F_p$  - fuel price

$g, h$  and  $i$  - coefficients known for each aircraft type and supposed economic assumption

$t_g$  - ground time

HSC- High Speed Cruise

$I_{AT}$  - income per flight

IOC - indirect operating cost

$IOC_{AT}$  - indirect operating cost per aircraft trip

$I_p$  - cabin load factor

$j, k, m$  and  $n$  - coefficients known for each particular aircraft type and for established network serviced by this aircraft type

$k_p$  - trip fuel correction factor

$k_t$  - trip time correction factor

$l_F$  - freight load factor

$LRC$  - Long Range Cruise

$P_{AT}$  - number of passengers per flight

$PF$  - passengers fare

$PR$  - profitability ratio

$R$  - trip distance

$R_A$  - maximum range to which maximum payload can be transported

$R_{av}$  - average trip distance

$RB$  - maximum range which can be attained by full fuel tanks

$RC$  - maximum range which can be attained by operating aircraft with full fuel tanks but with zero payload

$R_l$  - left limit of the usability range

$R_r$  - right limit of the usability range

$S_a$  - number of available seats

$t_b$  - block time

$TC_{AT}$  - total costs per each flight

$t_f$  - trip time or flight time

$t_g$  - time for taxing

$t_{io}$  - flight time required when carrying full payload

$UR$  - usability range

$W_E$  - Operating Empty Weight

$W_{Fb}$  - block fuel

$W_{F\theta}$  - required trip fuel for max. payload

$W_{Fg}$  - fuel used for taxing

$W_{Fl}$  - trip fuel

$W_g$  - mass of the aircraft with zero payload

$W_p$  - actual payload

$W_{p0}$  - maximum payload

$W_{pF}$  - maximum mass that could be transported in freight compartments after passengers have been loaded

## INTRODUCTION

Regional transports are designed to operate on air networks having the basic characteristics of short trip distances and low density passengers/cargo, i.e. small numbers of passengers per flight. Regional transports passenger capacity is from 10 to 100 seats and operate on routes from 350 to 1000 nautical miles (nm).

An air network operated by regional transports has the following characteristics (Kanafani & Ghobrial, 1982; MIT, 1973):

- connecting regional centers;
- operating on low density passengers/cargo flow services with minimum two frequencies per day;
- operating on high density passengers/cargo flow with more than two frequencies per day; and
- operating supplemental services whenever market demands in order to help bigger capacity aircraft already operating the same routes (Kanafani & Ghobrial, 1982; MIT, 1973).

Airlines owning regional transports have to find out what are the trip distances ( $R$ ) and what are cabin load factors ( $I_p$ ) that make particular aircraft operation efficient. Efficient operation of an airliner, in this paper, is defined by results that achieve a maximum yield/cost ratio.

Passengers, being the sole air transportation consumers, need to be transported to their destinations with low cost and with convenient time tables without any delays.

In order to meet passenger requirements providing low fares and high or required number of frequencies, airlines must constantly monitor operational costs and keep them low. It is obvious that costs of operating aircraft must be lower than yield obtained by transporting passengers and cargo. The requirement to achieve favorable yield/cost ratio must provide the answer to the question of which aircraft will best meet a specific air network (Simson, 1972). An air network is defined by the number of services, the trip distance of each service, and the number of flights (frequencies) per day and week.

## DETERMINATION OF OPERATING COSTS PER FLIGHT

Operating a commercial flight on a trip distance ( $R$ ) an airline would experience block time ( $t_b$ ) and block fuel ( $W_{Fb}$ ). Block time is a sum of the time required for taxing ( $t_g$ ) and trip time or flight time ( $t_f$ )

$$t_b = t_g + t_f \quad (1)$$

whereas block fuel ( $W_{Fb}$ ) is a sum of fuel used for taxing ( $W_{Fg}$ )(kg) and trip fuel ( $W_{Ft}$ )(kg),

$$W_{Fb} = W_{FG} + W_{Ff} \quad (2)$$

Both flight time and trip fuel represent time and fuel required for take-off, climb, cruise, descent and landing. For one particular aircraft type both flight time and trip fuel are directly proportional to the trip distance ( $R$ ) and may be expressed in equation form,

$$t_f = a + b \cdot R \quad (3)$$

$$W_{Ff} = c + d \cdot R \quad (4)$$

where coefficients  $a$  (Fh),  $b$  (Fh/NM),  $c$  (kg) and  $d$  (kg/NM) depend upon the type of cruise (HSC or LRC) and flight altitude ( $H$ ). Table 1, among other data, gives coefficients  $a$ ,  $b$ ,  $c$  and  $d$  for ISA conditions and High Speed Cruise for 15 aircraft types.

Commercial flights, in air transportation, are considered such flights in which payload (passengers and cargo) is transported to a distance- $R$ . The PAYLOAD-RANGE diagram shown on Figure 1 is defined for each aircraft type.

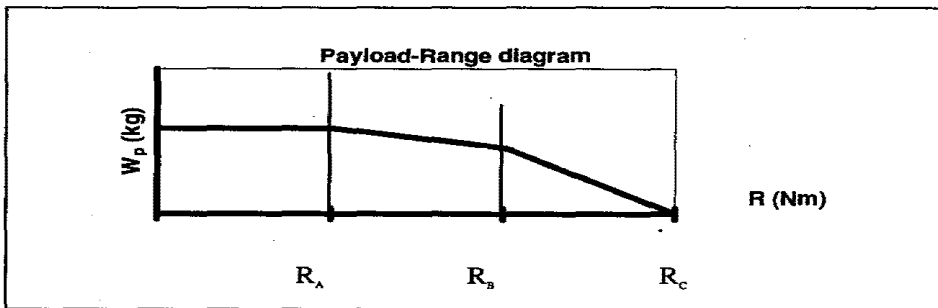


Figure 1. Payload-range diagram

One can easily note three characteristic ranges.

- Range  $R_A$  is the maximum range to which maximum payload can be transported;
- Range  $R_B$  is the maximum range which can be attained by full fuel tanks;
- Range  $R_C$  is the maximum range which can be attained by operating aircraft with full fuel tanks but without any payload.

Functional relation of the mass of payload ( $W_p$ ) versus range ( $R$ ) can be expressed in analytical forms as given below

$$W_p = W_{p0} \quad \text{for } 0 < R \leq R_A \quad (5a)$$

$$W_p = e - f \cdot R \quad \text{for } R_A < R \leq R_B \quad (5b)$$

$$W_p = e_1 - f_1 \cdot R \quad \text{for} \quad R_B < R \leq R_C \quad (5c)$$

where  $R$  (NM) represents range,  $W_{po}$  (kg) represents maximum payload,  $e$  (kg),  $f$  (kg/NM),  $e_1$  (kg) i  $f_1$  (kg/Nm) are coefficients which can be determined for each aircraft type and configuration, reserve fuel and flight regime. In Table 1, coefficients  $e$ ,  $f$ ,  $e_1$  and  $f_1$  are determined for the concerned aircraft on the basis of data given by manufacturers. See Table 3 for a list of the specific manuals researched.

In transporting the mass of payload  $W_p$  to the given or required distance  $R$ , the aircraft consumes fuel and flight time which both influence transportation costs. Airlines consider such cost as operational costs. The aim of each airline is to control and administer their traffic on the given air network and so try to accomplish minimum total costs per each flight ( $TC_{AT}$ ). Total cost per aircraft trip in air transport industry is usually split into:

- direct operating cost per aircraft trip -  $DOC_{AT}$  and
- indirect operating cost per aircraft trip -  $IOC_{AT}$

Direct operating costs  $DOC$  that depend on the trip include:

- flight crew,
- fuel,
- maintenance,
- hull insurance,
- depreciation, and
- finance. (Boeing Airplane Economic Group, 1994)

Indirect operating cost- $IOC$  are, by rule, independent on the trip distance or flight time and can be split into:

- airline related,
- passenger related, and
- cargo related. (Boeing Airplane Economic Group, 1994)

Indirect operating cost are estimated on the basis of aircraft capacity (seats and cargo), average trip distance for the network flown, type of traffic (domestic, international), expected passenger cabin and cargo compartments load factors, ticket sales commission, etc. (Boeing Airplane Economic Group, 1994).

Total cost per flight ( $TC_{AT}$ ) are obtained by adding direct operating costs ( $DOC_{AT}$ ) and indirect operating cost  $IOC_{AT}$

$$TC_{AT} = DOC_{AT} + IOC_{AT} \quad (6)$$

Direct operating cost per flight  $DOC_{AT}$  is linear function of the trip distance [3] and may be defined as:

$$DOC_{AT} = c_1 + c_2 \cdot R \quad (7)$$

where  $DOC_{AT}$  is given in U.S. dollars,  $c_1$  in U.S. dollars (USD) and  $c_2$  in U.S. dollars per nautical mile (USD/NM) and both  $c_1$  and  $c_2$  are known coefficients for one particular aircraft type and market environment data. Indirect operating costs -  $IOC_{AT}$  are determined for each aircraft type on the basis of average passenger cabin load factor -  $l_p$ , average trip distance  $R_{av}$ , as explained in by Simpson (1972) or they could be estimated as a percent of direct operating cost per aircraft trip  $DOC_{AT}$  (AEA, 1990).

To determine direct operating costs for a particular aircraft type it is necessary to define required block time -  $t_b$  and block fuel -  $W_{fb}$  for the anticipated trip distance -  $R$ , whereas for the estimation of indirect operating costs it is necessary to judge or to know passengers and average weight of cargo per flight as well as average trip distance on the network. Equations 1 through 4 for determination of the block time and block fuel are written for the case when transporting maximum payload. However, since the number of passengers and cargo weight are both, as a rule, less than maximum payload, it is necessary to perform a correction of the required flight time and fuel when the actual payload  $W_p$  is less than the maximum  $W_{p0}$  i.e.  $W_p < W_{p0}$ . It is known that a lighter aircraft consumes less fuel and, when flying HSC techniques, the flight time is less for the same trip distance. For corrections of required flight time  $t_f$  and trip fuel  $W_{FT}$  for selected trip distance by using reduced mass of payload  $W_p$ , the following correction parameters are introduced:  $k_t$  is the correction factor for flight time and  $k_p$  the correction factor for trip fuel. The correction factor of the flight time  $k_t$ , represents the ratio between time required -  $t_f$  when carrying a reduced mass of payload  $W_p$  and the time  $t_{f0}$  required when carrying a full payload  $W_{p0}$  for the same trip distance  $R$  or

$$k_t = \frac{t_f}{t_{f0}} \quad (8)$$

$$t_f = k_t \cdot t_{f0} \quad (8a)$$

The trip fuel coefficient correction  $k_f$  is the ratio between required trip fuel -  $W_{Ff}$  to transport a reduced mass of payload -  $W_p$  and the required trip fuel  $W_{Ff0}$  to transport a full payload  $W_{p0}$  over the same trip distance.

$$k_f = \frac{W_{Ff}}{W_{Ff0}} \quad (9)$$

$$W_{Ff} = k_f \cdot W_{Ff0} \quad (9a)$$

Correction coefficients ( $k_t$  and  $k_f$ ) are both nondimensional units. Values for ( $t_{f0}$  and  $W_{Ff0}$ ) are determined by equations 3 and 4. Numerical values of the coefficients ( $k_t$  and  $k_f$ ) for one particular aircraft type are determined by using statistical methods for determining trip fuel and time, and for the number of different masses of payload (from  $l_p = 0.1$  to  $l_p = 1.0$ ) for selected trip distances based upon the Performance Manual and they have the following form.

$$k_t = \alpha \cdot (W_E + l_p \cdot S_a \cdot 91 + l_F \cdot W_{pF})^\beta \quad (10)$$

$$k_F = \gamma \cdot (W_E + l_p \cdot S_a \cdot 91 + l_F \cdot W_{pF})^g \quad (11)$$

where  $\alpha$ ,  $\beta$ ,  $g$  and  $d$  represent statistically determined coefficients for one particular aircraft type using Performance Manual.  $W_E$  (kg) is Operating Empty Weight,  $l_p$  is passenger cabin load factor,  $S_a$  number of available seats,  $l_F$  freight load factor,  $W_{pF}$  (kg) is maximum mass that could be transported in freight compartments after passengers have been loaded. It is assumed that one passenger mass together with baggage is 91 kg. So this gives

$$W_{pF} = W_{p0} - S_a \cdot 91 \quad (12)$$

Numerical values of coefficients  $\alpha$ ,  $\beta$ ,  $g$  and  $d$  for ISA condition and HS cruise are given in Table 5 for 15 different aircraft types. Figures 2a and 2b are graphic representations of  $t_f(R)$  and  $W_{FF}(R)$  for passenger cabin load factors  $l_{p1} < l_{p2} < l_{p3}$ .

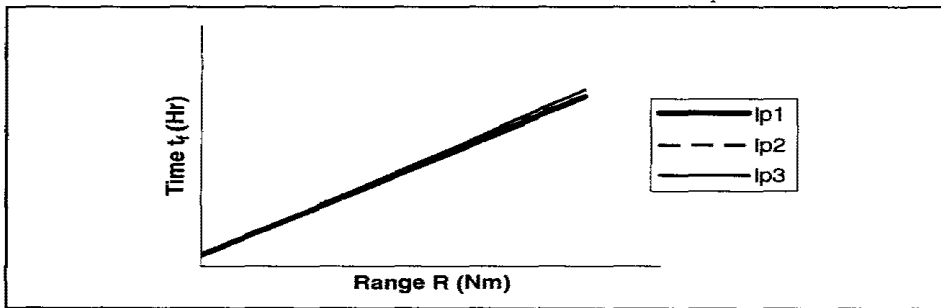


Figure 2a. Representation of trip time for different passenger load factors

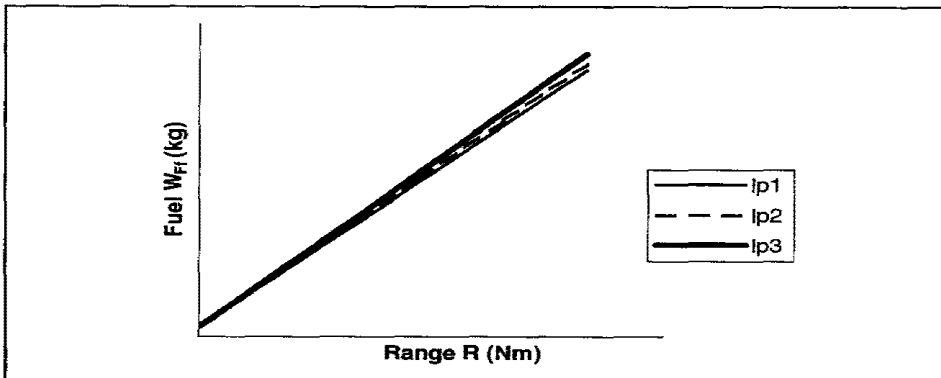


Figure 2b. Representation of trip fuel for different passenger load factors

Diagrams as shown on Figures 2a and 2b clearly indicate that both trip time and trip fuel determined by equations 8 and 9 are linear functions of trip distance and that the higher the mass of payload  $W_p$  and hence higher the mass of the aircraft  $W_g$ , more time and more fuel is required to fly selected distance. Using equations (8) and (9) an assembly of straight lines are obtained which are used to determine required trip time -  $t_f$  and trip fuel -  $W_{Ff}$  not only as function of trip distance  $R$  but also as functions of payload mass which are pondered by using correction coefficients  $k_T$  and  $k_F$  more exactly by coefficients of passenger cabin load factor -  $l_p$  and freight compartment load factor  $l_F$ .

Using methods to estimate the total costs per flight  $TC_{AT}$  adapted for both turbo jet and turboprop aircraft, as well as equations 8 through 11 to determine required trip time and fuel with newly introduced correction coefficients  $k_t$  and  $k_F$  for selected trip distance -  $R$  it is possible to determine total costs per flight  $TC_{AT}$  versus trip distance -  $R$  versus mass of payload -  $W_p$ . So now we have the equation

$$TC_{AT} = \Phi(R, W_p) \quad (13)$$

Direct operating costs per flight  $DOC_{AT}$ , are depending on trip time and fuel but also on trip distance  $R$  and payload mass  $W_p$ .

$$DOC_{AT} = \Phi_1(R, W_p) \quad (14)$$

This can be written in the form of following equation

$$DOC_{AT} = g + h \cdot t_g + i \cdot t_f + F_p \cdot W_{Fg} + F_p \cdot W_{Ff} \quad (14a)$$

$$DOC_{AT} = g + h \cdot t_g + i \cdot k_t \cdot t_g + F_p \cdot W_{Fg} + F_p \cdot k_F \cdot W_{Ff} \quad (14b)$$

where  $DOC_{AT}$  (USD) represent direct operating cost per flight,  $g$  (USD),  $h$  (USD/Hr) and  $i$  (USD/Hr) are coefficients known for each aircraft type and economic assumption (Table 1),  $t_g$  (Hr) ground time,  $t_f$  flight time (Hr),  $F_p$  (USD/kg) fuel price,  $W_{Fg}$  (kg) ground fuel and  $W_{Ff}$  (kg) flight fuel.

Coefficients given in equations (14a) and (14b) depend on aircraft characteristics and economic assumptions under which the traffic is being executed and they are determined by modified methods (AEA, 1990; Simpson, 1972). The following conditions should be noted,

1. coefficient  $g$  depends on the number of aircraft of the same type in the fleet, value of spare parts (aircraft and engine), of the power plant parameters, aircraft operating empty mass, maintenance labor rate for aircraft structure and power plant and upon burden.
2. coefficient  $h$  depends on total investment per aircraft, annual utilization, depreciation period for aircraft and equipment (number of years and depreciation rate), interest rate and time to pay off the credit, insurance rate and aircraft take-off mass.



**Table 1**  
**Entry Economic Assumptions That Influence Traffic**

Economic Assumptions	
Number of aircraft	5
Deprecation period (years)	10
Residual value (%)	15
Financial period (years)	5
Interest rate (%)	6,75
Insurance (%)	0,75
Labor rate (USD/Mh)	50
Burden (%)	200
Crew utilization (Bh/Month)	65
Fuel price (USD/kg)	0,215
Average distance (NM)	250

3. coefficient  $i$  depends from one side from total investment per aircraft, annual utilization, depreciation period (number of years and depreciation rate), interest rate and time to pay off the credit, insurance rate and aircraft take-off mass, and from the other side, from number of aircraft in the fleet, aircraft operating empty mass, power plant parameters, labor rate for maintenance of aircraft and power plant and finally burden.

Indirect operating cost per flight  $IOC_{AT}$  for established network having average trip distance  $R_{av}$ , depends on the payload mass  $W_p$ , and for an aircraft type could be written as:

$$IOC_{AT} = \Phi_2 (W_p) \quad (15)$$

$$IOC_{AT} = \Phi_2 (W_p) \quad (15a)$$

where  $IOC_{AT}$  (USD) represent indirect operating cost per flight, and  $j$  (USD),  $k$  (USD),  $m$  (USD) and  $n$  (USD) are coefficients which are known for each particular aircraft type and for established network serviced by this aircraft type. These are presented in Table 2.

Coefficients in equation 15 are determined by using modified methods and the following conditions should be noted.

1. Coefficient  $j$  depends on aircraft empty mass and mass of maximum payload.
2. Coefficients  $k$  and  $m$  depend on number of passenger seats, and maximum mass of freight that can be loaded in freight compartments, average trip distance on the network flown by the same aircraft type, maximum take-off mass, sales commission for selling transport capacity (passenger seats and cargo).
3. Coefficient  $n$  represents cost for tanking fuel at departure airport.

To estimate total cost per flight  $TC_{AT}$  by equations 13, 14, and 15 one must define entry economic assumptions which influence traffic, as shown in Table 1. This information is required in addition to knowledge of specific aircraft as presented in Table 3.

Table 6 contains coefficients ( $g, h, i, j, k, m$  and  $n$ ) required for estimation of total cost per flight for fifteen regional transports. Since both trip time  $t_f$  and trip fuel  $W_{Ff}$  are linear function of the trip distance  $R$  as shown by equations 8 and 9 and by figures 2a and 2b, it means that total cost per flight is also linear function of trip distance  $R$  but depends upon payload mass -  $W_p$  as shown in the equation below,

$$TC_{AT} = \Phi_1 (R, W_p) + \Phi_2 (W_p) \quad (16)$$

Figure 3 shows total cost per flight  $TC_{AT}$  versus trip distance -  $R$  for different values of payload mass -  $W_p$  expressed by passenger cabin load factor coefficient -  $l_p$  where  $l_{p1} < l_{p2} < l_{p3}$  as shown here.

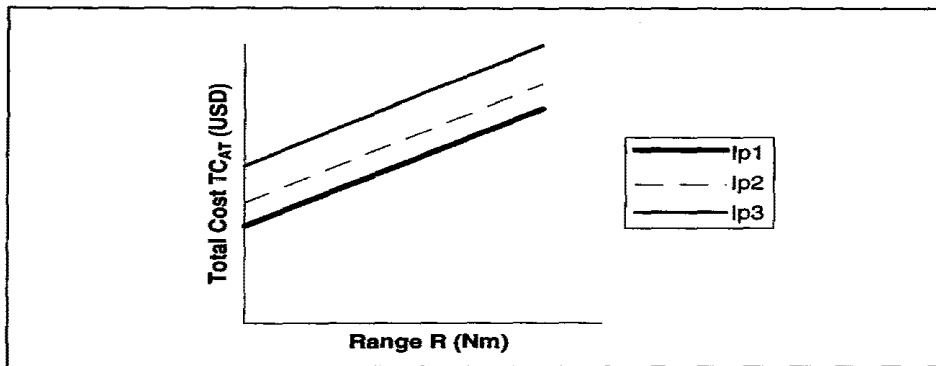


Figure 3. Total cost per flight for different passenger load factors

The difference between standard estimation of operating cost and method described above lies in introduction of correction coefficients  $k_t$  and  $k_F$  which both depend on payload mass -  $W_p$ . By doing so we have, instead of a single straight line representing total cost  $TC_{AT}$  versus trip distance  $R$ , an assembly of straight lines (figure 3) representing a nomogram for cost estimation. This renders possible more precise estimation of operating cost. So, for instance, a turbo-prop aircraft having a 50 passenger seat capacity ( $S_a = 50$ ) over a trip distance ( $R = 250 \text{ NM}$ ), and if the coefficient of passenger load factor is  $l_p = 0.7$  or  $PA_{AT} = 35$ , then the planned total cost per flight can be estimated to be USD 3423 which is for USD 321 or 9.38 percent less in comparison with an estimation using the standard method (based upon transportation of full payload all the time).

For efficient planing, it is necessary to know, besides total cost per flight  $TC_{AT}$ , value of unit cost-  $t.c.$  Unit cost is defined as total cost per flight  $TC_{AT}$  divided by unit of transportation work ( payload range) or

$$t.c. = \frac{TC_{AT}}{W_p \cdot R} \quad (17)$$

or

$$t.c. = \frac{g + h \cdot t_g + i \cdot t_f + F_p \cdot W_{FG} + F_p \cdot W_{FJ} + j + k + m + n}{W_p \cdot R} \quad (17a)$$

$$t.c. = \frac{g + h \cdot t_g + i \cdot k_i \cdot t_{j0} + F_p \cdot W_{FG} + F_p \cdot k_F \cdot W_{FJ} + j + k + m + n}{W_p \cdot R} \quad (17b)$$

where  $t.c.$  (USD/kg/NM) represent operational cost per seat equivalent freight per NM;  $TC_{AT}$  (USD)- total cost per flight determined by use of equations 14 and 15;  $W_p = l_p \cdot S_a \cdot 91 + l_F \cdot W_{pF}$  is the mass of the payload (kg) determined by Figure 6 and/or by equation 5.  $R$  (NM) is the trip distance. Dependence of change in unit cost -  $t.c.$  versus trip distance  $R$  is shown in Figure 4.

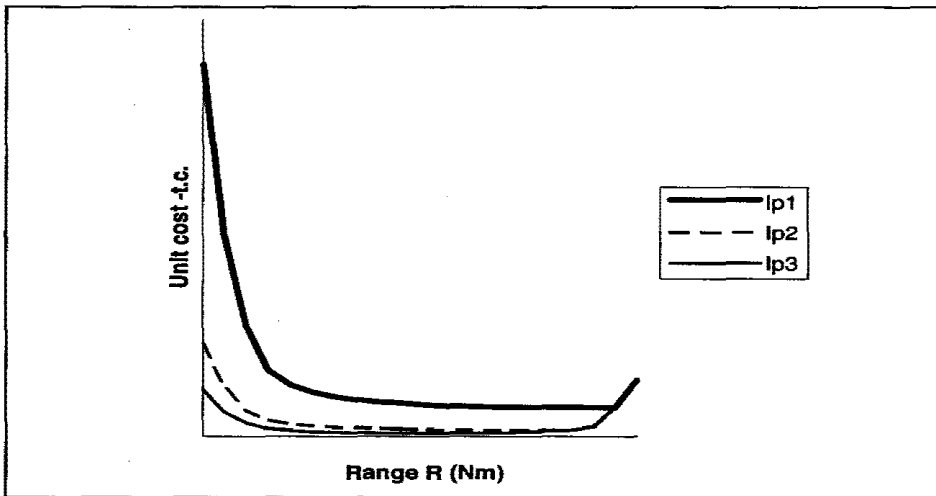


Figure 4. Dependence of change in unit cost for different passenger load factors

Unit cost  $t.c.$  (same as total cost per flight  $TC_{AT}$ ) depends not only on the trip distance  $R$  but also on the mass of payload  $W_p$  so, as a consequence, instead of one single line we have assembly of lines each for one particular value of payload mass. Knowing unit costs values it is possible to compare two or more different aircraft types to be used on established air network. By using this advantage it is possible to define tariff policy, etc.

## DETERMINATION OF REGIONAL TRANSPORT USABILITY INTERVAL

For airlines it is important to know what are total cost per flight but besides this they should be able to predict what are trip distances  $R$  that their airlines can economically operate if passengers and cargo flows are known (expressed by coefficients  $l_p$  and  $l_f$ ). Rational operation then can be defined as range interval  $\Delta R$  in which airplanes can economically operate by transporting known or anticipated passengers and cargo flows. Such interval can be defined as useful range ( $UR$ ).

$$UR = (R_l, R_r) \quad (18)$$

Left side limits represent minimum range and right side limits represent maximum range. Intervals within such limits are usually defined as the useful range in which it is possible to operate economically.

Criteria used to determine useful range interval -  $UR$  in this paper are the minimum operational costs or the maximum profit-ratio of income per flight-  $I_{AT}$  over costs per flight-  $TC_{AT}$ .

In the second criteria above, total costs per flight ( $TC_{AT}$ ) have been defined by using equations 14, 15 and 16. The results are shown in figure 3. Since total costs per flight are represented as an assembly of straight lines versus trip distance  $R$  it means that it is not possible to determine trip distance for the minimum cost except when the trip distance equal to zero. This solution is of course not usable.

Unit operational costs determined by equation 17 and shown in Figure 4 for a unit mass of payload  $W_p$  are decreased by increasing range until the point  $R_A$  (range to which maximum mass of payload can be transported). At the range  $R_A$  unit costs have the minimum value and after further increases in range, the unit costs start to raise again. This is logical due to the reduced mass of payload being transported as shown by equation 5 and presented in Figure 1. This means that a criteria of minimum unit costs can not be used to determine useful range, because interval is reduced to one single point  $R_A$ . It is known from practical operation that transport category aircraft and especially regional transports operate on ranges considerably less than  $R_A$ , or:

$$\min(t.c) \rightarrow R = R_A \quad (19)$$

The aim of operating an airline is to create income from transported passengers and/or cargo. Income per flight is obtained by the number of passengers carried ( $P_{AT}$ ) and airfares applied ( $PF$ ).

The number of passengers per flight  $P_{AT}$  is represented by the equation

$$P_{AT} = l_p \cdot S_a \quad (20)$$

Passengers fares ( $PF$ ) basically depends upon the trip distance-  $R$  (NM) and could be represented as

$$PF = o \cdot R^p \quad (21)$$

where  $PF$  (USD) represents the passenger fare on the trip distance  $R$  (NM) and  $o$  (USD) and  $p$  are statistically determined coefficients which depend upon the type of operation (domestic or international), and the quality of transportation (first class, tourist class, reduced fares). In this paper coefficients have been determined using AIR INTER GROUP AIR FRANCE fares in 1996 for Y class so that their values are  $o = 2.492$  (USD),  $p = 0.752$  and correlation coefficient  $r = 0.706$ . Knowing all this, the income per aircraft trip  $I_{AT}$  can be expressed as the following equations.

$$I_{AT} = P_{AT} \cdot PF \quad (22)$$

$$I_{AT} = I_p \cdot S_a \cdot o \cdot R^p \quad (22a)$$

where  $I_{AT}$  (USD) represent income per aircraft trip,  $I_p$  (percent) passenger cabin load factor,  $S_a$  number of passenger seats in the aircraft,  $R$  (NM) trip distance,  $o$  (USD) and  $p$  statistical coefficients depending upon airline fare policy and type of fare. Therefore income per aircraft trip depends upon number of passengers  $P_{AT}$  or mass of payload carried  $W_p = I_p \cdot S_a \cdot 91$ . The number 91 represents the mass of a single passenger with baggage and  $R$  trip distance. So it can be written as follows,

$$I_{AT} = \Phi_3(W_p \cdot R) \quad (23)$$

Each airline works hard to have more income than operating costs, or at least to equalize both ( $TC_{AT}$   $I_{AT}$ ). To estimate whether the operation is economical, the ratio between income per aircraft trip  $I_{AT}$  and total operating cost per aircraft trip  $TC_{AT}$  may be used. This ratio is named the profitability ratio ( $PR$ ) and can be determined by equations 13, 15 and 16.

$$PR = \frac{I_{AT}}{TC_{AT}} = \frac{\Phi_3(W_p, R)}{\Phi_1(W_p, R) + \Phi_2(W_p)} = \Phi_4(W_p, R) \quad (24)$$

or

$$PR = \frac{I_p \cdot S_a \cdot o \cdot R^p}{g+h \cdot t_g + i \cdot t_f + F_p \cdot W_{Fg} + F_p \cdot W_{Ff} + j+k+m+n} \quad (25)$$

$$PR = \frac{I_p \cdot S_a \cdot o \cdot R^p}{g+h \cdot t_g + i \cdot k_i \cdot t_{fo} + F_p \cdot W_{Fg} + F_p \cdot k_{Ff} \cdot W_{Ffo} + j+k+m+n} \quad (25a)$$

Diagram of profitability ratio  $PR$  versus trip distance  $R$  and number of passengers per flight  $P_{AT}$  or passenger cabin load factor is shown in Figure 5.

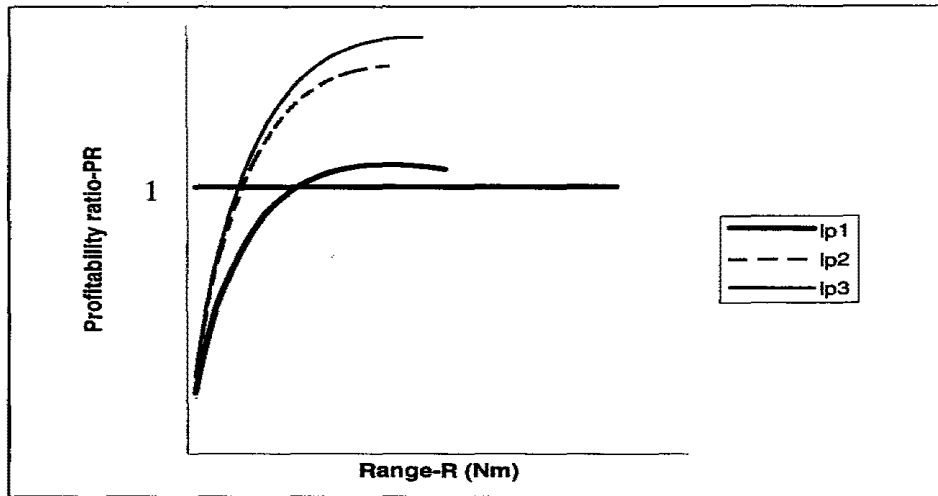


Figure 5. Profitability ration versus trip distance for different passenger load factors

Both Figure 5 and equation 25 show that increasing trip distance  $R$  and/or the mass of the payload (number of passengers) per flight increases the profitability ratio.

If we accept criteria  $PR > 1$  it is then possible to determine the interval of useful range  $UR$  (i.e. left  $R_l$  and right  $R_r$ ) and limits of the range. Left and right limits of the useful range are determined as follows. The interval of useful range is determined for a value of payload mass  $W_p = const.$  or  $I_p = const.$  and for the pre-determined economic assumptions as listed in Table 1. The left limit of the useful range interval  $R_l$  is determined from equation 25 by setting the profitability ratio equal to one.

$$PR = 1.0 \rightarrow R_l(Nm) \quad (26)$$

The right limit of the useful range interval  $R_r$  is determined from the condition that constant mass of payload  $W_p = const.$  is transported to such a distance to achieve max.  $PR$ . For one particular value of the mass of payload  $W_p$ , the maximum value of  $PR$  is determined from equation 25. The maximum possible distance to which payload  $W_p$  can be transported is  $R$ , as defined by equations 5b and 5c i.e.

$$\max.PR \rightarrow R_r = \frac{e - I_p \cdot S_a \cdot 91}{f} \quad (27a)$$

or

$$\max.PR \rightarrow R_r = \frac{e_1 - I_p \cdot S_a \cdot 91}{f_1} \quad (27b)$$

where  $e$  (kg),  $f$  (kg/NM),  $e_i$  (kg) and  $f_i$  (kg/NM) are coefficients determined for each aircraft type based upon payload range diagram (Figure 1) as shown in Table 3. Using equations 26 and 27, the useful range  $UR$  or interval of rational range from the economical point of view can be determined by:

$$UR = \Delta R = R_r - R_l \quad (28)$$

The interval of useful ranges ( $UR$ ) for passenger cabin load factors of  $l_p = 0.5$  and  $l_p = 0.7$  and high speed flight conditions is given for 15 regional transports in Table 2 by using equations 26, 27 and 28.

**Table 2**  
Intervals of Useful Ranges for Aircraft at Two Values of Cabin Load Factors

	$Sa$	$l_p=0,5$			$l_p=0,7$		
		$R_l$ (NM)	$R_r$ (NM)	$UR$ (NM)	$R_l$ (NM)	$R_r$ (NM)	$UR$ (NM)
Do 228	19	236	1090	854	128	867	739
1900D	19	327	1227	900	154	980	826
SD 330	30	250	626	376	122	616	494
Do 328	30	193	1552	1359	115	1327	1212
SF 340	34	188	1370	1182	110	1346	1236
ATR 42	46	141	2456	2315	88	2430	2342
F 50	50	134	1508	1374	85	1465	1380
Saab 2000	50	159	1240	1081	100	1221	1121
ATR 72	66	109	2282	2173	72	2243	2171
Dash8-400A	70	119	1331	1212	79	1318	1239
CRJ	50	166	1303	1137	104	1256	1152
CRJ-700	70	164	2569	2404	104	2391	2287
F 70	79	132	1748	1616	87	1656	1569
F 100	105	102	1672	1570	70	1550	1480
A 319	124	133	3231	3098	88	2720	2632

Results given in Table 2 may be used as a base to determine the trip distances to be operated by different aircraft. Figures given take into consideration aircraft capacities and performances and are presented for two values of cabin load factors. By increasing the mass of payload  $W_p$  or passenger cabin load factor  $l_p$ , the left limit of the usability range moves towards shorter ranges which means that the aircraft could be used economically on shorter trip distances.

For airlines, it is significant to define the minimum trip distance operation which is economically justifiable. This minimum trip distance is the left limit of the trip distance interval ( $R$ ) obtained from the condition when  $PR=1.0$ . As already stated, limits of the interval are not fixed values but they do depend on the mass of the payload for the defined economic assumptions (Table 1).

Left limits of the usability range versus passenger cabin load factor  $l_p$  is shown on Figure 6 for two aircraft of the same capacity (70 passengers seats). One airplane is the propjet Dash 8-400A while the other is the turbojet CRJ-700.

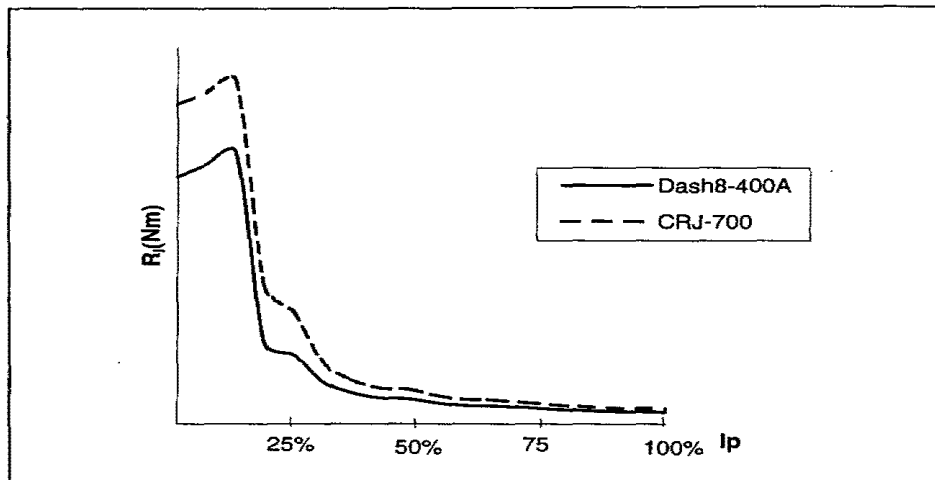


Figure 6. Left limits of usability range for Dash 8-400A and the CRJ-700

Results shown in Figure 6 confirm the known supposition that, for the same capacity, turboprop aircraft are more economical on short trip distances than pure turbo jet aircraft. Both in table T3a and T3b give left limit of the usability range  $R_i$  versus number of passengers per flight  $P_{AT}$  (or mass of payload  $W_p$ ) for 15 aircraft types considered in this paper. They were examined for High Speed flight and condition that  $PR=1$

The following conclusions can be drawn from Table 3:

1. for one type and capacity of aircraft, an increase in the number of passengers per flight  $P_{AT}$  reduces the range of economically operated ( $PR > 1.0$ ) trip distances.
2. for a predetermined number of passengers per flight (data typical for a network serviced by the operator)  $P_{AT} = const.$  it can be shown that the more seats that exist in the aircraft, the longer the trip distance is required to operate economically with  $PR=1.0$ . So for an aircraft with 30 passenger seats (Do328), the minimum range for economical operation is  $R_i=75 NM$  whereas for an aircraft with 50 passenger seats (Saab 2000)  $R_i=122 NM$  and for an aircraft with 70 passenger seats  $R_i=149 NM$ .
3. turbojet powered aircraft, by the rule for the same economical assumption for  $PR > 1.0$ , require longer ranges to operate economically as compared with turbo prop powered aircraft.



**Table 3**  
**Left Limits of Usability Range for Aircraft by Number of Passengers**

<i>P<sub>AT</sub></i>	<i>Do228</i>	<i>1900D</i>	<i>SD 330</i>	<i>Do 328</i>	<i>SF 340</i>	<i>ATR 42</i>	<i>F 50</i>	<i>Saab 2000</i>	<i>ATR 72</i>	<i>Dash8-400A</i>	<i>CRJ</i>	<i>CRJ-700</i>	<i>F 70</i>	<i>F 100</i>	<i>A 319</i>
5	1065,1	1004,9	729,2	1236,5	1021,8	962,3	1130,6	1240,3	989,6	1181,7	1145,4	1524,1	1348,5	1325,4	1451,5
10	209,4	281,2	796,3	581,5	1099,6	1030,3	1199,6	1311,8	1048,2	1243	1206,4	1583,8	1404,3	1375,5	1493,8
15	107,2	126,7	250,5	193,6	245,1	375,4	494,3	530,3	1107,3	1303,2	1267	1642,6	1460,8	1426,6	1532,1
19	80	91,7	145	132,4	154	200,1	246,3	264,8	305,7	386,5	345,8	659,6	719,6	1466,9	1562,8
20			132	123	141,8	180,5	221	237,5	266,6	330,5	300,9	534,2	557,5	1027,3	1570,4
25			94	92,5	102,8	124,4	148,7	159,3	166,8	202,2	187,6	296,9	291,4	363,5	1609,6
30			74,7	75,4	82,3	96,7	114,1	121,8	124,6	148,8	139,3	210,3	202,4	239,1	597,7
34					71,7	83	97,1	103,5	104,5	123,5	116,4	171,1	164,9	190,3	398,5
35						80,2	93,7	99,8	100,6	118,8	111,9	163,9	157,5	181,7	369,4
40						69,4	80,3	85,3	85,3	100	94,5	135,6	129,9	147,2	273,3
45						61,6	70,8	75	74,6	86,8	82,4	116	111	124,9	217,3
46						60,3	69,3	73,3	72,9	84,7	80,5	112,9	108,1	121	209,4
50							63,8	67,4	66,9	77,3	73,6	102	97,6	108,7	182,4
55									60,9	69,9		91,3	87,4	96,6	157,6
60									56,2	64,2		83	79,5	87,5	138,8
65									52,3	59,6		76,4	73,1	80,1	124,6
66									51,7	58,7		75,2	72	78,8	122,1
70										55,7		70,9	67,9	74	113,3
75													63,6	69,1	103,9
79													60,6	65,7	97,7
80														64,9	96,3
85														61,3	89,9
90														58,2	84,3
95														55,5	79,6
100														53,2	75,5
105														51	71,8
110															68,6
115															65,8
120															63,2
125															61,3

Note: Examined for high speed flight where the profitability ration = 1.0

## CONCLUSION

The level of operational costs depends on economic assumptions but it also depends on trip time and trip fuel. In this paper trip time  $k_t$ , and trip fuel  $k_F$  correction coefficients are introduced by estimating the influence of payload mass  $W_p$  and  $W_{F1}$  on costs per flight  $TC_{AT}$ . So it is now possible to predict costs per flight more precisely and therefore predict them not only depending on the trip distance  $R$ , but also on the payload mass  $W_p$  through the use of coefficients  $I_p$  and  $I_f$ .

Introduction of usability range  $UR$  interval in which it is economical to fly if the condition  $PR > 1.0$  is suggested. Usability range, therefore, for defined economic assumptions, depend only on the payload mass i.e. number of passengers per flight  $P_{AT}$ . Proposed methods to estimate usability interval and especially its left limit  $R_l$  (tables 2 and 3) may be used to anticipate aircraft capacity.

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- Boeing Airplane Economic Group. (1994, May). *Operating Cost Ground Rules*.
- AEA. (1990). Short-Medium Range Aircraft, DOC Calculation AEA, G(T)5656 26.09.90

**Table 4**  
**Sources of Manufacturers Information of Aircraft Studied**

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Dornier 228-212 Specification Appendix C2 Performance PED 000E Issue3B, Feb90
Beechcraft 1900D Airliner Ref. Report AM95-1900D, January 1995
Flight Planning and Operating Data Shorts 330", Crew Manual Sept. 1977
Dornier 328 Standard Specification, ref.AVS 001A, Feb.1990, Issue2
Saab 340 Performance Planning Guide, ref.SOO2.201 , August 1988
Performance Data for ATR 42 ref. A/DC/ET No. 732, May 1985 Issue 1
Fokker 50 Performance Information Based on:PDI-83-31, Issue3 , May 1985
Saab 2000 Performance Engineers Handbook Reg.No. 73ADS0394 Nov1995
Performance Data for ATR 72 ref. A/DC/ET No. 757 Nov.1985 Issue2
Dash 8Q Series 400 Program Overview ref. ASC072.A, March 1997
Performance Data CRJ Memo No: MAA-601R107F, Feb, 1993 Issue:F
Canadair Regional Jet Series 700, Program Overview ref. ASC074.AA May 1997
Fokker 100 Performance Information Based on:PDI-83-25, Issue2, Nov. 1983
Performance Manual A320 ref. P2210 Issue2 Nov. 1991

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**Table 5**  
**Specification for Aircraft Studied**

	<i>Do 228 100/HS</i>	<i>1900D 200/HS</i>	<i>SD 3-30 100/HS</i>	<i>Do 328 250/HS</i>	<i>SF 340 200/HS</i>	<i>ATR 42 200/HS</i>	<i>F 50 250/HS</i>
Price (MUSD)	3.900.000	5.950.000	8.050.000	10.100.000	11.850.000	13.200.000	14.250.000
$W_o$ (kg)	6400	7688	10387	12500	12925	15750	20820
$W_c$ (kg)	3739	4815	6805	8175	8034	9973	12474
$S_a$	19	19	30	30	34	46	50
$W_{PF}$ (kg)	83	260	65	474	663	341	1120
$W_{po}$ (kg)	1812	1989	2795	3204	3757	4527	5670
$t_p/W_{Fg}$ (Hr/kg)	0,167/15	0,167/41	0,167/41	0,167/56	0,167/44	0,167/57	0,167/74
$R_A$ (NM)	480	350	87	420	420	400	830
$R_B$ (NM)	1260	1225	610	1340	1340	2480	1415
a (Hr)	0,048	0,0033	0,074	0,084	0,084	0,085	0,134
b (Hr/NM)	0,00427	0,0038	0,00546	0,00360	0,00360	0,00378	0,00347
c (kg)	24.711	55.330	4.780	38.526	38.526	45.443	109.124
d (kg/NM)	1.375	1.333	2.223	1.643	1.643	1.897	1.865
e (kg)	2.557.846	2421,8	2888,155	4415,304	4415,304	5147,577	7684,701
f (kg/NM)	-1.554	-1.236	-1.071	-1.567	-1.567	-1.551	-2.427
e1 (kg)	19500,000	28683,875	36318,750	36782,778	36782,778	81900,000	34318,750
f1 (kg/Nm)	-15.000	-22.675	-55.875	-25.722	-25.722	-32.500	-21.250
$\alpha$	-	0,439	0,409	0,195	0,195	0,352	0,363
$\beta$	-	0,1030	0,0980	0,1743	0,17430	0,109	0,103
$\chi$	0,75710	0,27100	0,33372	0,20660	0,20660	0,25246	0,08945
$\delta$	0,032	0,142	0,120	0,168	0,168	0,143	0,246

Table 5 – continued  
Specification for Aircraft Studied

	<i>Saab 2000</i> 250/HS	<i>ATR 72</i> 200/HS	<i>Dash 8Q-400A</i> 250/HS	<i>Canadair RJ-100</i> 310/HS	<i>CRJ-Series 700</i> 310/HS	<i>F 70</i> 310/HS	<i>F 100</i> 310/HS	<i>A319</i> 310/HS
Price (MUSD)	16.300.000	16.900.000	19.500.000	20.900.000	25.050.000	23.800.000	27.200.000	43.650.000
$W_o$ (kg)	22800	21500	27329	21523	32885	39915	44450	64000
$W_e$ (kg)	13800	12200	16537	13653	19595	22673	26119	40100
$S_a$	50	66	70	50	70	79	105	124
$W_{pF}$ (kg)	1350	1494	1474	939	2157	2113	1066	5616
$W_{po}$ (kg)	5900	7500	7844	5489	8527	9302	10621	16900
$t_g/W_{Fg}$ (Hr/kg)	0,167/124	0,167/95	0,167/137	0,167/141	0,167/141	0,167/251	0,167/251	0,167/131
$R_A$ (NM)	770	560	523	510	960	1206	900	685
$R_B$ (NM)	1190	2240	1307	1230	2483	1450	1450	3400
$a$ (Hr)	0,096	0,082	0,108	0,107	0,26000	0,080	0,0810	0,193
$b$ (Hr/NM)	0,00271	0,00364	0,00292	0,00213	0,00214	0,00230	0,00230	0,00218
$c$ (kg)	52.070	55.757	127.333	104.440	136.200	391.700	346.460	581.160
$d$ (kg/NM)	2.771	2.162	2.983	2.916	3.276	4.714	4.943	5.485
$e$ (kg)	8283,333	8566,667	9392,320	6714,342	11255,720	14497,545	14497,545	19929,138
$f$ (kg/NM)	-3.095	-1.905	-2.960	-2,451	-2.842	-4.307	-4.307	-4.422
$e1$ (kg)	59340,000	73100,000	132164,421	27652,631	33477,871	30915,500	30915,500	81574,184
$f1$ (kg/Nm)	-46.000	-30.714	-96.895	-19,474	-11.792	-15.630	-15.630	-22.553
$\alpha$	0,470	0,388	0,463	0,427	0,421	0,675	0,675	0,852
$\beta$	0,0076	0,095	0,077	0,086	0,087	0,038	0,038	0,014
$\chi$	0,16260	0,29911	0,16499	0,17409	0,17665	0,02995	0,02995	0,01357
$\delta$	0,184	0,122	0,182	0,177	0,175	0,336	0,336	0,392

**Table 6**  
**Specifications for Aircraft Studied**

	<i>Do 228</i> <i>100/HS</i>	<i>1900D</i> <i>200/HS</i>	<i>SD 3-30</i> <i>100/HS</i>	<i>Do 328</i> <i>250/HS</i>	<i>SF 340</i> <i>200/HS</i>
g (USD)	69.239	79.793	79.927	93.417	89.984
h (USD/Fh)	57,504+549705/U	63,257+838652,5/U	73,970+1134647/U	81,447+1423595/U	82,876+1670257/U
i (USD/Fh)	247,432+549705/U	263,401+838652,5/U	295,986+1134647/U	332,261+1423595/U	331,555+1670257/U
j (USD)	100.737	120.770	165.468	193.909	200.489
k (USD)	395,919* $l_p$ +8,775* $l_F$	395,922* $l_p$ +27,489* $l_F$	625,140* $l_p$ +6,872* $l_F$	625,140* $l_p$ +50,114* $l_F$	708,492* $l_p$ +70,097* $l_F$
m (USD)	453.016	482.201	561.235	599.586	616.119
n (USD)	1,946+0,067*R	4,720+0,065*R	2,243+0,109*R	5,949+0,071*R	4,044+0,080*R
	<i>ATR 42</i> <i>200/HS</i>	<i>F 50</i> <i>250/HS</i>	<i>Saab 2000</i> <i>250/HS</i>	<i>ATR 72</i> <i>200/HS</i>	<i>Dash 8Q-400A</i> <i>250/HS</i>
g (USD)	99.951	101.666	173.140	110.211	141.237
h (USD/Fh)	91,848+1860540/U	106,192+2008537/U	111,329+2297485/U	107,982+2382055/U	122,329+2748525/U
i (USD/Fh)	367,509+1860540/U	411,142+2008537/U	484,039+2297485/U	419,673+2382055/U	453,295+2748525/U
j (USD)	243.796	302.049	326.927	326.903	401.750
k (USD)	958,548* $l_p$ +36,053* $l_F$	1041,9* $l_p$ +118,414* $l_F$	1041,9* $l_p$ +142,731* $l_F$	1375,308* $l_p$ +157,956* $l_F$	1458,660* $l_p$ +155,841* $l_F$
m (USD)	683.022	763.651	789.549	802.392	878.753
n (USD)	5,020+0,093*R	8,973+0,091*R	11,756+0,118*R	7,387+0,106*R	12,952+0,146*R
	<i>Canadair RJ-100</i> <i>310/HS</i>	<i>CRJ-Series 700</i> <i>310/HS</i>	<i>F 70</i> <i>310/HS</i>	<i>F 100</i> <i>310/HS</i>	<i>A319</i> <i>310/HS</i>
g (USD)	190.003	225.203	227.954	238.276	324.918
h (USD/Fh)	108,042+2945855/U	134,686+3530797/U	148,962+3354610/U	157,535+3833840/U	190,413+6152467/U
i (USD/Fh)	497,243+2945855/U	673,662+3530797/U	658,921+3354610/U	677,393+3833840/U	831,047+6152467/U
j (USD)	318.010	461.560	523.160	599.336	923.212
k (USD)	1041,9* $l_p$ +99,278* $l_F$	1458,660* $l_p$ +228,053* $l_F$	1646,202* $l_p$ +223,401* $l_F$	2187,990* $l_p$ +112,705* $l_F$	2583,912* $l_p$ +593,763* $l_F$
m (USD)	771.928	941.302	1024,928	1102,628	1306,814
n (USD)	12,026+0,143*R	13,583+0,160*R	31,492+0,231*R	29,275+0,242*R	34,896+0,269*R